hw4: answer sheet

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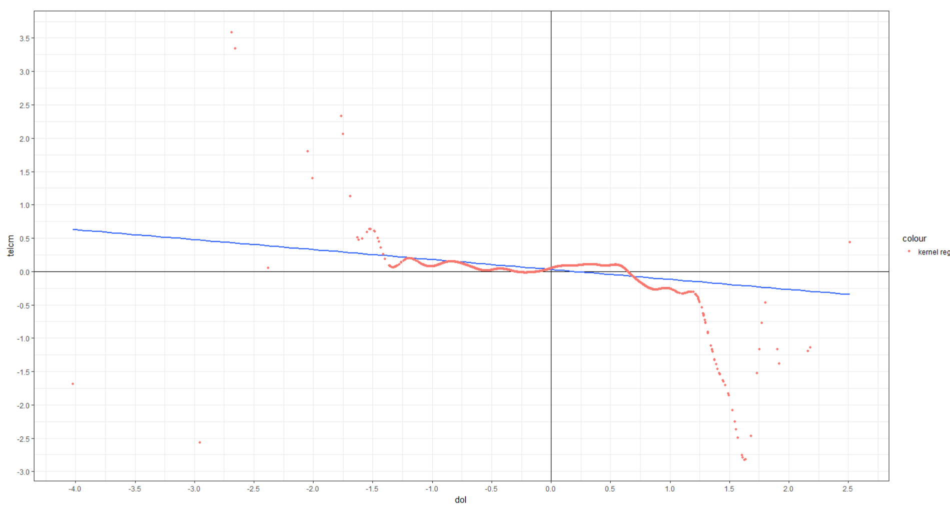
1. By means of Nadaraya-Watson, we estimate the functional relation between the expected continuous return on the telcm portfolio and the dollar appreciation rate:

$$R_t^{telcm} = f\left(R_t^{dol}\right) + u_t$$

This non-parametric estimator guesses y(x_t) as an average of y in the vicinity of x_t. As bandwidth we choose:

$$h_x = std(x_t)1.06T^{-1/5}$$

since it is the optimal choice (in mean squared error terms) if the data is normally distributed and the N(0,1) kernel is used. Here, we report the plot of the fitted values along with the OLS curve.



As visible from the picture, the estimated absolute effect on telcm is larger when the dollar appreciates by 1%, i.e. when the trade weighted dollar index increases.

2. The following table reports whether we should see positive or negative returns for different values of the dollar return:

Return of *dol*, in %	-1	-0.5	0	+0.5	+1
Sign of *telcm* return	+	+	+	+	-

This is inferred by looking at the sign of the red line at the relevant *dol* values.

3. When we increase the bandwidth, we consider a larger neighborhood around x, i.e. we compute the average y_t across a larger number of values. If $h \to \infty$ then the whole sample is considered and the $\hat{b}(x)$ becomes the sample average of y_t. Therefore, by choosing a higher bandwidth we make the plot smoother, i.e. less volatile.

4. We now move to maximum likelihood estimation and we assume that the true model of the relation between *telcm* and *mkt* is:

$$R_t^{telcm} = \beta_0 + \beta_1 R_t^{mkt} + u_t$$

$$u_t \sim L(0, s)$$

Where L is the zero-mean Laplace distribution, parametrized with the scale parameter s:

$$p(u_t) = \frac{1}{2s} \exp\left(-\frac{|u_t|}{s}\right)$$

By maximizing the log-likelihood function we obtained the following parameters:

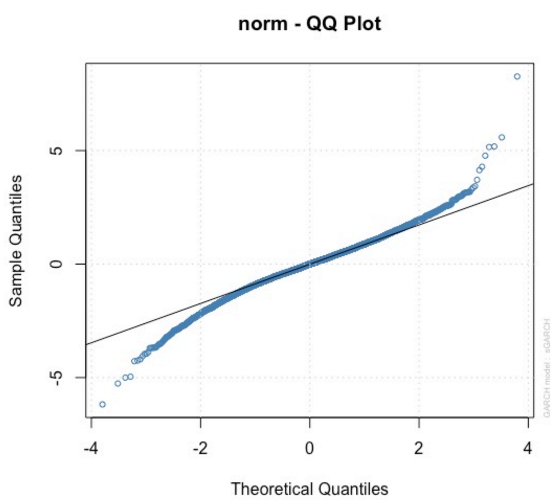
Coefficient	β_0	β_1
estimate	-0.012	0.921
std. error	0.007	0.009

5. In order to test for ARCH effects (autoregressive conditional heteroscedasticity), we would normally study the series of the squared residuals of the model. This could be done by means of a Ljung-Box statistic (the null hypothesis is that the first m lags of the ACF of the series of the squared residuals are zero) or the Engle's test (the null hypothesis is that the coefficients of an autoregressive process on the squared residuals are zero). However, alternatively, it is also possible to analyze the squared returns of the series. Therefore, we test for ARCH effects by performing a Ljung-Box test with 12 lags on the squared returns of *telcm*, i.e. we run a test for the joint significance of the autocorrelation coefficients.

