Lab_1 Problem 1.3: Test Assumptions

QUESTION: For the four following questions, your task is to evaluate the assumptions for the given test. It is not enough to say that an assumption is met or not met; instead, present your evidence in the form of background knowledge, visualizations, and numerical summaries. If you produce a histogram as part of your evaluation, be sure to consider what the most appropriate bin width is. The test that we ask you to evaluate may or may not be the most appropriate test for the scenario. Because the goal of this task is to evaluate whether the data satisfies the assumptions necessary for the test to provide meaningful results, you do not need perform the test (you may perform the test, but we will not be marking for the test results).

1.3.1 World Happiness

QUESTION: The file datasets/Happiness_WHR.csv is subsetted from the World Happiness Report, a yearly publication that uses data from the Gallup World Poll surveys. The variable life ladder is a measure of happiness, described in the FAQ as follows: This is called the Cantril ladder: it asks respondents to think of a ladder, with the best possible life for them being a 10, and the worst possible life being a 0. They are then asked to rate their own current lives on that 0 to 10 scale. The rankings are from nationally representative samples,for the years 2018-2020. You would like to know whether people in countries with high GDP per capita (higher than the mean) are more happy or less happy than people in countries with low GDP (lower than the mean).

Solution

Test Assumptions for a Two Sample T-Test

1. Metric scale (In particular, the t-test is not valid for variables which only have an ordinal structure)
2. IID data
3. No major deviations from normality, considering the sample size (In particular, the t-test is invalid for highly skewed distributions when sample size is larger than 30. It may also be invalid for very highly skewed distributions at higher sample sizes)

Metric Scale

Upon evaluating the dataset happinees_WHR.csv, although the evaluation was on a scale of 1-10, the Cantril Ladder is an example of a Likert Scale meaning that, in this problem, we have a non-metric scale. In other words the values used in this dataset rely upon opinion (qualitative measurement) rather than a quantitative measurement. Therefore a two sample t-test fails on this assumption. View the query below to get an understanding of the happinees_WHR.csv dataset.

```r
library(readr)
happiness_WHR <- read_csv("happiness_WHR.csv")
```

```
## Rows: 239 Columns: 11

## ── Column specification ────────────────────────────────────────────────────────
## Delimiter: ","
## chr  (1): Country name
## dbl (10): year, Life Ladder, Log GDP per capita, Social support, Healthy lif...

##
## ℹ Use `spec()` to retrieve the full column specification for this data.
## ℹ Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```r
summary(happiness_WHR)
```

```
##  Country name            year       Life Ladder    Log GDP per capita
##  Length:239         Min.   :2019   Min.   :2.375   Min.   : 6.966    
##  Class :character   1st Qu.:2019   1st Qu.:4.971   1st Qu.: 8.827    
##  Mode  :character   Median :2019   Median :5.768   Median : 9.669    
##                     Mean   :2019   Mean   :5.678   Mean   : 9.584    
##                     3rd Qu.:2020   3rd Qu.:6.428   3rd Qu.:10.527    
##                     Max.   :2020   Max.   :7.889   Max.   :11.648    
##                                                    NA's   :13        
##  Social support   Healthy life expectancy at birth Freedom to make life choices
##  Min.   :0.4200   Min.   :48.70                    Min.   :0.3850              
##  1st Qu.:0.7590   1st Qu.:62.00                    1st Qu.:0.7360              
##  Median :0.8480   Median :67.20                    Median :0.8220              
##  Mean   :0.8256   Mean   :65.84                    Mean   :0.8038              
##  3rd Qu.:0.9175   3rd Qu.:70.45                    3rd Qu.:0.8910              
##  Max.   :0.9830   Max.   :77.10                    Max.   :0.9700              
##                   NA's   :8                        NA's   :2                   
##    Generosity       Perceptions of corruption Positive affect  Negative affect
##  Min.   :-0.28900   Min.   :0.0700            Min.   :0.3220   Min.   :0.0830  
##  1st Qu.:-0.11900   1st Qu.:0.6470            1st Qu.:0.6450   1st Qu.:0.2250  
##  Median :-0.04500   Median :0.7780            Median :0.7330   Median :0.2830  
##  Mean   :-0.01524   Mean   :0.7168            Mean   :0.7149   Mean   :0.2891  
##  3rd Qu.: 0.07400   3rd Qu.:0.8480            3rd Qu.:0.7900   3rd Qu.:0.3440  
##  Max.   : 0.56100   Max.   :0.9630            Max.   :0.8910   Max.   :0.5320  
##  NA's   :14         NA's   :14                NA's   :2        NA's   :2
```

IID Data

Based on background knowledge of this study, each country is fairly independent of one another and therefore appears to fulfill this test assumption. Going one step further, there are no apparent violations of independence, some examples being clustering of data, in geographical regions, school cohorts, or families, strategical interaction, like competition among sellers or imitation of a species, or autocorrelation were one time period may affect the next. The data also appears to be identically distributed as will be shown in the plots below. This assumption for the data is meet, but as stated previously, a t-test would not be a wise hypothesis test to use for a non-metric statistical analysis.

