Tax Progressivity as a Stabilizer of labor Income*

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Abstract

In this article we use Italian administrative data to study the role that a progressive income tax can play in redistributing cyclical risk from low to high wage workers and reduce the volatility of aggregate employment. We do this by developing and estimating a frictional model of the labor market with heterogeneous workers, aggregate shocks and a non-linear tax schedule. Our results show that eliminating income tax progressivity in Italy while maintaining the tax revenue fixed would come at the expense of the majority of workers. The current system of marginal tax rates is effective at reallocating cyclical income risk from low to high wage workers and reduces aggregate employment volatility by 18.5% compared to a counter-factual flat rate system.

Keywords:

JEL codes:

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1 Introduction

There is robust evidence that income risk is not evenly spread. Considering income earned from labor, low wage workers tend to face a more volatile income process, experiencing more frequent and on average longer unemployment spells[1]. They also tend to be more sensitive to aggregate shocks, facing higher cyclical income risk[2]. At the same time, an increasing number of works has argued that the cost of income fluctuations is not homogeneous across individuals. Both the long-run persistence of income losses and their welfare costs is found to be higher for low income workers[3]. Recessions then exacerbate a situation where those more exposed to income fluctuations are also those more sensitive to their realisations. Policies that reduce and redistribute income risk across workers can thus prove effective at reducing the social and long-term cost of recessions.

Several government policies have the potential to even out cyclical and idiosyncratic risk. In this work we focus on the role played by a progressive income tax. Keeping total tax revenues fixed, a more progressive income tax reallocates the burden of taxation away from workers at higher risk of cyclical job loss, increasing the profitability of their jobs and reducing their likelihood of being laid-off. If the cost of financing the government's revenue is shifted towards jobs generating higher private surplus and therefore facing lower risk of destruction, a progressive income tax can both reallocate risk away from low income workers and stabilise aggregate employment. Our aim in this paper is to assess this redistributing and stabilizing role of income tax progressivity. In doing so, we take a structural approach and consider the equilibrium effect of a progressive income tax in a frictional labor market with heterogeneous workers.

We start by exploiting Italian administrative data to document the properties of income risk. We follow the literature and approximate income risk by the observed dispersion in income changes conditional on past wages. In line with previous research we find that risk is higher for low wage individuals, with the bulk of this volatility resulting from unemployment risk. We then show that the heterogeneity in income risk is exacerbated by the business cycle, with low wage individuals facing on average higher cyclical risk.

[1]See for example Guvenen et al. (2014) and Karahan et al. (2018)

[2]See for example Guvenen et al. (2017b). Sensitivity of income to the aggregate state of the economy is often found to be U-shaped with respect to past earnings, decreasing with past earnings up to the the very top of the earning distribution before increasing again for top earners.

[3]The higher persistence of income losses for low income workers is mostly due to a higher cost of non-employment and job displacement, see for example Guvenen et al. (2017a) and Schmieder et al. (2018). The higher welfare cost of income losses is associated to their limited ability to self-insure (Blundell et al. 2008).

In particular, we show that the three first moments of the distribution of income changes are more cyclical for low wage individuals.

In order to study the effect of a progressive income tax on the equilibrium in the labor market we rely on a search and matching theoretical framework. We develop and estimate a search and matching model where heterogeneous workers search for firms in a frictional labor market and firm-worker matches are subject to aggregate productivity shocks. The model can reproduce the heterogeneous sensitivity of different workers to the business cycle, with low-skill workers being affected the most. Importantly, we introduce in our model an income tax schedule that is calibrated to match the shape of its empirical counterpart.

One difficulty in solving our model lies in the fact that the relevant aggregate state is infinite-dimensional. We adopt a projection-and-perturbation approach based on Reiter (2009) to solve and simulate our model, which we estimate using data from 1977 to 2012. We use our quantitative model to investigate the impact of income taxes on the determination of income risk. Our structural approach can capture the general equilibrium effect of income tax policies, allowing us to assess the effect of income tax progressivity beyond its direct effect on the difference between net and gross income. This distinction is important. Considering only the direct net-gross income effect of the tax would in fact neglect its effect on job creation, job destruction and equilibrium wage determination.

In performing our counter-factual exercise we focus on two sets of outcomes. First, we look at the re-distributive role of income tax progressivity and study how it reallocates cyclical risk from low to high income workers. While the direct effect of a more progressive taxation on match surpluses is likely to reduce the risk faced by low income workers, its effect on job creation can go either ways. In performing this analysis we exploit our model to isolate the equilibrium effects of taxation from its direct effect on the net income. Second, we study the stabilizing role of income tax progressivity and investigate its effect on the average level and volatility of aggregate employment. If low skilled workers form matches with lower surplus, for a given level of aggregate productivity reallocating the tax burden towards high income jobs can reduce the number of unfeasible matches in the economy and reduce aggregate unemployment. By reallocating the tax burden away from low to high wage workers, a progressive taxation can therefore increase aggregate employment and stabilise its cyclical fluctuations.

In our analysis of tax progressivity we use our model to compare two economies: one where a stepwise system of marginal tax rates approximating the current Italian system is in place and one with a tax-revenue-equivalent constant rate. Our estimated model suggests that in order to guarantee an equivalent tax revenue in the long-run, the government would have to impose a 29% flat rate, imposing an additional burden on around 87% percent of the working population[4]. The progressive tax system currently in place is shown to play an important role in reducing the cyclical fluctuation in the income of low wage workers. Our exercise shows that both the time-series volatility and time-series left-skewness of the type-specific average income are substantially reduced for workers in the bottom half of the skill distribution. We also show that the equilibrium effect of taxation on the determination of gross income plays an important role in strengthening the re-distributive effects of the progressive system. Finally, our simulations shows that the progressive tax schedule acts as a stabiliser of aggregate employment, reducing its cyclical volatility by 18.5% and decreasing the long-run unemployment rate by around 0.8 percentage points.

In our work we do not perform welfare analysis nor derive the optimal degree of tax progressivity. While we believe that these are interesting exercises, they require a more detailed representation of preferences and savings decisions, which would make the estimation of our model of the labor market unfeasible. We instead perform a positive analysis of income tax progressivity, assessing its effect on the cyclical dynamics of income and on aggregate employment. In this sense our approach is similar to that in Blundell et al. (2015) and McKay and Reis (2016b).

Related Literature The nature of cyclical income risk and its distributional consequences have been the object of renewed interests in recent years, thanks in part to the availability of extensive administrative data on workers histories. Our descriptive analysis of income changes is related to a set of works discussing the pro-cyclical first moment, counter-cyclical dispersion (Storesletten et al. (2004)) and pro-cyclical skewness of income changes (Guvenen et al. (2014)). Recent works on these empirical regularities include Busch and Ludwig (2017) on German and US household data, Angelopoulos et al. (2019) on UK data, Harmenberg and Sievertsen (2017) on Danish data and Hoffmann and Malacrino (2019) for Italy[5]. In line with our descriptive results, this literature finds the skewnees and mean

[4]As we discuss in appendix F, the counter-factual exercise is carried out without considering a change in the set of tax deductions currently in place.

[5]Contrary to our descriptive analysis Hoffmann and Malacrino (2019) do not desegregate by worker type and focus on the relative importance of hours worked and hourly wage in determining the cyclical

of income changes to be pro-cyclical and the dispersion to be either counter-cyclical or acyclical, with these cyclical properties being stronger for low wage workers[6]. Hoffmann and Malacrino (2019) show evidence that these cyclical patterns are explained mostly by changes happening at the extensive (hours worked) rather than intensive margin (hourly wage).

An important stream of research have investigated the role of taxes and transfers in insuring against income risk. The literature often find an important role of taxes and transfers in attenuating income shocks, especially for low skilled workers (Blundell et al. 2015). Our work is more closely related to the analysis in Busch and Ludwig (2017) and Busch et al. (2018), who find an important role of taxes and transfers in attenuating the cyclicality in the first three moments of distribution of income shocks. Compared to these works, we focus on the first moment of the distribution and we micro-found the labor income process within a frictional model of the labor market. Our approach has the advantage of taking into account the equilibrium effects of taxes, allowing us to asses their role in altering the cyclicality of income risk.

The role of tax progressivity as an automatic stabiliser of the economy has been the object of several works in the literature. Most recently, McKay and Reis (2016b) quantitatively study the role of automatic stabilizers in the US and finds an important role for taxes-transfers and social insurance. McKay and Reis (2016a) study the optimal level of unemployment insurance and tax progressivity when stabilization concerns are taken into account [7]. To the best of our knowledge our paper is the first to analyse the stabilisation role of tax progressivity in a search and matching model with heterogeneous workers and aggregate shocks.

Our focus on the implication of progressive taxation in reallocating labor income risks across workers contributes to an expanding literature employing heterogeneous agents search and matching models of the labor market to evaluate the impact of government policies. A first group of works has used this class of models to evaluate the steady-state effect of labor market policies policies. Breda et al. (2016) and Pizzo (2018) evaluates the

patterns of income changes.

[6]There is not a strong consensus in the literature on the cyclicality of the dispersion in income changes. Guvenen et al. (2014) and Angelopoulos et al. (2019) finds no significant cyclicality in the second moment while Storesletten et al. (2004), Busch and Ludwig (2017) show evidence of a counter-cyclical second moment with Hoffmann and Malacrino (2019) finding a somewhat mild cyclicality.

[7]The set of works on automatic stabilisers is vast. Other recent works include Auerbach and Feenberg (2000), Mattesini and Rossi (2012) and Larch et al. (2013), among the others.

effects of payroll taxes deduction on wages and employment. Engbom and Moser (2018) evaluates the effect of an increase in the minimum wage in Brazil on income inequality. Our paper is more closely related to works evaluating government policies using search and matching models featuring heterogeneity and aggregate shocks. In this sense our work relates to Murtin and Robin (2018) who analyse the effect of a set of labor market reforms on unemployment dynamics and Lise et al. (2018) who study the effect of the minimum wage on cyclical employment and sorting.