Test statistic with the p-value	5795.8 (0.000)
Conclusion	Reject the null hypothesis that the squared returns are independently distributed and conclude in favor of the presence of ARCH effects

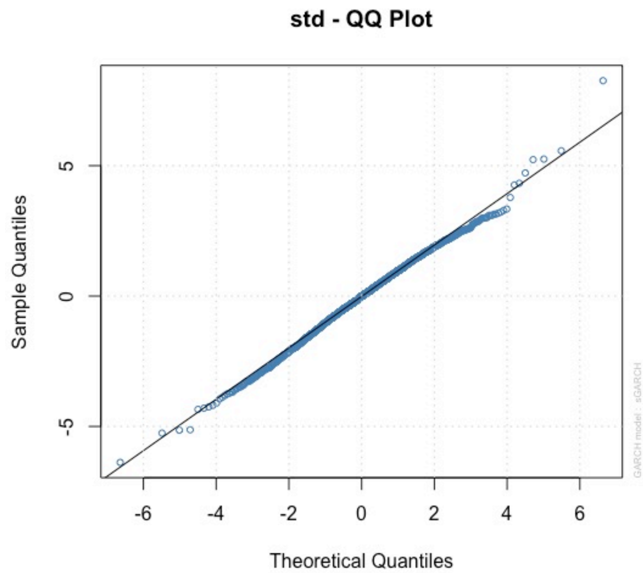
The null hypothesis of the test is that the data are independently distributed, i.e. the autocorrelations are zero. The Ljung-Box test statistic can be compared to the critical value of 5.226 for a 5% significance level and 12 degrees of freedom. Since the statistic is greater than this value, and since the p-value is zero, we reject the null hypotheses and we conclude in favor of ARCH effects.

6. We estimate an ARCH(5) model on the returns of telcm. Here, we report the QQ plot:



As visible from the graph, the residuals have too fat tails compared to a standard normal distribution, therefore violating the initial assumption and suggesting a different distribution, such as the t-student's, which is known to have fatter tails.

7. We now re-estimate the same model with a t-distribution for the residuals, instead of a normal one.



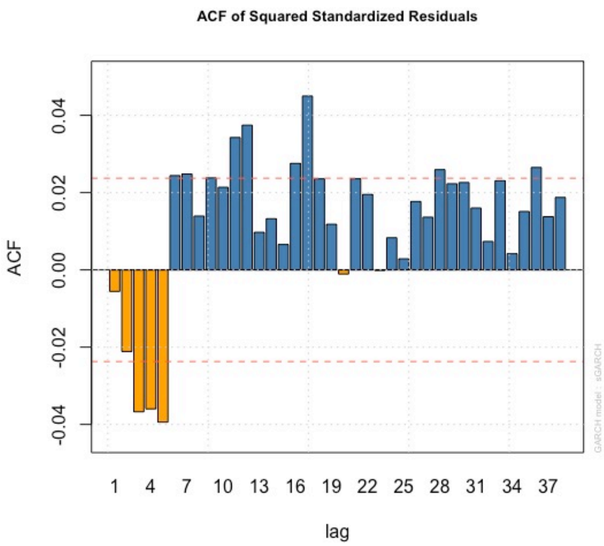
As visible from the plot, the t-distribution is a much better description of the residuals.