No Major Deviations from Normality

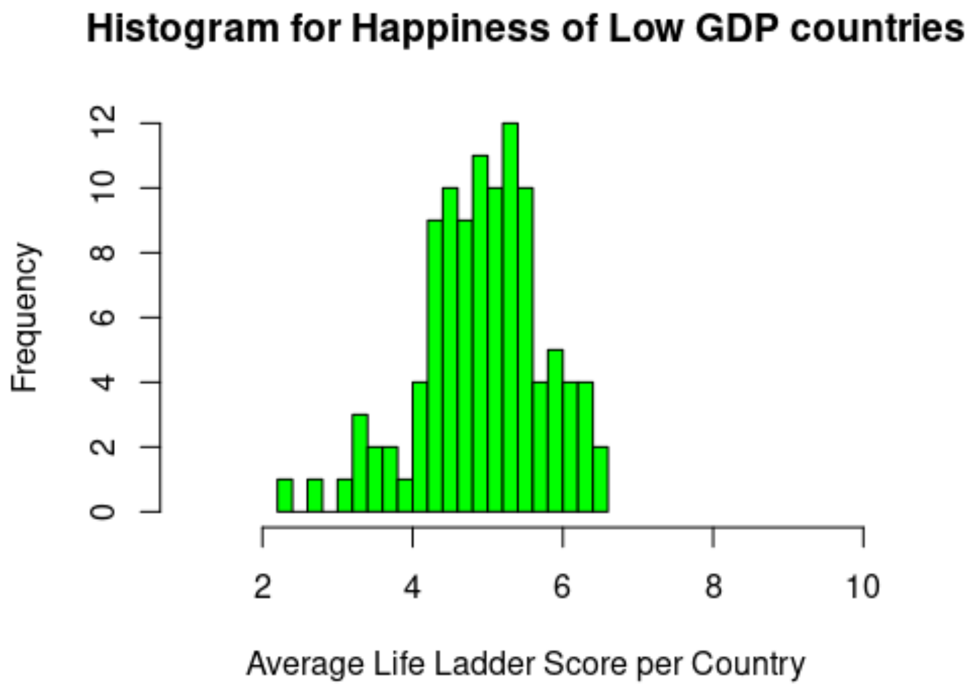
For further clarification, this assumption would not be met if the distribution was highly skewed for distributions when the sample size is larger than 30. In this problem, our data is slightly left skewed, although it does not meet the criteria for highly skewed data and therefore this assumption is meet. Please see the histograms below.

The steps taken here to create the histograms were to first separate the data based off of high and low GDP as done by splitting across the mean GDP of the given 239 countries.

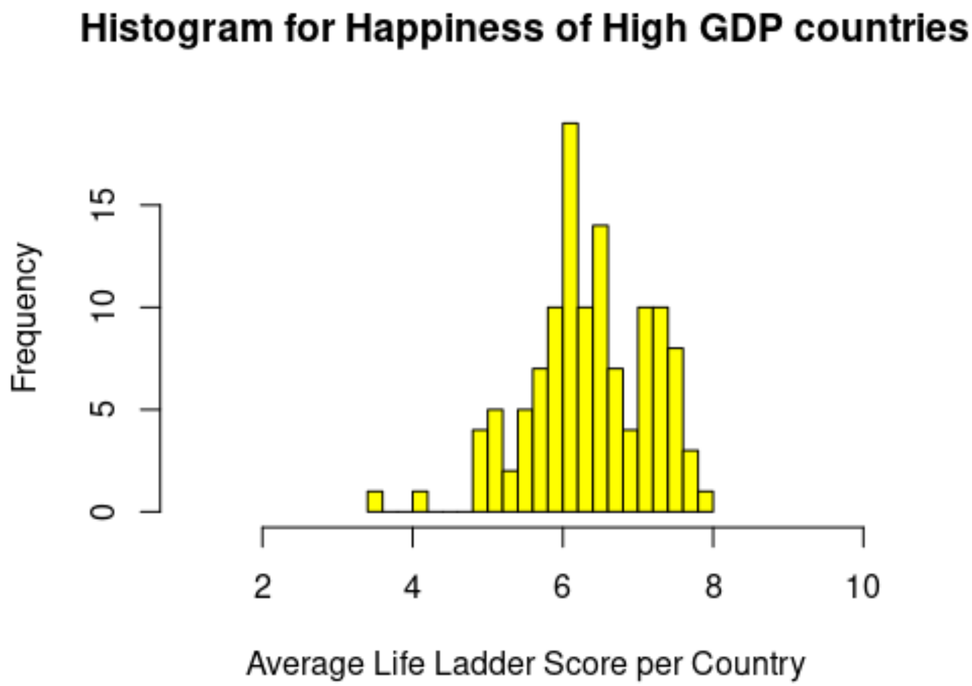
```
m <- mean(happiness_WHR$`Log GDP per capita`, na.rm = TRUE)
h_GDP <- subset(happiness_WHR, subset = happiness_WHR$`Log GDP per capita` >
m)
l_GDP <- subset(happiness_WHR, subset = happiness_WHR$`Log GDP per capita` <
m)
```

Next, two histograms were made and show the left skewness of the distribution for high and low GDP countries.

```
hist(l_GDP$`Life Ladder`,
     main="Histogram for Happiness of Low GDP countries",
     xlab="Average Life Ladder Score per Country",
     col="green",
     xlim=c(1,10),
     breaks=15)
```




```r
hist(h_GDP$`Life Ladder`,
     main="Histogram for Happiness of High GDP countries",
     xlab="Average Life Ladder Score per Country",
     col="yellow",
     xlim=c(1,10),
     breaks=20)
```



Lastly, a simple unpaired t-test was run to see what the results would have been and given the p-value for this example, it would have been likely that the null hypothesis would have been rejected.

```
t.test(h_GDP$`Life Ladder`, l_GDP$`Life Ladder`)

##
##  Welch Two Sample t-test
##
## data:  h_GDP$`Life Ladder` and l_GDP$`Life Ladder`
## t = 13.251, df = 218.47, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  1.218086 1.643727
## sample estimates:
## mean of x mean of y
##  6.355488  4.924581
```

In conclusion the Two Sample T-Test would not be the appropriate hypothesis test based on the failed Metric Scale assumption. The other two assumptions are met.

