Lab 1

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February 2022

Exercise 1: Professional magic

1) What is the type I error of the test?

A type I error occurs when the null hypothesis is true but we reject it. Its associated probability is $\alpha = P(\text{reject } H_0 | H_0 \text{ is true})$.

In this case, we assume that $H_0 : p = \frac{1}{2}$ and we set the rejection region when the test statistic $S = X_1 + Y_1 + X_2 + Y_2 + X_3 + Y_3$ is either 0 or 6. In terms of type I error, this means calculating the probability that S takes the values 0 or 6, assuming that $p = 1/2$, or $P(S = 0 \cup S = 6 | p = \frac{1}{2})$.

We first need to notice that $S = 0$ or $S = 6$ will happen only when all X_i and Y_i are the same. All variables need to take the value 0 or all the variables need to be 1. From the joint distribution, we can see that $P(X_i = Y_i = 0) = P(X_i = Y_i = 1) = p/2$. The probability of either of these things for happening is just the sum of both, so $P(X_i = Y_i) = p$.

So, $P(S = 0 \cup S = 6)$ can be written as:

$$P(X_1 = Y_1 \cap X_2 = Y_2 \cap X_3 = Y_3)$$

Because we know that each pair is independent of each other:

$$P(X_1 = Y_1) \cdot P(X_2 = Y_2) \cdot P(X_3 = Y_3) = p \cdot p \cdot p = p^3$$

Finally, to calculate α we need to assume that H_0 is true:

$$\boxed{\alpha = P(S = 0 \cup S = 6 | p = 1/2) = (1/2)^3 = 1/8 = 12.5\%}$$

2) What is the power of the test for $H_a : p = 3/4$?

Power means supporting H_a assuming H_a is true, its associated probability is $1 - \beta = P(\text{support } H_a | H_a \text{ is true})$, where β is the probability of a type II error.

This means the probability of our statistic falling in the rejection region assuming that $p = 3/4$, which expressed in terms of probability is the same as saying $(1-\beta) = P(S = 0 \cup S = 6 | p = 3/4)$.

Because we already know that $P(S = 0 \cup S = 6) = p^3$, we only need to assume that $p = 3/4$ to get the desired probability:

$$\boxed{\text{Power} = (1 - \beta) = P(S = 0 \cup S = 6 | p = 3/4) = (3/4)^3 \approx 42.2\%}$$