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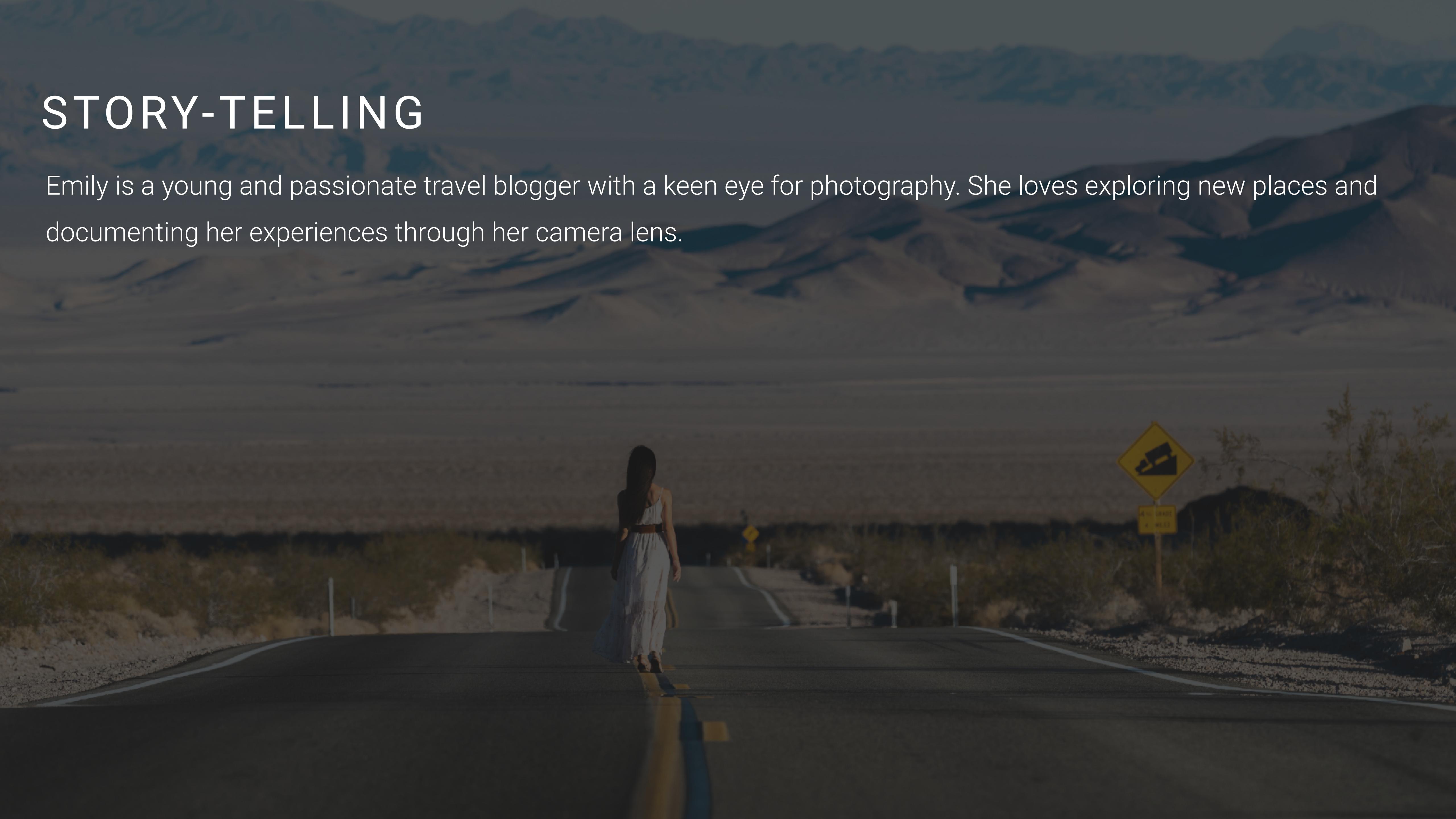
# STRATEGIC INTERACTIONS AMONG SOCIAL MEDIA CONTENT CREATORS

NICOLÒ PAGAN,  
UNIVERSITY OF ZÜRICH

WORKSHOP ON "GAMES ON NETWORKS":  
NATIONAL UNIVERSITY OF SINGAPORE, APRIL 5, 2023

# STORY-TELLING

Emily is a young and passionate travel blogger with a keen eye for photography. She loves exploring new places and documenting her experiences through her camera lens.



# STORY-TELLING

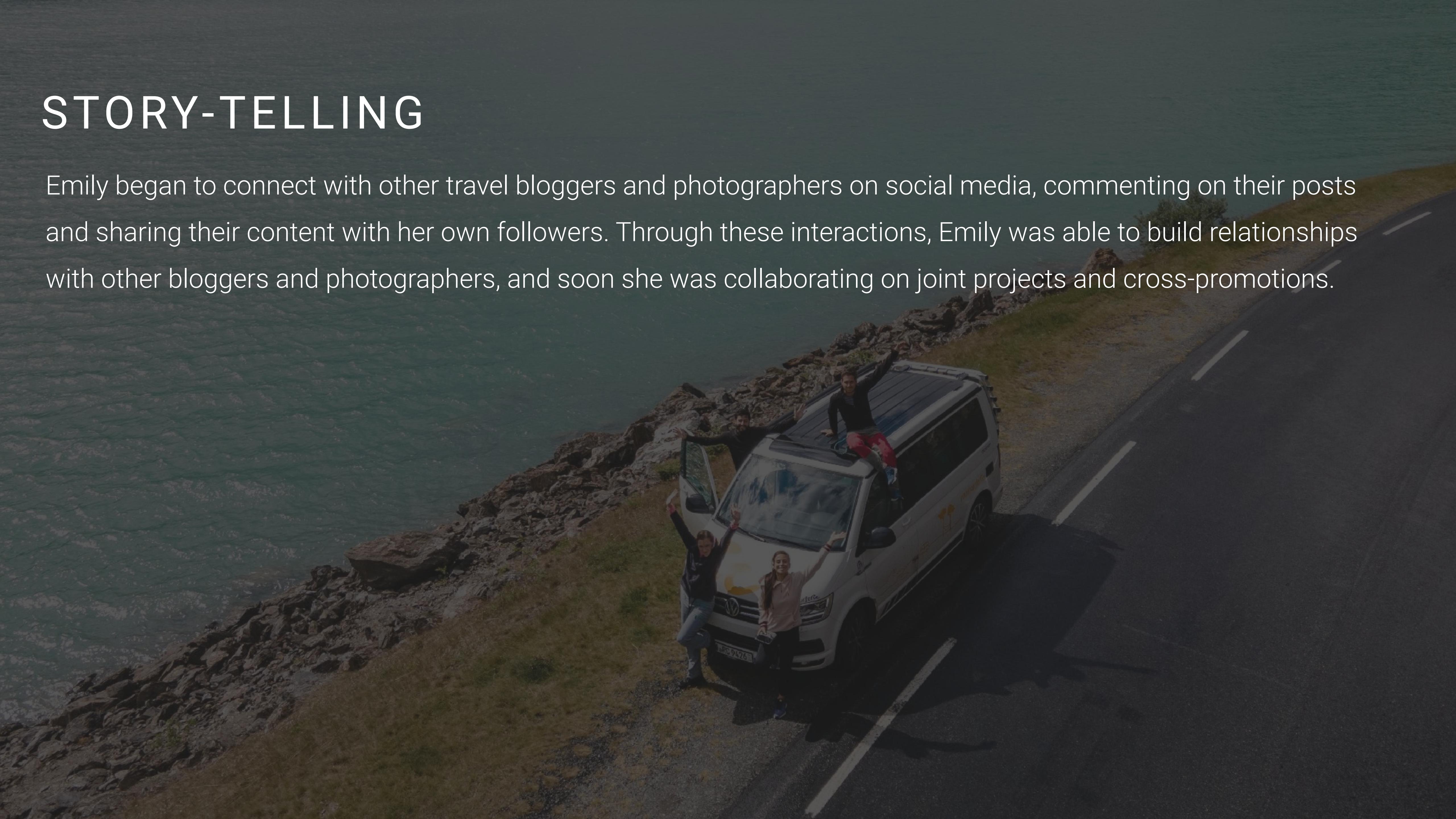
When Emily first started her travel blog, she struggled to gain traction. She had a few followers, but her content wasn't reaching a wider audience.

That's when she decided to try a new approach.



# STORY-TELLING

Emily began to connect with other travel bloggers and photographers on social media, commenting on their posts and sharing their content with her own followers. Through these interactions, Emily was able to build relationships with other bloggers and photographers, and soon she was collaborating on joint projects and cross-promotions.



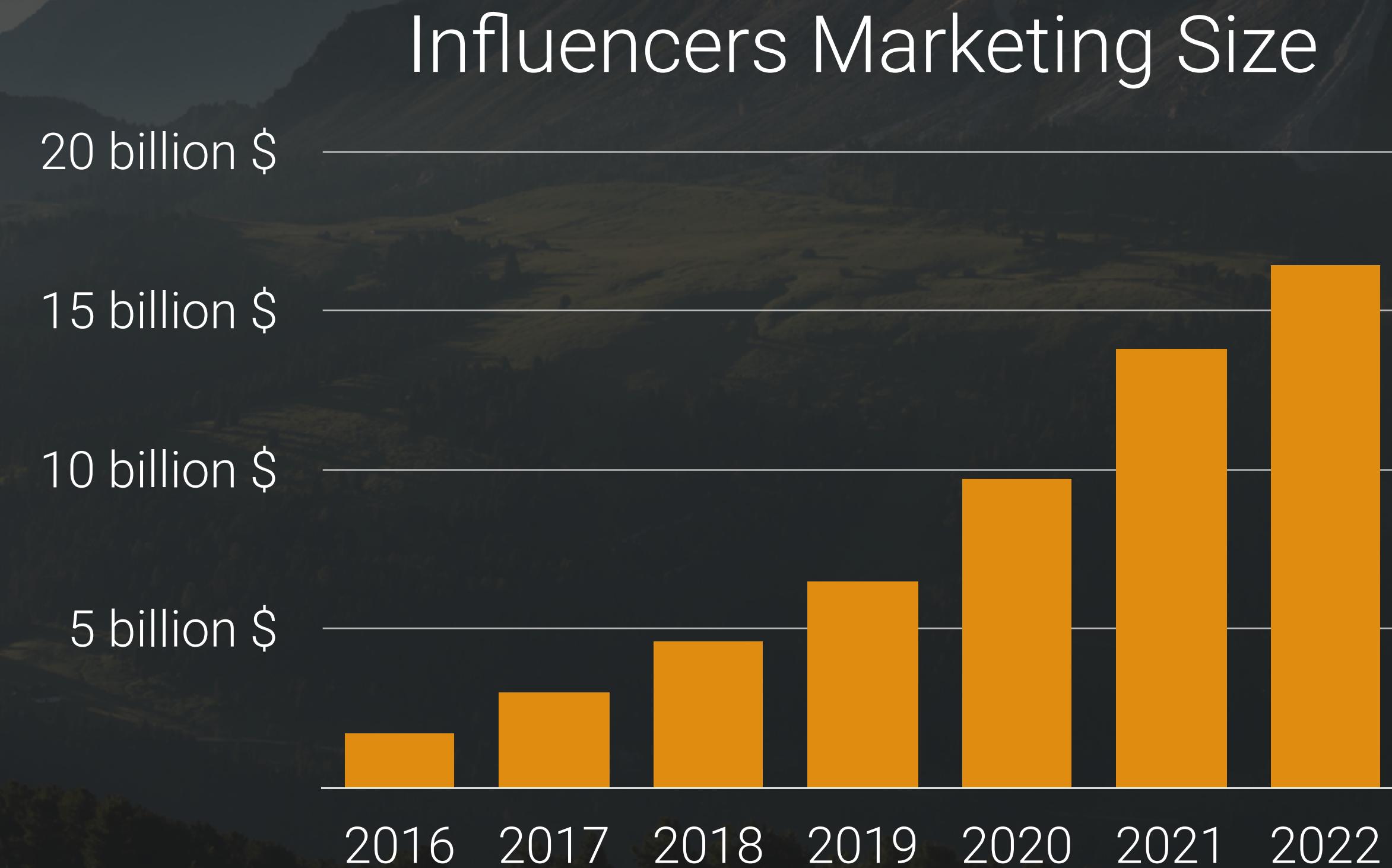
# MOTIVATION

- Social media platforms are increasingly being leveraged as a means of making a living.



# MOTIVATION

- ▶ Social media platforms are increasingly being leveraged as a means of making a living.
- ▶ The Creator Economy has seen tremendous growth over the past decade with more than 50 million content creators



Source: Influencer Marketing Hub

# MOTIVATION

- ▶ Social media platforms are increasingly being leveraged as a means of making a living.
- ▶ The Creator Economy has seen tremendous growth over the past decade with more than 50 million content creators
- ▶ However, social media platforms do not always provide **FAIR OPPORTUNITIES** and **COMPENSATION** for these creators.

For example: there exists 30% gender earning gap, and a 35% racial earning gap [1-3].

[1] IZEA. 2022. The State of Influencer Equality: 2022 Report. <https://izea.com/resources/insights/2022-state-of-influencer-equality/>

[2] MSL. 2021. MSL Study Reveals Racial Pay Gap in Influencer Marketing <https://www.mslgroup.com/whats-new-at-msl/msl-study-reveals-racial-pay-gap-influencer-marketing>

[3] Amy Pei, Yakov Bart, Koen Pauwels, and Kwong Chan. 2022. Racial Pay Gap in Influencer Marketing. Available at SSRN 4156872 (2022)

# RESEARCH QUESTIONS

1. Do Social media platforms offer a **FAIR** place, meaning that they reward Content Creators proportionally to their efforts?
2. What is the role of the **RECOMMENDATION SYSTEMS** in allocating fair outcomes?
3. Which **STRATEGIES** can Content Creators apply to improve their outcome?

# INDEX

INTRO ON NETWORK FORMATION MODELS

QUALITY-BASED NETWORK FORMATION MODEL

MERITOCRATIC ANALYSIS OF UNIFORM RANDOM RECOMMENDATION

EFFECT OF RECOMMENDER SYSTEMS EXPLORATION

GAMES ON NETWORKS

QUALITY CHANGE

COALITIONS

SUMMARY & OUTLOOK

# INDEX

INTRO ON NETWORK FORMATION MODELS

QUALITY-BASED NETWORK FORMATION MODEL

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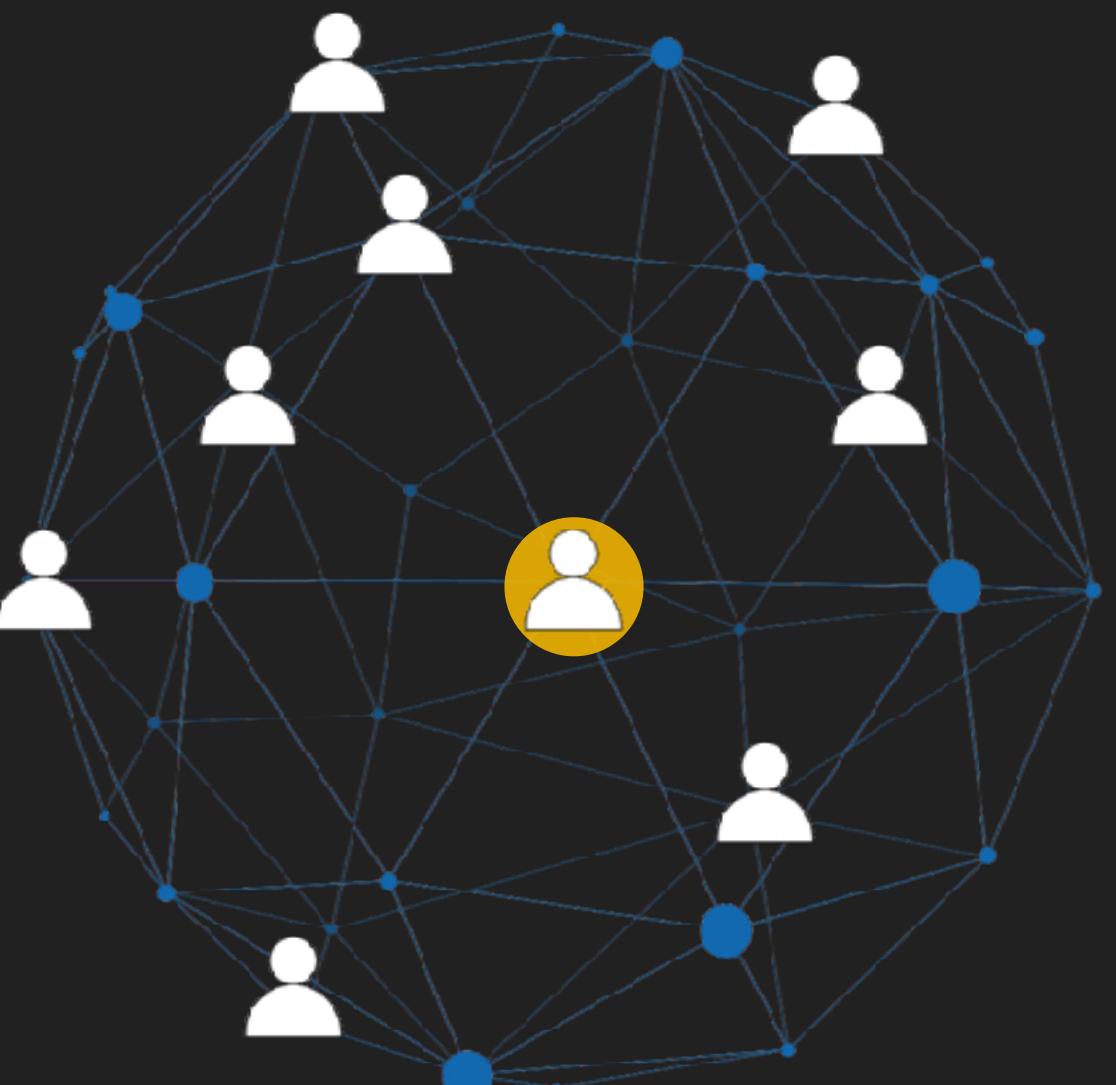
COALITIONS

SUMMARY & OUTLOOK

# NETWORK FORMATION MODELS IN THE LITERATURE

PROBABILISTIC  
INTERACTIONS

USERS are NOT UTILITARIAN and follow  
STATISTICAL RECOMMENDATIONS



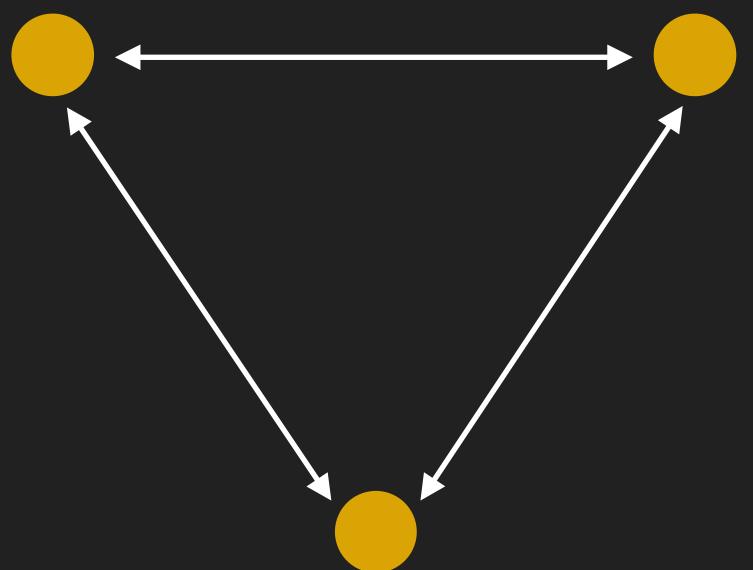
SOCIO-STRATEGIC  
INTERACTIONS

USERS are UTILITARIAN and driven by  
sociological principles

## SOCIO-STRATEGIC INTERACTIONS



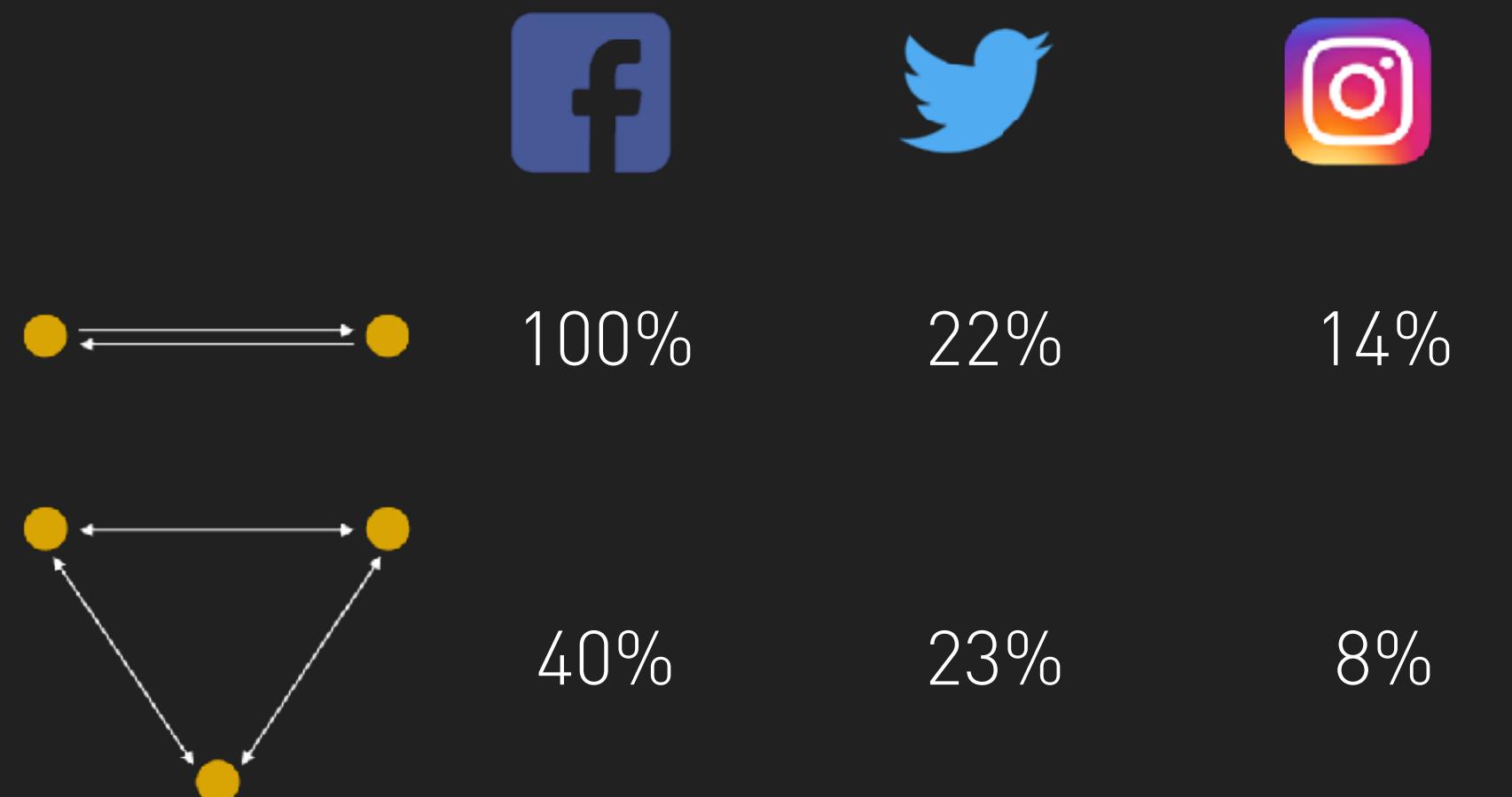
Real-world social connections tend to be  
RECIPROCATED and in TRIADIC STRUCTURES

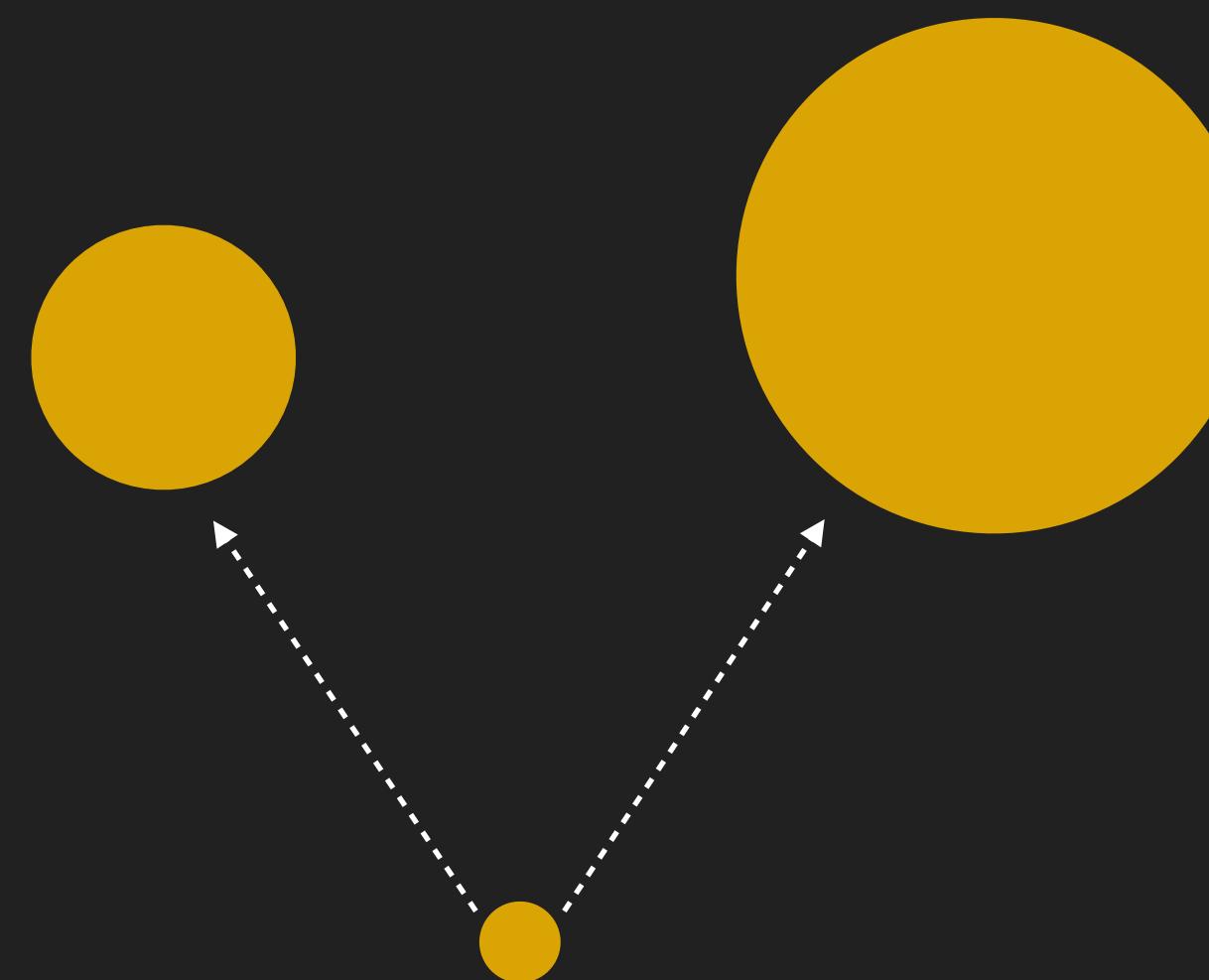
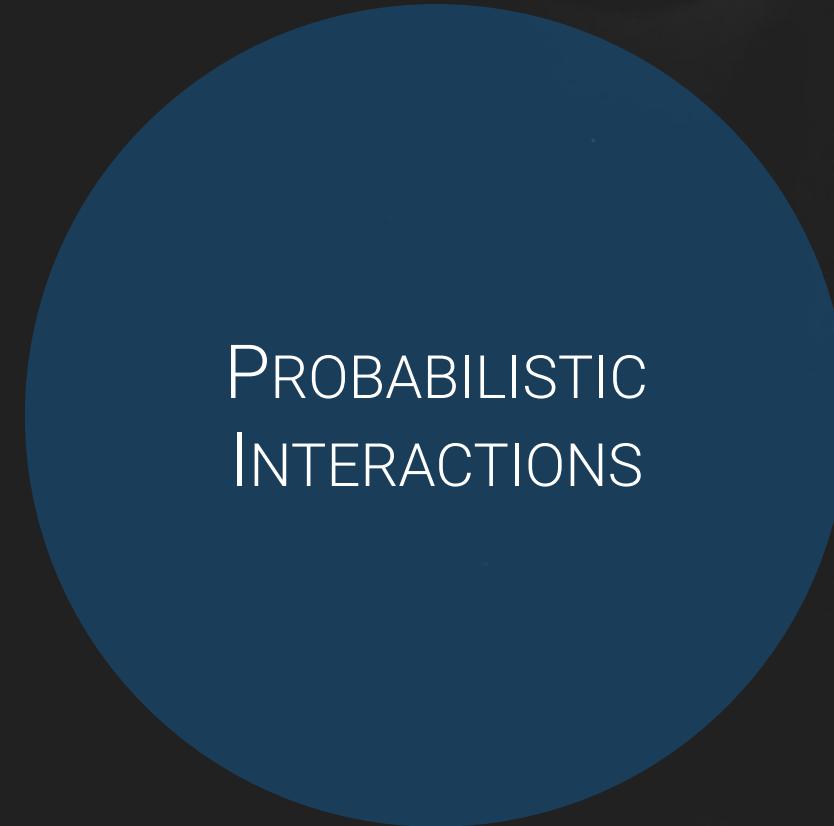