1.3.2: Legislators

QUESTION: The file datasets/legislators-current.csv is taken from the congress-legislators project on Github. You would like to test whether Democratic or Republican senators are older. List all assumptions for a Wilcoxon rank-sum test (using the Hypothesis of Comparisons). Then evaluate each assumption, presenting evidence based on your background knowledge, visualizations, and numerical summaries.

Solution

Test Assumptions for a Wilcoxon Rank-Sum Test (Hypothesis of Comparisons)

1. Ordinal Scale
2. IID Data

Null Hypothesis: The probability that a draw from X ranks higher than a draw from Y is the same as the probability that a draw from Y ranks higher than a draw from X.

$$P(X > Y) = P(X < Y)$$

Ordinal Scale

Based on background knowledge, ordinal scale measures data of categorical nature where ordered categories and the distances between the categories are not known. Since in this problem we are measuring age given by the birthday of each congressman, and age would be considered a metric variable, this test would fail on this assumption and not be a viable option for statistical analysis. View a summary of the data below.

```
library(readr)
legislators_current <- read_csv("legislators-current.csv")

## Rows: 538 Columns: 34

## ── Column specification ────────────────────────────────────────────────────────
## Delimiter: ","
## chr  (26): last_name, first_name, middle_name, suffix, nickname,
full_name, ...
## dbl   (6): district, senate_class, cspan_id, govtrack_id, votesmart_id,
icps...
## lgl   (1): washington_post_id
## date  (1): birthday

##
## ℹ Use `spec()` to retrieve the full column specification for this data.
## ℹ Specify the column types or set `show_col_types = FALSE` to quiet this
message.

summary(legislators_current)

##   last_name          first_name         middle_name           suffix
##  Length:538         Length:538         Length:538         Length:538
##  Class :character   Class :character   Class :character   Class :character
##  Mode  :character   Mode  :character   Mode  :character   Mode  :character
```

```
##
##
##
##
##    nickname          full_name           birthday            gender
##  Length:538         Length:538         Min.   :1933-06-09   Length:538
##  Class :character   Class :character   1st Qu.:1953-03-30   Class
:character
##  Mode  :character   Mode  :character   Median :1961-03-05   Mode
:character
##                                        Mean   :1961-11-29
##                                        3rd Qu.:1970-08-14
##                                        Max.   :1995-08-01
##
##      type              state              district       senate_class
##  Length:538         Length:538         Min.   : 0.000   Min.   :1.00
##  Class :character   Class :character   1st Qu.: 3.000   1st Qu.:1.00
##  Mode  :character   Mode  :character   Median : 6.000   Median :2.00
##                                        Mean   : 9.984   Mean   :2.01
##                                        3rd Qu.:13.000   3rd Qu.:3.00
##                                        Max.   :53.000   Max.   :3.00
##                                        NA's   :100      NA's   :438
##     party               url              address             phone
##  Length:538         Length:538         Length:538         Length:538
##  Class :character   Class :character   Class :character   Class :character
##  Mode  :character   Mode  :character   Mode  :character   Mode  :character
##
##
##
##
##  contact_form         rss_url            twitter            facebook
##  Length:538         Length:538         Length:538         Length:538
##  Class :character   Class :character   Class :character   Class :character
##  Mode  :character   Mode  :character   Mode  :character   Mode  :character
##
##
##
##
##    youtube           youtube_id        bioguide_id         thomas_id
##  Length:538         Length:538         Length:538         Length:538
##  Class :character   Class :character   Class :character   Class :character
##  Mode  :character   Mode  :character   Mode  :character   Mode  :character
##
##
##
##
##  opensecrets_id        lis_id             fec_ids            cspan_id
##  Length:538         Length:538         Length:538         Min.   :    260
##  Class :character   Class :character   Class :character   1st Qu.:  45591
##  Mode  :character   Mode  :character   Mode  :character   Median :  79718
```

```
##                                                             Mean   : 543374
##                                                             3rd Qu.:1003305
##                                                             Max.   :9275683
##                                                             NA's   :156
##   govtrack_id      votesmart_id    ballotpedia_id     washington_post_id
##  Min.   :300018   Min.   :   119   Length:538         Mode:logical
##  1st Qu.:412199   1st Qu.: 22411   Class :character   NA's:538
##  Median :412570   Median : 52964   Mode  :character
##  Mean   :412042   Mean   : 75411
##  3rd Qu.:412772   3rd Qu.:133024
##  Max.   :456862   Max.   :188334
##                   NA's   :62
##    icpsr_id     wikipedia_id
##  Min.   :14066   Length:538
##  1st Qu.:21106   Class :character
##  Median :21564   Mode  :character
##  Mean   :24264
##  3rd Qu.:21972
##  Max.   :94659
##  NA's   :77
```

IID Data

For this assumption, each X_i has to be drawn from the same distribution, each Y_i has to be drawn from the same distribution, and all X_i and Y_i are mutually independent. There are no apparent violates of independents, some examples violations are clustering of data, in geographical regions, school cohorts, or families, strategical interaction, like competition among sellers or imitation of a species, or autocorrelation were one time period may affect the next. The data also appears to be identically distributed as will be shown in the plots below. This assumption for the data is meet, but as stated previously, the Hypothesis of Comparison version of the Wilcoxon Rank-Sum would not be a wise test to use for a metric statistical analysis.

