2.1 Multivariate Inheritance & Response to Selection

Stevan J. Arnold
Department of Integrative Biology
Oregon State University

Thesis

- The statistical approach that we used for a single trait can be extended to multiple traits.
- The key statistical parameter that emerges is the G-matrix.
- The G-matrix affects the response of the multivariate mean to selection and drift.

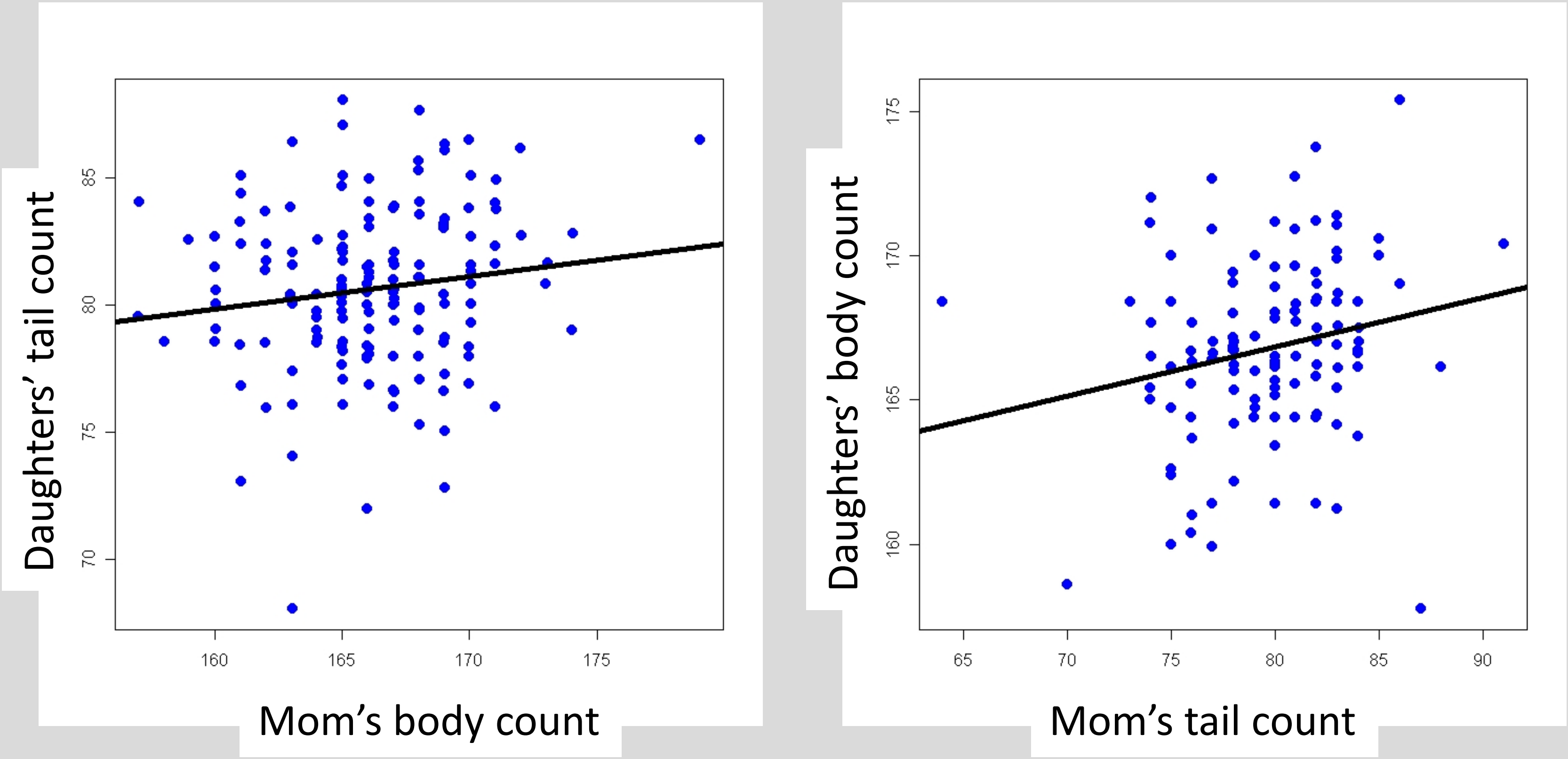
Outline

1. Multivariate resemblance between parents and offspring is captured by the G-matrix.
2. Our model of inheritance is multivariate.
3. Some examples.
4. The G-matrix is affected by opposing forces.
5. The G-matrix affects the evolution of the multivariate mean.

