Learning strategic behavior in social and economic networks

Nicolò Pagan [1]   Florian Dörfler [1]

[1] Automatic Control Laboratory, ETH Zürich, Switzerland

Complex Networks 2018

7^{th} International Conference on Complex Networks and Their Applications

December, 11-13 2018, Cambridge, UK

Motivation

Observations

- Actors decide with whom they want to interact.

Motivation

Observations

- Actors decide with whom they want to interact.
- Network positions provide benefits to the actors.

Forbes

3,853 views | Sep 11, 2017, 10:09am

Using Social Networks To Advance Your Career

Adi Gaskell Contributor

Shutterstock

"It's not what you know, it's who you know" is one of those phrases

Motivation

Observations

- Actors decide with whom they want to interact.
- Network positions provide benefits to the actors.

Motivation

Observations

- Actors decide with whom they want to interact.
- Network positions provide benefits to the actors.

ETH zürich

Motivation

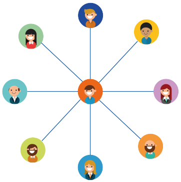
Observations

- Actors decide with whom they want to interact.
- Network positions provide benefits to the actors.

Social Network positions' benefits

Social Network positions' benefits

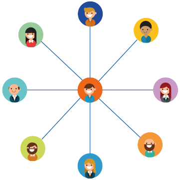
Social Influence



The more people we are connected to, the more we can influence them.

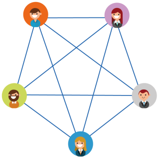
Social Network positions' benefits

Social Influence



The more people we are connected to, the more we can influence them.

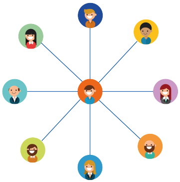
Social Support



The more our friends' friends are our friends, the safer we feel.
[Heider (1946); Coleman (1990)]
(Structural Balance theory)

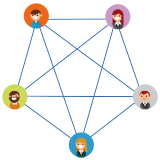
Social Network positions' benefits

Social Influence



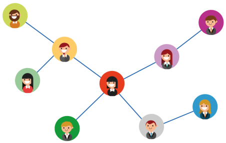
The more people we are connected to, the more we can influence them.

Social Support



The more our friends' friends are our friends, the safer we feel.
[Heider (1946); Coleman (1990)]
(Structural Balance theory)

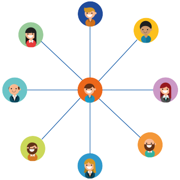
Brokerage



The more we are on the path between people, the more we can control.
[Burt (1992)]
(Structural Holes theory)

Social Network positions' benefits

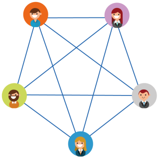
Social Influence



The more people we are connected to, the more we can influence them.

Degree Centrality

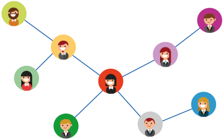
Social Support



The more our friends' friends are our friends, the safer we feel.
[Heider (1946); Coleman (1990)]
(Structural Balance theory)

Clustering Coefficient

Brokerage



The more we are on the path between people, the more we can control.
[Burt (1992)]
(Structural Holes theory)

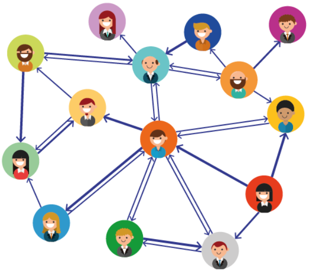
Betweenness Centrality

ETH zürich

Social Network Formation Model

Ingredients

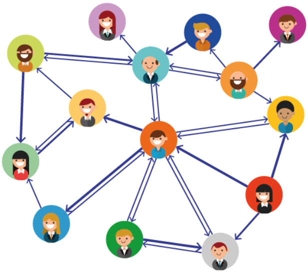
- **Directed weighted** network $\mathcal{G}$ with $\mathcal{N} = \{1, \ldots, N\}$ agents.



Social Network Formation Model

Ingredients

- **Directed weighted** network $\mathcal{G}$ with $\mathcal{N} = \{1, \dots, N\}$ agents.
- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among i and j **from i's point of view**.

