From individuals decisions to emerging social structure

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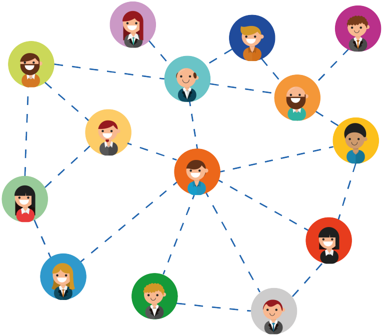
How do social networks form?



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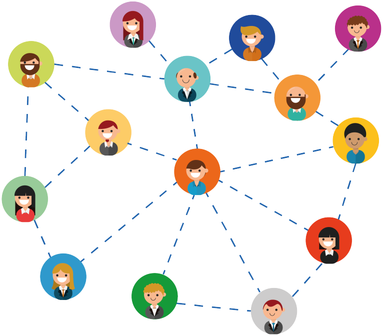

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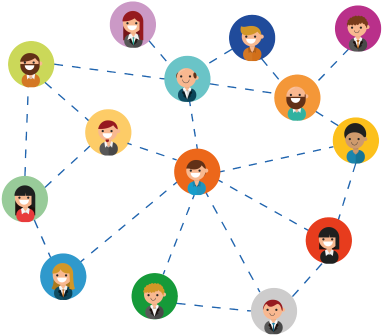

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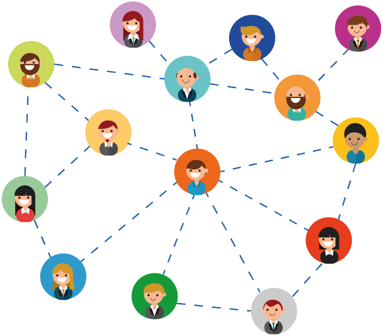

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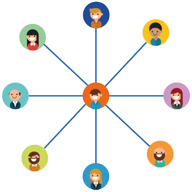

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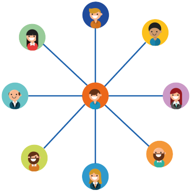

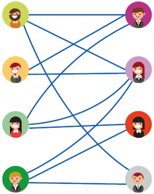

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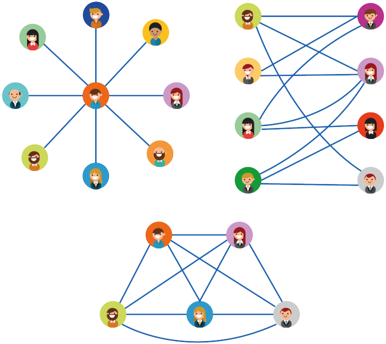

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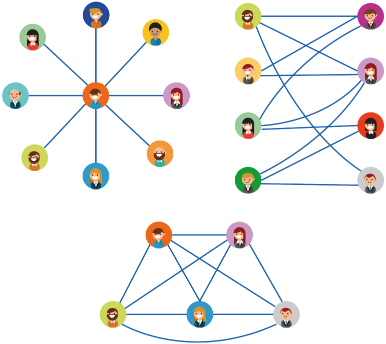

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Individual incentives: why do we form social ties?

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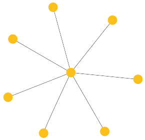

Degree Centrality

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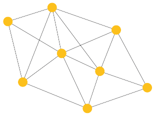

High Clustering

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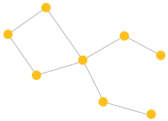

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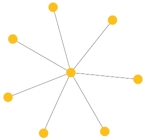

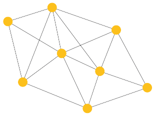

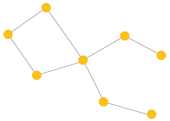

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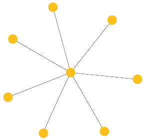

Degree Centrality

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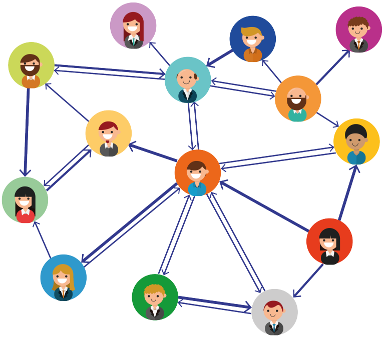
Betweenness Centrality

Closed triads have **positive externalities** (**Structural Balance theory**) [Cartwright and Harary (1956)]

Close triads have **negative externalities** (**Structural Holes theory**) [Burt (1992)]

Social Network Formation Model



- **Directed weighted** network $\mathcal{G}$ with $N \geq 3$ agents.