1. Multivariate resemblance

Traits can run together in families

Animation 1



2. A Model for Multivariate Resemblance

a. A model for phenotypic value

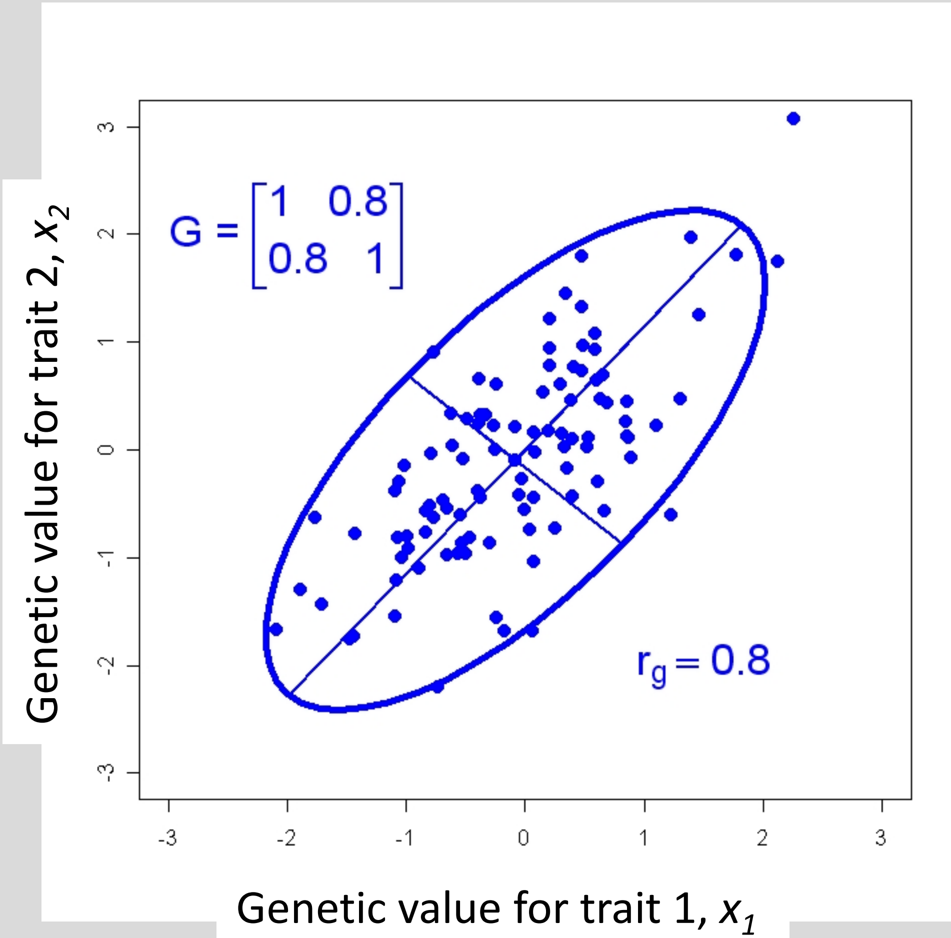
phenotypic value $$z = x + e = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

phenotypic mean $$\bar{z} = \bar{x} + \bar{e} = \begin{bmatrix} \bar{z}_1 \\ \bar{z}_2 \end{bmatrix} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} + \begin{bmatrix} \bar{e}_1 \\ \bar{e}_2 \end{bmatrix}$$

phenotypic var/covar $$P = G + E = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} + \begin{bmatrix} E_{11} & E_{12} \\ E_{12} & E_{22} \end{bmatrix}$$