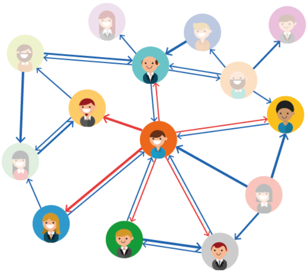


Social Network Formation Model

Ingredients

- **Directed weighted** network $\mathcal{G}$ with $\mathcal{N} = \{1, \dots, N\}$ agents.
- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among i and j **from i's point of view**.
- A typical action of agent i is:

$$a_i = [a_{i1}, \dots, a_{i,i-1}, a_{i,i+1}, \dots, a_{iN}] \in \mathcal{A} = [0, 1]^{N-1},$$



ETH zürich

Social Network Formation Model

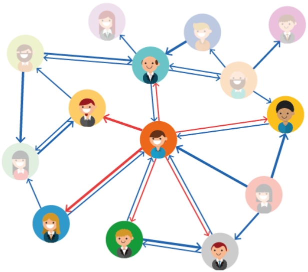
Ingredients

- **Directed weighted** network $\mathcal{G}$ with $\mathcal{N} = \{1, \ldots, N\}$ agents.
- $a_{ij} \in [0, 1]$ quantifies the importance of the friendship among i and j **from i's point of view**.
- A typical action of agent i is:

$$a_i = [a_{i1}, \ldots, a_{i,i-1}, a_{i,i+1}, \ldots, a_{iN}] \in \mathcal{A} = [0, 1]^{N-1},$$

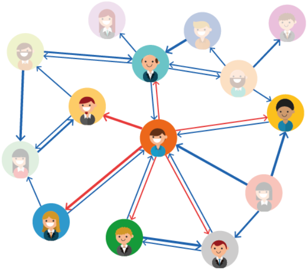
- **Rational** agents: every agent i is endowed with a payoff function V_i and is looking for

$$a_i^\star \in \arg\max_{a_i \in \mathcal{A}} V_i(a_i, \mathbf{a_{-i}})$$



Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = t_i(a_i, \mathbf{a_{-i}}) + u_i(a_i, \mathbf{a_{-i}}) - c_i(a_i),$$

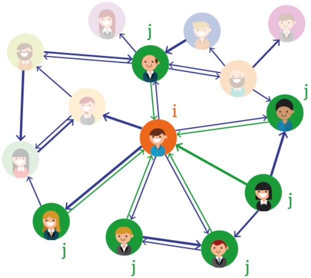


Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = t_i(a_i, \mathbf{a_{-i}}) + u_i(a_i, \mathbf{a_{-i}}) - c_i(a_i),$$

- **Social influence** on friends,

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji}$$



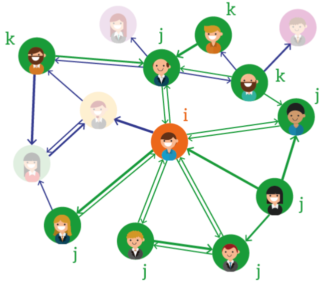
ETH zürich

Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = t_i(a_i, \mathbf{a_{-i}}) + u_i(a_i, \mathbf{a_{-i}}) - c_i(a_i),$$

- **Social influence** on friends, on friends of friends, with $\delta_i \in [0,1]$:

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}}$$



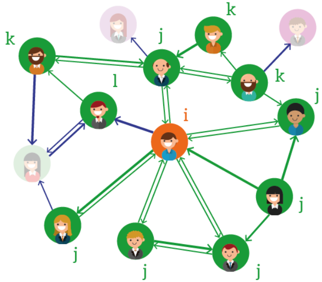
Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = t_i(a_i, \mathbf{a_{-i}}) + u_i(a_i, \mathbf{a_{-i}}) - c_i(a_i),$$

- **Social influence** on friends, on friends of friends, ... with $\delta_i \in [0,1]$:

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}},$$

[Jackson and Wolinsky (1996)]