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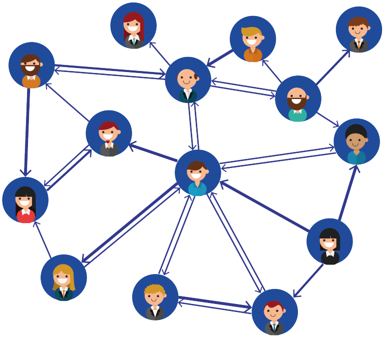

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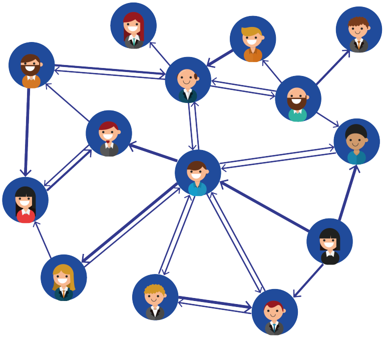

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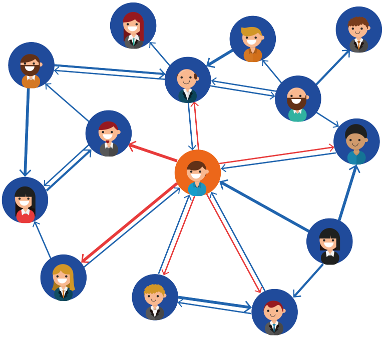

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- Let $\mathbf{a_{ij}} \in [0, 1]$ quantify the importance of the friendship among i and j **from i's point of view**.
- A typical action of each agent i is:

$$a_i = [a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}] \in \mathcal{A} = [0, 1]^{N-1},$$

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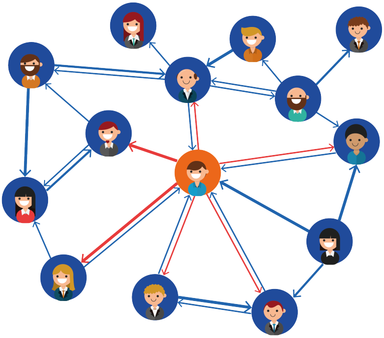

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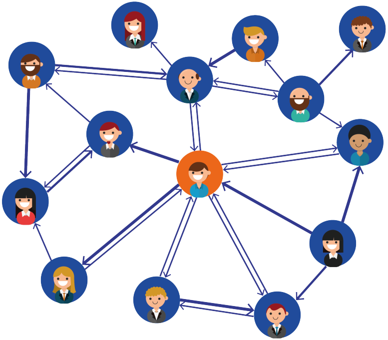
- **Rational** agents: i looks for

$$a_i^\star = \arg\max_{a_i \in \mathcal{A}} V_i(a_i, \mathbf{a_{-i}})$$

From individual incentives to the Payoff Function

$$V_i(a_i, \mathbf{a_{-i}}) =$$



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$$V_i(a_i, \mathbf{a_{-i}}) = \quad P_i(a_i, \mathbf{a_{-i}})$$

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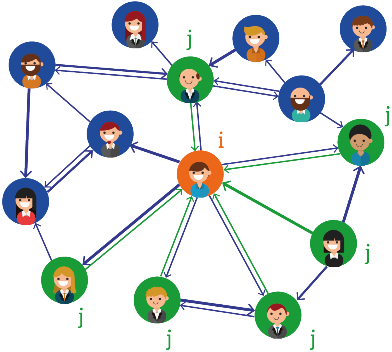

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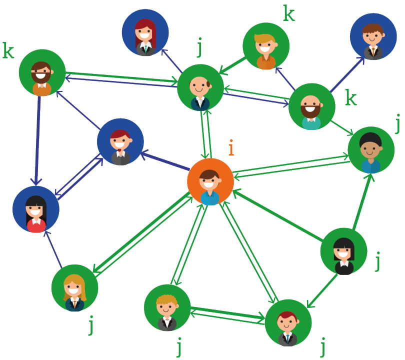

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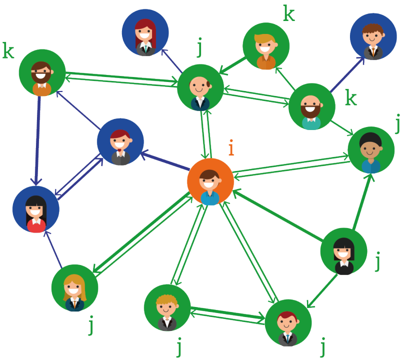