Finally, our methodology builds on works that have extended and estimated standard search and matching model with heterogeneous agents in environments featuring aggregate productivity shocks and in particular to Robin (2011) and Lise and Robin (2017). Compared to these studies we introduce in our model a non-linear income tax schedule, which allows us to study the role of income tax progressivity in the allocation of cyclical risk among heterogeneous workers.

Outline The rest of the paper is organised as follows. In section 2 we present empirical evidence on cyclical income risk in Italy. In section 3 we lay down our theoretical model. In section 4 we discuss the role of a progressive income tax in determining income and aggregate employment. In section 5 we present our estimation approach. In section 6 we describe the results of our counter-factual analysis before concluding in section 7.

2 Descriptive Evidence

In this section, we use Italian administrative data to provide evidence on the distribution of income risk, its dependence on a worker's type and its cyclical properties.

2.1 Data

We use Italian matched employer-employee data to obtain information on wages, separation rates and job-to-job movements. In particular, we use the file LoSai made available by the Italian Ministry of Labor and INPS for the period 1977-2012.[8] Our dataset contains data on employment spells, gross wage, the number of weeks worked, firm and worker

[8] We use the file "Estratti Conto", which contains records on each transaction made between the social security institute (INPS) and the worker. From this set of data we select the records that corresponds to labor market events. The advantage of this file compared to the file "Dipendenti" is that it contains information on the exact date of start and end of the event.

unique ID for around 1/15th of the workers.[9] Exploiting the information contained in this dataset we construct annual series on wage and income changes. See section D in the Appendix for further details on the selection of the data we used in our empirical applications.

We use OECD quarterly seasonally adjusted data on GDP per employed person from 1981 to 2017 as a measure of productivity.[10]. We use ISTAT quarterly series on aggregate unemployment rate for the period 1977-2017 [11] We use income tax data from the OECD database from 1983 to 2016 to construct the tax schedule. Finally, we use ISTAT quarterly series on job vacancies rate for the period 2004-2017[12].

2.2 Income Risk

We start by presenting the long-run distribution of income shocks for workers at different levels of the income distribution. Ideally, in order to measure the degree of cyclical risk faced by each worker in the economy we would need to compute statistics that condition on their entire dimensions of heterogeneity. As these are not observed we follow the literature and compute statistics on the distribution of income changes conditional on past-average wages[13]. Our variable of interests is the differences in the log of annual labor income. Defining labor income for a worker i at time t as $Y_{i,t}$ we define an income shock at time t as [14]

$$\Delta y_{i,t} = log(Y_{i,t}) - log(Y_{i,t-1})$$

we then classify workers into categories based on the five-year average of their past weekly wage

$$x_{i,t} = \frac{1}{5} \sum_{j=t-6}^{t-1} \frac{Y_{i,j}}{W_{i,j}}$$

where $W_{i,t}$ are the number of weeks worked in a year. In the rest of the paper we refer to $\Delta y_{i,t}$ conditional on a worker's type $x_{i,t}$ as income risk. Given that we only observed the realised change in wages, we implicitly assume that the variation in $\Delta y_{i,t}$ that is not

[9]The dataset includes workers born on the 1st or 9th day of each month.

[10]Series ULC_EEQ_31102017104913339.

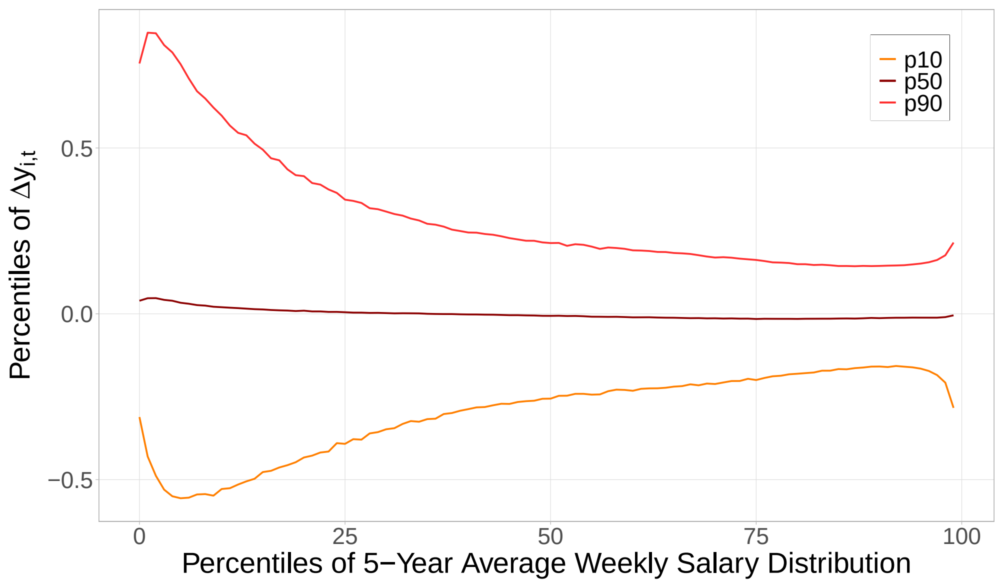
[11]This series includes worker (males and females) for the age category 15-64.

[12]Series are obtained from the Vela survey.

[13]This is the approach used, for example, in Guvenen et al. (2017a). Our model on the other hand will allows us to condition on a worker's type. In analysing our simulated data we will therefore focus on a set of statistics that more closely capture income risk and that are related to the time-series properties of income, conditional on a worker's type.

[14]Appendix D gives more detail on how we select our sample and on how we compute $Y_{i,t}$. To compute the statistics in this section we use gross income after payroll contributions are payed and before income taxes.

Figure 1: Yearly log income changes as a function of past income



Notes: The graph plots the 10th, 50th and 90th percentile of the distribution of log income changes (y-axis) by percentile of the distribution of 5-years average past weekly wages (x-axis). Dataset are from the LoSai data pooled over the period 1977-2012.

predicted by $x_{i,t}$ is not predictable given the worker's information set.

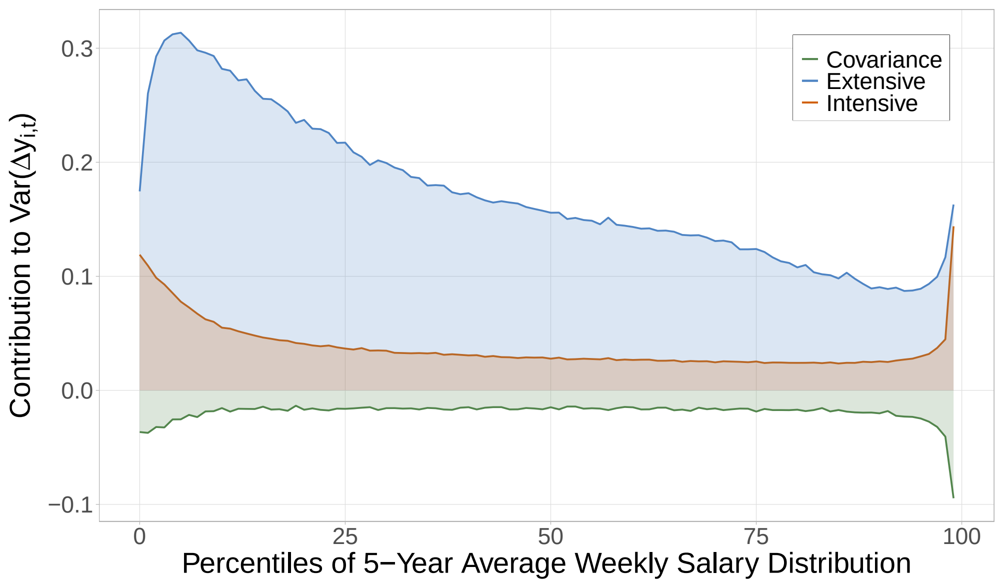
Cross Sectional Properties Figure 1 plots the 10th, 50th and 90th percentile of $\Delta y_{i,t}$, on the y-axis, by percentile of $x_{i,t}$, on the w-axis. Moving from the left to the right of the x-axis it is therefore possible to observe the evolution of the (empirical) distribution of $\Delta y_{i,t}$ as past wages increase. It is clear form the graph that the distribution of $\Delta y_{i,t}$ is not homogeneous across workers' types, with low wage workers experiencing a much higher dispersion in income changes. Workers at the 10th percentile of the distribution of $x_{i,t}$ see a dispersion in $\Delta y_{i,t}$ (as measured by the inter-decile range) 3.86 times higher than that seen among workers at the 90th percentile of past wages.

Figure 2 decomposes the variance of $\Delta y_{i,t}$ by percentile of $x_{i,t}$, where

$$Var(\Delta y_{i,t}) = Var(\Delta w_{i,t}) + Var(\Delta h_{i,t}) + 2Cov(\Delta w_{i,t}, \Delta h_{i,t})$$

with $\Delta w_{i,t}$ being the log-change in weekly wage (intensive margin) and $\Delta h_{i,t}$ the log-change in weeks worked (extensive margin). The decomposition shows that most of the dispersion in $\Delta y_{i,t}$ comes from changes at the extensive margin, which explains around 4 times more of the total variance in $\Delta y_{i,t}$ compared to changes at the intensive margin

Figure 2: Decomposition yearly income changes: extensive and intensive margins



Notes: The graph plot the decomposition of the variance of log-income changes into the contribution of log-changes in weeks worked (blue area), log-changes in weekly wages (red area) and their covariance (green area). Data are from the LoSai dataset pooled over the period 1977-2012.

for workers between the 10th and the 90th percentile of $x_{i,t}$ and slightly less for workers in the bottom and top decile[15]. As a result most of the differences in the variance of $\Delta y_{i,t}$, comes from the intensive margin, suggesting that employment-to-unemployment (EU) and unemployment-to-employment (UE) transitions are key in determining the heterogeneity of income risk across types. Taking into account these margins of adjustment seems therefore key in assessing policies aimed at reducing or smoothing income risk among workers.