ETH zürich

Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = t_i(a_i, \mathbf{a_{-i}}) + u_i(a_i, \mathbf{a_{-i}}) - c_i(a_i),$$

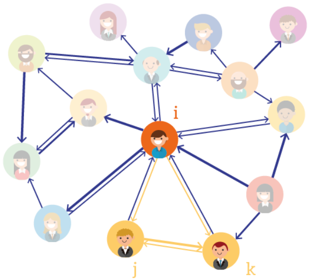
- **Social influence** on friends, on friends of friends, ... with $\delta_i \in [0,1]$:

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}},$$

[Jackson and Wolinsky (1996)]

- **Clustering coefficient**: number of closed triads: $u_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right)$,

[Burger and Buskens (2009)]



ETH zürich

Payoff function

$$V_i(a_i, \mathbf{a_{-i}}) = t_i(a_i, \mathbf{a_{-i}}) + u_i(a_i, \mathbf{a_{-i}}) - c_i(a_i),$$

- **Social influence** on friends, on friends of friends, ... with $\delta_i \in [0, 1]$:

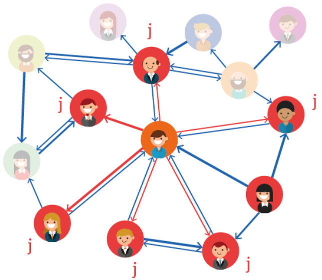
$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}},$$

[Jackson and Wolinsky (1996)]

- **Clustering coefficient**: number of closed triads: $u_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right)$,

[Burger and Buskens (2009)]

- **Cost** of maintaining ties: $c_i(a_i) = \sum_{j \neq i} a_{ij}$.



ETH zürich

Payoff function

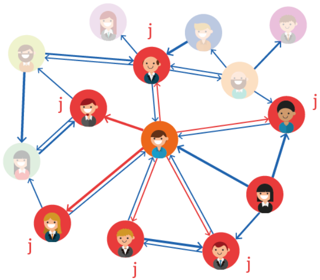
$$V_i(a_i, \mathbf{a_{-i}} | P_i) = \alpha_i t_i(a_i, \mathbf{a_{-i}}) + \beta_i u_i(a_i, \mathbf{a_{-i}}) - \gamma_i c_i(a_i), \qquad \alpha_i \geq 0, \beta_i \in \mathbb{R}, \gamma_i > 0$$

$$P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$$

- **Social influence** on friends, on friends of friends, ... with $\delta_i \in [0,1]$:

$$t_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ji} + \underbrace{\delta_i \sum_{k \neq j} \sum_{j \neq i} a_{kj} a_{ji}}_{\text{paths of length 2}} + \underbrace{\delta_i^2 \sum_{l \neq k} \sum_{k \neq j} \sum_{j \neq i} a_{lk} a_{kj} a_{ji}}_{\text{paths of length 3}},$$

  [Jackson and Wolinsky (1996)]



- **Clustering coefficient**: number of closed triads: $u_i(a_i, \mathbf{a_{-i}}) = \sum_{j \neq i} a_{ij} \left(\sum_{k \neq i,j} a_{ik} a_{kj} \right)$,

  [Burger and Buskens (2009)]

- **Cost** of maintaining ties: $c_i(a_i) = \sum_{j \neq i} a_{ij}$.

ETH zürich

Nash Equilibrium

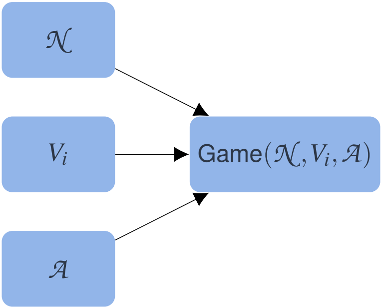
Definition (Nash equilibrium).
The network $\mathcal{G}^{\star}$ is a NE if for all agents i,

$$V_i\left(a_i, \mathbf{a_{-i}}^{\star} | P_i\right) \leq V_i\left(a_i^{\star}, \mathbf{a_{-i}}^{\star} | P_i\right), \forall a_i \in \mathcal{A}.$$