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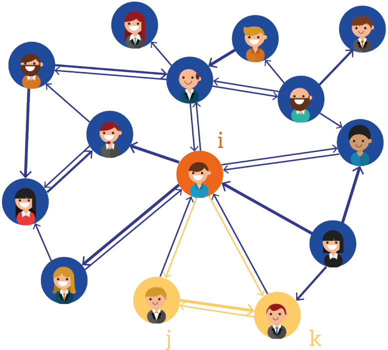

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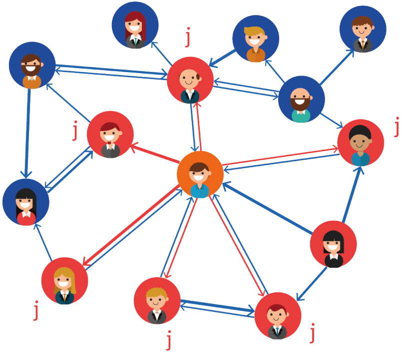

From individual incentives to the Payoff Function

$$V_i(a_i, \mathbf{a_{-i}}) = \alpha P_i(a_i, \mathbf{a_{-i}}) + \beta B_i(a_i, \mathbf{a_{-i}}) - \gamma C_i(a_i), \qquad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0$$

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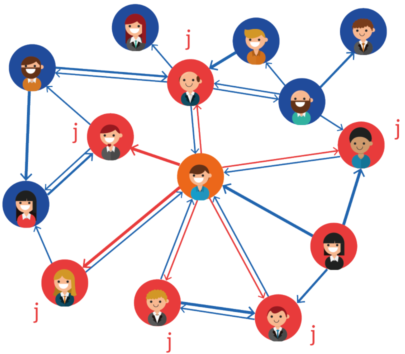
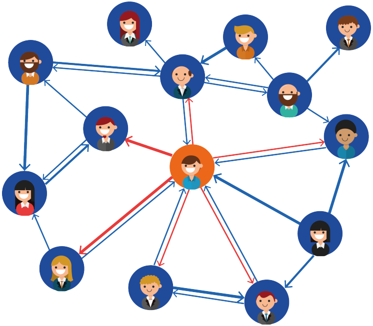


Nash stability



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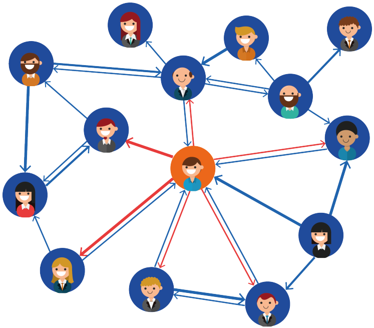

Definition (Nash equilibrium, NE).
The network $\mathcal{G}^\star$ is a NE if for all agents i

$$V_i\left(a_i, \mathbf{a_{-i}}^\star\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star\right), \forall a_i \in \mathcal{A}.$$

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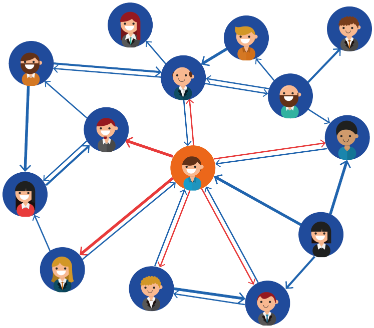

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Remark (Payoff function).

$$\begin{aligned} V_i(a_i, \mathbf{a_{-i}}) = & \alpha P_i(a_i, \mathbf{a_{-i}}) + \quad \beta B_i(a_i, \mathbf{a_{-i}}) - \quad \gamma C_i(a_i), \\ & \alpha \geq 0, \qquad\qquad \beta \in \mathbb{R}, \qquad\qquad \gamma \geq 0. \end{aligned}$$

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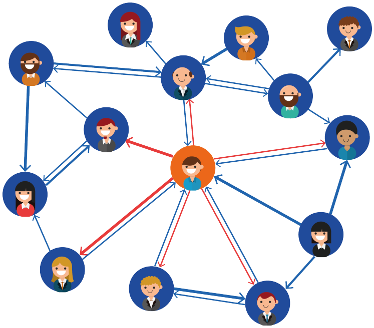

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Stability conditions depend on:

- network $\mathcal{G}^\star$: $\{a_i^\star, i \in \mathcal{N}\}$,
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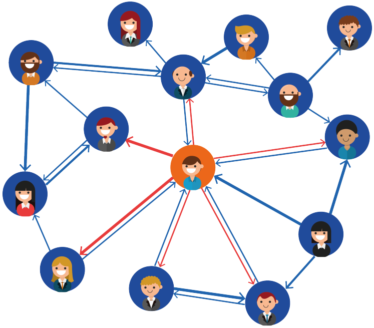

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Question: For which parameters is a certain network stable?