2. A Model for Multivariate Resemblance

c. The G-matrix describes a cloud of genetic values



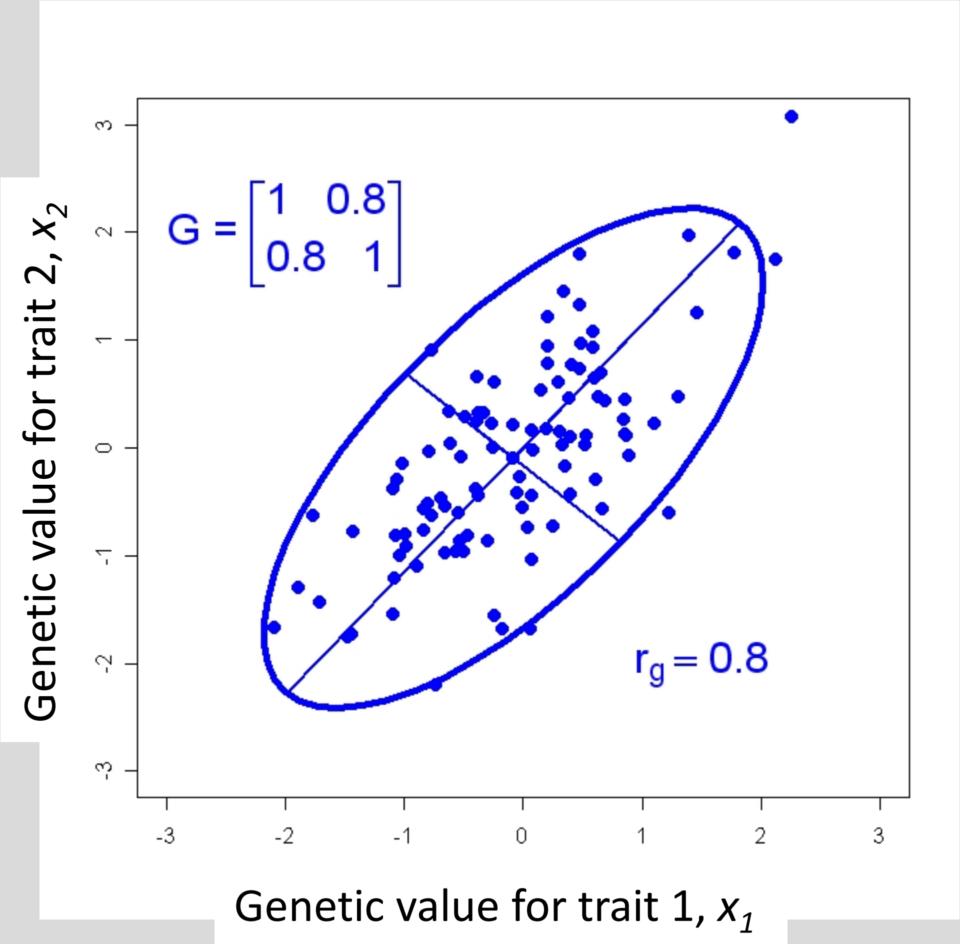
Causes of genetic covariance:

Pleiotropy

Linkage Disequilibrium

2. A Model for Multivariate Resemblance

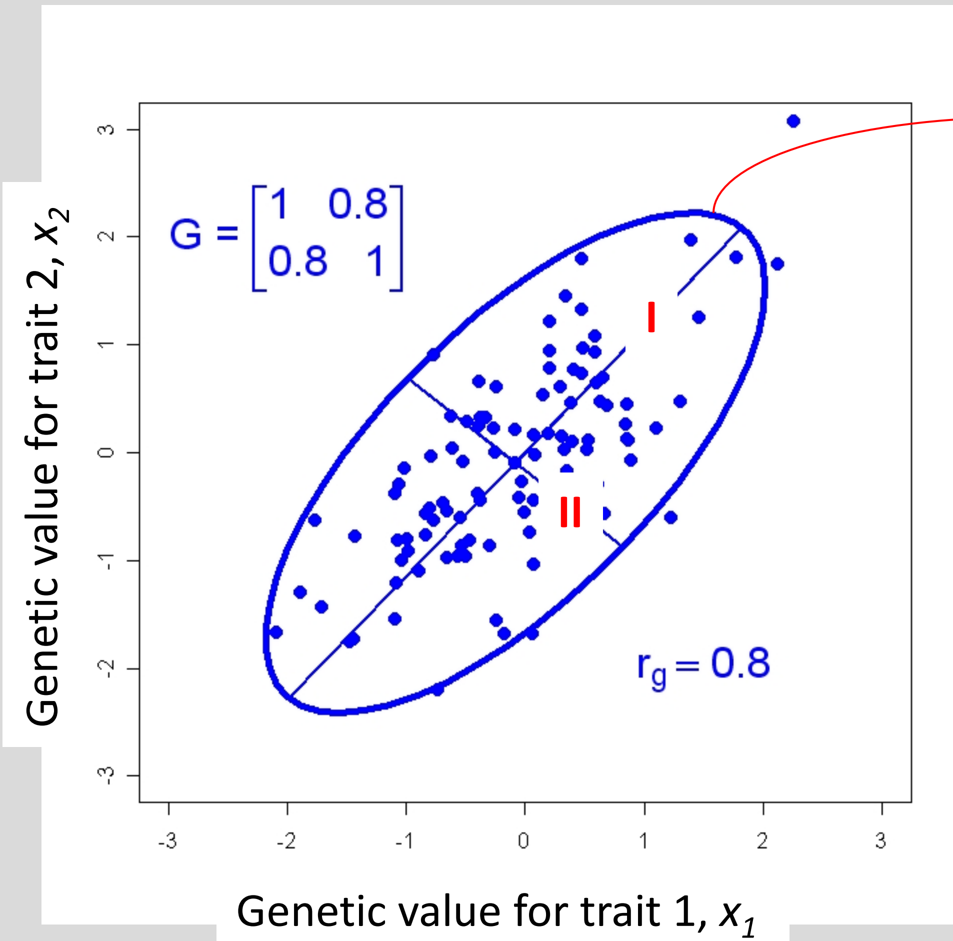
c. The G-matrix describes a cloud of genetic values