Cyclical Properties We now turn to the cyclical properties of income risk for different types of workers. We compute a set of statistics on the distribution of $\Delta y_{i,t}$ for $t \in [1982, 2012]$ and study how they evolve with the aggregate state of the economy as measured by the cyclical component of (log) total GDP[16]. We focus here on measures of location (mean), dispersion (inter-quartile range) and skewness (Bowley's measure)[17]. To

[15] A small negative covariance between the two terms is also found in Hoffmann and Malacrino (2019) and it may be partially due to mis-measurement of the extensive margin variable (weeks worked).

[16] We use a smoothing parameter of 100.

[17] Inter-quartile range and Bowley's measure are robust to outlyers. Bowley skewness is defined as $\frac{F^{-1}(0.75)+F^{-1}(0.25)-2F^{-1}(0.5)}{F^{-1}(0.75)-F^{-1}(0.25)}$. We HP-filter all three measures with a smoothing parameter of 100 and remove

Table 1: Co-movements with aggregate output

	Aggregated	Quartile of Past Weekly Wage: 1st	2nd	3rd	4th
Mean	0.633	0.869	0.603	0.555	0.444
	(0.210)	(0.272)	(0.238)	(0.21)	(0.168)
Dispersion (IDR)	-0.298	-0.991	-0.612	-0.199	0.181
	(0.322)	(0.417)	(0.429)	(0.325)	(0.183)
Skewness (Bowley)	0.96	2.121	0.892	0.641	0.309
	(0.454)	(0.84)	(0.567)	(0.478)	(0.271)

Note: The table presents the OLS coefficient of log annual GDP on the first three moments of the distribution of log income changes by quartiles of the income distribution. The series used in the estimation are hp-filtered with a smoothing parameter 100. Data are from the LoSai dataset for the period 1981-2012. Standard errors are reported in brackets.

describe the cyclical properties of these statistics we then run an OLS of our descriptive statistics on the contemporary level of (cyclical) GDP. We do this by quartile of $x_{i,t}$ and present the results in Table 1.

Three facts on cyclical income risk are evident from 1: the mean income shock is pro-cyclical; the dispersion of income shocks is mildly counter-cyclical; the skewness of income shocks is pro-cyclical (left-skewness is counter-cyclical). What is also clear from Table 1 is that the magnitude of these cyclical properties varies considerably across workers' types, with a cyclical sensitivity always monotonically decreasing in the average of past wages. The effect of a one percent drop in the GDP on the average growth rate of income for workers in the bottom quartile of past wage is almost twice as big as the effect on workers in the top quartile. The same is true for the skewness, where the effect at the first quartile is almost seven times higher than the effect at the fourth quartile. Differences are also clearly visible when considering the counter-cyclicality of dispersion. The inter-decile range is strongly counter-cyclical for low wage workers, while pro-cyclical for worker in the top quartile of $x_{j,t}$ (though the coefficient is not significant)[18]. Taken together this evidence suggests that low wage workers are on average more sensitive to the aggregate state of the economy compared to high wage workers.

In the rest of the paper we will focus on cyclical first moment risk: cyclical fluctuations

the trend component.

[18]The fact that dispersion is found to be significantly cyclical only for low wage workers might in part explain the lack of consensus in the literature on the cyclicality of the second moment.

in the first moment of the distribution of income conditional on a worker's type. The evidence presented above suggests that the unequal sensitivity of labor income to the aggregate conditions of the economy is a robust business cycle fact that involves the entire distribution of income. Aggregate shocks are transmitted disproportionately more to low wage workers, who tend to face higher cyclical income risk. Works estimating the cost of income shocks have also shown them to be higher for low income workers, which have also been found to be less able to self-insure. Redistributing cyclical income risk away from low wage workers can therefore have important implications for aggregate welfare.

3 Model

The previous section has underlined that workers differ in the intensity of income risk they face along the business cycle. This section presents a frictional model of the labor market, which can rationalize the patterns we observe in the data. We introduce different types of workers in order to model the heterogeneity in terms of exposure to labor income shocks. The model can be seen as limit case of the model of Lise and Robin (2017), with a single type of firm rather than a continuum of types and a tax on wages. Alternatively, one may see the following model as an extension of the model Robin (2011), in which the job meeting rate is time-varying, rather than equal to its steady-state value, and a tax on labor income is levied.

3.1 Agents and Timing

We make the assumption that the economy is populated by a continuum of infinitely-lived workers differing in their "ability", indexed by x. The ability level is fixed and measures a worker's productivity. We make the assumption that there is a continuum of firms having access to the same technology. The measures of workers and firms is normalized to one. The distribution of ability x is exogenous and denoted by $\ell(x)$. The measure of unemployed workers of type x at time t, denoted by $u_t(x)$ is endogenous. It is determined by the number of vacancies posted by firms, their hiring decision and the search effort of worker.

When opening v vacancies, firms incur an exogenous cost $c(v)$. We assume that there are no barriers to entry. Hence, the total number of vacancies is determined by a zero-profit condition. To introduce business cycle fluctuations, we make the assumption that agents face an aggregate productivity uncertainty. A the beginning of each period, the aggregate state of the economy changes from z_{t-1} to z_t according to the Markov transition probability $\pi(z_{t-1}, z_t)$.

We make the assumption that search on the labor market is random. That is, firms and workers randomly search for potential partners. The type of a worker is revealed during an interview, but there is no screening device that would allow firms to filter applicants in the first place. We also make the simplifying assumptions, which are common in the literature, that (i) each firm can only hire one worker (ii) workers can apply to only one job per period.

3.2 Labor Contracts

This section describes the assumptions we make regarding the contractual agreement between firms and workers. Labor contracts are such that a firm commits to pay a fixed wage w to a worker. We make the assumption that contracts are renegotiable by mutual consent only, as in Postel-Vinay and Turon (2010). Let $S(x, w, \Gamma_t)$ denote the total surplus of a match (job), where the variable Γ_t is the aggregate state variable. The aggregate state variable contains at least the aggregate productivity level z_t. By definition, the total surplus writes:

$$S(x, w, \Gamma_t) = W_t(x, w, \Gamma_t) - U(x, \Gamma_t) + \Pi(x, w, \Gamma_t) - V(\Gamma_t) \tag{1}$$

where $W(x, w, \Gamma_t)$ denotes the value of being employed at time t for a worker of type x, with a wage w; $U(x, \Gamma_t)$ denotes the value of being unemployed at time t for a worker of type x; $\Pi(x, w, \Gamma_t)$ denotes the value of a job to a firm and $V(\Gamma_t)$ denotes the value of an unfilled vacancy to a firm.

Because we make the assumption that firms can freely participate in the labor market, firms enter the labor market and post vacancies as long as long as it is profitable to do so. In equilibrium, the value of any unfilled vacancy is equal to zero:

$$V(\Gamma_t) = 0. \tag{2}$$

We make the assumption that both workers and firms are fully rational and forward-looking. This assumption implies that only labor contracts that improve the situations of both parties with respect to their respective outside options exist. Hence, a labor contract offering a wage w at time t must satisfy the conditions:

$$\begin{cases} W(x, w, \Gamma_t) & \geq U(x, \Gamma_t) \\ \Pi(x, w, \Gamma_t) & \geq 0. \end{cases} \tag{3}$$

We assume that unemployed workers have zero bargaining power. Let $\phi_t^0(x)$ denote the wage offered to an unemployed worker of type x when hired at time t. By definition, the starting wage $\phi_t^0(x)$ is such that the worker receives exactly her outside option:

$$W(x, \phi_t^0(x), \Gamma_t) = U(x, \Gamma_t) \tag{4}$$

Labor contracts can be renegotiated by mutual consent only. That is, only credible threats to the existing contract are considered by agents. Credible threats are of two types.

Firstly, a worker may be contacted by another firm willing to offer a better wage. When such an event happens, a Bertrand competition between the poaching and incumbent firms leads the worker to receive the promotion wage $w = \phi_t^1(x)$. The promotion wage is defined as the wage that gives the worker the total surplus of a match:

$$W(x, \phi_t^1(x), \Gamma_t) = U(x, \Gamma_t) + S(x, \phi_t^1(x), \Gamma_t) \tag{5}$$

Secondly, a productivity shock might break the joint rationality condition (3). When the aggregate productivity shock is such that condition (3) no longer holds, three situations may occur. First, if the shock is such that the joint surplus of a match $S_t(x, w)$ is negative at the *smallest wage a worker is willing to accept*, which is by definition equal to $\phi_t^0(x)$, the match is destroyed and the worker becomes unemployed. Second, if a worker has a credible threat to quit, a renegotiation occurs and the worker obtains a wage increase to the starting wage. That is, if $S(x, w, \Gamma_t) > 0$ and $W(x, w, \Gamma_t) < U(x, \Gamma_t)$, to wage is re-negotiated to $w = \phi_t^0(x)$. Third, if a firm has a credible threat to break the match, which happens when $S(x, w, \Gamma_t) > 0$ and $\Pi(x, w, \Gamma_t) < 0$, the new wage re-negotiated down to the promotion wage $w = \phi_t^1(x)$.

3.3 Meeting Technology

This subsection describes the assumptions we make regarding how firms and workers meet. At the beginning of period t, a measure $u_t(x)$ of unemployed workers of type x and a measure $h_t(x, w)$ of workers of type x with a wage w are inherited from period $t-1$. We abstract from movements in and out the labor force by assuming that workers either employed or looking for a job. As a result, the following accounting identity holds:

$$u_t(x) + \int h_t(x, w) dw = \ell(x) \tag{6}$$

At the beginning of period t, the aggregate state of the economy changes from z_{t-1} to z_t. We assume that the separation phase occurs first. At the new aggregate state z_t, some jobs are no longer profitable and are destroyed. Some jobs are also destroyed exogenously with probability δ. Endogenous job destruction captures the unemployment caused by business cycle fluctuations, while δ represents the residual level of frictional unemployment. Then, the meeting phase takes place. Both unemployed and employed workers search for jobs. We assume that each unemployed worker search with an intensity normalized to 1, while employed workers search with an intensity s. The aggrregate search effort in period t is a linear aggregation of individuals' search effort:

$$L_t = \int_0^1 u_{t+}(x)dx + s\int_0^1 \int h_{t+}(x,w)dwdx, \tag{7}$$

In equation 7, $u_{t+}(x)$ denotes the measure of unemployed workers of type x after the separation phase and $h_{t+}(x,w)$ is the measure of employed workers of type x with a wage w after separation.