Network Motifs

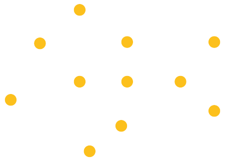
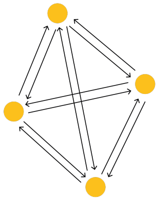


Figure: Empty Network



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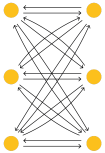


Figure: Complete Balanced Bipartite Network

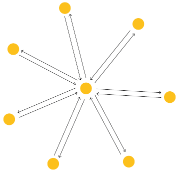
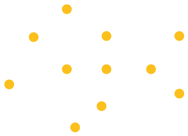


Figure: Star Network

Empty/Complete Network stability regions

Empty Network



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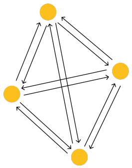

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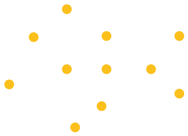
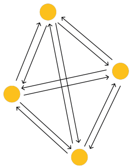


Theorem . *The empty network $\mathcal{G}^{EN}$ is always a Nash equilibrium.*

Complete Network



Theorem . *Let $\mathcal{G}^{CN}$ be a complete network. Define*

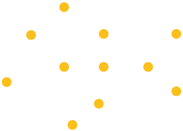
$$\gamma_{NE} := \begin{cases} \frac{\alpha\delta(1+\delta(2N-3))}{d(N-1)^{d-1}} + \frac{2\beta(N-2)}{\max(d,2)(N-1)^{d-1}}, & \text{if } \beta \geq 0 \\ \frac{\alpha\delta(1+\delta(2N-3))}{d(N-1)^{d-1}} + \frac{2\beta(N-2)}{d(N-1)^{d-1}}, & \text{if } \beta < 0 \end{cases}$$

then $\mathcal{G}^{CN}$ is a NE if and only if $\gamma \leq \gamma_{NE}$.

Remark (Payoff function).
$$V_i(a_i,\mathbf{a_{-i}}) = \alpha P_i(a_i,\mathbf{a_{-i}}) + \beta B_i(a_i,\mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$$

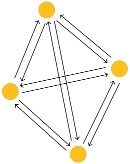
Empty/Complete Network stability regions

Empty Network



Theorem . *The empty network $\mathcal{G}^{EN}$ is always a Nash equilibrium.*

Complete Network



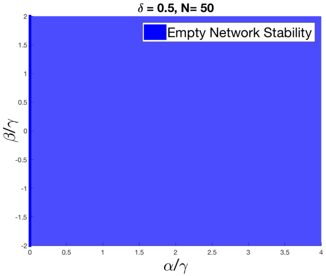
Theorem . *Let $\mathcal{G}^{CN}$ be a complete network. $\mathcal{G}^{CN}$ is a NE if and only if*

$$\frac{\beta}{\gamma} \geq \max\left\{\frac{\max\{d,2\}}{2d\,(N-2)}\left(d\,(N-1)^{d-1} - \delta\,(1+\delta(2N-3))\,\frac{\alpha}{\gamma}\right),\right.$$
$$\left.\frac{1}{2\,(N-2)}\left(d\,(N-1)^{d-1} - \delta\,(1+\delta(2N-3))\,\frac{\alpha}{\gamma}\right)\right\}.$$

Remark (Payoff function).
$V_i(a_i,\mathbf{a}_{-\mathbf{i}}) = \alpha P_i(a_i,\mathbf{a}_{-\mathbf{i}}) + \beta B_i(a_i,\mathbf{a}_{-\mathbf{i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$

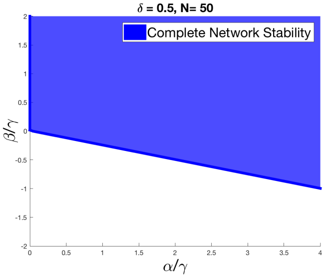
Empty/Complete Network stability regions

Empty Network



- No agent has a selfish incentive to create a link $\rightarrow$ **always a NE**.

Complete Network



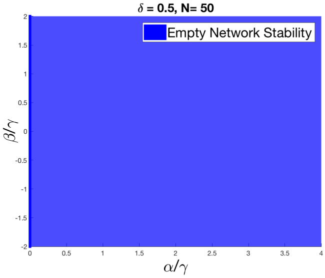
- Piecewise linear relation between $\frac{\beta}{\gamma}$ and $\frac{\alpha}{\gamma}$.