For the analysis of this dataset, a simple extraction of the each congressman's birth year subtracted from the current year yields their age.

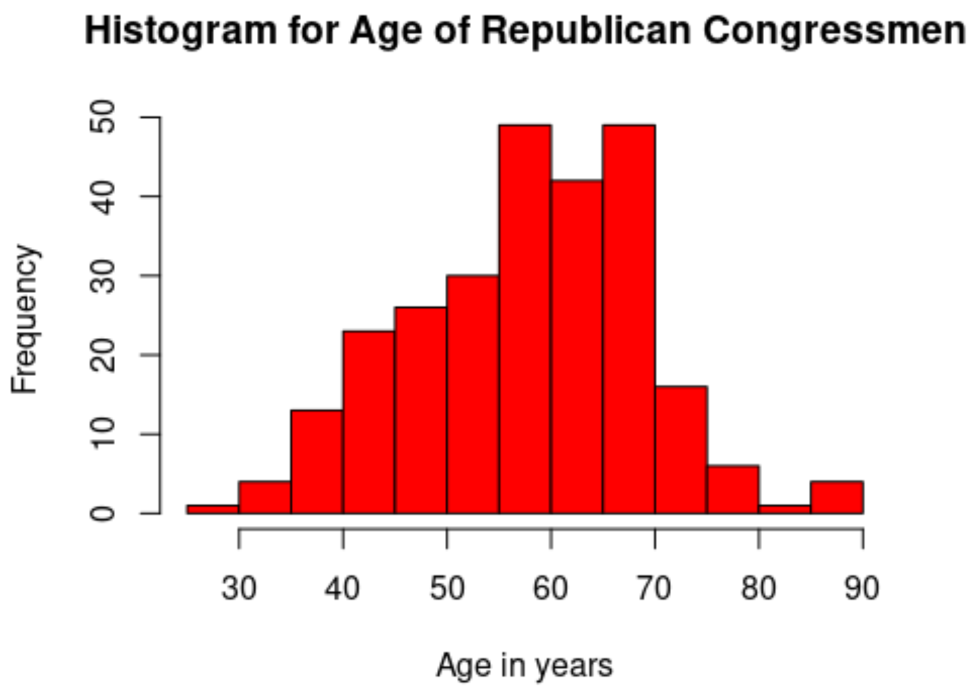
```
a <- as.POSIXct(legislators_current$birthday, format = "%Y-%m-%d")
year <- strtoi(format(a, format = "%Y"))
legislators_current$year = year
legislators_current$age <- 2021 - legislators_current$year
```

Next, subsetting the data by Republicans and Democrats in rep and dem respectively.

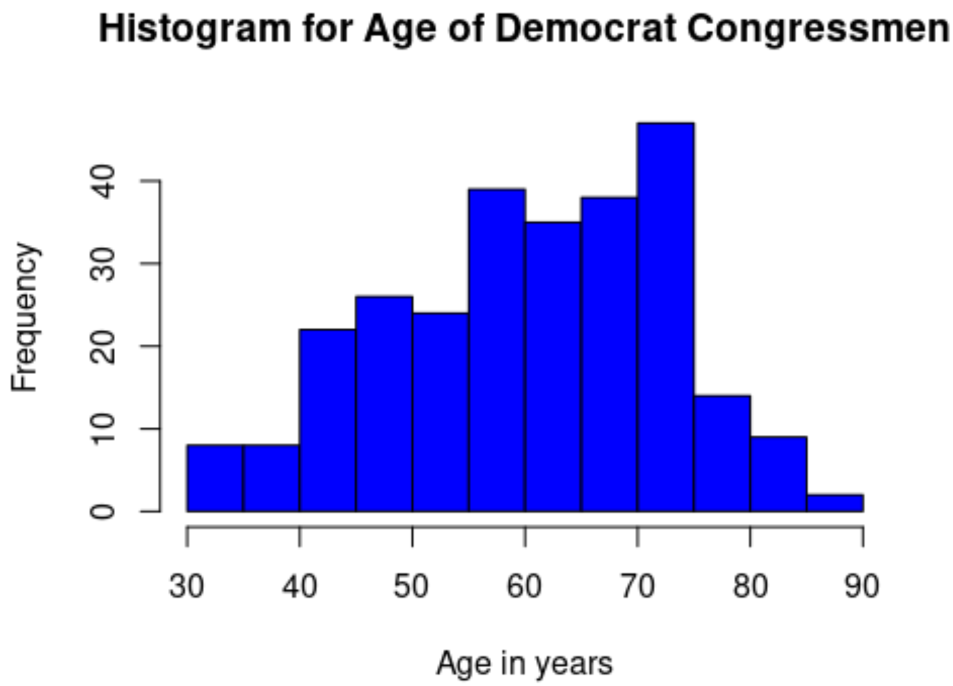
```
legislators_current$party[legislators_current$party == 'Independent'] = NA
rep <- subset(legislators_current, subset = legislators_current$party ==
"Republican")
dem <- subset(legislators_current, subset = legislators_current$party ==
"Democrat")
```