Nash Equilibrium

Definition (Nash equilibrium).
The network $\mathcal{G}^\star$ is a NE if for all agents i,

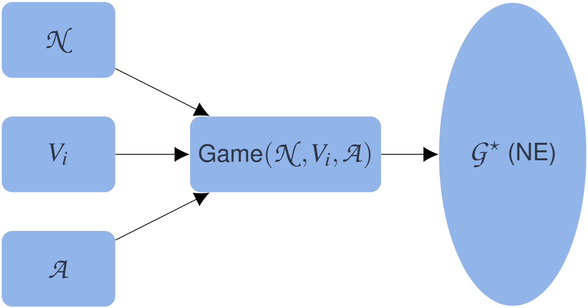
$$V_i\left(a_i, \mathbf{a_{-i}}^\star | P_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star | P_i\right), \forall a_i \in \mathcal{A}.$$



Nash Equilibrium

Definition (Nash equilibrium).
The network $\mathcal{G}^\star$ is a NE if for all agents i,

$$V_i\left(a_i, \mathbf{a_{-i}}^\star | P_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star | P_i\right), \forall a_i \in \mathcal{A}.$$

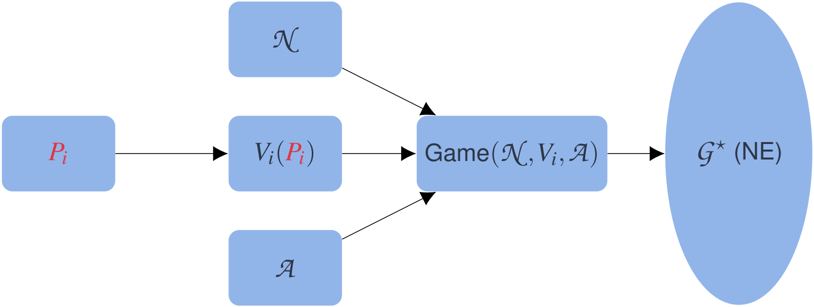


ETH zürich

Nash Equilibrium

Definition (Nash equilibrium).
The network $\mathcal{G}^\star$ is a NE if for all agents i,

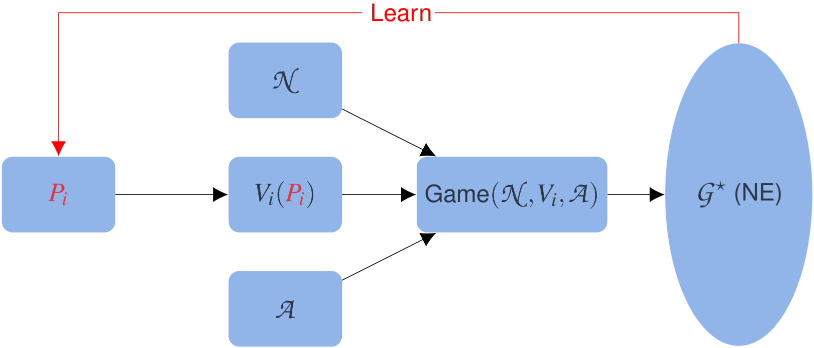
$$V_i\left(a_i, \mathbf{a_{-i}}^\star | P_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star | P_i\right), \forall a_i \in \mathcal{A}.$$



Nash Equilibrium

Definition (Nash equilibrium).
The network $\mathcal{G}^\star$ is a NE if for all agents i,

$$V_i\left(a_i, \mathbf{a_{-i}}^\star | P_i\right) \leq V_i\left(a_i^\star, \mathbf{a_{-i}}^\star | P_i\right), \forall a_i \in \mathcal{A}.$$



Question: For which individual parameters P_i is $\mathcal{G}^\star$ in equilibrium?

Homogeneous agents

Assumption

Individual preferences $P_i = P$, for all agents i.

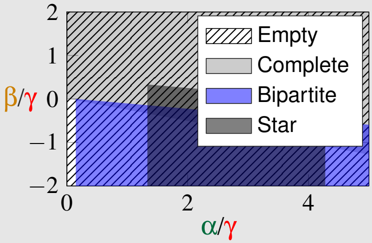
Homogeneous agents

Assumption

Individual preferences $P_i = P$, for all agents i.