$$r_g = G_{12}/\sqrt{G_{11}}G_{22}$$

2. A Model for Multivariate Resemblance

c. The G-matrix describes a cloud of genetic values



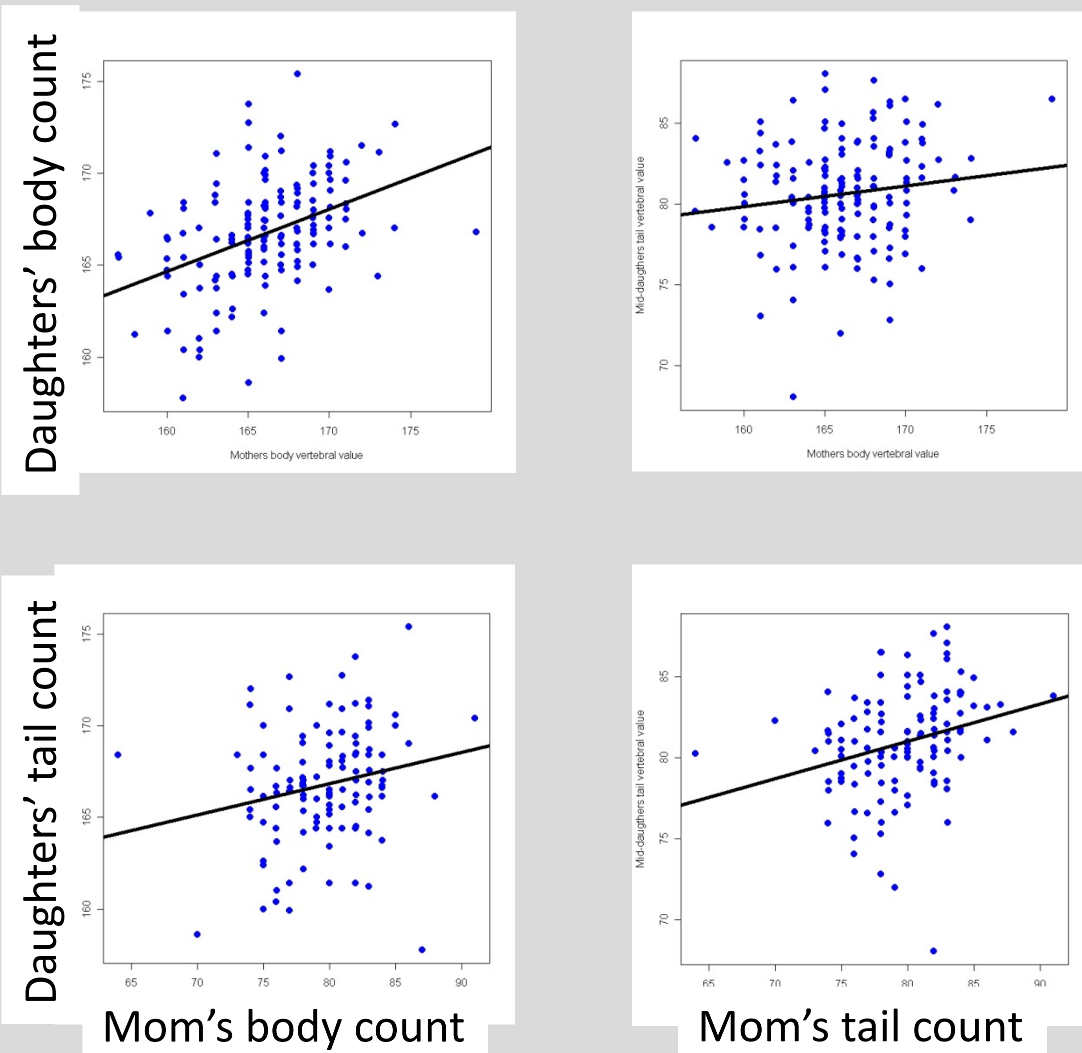
95% confidence ellipse

First principal component

Second principal component

3. Some examples

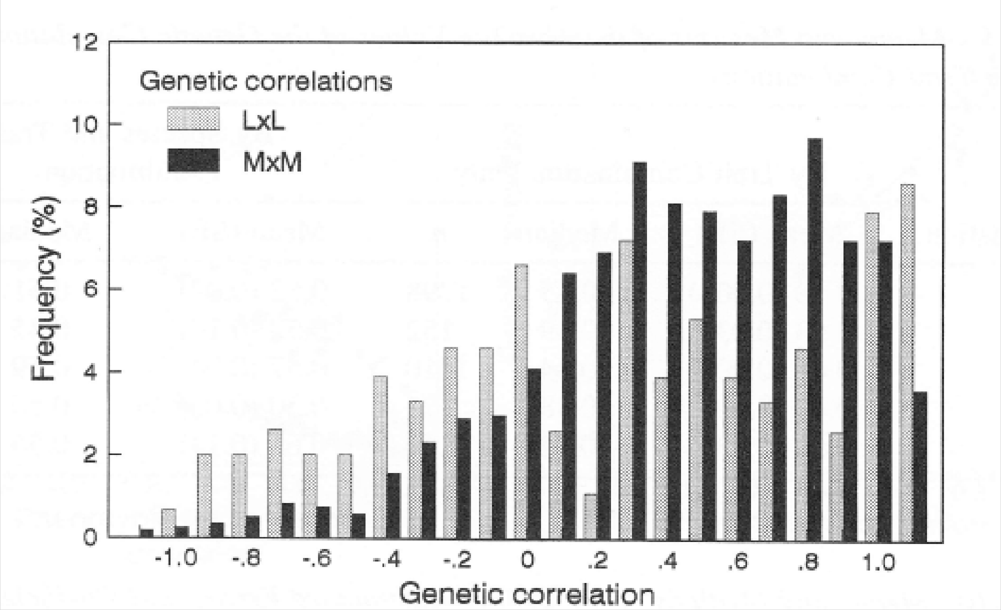
a. Mother-daughter resemblance in vertebral counts in garter snakes



$$G = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} = \begin{bmatrix} 8.17 & 3.78 \\ 3.78 & 8.16 \end{bmatrix}$$