Once the search effort is realized, firms create a measure v_t of job opportunities. The total measure of meetings at time t is given by $M_t = M(L_t, V_t)$. The probability that an unemployed worker contact a vacancy in period t is given by $\lambda_t = M_t/L_t$. The probability that an employed searcher contacts a vacancy in period t is given by $s\lambda_t$. Let $q_t = M_t/V_t$ denote the probability per unit of recruiting effort v_t that a firm contacts any searching worker. After meeting occurs, if a worker and a firm have a mutual interest in working together, a labor contract is signed and production begins.

3.4 Bellman equations

Unemployed workers enjoy a flow of utility $b(x)$, measure the value of leisure. They also receive a lump-sum transfer l_t from the government. In period t, they meet a vacant position with probability λ_t. The value of being unemployed at time t for a worker of type x can be expressed as:

$$U(x,\Gamma_t) = b(x) + l_t + \frac{1}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\left[(1-\lambda_{t+1})U(x,\Gamma_{t+1}) + \lambda_{t+1}W(x,\phi^0_{t+1}(x),\Gamma_{t+1})\right] \tag{8}$$

Because we maintained the assumption that unemployed workers are only offered their reservation wage, equation (8) simplifies to:

$$U(x) = b(x) + l_t + \frac{1}{1+r}U(x) \tag{9}$$

The assumption of zero bargaining power makes the value of unemployment independent from the job meeting rate. If the flow value of being unemployed is independent from the aggregate state of the economy, the value of unemployment does not depend on the rest of the economy, as illustrated in equation (9).

Before writing the Bellman equation for the value of a job, let us make the observation that if $S(x, \phi_t^0(x), \Gamma_t)$ is greater or equal to 0, then the job is feasible in the sense that the joint rationality condition (3) holds. To see that, using the zero-profit condition and the definition of the starting wage (4), the joint surplus of a match with wage $\phi_t^0(x)$ writes:

$$S(x, \phi_t^0(x), \Gamma_t) = \Pi(x, \phi_t^0(x), \Gamma_t) \tag{10}$$

At the wage $\phi_t^0(x)$, the first line in (3) is met by definition of the starting wage. If $S(x, \phi_t^0(x), \Gamma_t) \geq 0$, then the second condition of (3) is also met, as made obvious by equation (10). A wage below $\phi_t^0(x)$ would break the match, as the worker would be better off unemployed. But if $S(x, \phi_t^0(x), \Gamma_t)$ is greater or equal to 0, there may exist higher wages compatible with the joint rationality condition (3).

The Bellman equation for the value of a job with a wage w to a worker of type x writes:

$$\begin{aligned} W(x, w, \Gamma_t) = \big(1 - \tau(w)\big)w + l_t + \frac{1}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\Big[&\delta U(x) \\ &+ (1-\delta)\mathbb{1}\left\{S(x, w, \Gamma_{t+1}) < 0\right\}\left[N(x, \Gamma_{t+1})\right] \\ &+ (1-\delta)\mathbb{1}\left\{S(x, w, \Gamma_{t+1}) \geq 0\right\}\left[s\lambda_{t+1}\big(U(x) + S(x, \phi_{t+1}^1(x), \Gamma_{t+1})\big)\right. \\ &\left.+ (1 - s\lambda_{t+1})\tilde{N}(x, w, \Gamma_t)\right]\Big] \end{aligned} \tag{11}$$

The flow value in equation (11) is the after-tax wage $\big(1 - \tau_w(w)\big)w$ plus a lump-sum transfer l_t. The continuation value is composed of four elements. Firstly, the job might be destroyed with probability δ, in which case the worker receives the value of unemployment. Secondly, while the job may not be profitable at the wage w inherited from last period, a wage renegotiation might restore job-feasibility. The intra-firm renegotiation that may follow is captured by the function $N(x, \Gamma_t)$ as follows:

$$N(x, \Gamma_t) = \begin{cases} W(x, \phi_t^1(x), \Gamma_t) & \text{if } S(x, \phi_t^0(x), \Gamma_t) \geq 0 \\ 0 & \text{if } S(x, \phi_t^0(x), \Gamma_t) < 0 \end{cases} \tag{12}$$

Equation (12) states that, if there exists at least one wage that makes the job feasible, the worker has full bargaining power and obtains the promotion wage $\phi_t^1(x)$. The third line captures the possibility that a worker meets with another firm, while being employed. Bertrand competition between the incumbent and the poaching firm implies that a series of offer and counter-offers is initiated. Independently of whether the worker changes job or stays, the worker receives the promotion wage. The fourth line takes into consideration intra-firm renegotiation for a worker who has no meeting with an alternative employer. Depending on aggregate productivity and on the state of the labor market, the different renegotiation possibilities are captures by the function $\tilde{N}(x, w, \Gamma_t)$, defined by:

$$\tilde{N}(x, w, \Gamma_t) = \begin{cases} W(x, w, \Gamma_t) & \text{if } 0 \leq W(x, w, \Gamma_t) - U(x) \leq S(x, w, \Gamma_t) \\ W(x, \phi_t^1(x), \Gamma_t) & \text{if } W(x, w, \Gamma_t) - U(x) > S(x, w, \Gamma_t) \\ W(x, \phi_t^0(x), \Gamma_t) & \text{if } W(x, w, \Gamma_t) - U(x) < 0 \end{cases} \tag{13}$$

The first line in (13) states that if both the worker and the firm have no credible threat to break the match, the match persists at the current wage w. The second line states that if the firm has a credible threat to break the match, the wage is re-bargaining down to $\phi_t^1(x)$. The third line states that if the worker is better off unemployed, she renegotiate her wage up to $\phi_t^0(x)$.

The expression for the worker's surplus, defined as $\Delta(x, w, \Gamma_t) \equiv W(x, w, \Gamma_t) - U(x)$, can be expressed as:

$$\begin{aligned} \Delta(x, w, \Gamma_t) = & \left[1 - \tau(w)\right] w - b(x) \\ & + \frac{1-\delta}{1+r} \mathbb{E}_{\Gamma_{t+1}|\Gamma_t} \Big[\mathbb{1}\left\{S(x, w, \Gamma_{t+1}) < 0\right\} R^w(x, \Gamma_{t+1}) \\ & + \mathbb{1}\left\{S(x, w, \Gamma_{t+1}) \geq 0\right\} \left[s\lambda_{t+1} S(x, \phi_{t+1}^1(x), \Gamma_{t+1}) \right. \\ & \left. + (1 - s\lambda_{t+1}) A^w(x, w, \Gamma_{t+1}) \right] \Big] \end{aligned} \tag{14}$$

where the functions $R^w(x, \Gamma_{t+1})$ and $A^w(x, w, \Gamma_{t+1})$ keep track of the different intra-firm renegotiations that may happen when at least one of two parties has a credible reason to break the match:

$$R^w(x, \Gamma_t) = \begin{cases} S(x, \phi_t^1(x), \Gamma_t) & \text{if } S(x, \phi_t^0(x), \Gamma_t) \geq 0 \\ 0 & \text{if } S(x, \phi_t^0(x), \Gamma_t) < 0 \end{cases} \tag{15}$$

$$A^w(x, w, \Gamma_t) = \begin{cases} \Delta(x, w, \Gamma_t) & \text{if } 0 \leq \Delta(x, w, \Gamma_t) \leq S(x, w, \Gamma_t) \\ S(x, \phi_t^1(x), \Gamma_t) & \text{if } \Delta(x, w, \Gamma_t) > S(x, w, \Gamma_t) \\ 0 & \text{if } \Delta(x, w, \Gamma_t) < 0 \end{cases} \tag{16}$$

By writing the Bellman equation for the value of a job to a firm and rearranging (see section A of the Appendix), on obtains the following equation for the total surplus of a match:

$$
\begin{aligned}
S(x,w,\Gamma_t) &= p(x,z_t) - \tau_w(w)w - b(x) \\
&+ \frac{1-\delta}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\Big[\mathbb{1}\left\{S(x,w,\Gamma_{t+1})<0\right\}R^w(x,\Gamma_{t+1}) \\
&+ \mathbb{1}\left\{S(x,w,\Gamma_{t+1})\geq 0\right\}\left[s\lambda_{t+1}S(x,\phi^1_{t+1}(x),\Gamma_{t+1})\right. \\
&\left.+ (1-s\lambda_{t+1})\big(A(x,w,\Gamma_{t+1})\big)\right]\Big]
\end{aligned} \tag{17}
$$

where the function $R^w(x,\Gamma_{t+1})$ is defined above and $A(x,w,\Gamma_{t+1})$ is a function that also keeps track of the different intra-firm renegotiation possibilities:

$$
A(x,w,\Gamma_t) = \begin{cases} S(x,w,\Gamma_t) & \text{if } 0 \leq \Delta(x,w,\Gamma_t) \leq S(x,w,\Gamma_t) \\ S(x,\phi^1_t(x),\Gamma_t) & \text{if } \Delta(x,w,\Gamma_t) > S(x,w,\Gamma_t) \\ S(x,\phi^0_t(x),\Gamma_t) & \text{if } \Delta(x,w,\Gamma_t) < 0 \end{cases} \tag{18}
$$

It is important to observe that the value functions $\Delta(x,w,\Gamma_t)$ and $S(x,w,\Gamma_t)$ are sufficient to determine both job feasibility and the evolution of wages.[19] At each period, the starting and the promotion wages are implicitly defined by:

$$
\begin{aligned}
\Delta(x,\phi^0_t(x),\Gamma_t) &= 0 \\
\Delta(x,\phi^1_t(x),\Gamma_t) &= S(x,\phi^1_t(x),\Gamma_t)
\end{aligned} \tag{20}
$$