Result

Analytic characterization of individual behavior, based on some specific network motifs.



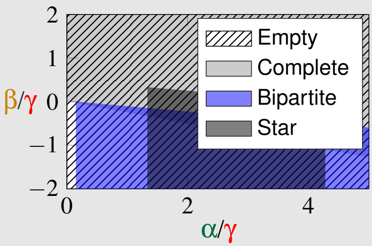
Homogeneous agents

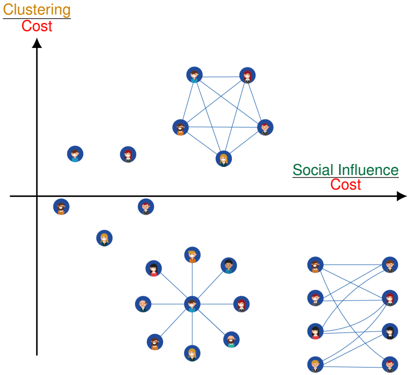
Assumption

Individual preferences $P_i = P$, for all agents i.

Result

Analytic characterization of individual behavior, based on some specific network motifs.





Bala and Goyal (2000), Buechel and Buskens (2013), Buechel and Buskens (2013), Buskens and van de Rijt (2008).

Heterogeneous Agents

Let the unkown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$.
Define:

$$\Phi_i(a_i, P_i) = V_i\left(a_i, a^\star_{-i} | P_i\right) - V_i\left(a^\star_i, a^\star_{-i} | P_i\right).$$

ETH zürich

Heterogeneous Agents

Let the unkown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$.
Define:

$$\Phi_i(a_i, P_i) = V_i\left(a_i, a^\star_{-i} | P_i\right) - V_i\left(a^\star_i, a^\star_{-i} | P_i\right).$$

Note: $\Phi_i(a_i, P_i) > 0$ if P_i violates the Nash equilibrium condition.

Heterogeneous Agents

Let the unkown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$.
Define:

$$\Phi_i(a_i, P_i) = V_i\left(a_i, a^\star_{-i} | P_i\right) - V_i\left(a^\star_i, a^\star_{-i} | P_i\right).$$

Note: $\Phi_i(a_i, P_i) > 0$ if P_i violates the Nash equilibrium condition.

Integrate over the all action space, taking into account only the violations.

$$\Psi_i(P_i) = \int_{\mathcal{A}} \max\{0, \Phi_i(a_i, P_i)\} \, \mathrm{d}a_i.$$

ETH zürich

Heterogeneous Agents

Let the unkown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$.
Define:

$$\Phi_i(a_i, P_i) = V_i\left(a_i, a^\star_{-i} | P_i\right) - V_i\left(a^\star_i, a^\star_{-i} | P_i\right).$$

Note: $\Phi_i(a_i, P_i) > 0$ if P_i violates the Nash equilibrium condition.

Integrate over the all action space, taking into account only the violations.

$$\Psi_i(P_i) = \int_{\mathcal{A}} \max\left\{0, \Phi_i(a_i, P_i)\right\} \mathrm{d}a_i.$$

Minimize the violations

$$\hat{P}_i \in \arg\min_{P_i \in \mathcal{P}} \left\{\Psi_i(P_i)\right\}$$

ETH zürich

Heterogeneous Agents

Let the unkown heterogeneous individual preferences set $P_i = \{\alpha_i, \beta_i, \gamma_i, \delta_i\}$.
Define:

$$\Phi_i(a_i, P_i) = V_i\left(a_i, a^\star_{-i} | P_i\right) - V_i\left(a^\star_i, a^\star_{-i} | P_i\right).$$

Note: $\Phi_i(a_i, P_i) > 0$ if P_i violates the Nash equilibrium condition.

