Panoramic Images: Cylindrical Projection

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1. Panoramic Image Representation

Various projection methods can be used in order to represent a panoramic image. The most common solution is to project the image inside a cylindrical surface. This technique is able to represent the full horizontal field of view but not the vertical one. For this reason is not suited to panoramic images with a large vertical extension and does not allow looking directly up or down. This issue can be solved by using a spherical projection that corrects both the horizontal and vertical distortion and allows looking in every direction, but is also more complex to compute.

Figure 1: Cylindrical Panorama

Figure 2: Spherical Panorama

It is also possible to project the panorama inside a cube and then store the images corresponding to the 6 faces of the cube. The visualization software will then recombine the images obtaining a result similar to the one of the spherical projection. As for the spherical projection, there is no distortion and it is possible to look in every direction but the sampling is less uniform. Finally, it is also possible to store the image as it is. This is the fastest and simplest approach but produces distorted images and works only for field of views smaller than 120° in both directions.

Figure 3: Cubic panorama (images projected on each of the 6 faces)

Projection Type	Horizontal distortion	Horizontal Angle	Vertical distortion	Vertical Angle
Cylindrical	No	360°	Yes	~120°
Spherical	No	360°	No	360°
Cubic	No	360°	No	360°
Planar	Yes	~120°	Yes	~120°

Table 1: Types of projection

2. Cylindrical projection

In order to build a cylindrical panorama we start from taking a set of pictures from the same viewpoint by rotating the camera in the horizontal direction with a constant rotation step. The rotation step must be smaller than the area covered by a picture in order to have the images partially superimposed. This allows to join them, e.g., by locating common features (after the projection each couple of pictures is related by a simple translation). Each picture is then projected on the cylinder and covers a particular angular sector of it.

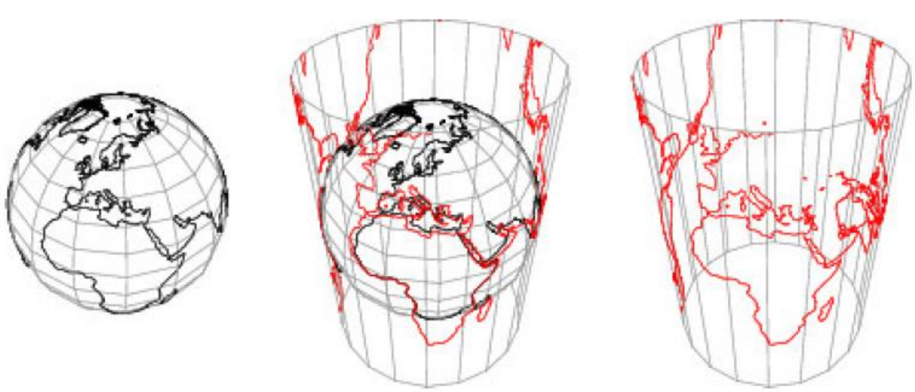

Figure 4: Cylindrical projection example

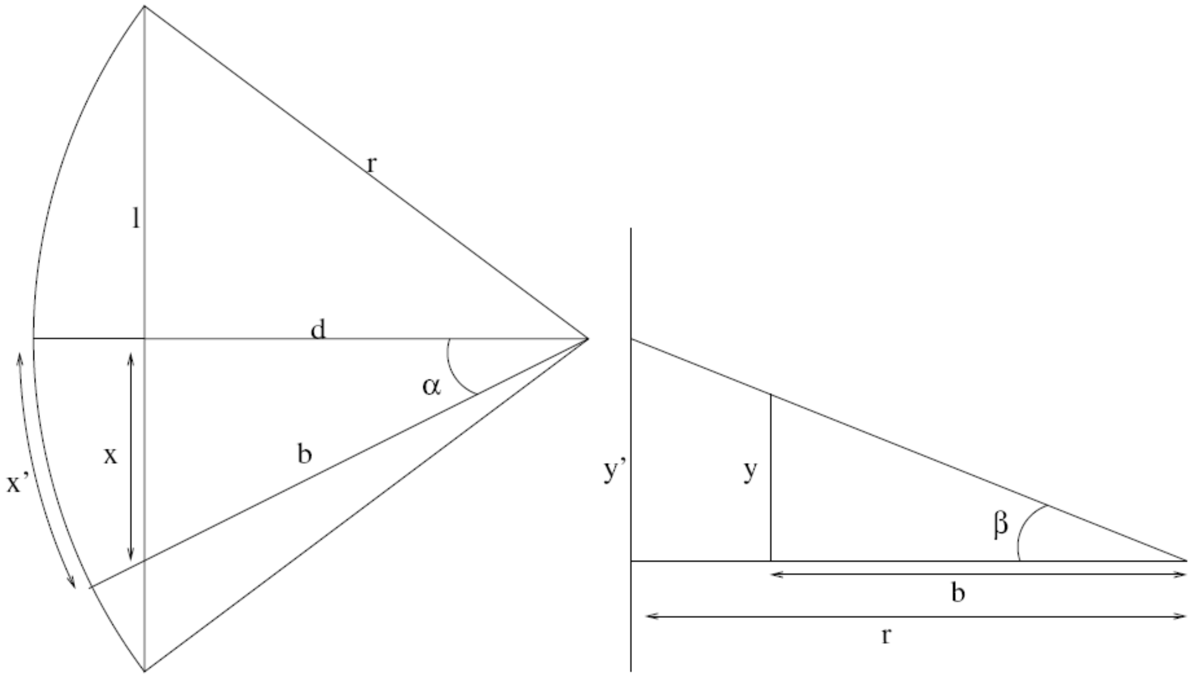

Figure 5: Cylindrical projection

Referring to Figure 5 and using the pixel as the measurement unit, consider the following measures:

- *r* : the radius of the cylinder,
- *l* : half of the horizontal size of each picture (input parameter, equal to *image_width* / 2)
- *d :* the distance between the camera viewpoint and the picture center (can be computed from *l* and the camera FOV)
- α : the angle between the segment joining the generic point *x* with the projection center.

Let us call *(x,y)* the coordinates of a point on the picture and with *(x',y')* its mapping on the cylinder, by using the equations:

$$r = \sqrt{l^2 + d^2} \text{ and } x' = r \cdot \alpha$$

we obtain:

$$x = d\ tan(\alpha) = d\ tan\left(\frac{x'}{r}\right)$$

In order to compute *y* the relations between the similar triangles in Figure 5b can be used together with the relation *b = d / cos(α)*, thus obtaining:

$$\frac{y}{y'} = \frac{b}{r} \quad \Rightarrow \quad y = y'\frac{b}{r} \quad \Longrightarrow \quad y = y'\frac{d}{r\cos(\frac{x'}{r})}$$

Finally by solving with respect to *x'* and *y'* the equations for the cylindrical projection can be derived:

$$x' = r\ arctan\left(\frac{x}{d}\right)$$

$$y' = y\frac{r}{d}\cos\left(\frac{x'}{r}\right)$$

In this way, we obtain a planar image corresponding to the projection of the picture on the cylinder. This makes the image superposition much easier since the relation between the various images on the cylinder is a simple translation. This is one of the main advantages of using the cylindrical projection.