3.5 Vacancy Creation and Labor Market Flows

To close the model, let us characterize vacancy creations and the flow equations determining employment dynamics. Each period, firms can advertise of v jobs through job placement agencies at a price $c(v)$. To insure existence and uniqueness of v_t, we assumed that $c(.)$ is an increasing and convex function. In equilibrium, firms choose v such that marginal cost and the expected returns of creating v vacancies are equal:

$$
c'(v_t) = q_t J_t \tag{21}
$$

[19] If there is no tax on wages, the total value of a match in equation (17) simplifies to the value function in Robin (2011):

$$
S(x,z_t) = p(x,z_t) - b(x) + \frac{1-\delta}{1+r}\mathbb{E}_{z_{t+1}|z_t}\Big[\max\left(S(x,z_t),0\right)\Big] \tag{19}
$$

When a firm hires from the pool of unemployed workers, it captures all the value possible by setting the wage to the reservation wage. When a firm poaches a worker from another firm, it must pay the promotion wage (leaving the poaching firm with zero profits). Hence, the expected value of a contact for a firm writes:

$$J_t = \int_0^1 \frac{u_{t+}(x)}{L_t} \max\left(S(x, \phi_t^0(x), \Gamma_t), 0\right) dx \tag{22}$$

The measure of employed workers $h_{t+}(x)$ in the sub-period t_+ is equal to the measure of employed at the end of the period t, minus the jobs both exogenously and endogenously destroyed:

$$h_{t+}(x) = (1-\delta)\mathbb{1}\{S_t(x, \phi_t^0(x)) \geq 0\} h_t(x). \tag{23}$$

At this point, it should be clear that the state variable Γ_t contains the aggregate productivity level z_t and the distribution of employment across types $h_t(x)$. The distribution of employment across types enters the state variable because it impacts the aggregate search effort of workers (equation (7)). It also enters the expected value of a contact for firms (equation (22)). Hence, the job meeting rate λ_t depends on $h_t(x)$.

To solve the model in the presence of an infinite-dimensional state variable, we use the projection-and-perturbation method of Reiter (2009). This method relies on three steps. First, we solve for the non-stochastic steady-state of the model using an iterative scheme. Secondly, we numerically differentiate the equilibrium objects around the non-stochastic steady-state, which results in a linear rational expectation model. Thirdly, we use the method of Sims (2002) to obtain a state-space representation of the linearized model. Details are presented in section C of the Appendix.

4 The Role of Income Tax Progressivity

What is the effect of an income tax on the labor market? How does changing its degree of progressivity affect its equilibrium and how does it impact the income process of workers of different types? First note from equation (17) that, for a given wage w, the total private surplus of a match $S(x, w, \Gamma_t)$ is decreasing in the average tax rate $\tau_w(w)$. Second, the private and worker's surplus are both strictly increasing in a worker's type x. Third, the starting and promotion wages are both strictly increasing in the worker's type x[20]. Finally, as long as the average tax rate is increasing with w, a match is unfeasible if the

[20]This is true as long as the function $b(x)$ is strictly increasing in x and the marginal tax rate is is always below 100%.

surplus is negative at the lowest acceptable (net) wage for the worker $\phi_t^0(x)$, as stated in equation (15).

The first effect of increasing tax progressivity is therefore to increase, for any given aggregate productivity level, the surplus at $\phi_t^0(x)$ for low x workers relative to high x workers, effectively reallocating unemployment risk away from low x types. Second, tax pogressivity increases the maximum feasible wage $\phi_t^1(x)$ for low x types relative to high x types, increasing in relative terms their promotion wage. These first two channels effectively reduce left tail risk (i.e. big drops in income) and increase the size of positive shocks (i.e. match formation rate and promotion wages) for low x workers relative to high x workers, redistributing third moment risk.

Next, note that having higher match surpluses, high x workers form matches that remain profitable under a wider range of aggregate productivity levels compared to matches formed by low x types. Reallocating tax burden from low to high x workers has therefore the potential to increase the total amount of feasible matches during periods of low aggregate productivity. In this sense, on top of its re-distributive aspect, a progressive taxation can play a role in stabilising total unemployment and reduce the average unemployment risk.

Lastly, note from equation (21) and (22) that in our model vacancy creation is a function of the expected value of filling the vacancy in the labor market and therefore a function of the entire distribution of surpluses across worker types. By reallocating the tax dead-weight across match types, changing the degree of progressivity can affect the expected return from posting vacancies and therefore, ultimately affect the market tightness and the number of jobs created in the economy.

5 Parametrization and Estimation

This section describes our parametrization and how we estimate the model via the simulated method of moments using Italian data. Our parametrization is standard relative to the search-and-matching literature. We discuss identification and present the fit of the model.

5.1 Parametrization

For the value added at the match level, we use a polynomial of the form, were the aggregate productivity enters multiplicatively:

$$p(x,z) = z(p_1 + p_2 x + p_3 x^2) \tag{24}$$

The flow value of being unemployed is defined as a constant fraction of value added when the aggregate state of the economy is at its neutral level:

$$b(x) = \gamma \times p(x,1) \tag{25}$$

For the distribution of workers types, we assume that it follows a beta distribution with shape parameters α and β.

$$\ell(x) = \mathrm{B}(x|\nu,\mu) \tag{26}$$

For the cost of posting v vacancies, we use a power function of the form:

$$c(v) = \frac{c_0 v^{1+c_1}}{1+c_1}, \tag{27}$$

For the matching function, we use a Cobb-Douglas specification, which is standard in the search-and-matching literature:

$$M = \min\{\alpha L^{\omega} V^{1-\omega},\, L,\, V\}, \tag{28}$$

Combining assumptions (27) and (28) leads to the following expression for the market tightness for an interior solution:

$$\theta_i = \Big(\frac{1}{L}\big(\frac{\alpha J}{c_0}\big)^{\frac{1}{c_1}}\Big)^{\frac{c_1}{c_1+\omega}} \tag{29}$$

The corresponding number of vacancies created is given by:

$$V = \Big(\frac{\alpha}{\theta^{\omega}}\frac{J}{c_0}\Big)^{1/c_1} \tag{30}$$

For the aggregate productivity process, we make the assumption that it follows an $AR(1)$ process of the form:

$$log(z_t) = \rho log(z_{t-1}) + \varrho\sqrt{1-\rho^2}\varepsilon_t;\ \varepsilon_t \sim \mathcal{N}(0,1), \tag{31}$$

We use the following non-linear function for the tax schedule on wages:

$$\tau(w) = \sum_{i=1}^{T} \mathbb{1}\{w > \bar{\tau}_{i-1}\}(w - \bar{\tau}_{i-1})(\tau_i - \tau_{i-1}) \tag{32}$$

Table 2: Income Tax - Italy 2007-2018

	Income Interval	Marginal Tax Rate
1st Bracket	0 - 15k	23%
2nd Bracket	15k - 28k	27%
3rd Bracket	28k - 55k	38%
4th Bracket	55k - 75k	41%
5th Bracket	above 75k	43%

Notes: This Table displays the marginal tax rate for each income interval in Italy, for the period 2007-2018. Tax deductions and regional taxes are omitted.

where $\bar{\tau}$ and τ are thresholds and marginal tax rates respectively, with $\bar{\tau}_0 = 0$ and $\tau_0 = 0$. One difficulty lies in converting the empirical thresholds and marginal tax rates into ones that make sense within our model. We use both the empirical and the simulated distribution of wages to achieve such a conversion (see section F of the Appendix).

5.2 Estimation

In the present setting, the likelihood function is untractable. This is why we estimate the model using the Simulated Method of Moments (SMM). By using the projection-and-perturbation approach developed in Reiter (2009), we are able to solve and simulate the model efficiently, which makes the estimation feasible. One evaluation of the *GMM objective function* involves solving for the non-stochastic steady-state of the model, linearizing around the non-stochastic steady-state, solving for the linearized rational expectation model and calculated moments using simulated data. The entire process takes less than a minute on a standard personal computer.

We choose to fit moments capturing long-run unemployment and unemployment by duration. The long-run unemployment is informative on the exogenous job destruction rate δ. Unemployment by duration is informative on the distribution of worker types. We also include moments related to the job vacancy rate[21]. The job vacancy rate is informative on parameters related to the cost of posting vacancies. We also include moments on Italian GDP, which is informative on the production function.

We calibrate some of our parameters and make simplifying assumptions to make the estimation feasible. In particular, we do not take into consideration variations in the tax schedule. A model with time-varying tax schedule would generate additional complexities, because one would need to take into consideration the expectation of agents relative

[21]The job vacancy rate is defined as: $JVR \equiv \frac{\text{vacancies}}{(\text{occupied positions}+\text{vacancies})}$

Table 3: Model Parameters

Calibrated			*Estimated*		
Parameter	Value	Source	Parameter	Value	Description
r	0.05	Lise, Robin (2017)	c_0	0.0063342593	cost of vacancies
threshold 1	29.4th quantile	Calibration	c_1	0.8201400264	cost of vacancies
threshold 2	76th quantile	Calibration	p_1	-0.6829041751	production function
threshold 3	95.5th quantile	Calibration	p_2	-3.5208027169	production function
threshold 4	98th quantile	Calibration	p_3	9.4237581222	production function
τ_1	23%	Tax data (2007-2012)	α	0.4989120534	matching function
τ_2	27%	Tax data (2007-2012)	s	0.2729833373	search effort
τ_3	38%	Tax data (2007-2012)	a_0	0.0912183629	AR(1) parameter
τ_4	41%	Tax data (2007-2012)	a_1	0.8600589669	AR(1) parameter
τ_5	43%	Tax data (2007-2012)	ν	209.8907695827	worker types
			μ	167.6475407856	worker types
			δ	0.1007817879	exogenous job destruction
			ω	0.7034116274	matching function
			γ	0.6083395126	home production

Notes: The table displays our calibrated parameters.

future changes in the tax schedule. Instead, we set the tax schedule to fit the Italian empirical annual income tax schedule for the period 2007-2012 (see Table 23)[22]. We do this by matching the empirical tax thresholds on the HP-filtered quantiles of the empirical annual wage distribution, averaged over the period 2007-2012 (we omit tax deductions). Tables 3 display our calibrated and estimated parameters respectively.

