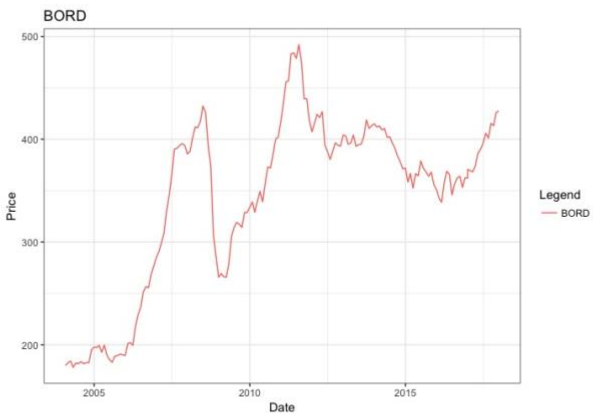
hw1: answer sheet

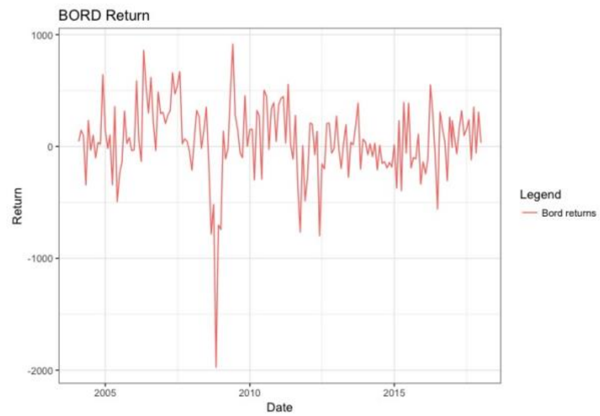
2. We computed the average monthly logarithmic returns (in percentage terms):

	mean_return_SMIUSD	mean_return_GOLD
Mean	0.446	0.678

We have calculated them as the mean of the time series of continuous returns over the relevant horizon.

3. Plots of the BORD level (left plot) and the BORD return (right plot):





4. T-test:

t-stat	1.961
Reject at the 5% significance level?	We tested the null hypothesis that the average monthly return of BOARD is 0: H_0: true mean_return_BORD = 0. Assuming that the estimates are normally distributed (which is a good approximation given the size of the sample) then: mean_return_BORD ~ N [0, Var(return_BORD)] Transforming it to a standard normal variable, it is possible to compare the value of the t-statistic to the relevant value of such a distribution: $\frac{\text{mean\_return\_BORD} - 0}{\text{Sd(mean\_return\_BORD)}} \sim N[0, 1]$ By looking at the p-value (0.052) and the t-statistic (1.961), we do not have sufficient evidence to reject the null hypothesis that the average monthly return of BOARD is statistically different from 0. In other words, the observed mean_return_BORD is a likely outcome under the null hypothesis. This result was derived using the t-test function available in R. Moreover, to check that the value obtained was indeed correct, we performed the computation of the t-statistic also step by step.

5. We defined the 95% confidence interval around the mean value of GOLD return and we ran a two-sided paired test for GOLD and USRF returns:

95% confidence interval	We constructed a confidence interval around the point estimate: [mean_return_GOLD±1.96*Sd(mean_return_GOLD)] Obtaining the following values for the lower and upper bounds respectively: -0.105% and 1.461%.
Conclusion (risk-free rate comparison)	Since the average monthly return of the risk-free asset is 0.1%, which lies inside the GOLD return confidence interval, we cannot reject the null hypothesis that the two average returns are equal. An alternative way to come up with the same conclusion is to run a two-sided paired test, obtaining a p-value of 0.146, which means that we fail to reject the null hypothesis that the two means are equal, at a 5% significance level.

6. We computed the highest correlation coefficients for BORD and GOLD:

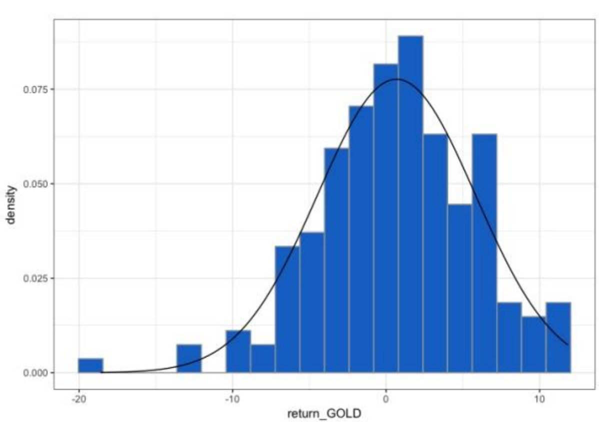
Asset most correlated with *BORD* (with the corresponding correlation)	The return of BORD is most correlated with the return of MSCIE: 0.554
Asset most correlated with *GOLD* (with the corresponding correlation)	The return of GOLD is most correlated with the return of BORD: 0.276