## SOCIO-STRATEGIC INTERACTIONS

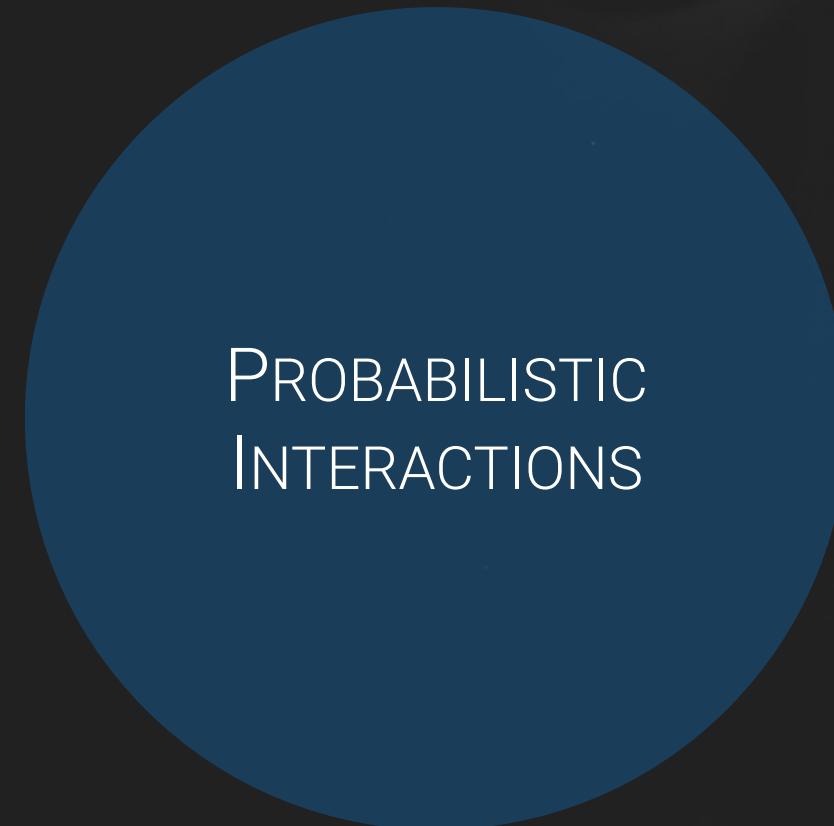
These motifs are NOT significantly present on DIRECTED SOCIAL NETWORKS because people can follow STRANGERS



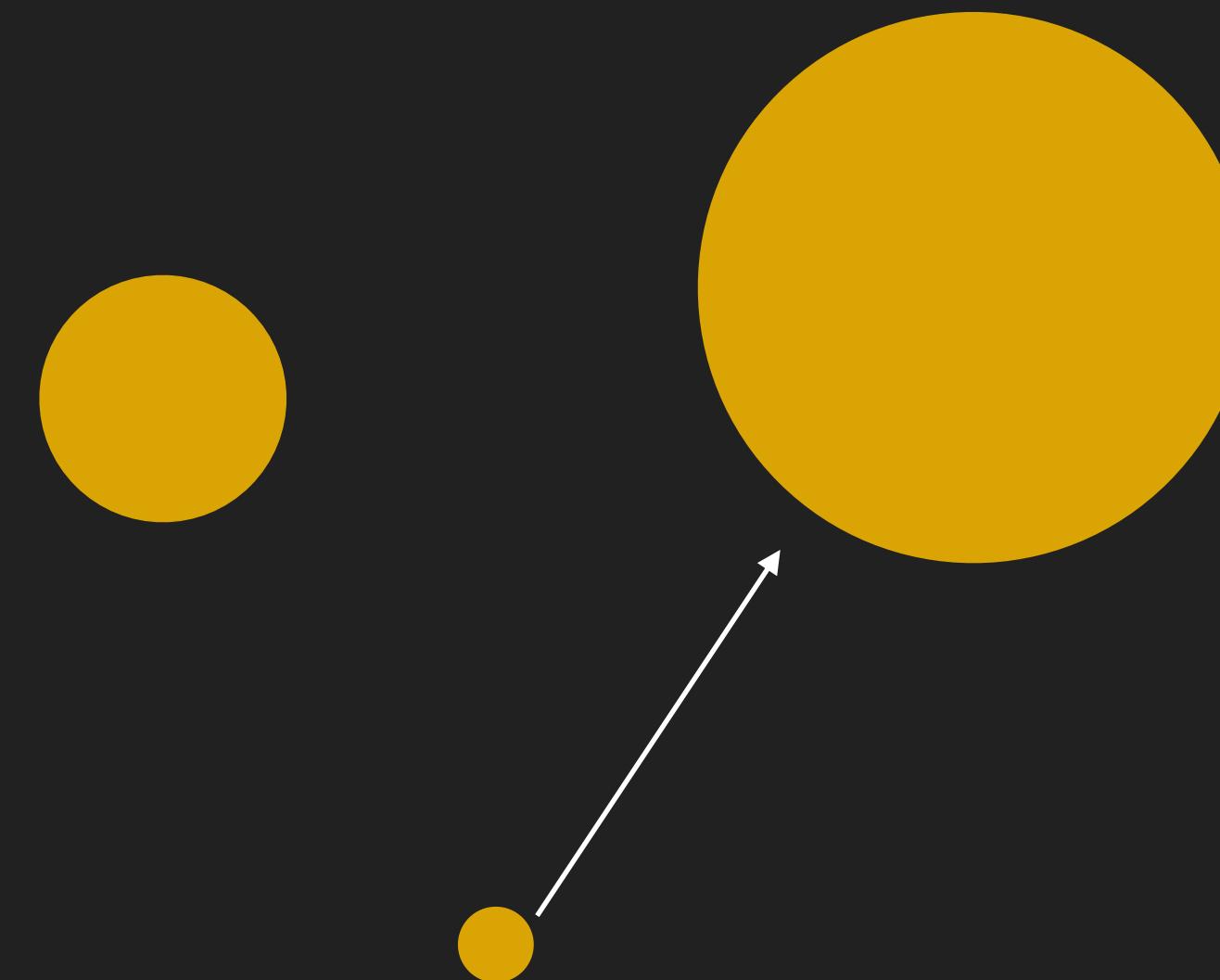


### PREFERENTIAL ATTACHMENT MODEL:

The probability of following an other user is proportional to their number of followers



PREFERENTIAL ATTACHMENT MODEL:  
The probability of following an other user is proportional to their number of followers



Leads to the "RICH-GET-RICHER" effect  
but leaves no space to individual DECISION-MAKING

# NETWORK FORMATION MODELS IN THE LITERATURE

SOCIO-STRATEGIC  
INTERACTIONS

BOTH APPROACHES ARE NOT  
SUITABLE TO MODEL TODAY'S  
ONLINE SOCIAL NETWORKS:

SEEKERS Vs CONTENT CREATORS  
LINK DIRECTIONALITY  
RECOMMENDATION SYSTEMS

PROBABILISTIC  
INTERACTIONS

USERS are NOT UTILITARIAN and follow  
STATISTICAL RECOMMENDATIONS

USERS are UTILITARIAN and driven by  
sociological principles

# NETWORK FORMATION MODELS IN THE LITERATURE

SOCIO-STRATEGIC  
INTERACTIONS

WE NEED A DIFFERENT MODEL  
THAT CONSIDERS THE  
QUALITY DIMENSION

PROBABILISTIC  
INTERACTIONS

USERS are NOT UTILITARIAN and follow  
STATISTICAL RECOMMENDATIONS



PROBABILISTIC  
INTERACTIONS

QUALITY-BASED  
INTERACTIONS

The RECOMMENDATION SYSTEM  
governs the “VISIBILITY” of the CONTENT CREATORS

SEEKERS maximise the “QUALITY” of  
the content they want to receive

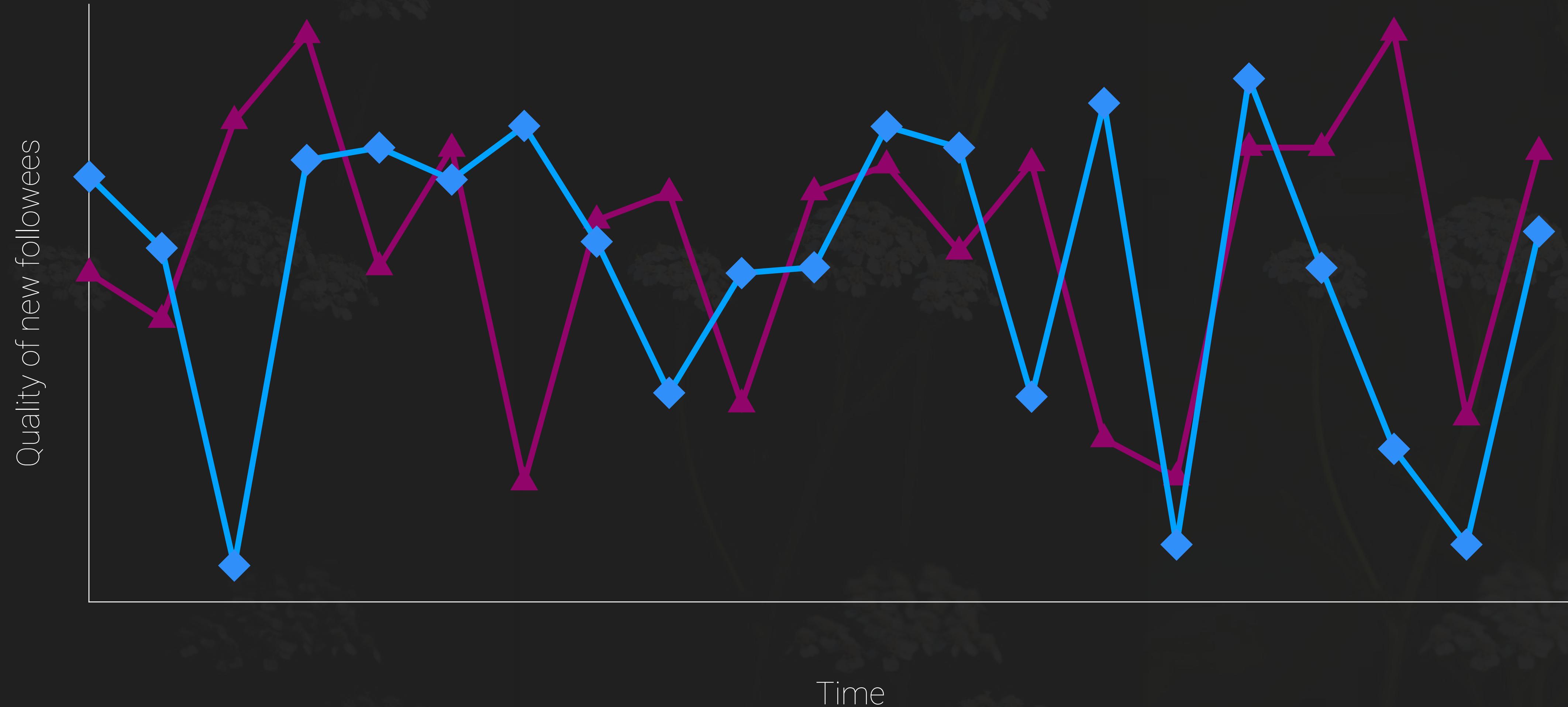
SOCIO-STRATEGIC  
INTERACTIONS



DO USERS TRY TO OPTIMISE  
QUALITY RECEIVED OVER TIME?

Random Walk 1  
Random Walk 2

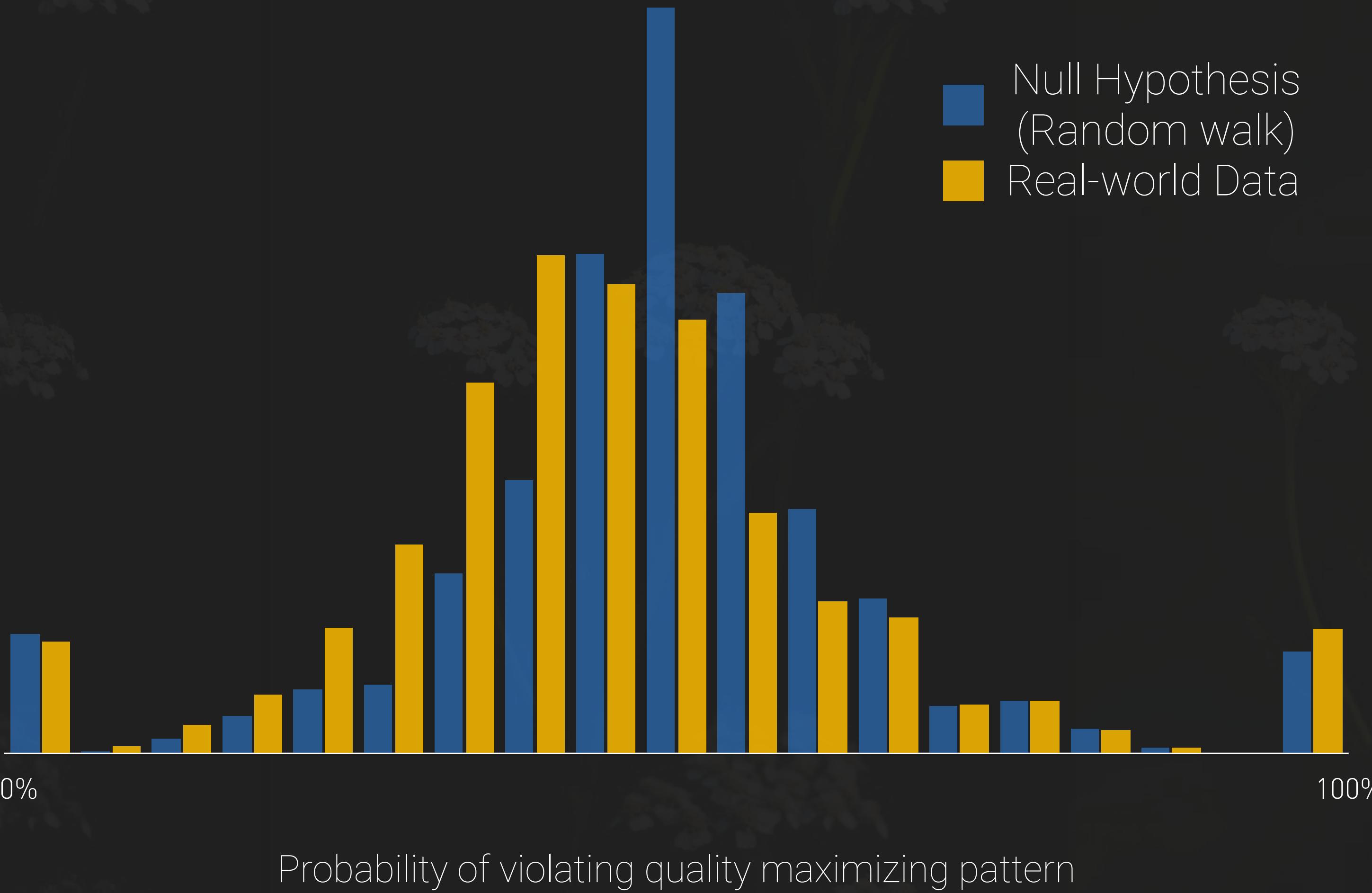
## TEMPORAL QUALITY PATTERN



# TEMPORAL QUALITY PATTERN



# TWITTER DATA-SET ON NETWORK SCIENTISTS





# DO USERS TRY TO OPTIMISE THE QUALITY RECEIVED OVER TIME?

YES, USERS TEND TO FOLLOW  
OTHER USERS ACCORDING TO  
INCREASING LEVELS OF  
USER GENERATED CONTENT  
(UGC)-QUALITY



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# THE QUALITY-BASED NETWORK FORMATION MODEL

Seeker



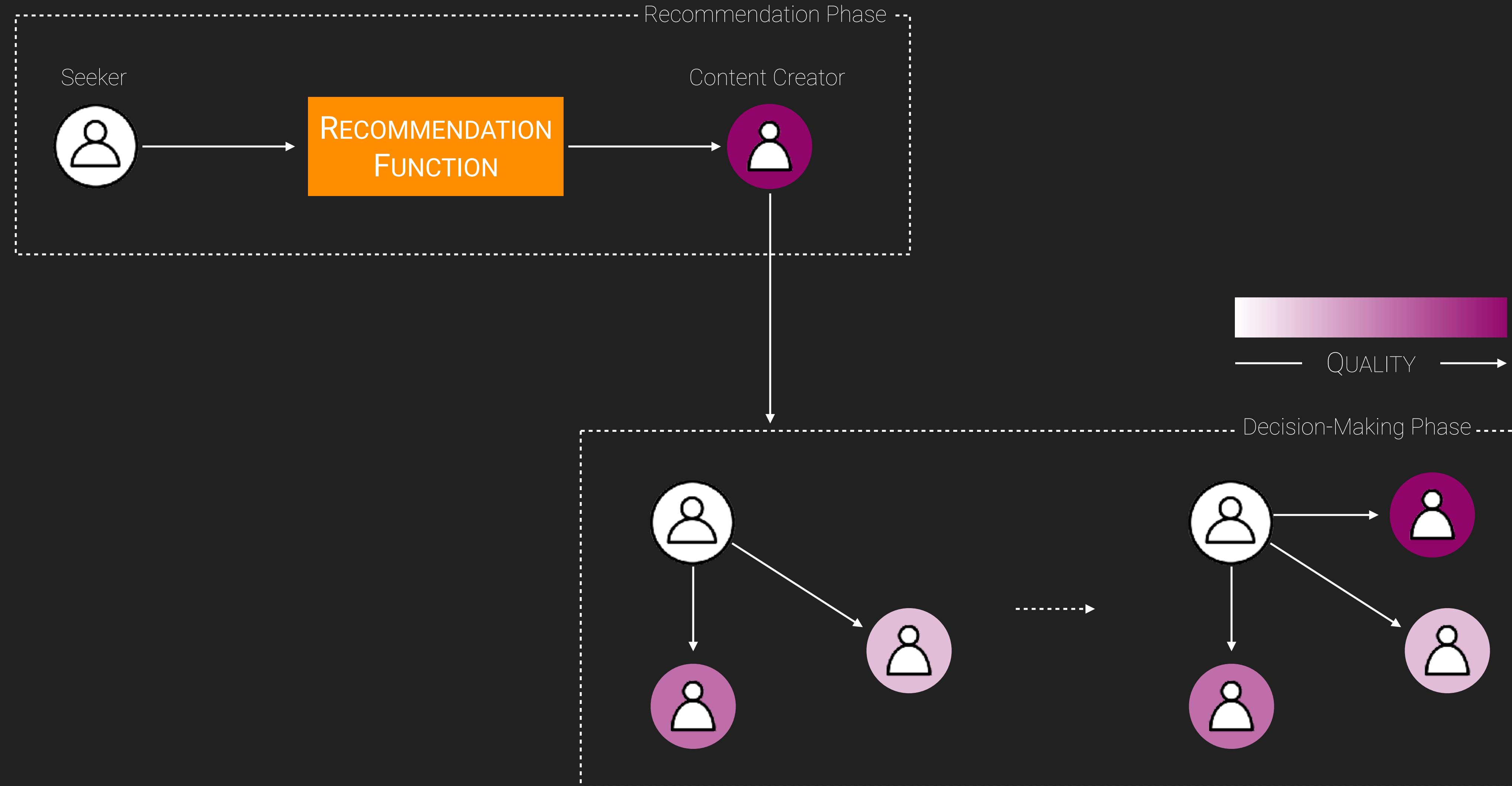
Content Creator



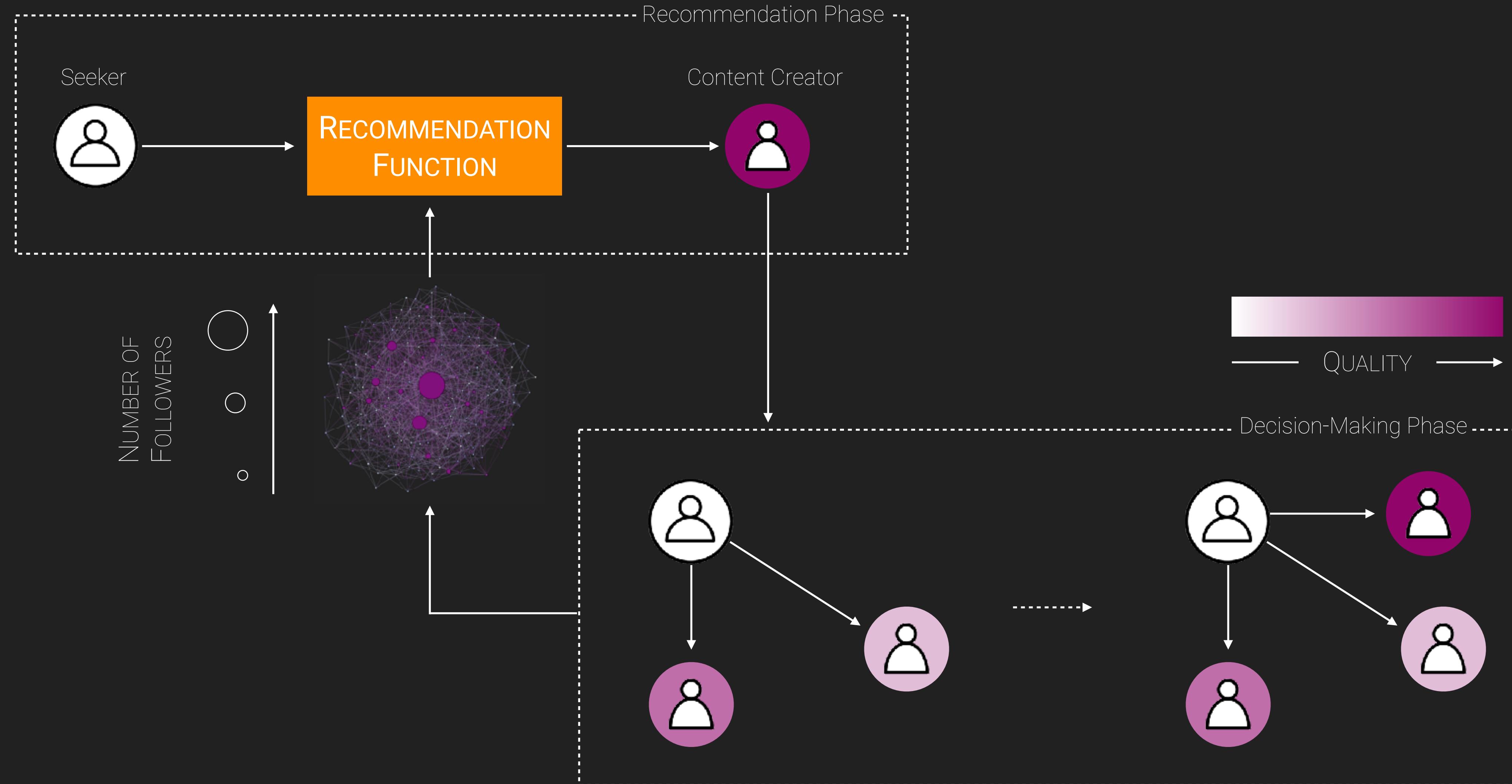
# THE QUALITY-BASED NETWORK FORMATION MODEL



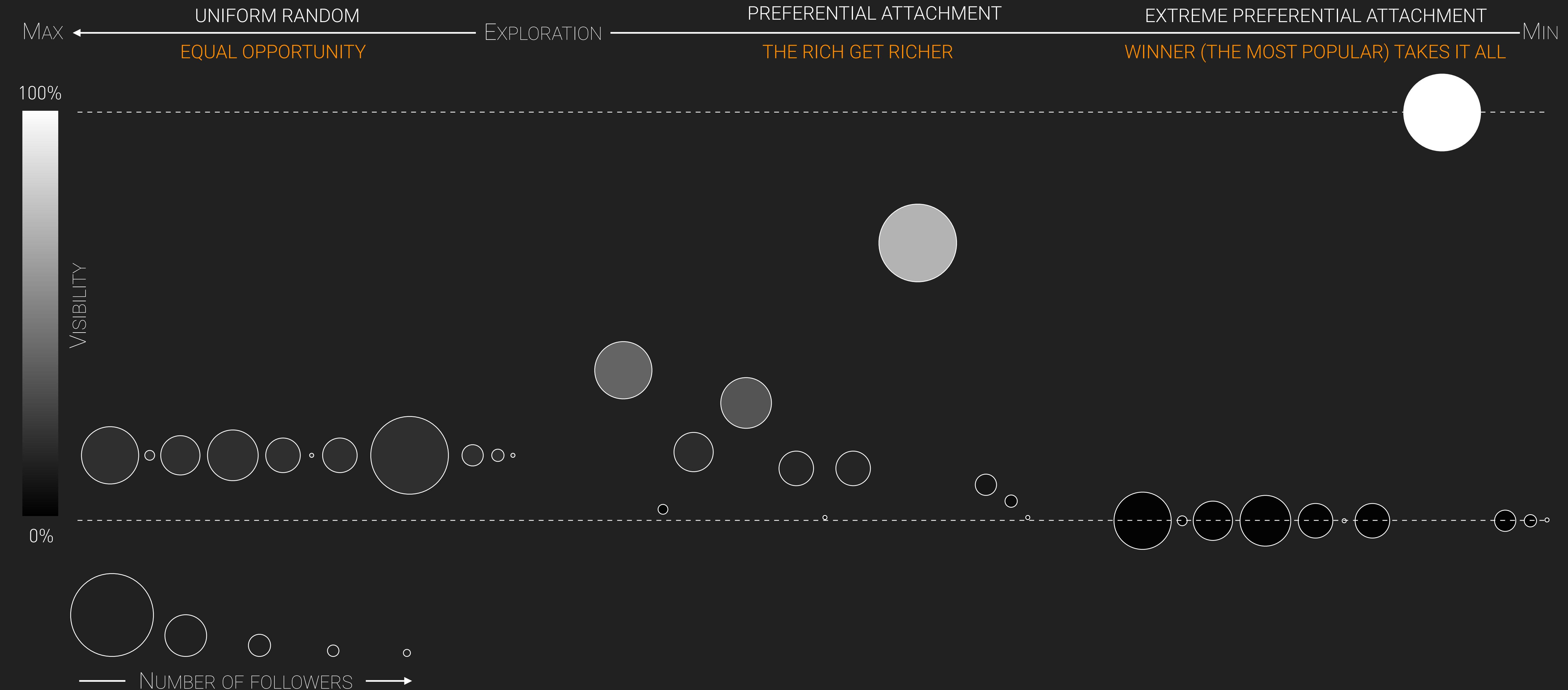
# THE QUALITY-BASED NETWORK FORMATION MODEL