And finally, plotting the resulting of ages for Republicans and Democrats.

```r
hist(rep$age,
     main="Histogram for Age of Republican Congressmen",
     xlab="Age in years",
     col="red",
     breaks=15)
```




```r
hist(dem$age,
     main="Histogram for Age of Democrat Congressmen",
     xlab="Age in years",
     col="blue",
     breaks=15)
```



And Finally below, I have the Wilcoxon Rank-Sum Test. I'm not sure how to make sure that I'm evaluating the Hypothesis of Comparison version.

```
wilcox.test(rep$age, dem$age)

##
##  Wilcoxon rank sum test with continuity correction
##
## data:  rep$age and dem$age
## W = 31022, p-value = 0.006447
## alternative hypothesis: true location shift is not equal to 0
```

In conclusion, the Wilcoxon Rank-Sum Test (Hypothesis of Comparisons) would not be the appropriate hypothesis test based on the failed Ordinal Scale assumption. The other assumptions are met.

1.3.3 Wine and Health

QUESTION: The dataset wine can be accessed by installing the wooldridge package. install.packages("wooldridge") library(wooldridge) ?wine wine It contains observations of variables related to wine consumption for 21 countries. You would like to use this data to test whether countries have more deaths from heart disease or from liver disease. List all assumptions for a signed-rank test. Then evaluate each assumption, presenting evidence based on your background knowledge, visualizations, and numerical summaries.

Solution

Test Assumptions for Wilcoxon Signed-Rank Test

1. Metric Scale
2. IID Data
3. The distribution of the difference $(X - Y)$ is symmetric around the same mean.

Metric Sacle

For clairification, the X and Y measured here have to be measured on the same scale since we are using a paired test. For this question, both the liver and heart deaths are on the same scale of 100,000 deaths. Therefore this assumption is met. This will be shown in the dataset below.

```
install.packages("wooldridge")

## Installing package into '/usr/local/lib/R/site-library'
## (as 'lib' is unspecified)

library(wooldridge)
?wine
wine

##          country alcohol deaths heart liver
## 1      Australia     2.5    785   211  15.3
## 2        Austria     3.9    863   167  45.6
## 3       Belg/Lux     2.9    883   131  20.7
## 4         Canada     2.4    793   191  16.4
## 5        Denmark     2.9    971   220  23.9
## 6        Finland     0.8    970   297  19.0
## 7         France     9.1    751    71  37.9
## 8        Iceland     0.8    743   211  11.2
## 9        Ireland     0.7   1000   300   6.5
## 10        Israel     0.6    834   183  13.7
## 11         Italy     7.9    775   107  42.2
## 12         Japan     1.5    680    36  23.2
## 13   Netherlands     1.8    773   167   9.2
## 14   New Zealand     1.9    916   266   7.7
## 15        Norway     0.8    806   227  12.2
## 16         Spain     6.5    724    86  36.4
## 17        Sweden     1.6    743   207  11.2
## 18   Switzerland     5.8    693   115  20.3
## 19            UK     1.3    941   285  10.3
## 20            US     1.2    926   199  22.1
## 21  West Germany     2.7    861   172  36.7
```

IID Data

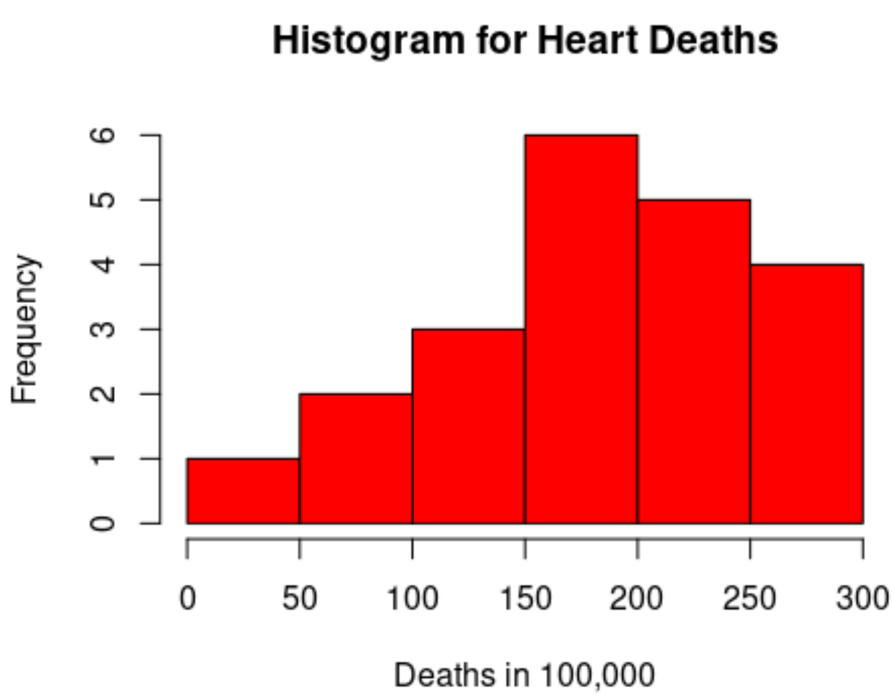
For further clarification of this assumption, each pair (X_i, Y_i) has to be drawn from the same distribution independently of all other pairs. Even though this problem's has a small dataset, it appears to be independent due to each observation being a different country and the data also appears to be identically distributed as will be shown in the plots below. In this case, There are no apparent violates of independents, some examples being clustering of data, in geographical regions, school cohorts, or families, strategical interaction, like competition among sellers or imitation of a species, or autocorrelation were one time period may affect the next. This assumption for the data is meet and thus far, the Wilcoxon Ranked-Summed Test appears to be a viable test.