Remark (Payoff function).

$$V_i(a_i, \mathbf{a_{-i}}) = \alpha P_i(a_i, \mathbf{a_{-i}}) + \beta B_i(a_i, \mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$$

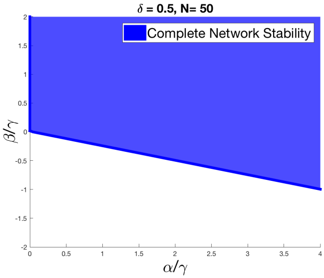
Empty/Complete Network stability regions

Empty Network



- No agent has a selfish incentive to create a link $\rightarrow$ **always a NE**.

Complete Network



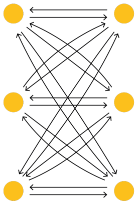
- Complete network stability is correlated with **large** values of β (**Bonding capital / high clustering**).

Remark (Payoff function).

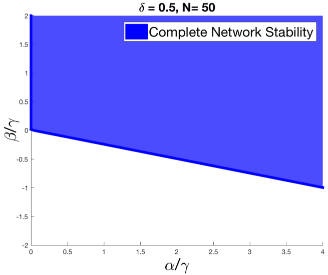
$V_i(a_i, \mathbf{a_{-i}}) = \alpha P_i(a_i, \mathbf{a_{-i}}) + \beta B_i(a_i, \mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$

Bipartite/Complete Networks stability regions

Bipartite Network



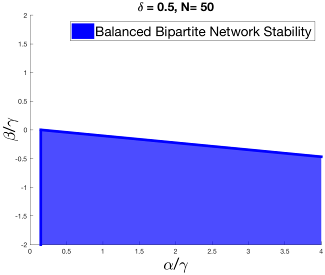
Complete Network



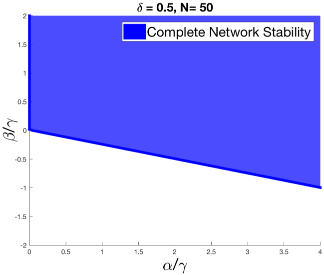
- Complete network stability is correlated with **large** values of β (**Bonding capital / high clustering**).

Bipartite/Complete Networks stability regions

Bipartite Network



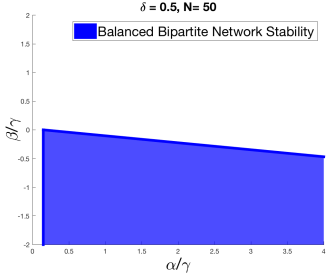
Complete Network



- Complete network stability is correlated with **large** values of β (**Bonding capital / high clustering**).

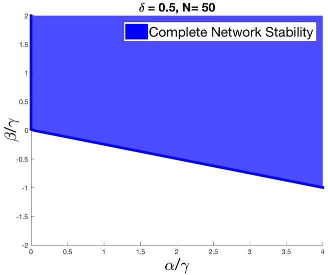
Bipartite/Complete Networks stability regions

Bipartite Network



- Bipartite network stability is correlated with **small** values of β (**Bridging capital** / **betweenness centrality**).

Complete Network

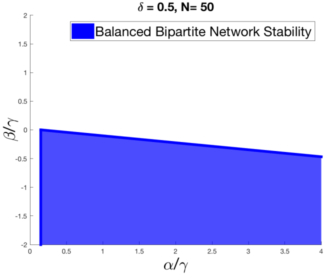


- Complete network stability is correlated with **large** values of β (**Bonding capital** / **high clustering**).

Remark (Payoff).
$V_i(a_i, \mathbf{a_{-i}}) = \alpha P_i(a_i, \mathbf{a_{-i}}) + \beta B_i(a_i, \mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$

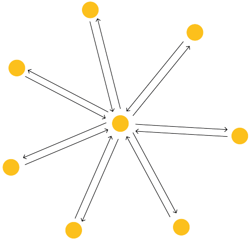
Bipartite/Star Networks stability regions

Bipartite Network



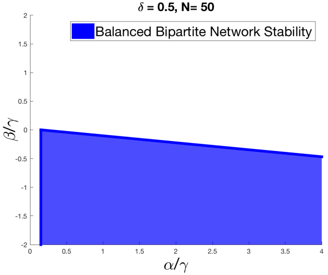
- Bipartite network stability is correlated with **small** values of β (**Bridging capital** / **betweenness centrality**).

Star Network



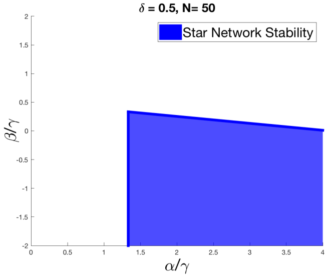
Bipartite/Star Networks stability regions

Bipartite Network



- Bipartite network stability is correlated with **small** values of β (**Bridging capital / betweenness centrality**).

