Social network formation: from individuals incentives to systemic stability

Nicolò Pagan [1] Florian Dörfler [1]

[1] Automatic Control Laboratory, ETH Zürich, Switzerland

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Social Networks

Social Networks provide benefits

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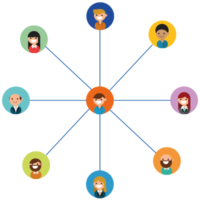
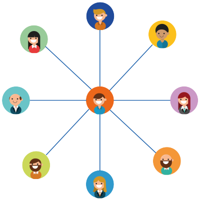
Social Influence



The more people we are connected to, the more we can influence them.

Social Networks provide benefits

Social Influence



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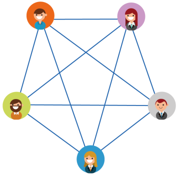
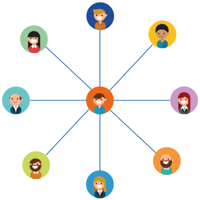
Safety



The more our friends' friends are our friends, the safer we feel.
[Heider (1946); Coleman (1990)]
(Structural Balance theory)

Social Networks provide benefits

Social Influence



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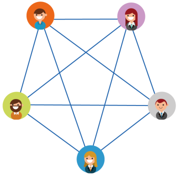

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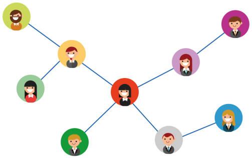
Efficacy



The more we are on the path between people, the more we can control.
[Burt (1992)]
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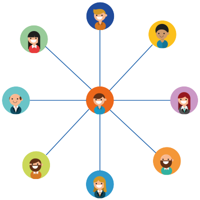
Social Networks provide benefits

Social Influence



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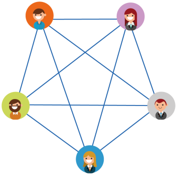
Degree Centrality

Safety



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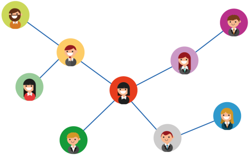
Clustering Coefficient

Efficacy



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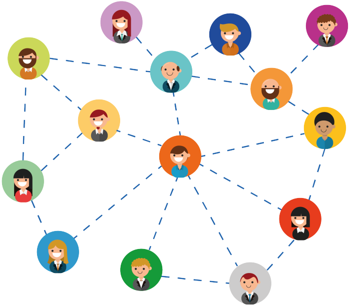
Betweenness Centrality

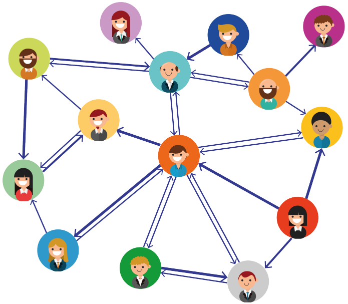
Network Formation



Analysis of the co-evolution of individual behavior and network topology.

- Probabilistic approach
- Game Theoretical approach (Strategic Network Formation)

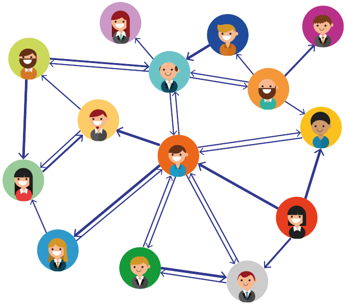
Social Network Formation Model



Ingredients

- **Directed weighted** network $\mathcal{G}$ with $N \geq 3$ agents.

Social Network Formation Model



Ingredients

- **Directed weighted** network $\mathcal{G}$ with $N \geq 3$ agents.
- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among i and j **from i's point of view**.