Integrate over the all action space, taking into account only the violations.

$$\Psi_i(P_i) = \int_{\mathcal{A}} \max\left\{0, \Phi_i(a_i, P_i)\right\} \mathrm{d}a_i.$$

Minimize the violations

$$\hat{P}_i \in \arg\min_{P_i \in \mathcal{P}} \left\{\Psi_i(P_i)\right\} \iff \hat{P}_i \in \arg\max_{P_i \in \mathcal{P}} \left\{L_i(P_i)\right\},$$

where $L_i(P_i) = -\Psi_i(P_i)$ is the likelihood function.

Behavioral analysis on real-world networks

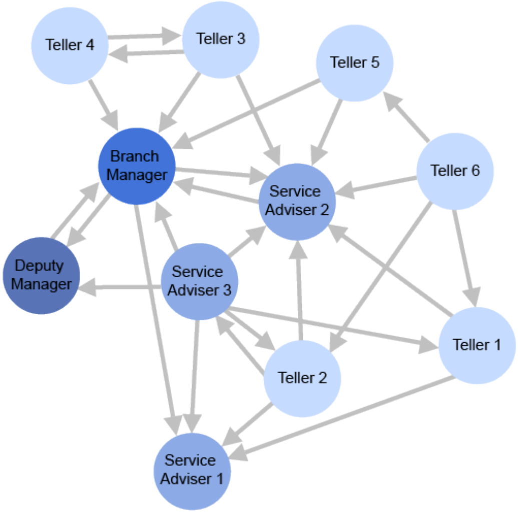
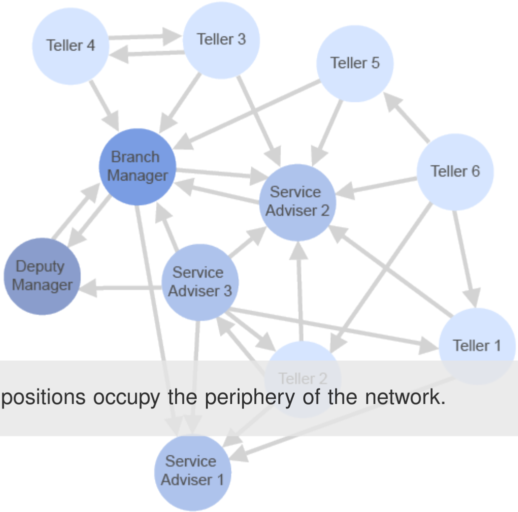


Figure: Australian bank data set, Pattison et al. (2000).

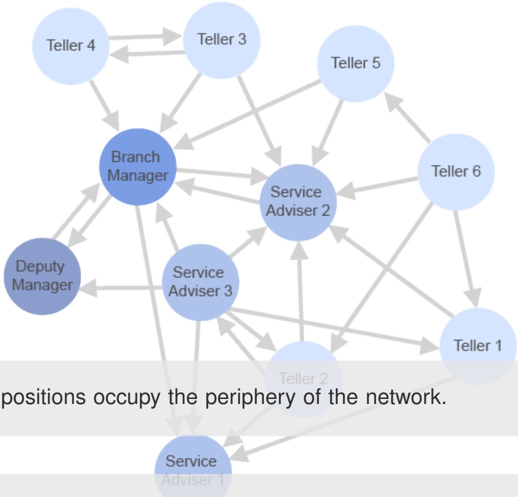
ETH zürich

Behavioral analysis on real-world networks



Low hierarchical positions occupy the periphery of the network.

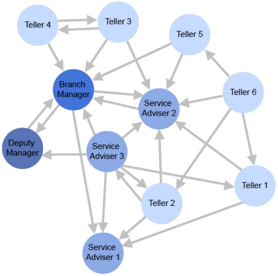
Behavioral analysis on real-world networks



Low hierarchical positions occupy the periphery of the network.

Presence of star-like motifs embedded in the network (e.g. Branch Manager).

Behavioral analysis on real-world networks



- Competitive behavior and reciprocity of high-ranking positions.
- Low-ranking positions inclined towards social support.

