Emergence of scaling on the followers of social media influencers

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Abstract

In recent years, on-line social networks such as Twitter or Instagram have seen the emergence of social media influencers, i.e., people who have built a sizeable set of followers. Thanks to their network position, social media influencers can virally spread information and opinions to their audience, having a huge impact on, e.g., social media marketing and information spreading. Thus, it became important to understand (i) how such nodes emerge and (ii) what are the properties of the resulting network structure. For instance, how social influencers obtain large audience (high indegree) while maintaining ordinary outdegree? Motivated by the above considerations, we develop a strategic network formation model to study the underlying dynamics of individuals in networks, assuming they control their ties in order to maximise a certain well-defined payoff function. We consider a directed unweighted network of n actors and we assume that each actor controls her outgoing ties, i.e., the set of followees. Moreover, each actor i is endowed with an attribute $q_i \in [0,1]$ that denotes the quality of the content generated by i. More precisely, q_i is the expectation of Q_i, a Bernoulli random variable describing the probability of followers liking i's content. We then assume that actors aim at maximizing the quality of the content they receive from their followee. In other words, let a_{ij} denote the tie between i and j, then $V_i = \max_{j \neq i} (q_j * a_{ij})$ reflects the maximum quality received by agent i.

We consider a sequential game starting from the empty network, in which at each step two distinct actors i and j are uniformly randomly selected, and i can decide whether to follow j. We prove that sequential best response dynamic converges almost surely to a pure Nash equilibrium of the game. Even though there are multiple Nash equilibria, we are able to provide statistical analysis of the indegree and outdegree of the nodes at equilibrium. We show that the probability distribution of the outdegree is uniform across all the nodes. Moreover, we show that the expected outdegree of the nodes has logarithmic growth in the number of nodes n, thus it remains relatively low compared to the network size, as in typical real world social networks. Conversely, the indegree of node i follows a Poisson binomial distribution whose expectation decreases with the rank i of the node's quality q_i. More precisely, the expected indegree of the highest quality node is $N-1$ (every other node follows her), whereas the i^{th}-highest quality node only has N/i expected followers. This leads to a scaling property of the expected indegree of the nodes with respect to their ranking in quality, as shown in the figure for $n = 51$. Preliminary results on the experimental evidence of this property has been found when comparing the most famous Instagram influencers of certain categories, e.g., travel, fashion and food bloggers.