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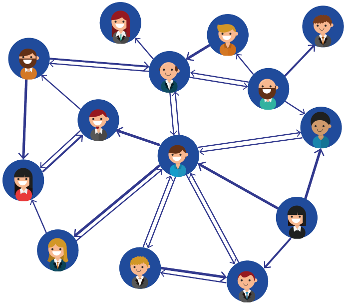

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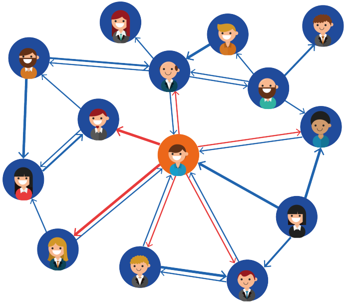

- **Directed weighted** network $\mathcal{G}$ with $N \geq 3$ agents.
- $a_{ij} \in [0,1]$ quantifies the importance of the friendship among i and j **from i's point of view**.
- **Homogeneous** agents.
- A typical action of agent i is:

$$a_i = [a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}]$$

living in the action space $\mathcal{A} = [0,1]^{N-1}$.

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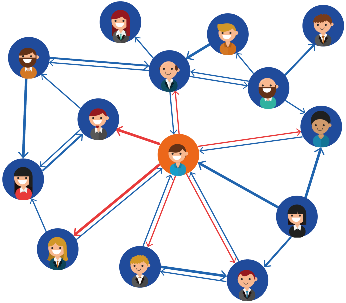

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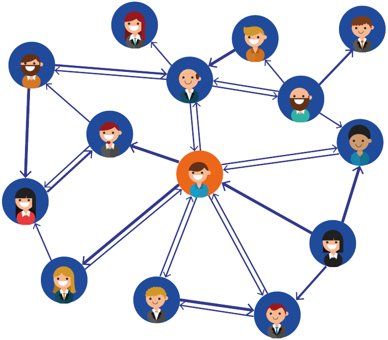
- **Rational** agents: i is endowed with a payoff function V_i and is looking for

$$a_i^\star \in \arg\max_{a_i \in \mathcal{A}} V_i(a_i, \mathbf{a_{-i}})$$

From individual incentives to the payoff function

$$V_i(a_i, \mathbf{a_{-i}}) =$$

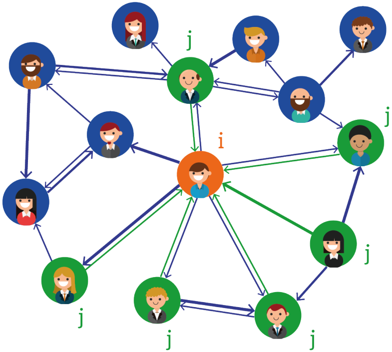


From individual incentives to the payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \quad t_i(a_i, \mathbf{a_{-i}})$$

- **Social influence** on friends,

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji}$$

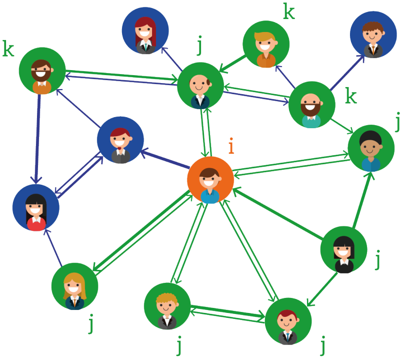


From individual incentives to the payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \quad t_i(a_i, \mathbf{a_{-i}})$$

- **Social influence** on friends, on friends of friends, with $\delta \in [0,1]$:

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji} + \delta \underbrace{\sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}}$$



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$$V_i(a_i, \mathbf{a_{-i}}) = \quad t_i(a_i, \mathbf{a_{-i}})$$

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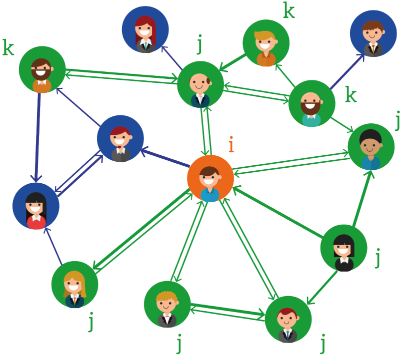

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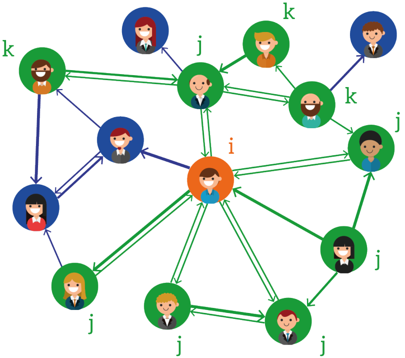
[Jackson and Wolinsky (1996)]