The fit of the model is presented in Table 4. The model does a good job at fitting the long-run unemployment rate and the job vacancy rate. The model also captures well important correlations between the aggregate values for unemployment, job vacancy rate and GDP. In terms of fitting standard deviations and auto-correlations, the fit of the model is satisfactory, except for long-term unemployment rates which are more auto-correlated and more volatile in the data.

6 Model Implications and Counter-Factual Exercises

This section describes a counter-factual experiment in which we modify tax progressivity, while keeping the government's revenues constant. This section shows that tax progressivity may be used to smooth business cycle fluctuations and reallocate cyclical income

[22]In this paper we focus only on the system of marginal rates reported in 23. For simplicity we do not considered and local tax rates. We also do not consider the set of tax deductions applicable to the *IRPEF* income tax. While these deductions substantially increase the overall degree of progressivity of the tax system. Tax deductions of this type are usually supported by most of the authors in favour of the application of a flat tax rate, see for example Hall and Rabushka (2013). We also need to convert the tax thresholds which are based on income into their wage equivalent thresholds from our model. We describe how we do this in Appendix F.

Table 4: Empirical and simulated moments

Moment	Data	Model
$\overline{\mathbb{E}}U$	0.090	0.090
$\overline{\mathbb{E}}U_{1+}$	0.084	0.061
$\overline{\mathbb{E}}U_{3+}$	0.072	0.060
$\overline{\mathbb{E}}JVR$	-0.394	-0.451
$corr(JVR, GDP)$	0.896	0.915
$corr(JVR, U)$	-0.579	-0.345
$corr(U, GDP)$	-0.421	-0.954
$std(JVR)$	0.194	0.089
$std(U)$	0.086	0.086
$std(U_{1+})$	0.074	0.013
$std(U_{3+})$	0.076	0.005
$autocorr(GDP)$	0.840	0.832
$autocorr(JVR)$	0.850	0.887
$autocorr(U)$	0.675	0.905
$autocorr(U_{1+})$	0.387	0.070
$autocorr(U_{3+}$	0.422	-0.008

Notes: This Table displays moments based on empirical data (first column) and on simulated date (second column). $\overline{\mathbb{E}}(x)$ is used denotes the empirical average for the variable x. $corr(x, y)$ is used to denote the empriical covariance between x and y. $std(x)$ denotes the empirical standard deviation for the variable x. $autocorr(x)$ denotes the empirical autocovariance of x. U is the aggregate unemployment rate; U_{1+} is the unemployment rate for one month or more; U_{3+} is the unemployment rate for 3 months or more; JVR it the job vacancy rate; GDP is gross domestic product.

risk. We explain the main mechanism behind this finding.

6.1 A flat tax experiment

The counterfactual experiment we run is based on the idea that a government may want to redistribute the tax burden across workers, while not changing its total revenue. In particular, we study the replacement of the Italian stepwise tax schedule for the period 2007-2018 by a "flat tax"[23]. Our experiment can be seen as a special case of what has been proposed by several economists, in particular Hall and Rabushka (2013). The authors propose the replacement of all existing federal income and corporation income taxes by a single tax rate applied to labor income above a given threshold, and all (gross) capital

[23] In our exercise we do not consider the set of tax deductions and local income taxes that applicable on top of the stepwise marginal tax rate shown in table (see the Appendix). In important to note that the tax thresholds in our model are based on the wage rather than on annual income as in the data. While this has no impact on the flat tax economy (as there is no threshold), it should be seen as only an approximation of the current stepwise income tax schedule in Italy.

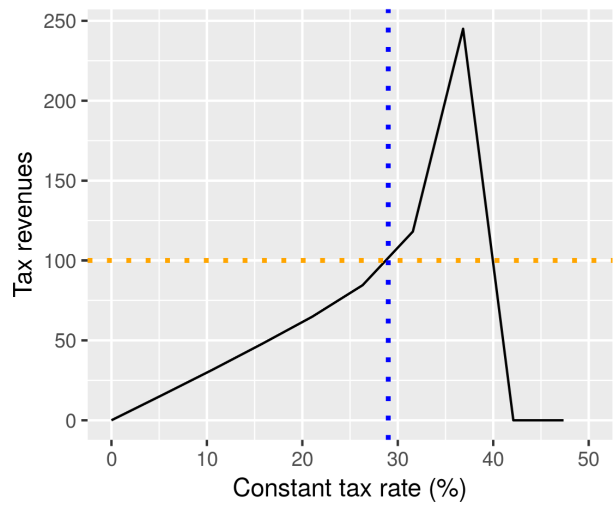
income after full investment deductibility. In our model, we do not have capital and for simplicity we ignore the possibility of a constant for tax rate that may start to apply only after a given threshold. While we cannot capture the distortions of taxes on capital accumulation, our model is well suited to study how changes in labor income taxation alters the equilibrium in the labor market.

We proceed by first determining the value of the tax neutral flat tax. Firstly, we calculate the total value of taxes collected in the economy with a stepwise tax function (our baseline estimated model). Secondly, we solve and simulate the model with different tax rates. Results are presented in Figure 3a: there are two constant tax rates that are such that the government's revenue is left unchanged. We select the smallest value, which is equal to 29%. While it is theoretically feasible that this constant tax rate is lower than the smaller marginal tax rate of the stepwise tax schedule, through an increase in job creation and in the total number of matches in the economy, this is not the case here. The estimated model indeed suggests that the counter-factual tax schedule has a positive effect on the return to vacancy creation. Compared to the baseline economy, firms in the counter-factual economy are in fact found to post more vacancies relative to the number of unemployed. The vacancy to unemployment ratio is estimated to be 13% higher in the flat tax economy. However, while this higher vacancy posting ratio leads to a higher job meeting rate, it fails to translate into a higher unemployment-to-employment (U2E) transition rate. The average U2E transition rate actually falls by 12% in the counter-factual economy. By rising the tax rate applied to low wages, the flat tax reduces the surplus from low types matches. As low type workers are over-represented in the pool of unemployed workers, the flat tax reduces the *job matching rate* conditional on meeting a vacancy and dominates the positive effect on the job meeting rate.

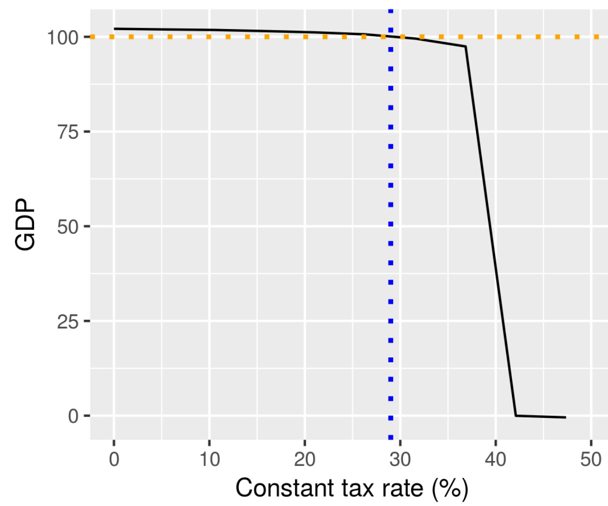
The positive effect on vacancy posting is therefore far from being able to finance a generalised decrease in the tax rate. To offset the losses of taxing less the high earners, the government has to tax more the low earners (see Figure 3c). As a result, most of the workers end up paying more than in the baseline economy, with our simulations suggesting that around 87% of the workers in the economy see an increase in their average tax rate.

Finally, in Figure 3b we plot the level of total GDP in the economy as a function of the constant tax rate (x-axis). GDP is maximized by setting the constant tax rate to 0. This is not a surprise given that taxes have no purpose in this economy.

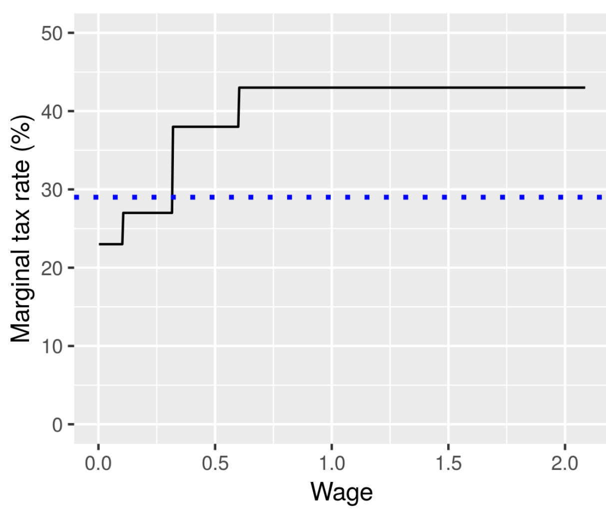
Figure 3: Revenue-neutral flat tax



(a) Total tax revenues as a function of the constant tax rate



(b) Total GDP as a function of the constant tax rate



(c) Stepwise tax schedule and flat tax schedule

Notes: This figure shows the links between the constant tax rate $\overline{\tau}$ of the flat tax and other economic variables. Panel 3a shows the total amount of taxes collected as a function of the constant rate rate $\overline{\tau}$. Values are normalized so that the total amount of taxes collected with the baseline economy (with a step-wise tax schedule), represented by the horizontal orange line, is equal to 100. The vertical blue line represents the revenue-neutral flat tax rate.
Panel 3b shows the GDP as a function of the constant rate rate $\overline{\tau}$. Values are normalized so that the GDP in the baseline economy (stepwise tax schedule) is equal 100, represented by the horizontal orange line. The vertical blue line represents the revenue-neutral flat tax rate. Panel 3c displays the stepwise tax schedule (in black) and the revenue-neutral flat tax rate (in blue).