Star Network

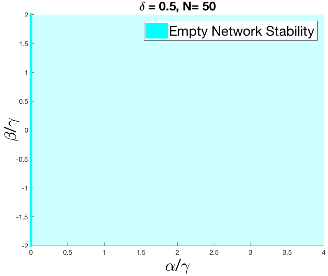


- Star network stability is correlated with **large** values of α (**Popularity capital / degree centrality**).

Remark (Payoff).

$V_i(a_i, \mathbf{a_{-i}}) = \alpha P_i(a_i, \mathbf{a_{-i}}) + \beta B_i(a_i, \mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$

Phase diagram

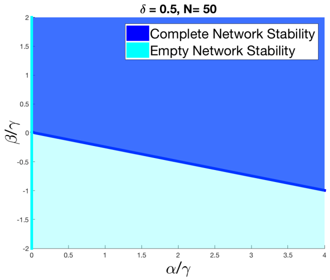


Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \alpha \left(\sum_{j \neq i} a_{ji} + \delta \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji} + \delta^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji} \right)$$

$$+ \beta \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i, j} a_{ik} a_{kj} \right) - \gamma \sum_{j \neq i} a_{ij}, \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0$$

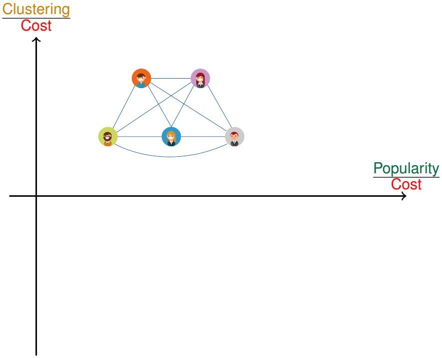
Phase diagram



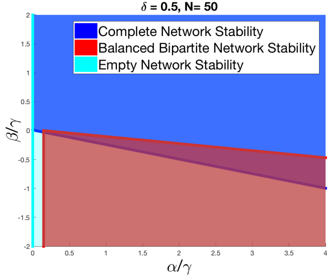
- **Complete network** stability and **high clustering** [Buechel and Buskens (2013)],

Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = \alpha \left(\sum_{j \neq i} a_{ji} + \delta \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji} + \delta^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji} \right) + \beta \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right) - \gamma \sum_{j \neq i} a_{ij}, \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0$$



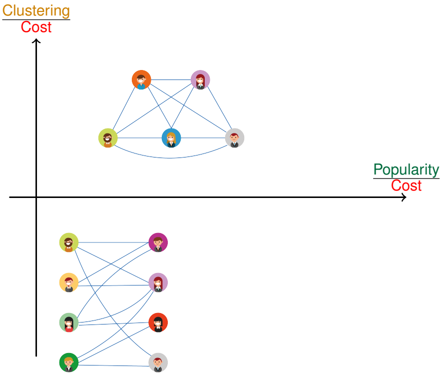
Phase diagram



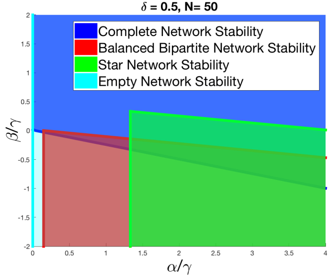
- **Complete network** stability and **high clustering** [Buechel and Buskens (2013)],
- **Balanced Bipartite network** stability and **betweenness centrality** [Buskens and van de Rijt (2008)],

Payoff function

$$V_i(a_i,\mathbf{a}_{-\mathbf{i}}) = \alpha\left(\sum_{j\neq i} a_{ji} + \delta\sum_{k\neq j}\sum_{j\neq i} a_{kj}a_{ji} + \delta^2 \sum_{l\neq k}\sum_{k\neq j}\sum_{j\neq i} a_{lk}a_{kj}a_{ji}\right)$$
$$+\beta\sum_{j\neq i} a_{ij}\left(\sum_{k\neq i,j} a_{ik}a_{kj}\right) - \gamma\sum_{j\neq i} a_{ij}, \quad \alpha\geq 0,\ \beta\in\mathbb{R},\ \gamma\geq 0$$



Phase diagram

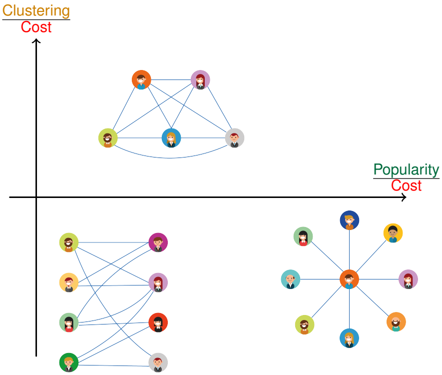


- **Complete network** stability and **high clustering** [Buechel and Buskens (2013)],
- **Balanced Bipartite network** stability and **betweenness centrality** [Buskens and van de Rijt (2008)],
- **Star network** stability and **degree centrality** [Bala and Goyal (2000)].