From individual incentives to the payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \quad t_i(a_i, \mathbf{a_{-i}}) + \quad u_i(a_i, \mathbf{a_{-i}})$$

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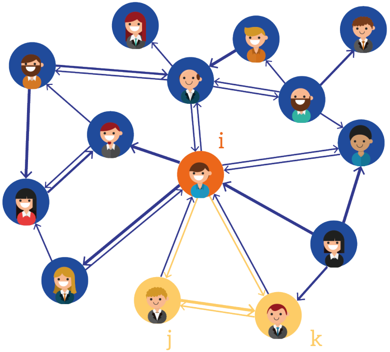
[Jackson and Wolinsky (1996)]

- **Clustering coefficient**: number of closed triads:
$u_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right),$



From individual incentives to the payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \quad t_i(a_i, \mathbf{a_{-i}}) \pm \quad u_i(a_i, \mathbf{a_{-i}})$$

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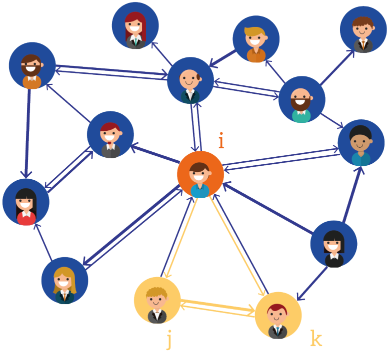

From individual incentives to the payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \quad t_i(a_i, \mathbf{a_{-i}}) \pm \quad u_i(a_i, \mathbf{a_{-i}}) - \quad c_i(a_i),$$

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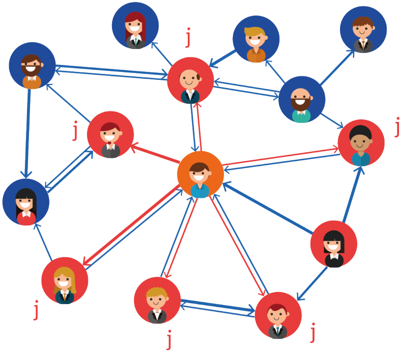
- **Cost** of maintaining ties:

$$c_i(a_i) = \sum_{j \neq i} a_{ij}.$$



From individual incentives to the payoff function

$$V_i(a_i,\mathbf{a_{-i}}) = \alpha t_i(a_i,\mathbf{a_{-i}}) + \beta u_i(a_i,\mathbf{a_{-i}}) - \gamma c_i(a_i), \qquad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0$$

- **Social influence** on friends, on friends of friends, ... with $\delta \in [0,1]$:

$$t_i(a_i,\mathbf{a_{-i}}) = \sum_{j\neq i} a_{ji} + \underbrace{\delta \sum_{k\neq j}\sum_{j\neq i} a_{kj}a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta^2 \sum_{l\neq k}\sum_{k\neq j}\sum_{j\neq i} a_{lk}a_{kj}a_{ji}}_{\text{paths of length 3}},$$

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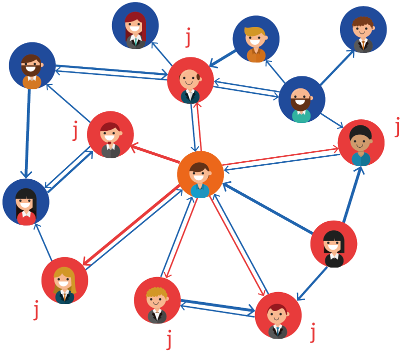
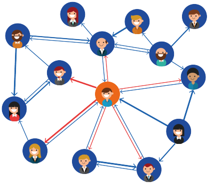


Stability Definitions

Nash equilibrium (NE)



$$V_i\left(\textcolor{red}{a_i}, \mathbf{a_{-i}}^{\star}\right)$$

Stability Definitions

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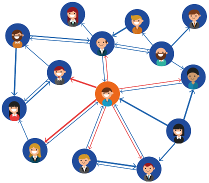