For the data visualization, two histograms of heart deaths and liver deaths were made in the plots below.

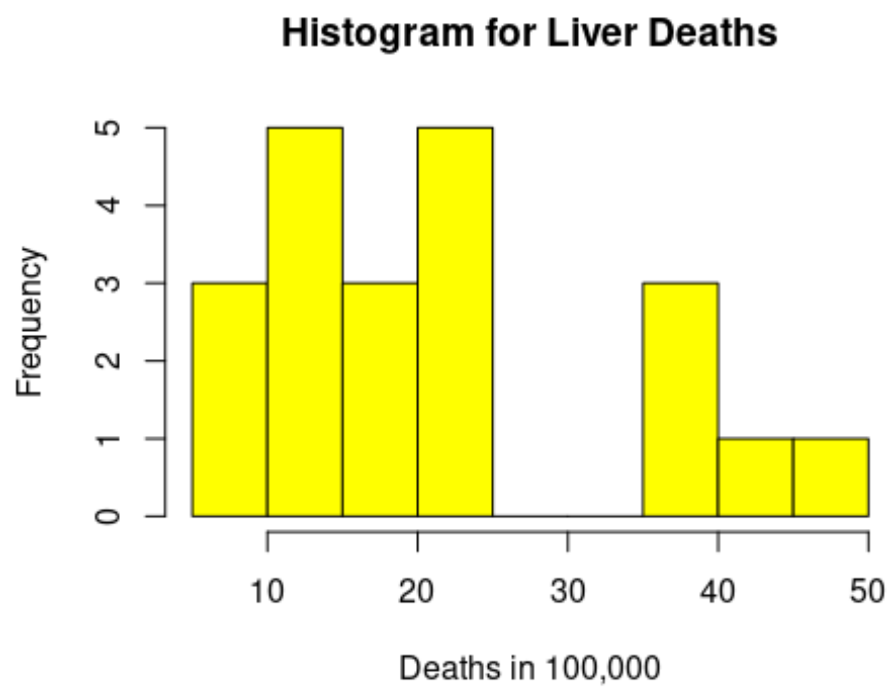
```
liver_m <- mean(wine$liver)
heart_m <- mean(wine$heart)
wine$difference <- wine$heart - wine$liver
(heart_m - liver_m)

## [1] 162.2524

hist(wine$heart,
     main="Histogram for Heart Deaths",
     xlab="Deaths in 100,000",
     col="red")
```



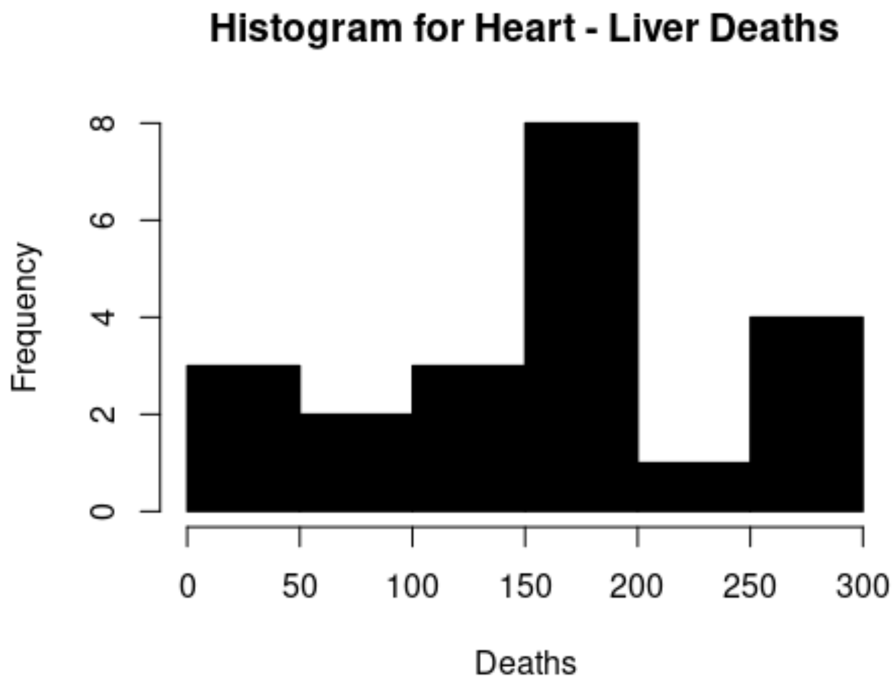

```
hist(wine$liver,
     main="Histogram for Liver Deaths",
     xlab="Deaths in 100,000",
     col="yellow")
```



