```
-----------------------------------------------------------------------------------------------
Logic in Computer Science, April 20th, 2023. Time: 1h45min. No books or lecture notes
allowed.
-----------------------------------------------------------------------------------------------
 - Insert your answers on the dotted lines ... below, and only there.
 - When finished, upload this file with the same name: exam.txt
 - Use the text symbols:     &     v    -       ->          |=        A       E
     for                   AND   OR   NOT   IMPLIES   "SATISFIES"  FORALL  EXISTS  etc.,
like in:
    I |=  p & (q v -r)     (the interpretation I satisfies the formula p & (q v -r)  ).
   You can write  not (I |= F)  to express "I does not satisfy F",  or
                  not (F |= G)  to express "G is not a logical consequence of F"
   Also you can use subindices with "_". For example write x_i to denote x-sub-i.
-----------------------------------------------------------------------------------------------


Problem 1. (2.5 points).

1a) Let F, G, H be formulas. Prove that if F v G |= H then F & -H is unsatisfiable,
    using only the definition of propositional logic.

Answer:
        F v G |= H                                ==>   [by def. of logical consequence]
    AI, if I |= F v G then I |= H                 ==>   [by def. of |=]
    ...

1b) Is it true that, for any two propositional formulas F and G, if F |= G
    and G is satisfiable, then F is satisfiable?  If it is, prove it.
    If it is not, give a concrete counterexample (and check it is so).

Answer:
    ...


-----------------------------------------------------------------------------------------------

Problem 2. (2.5 points).

2a) Write all clauses (as disjuctions of literals) obtained by applying Tseitin’s
    transformation to the formula (p & (q v -r)) v q.
    Use auxiliary variables named e0, e1, e2, ... (where e0 is for the root).

Answer:
    ...

2b) Prove that it is not true that for any propositional formula F, F and Tseitin(F) are
    logically equivalent.

Answer:
    ...

2c) Is 3-SAT NP-complete? Explain your answer very briefly, using the fact that SAT
    (deciding the satisfiability of an arbitrary propositional formula F) is NP-complete

Answer:
    ...


-----------------------------------------------------------------------------------------------

Problem 3. (2.5 points).

3) Given S a set of clauses (CNF) over *n* propositional symbols,
   and Resolution the deductive rule:

                 p v C         -p v D
                 ---------------------     for some symbol p
                         C v D
```

Let Res(S) be its closure under resolution. For each one of the following cases, indicate whether Res(S) is infinite or finite, and, if finite, express an accurate upper bound on its size in terms of n. Very briefly explain why. Use the notation a^b to express exponentiation.

3a) S is a set of Horn clauses.

Answer:
...

3b) If clauses in S have at most two literals.

Answer:
...

3c) S is an arbitrary set of propositional clauses.

Answer:
...

Problem 4. (2.5 points).

4a) Prove that for any propositional formula F (that is, built up with AND (&), OR (v) and NOT (-) connectives), there exists a logically equivalent formula G which is built up with exclusively NAND connectives (and propositional symbols).

We recall that the NAND connective is defined as NAND(x, y) = -(x & y).

Answer:
...

4b) Let P be a fixed set of propositional symbols.
Given two interpretations I, I': P -> {0, 1}, we write
I <= I' iff I(p) <= I'(p) for all p in P.
We say a formula F is MONOTONIC iff I <= I' implies that eval_I(F) <= eval_I'(F).

Prove that any propositional formula F built up only with AND & and OR v connectives is monotonic. Hint: use induction on F.

Answer:
...