C 1. for all agents i,

$$V_i\left(a_i, \mathbf{a_{-i}}^\star\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star\right), \forall a_i \in \mathcal{A}.$$

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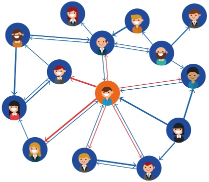

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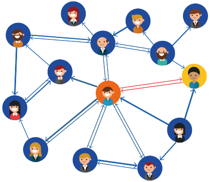
Pairwise-Nash equilibrium (PNE)



$$V_i\left(a_{ij}, a_{ji}, \mathbf{a_{-(i,j)}}^\star\right),\ V_j\left(a_{ji}, a_{ij}, \mathbf{a_{-(i,j)}}^\star\right)$$

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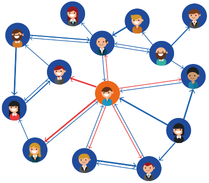

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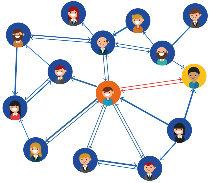

C 2. for all agents i and j, and $\forall a_{ij} \in [0,1]$,

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C 3. for all pairs (i,j), and $\forall\left(a_{ij}, a_{ji}\right) \in [0,1]^2$,

$$V_i\left(a_{ij}, a_{ji}, \mathbf{a}_{-(\mathbf{i},\mathbf{j})}{}^{\star}\right) > V_i\left(a_{ij}{}^{\star}, a^{\star}_{ji}, \mathbf{a}_{-(\mathbf{i},\mathbf{j})}{}^{\star}\right)$$

$$\Downarrow$$

$$V_j\left(a_{ji}, a_{ij}, \mathbf{a}_{-(\mathbf{i},\mathbf{j})}{}^{\star}\right) < V_j\left(a_{ji}{}^{\star}, a^{\star}_{ij}, \mathbf{a}_{-(\mathbf{i},\mathbf{j})}{}^{\star}\right).$$

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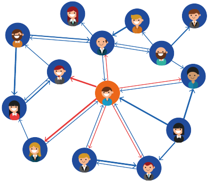

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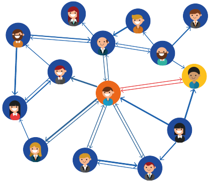
C 1. and **C 2.** are selfish conditions.
C 3. is a cooperative (Pareto optimality) condition.

Pairwise-Nash equilibrium (PNE)



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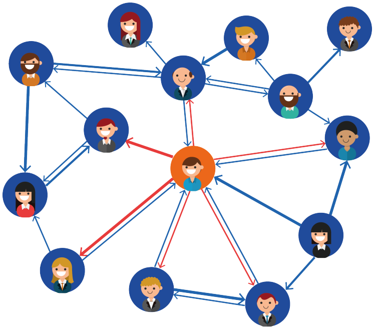
Stability Conditions



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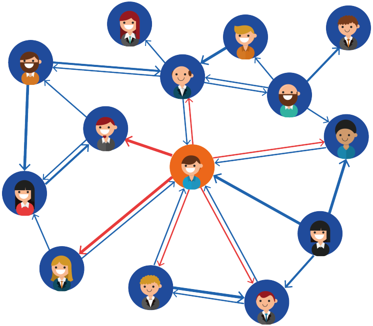

Definition (Nash equilibrium).
The network $\mathcal{G}^\star$ is a NE if for all agents i

$$V_i\left(a_i, \mathbf{a_{-i}}^\star\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star\right), \forall a_i \in \mathcal{A}.$$

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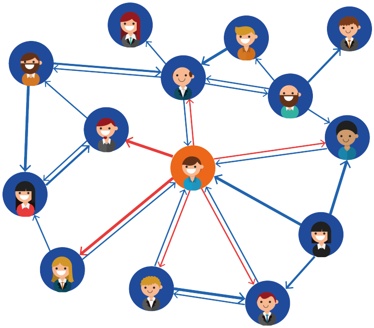

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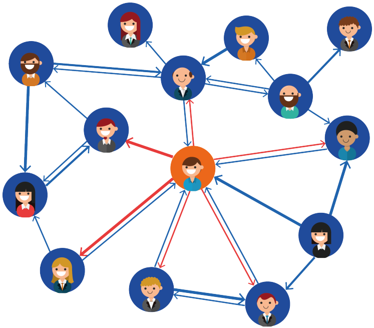
Stability conditions depend on:

- network $\mathcal{G}^\star$: $\{a_i^\star, i \in \mathcal{N}\}$,
- parameters: $\{\alpha, \beta, \gamma, \delta, N\}$.