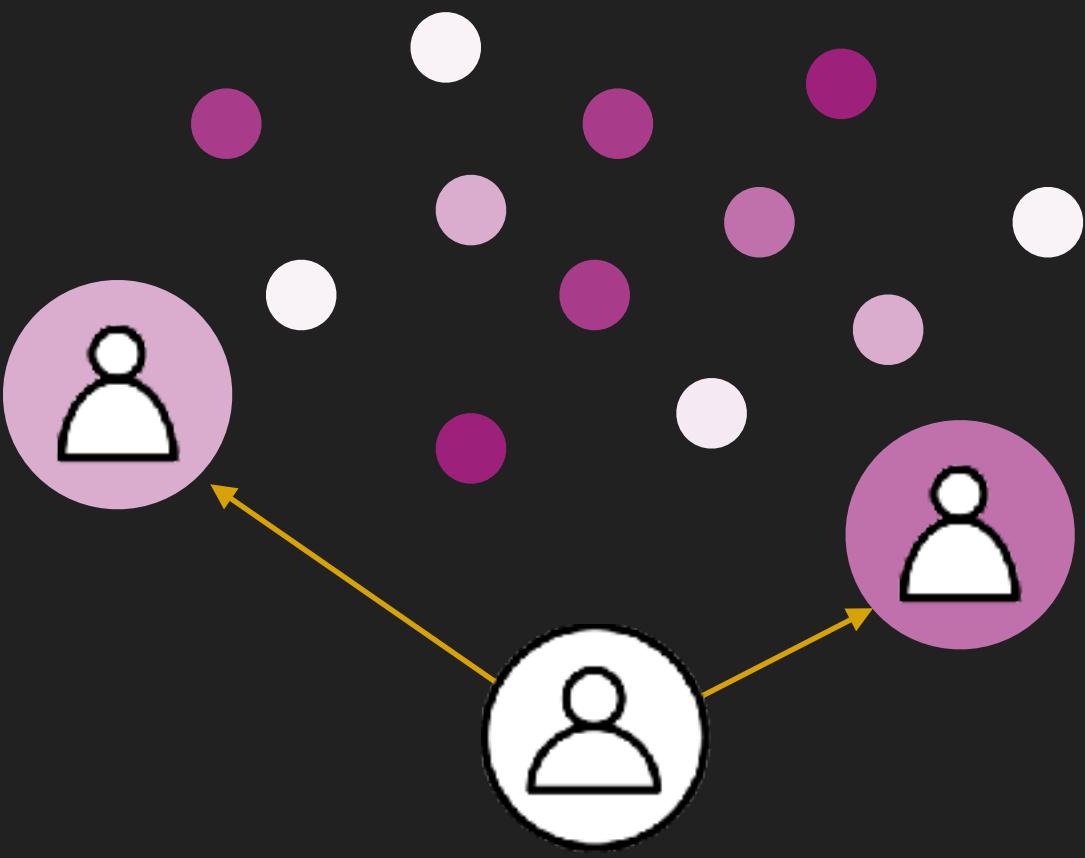
# THE QUALITY-BASED NETWORK FORMATION MODEL



# STEP 1: RECOMMENDATION FUNCTIONS



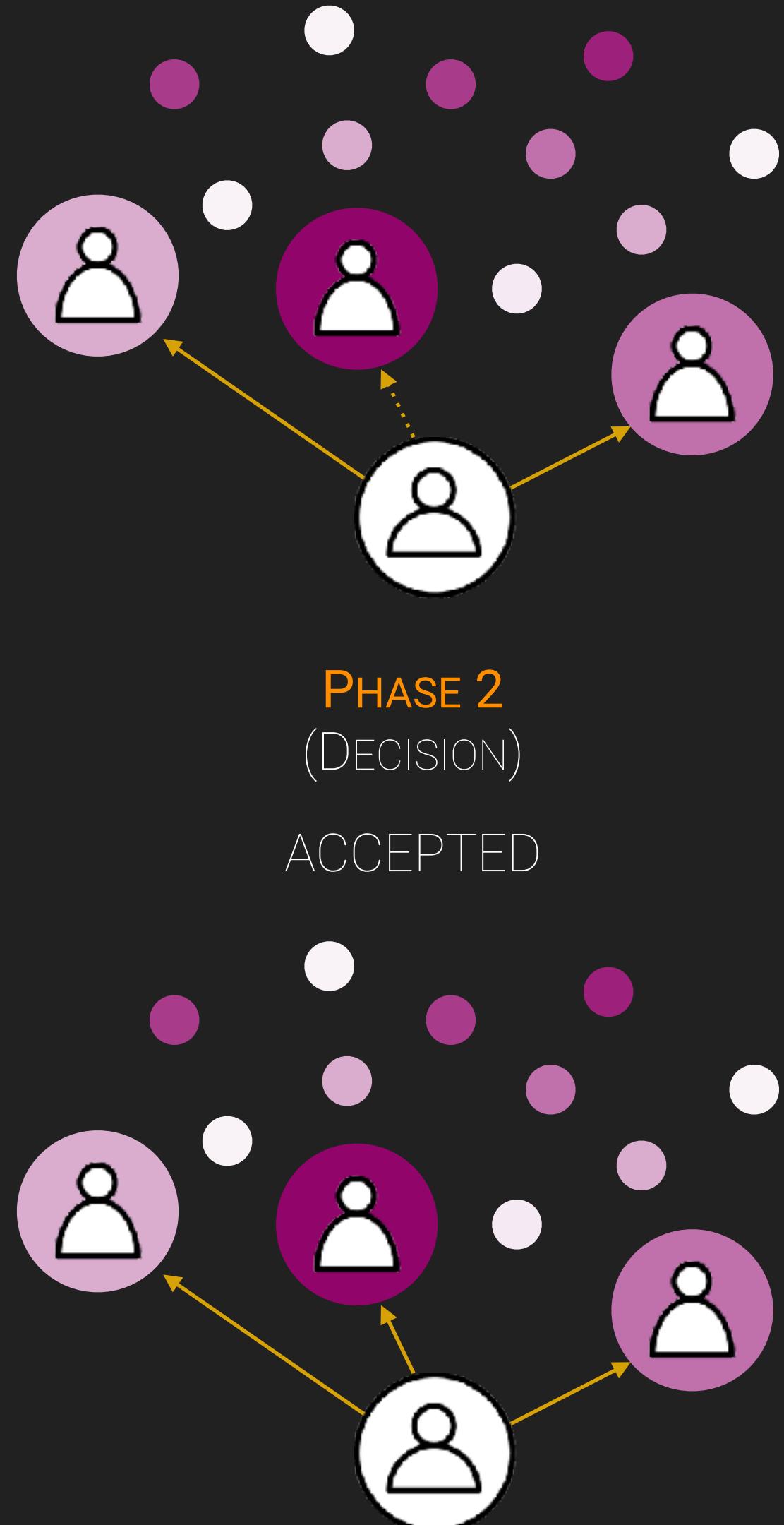
## STEP 2: STRATEGIC DECISION



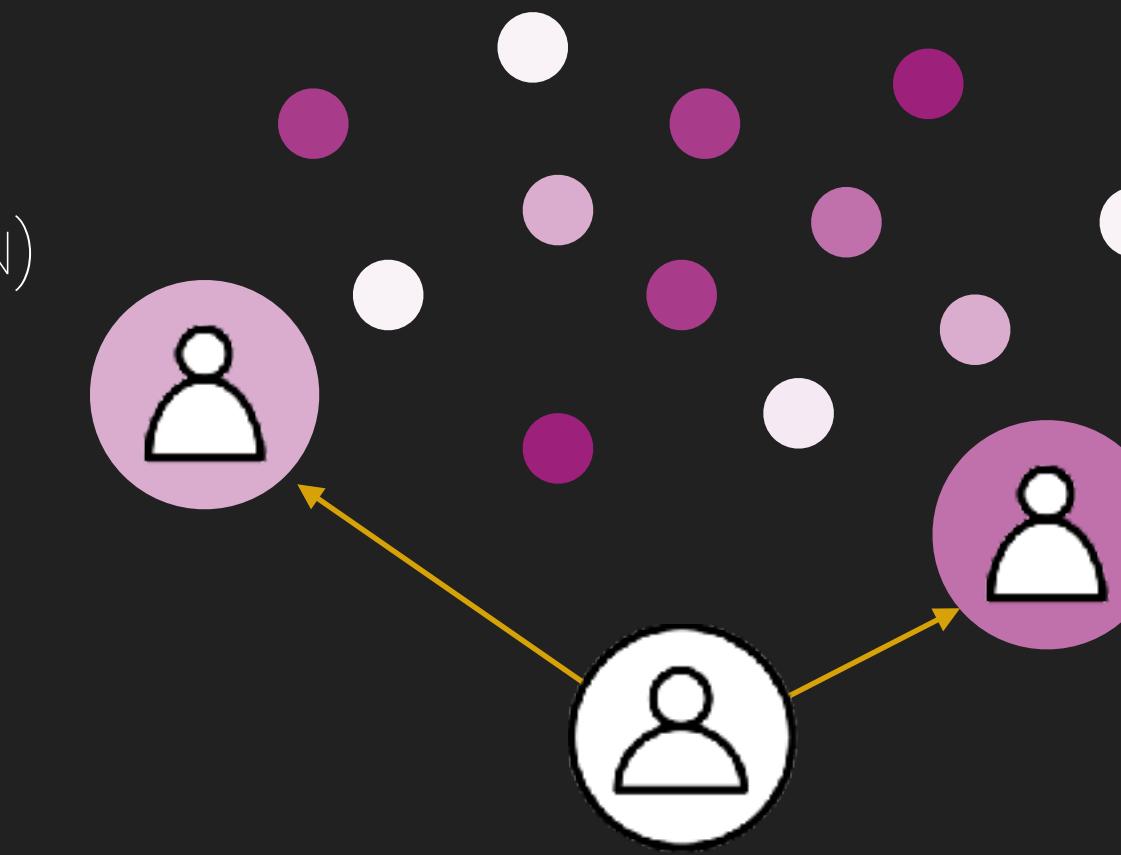
# STEP 2: STRATEGIC DECISION



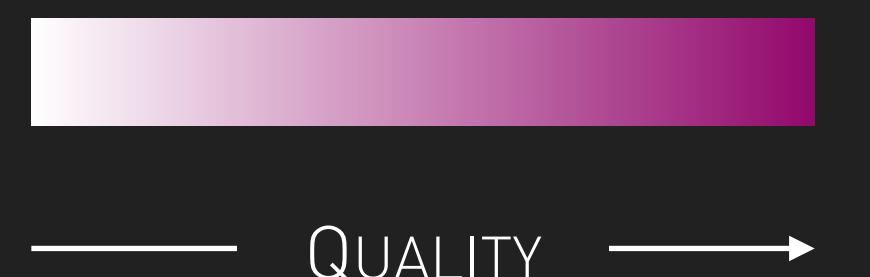
# STEP 2: STRATEGIC DECISION



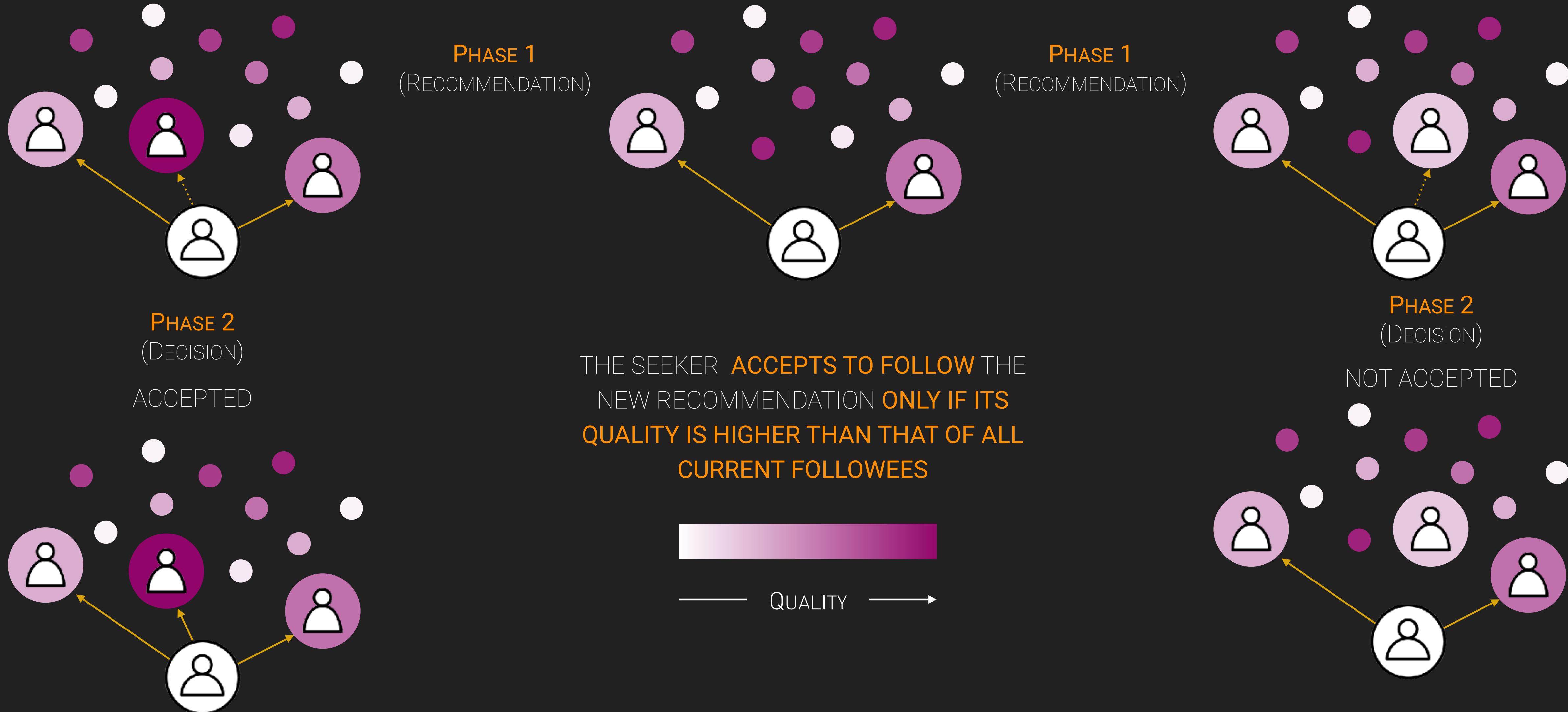
PHASE 1  
(RECOMMENDATION)



THE SEEKER **ACCEPTS TO FOLLOW THE**  
NEW RECOMMENDATION **ONLY IF ITS**  
**QUALITY IS HIGHER THAN THAT OF ALL**  
**CURRENT FOLLOWEES**



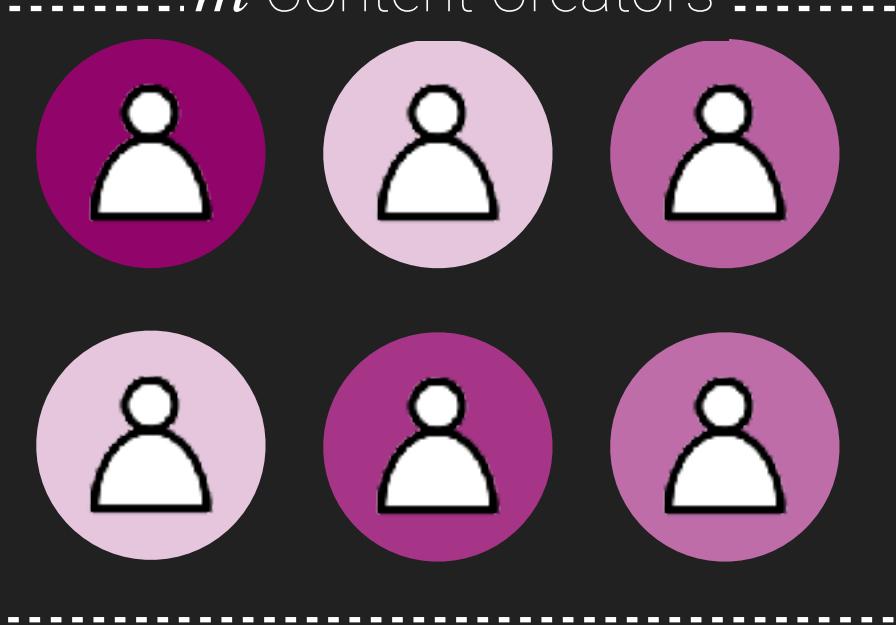
# STEP 2: STRATEGIC DECISION



***n*** Seekers

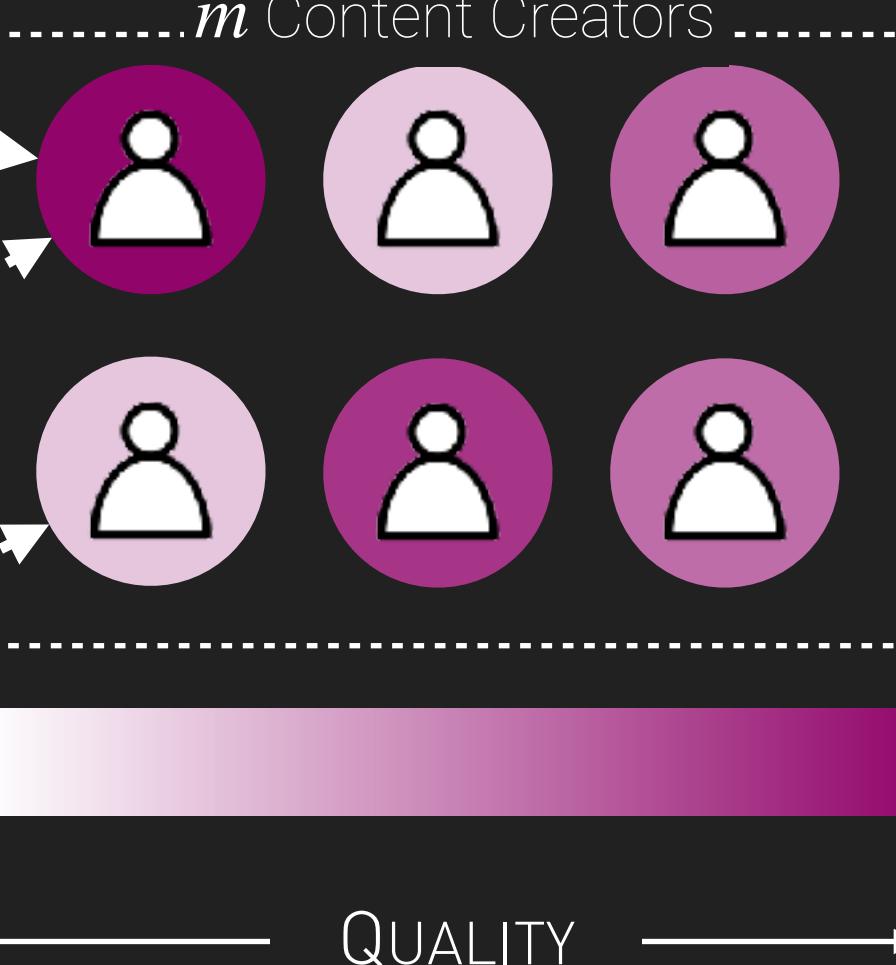
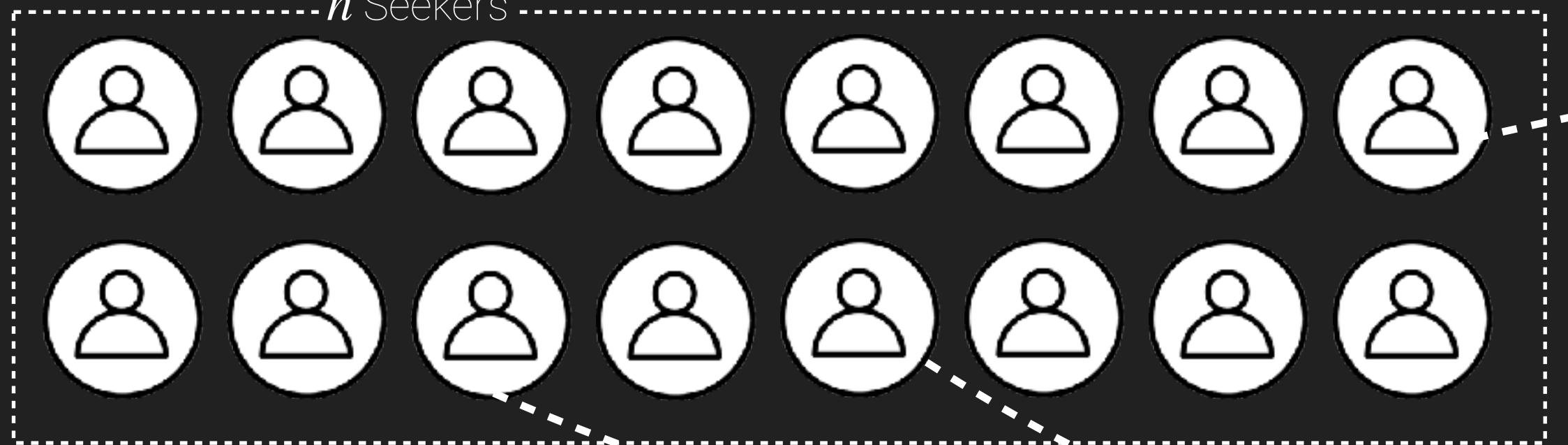


***m*** Content Creators



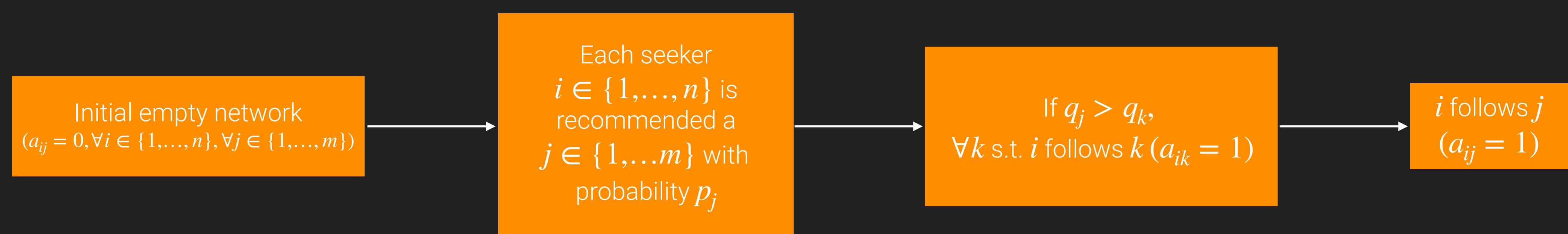
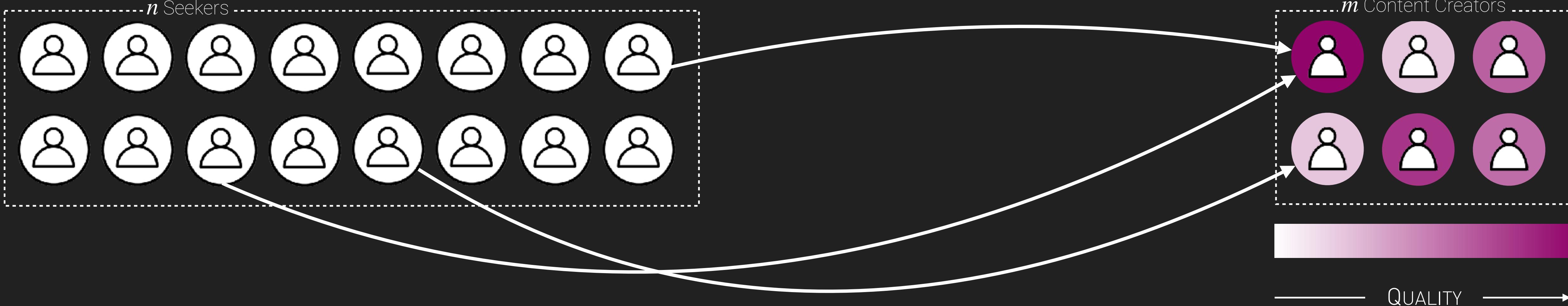
QUALITY

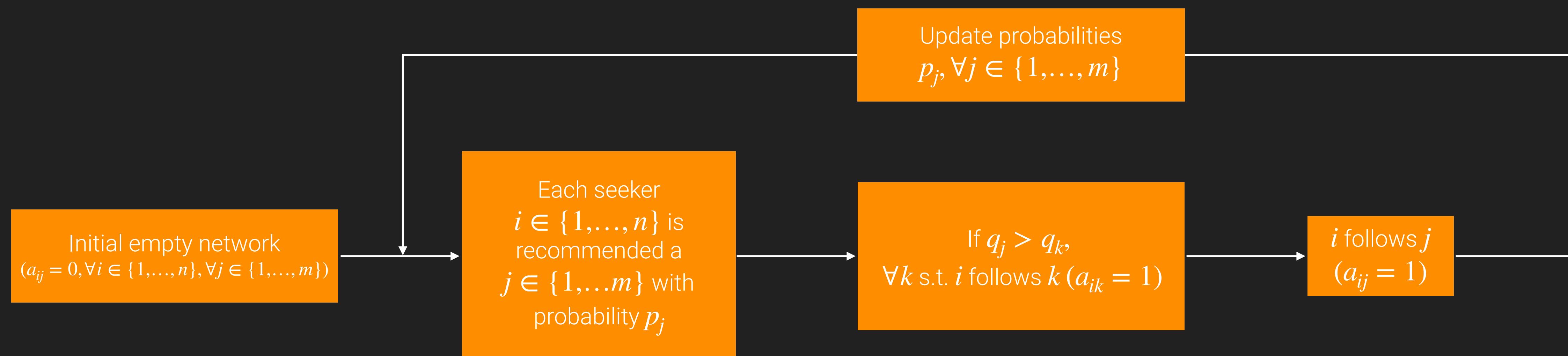
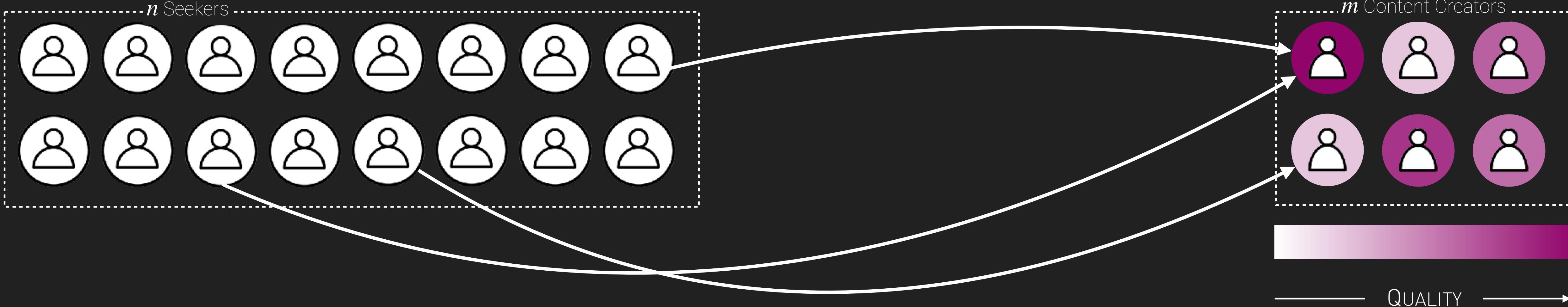
Initial empty network  
 $(a_{ij} = 0, \forall i \in \{1, \dots, n\}, \forall j \in \{1, \dots, m\})$

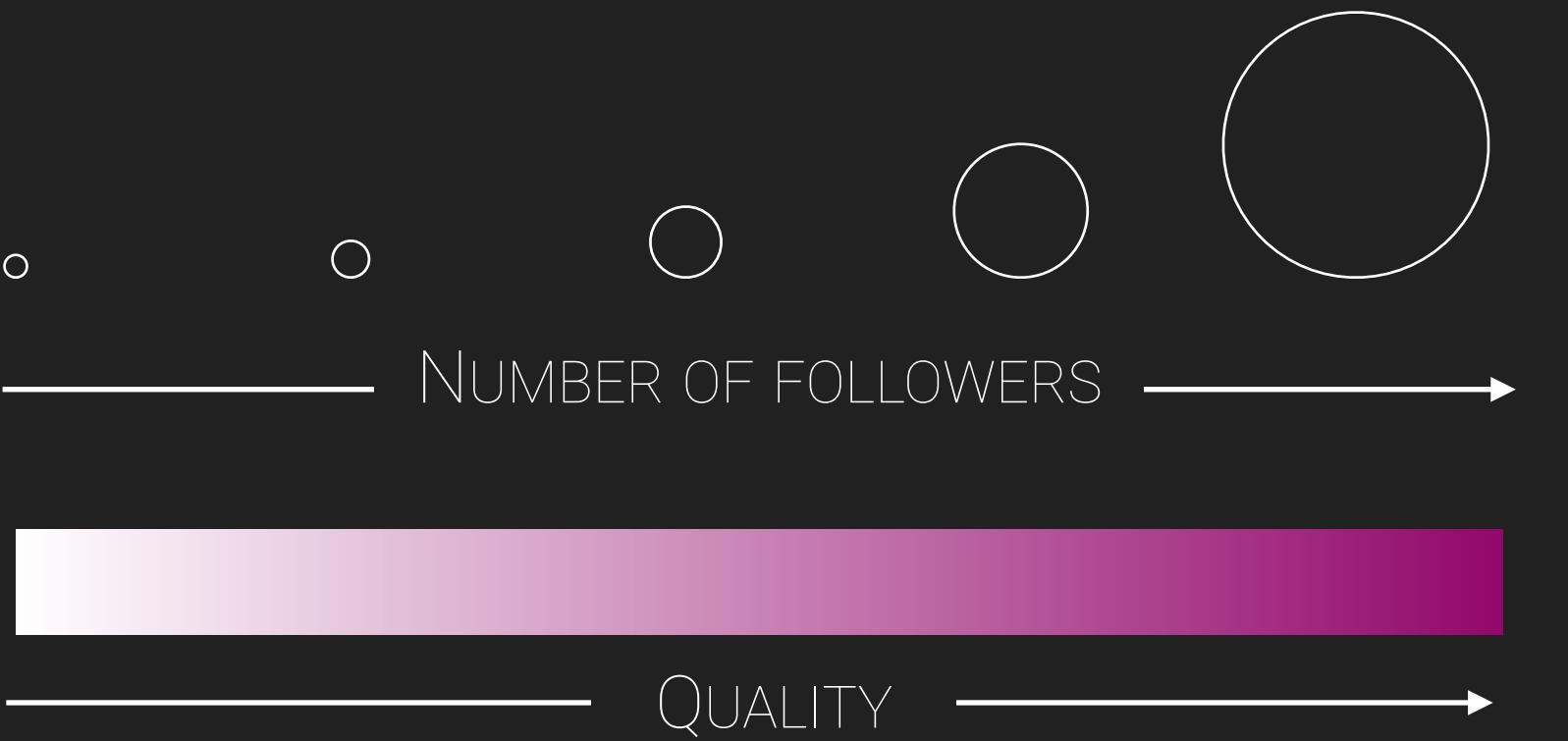


Initial empty network  
 $(a_{ij} = 0, \forall i \in \{1, \dots, n\}, \forall j \in \{1, \dots, m\})$

Each seeker  
 $i \in \{1, \dots, n\}$  is recommended a  
 $j \in \{1, \dots, m\}$  with probability  $p_j$







MERITOCRACY: IS THE NUMBER OF FOLLOWERS  
CORRELATED WITH THE QUALITY?