The Distribution of the difference $(X - Y)$ is the Same Around Some Mean

Using the data visualization below, we have taken the difference of Heart and Liver deaths over the 21 different observations and take the difference of their means (162) as the point for the data to be symmetrical around. The data below shows that even with this small dataset it appears that the data meets this assumption being symmetrical around the mean of 162.

```
hist(wine$difference,
     main="Histogram for Heart - Liver Deaths",
     xlab="Deaths",
     col="black")
```



The test was ran below to see what the results might be. Due to the low p-value, we would be inclined to reject the null hypothesis.

```
wilcox.test(wine$heart, wine$liver, paired=TRUE)

##
##  Wilcoxon signed rank exact test
##
## data:  wine$heart and wine$liver
## V = 231, p-value = 9.537e-07
## alternative hypothesis: true location shift is not equal to 0
```

In conclusion, all of the assumptions have been met for the Wilcoxon Signed-Rank Test and it appears to be a viable hypothesis test to use for this dataset.

1.3.4 Attitudes Towards Religion

QUESTION:The file datasets/GSS_religion is a subset of data from the 2004 General Social Survey (GSS). The variables prottemp and cathtemp are measurements of how a respondent feels towards protestants and towards Catholics, respectively. The GSS questions are phrased as follows: I'd like to get your feelings toward groups that are in the news these days. I will use something we call the feeling thermometer, and here is how it works: I'll read the names of a group and I'd like you to rate that group using the feeling thermometer. Ratings between 50 degrees and 100 degrees mean that you feel favorable and warm toward the group. Ratings between 0 degrees and 50 degrees mean that you don't feel favorable toward the group and that you don't care too much for that group. If we come to a group whose name you Don't recognize, you don't need to rate that group. Just tell me and we'll move on to the next one. If you do recognize the name, but you don't feel particularly warm or cold toward the group, you would rate the group at the 50 degree mark. How would you rate this group using the thermometer? You would like to test whether the US population feels more positive towards Protestants or towards Catholics. List all assumptions for a paired t-test. Then evaluate each assumption, presenting evidence based on your background knowledge, visualizations, and numerical summaries.

Solution

Test Assumptions Paired t-Test:

1. Metric Scale
2. IID Data
3. No major deviations from normality, considering the sample size.

Null Hypothesis: The expectation of X equals the expectation of Y.

Metric Scale

For further clarification, the t-test is not valid for variables which only have an ordinal structure. In this case, the feeling thermometer used to determines ones feelings towards Catholics or Protestants would fall under the Likert Scale and also be considered of ordinal structure. Therefore, the test would fail under this assumption. A summary of the given data is shown below as supporting evidence.

```
library(readr)
GSS <- read_csv("GSS_religion.csv")

## New names:
## * `` -> ...1

## Rows: 802 Columns: 5

## ── Column specification ────────────────────────────────────────────────────────

## Delimiter: ","
## dbl (5): ...1, year, id, prottemp, cathtemp

##
## ℹ Use `spec()` to retrieve the full column specification for this data.
## ℹ Specify the column types or set `show_col_types = FALSE` to quiet this
message.

summary(GSS)

##       ...1            year            id             prottemp
##  Min.   :  1.0   Min.   :2004   Min.   :   4.0   Min.   :  0.00
```

```
##   1st Qu.:201.2   1st Qu.:2004   1st Qu.: 728.8   1st Qu.: 50.00
##   Median :401.5   Median :2004   Median :1373.5   Median : 60.00
##   Mean   :401.5   Mean   :2004   Mean   :1381.9   Mean   : 65.56
##   3rd Qu.:601.8   3rd Qu.:2004   3rd Qu.:2053.5   3rd Qu.: 85.00
##   Max.   :802.0   Max.   :2004   Max.   :2808.0   Max.   :100.00
##      cathtemp
##   Min.   :  0.00
##   1st Qu.: 50.00
##   Median : 60.00
##   Mean   : 63.16
##   3rd Qu.: 85.00
##   Max.   :100.00
```