Remark (Payoff function).

$$\begin{aligned} V_i(a_i, \mathbf{a_{-i}}) = & \alpha t_i(a_i, \mathbf{a_{-i}}, \delta) + & \beta u_i(a_i, \mathbf{a_{-i}}) - & \gamma c_i(a_i), \\ & \alpha \geq 0, \delta \in [0,1], & \beta \in \mathbb{R}, & \gamma > 0. \end{aligned}$$

Stability Conditions



Definition (Nash equilibrium).
The network $\mathcal{G}^\star$ is a NE if for all agents i

$$V_i\left(a_i, \mathbf{a_{-i}}^\star\right) \le V_i\left(a_i^\star, \mathbf{a_{-i}}^\star\right), \forall a_i \in \mathcal{A}.$$

Stability conditions depend on:

- network $\mathcal{G}^\star$: $\{a_i^\star, i \in \mathcal{N}\}$,
- parameters: $\{\alpha, \beta, \gamma, \delta, N\}$.

Remark (Payoff function).

$$\begin{aligned} V_i(a_i, \mathbf{a_{-i}}) = &\alpha t_i(a_i, \mathbf{a_{-i}}, \delta) + &&\beta u_i(a_i, \mathbf{a_{-i}}) - &&\gamma c_i(a_i), \\ &\alpha \ge 0, \delta \in [0,1], &&\beta \in \mathbb{R}, &&\gamma > 0. \end{aligned}$$

Question: For which parameters is a certain topology $\mathcal{G}^\star$ stable?

Network Topologies

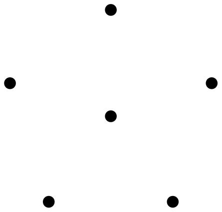


Figure: Empty Network

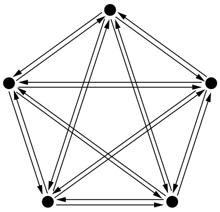


Figure: Complete Network

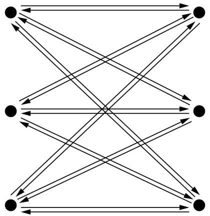


Figure: Complete Balanced Bipartite Network

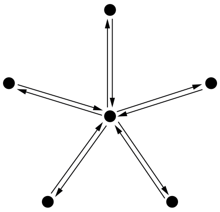
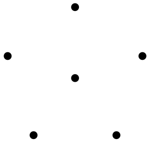
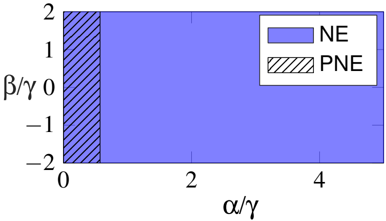


Figure: Star Network

Empty Network stability





Theorem .

- *$\mathcal{G}^{EN}$ is always a NE.*
- *$\mathcal{G}^{EN}$ is a PNE if and only if*

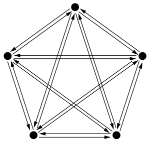
$$\frac{\alpha}{\gamma} \leq \frac{1}{(1+\delta+\delta^2)}.$$

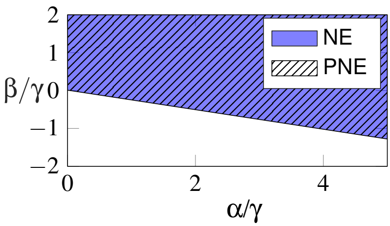
- No agent has a selfish incentive to create a link $\rightarrow$ **always a NE**.
- Cooperation reduces the region of pairwise-Nash equilibrium.