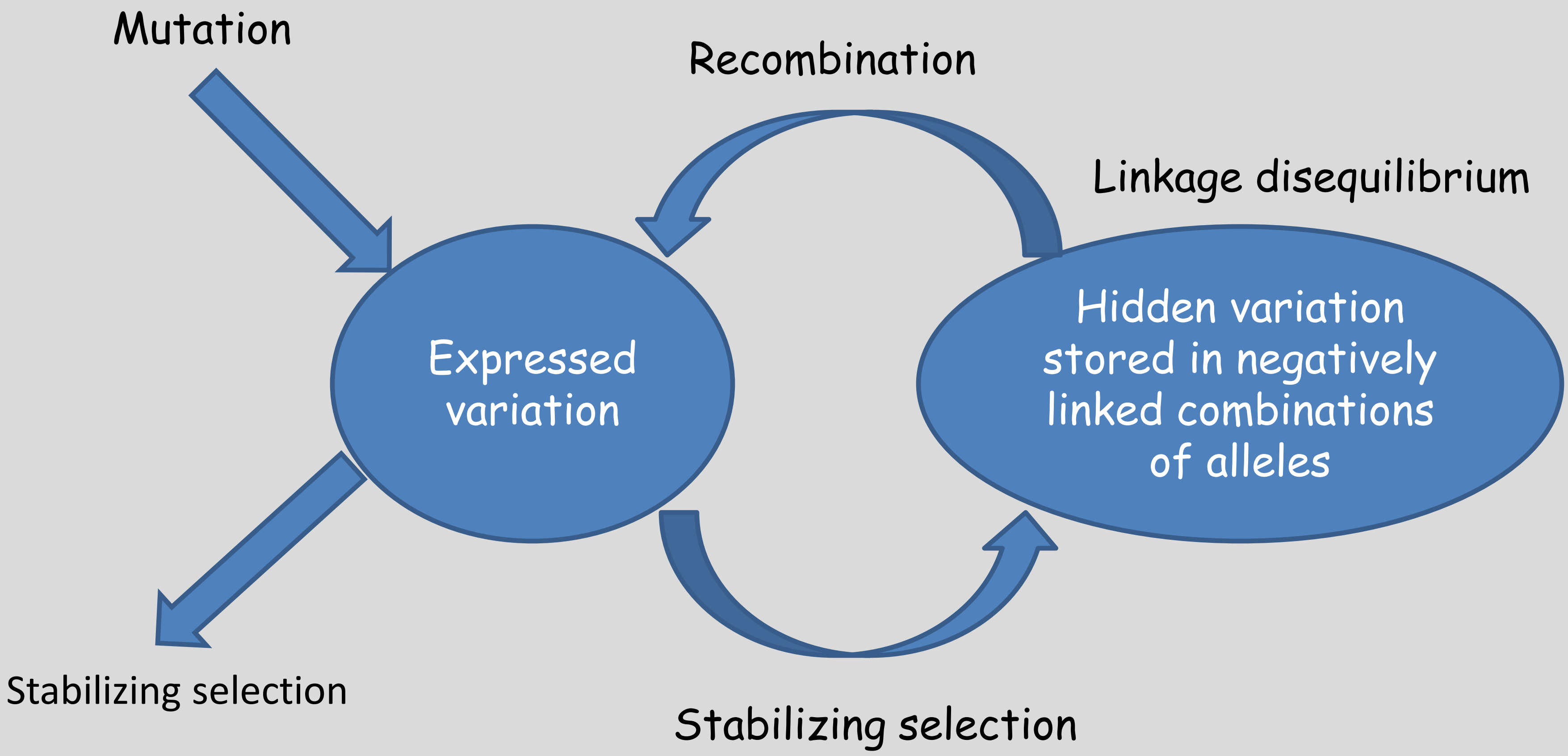
3. Some examples

b. Prevalence of genetic correlation



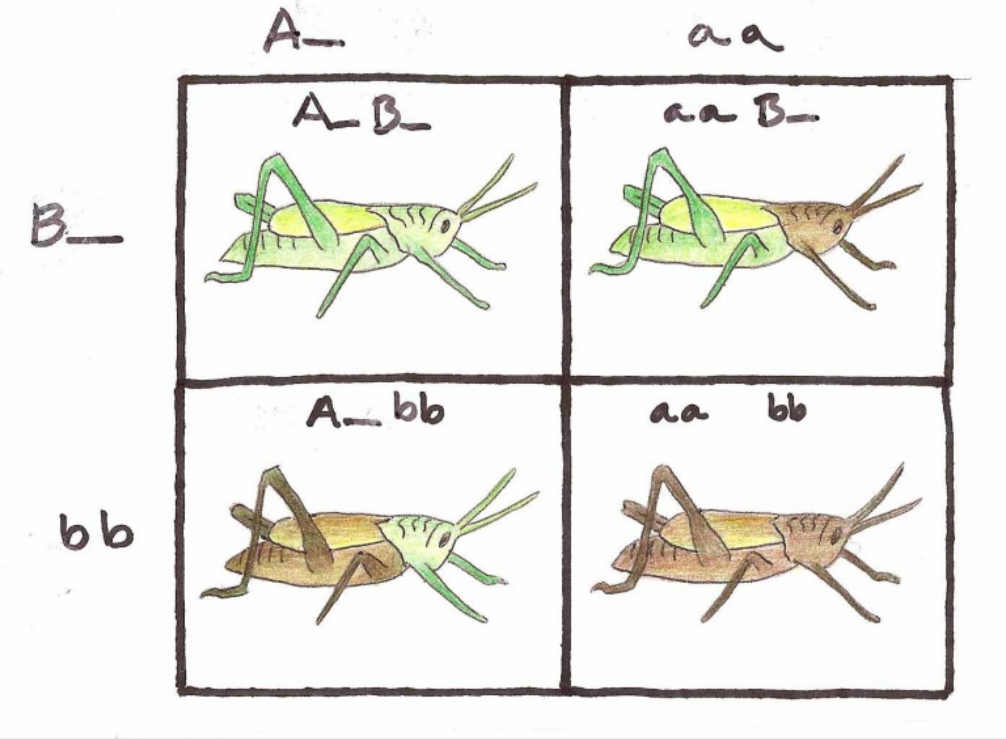
4. Why don't we run out of