Lab 1

Group 2: Ethan Duncan, Jeremy Lan, Nicolas Loffreda

February 2022

Exercise 1: Professional magic

1) What is the type I error of the test?

A type I error occurs when the null hypothesis is true but we reject it. Its associated probability is $\alpha = P(\text{reject } H_0 | H_0 \text{ is true})$.

In this case, we assume that $H_0 : p = \frac{1}{2}$ and we set the rejection region when the test statistic $S = X_1 + Y_1 + X_2 + Y_2 + X_3 + Y_3$ is either 0 or 6. In terms of type I error, this means calculating the probability that S takes the values 0 or 6, assuming that $p = 1/2$, or $P(S = 0 \cup S = 6 | p = \frac{1}{2})$.

We first need to notice that $S = 0$ or $S = 6$ will happen only when all X_i and Y_i are the same. All variables need to take the value 0 or all the variables need to be 1. From the joint distribution, we can see that $P(X_i = Y_i = 0) = P(X_i = Y_i = 1) = p/2$. The probability of either of these things for happening is just the sum of both, so $P(X_i = Y_i) = p$.https://www.overleaf.com/project/621183daabca0fb2ff2eee82

So, $P(S = 0 \cup S = 6)$ can be written as:

$$P(X_1 = Y_1 \cap X_2 = Y_2 \cap X_3 = Y_3)$$

Because we know that each pair is independent of each other:

$$P(X_1 = Y_1) \cdot P(X_2 = Y_2) \cdot P(X_3 = Y_3) = p \cdot p \cdot p = p^3$$

Finally, to calculate α we need to assume that H_0 is true:

$$\boxed{\alpha = P(S = 0 \cup S = 6 | p = 1/2) = (1/2)^3 = 1/8 = 12.5\%}$$

2) What is the power of the test for $H_a : p = 3/4$?

Power means supporting H_a assuming H_a is true, its associated probability is $1 - \beta = P(\text{support } H_a | H_a \text{ is true})$, where β is the probability of a type II error.

This means the probability of our statistic falling in the rejection region assuming that $p = 3/4$, which expressed in terms of probability is the same as saying $(1-\beta) = P(S = 0 \cup S = 6 | p = 3/4)$.

Because we already know that $P(S = 0 \cup S = 6) = p^3$, we only need to assume that $p = 3/4$ to get the desired probability:

$$\boxed{\text{Power} = (1 - \beta) = P(S = 0 \cup S = 6 | p = 3/4) = (3/4)^3 \approx 42.2\%}$$

Exercise 2: Wrong Test, Right Data

1) What are the consequences of violating the metric scale assumption, if a paired t-test was used on this Likert data?

To accurately perform a paired t-test test on this data, one would need to assume that the response levels of the ordinal Likert variables are equally spaced apart. However, it's difficult to justify that the difference between "strongly agree" and "agree" is the same as the difference between "agree" and "neutral", and so on.

We also might struggle to draw any practical significance from the results. **A paired t-test is ultimately testing a difference in means between 2 observations on one set of individuals, but means do not make sense on Likert data.**

2) What would you propose to remedy this problem?

When considering a dependent test on this ordinal Likert data, a **sign test** may be a good option. The sign test does not require metric structure, but only records if the change in each paired case was positive or negative. The downside with the sign test is that we lose statistical power as we discard the magnitude of the changes in the paired data - a large sample size may be required to detect a statistically significant effect.

Other non-parametric dependent tests like the Wilcoxon signed-rank test may be more powerful, but still would require the same metric-scale assumptions that we are trying to avoid.