Remark (Payoff function).

$V_i(a_i, \mathbf{a_{-i}}) = \alpha t_i(a_i, \mathbf{a_{-i}}, \delta) + \beta u_i(a_i, \mathbf{a_{-i}}) - \gamma c_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0.$

Complete Network stability





Theorem . *Define*

$$\bar{\gamma}_{NE} := \begin{cases} \alpha\delta\,(1+\delta(2N-3)) + \beta\,(N-2)\,, & \text{if } \beta > 0 \\ \alpha\delta\,(1+\delta(2N-3)) + 2\beta\,(N-2)\,, & \text{if } \beta \leq 0, \end{cases}$$

$$\bar{\gamma}_{PNE} := \alpha\delta\,(1+\delta(2N-3)) + 2\beta\,(N-2)\,,$$

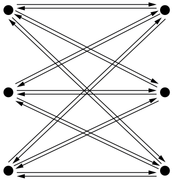
then,

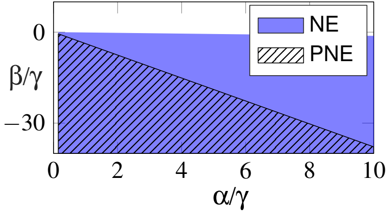
- $\mathcal{G}^{CN}$ *is a NE* $\iff \gamma \leq \bar{\gamma}_{NE}$,
- $\mathcal{G}^{CN}$ *is a PNE* $\iff \gamma \leq \bar{\gamma}_{PNE}$.

Remark (Payoff function).
$V_i(a_i, \mathbf{a_{-i}}) = \alpha t_i(a_i, \mathbf{a_{-i}}, \delta) + \beta u_i(a_i, \mathbf{a_{-i}}) - \gamma c_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0.$

- Complete network stability is correlated with **large** values of β (**high clustering**).
- Phase transition: Selfish agents prefer to drop all the links simultaneously.

Balanced Complete Bipartite Network stability





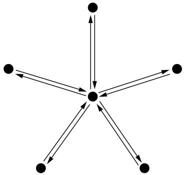
At equilibrium,

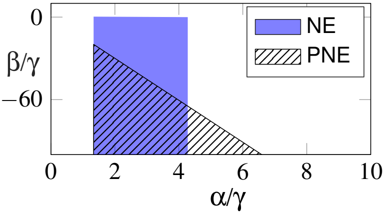
- ties across different node partitions should not be dismissed;
- ties within the same node partition should not be initiated.

- Balanced Complete Bipartite network stability is correlated with **negative** values of β (**betweenness centrality**).
- Cooperation reduces the region of pairwise-Nash equilibrium.

Remark (Payoff function).
$V_i(a_i, \mathbf{a}_{-\mathbf{i}}) = \alpha t_i(a_i, \mathbf{a}_{-\mathbf{i}}, \delta) + \beta u_i(a_i, \mathbf{a}_{-\mathbf{i}}) - \gamma c_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0.$

Star stability





At equilibrium,

- ties from the central node should not be dismissed;
- ties to the central node should not be dismissed;
- ties between peripheral nodes should not be initiated.

- Star network stability needs **negative** values of β (**betweenness centrality**) and **large** values of α (**degree centrality**). However, an upper bound on α exists (for NE).
- Cooperation reduces the region of pairwise-Nash equilibrium.

Remark (Payoff function).

$V_i(a_i, \mathbf{a_{-i}}) = \alpha t_i(a_i, \mathbf{a_{-i}}, \delta) + \beta u_i(a_i, \mathbf{a_{-i}}) - \gamma c_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0.$

