
Logic in Computer Science, November 9th, 2021. Time: 1h30min. No books or lecture notes allowed.

-Insert your answers on the dotted lines ... below, and only there.
-Do NOT modify the problems or the @nota lines.
-When finished, upload this file with the same name: exam.txt
-Use the text symbols: & v - -> |= A E
for AND OR NOT IMPLIES "SATISFIES" FORALL EXISTS
etc., like in:
I |= p & (q v -r) (the interpretation I satisfies the formula p & (q v -r)).
You can write subindices using "_". For example write x_i to denote x-sub-i.

Problem 1. (3 points). @n@nota1:

1a)
Let F and G be propositional formulas such that F is satisfiable and F -> G is also satisfiable.
Is it true that G is satisfiable? Prove it using only the definitions of propositional logic.

...

1b)
Let F and G be propositional formulas such that F is a tautology.
Is it true that F & G is logically equivalent to G?
Prove it using only the definitions of propositional logic.

...

Problem 2. (3 points). @n@nota2:

A 1in3-constraint is an expression of the form 1in3(lit1, lit2, lit3) where lit1,lit2 and lit3 are literals.
An interpretation I satisfies 1in3(lit1, lit2, lit3) if it satisfies EXACTLY ONE of lit1, lit2 and lit3.
The 1in3-SAT problem is the problem of deciding the satisfiability of a conjunction (AND) of 1in3-constraints.
For example,
1in3(x,y,z) & 1in3(-x,-y, z) & 1in3(-x,y,-z) is satisfiable (if I(x)=1, I(y)=0, I(z)=0 then I is a model)
but
1in3(x,y,z) & 1in3(-x,-y,-z) is unsatisfiable.

2a) Is 1in3-SAT in NP? Explain in a few words why.

...

2b) Let C be a normal 3-SAT clause l1 v l2 v l3, where l1,l2,l3 are literals over variables x,y,z.
Let F be: 1in3(-l1,a,b) & 1in3(l2,b,c) & 1in3(-l3,c,d) (here a,b,c,d are variables).
Check for each one of the 7 possible models I of C that then F has a model I' such
that I' "extends" I, that is I(x)=I'(x), I(y)=I'(y), I(z)=I'(z).

Similarly, check that for the (unique) I that is NOT a model of C, there is no model I' of F extending I

```
    (and therefore every model I' of F extends a model I of C).

...


2c) Is 1in3-SAT NP-complete? Explain very briefly why. Hint: use 2a) and 2b).

...


-------------------------------------------------------------------------------------------------

Problem 3. (3 points).                                    @n@nota3:
For each one of the following problems, show that it is polynomial by expressing
it as (or
reducing it to) a polynomial version of SAT.  Be very brief: just give the needed
SAT variables and
clauses and say which polynomial SAT problem it is.  If there is no such
reduction, just write: "Not possible".

3a) 2-coloring: given an undirected graph G and 2 colors,
can we assign a color to each node of G such that adyacent nodes get different
colors?

...

3b) 3-coloring.

...

3c) Amazon. Assume
    M is a list of Amazon products we MUST buy.
    P is a list of pairs (p,p') of products that are incompatible: we cannot buy p
and also p'.
    R is a list of rules of the form "S needs p", indicating that, if we buy
          all products in the set of products S, then we must also buy the product
p.
    Given M,P,R, can we buy a set of products satisfying the requirements of M,P,R?

...

-------------------------------------------------------------------------------------------------

Problem 4. (1 point ).                                    @n@nota4:

4) UNIQUE-SAT is the problem of determining whether a given set of clauses S has
exactly one model.
Explain very briefly how you would use a SAT solver to decide UNIQUE-SAT.

...

-------------------------------------------------------------------------------------------------
```