additive genetic variance and covariance?

a. Mutation-Selection Balance



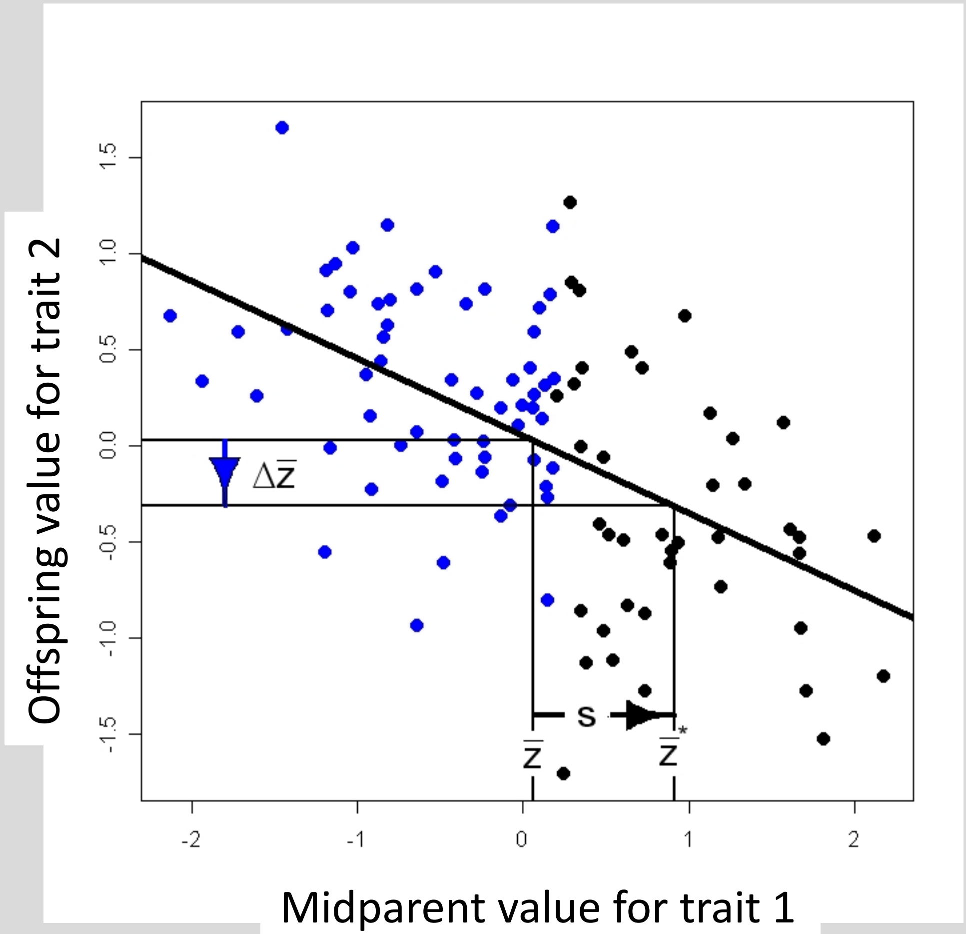
4. Why don't we run out of additive genetic covariance?