Phase diagram

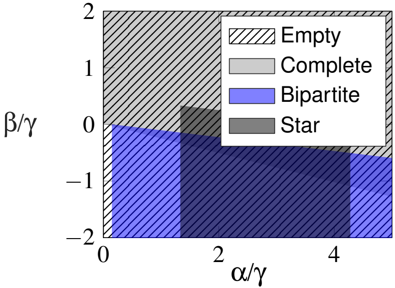


Figure: Nash Equilibrium

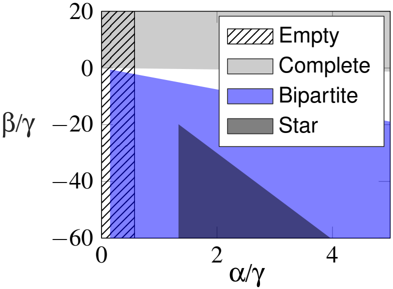
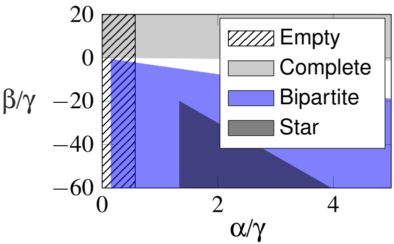


Figure: Pairwise-Nash Equilibrium

- Different stable topologies can co-exist in the state space.
- Cooperation between agents often reduces the stability region.

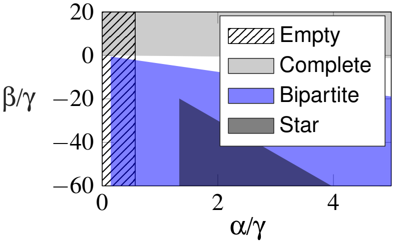
Individual incentives and Pairwise-Nash stable topologies



Payoff function

$$V_i(a_i,\mathbf{a_{-i}}) = \alpha\left(\sum_{j\neq i} a_{ji} + \delta \sum_{k\neq j}\sum_{j\neq i} a_{kj}a_{ji} + \delta^2 \sum_{l\neq k}\sum_{k\neq j}\sum_{j\neq i} a_{lk}a_{kj}a_{ji}\right)$$
$$+\beta \sum_{j\neq i} a_{ij}\left(\sum_{k\neq i,j} a_{ik}a_{kj}\right) - \gamma \sum_{j\neq i} a_{ij}, \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0$$

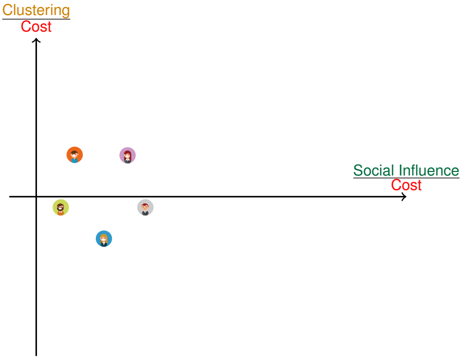
Individual incentives and Pairwise-Nash stable topologies



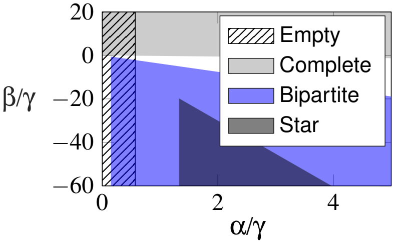
- **Empty network** stability and **high cost** [Bala and Goyal (2000); Buechel and Buskens (2013)],

Payoff function

$$V_i(a_i,\mathbf{a_{-i}}) = \alpha\left(\sum_{j\neq i} a_{ji} + \delta\sum_{k\neq j}\sum_{j\neq i} a_{kj}a_{ji} + \delta^2\sum_{l\neq k}\sum_{k\neq j}\sum_{j\neq i} a_{lk}a_{kj}a_{ji}\right)$$
$$+\beta\sum_{j\neq i} a_{ij}\left(\sum_{k\neq i,j} a_{ik}a_{kj}\right) - \gamma\sum_{j\neq i} a_{ij}, \quad \alpha\geq 0,\ \beta\in\mathbb{R},\ \gamma>0$$