	return_SP500	return_SMIUSD	return_MSCIE	return_GOLD	return_BORD
return_SP500	1.00000000	0.5441094	0.6713961	0.08195179	0.4772069
return_SMIUSD	0.54410943	1.0000000	0.8924280	0.14230975	0.4593084
return_MSCIE	0.67139611	0.8924280	1.0000000	0.15622097	0.5540938
return_GOLD	0.08195179	0.1423097	0.1562210	1.00000000	0.2763825
return_BORD	0.47720693	0.4593084	0.5540938	0.27638254	1.0000000

7. Among the five risky assets, we identified the one with the lowest excess kurtosis:

Asset with the lowest excess kurtosis (with the corresponding coefficient)	While all the assets have a positive excess kurtosis, the return of GOLD has the lowest one, with a value of 0.586

Paste your histogram here and comment.



From a graphical analysis, the return of GOLD does not seem to be normally distributed over the subsample considered. To confirm this intuition, we also ran a Jarque-Bera normality test: with a JB value of 6.345 and a p-value of 0.042, we can reject the hypothesis that the returns are normally distributed at a 5% significance level.

8. We determined which risky asset is farthest from a normal distribution:

Asset	The SP500 is the asset which is farthest from a normal distribution, since it has the highest absolute value for both excess kurtosis and skewness, respectively 10.407 and 2.034.
JB test statistic	The JB value is 899.766 and the p-value is 0.000, which means that we can reject the null hypothesis of normality.

9. We found the following for an equally weighted portfolio of all five risky assets:

Expected return	0.487%
Sharpe ratio	0.121

The portfolio expected return is the mean over the subsample considered of the monthly portfolio returns, computed as the equally weighted average of the log returns of the risky assets. The Sharpe ratio instead, uses the expected excess return (i.e. the expected portfolio return minus the risk free rate), scaled by the standard deviation.

10. To test if the expected return of the equally weighted portfolio is statistically different from zero, we computed a 95% confidence interval around the point estimate. We did so both by means of the t.test function:

Lower c. band	Expected Return	Higher c. band
0.000%	0.487%	0.974%

Conclusion:
Since the lower bound of the confidence interval is zero, we fail to reject the null hypothesis that the expected return of the portfolio considered is zero. We could have reached an identical conclusion by calculating the confidence interval around zero (i.e. the null hypothesis) and observing that the expected return of the portfolio does not lie outside this interval. Finally, it would have been possible to come with this conclusion also by looking at t-stat and p-value.
It is important to notice that the lower bound of the confidence interval coincides (to the significant degree of approximation chosen) with the value of zero. This means that, although we do not reject the null hypothesis with a 95% confidence level, a slightly lower confidence level (or equivalently a slightly higher significance level) would have led to the rejection. This is where the tradeoff between type II error and type I error comes into play.

11. To perform the following tasks, we reconsidered the whole sample from 1962:01 to 2017:12 (remember that all the previous questions were answered on a subset of the time series). Moreover, we included a new variable in our data set: a dummy variable which distinguishes between years with a Democrat President and years with a Republican one.

(c) We computed the average total return of the SP500, inclusive of dividends, for the dates when a Democrat president was in office. To do so, we first computed the log of the SP500 price plus the monthly dividend all divided by the lagged SP500 price; then, we averaged out this measure over the years where the dummy variable was 1:

Average total return	0.011%

(d) We then repeated the process for a Republican president (dummy equal to 0):

Average total return	0.005%

(e) Finally, we implemented a two-sample t-test to compare the two means:

t-test	-2.018
Conclusion	The means are different

Before testing the null hypothesis that the two time series have the same mean, we studied their variances using the var function of R. After finding that the two variances are statistically different, we ran the two-sample t-test to verify whether the average return of the SP500 differs significantly according to the political party in power. This test assumes that both samples are random, independent and come from normally distributed populations.
Using a two-sided test, we obtained a p-value of 0.043 and a t-stat of -2.024, which allowed us to reject the null hypothesis that the mean values are equal, in favor of the alternative hypothesis that the two means are different. We went further to examine under which party the SP500 performed better on average. To do so, we implemented two one-sided tests, one left sided and one right sided. We found that the former returned a p-value of 0.022 and the latter a p-value of 0.978. We can conclude that the SP500 performed statistically better under the Democrats than under the Republicans.