# INDEX

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COALITIONS

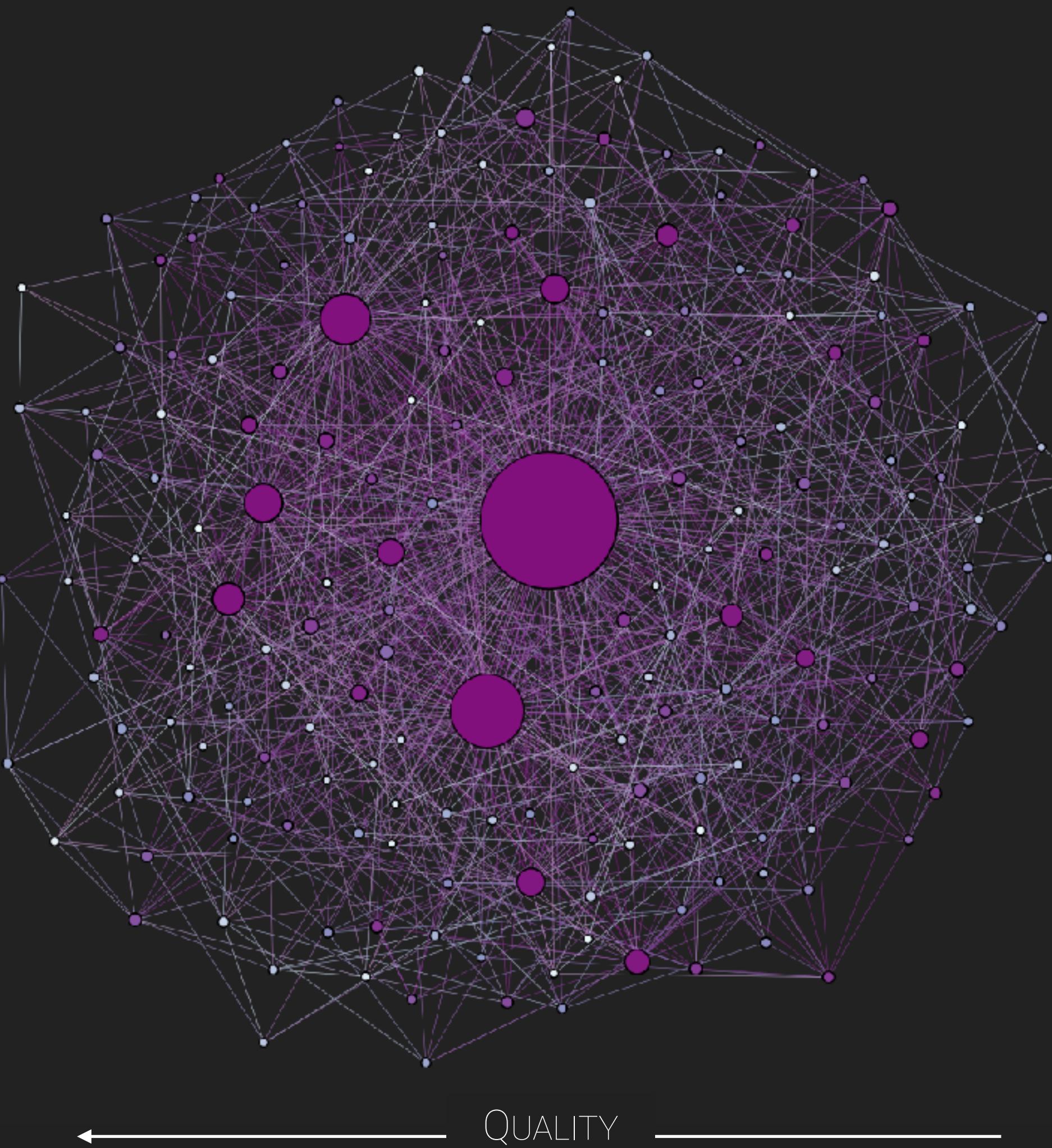
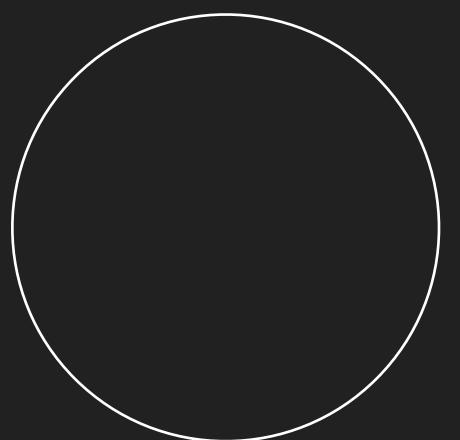
SUMMARY & OUTLOOK

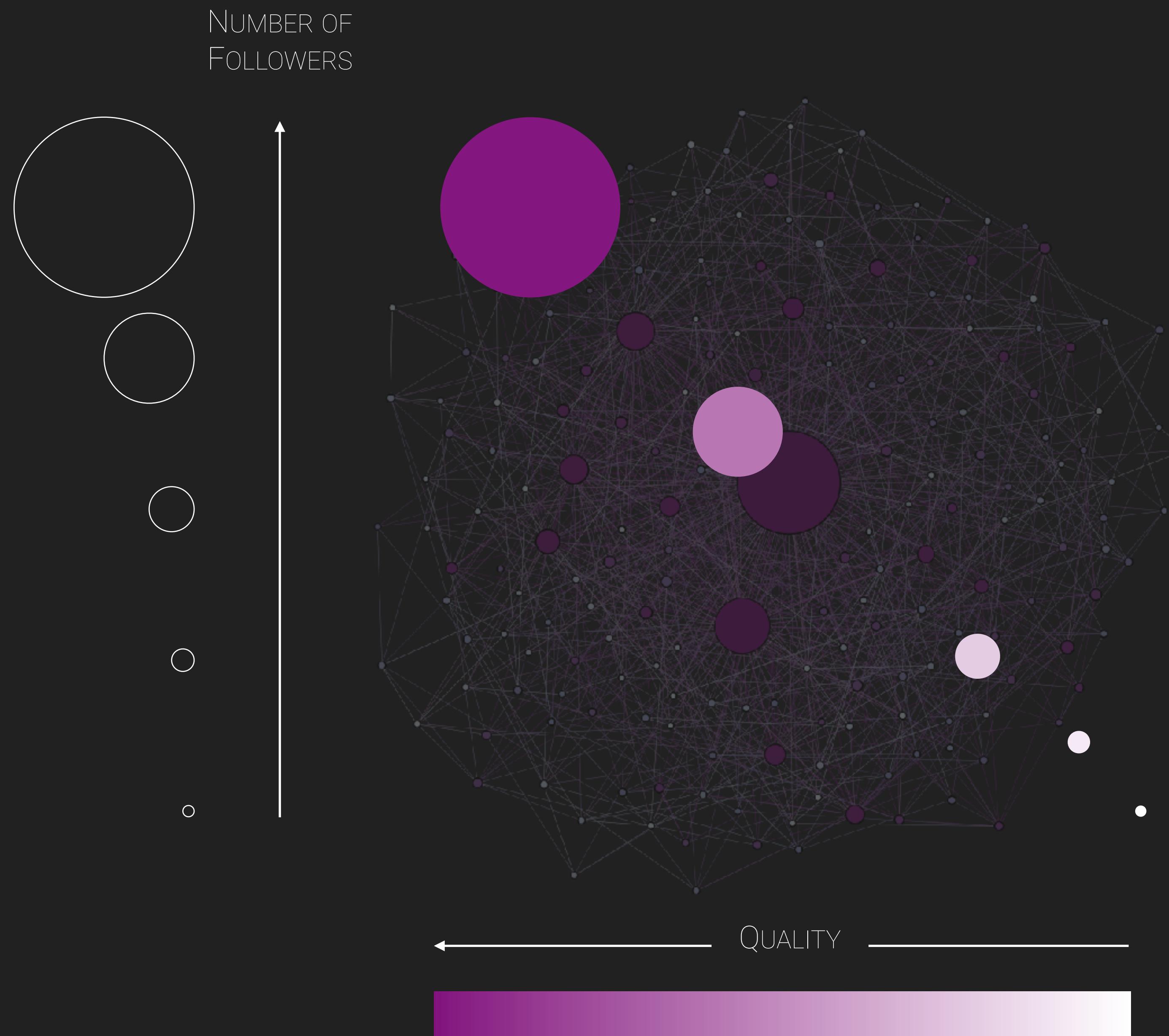
# CONVERGENCE RESULTS

## THEOREM.

For any quality vector  $\mathbf{q} = [q_1, \dots, q_m]$ , the network reaches an equilibrium almost surely.  
At equilibrium, every seeker follows the best Content Creator.

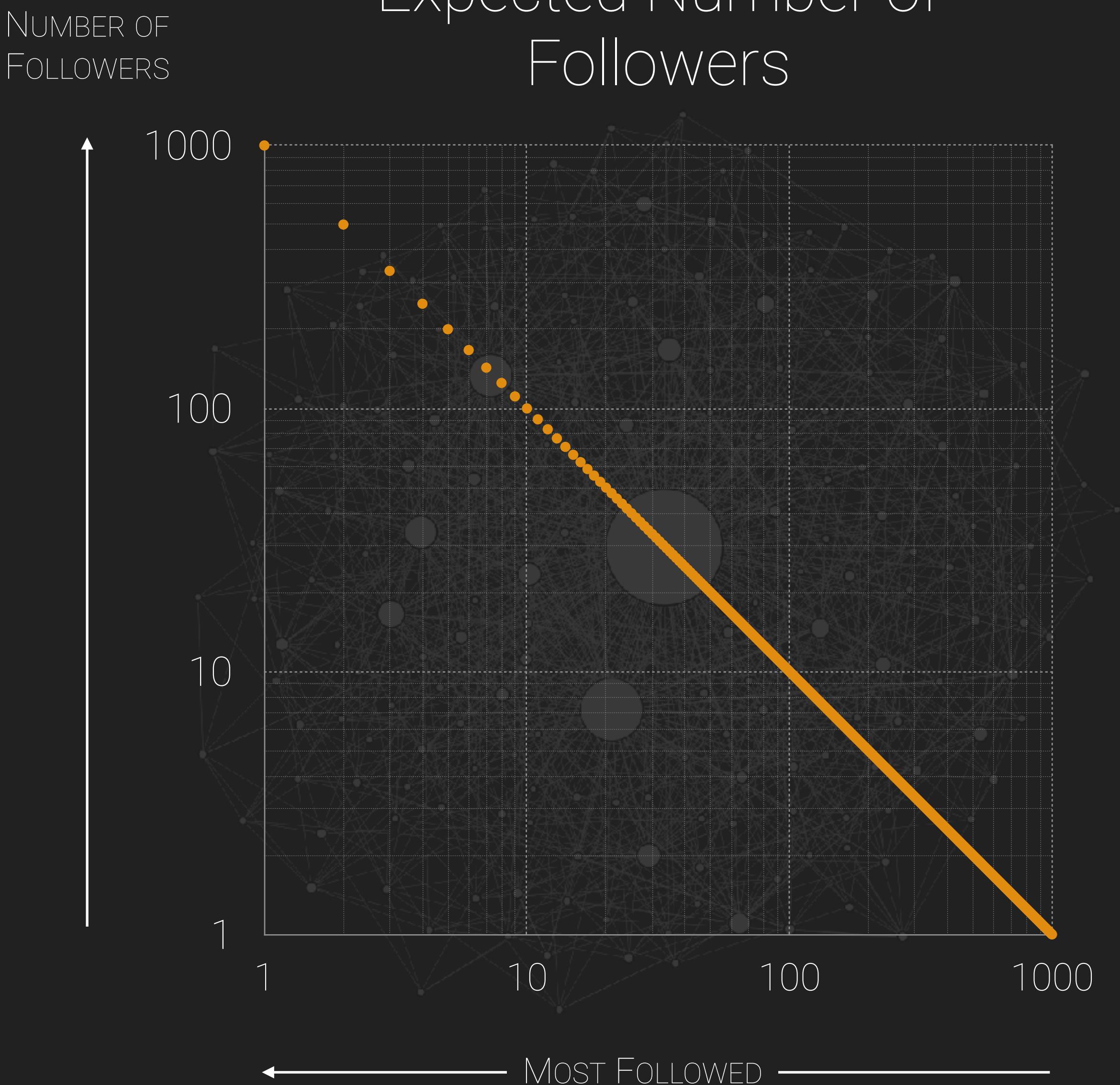
NUMBER OF  
FOLLOWERS





NUMBER OF FOLLOWERS  
CORRELATES WITH UGC-QUALITY:  
THE HIGHEST THE QUALITY, THE  
HIGHEST THE NUMBER OF  
FOLLOWERS.

# Expected Number of Followers

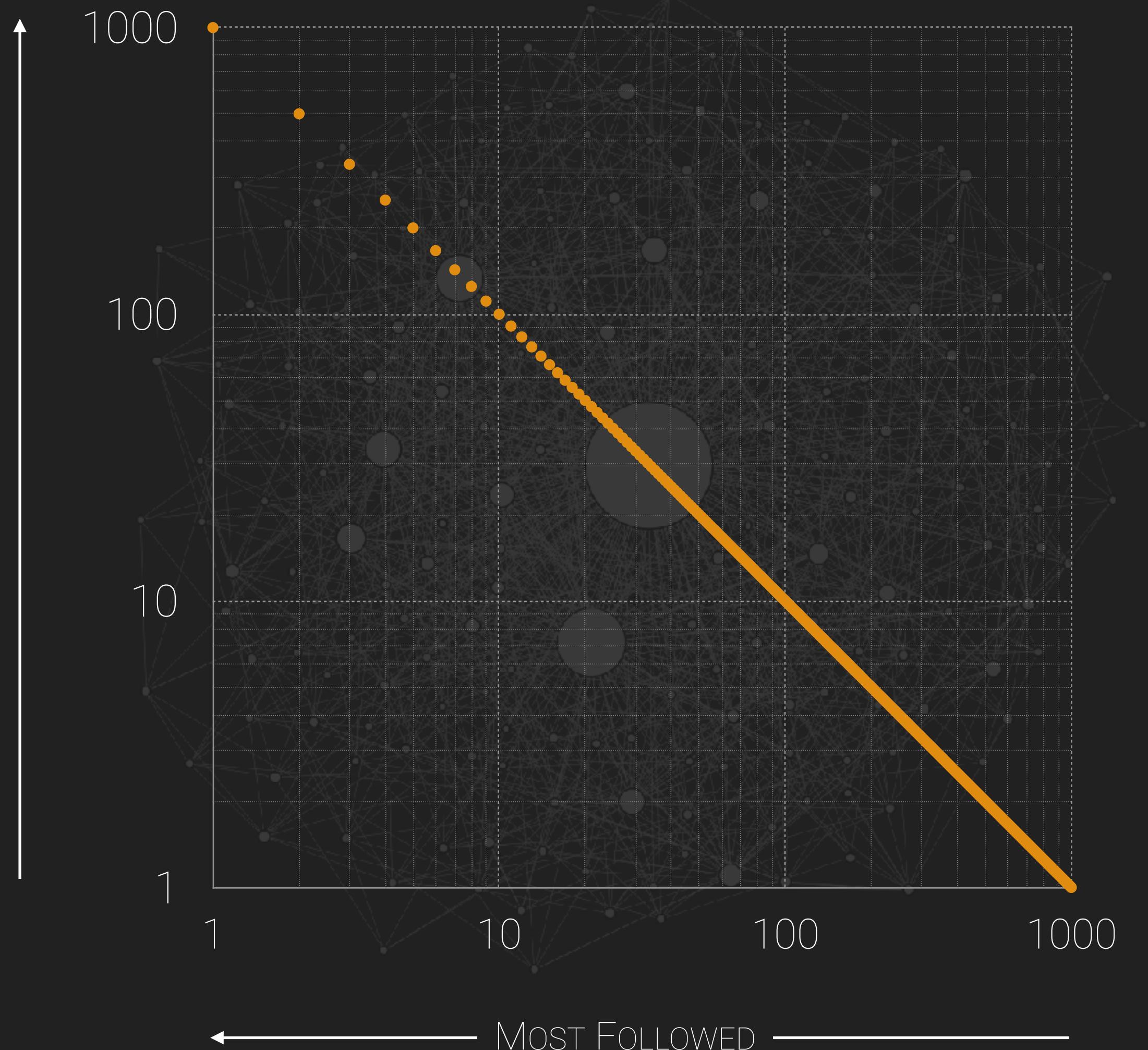


# 1<sup>ST</sup> MOST FOLLOWED USER HAS:

- ▶ 2 TIMES AS MANY FOLLOWERS AS THE 2<sup>ND</sup>
  - ▶ 3 TIMES AS MANY FOLLOWERS AS THE 3<sup>RD</sup>
  - ▶ 4 TIMES AS MANY FOLLOWERS AS THE 4<sup>TH</sup>
  - ▶ ...

# Expected Number of Followers

NUMBER OF FOLLOWERS



INTUITIVELY...

# MODEL VALIDITY

In [1], we fully validated the model by studying:

- ▶ Indegree distribution
- ▶ Outdegree distribution
- ▶ Overlap Index (similarities between followers' lists)

Furthermore, we collected and analysed several data-sets on Twitch.

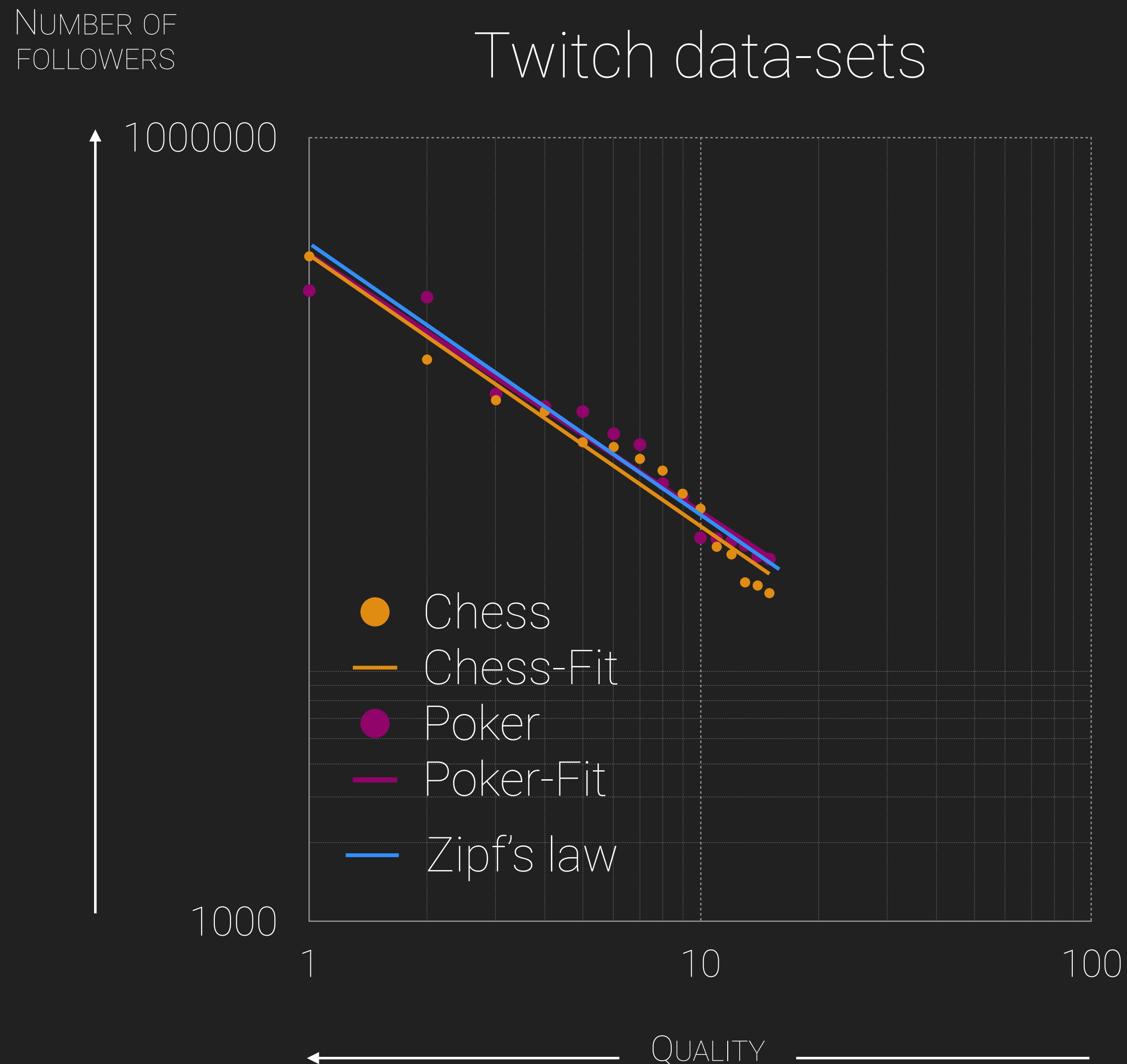
**THEOREM.**

At equilibrium, the CC's expected indegree (when indexed by decreasing quality) follows a Zipf's law:  
 $\mathbb{E}(d_i^{in,\star}) = \frac{n}{i}$