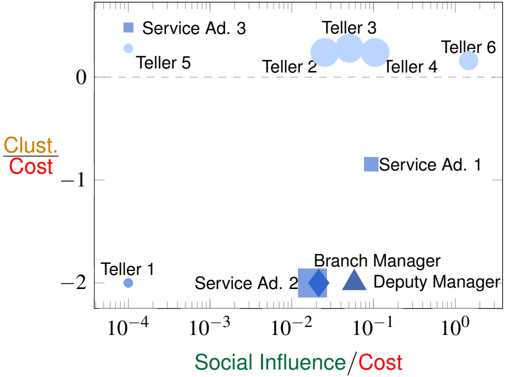
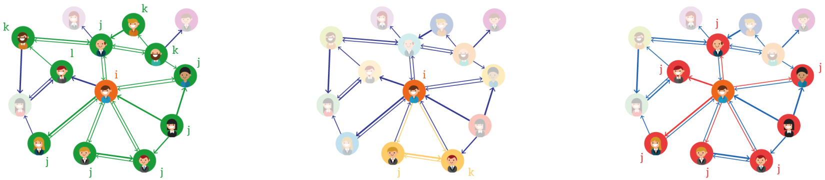


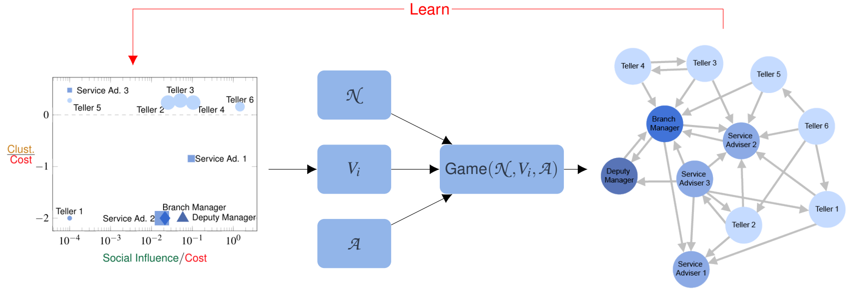
Figure: Maximum likelihood estimate of strategic behavior $\hat{P}_i$.

$$V_i(a_i, \mathbf{a}_{-\mathbf{i}} | P_i) = \alpha_i t_i(a_i, \mathbf{a}_{-\mathbf{i}}) + \beta_i u_i(a_i, \mathbf{a}_{-\mathbf{i}}) - \gamma_i c_i(a_i), \qquad \alpha_i \geq 0, \beta_i \in \mathbb{R}, \gamma_i > 0.$$

Summary



$$V_i(a_i, \mathbf{a_{-i}}|P_i) = \alpha_i t_i(a_i, \mathbf{a_{-i}}) + \beta_i u_i(a_i, \mathbf{a_{-i}}) - \gamma_i c_i(a_i), \qquad \alpha_i \geq 0,\ \beta_i \in \mathbb{R},\ \gamma_i > 0.$$



pagann@ethz.ch, people.ee.ethz.ch/∼pagann/

ETH zürich

Bala, V. and Goyal, S. (2000). A Noncooperative Model of Network Formation. *Econometrica*, 68(5):1181–1229.

Buechel, B. and Buskens, V. (2013). The Dynamics of Closeness and Betweenness. *The Journal of Mathematical Sociology*, 37(July):159–191.

Burger, M. J. and Buskens, V. (2009). Social context and network formation: An experimental study. *Social Networks*, 31(1):63–75.

Burt, R. S. (1992). Structural hole. *Harvard Business School Press, Cambridge, MA*.

Buskens, V. and van de Rijt, A. (2008). Dynamics of Networks if Everyone Strives for Structural Holes. *American Journal of Sociology*, 114(2):371–407.

Coleman, J. (1990). Foundations of social theory. *Cambridge, MA: Belknap*.

Heider, F. (1946). Attitudes and cognitive organization. *The Journal of psychology*, 21(1):107–112.

Jackson, M. O. and Wolinsky, A. (1996). A Strategic Model of Social and Economic Networks. *Journal of Economic Theory*, 71(1):44–74.

Pattison, P., Wasserman, S., Robins, G., and Kanfer, A. M. (2000). Statistical evaluation of algebraic constraints for social networks. *Journal of mathematical psychology*, 44(4):536–568.