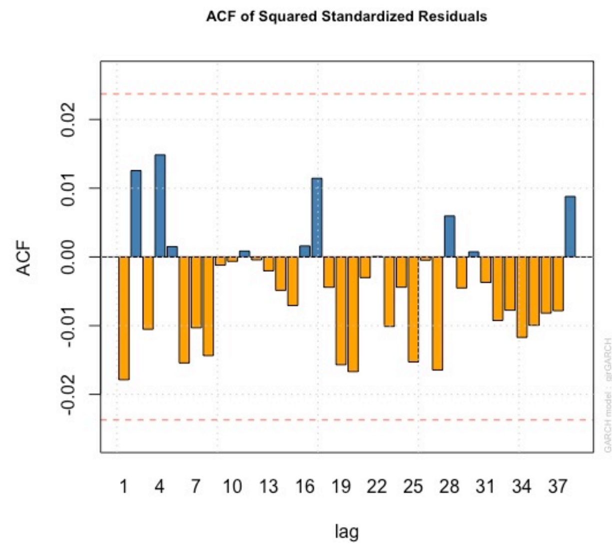
"Shape" coefficient	6.390
Comment	The Shape coefficient gives the degrees of freedom of the t-Student distribution. We know that the t distribution converges to the normal when the degrees of freedom go to infinity and that the two are approximately equal for df = 30 and higher. Given the low value of approximately 6 that we obtained, we can conclude that the distribution of u_t is far from normal and is better modeled by the t-distribution.

8. Now we assume a t-student distribution for the innovations and we estimate an asymmetric GARCH.

Relevant Coefficient (with its standard error)	0.089 (0.013)
Interpretation	Given the parametrization in the *rugarch* package (δ takes a value of 1 if the previous shock was negative) and given the estimated positive gamma coefficient, we conclude that negative return news has a bigger impact on future volatility than positive news. Given a negative return on the previous day, variance goes up. Finally, given the p-value of 0 and the t-statistic of 7.010 for the gamma coefficient, we can conclude that such move is statistically significant at the 95% level.

9. A strong indication of ARCH effects is when the residuals are uncorrelated, but the squared residuals show autocorrelation. Therefore, to test the goodness of the model, we can check the standardized residuals. If the serial correlation structure in the conditional first two moments is correctly modeled, then there should be no autocorrelation in the standardized residuals ($z_t = u_t/\sigma_t$) and the standardized residuals squared, namely because the model aims at capturing the heteroscedasticity.

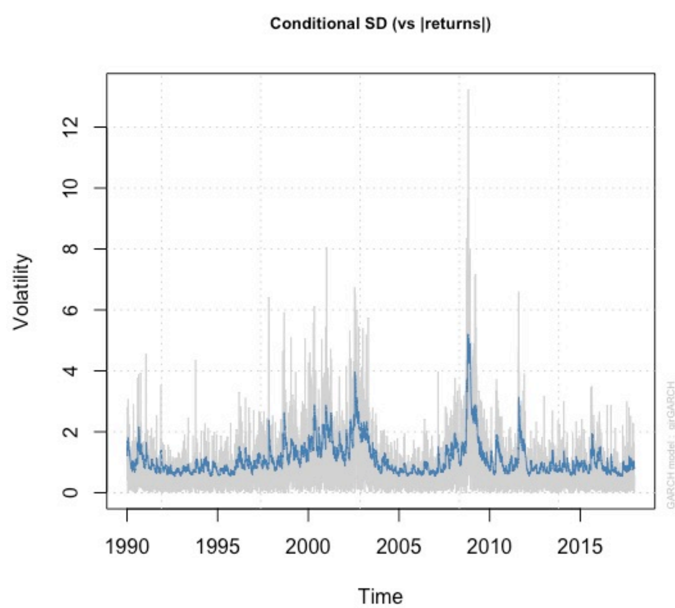




As visible from the two plots, while the ARCH(5) with t-student distributed residuals displays statistically significant autocorrelation at some lags, the asymmetric ARCH model does not. Therefore, we can conclude that the latter is a better descriptive model.

10. The 95% VaR is the negative of the 5% quantile of the distribution of returns and represents the maximum loss that is obtained with a probability of 95%. Assuming that the returns are conditionally normally distributed with zero mean and a volatility equal to the one estimated by model 3, we can infer the maximum 95% VaR by looking at the conditional volatility plot:

$$VaR_{95\%} = -(\mu - 1.64\sigma) \rightarrow maximum\ VaR\ is\ 1.65 * 5.207 = 8.54$$



11. We list the AIC coefficient for each of the three models:

	AIC
ARCH(5) – Normal Dist	2.927
ARCH(5) – T-Dist	2.877
GJR(1,1) – T-Dist	2.828

The Akaike information criterion can be used to select, within a set of the models, the best one for prediction, i.e. the one with the lowest AIC. In our specific case models can be ranked in ascending order based on their AIC as follows: GJR (1,1), ARCH (5) t-dist, ARCH (5) normal-dist. Since GJR (1,1) has the minimum AIC it could be appointed as the best one among the three models; however, given that the difference between the three AICs is modest, no informative conclusions can be drawn from this comparison.