**THEOREM.**

The seekers' expected outdegree is equals to the harmonic number  $m$ :

$$\mathbb{E}[d_N^{out,\star}] = \sum_{k=1}^{N-1} \frac{1}{k}.$$



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EFFECT OF RECOMMENDER SYSTEMS EXPLORATION

GAMES ON NETWORKS

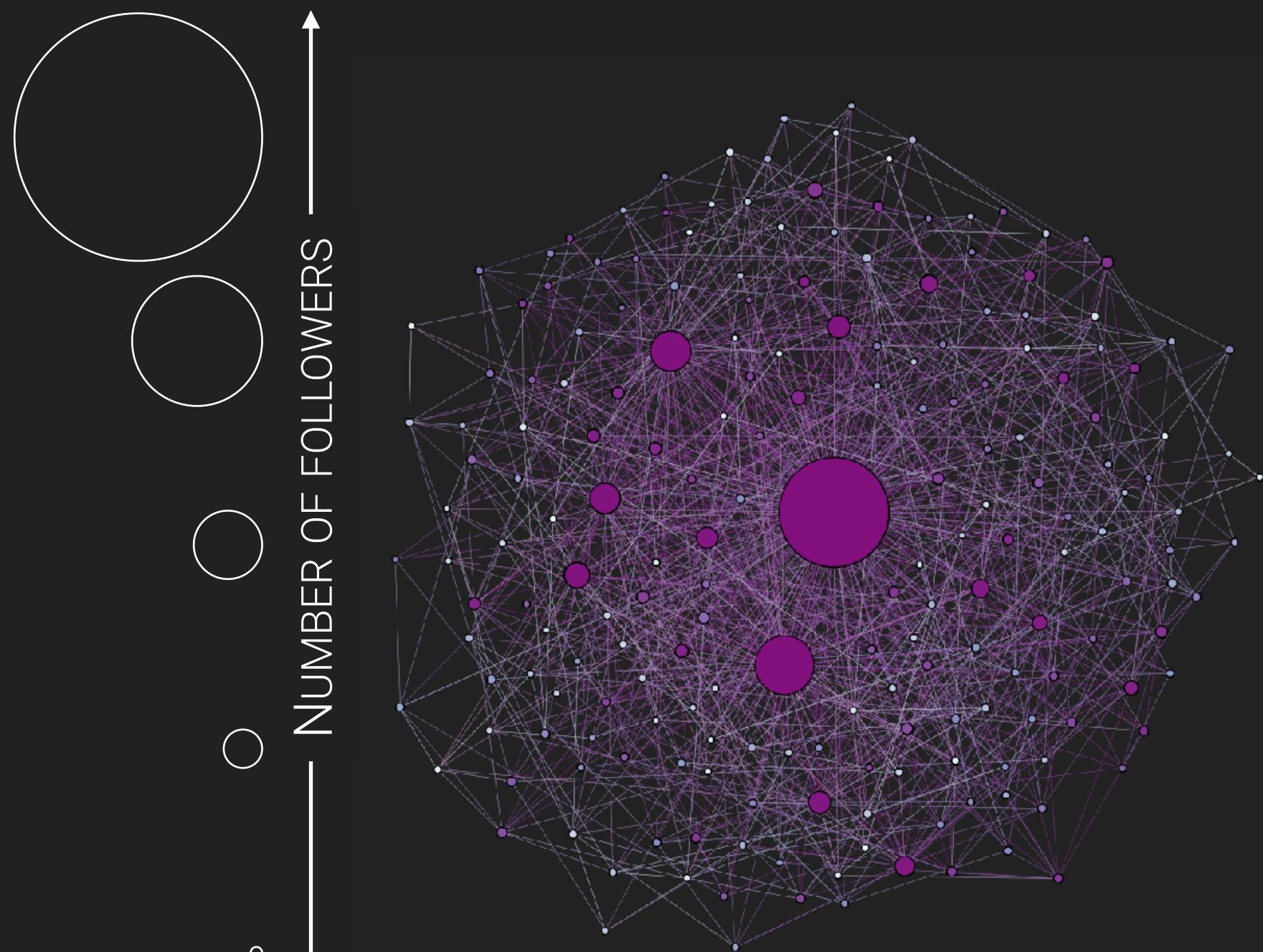
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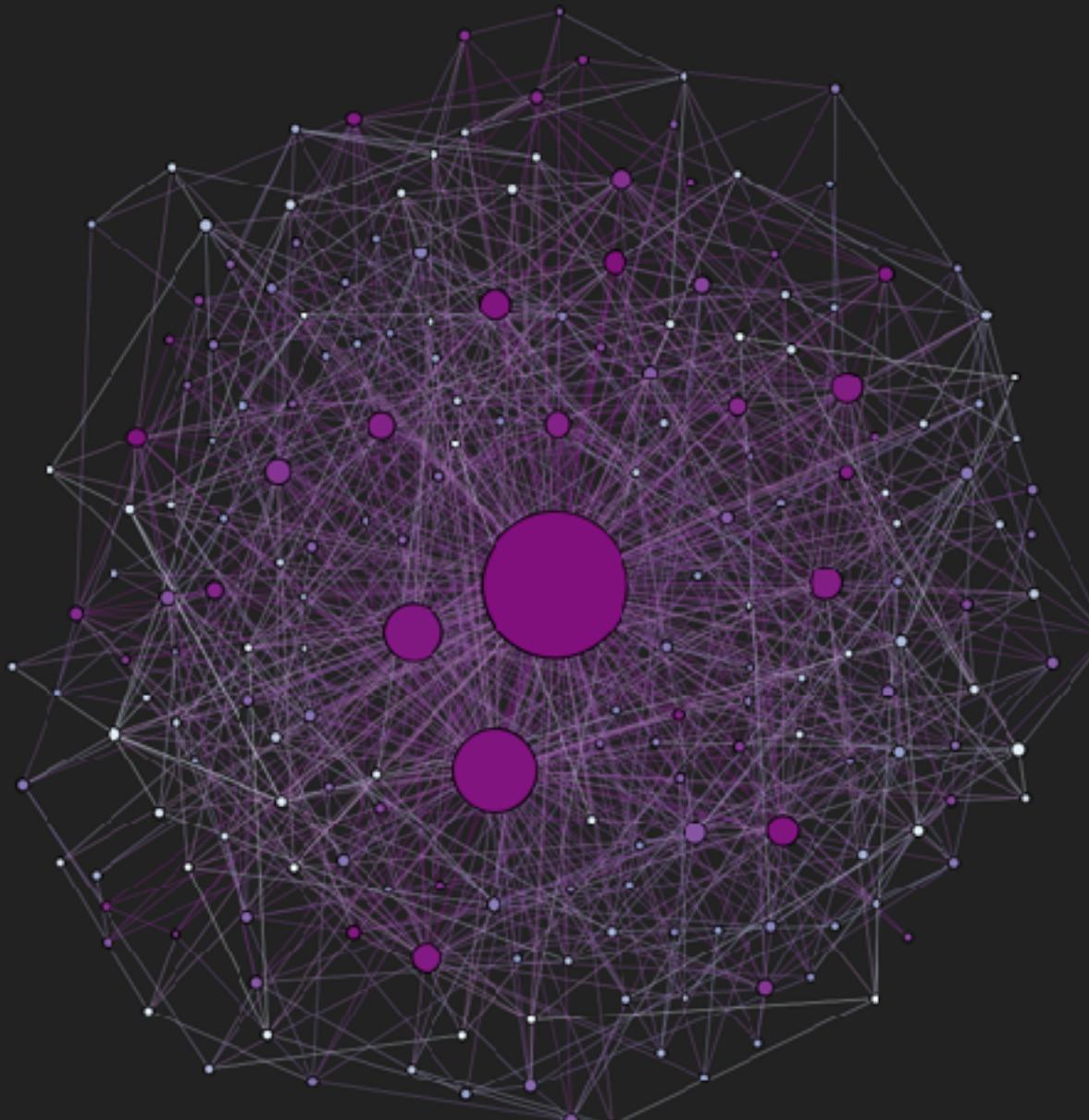
SUMMARY & OUTLOOK

Max ← Recommendation System's EXPLORATION → Min

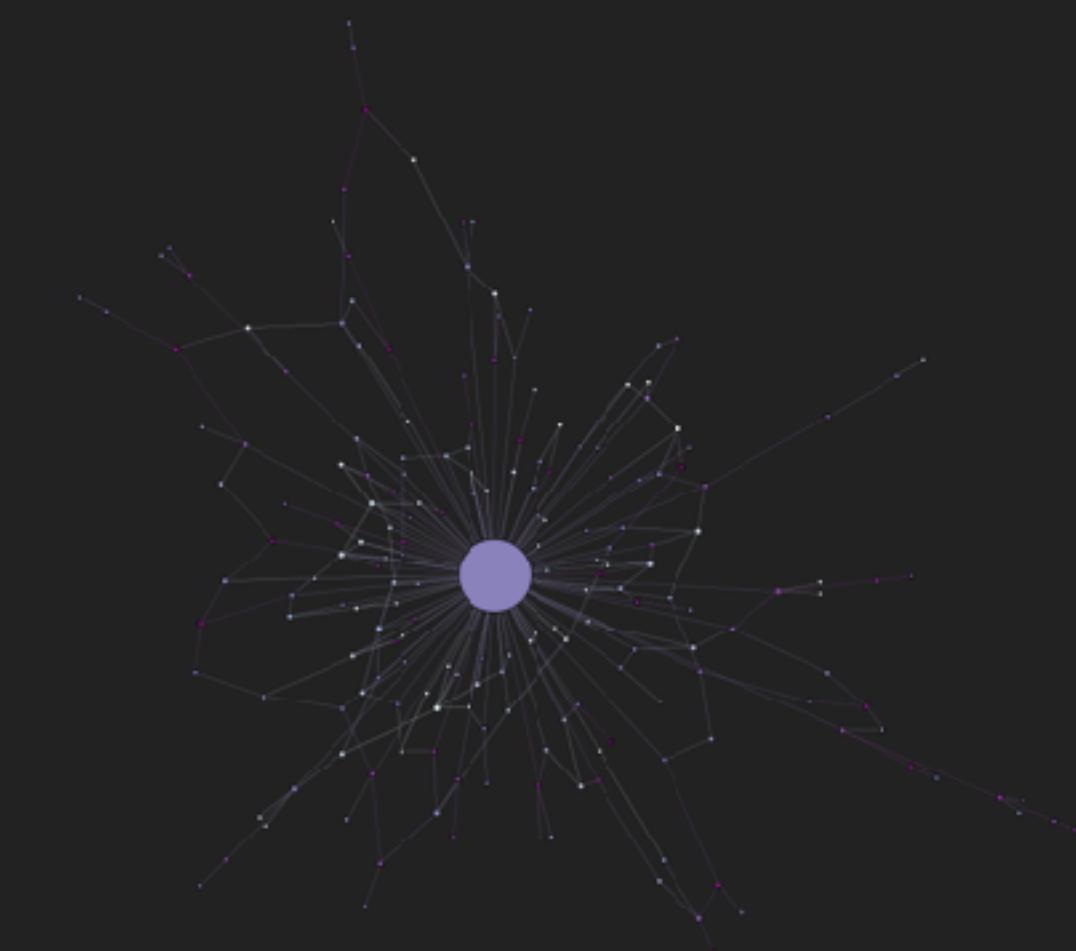
UNIFORM RANDOM



PREFERENTIAL ATTACHMENT



EXTREME PREFERENTIAL ATTACHMENT

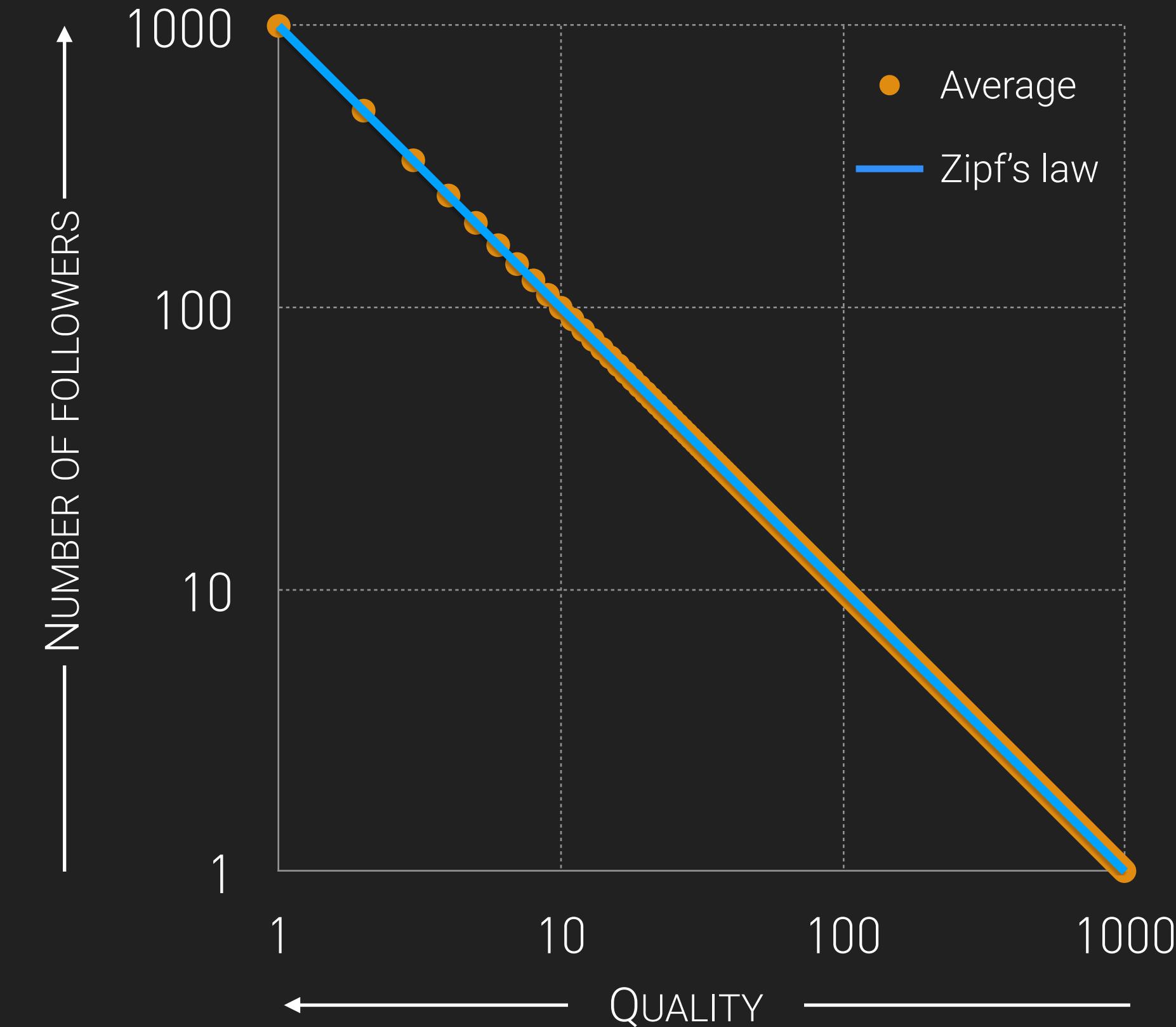


← QUALITY →

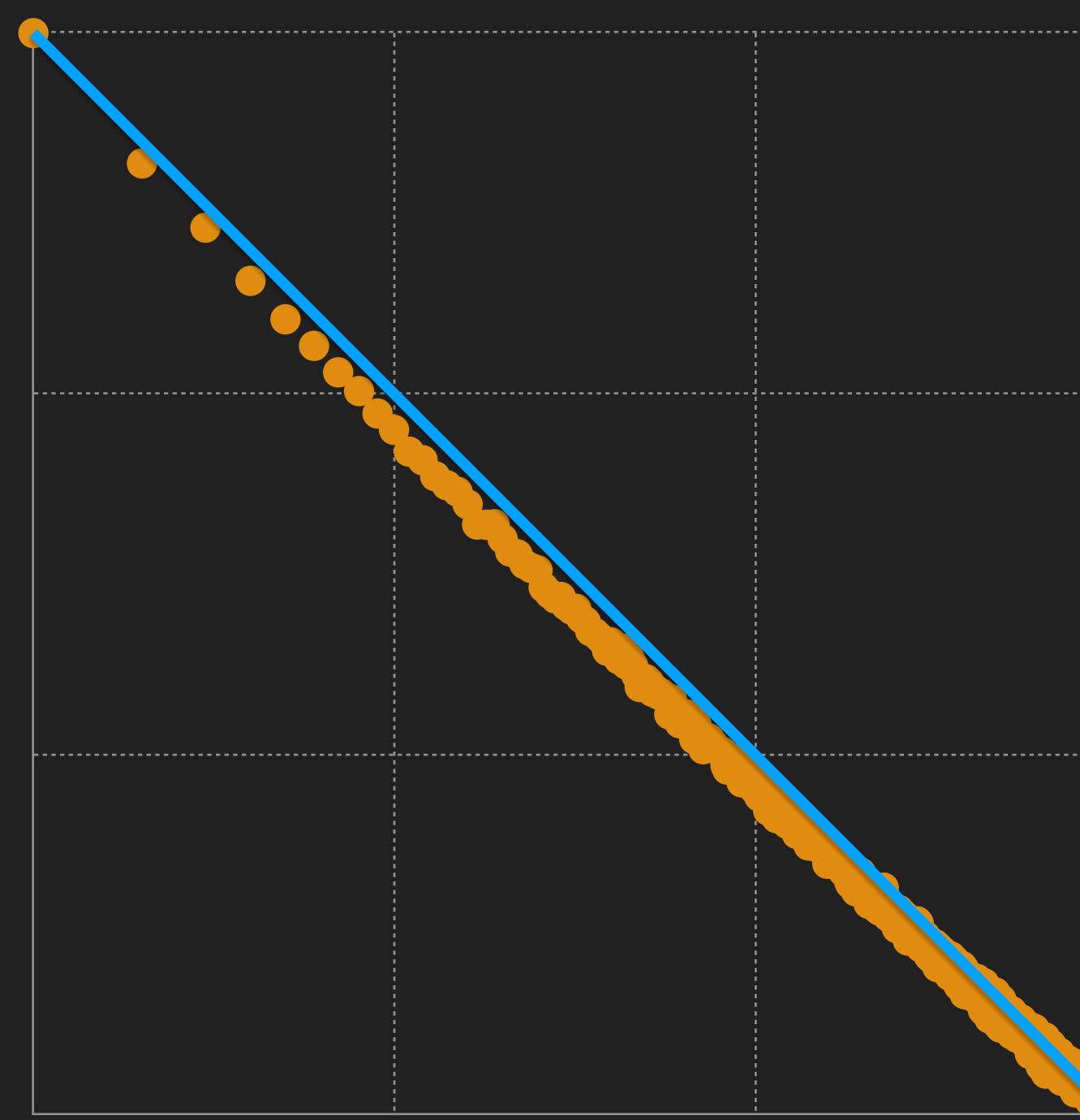
A horizontal color bar at the bottom, transitioning from dark purple on the left to white on the right, representing the range of node quality.

Max ← Recommendation System's EXPLORATION → Min

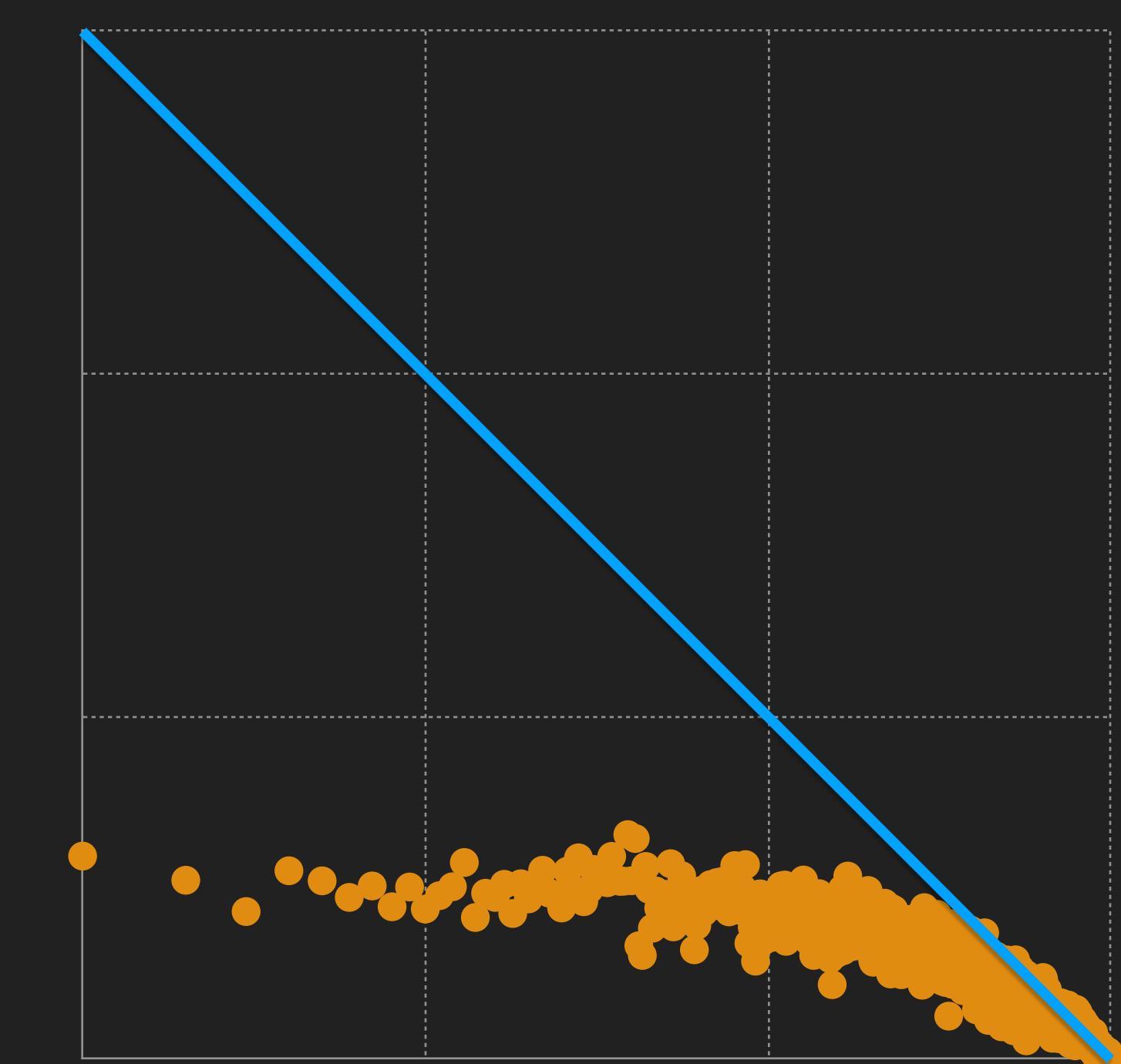
UNIFORM RANDOM



PREFERENTIAL ATTACHMENT



EXTREME PREFERENTIAL ATTACHMENT

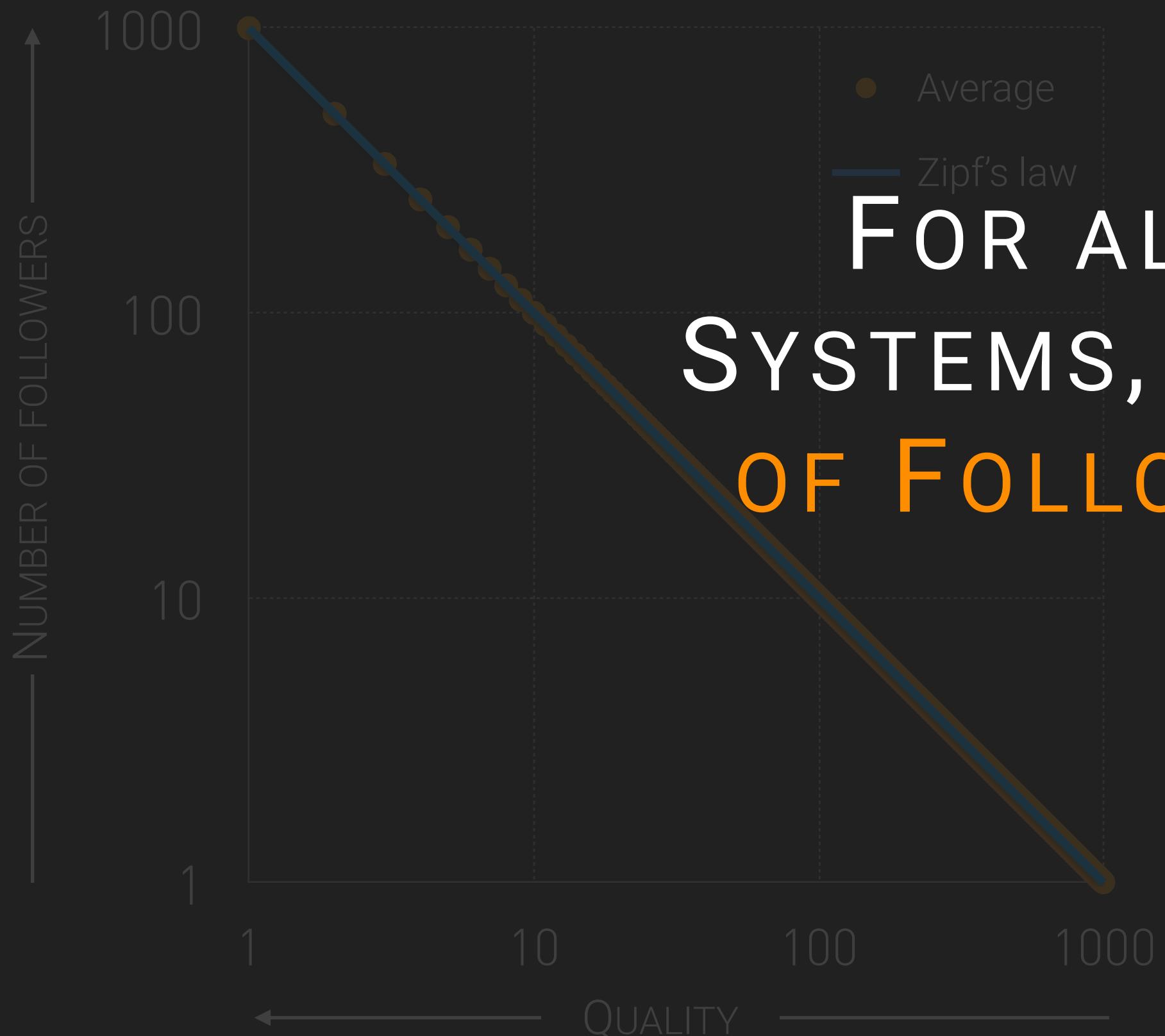


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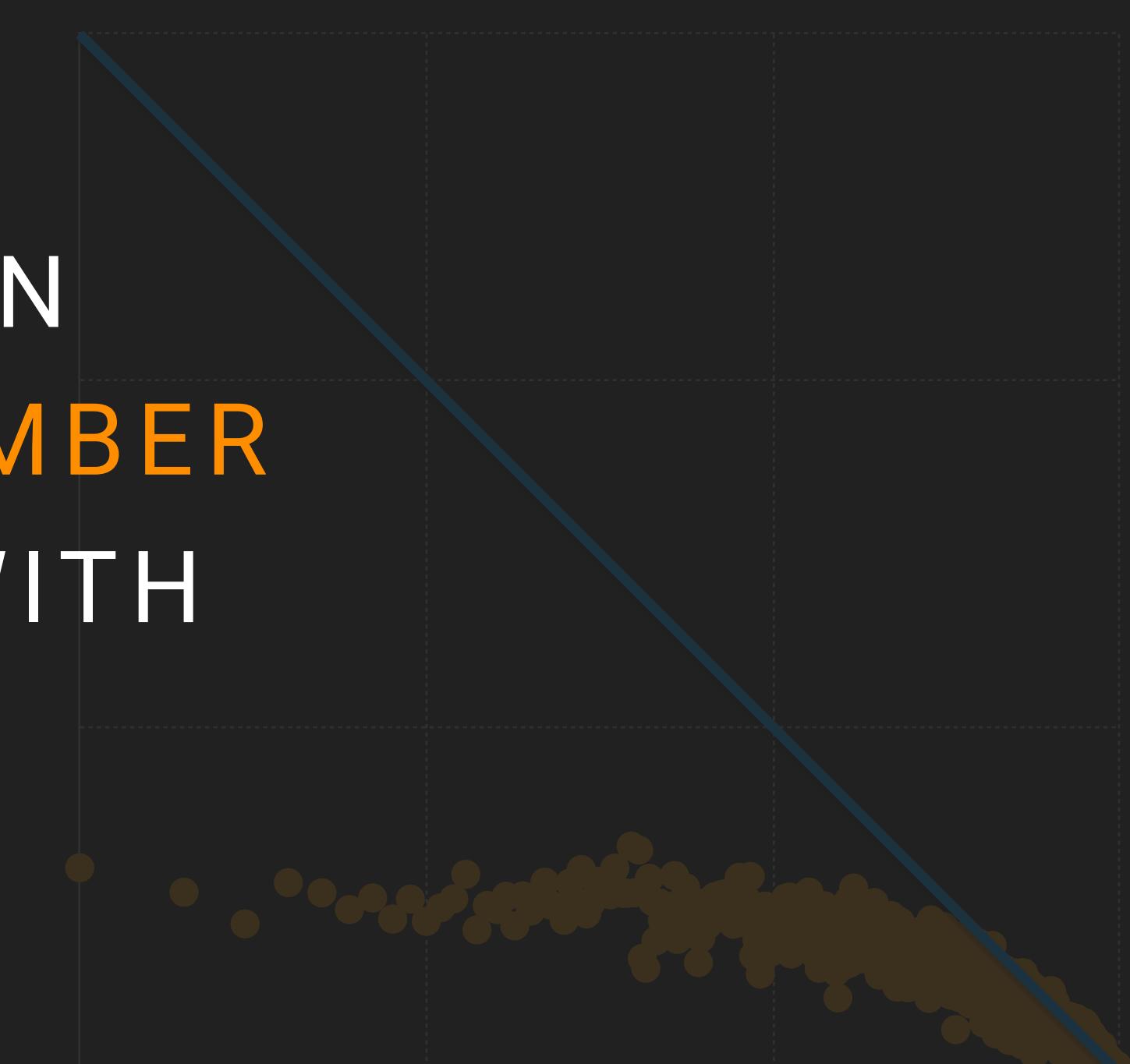
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PREFERENTIAL ATTACHMENT

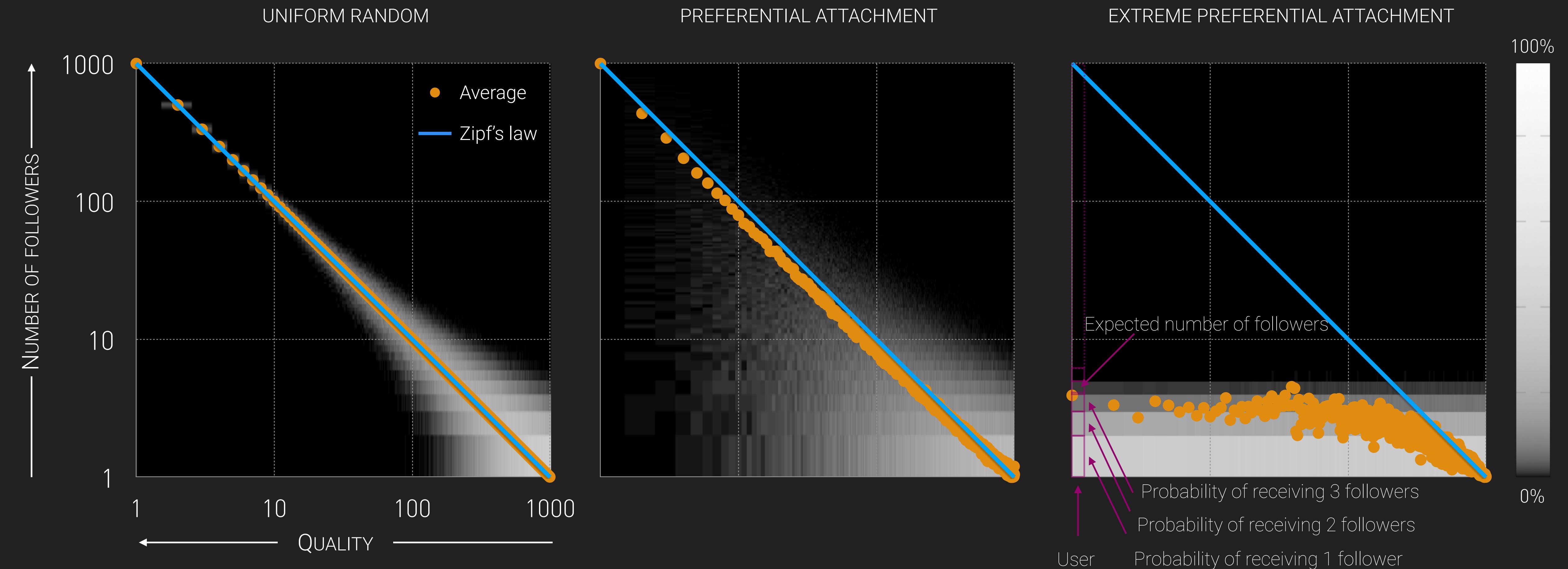
EXTREME PREFERENTIAL ATTACHMENT



FOR ALL RECOMMENDATION  
SYSTEMS, THE EXPECTED NUMBER  
OF FOLLOWERS INCREASES WITH  
QUALITY



Max ← Recommendation System's EXPLORATION → Min



[Pagan, N., Mei, W., Li, C. and Dörfler, F., 2021. A meritocratic network formation model for the rise of social media influencers. *Nature communications*, 12(1), p.6865.]

