hw2: answer sheet

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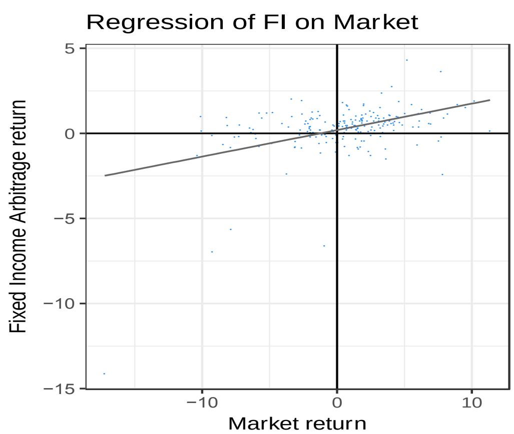
1. We ran a linear regression model of the monthly simple excess returns of the Fixed Income Arbitrage Hedge Fund Index on the S&P500 excess returns:

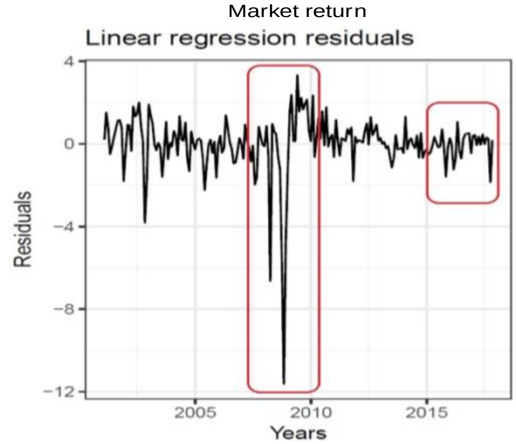
$$r_{FI,t} = \alpha_{FI} + \beta_{FI} r_{m,t} + u_t$$

	Coefficient	Standard error	t-stat	P-value
Alpha	0.192	0.101	1.904	0.058
Beta	0.156	0.023	6.635	0.000
R-squared	0.180			
Alpha significant at the 10% level?	Yes			



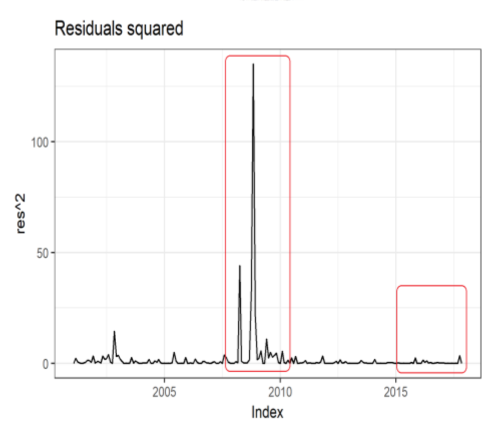
Both the alpha and the beta coefficients are statistically significant at the 10% level, as their t-stats are above the critical value of 1.645 and their p-values are below 10%. Economically, alpha measures the average excess return compared to an equally risky investment on the security market line: the positive value represents the active return of the fund, as a result of the strategy implemented. The beta instead is a measure of the systematic risk of a security: the value smaller than 1 indicates less than proportional fluctuation of the excess return of the fund in response to movements in the market excess return.

The coefficient of determination, R^2, is indicative of the strength of the linear relationship between the exogenous variable and the endogenous one, i.e. of the ability of the former to explain the movements of the latter.



2. The time series of the residuals of the above regression displays some apparent heteroscedasticity, which is verified with the time series of the squared residuals. Even though the least squares procedure remains consistent, this raises an issue for the classic formula of the standard error of the coefficient, as its derivation relies on three assumptions, including indeed the constant variance of the residuals.

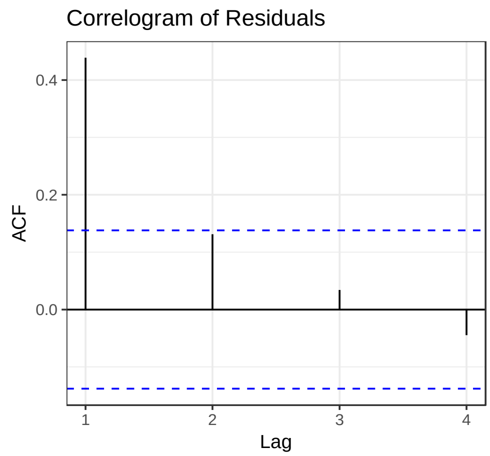
We therefore ran a White's test to assess the null hypothesis of homoscedasticity, against the alternative of hypothesis of the kind of heteroscedasticity that can be explained by the levels and squares of the regressors.



Test used	White's test of heteroscedasticity
Test statistic and p-value	t-stats = 88.89 and p-value = 0.000
Conclusion	The time series of residuals is heteroscedastic.

The p-value returned by the test allows us to reject the null hypothesis of homoscedasticity and to accept the alternative hypothesis of a form of heteroscedasticity where the volatility in the residuals is related to the regressors.