b. Correlational selection
– one kind of multivariate stabilizing selection –
can produce linkage disequilibrium



5. Changing the multivariate mean with selection

a. Genetic covariance causes selection on one trait to affect a correlated trait



5. Changing the multivariate mean with selection

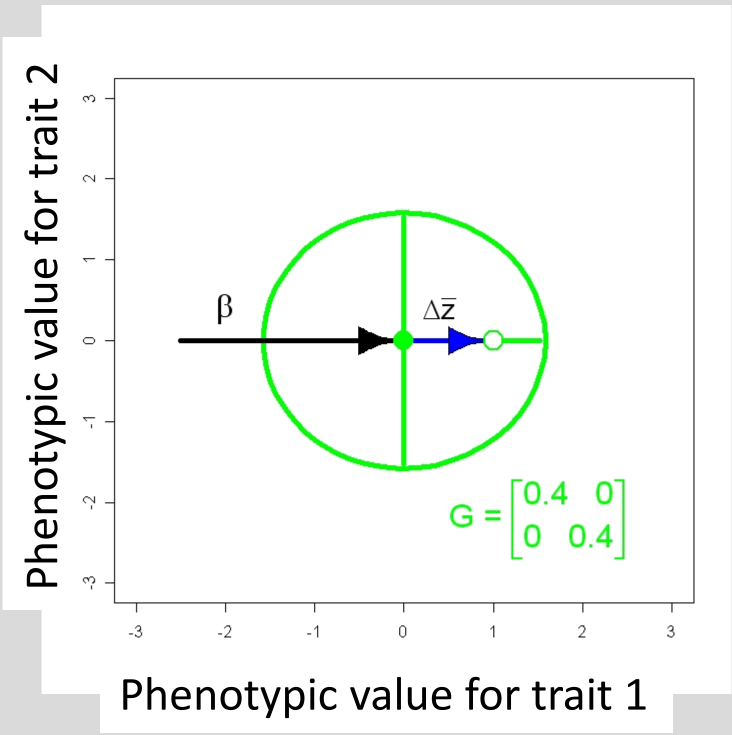
a. Direct and correlated responses to selection

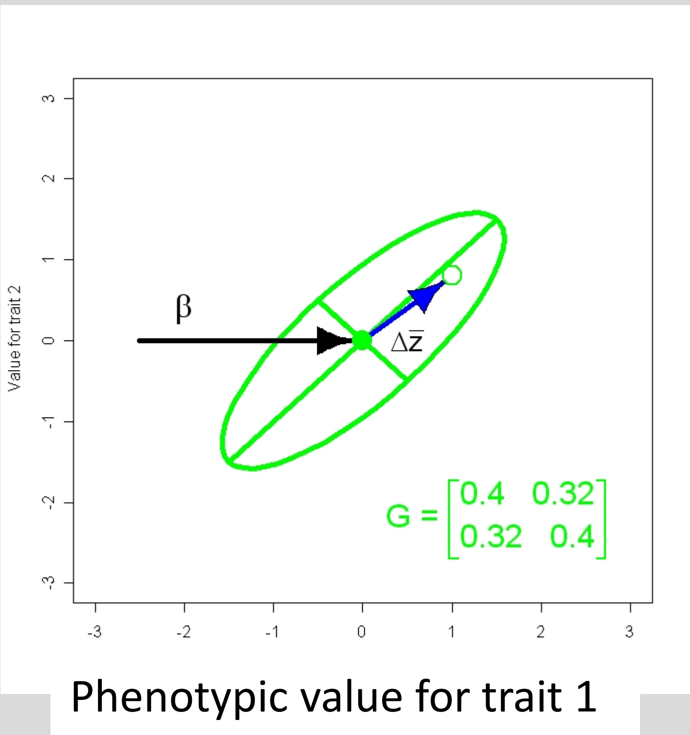
$$\Delta \bar{z} = GP^{-1}s = G\beta$$

$$\begin{bmatrix} \Delta \bar{z}_1 \\ \Delta \bar{z}_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{12} & G_{22} \end{bmatrix} \begin{bmatrix} \beta 1 \\ \beta 2 \end{bmatrix} = \begin{bmatrix} G_{11}\beta_1 + G_{12}\beta_2 \\ G_{12}\beta_1 + G_{22}\beta_2 \end{bmatrix}$$