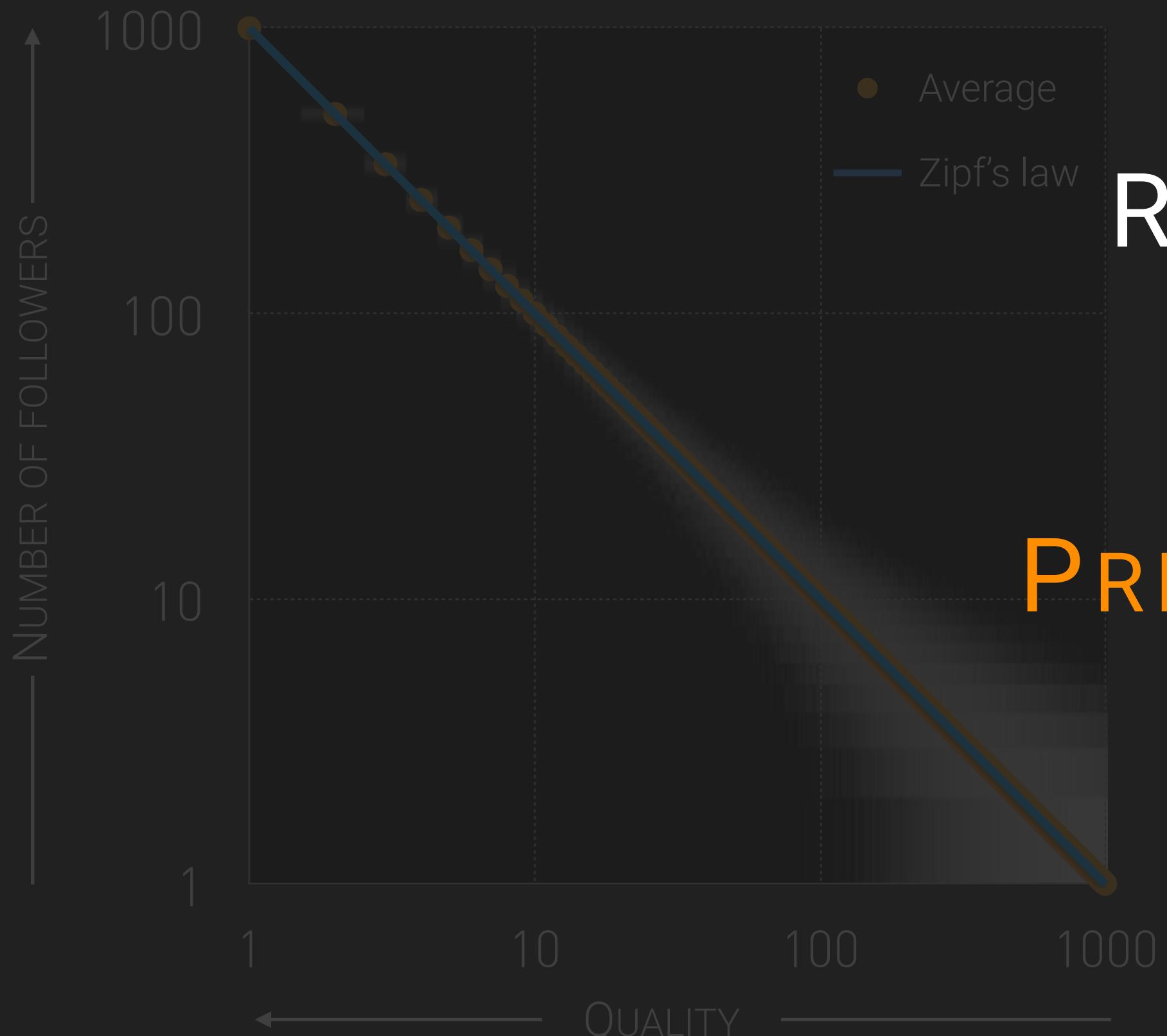
Ionescu, S., Pagan, N. and Hannák, A., 2023, January. Individual Fairness for Social Media Influencers. In *Complex Networks and Their Applications XI*.

Max ← Recommendation System's EXPLORATION → Min

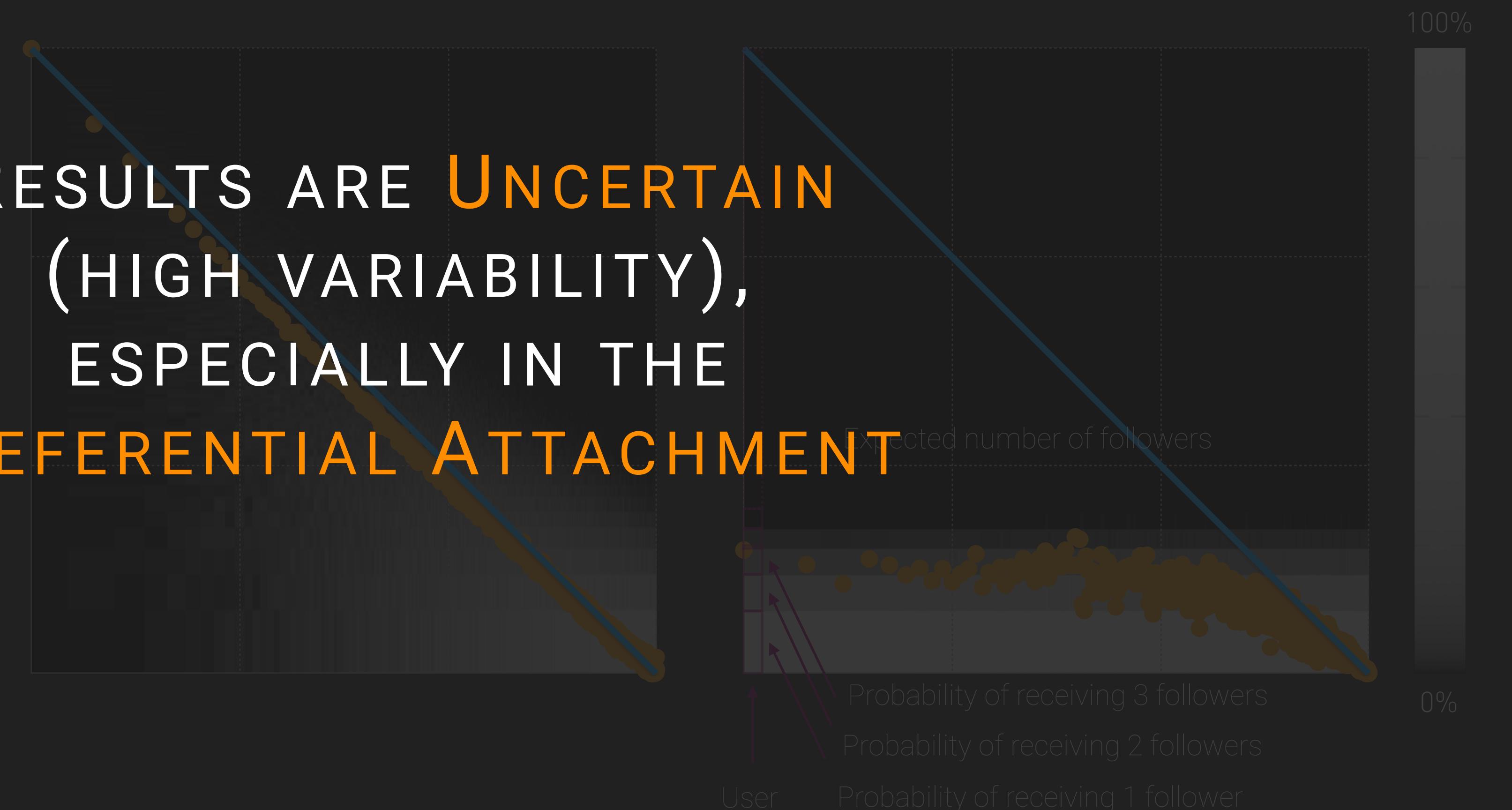
UNIFORM RANDOM

PREFERENTIAL ATTACHMENT

EXTREME PREFERENTIAL ATTACHMENT



RESULTS ARE UNCERTAIN  
(HIGH VARIABILITY),  
ESPECIALLY IN THE  
PREFERENTIAL ATTACHMENT



Max ← Recommendation System's EXPLORATION → Min

UNIFORM RANDOM

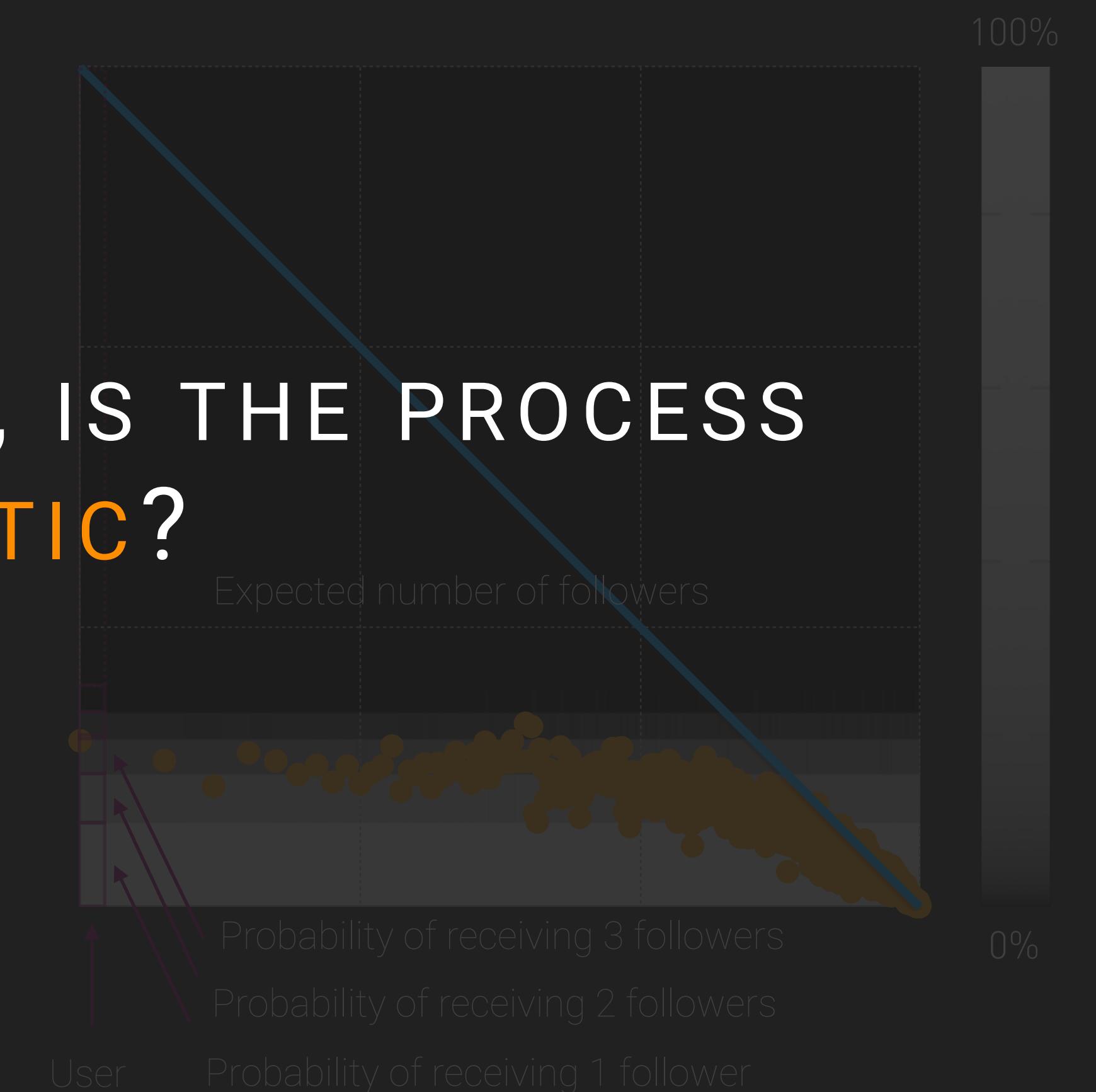


PREFERENTIAL ATTACHMENT

SINCE THE VARIABILITY IS LARGE, IS THE PROCESS  
ALWAYS MERITOCRATIC?

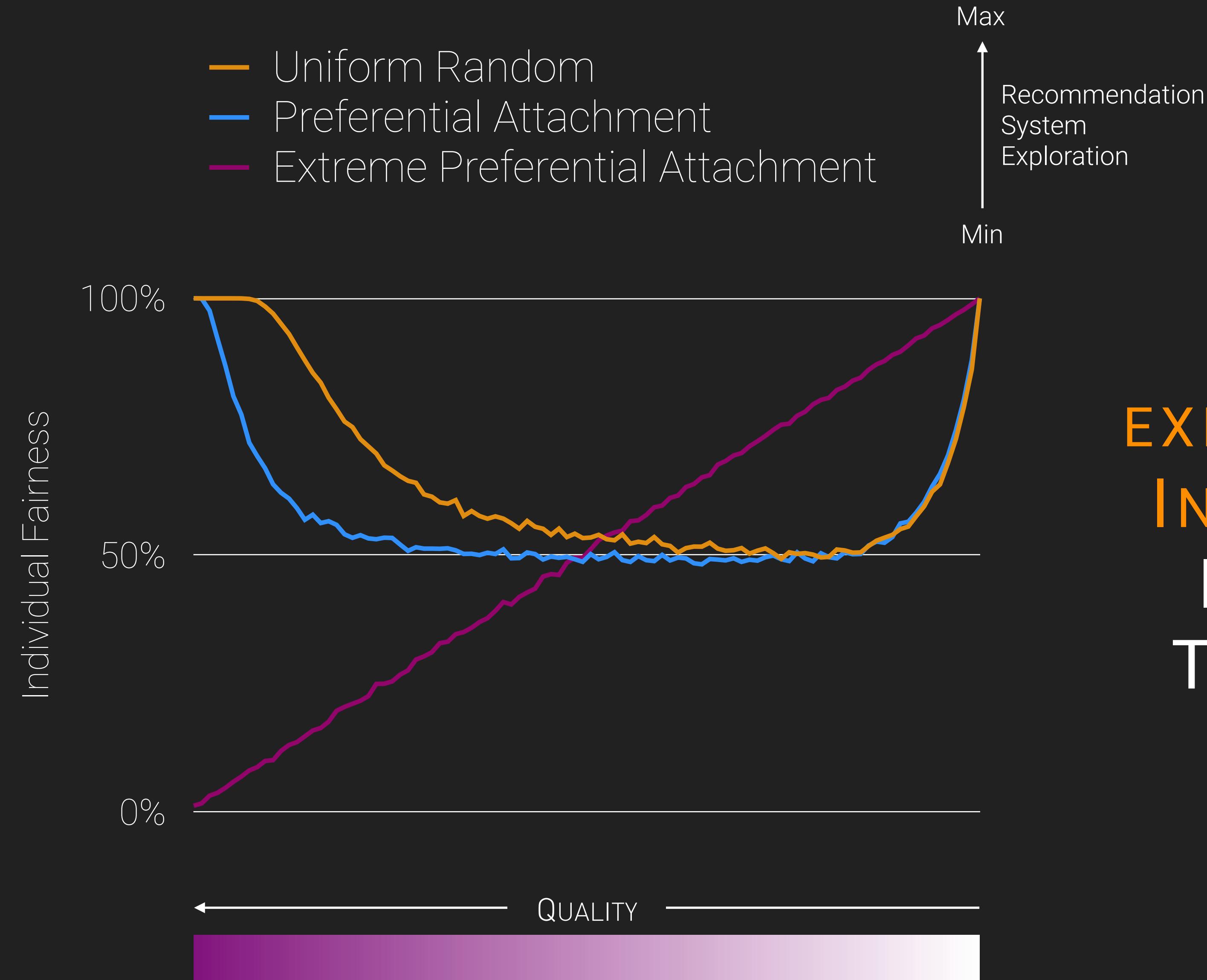


EXTREME PREFERENTIAL ATTACHMENT



## INDIVIDUAL FAIRNESS:

WHAT IS THE PROBABILITY THAT THE  $N^{\text{TH}}$   
QUALITY-RANKED INDIVIDUAL IS IN THE  
TOP- $N$  MOST FOLLOWED ONES?



**EXPLORATION IMPROVES  
INDIVIDUAL FAIRNESS,  
IN PARTICULAR FOR  
TOP-QUALITY NODES**

# INDEX

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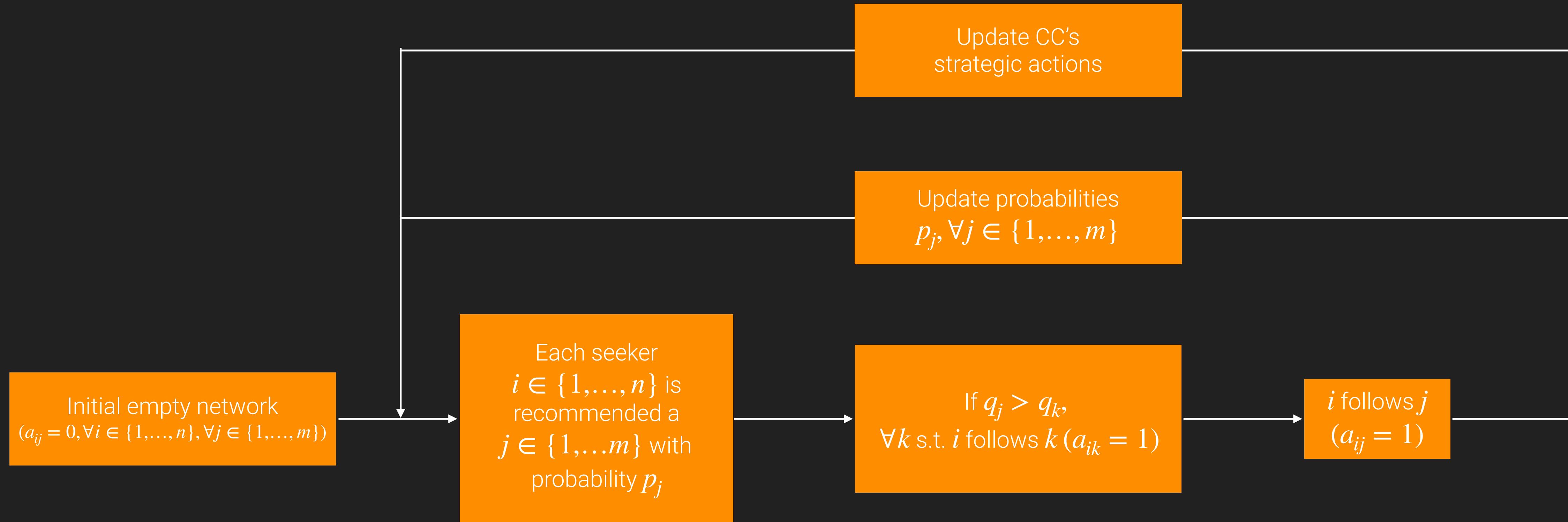
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COALITIONS

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# INDEX

INTRO ON NETWORK FORMATION MODELS

QUALITY-BASED NETWORK FORMATION MODEL

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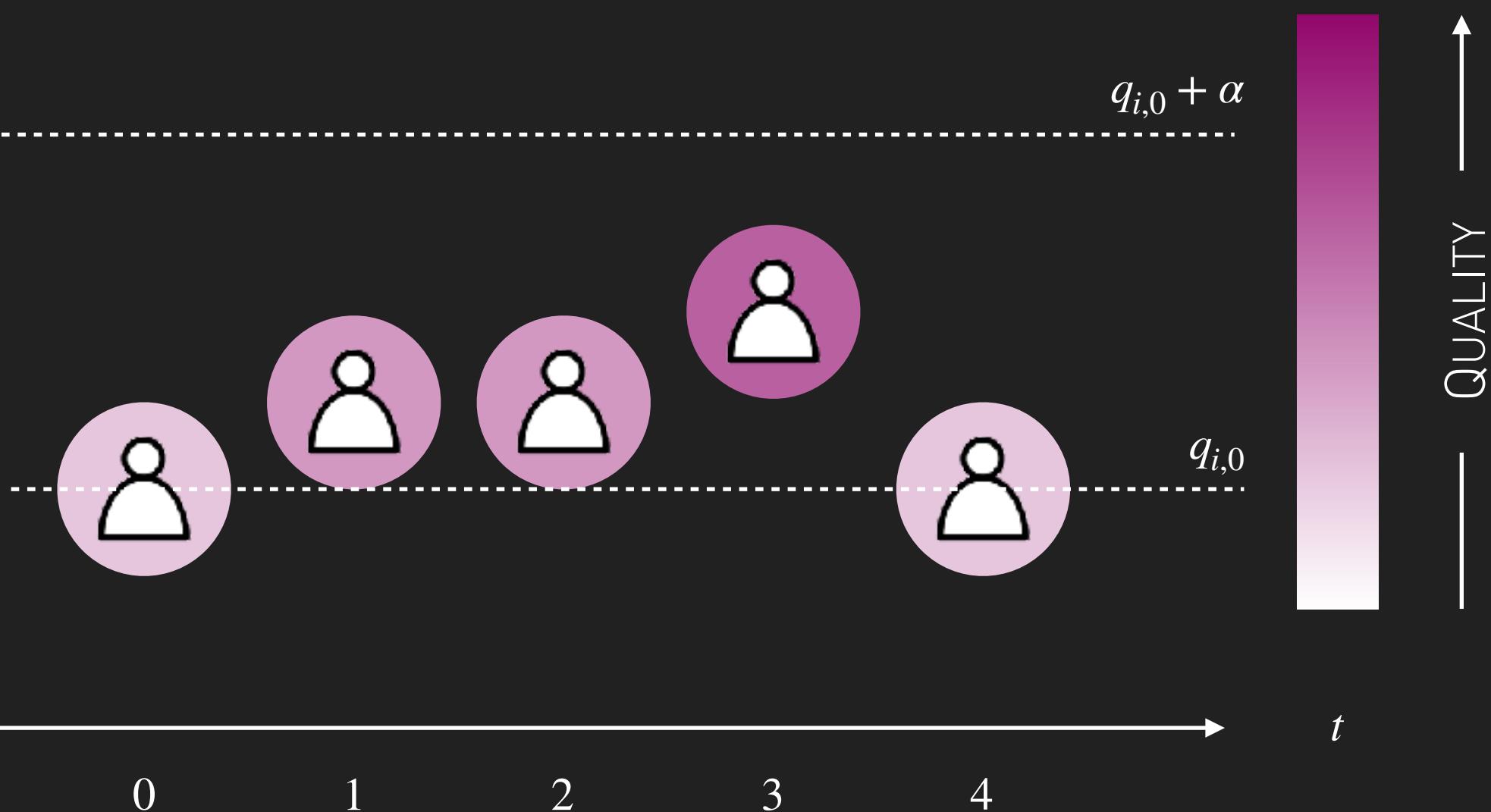
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# QUALITY GAME

Content Creators can purposefully change the **QUALITY** of their content by choosing:

- ▶ An action (quality)  $q_i(t) \in [q_{1,0}, q_{i,0} + \alpha]^\star$ , where  $\alpha$  is now the maximum deviation from the **INTRINSIC QUALITY**.



<sup>★</sup> : Again, we force  $q_i(t) \in [0,1]$ , as it is intended to be the probability a seeker will like the content.