Payoff function

$$V_i(a_i,\mathbf{a_{-i}}) = \alpha\left(\sum_{j\neq i} a_{ji} + \delta\sum_{k\neq j}\sum_{j\neq i} a_{kj}a_{ji} + \delta^2 \sum_{l\neq k}\sum_{k\neq j}\sum_{j\neq i} a_{lk}a_{kj}a_{ji}\right)$$

$$+\beta\sum_{j\neq i} a_{ij}\left(\sum_{k\neq i,j} a_{ik}a_{kj}\right) - \gamma\sum_{j\neq i} a_{ij}, \quad \alpha\geq 0, \beta\in\mathbb{R}, \gamma\geq 0$$



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Buechel, B. and Buskens, V. (2013). The Dynamics of Closeness and Betweenness. *The Journal of Mathematical Sociology*, 37(July):159–191.

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Burt, R. S. (1992). Structural hole. *Harvard Business School Press, Cambridge, MA*.

Buskens, V. and van de Rijt, A. (2008). Dynamics of Networks if Everyone Strives for Structural Holes. *American Journal of Sociology*, 114(2):371–407.

Cartwright, D. and Harary, F. (1956). Structural balance: a generalization of heider's theory. *Psychological review*, 63(5):277.

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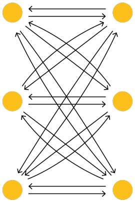
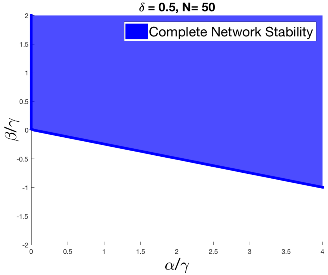
Bipartite/Complete Networks stability regions

Bipartite Network



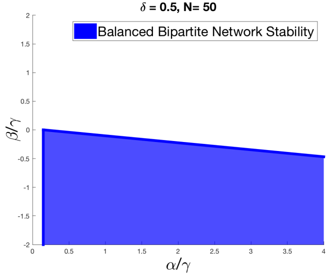
Complete Network

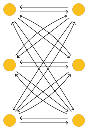


- Complete network stability is correlated with **large** values of β (**high clustering**).

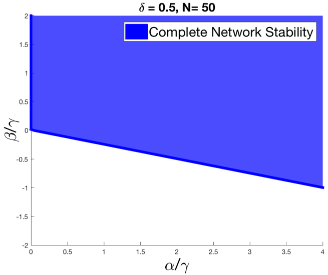
Bipartite/Complete Networks stability regions

Bipartite Network





Complete Network



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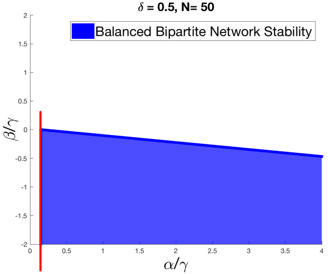
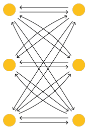
Bipartite/Complete Networks stability regions

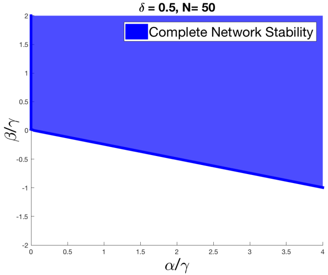
Bipartite Network





- $\frac{\alpha}{\gamma} \geq \cdots \rightarrow$ non destroying existing links across different partitions

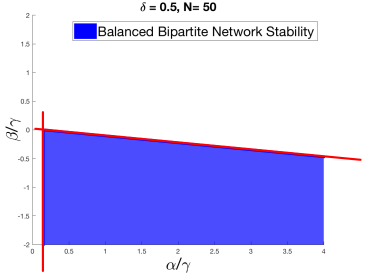
Complete Network

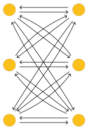


- Complete network stability is correlated with **large** values of β (**high clustering**).