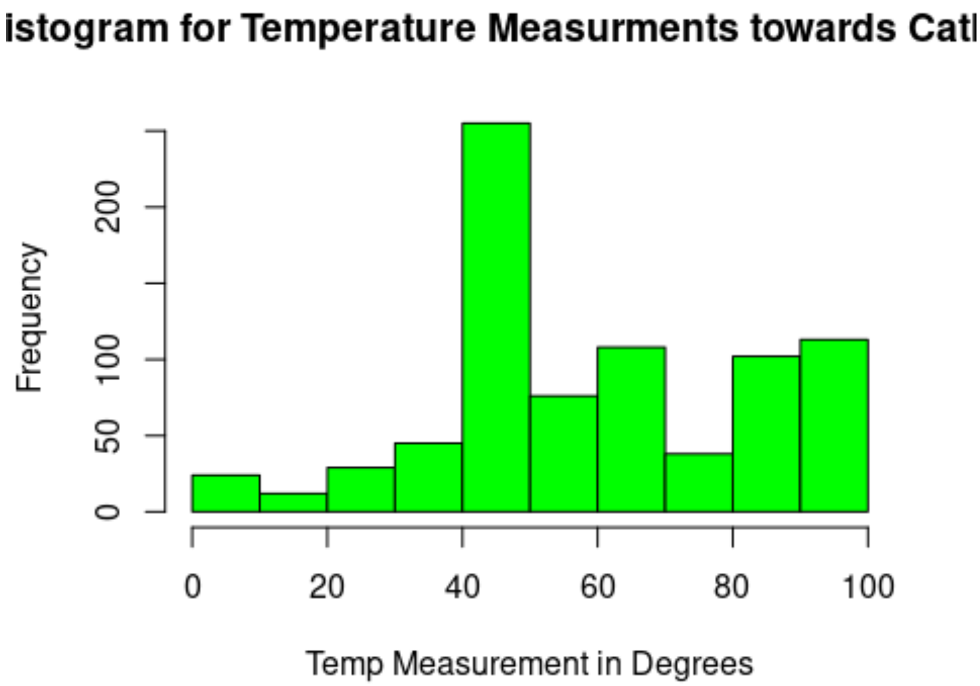
IID Data

Making use of background knowledge, each pair of measurements (X_i, Y_i) is drawn from the same distribution, independently of all other pairs. This data appears to have paring, independence and identical distribution are not violated. There are no apparent violates of independents, some examples being clustering of data, in geographical regions, school cohorts, or families, strategical interaction, like competition among sellers or imitation of a species, or autocorrelation were one time period may affect the next. The data also appears to be identically distributed as will be shown in the plots below. This assumption for the data is meet, but as stated previously, the Paired t-Test would not be a wise test to use for an ordinal scale statistical analysis.

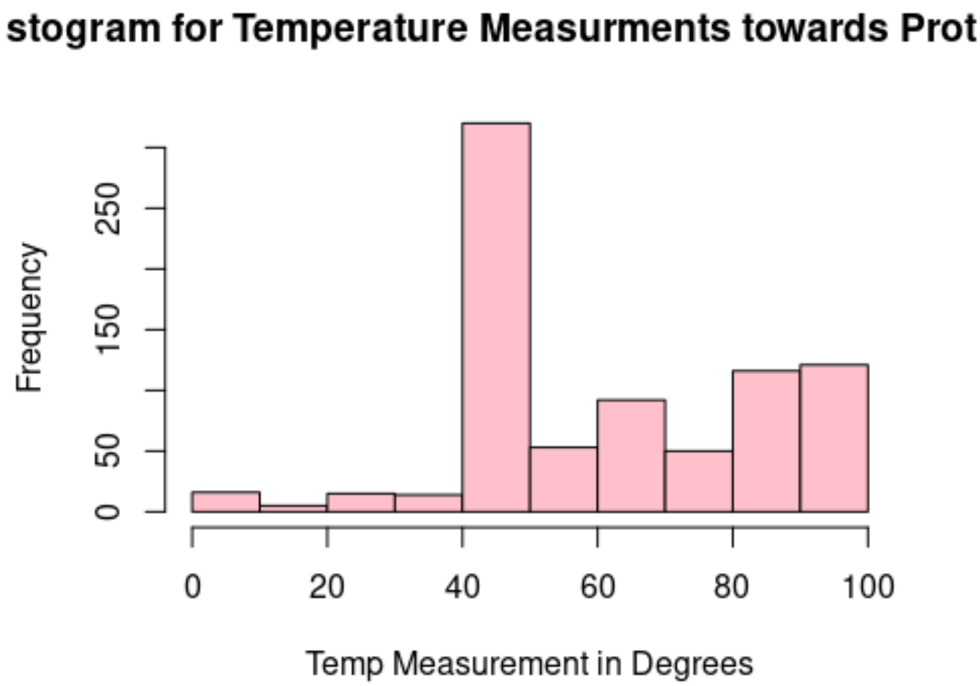
No Major Deviations from Noramlity, Considering the Sample Size

For clarification, the t-test is invalid for highly skewed distributions when sample size is larger than 30. It may also be invalid for very highly skewed distributions at higher sample sizes. This does not appear to be the case as shown in the graphs below. This assumption for the data is meet, but as stated previously, the Paired t-Test would not be a wise test to use for an ordinal scale statistical analysis. See the histograms below for as supporting evidence.

```
hist(GSS$cathtemp,
     main="Histogram for Temperature Measurments towards Catholics",
     xlab="Temp Measurement in Degrees",
     col="green")
```




```
hist(GSS$prottemp,
     main="Histogram for Temperature Measurments towards Protestant",
     xlab="Temp Measurement in Degrees",
     col="pink")
```



A Paired T-Test was run out of curiosity for the results.

```
t.test(GSS$prottemp, GSS$cathtemp, paired=TRUE)

##
##  Paired t-test
##
## data:  GSS$prottemp and GSS$cathtemp
## t = 2.9249, df = 801, p-value = 0.003543
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  0.7902444 4.0152419
## sample estimates:
## mean of the differences
##                2.402743
```

Finally, the Paired T-Test would not be the appropriate hypothesis test based on the failed Metric Scale assumption. The other assumptions are met.

In conclusion, through our analysis of the different scenarios where hypothesis tests could be run, the World Happiness statistical test would not be best evaluated with a Two-Sample T-Test, the Legislators problem would not be best evaluated with a hypothesis of comparisons version of the Wilcoxon Rank-Sum Test, the Wine and Health problem does fit all the assumptions for a Wilcoxon Signed-Rank Test and finally the Attitudes towards Religions problem does not fit the assumptions for a Paired T-Test.