# QUALITY GAME

Content Creators can purposefully change the **QUALITY** of their content by choosing:

- ▶ An action (quality)  $q_i(t) \in [q_{i,0}, q_{i,0} + \alpha]^\star$ , where  $\alpha$  is now the maximum deviation from the **INTRINSIC QUALITY**.
- ▶ They aim at maximising their utility function , given the current state of the network, i.e.,

$$q_i^\star(t) := \arg \max_{q_i \in Q_i} \left\{ U_i(q_i, q_{-i}(t-1), A(t-1)) \right\}, \text{ where}$$

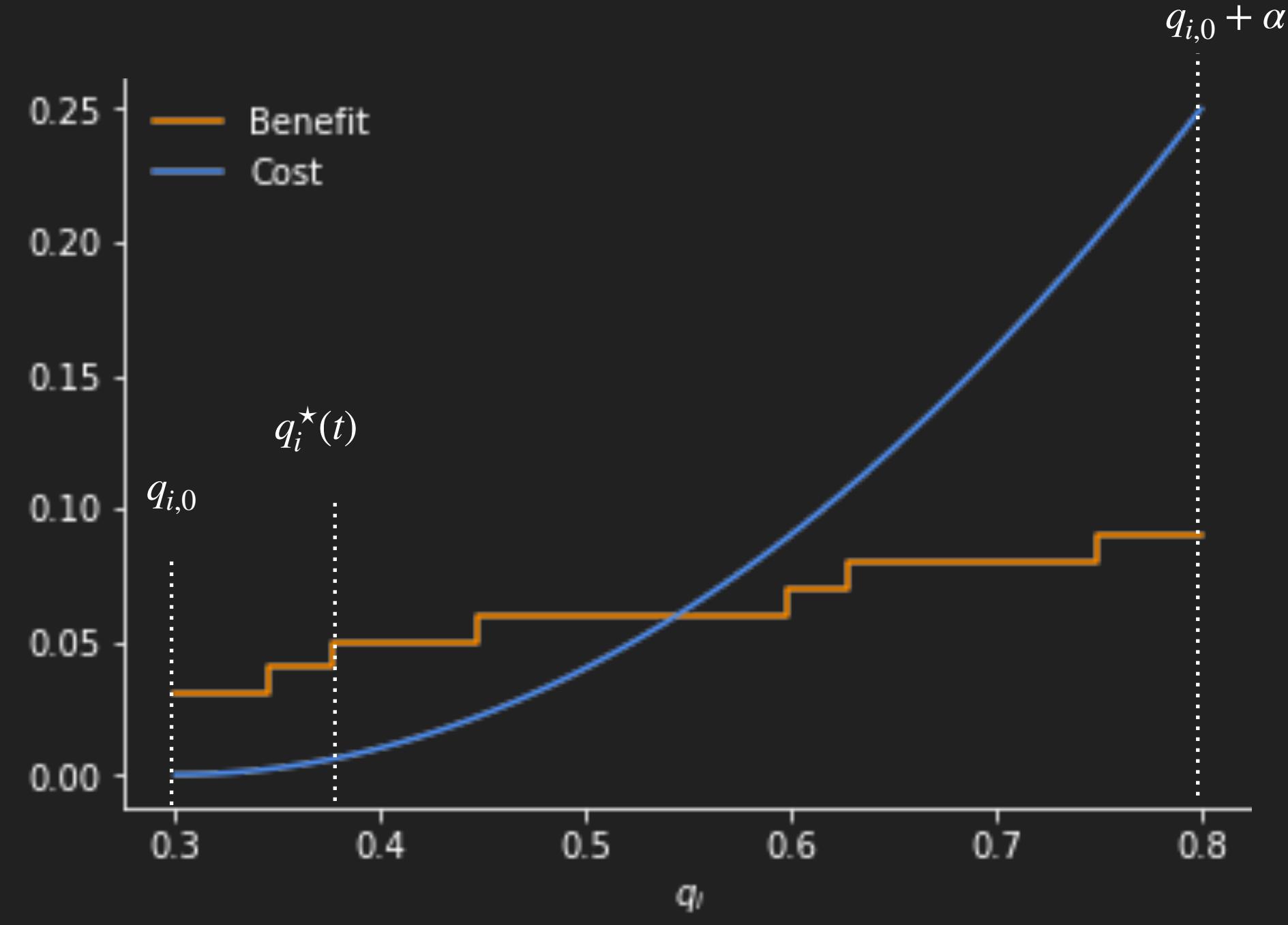
$$U_i(q_i, q_{-i}(t-1), A(t-1)) = b_i(q_i, q_{-i}(t-1), A(t-1)) - c_i(q_i), \text{ and}$$

$$b_i(\cdot) := \frac{1}{n} \left| \{j \in \{1, \dots, n\} \text{ s.t. } j \text{ does not follow } i, \text{ and } \forall k \text{ s.t. } j \text{ follows CC } k, q_k(t-1) < q_i\} \right|$$

measures the ratio of potential new followers,

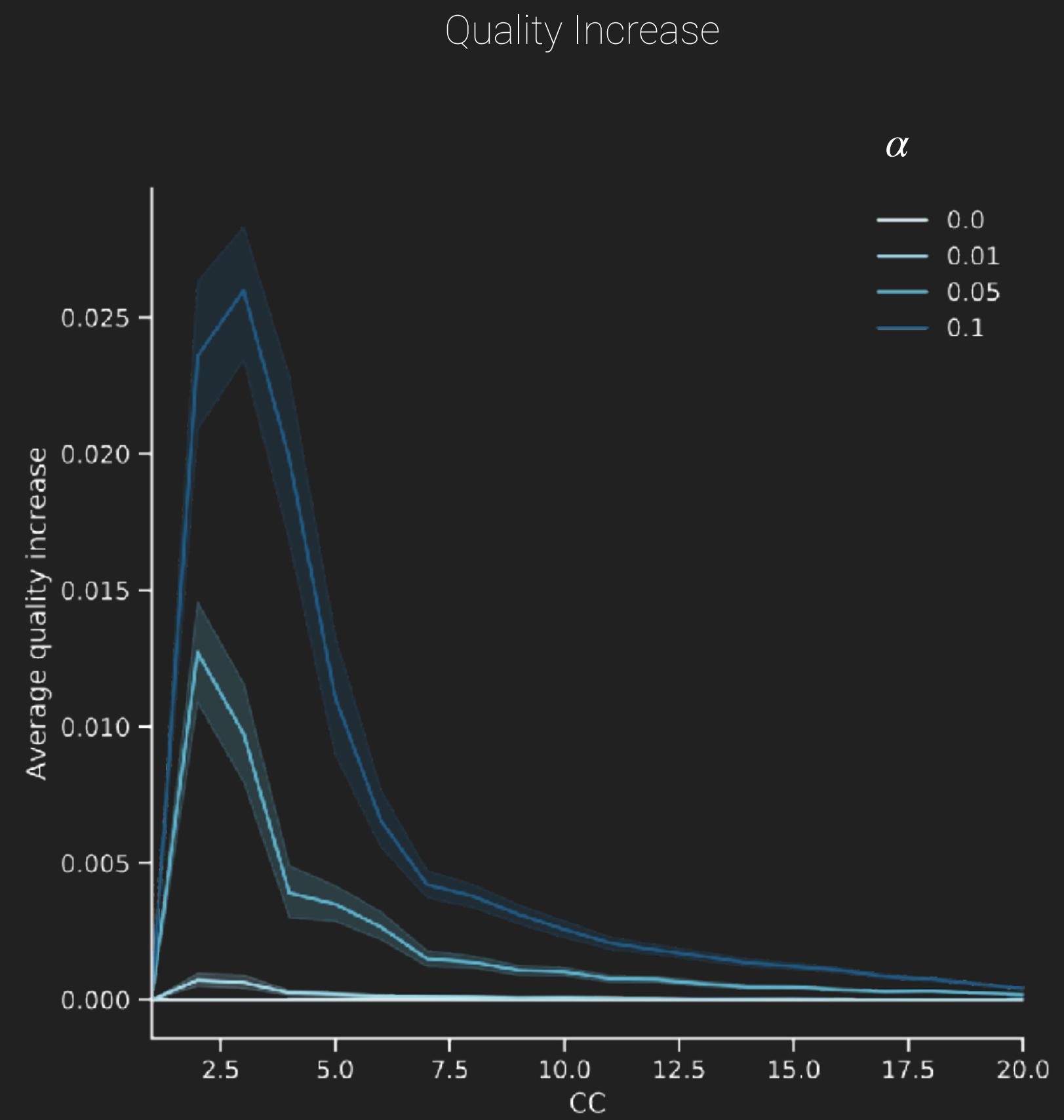
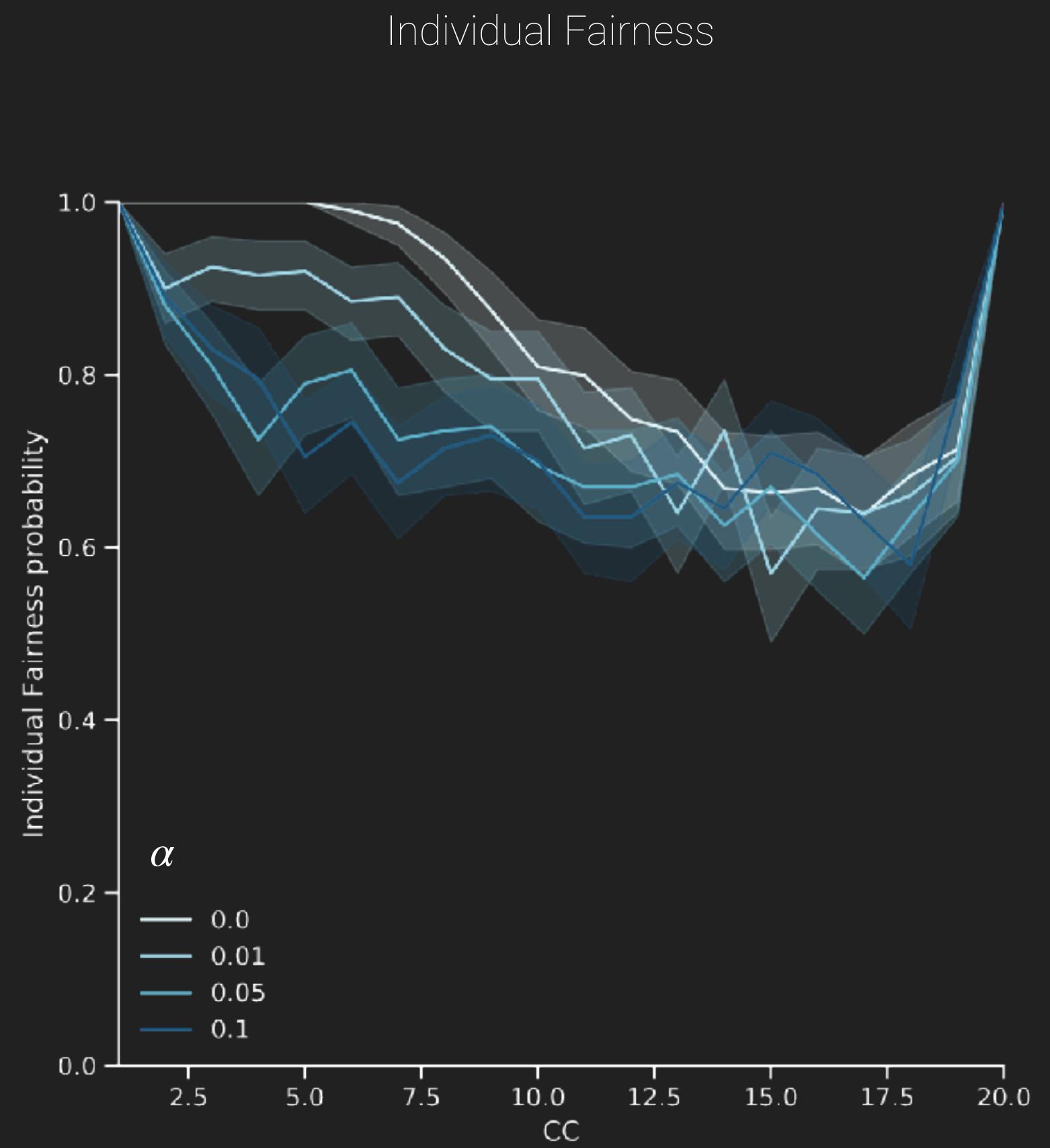
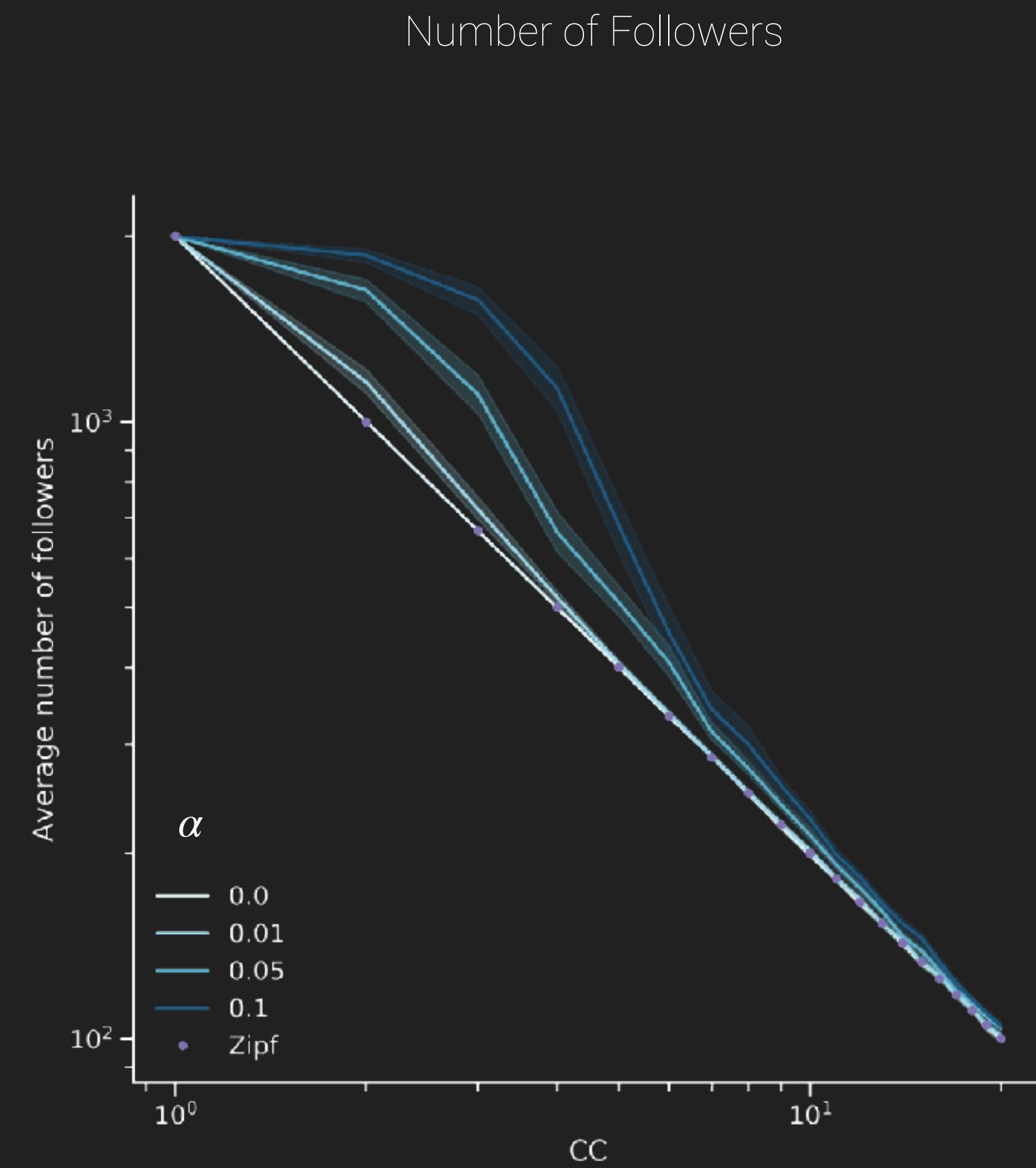
and  $c_i(q_i) := (q_i - q_{i,0})^2$ , models the cost of increasing the quality from the baseline **INTRINSIC QUALITY**.

$\star$  : We also force  $q_i(t) \in [0,1]$ , as it is intended to be the probability a seeker will like the content.



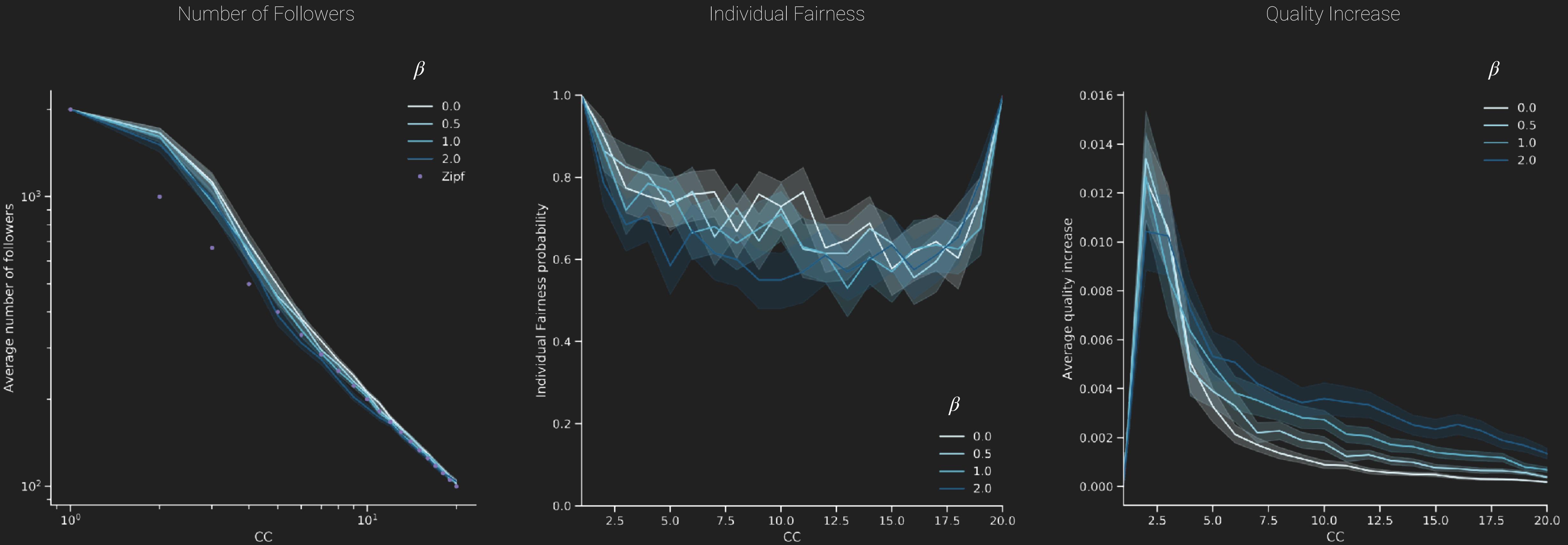
# QUALITY-GAME RESULTS

FIXED: UNIFORM RANDOM RECOMMENDATION



# QUALITY-GAME RESULTS

FIXED:  $\alpha = 0.05$



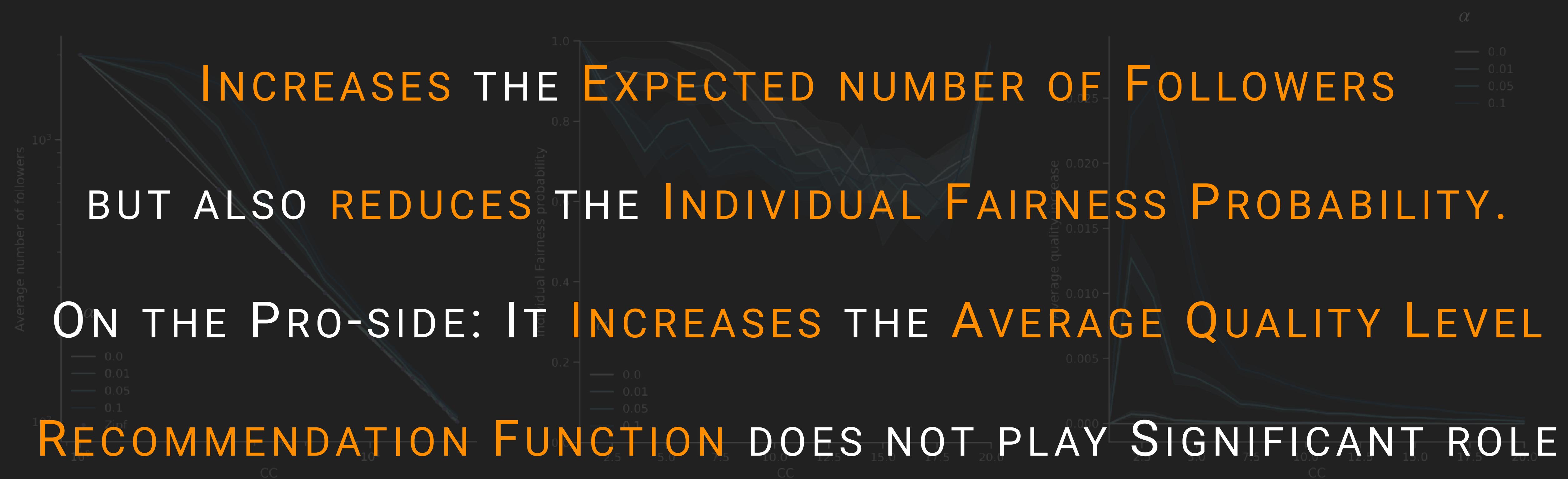
$$\mathbb{P}(i \text{ is recommended}) = \frac{(d_{in}^i)^\beta + 1}{\sum_{j=1}^m ((d_{in}^j)^\beta + 1)}$$

# QUALITY-GAME RESULTS

Number of Followers

## STRATEGIC PLAY IN QUALITY:

Quality Increase



# INDEX

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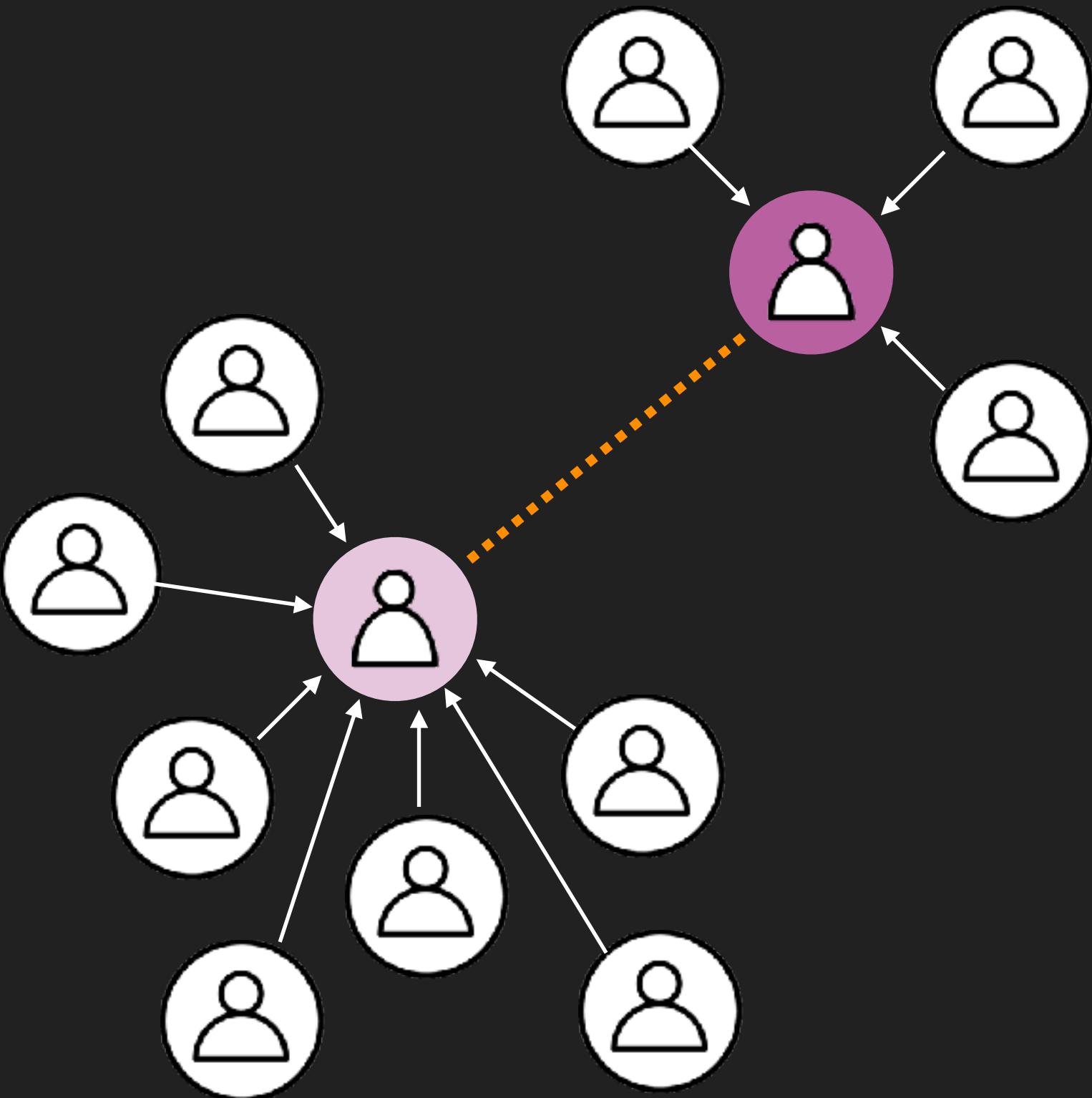
COALITIONS

SUMMARY & OUTLOOK

# COALITION GAME

Content Creators can purposefully create coalition with one another by choosing:

- ▶ Another content creator  $j \in \{1, \dots, m\}^*$ ,



\* :  $j = i$  means that the CC does not form a coalition.

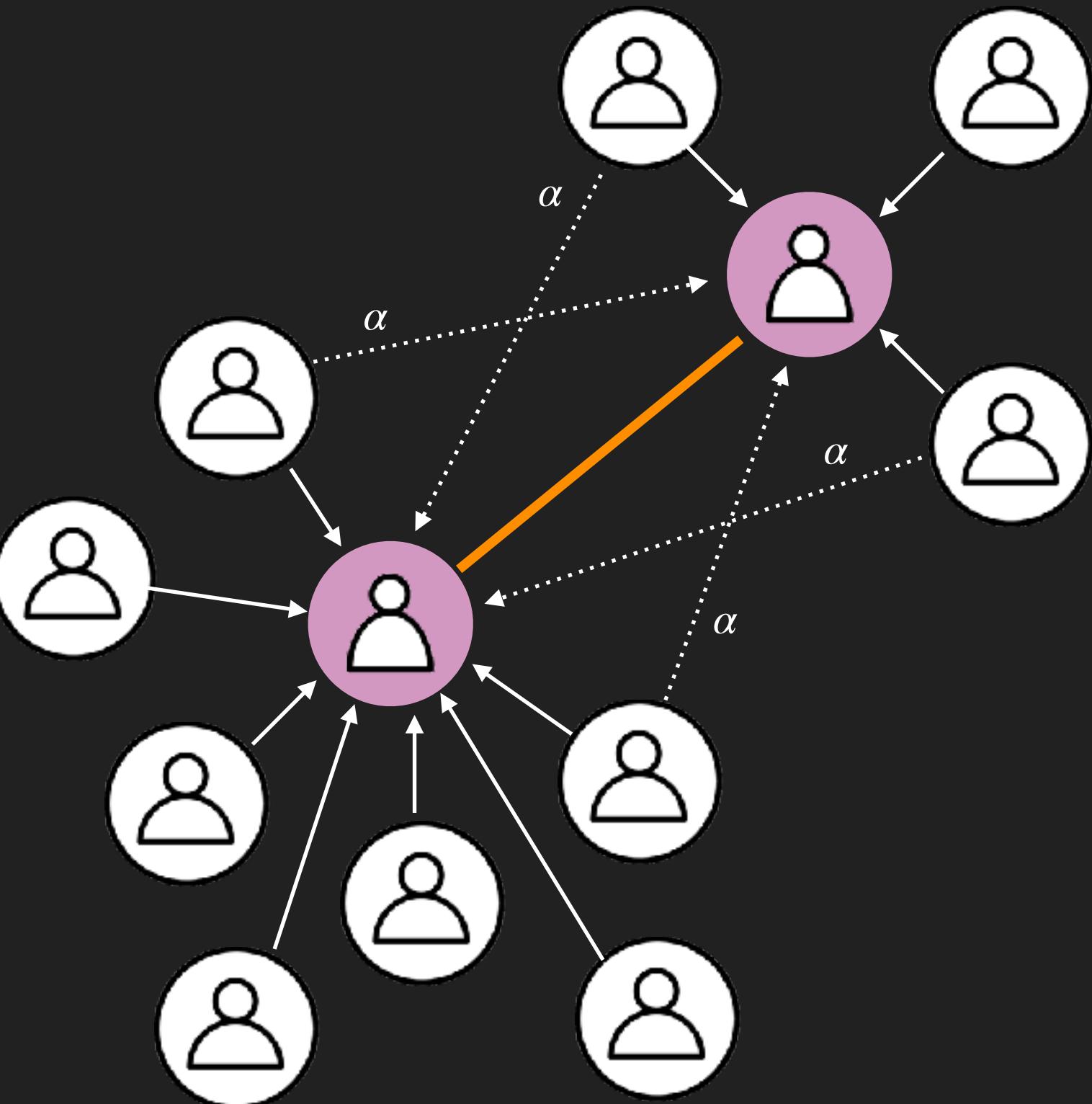
# COALITION GAME

Content Creators can purposefully create coalition with one another by choosing:

- ▶ Another content creator  $j \in \{1, \dots, m\}^*$ , such that:

$$q_i(t) = \frac{1}{2} (q_{i,0} + q_{j,0}).$$

Whenever such a **COALITION** is in place, with probability  $\alpha$  (strategic level), a follower of  $i$  (respectively,  $j$ ) will be recommended  $j$  (respectively,  $i$ ).