5. Changing the multivariate mean with selection

b. Response to selection as a pool shot

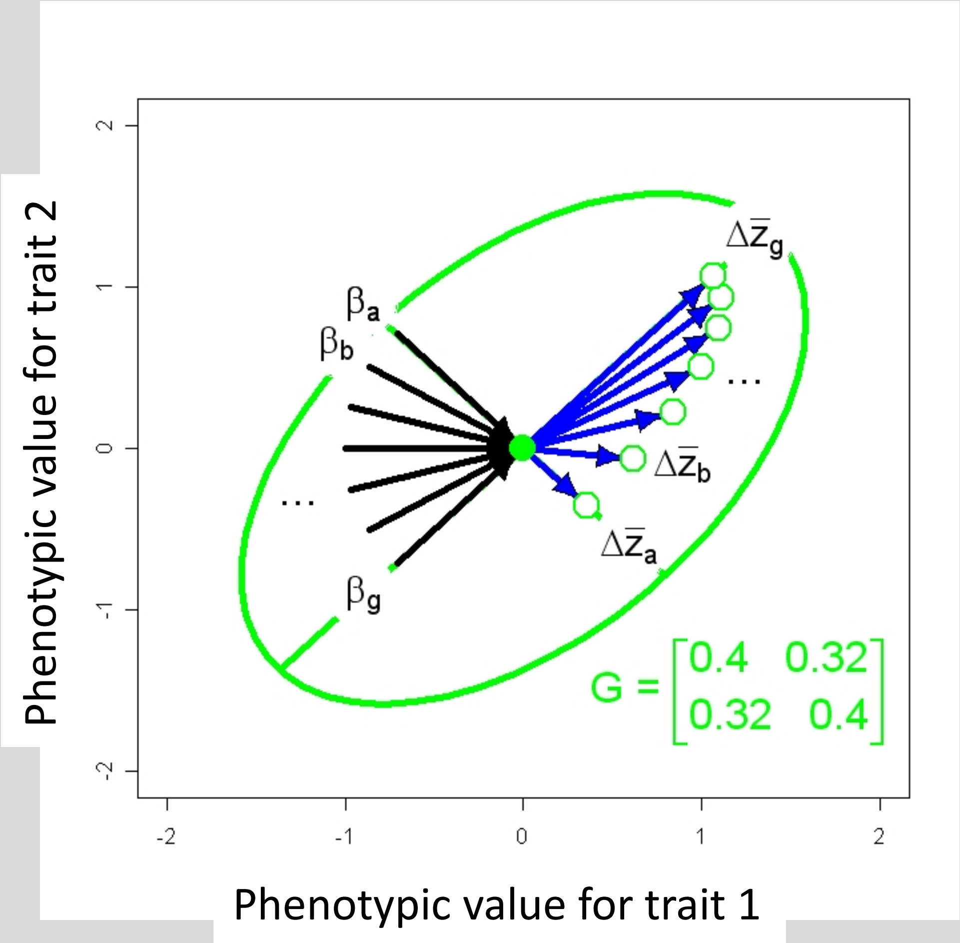




[Animation 2]

5. Changing the multivariate mean with selection

b. Response to selection as a pool shot, continued



Animation 3

What have we learned?

1. The additive genetic variance-covariance matrix, *G*, is the key to understanding multivariate resemblance between parents and offspring.

2. Consequently, the *G-matrix* is also the key to modeling multivariate responses to selection.

3. *G* induces correlated responses to selection that may be non-intuitive.

References

- Arnold, S. J. and P. C. Phillips. 1999. Hierarchial comparison of genetic variance-covariance matrices.II. Coastal-inland divergence in the garter snake, *Thamnophis elegans*. Evolution 53:1516-1527.
- Lande, R. 1979. Quantitative genetic analysis of multivariate evolution, applied to brain: body size allometry. Evolution 33: 402-416.
- Roff, D. A. 1997. Evolutionary Quantitative Genetics. Chapman & Hall.
- Lande, R. 1980. The genetic covariance between characters maintained by pleiotropic mutations. Genetics 94: 203-215.
- Lande, R. 1984. The genetic correlation between characters maintained by selection, linkage and inbreeding. Genetical Research Cambridge 44: 309-320.