Bipartite/Complete Networks stability regions

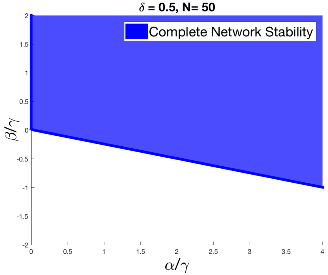
Bipartite Network





- $\frac{\beta}{\gamma} \leq \cdots \rightarrow$ non creating new links within the same partition

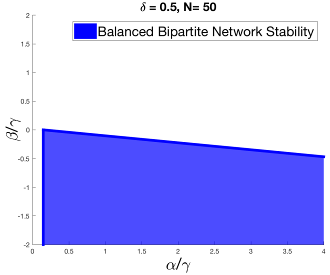
Complete Network



- Complete network stability is correlated with **large** values of β (**high clustering**).

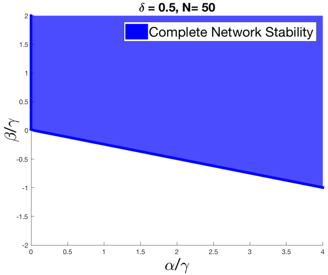
Bipartite/Complete Networks stability regions

Bipartite Network



- Bipartite network stability is correlated with **small** values of β (**high betweenness centrality**).

Complete Network

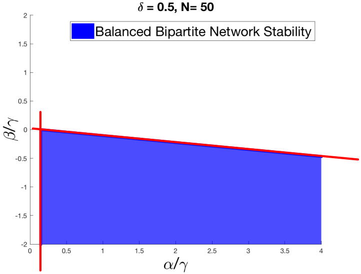


- Complete network stability is correlated with **large** values of β (**high clustering**).

Remark (Payoff).
$V_i(a_i,\mathbf{a_{-i}}) = \alpha P_i(a_i,\mathbf{a_{-i}}) + \beta B_i(a_i,\mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$

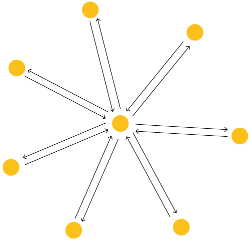
Bipartite/Star Networks stability regions

Bipartite Network



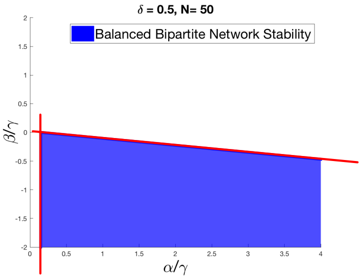
- Bipartite network stability is correlated with **small** values of β (**high betweenness centrality**).

Star Network



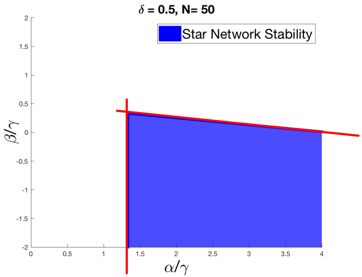
Bipartite/Star Networks stability regions

Bipartite Network



- Bipartite network stability is correlated with **small** values of β (**high betweenness centrality**).

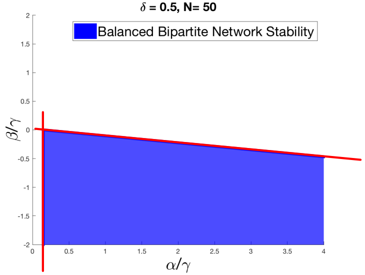
Star Network



- $\frac{\alpha}{\gamma} \geq \cdots \rightarrow$ non destroying existing links across different partitions
- $\frac{\beta}{\gamma} \leq \cdots \rightarrow$ non creating new links within the same partition

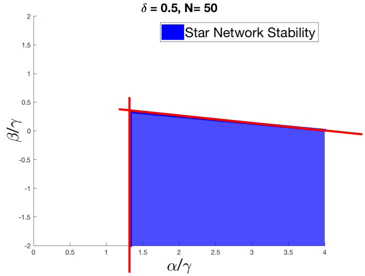
Bipartite/Star Networks stability regions

Bipartite Network



- Bipartite network stability is correlated with **small** values of β (**high betweenness centrality**).

Star Network



- Star network stability is correlated with **large** values of α (**high Popularity capital**).

Remark (Payoff).

$$V_i(a_i, \mathbf{a_{-i}}) = \alpha P_i(a_i, \mathbf{a_{-i}}) + \beta B_i(a_i, \mathbf{a_{-i}}) - \gamma C_i(a_i), \quad \alpha \geq 0, \beta \in \mathbb{R}, \gamma \geq 0.$$