3. The autocorrelation of the residuals is a further violation of the assumptions underlying the derivation of the standard expression for the variance of $\widehat{\beta_{FI}}$. Like before, the least squares procedure is consistent, but the variance is again wrong. More precisely, positively autocorrelated residuals result in an uncertainty level above the one estimated by OLS under iid assumptions.
The correlogram shows indeed the presence of positive autocorrelation of the residuals, decaying quite fast from above 40% at 1st lag, to below 5% at 3rd lag. The autocorrelation coefficient is negative at 4th lag.
We tested for autocorrelation at 1st lag by using the Durbin-Watson test. With a statistic of 1.122 and a p-value of 0.000 we rejected the null hypothesis of no autocorrelation in favor of the alternative hypothesis of positive autocorrelation.



DW statistic	1.122
Conclusion	There is autocorrel. at the 1st lag.

4. The presence of heteroscedasticity and autocorrelation does not result in biased parameters estimates and does not affect the consistency of the least squares procedure; rather, it only affects the standard error of the coefficients. In other words, the OLS estimates are no longer BLUE, as they are not the estimates with the lowest variance. The standard errors are biased, and this also results in biased test statistics. To address the first issue, it is possible to estimate White's covariance matrix, which is a heteroscedasticity consistent matrix. To address the second issue, it is possible to estimate the Newey-West covariance matrix, which is an autocorrelation consistent matrix. Both methods correct the standard error of the estimate and do not change in any way the estimate of the coefficient.

5. To compute the heteroscedasticity and autocorrelation consistent standard errors, we used the Newey-West estimator, which is capable of handling both problems. As expected, and as visible from the table, the estimates of the alpha and beta are indeed unchanged, while the standard errors increase: this results in lower t-stats for both estimates. More precisely, beta is now significant at the 10% level while alpha is not.

	Coefficient	Standard error	t-stat	P-value
Alpha	0.192	0.169	1.138	0.256
Beta	0.156	0.079	1.983	0.049
Alpha significant at the 10% level?	No			

6. The SURE approach allows to implement a formal test of the CAPM for several assets. In the first part, we set up a system of seemingly unrelated regressions (SURE) of the excess returns of FI, MULT, and ELS on the excess return of the market and then we derived the correlation matrix of the residuals.

Correlation Matrix	FI	MULT	ELS
FI	1.000	0.664	0.331
MULT	0.664	1.000	0.708
ELS	0.331	0.708	1.000

7. In the second part, we test the null hypothesis that all alphas are zero by means of a Chi-square test.

Test statistic	26.443
Critical value	6.251
Conclusion	Reject the null hypothesis H_0: $\alpha_{FI} = \alpha_{MULT} = \alpha_{ELS} = 0$

The t-statistic is well above the cutoff value of 6.251 and we can therefore reject the null hypothesis that all three coefficients are jointly equal to zero at the 10% significance level.

8. After trimming the data series to include only the observations after 2009:7, we re-estimated the CAPM on the Fixed Income Arbitrage Hedge Fund Index using robust standard errors. In order to do so, we first tested the subsample for heteroscedasticity and autocorrelation by means of the White's test and the Durbin-Watson test. With a p-value of 0.202 we failed to reject the null hypothesis of homoscedasticity of the residuals, while with a p-value of 0.000 and a statistic of 1.18 we rejected the null hypothesis of no autocorrelation, in favor of the alternative hypothesis of positive autocorrelation in the regressors. Therefore, we applied the Newey-West approach. This comes with the cost of estimating lots of parameters and added noise and uncertainty.

Test statistic	0.754
Critical value	1.960
Conclusion	We fail to reject the null hypothesis: H_0: $\alpha_{FI} = 0$

After re-estimating the alpha coefficient, we concluded that it was statistically different from 0.4 at a 5% significance level, as its t-stat is smaller than the relevant critical value.

9. We proceeded to the analysis of the returns of the Equity Long Short Fund Index. Running a CAPM, we found evidence in favor of the presence of heteroscedasticity (p-value = 0.001) but we failed to reject the null hypothesis of no autocorrelation (DB = 1.683). We draw the same conclusions when we run a Fama French five factor model (p-value = 0.001 and DW = 1.643). Therefore, we used once more the Newey-West approach, but, for both models, we excluded all lags setting m=0.

	Coefficient	St. err.	t-stat	P-value
CAPM alpha	0.195	0.094	2.074	0.039
5-factor model alpha	0.140	0.099	1.421	0.157
Conclusion	Alpha of CAPM is statistically significant; alpha of FF5 is not			

After running the two regression models, we found that the alpha coefficient of the CAPM is statistically significant as its p-value is above the critical value of 1.645, while the alpha coefficient of the five-factor model is not. This can also be seen from the p-values, respectively below and above the 10% level. This is likely due to the greater explanatory power of the more articulated model.

10. We estimated a Fama-French five factor model on the returns of FI, MULT and ELS with a SURE approach according to the same procedure described above. We report the variance covariance matrix of the residuals.

Covariance Matrix	FI	MULT	ELS
FI	1.999	1.061	0.606
MULT	1.061	1.196	0.980
ELS	0.606	0.980	1.612

11. Since from the above system we found two statistically insignificant and one strongly significant alpha coefficients, we run a Chi-square test of joint significance.

Test statistic	26.253
Critical value	7.815
Conclusion	Reject the null hypothesis H_0: $\alpha_{FI} = \alpha_{MULT} = \alpha_{ELS} = 0$

The t-statistic is well above the cutoff value of 7.815 and we can therefore reject the null hypothesis that all three coefficients are jointly equal to zero at the 5% significance level.