6.2 Distributive Effect on Income and Income Risk

The most direct effect of a more progressive tax is to redistribute resources from high to low wage workers. In this section we explore the re-distributive role of the stepwise tax schedule shown in table 23. We leverage on our structural approach to compute statistics that condition on a worker's unobserved type, allowing us to study how tax progressivity reallocates aggregate risk from low to high wage workers. Our analysis focuses on the cyclical properties of the first moment of the income distribution, conditional on a worker's type. Our model is well suited to study the cyclical dynamics of this conditional moment[24].

We compute the average type-specific income $\bar{Y}_{k,t}$ for each year in our sample and study its cyclical properties. The time-series of $\bar{Y}_{k,t}$ contains important information on the exposure of each type of worker to the business cycle and gives us a measure of the degree of cyclical risk (first-moment risk) that a worker faces. We derive this time-series in the baseline (stepwise) and counter-factual (flat tax) economy and compute its mean, standard deviation and skewness for both net and gross income. In performing our experiment we maintain the same process for the aggregate TFP shocks z_t in the two economies. To deal with zero income observations (unemployment), standard deviation and skewness are derived on the percent deviation from the long-run mean, $(\bar{Y}_{k,t} - \bar{Y}_k)/\bar{Y}_k$[25].

To asses the effect of tax progressivity at different points of the distribution of types we compute statistics for 5 types of workers. The first four are selected to be representative of the 4 quartiles of the distribution of types. We select the 12.5, 37.5, 62.5 and 87.5 percentiles of the distribution. Given that from the discussion in section 6.1 the constant tax rate is likely to benefit only workers in the very high portion of the type distribution, we also consider the type at the 97.5 percentile. In what follows we define these workers as type 1, 2, 3, 4 and 5, respectively.

Table 5 presents the results of our simulations. We first compute the (long-run) average income over our entire simulated period, $\bar{Y}_k$: the first moment of the time series of $\bar{Y}_{k,t}$. This statistics gives us a measure of the average gain and loss that each type of worker experience when moving from the baseline to the counter-factual tax system. The results show that the relative gains from a flat tax are increasing in a worker's type, with type

[24] On the other hand, our model is not well suited to study cyclical changes in higher-moment risk. As firms are homogeneous and workers' types are fixed over time, cyclical idiosyncratic differences conditional on a worker's type are small. The threshold level of z_t that guarantees the feasibility of a match is the same for all workers of the same type x. This leaves little room for those within-type differences that would make studying higher-moment risk an interesting exercise. Expanding our model to allow for firm heterogeneity and dynamic workers' types is an interesting avenue for future research.

[25] This gives us a way to compare the relative volatility across types without using logs, allowing us to deal with the presence of unemployment (zero income observations).

Table 5: Average Income by Type: Time-Series Variation

Type	Net Income					Gross Income				
	1	2	3	4	5	1	2	3	4	5
						Mean Income (Level)				
Stepwise	0.0241	0.122	0.184	0.263	0.382	0.0314	0.163	0.255	0.374	0.561
Flat	0.0225	0.113	0.163	0.267	0.398	0.0317	0.159	0.229	0.376	0.56
						Standard Deviation				
Stepwise	0.2237	0.0861	0.0623	0.111	0.1436	0.2237	0.0831	0.0576	0.1139	0.1464
Flat	0.2798	0.1023	0.0932	0.1235	0.151	0.2798	0.1023	0.0932	0.1235	0.151
						Skewness				
Stepwise	-2.25	0.47	0.56	-0.32	-0.19	-2.25	0.47	0.59	-0.32	- 0.18
Flat	-3.2	-0.17	-0.09	-0.31	-0.17	-3.2	-0.17	-0.09	-0.31	-0.17

Notes: The statistics are computed from the simulated samples in the baseline (stepwise) and counter-factual (29% flat tax rate) economy. The left side of the table reports statistics for the average type-specific net income, while the right side reports the same statistics for the average type-specific gross income. Type 1, 2, 3, 4 and 5 corresponds to workers at the 12.5, 37.5, 62.5, 87.5 and 97.5 percentile of the distribution of worker types.

1 and 2 losing around 7% of their long run average net income and workers of type 4 seeing a 1.5% gain. Only workers at the very top of the distribution see an significant positive effect, with type 5 gaining around 4.2% on their average net income. Comparison between the effect on net and gross income suggests that, for workers in the top and bottom quarters of the distribution, almost the entire effect on net income comes directly from the net-gross income difference. With the exception of type 3, the difference in gross average wages between the two tax system is in fact very small, suggesting that the net equilibrium effect of tax progressivity on the long-run average gross income is small[26].

Next we compute the difference in the volatility of average income over-time, calculated as the standard deviation in $(\bar{Y}_{k,t} - \bar{Y}_k)/\bar{Y}_k$. This is a measure of the overall responsiveness of a type's specific income to the cyclical fluctuations in the economic environment. It therefore gives us information on the degree of cyclical risk that each worker type faces. The results show that the constant tax rate has a clear positive effect on the volatility of income for low type workers, with type 1 seeing a 25% increase in the time-series standard deviation of its expected income. Interestingly, while the magnitude of the effect is decreasing in a worker's type, all 5 types see an increase in income volatility. This should not be seen as a counter-intuitive result. Let us consider workers of type 5. On one hand, given their high productivity, the unemployment probability of these highly productive workers is almost insensitive to the change in the tax rate[27]. On the

[26]As the rest of the results in table 5 suggest, this does not mean that the labor market equilibrium is not affected by the progressivity of taxation. As several forces are at play, the result simply suggests that the sum of these their effects is small.

[27]The unemployment probability for type 5 drops by less then 1% in the flat tax economy.

other hand, under a constant tax rate they loose the wage insurance channel coming from a progressive tax schedule[28]. Putting the two together, the second effect dominates, resulting in higher income volatility. Considering the difference between the effect on net and gross income, suggests that most of the change in volatility comes from equilibrium forces affecting the determination of the gross wage. Most of the reduction in income volatility under a progressive system is in fact already observable on the standard deviation in gross income. This comparison suggests that ignoring the effect of a progreesive tax on the determination of gross income would considerably understate its role as a tool to reduce the cyclical risk faced by low income workers.

Finally, we look at the time-series skewness of average income conditional on a worker's type. This measure gives us a way to gauge whether the increase in volatility is caused mainly by negative rather than positive shocks. Once again, the results show that the insurance effect of tax progressivity is decreasing in a worker's type. Left-tail aggregate risk, as measured by the time-series left (negative) skewness of average income, is reduced for workers up to type 3 and redistributed to type 4 and 5 who see a slight increase in their left skewness[29]. The result shows that the re-distributive effect on aggregate risk works mostly trough the re-allocation of cyclical negative income shocks away from low wage workers. Similarly to the volatility of average income, a comparison between the statistics computed on net and gross income shows that most of the re-distributive effect comes from changes in the determination of gross income (equilibrium effect).

Overall the results in this section show that a progressive income tax can play an important role in reallocating aggregate risk away from low wage workers, mostly by reducing the magnitude of the negative cyclical shocks to their income. The magnitude of the effect is also large. Going from a tax-revenue equivalent flat tax to the current Italian system increases the average net income of worker at the 12.5*th* percentile of the productivity distribution by around 7% and reduces by $1/5th$ the difference in income volatility between workers at the 12.5*th* and 87.5*th* percentile of the type distribution. The results also show that the equilibrium effects of a progressive tax on the determination of gross income are important. Considering only the effect on the difference between net and gross income would considerably understate its ability to reallocate risk across agents.

[28] Type 5 workers gain on average from a flat tax, but the volatility of their wage increase. This is simply due to the fact that under a progressive tax system when their are hit by a bad shock their average tax rate decreases with their wage, while it remains constant under a flat tax.

[29] As already discussed above we are here referring to the properties of aggregate risk as measured by the time series properties of the average income conditional on a worker's type. We are not referring to the cyclical left skewness in idiosyncratic risk (i.e. within type, cross-sectional distribution of income) studied in Guvenen et al. (2014).

Table 6: Counter-Factual: Aggregate Employment

	Aggregate Employment	
	Step Tax	Flat Tax
Mean	0.91	0.902
Standard Deviation (log)	0.053	0.065
Skewness (log)	-1.76	-1.91

Note: *Notes*: The table displays a set of statistics on the time-series properties of aggregate employment. The second column shows the values under a step tax (baseline model). The third column shows the values under a 29% flat tax (counter-factual model).

6.3 Aggregate Employment

Next, we look at the effect of our policy experiment on the aggregate dynamics of employment. As discussed in the previous sections, reallocating the tax burden away from low to high surplus workers can have a stabilising effect on the economy by reducing the average volatility of firm-workers matches.

We compute the share of workers employed at each point in time in the baseline and counter-factual economy, E_t, and study its cyclical properties in our simulated business cycle. Once again, we keep the realisations of z_t fixed in the baseline and counter-factual economies. We look at a set of statistics describing the time-series properties of E_t. The results are reported in table 6. We first look at the long-run average employment rate, which in our model is simply equal to one minus the unemployment rate[30]. As discussed in section 4, reallocating the tax burden away from low type workers who form low surplus match can increase the overall level of employment. The first row in table 6 supports this view. The average employment rate over the simulated periods is 0.8 percentage points higher (and the unemployment rate 0.8 points lower) in the baseline economy.

Next we look at the standard deviation of $log(E_t)$, a measure of the volatility in aggregate employment. As seen in section 4, by the increasing the set of feasible matches when the economy is hit by negative productivity shock, the reallocation of the tax burden towards high surplus matches can stabilise unemployment and reduce its volatility. The second row in table 6 shows that the baseline stepwise tax schedule plays an important role in stabilising aggregate employment. The standard deviation of $log(E_t)$ is 22% higher under the revenue equivalent flat tax. The stabilising role of tax progressivity is confirmed in the third row, which reports the time-series skewness of $log(E_t)$ in the two economies. The estimated skewness is 8% lower under a stepwise tax, suggesting

[30]In our model workers are either employed or unemployed as labor supply decisions are not modelled.

that stabilising role of the baseline tax schedule is asymmetric, working mostly trough a reduction in the negative movements of $log(E_t)$.