\* :  $j = i$  means that the CC does not form a coalition.

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- ▶ At each time-step, Content Creators are selected in random order, and they propose a coalition to their best match:

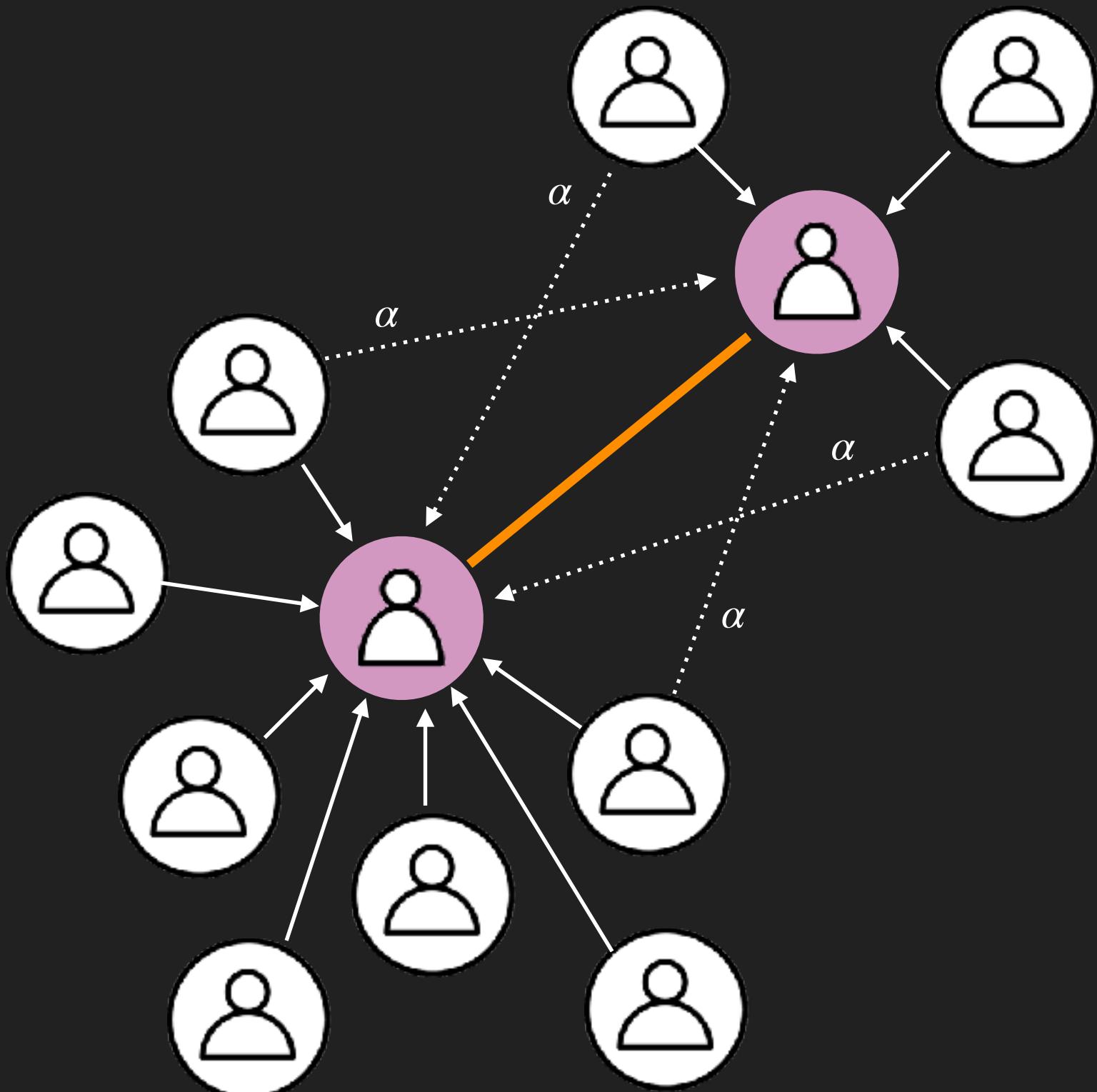
$$j^* \in \arg \max_{\{i, \dots, m\}, j \neq i} U_i(j, A(t-1)), \text{ where}$$

$$U_i(j, A(t-1)) := (1 - \alpha) \left| \left\{ k \in \{1, \dots, n\} \text{ s.t. } k \text{ does not follow } i, \text{ and } \forall l \text{ s.t. } k \text{ follows CC } l, q_l < \frac{1}{2} (q_{i,0} + q_{j,0}) \right\} \right| + \alpha \left| \{k \text{ follows } j \text{ but not } i\} \right|^{**}$$

measures the number of potentially available followers also thanks to a direct recommendation from the coalized CC.

\* :  $j = i$  means that the CC does not form a coalition.

\*\* : if  $j \neq i$ , otherwise, the second term does not appear.



# COALITION GAME

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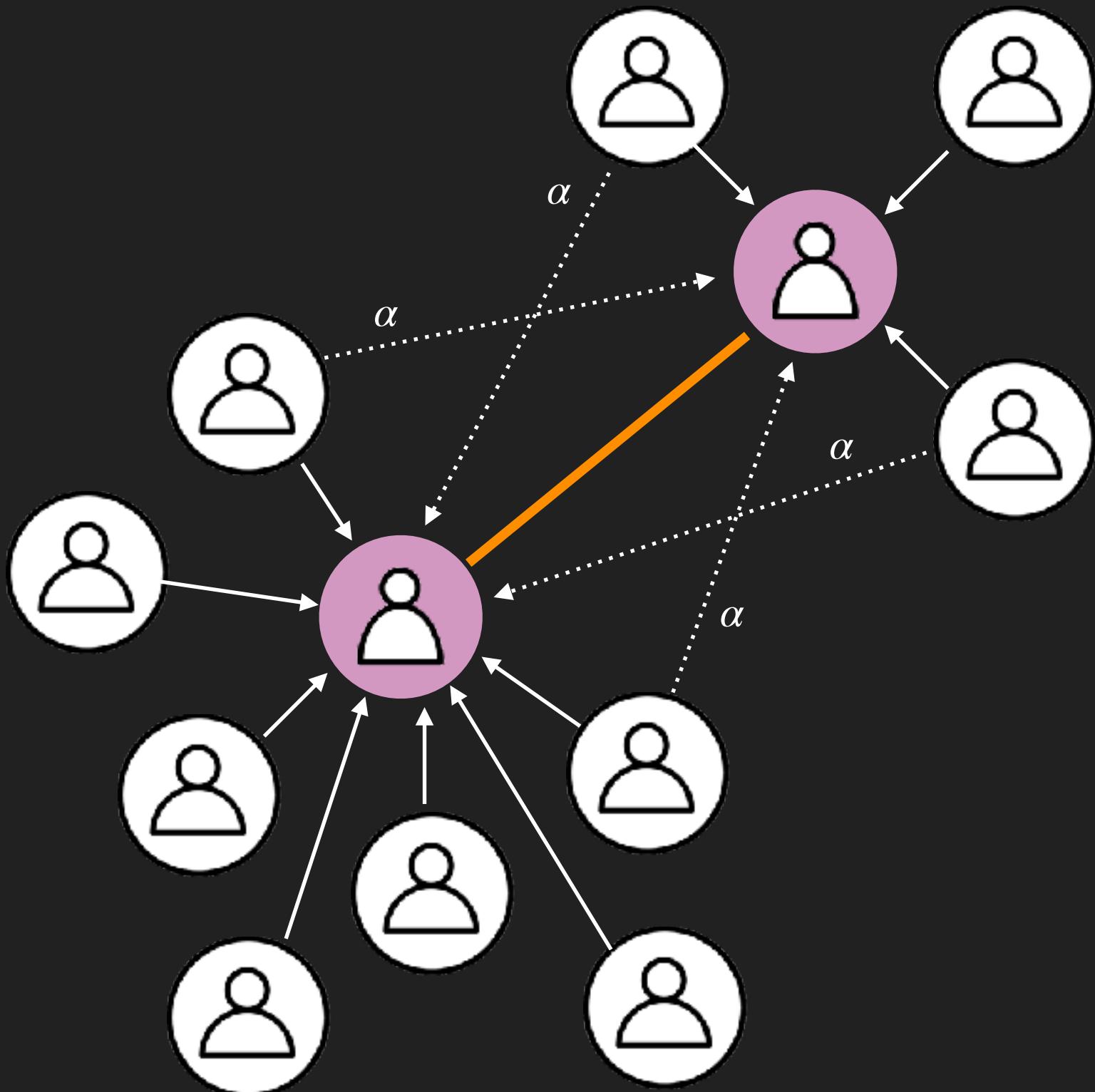
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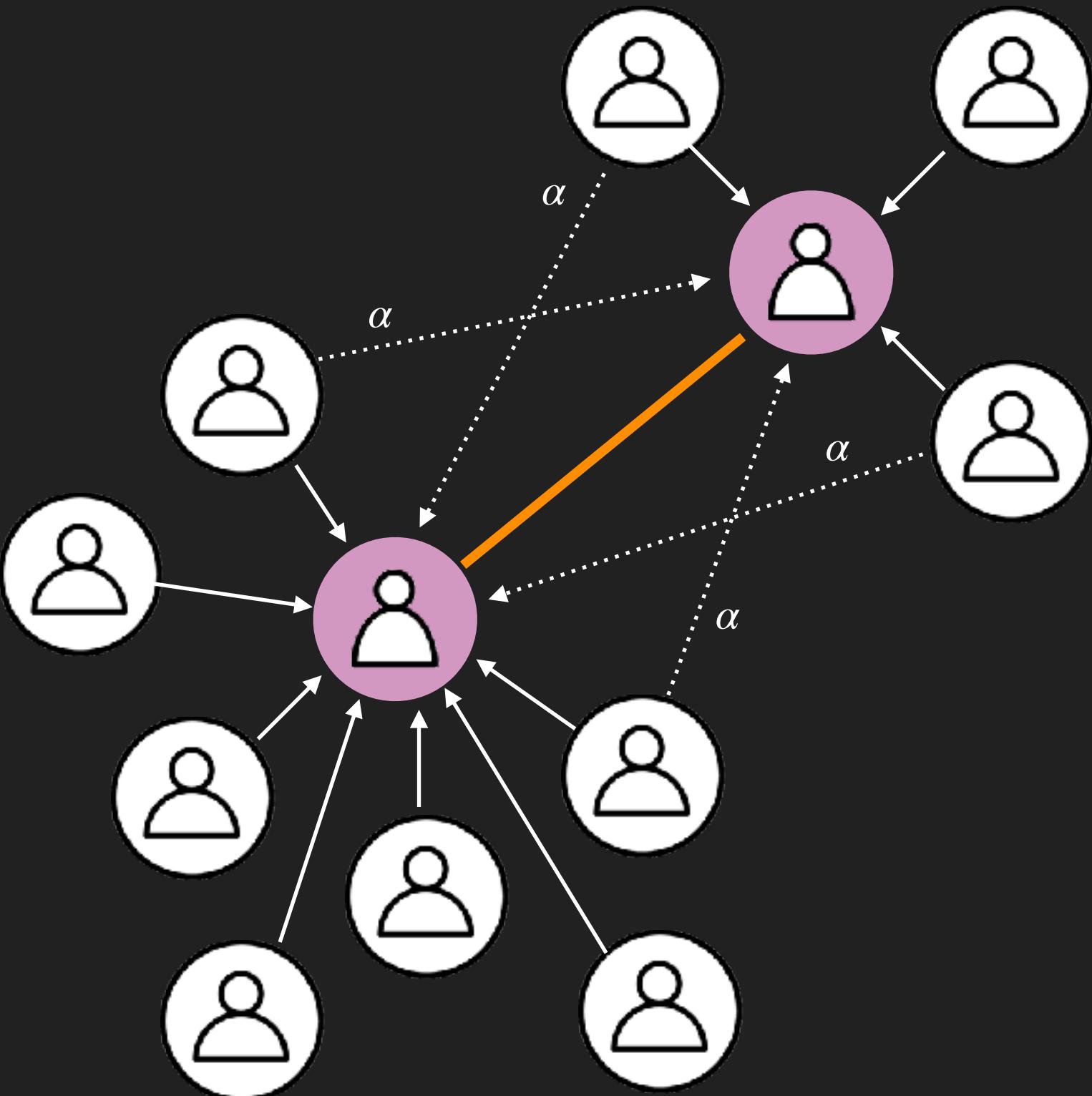
measures the number of potentially available followers also thanks to a direct recommendation from the coalized CC.

- ▶ Then,  $j^*$  can decide (based on a similar utility function) whether to form a coalition with  $i$ , or keep its current status.



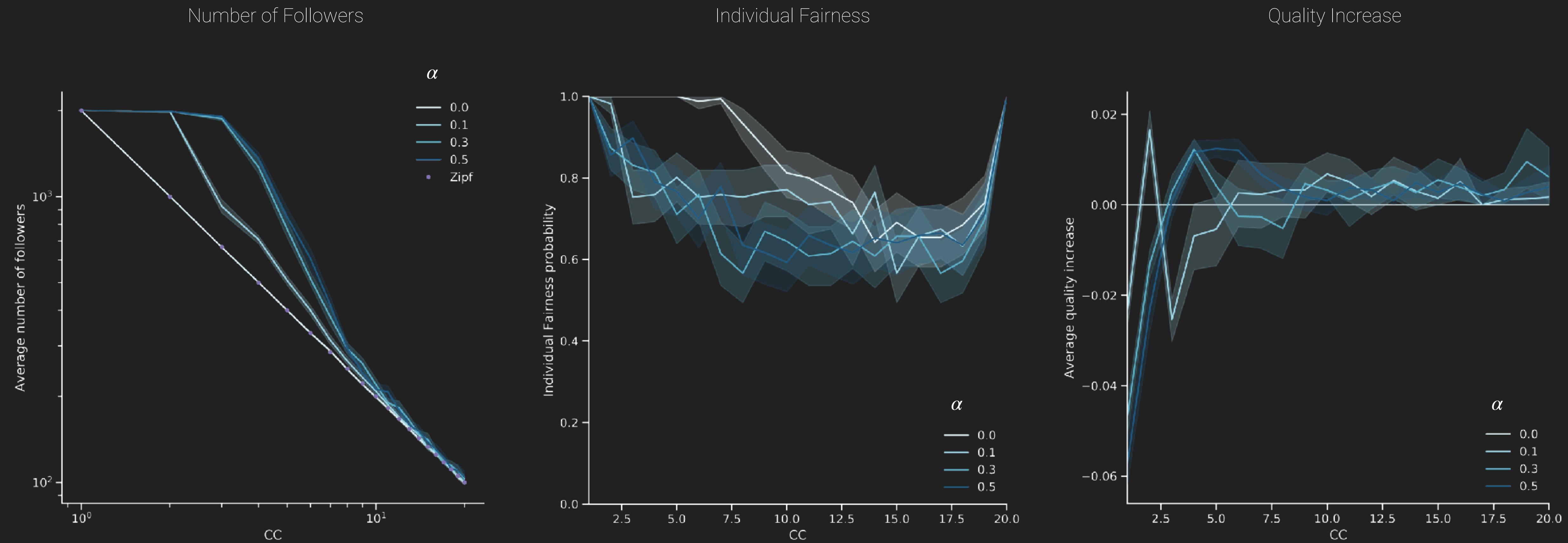
# COALITION GAME

- ▶ Connecting to a higher-quality Content Creators is always beneficial (compared to no coalition).
- ▶ Connecting to lower-quality Content Creators can be beneficial, provided that they have a large number of (non-overlapping) followers (as in the example on the right).



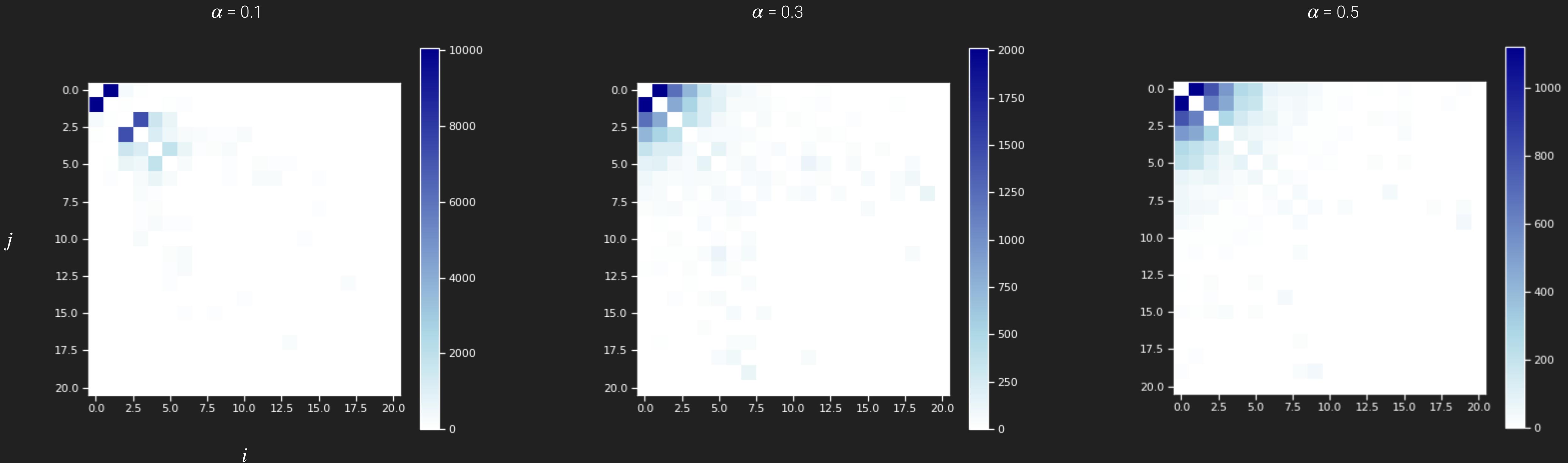
# COALITION-GAME RESULTS

FIXED: UNIFORM RANDOM RECOMMENDATION



# COALITION-GAME RESULTS

Coalition Frequency Matrices



# COALITION-GAME RESULTS

SIMILAR TO PREVIOUS ANALYSIS, STRATEGIC COALITIONS:

Number of Followers

Individual Fairness

Quality Increase

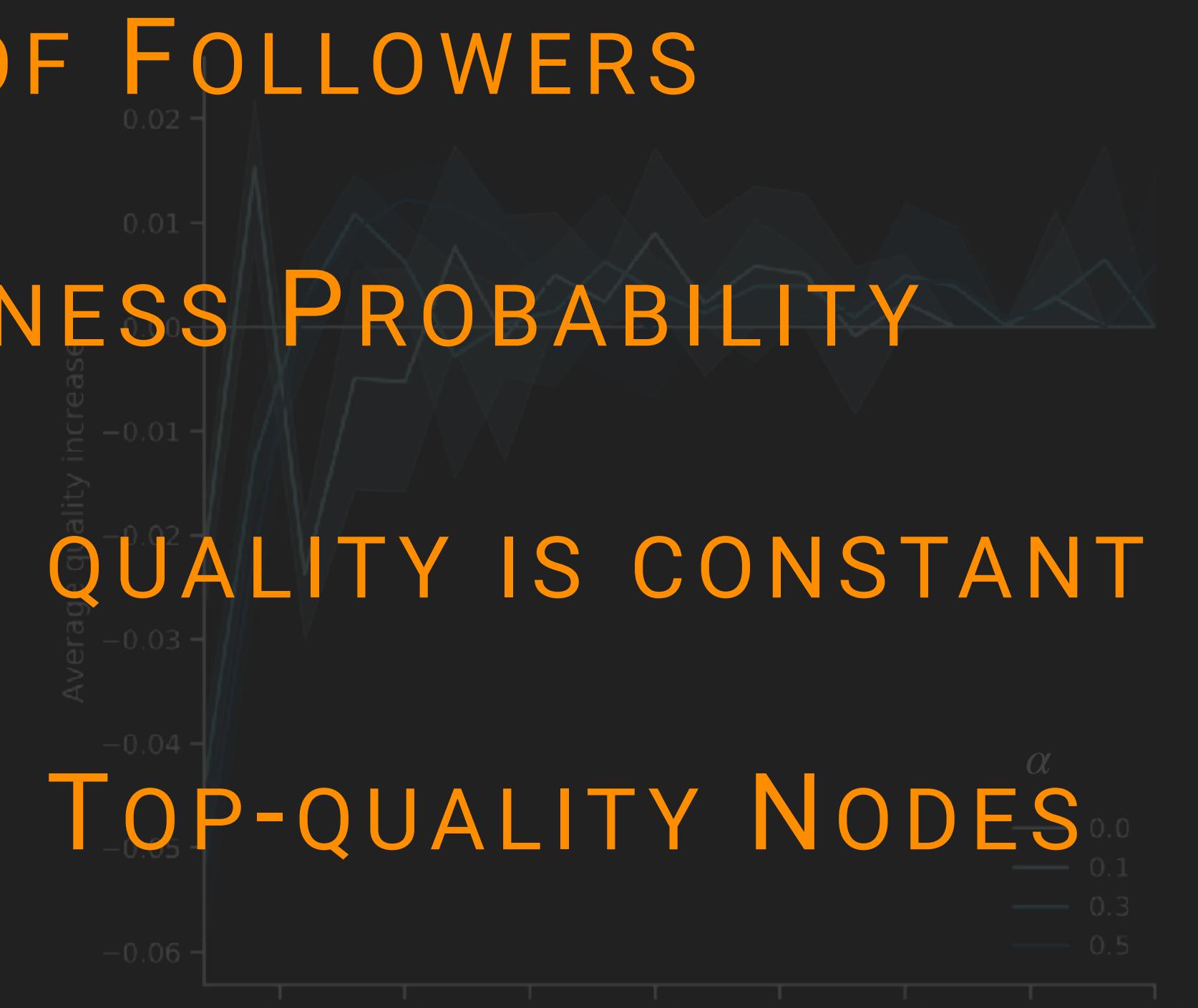
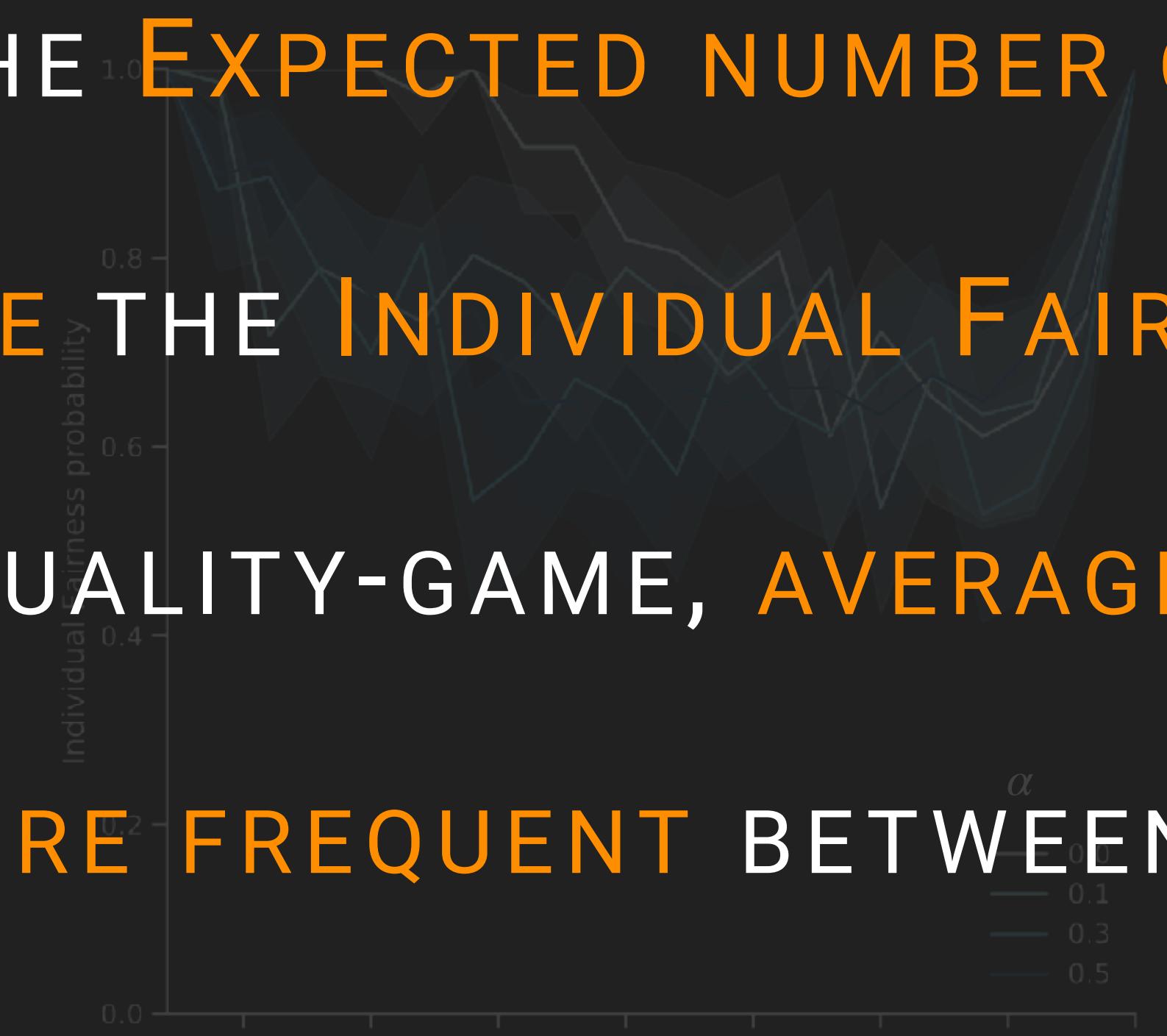
INCREASE THE EXPECTED NUMBER OF FOLLOWERS

BUT ALSO REDUCE THE INDIVIDUAL FAIRNESS PROBABILITY

DIFFERENTLY FROM QUALITY-GAME, AVERAGE QUALITY IS CONSTANT

COALITIONS ARE MORE FREQUENT BETWEEN TOP-QUALITY NODES

EFFECT OF RECOMMENDATION IS... WORK-IN-PROGRESS



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SUMMARY & OUTLOOK

# SUMMARY

- ▶ Social Media Platforms are **COMPETITIVE** environments
- ▶ **RECOMMENDATION SYSTEMS** play a key role in distributing **INDIVIDUAL FAIRNESS** to Content Creators
- ▶ **STRATEGIC QUALITY PLAY** decreases the probability of achieving individual fairness (while increasing the average engagement). However, it increases the average quality on the platform
- ▶ **STRATEGIC COALITIONS**, instead, do not improve the average quality on the platform (which is necessarily conserved). Similar to the other scenario, they decrease the probability of achieving individual fairness (while increasing the average engagement)

# OUTLOOK

- ▶ Explore the **COUPLED EFFECT** of Recommender Systems and CC strategic play.
- ▶ Study the effect on individual fairness when only a **LIMITED AMOUNT** of players act strategically
- ▶ Starting from the simulation results, set up a rigorous game-theoretical framework to derive **THEORETICAL INSIGHTS**.
- ▶ Add to the strategic play techniques the case of **CONTENT AGGREGATORS** (users that mainly re-post others' content)
- ▶ Perform a similar analysis when individuals are partitioned into different (**ADVANTAGED/DISADVANTAGED**) groups.



University of  
Zurich<sup>UZH</sup>

NICOLÒ PAGAN

UNIVERSITY OF ZÜRICH - SOCIAL COMPUTING GROUP

[nicolo.pagan@uzh.ch](mailto:nicolo.pagan@uzh.ch)

<https://nicolo-pagan.github.io/>