Individual incentives and Pairwise-Nash stable topologies



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Payoff function

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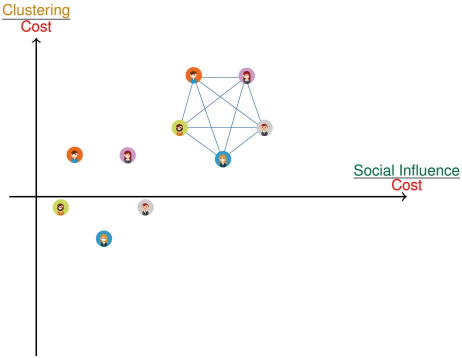
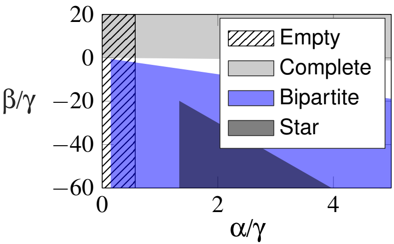
$$+ \beta \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right) - \gamma \sum_{j \neq i} a_{ij}, \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0$$



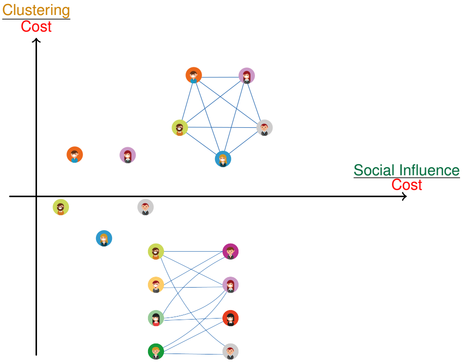
Individual incentives and Pairwise-Nash stable topologies



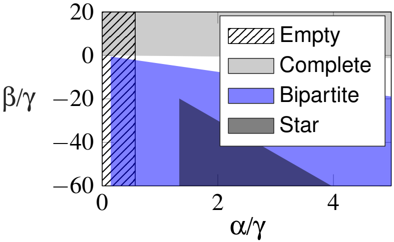
- **Empty network** stability and **high cost** [Bala and Goyal (2000); Buechel and Buskens (2013)],
- **Complete network** stability and **high clustering** [Buechel and Buskens (2013)],
- **Balanced Complete Bipartite network** stability and **betweenness centrality** [Buskens and van de Rijt (2008)],

Payoff function

$$V_i(a_i, \mathbf{a}_{-\mathbf{i}}) = \alpha \left(\sum_{j\neq i} a_{ji} + \delta \sum_{k\neq j} \sum_{j\neq i} a_{kj} a_{ji} + \delta^2 \sum_{l\neq k} \sum_{k\neq j} \sum_{j\neq i} a_{lk} a_{kj} a_{ji} \right) + \beta \sum_{j\neq i} a_{ij} \left(\sum_{k\neq i,j} a_{ik} a_{kj} \right) - \gamma \sum_{j\neq i} a_{ij}, \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma > 0$$



Individual incentives and Pairwise-Nash stable topologies



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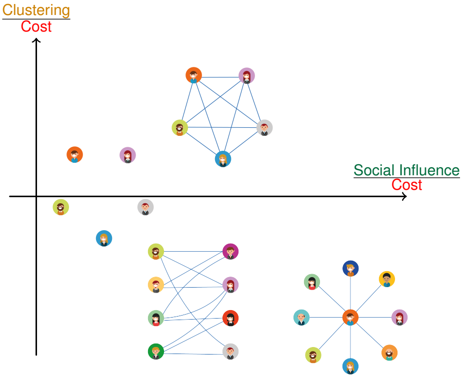
- **Balanced Complete Bipartite network** stability and **betweenness centrality** [Buskens and van de Rijt (2008)],
- **Star network** stability and **degree centrality** [Bala and Goyal (2000)].

Payoff function

$$V_i(a_i,\mathbf{a_{-i}}) = \alpha\left(\sum_{j\neq i} a_{ji} + \delta\sum_{k\neq j}\sum_{j\neq i} a_{kj}a_{ji} + \delta^2\sum_{l\neq k}\sum_{k\neq j}\sum_{j\neq i} a_{lk}a_{kj}a_{ji}\right)$$

$$+\beta\sum_{j\neq i} a_{ij}\left(\sum_{k\neq i,j} a_{ik}a_{kj}\right) - \gamma\sum_{j\neq i} a_{ij}, \quad \alpha \geq 0,\ \beta\in\mathbb{R},\ \gamma > 0$$



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