Overall, our analysis of the simulated time-series for the aggregate employment rate suggests that in the presence of frictions in the labor market, income tax progressivity is not just a tool to redistribute risk across workers and that it can also play an important role in stabilising the cyclical fluctuations in aggregate employment.

7 Conclusion

In this paper we use Italian administrative data to study the role of income tax progressivity in redistributing cyclical risk from low to high wage workers and reduce aggregate employment volatility. Our estimated model suggests that a progressive tax schedule is effective at both, reducing the risk gap between low and high type workers and reducing the overall level and volatility of the unemployment rate. Our results show that the effect of taxation on the equilibrium determination of gross income is important. Considering only its effect on the difference between net and gross income would substantially understate the insurance role of a progressive tax system.

Our counter-factual exercise suggests that substituting a progressive stepwise marginal tax schedule with a tax-revenue-equivalent constant tax rate would come at the expense of the majority of the working population. In the Italian case considered in this paper, substituting the current system of marginal tax rates while keeping the government's revenue fixed would require setting a 29% flat rate and would impose a considerable cost on low income workers.

In our exercise we have focused on the effect of tax progressivity on cyclical first moment risk. Our model is well suited to perform such an analysis. Nonetheless, as we have shown in our descriptive section, the business cycle has also implications for higher-moment risk. Studying the effect of a progressive taxation on the higher moments of the type-specific income distribution would require a model with firm heterogeneity and/or a dynamic worker type. We see this as an interesting avenue for future research.

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Appendix

A Bellman equations

This section derives the Bellman equations for the worker surplus and for the total surplus of a match.

A.1 Value of a job to workers

Using the definition of $\phi_t^1(x)$ and $\phi_t^0(x)$, the equation (11) can be rewritten as:

$$
\begin{aligned}
W(x,w,\Gamma_t) = & \big(1-\tau(w)\big)w + l_t \\
& + \frac{1}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\Big[\delta U(x) \\
& + (1-\delta)\mathbb{1}\left\{S(x,w,\Gamma_{t+1}) < 0\right\}\left[U(x) + R^w(x,w,\Gamma_{t+1})\right] \\
& + (1-\delta)\mathbb{1}\left\{S(x,w,\Gamma_{t+1}) \geq 0\right\}\left[s\lambda_{t+1}\big(U(x) + S(x,\phi_{t+1}^1(x),\Gamma_{t+1})\big)\right. \\
& \left. + (1-s\lambda_{t+1})(U(x) + A^w(x,w,\Gamma_{t+1}))\right]\Big]
\end{aligned} \tag{33}
$$

with

$$
R_t^w(x) = \begin{cases} S_t(x,\phi_t^1(x)) & \text{if } S_t(x,\phi_t^0(x)) \geq 0 \\ 0 & \text{if } S_t(x,\phi_t^0(x)) < 0 \end{cases} \tag{34}
$$

and

$$
A_t^w(x,w) = \begin{cases} W_t(x,w) - U_t(x) & \text{if } 0 \leq W_t(x,w) - U_t(x) \leq S_t(x,w) \\ S_t(x,\phi_t^1(x)) & \text{if } W_t(x,w) - U_t(x) > S_t(x,w) \\ 0 & \text{if } W_t(x,w) - U_t(x) < 0 \end{cases}
$$

A.2 Value of a Job for Firms

The Bellman equation for the value of a job to a firm writes:

$$
\begin{aligned}
\Pi(x,w,\Gamma_t) &= p(x,z_t) - w \\
&+ \frac{1}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\Big[(1-\delta)\mathbb{1}\left\{S(x,w,\Gamma_{t+1})<0\right\}\left[R^f(x,\Gamma_{t+1})\right] \\
&+ (1-\delta)\mathbb{1}\left\{S(x,w,\Gamma_{t+1})\geq 0\right\}\left[s\lambda_{t+1}\Pi(x,\phi^1_{t+1}(x),\Gamma_{t+1})\right. \\
&\left.+ (1-s\lambda_{t+1})A^f(x,w,\Gamma_{t+1})\right]\Big]
\end{aligned} \tag{35}
$$

In period t, firms enjoy a flow utility equal to production minus the wage paid to workers. The continuation value is composed of four elements. Firstly, with probability δ the match is exogenously destroyed. Secondly, an aggregate productivity shock might trigger an intra-firm renegotiation. If the match is no longer feasible at the current wage, the worker receives the promotion wage:

$$
R^f(x,\Gamma_t) = \begin{cases} \Pi(x,\phi^1_{t+1}(x),\Gamma_t) & \text{if } S(x,\phi^0_t(x),\Gamma_t)\geq 0 \\ 0 & \text{if } S(x,\phi^0_t(x),\Gamma_t)<0 \end{cases} \tag{36}
$$

Thirdly, a worker may meet with another firm. A worker meeting with an alternative employer reveals the meeting to her current employer, which triggers a Bertrand competition between the poaching and incumbent firms. The result of the series of offers and counteroffers is that the worker receives the promotion wage. Fourthly, the worker may not meet with another firm. In this situation, the wage may also be renegotiated within the firm to restore job feasibility. Intra-firm renegotiation is captured by the function $A^f(x,\Gamma_{t+1})$, defined as:

$$
\nabla w \text{ such that } S(x,w,\Gamma_{t+1}) \geq 0:
$$

$$
A^f(x,w,\Gamma_t) = \begin{cases} \Pi(x,w,\Gamma_t) & \text{if } 0 \leq W(x,w,\Gamma_t) - U(x) \leq S(x,w,\Gamma_t) \\ \Pi(x,\phi^1_t(x),\Gamma_t) & \text{if } W(x,w,\Gamma_t) - U_t(x) > S(x,w,\Gamma_t) \\ \Pi(x,\phi^0_t(x),\Gamma_t) & \text{if } W(x,w,\Gamma_t) - U(x) < 0 \end{cases} \tag{37}
$$

Using the definition of $\phi^1_{t+1}(x)$ and $\phi^0_{t+1}(x)$, equation (35) can be rewritten more compactly as:

$$
\Pi(x,w,\Gamma_t) = p(x,z_t) - w + \frac{1-\delta}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\left[(1-s\lambda_{t+1})A^f(x,w,\Gamma_t)\right] \tag{38}
$$

with

$$A^f(x,w,\Gamma_t) = \begin{cases} \Pi(x,,\Gamma_t w) & \text{if } 0 \leq W(x,w,\Gamma_t) - U(x) \leq S(x,w,\Gamma_t) \\ 0 & \text{if } W(x,w,\Gamma_t) - U(x) > S(x,w,\Gamma_t) \\ S(x,\phi_t^0(x),\Gamma_t) & \text{if } W(x,w,\Gamma_t) - U(x) < 0 \end{cases} \tag{39}$$

A.3 The Total Surplus of a Match

Let $\Delta(x,w,\Gamma_t)$ denote the surplus of a worker of type x with wage w. $\Delta(x,w,\Gamma_t)$ is by definition equal to $W(x,w,\Gamma_t) - U(x)$. Equations (9) and (33) imply that:

$$\begin{aligned} \Delta(x,w,\Gamma_t) = & \left[1-\tau(w)\right]w - b(x) \\ & + \frac{1-\delta}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\Big[\mathbb{1}\left\{S(x,w,\Gamma_{t+1}) < 0\right\} R^w(x,\Gamma_{t+1}) \\ & + \mathbb{1}\left\{S(x,w,\Gamma_{t+1}) \geq 0\right\} \left[s\lambda_{t+1}S(x,\phi_{t+1}^1(x),\Gamma_{t+1}) \right. \\ & \left. + (1-s\lambda_{t+1})A^w(x,w,\Gamma_{t+1})\right]\Big] \end{aligned} \tag{40}$$

where the functions $R^w(x,\Gamma_{t+1})$ and $A^w(x,w,\Gamma_{t+1})$ keep track of the different intra-firm renegotiations that may happen when at least one of two parties has a credible reason to break the match:

$$R^w(x,\Gamma_t) = \begin{cases} S(x,\phi_t^1(x),\Gamma_t) & \text{if } S(x,\phi_t^0(x),\Gamma_t) \geq 0 \\ 0 & \text{if } S(x,\phi_t^0(x),\Gamma_t) < 0 \end{cases} \tag{41}$$

$$A^w(x,w,\Gamma_t) = \begin{cases} \Delta(x,w,\Gamma_t) & \text{if } 0 \leq \Delta(x,w,\Gamma_t) \leq S(x,w,\Gamma_t) \\ S(x,\phi_t^1(x),\Gamma_t) & \text{if } \Delta(x,w,\Gamma_t) > S(x,w,\Gamma_t) \\ 0 & \text{if } \Delta(x,w,\Gamma_t) < 0 \end{cases} \tag{42}$$

Equations (40) and (38) imply that the total surplus of a match writes:

$$\begin{aligned} S(x,w,\Gamma_t) = & \, p(x,z_t) - \tau_w(w_t)w - b(x) \\ & + \frac{1-\delta}{1+r}\mathbb{E}_{\Gamma_{t+1}|\Gamma_t}\Big[\mathbb{1}\left\{S(x,w,\Gamma_{t+1}) < 0\right\} R^w(x,\Gamma_{t+1}) \\ & + \mathbb{1}\left\{S(x,w,\Gamma_{t+1}) \geq 0\right\} \left[s\lambda_{t+1}S(x,\phi_{t+1}^1(x),\Gamma_{t+1}) \right. \\ & \left. + (1-s\lambda_{t+1})\big(A(x,w,\Gamma_{t+1})\big)\right]\Big] \end{aligned} \tag{43}$$

where the function $R^w(x, \Gamma_{t+1})$ is defined above and $A(x, w, \Gamma_{t+1})$ is a function that also keeps track of the different intra-firm renegotiation possibilities:

$$A(x, w, \Gamma_t) = \begin{cases} S(x, w, \Gamma_t) & \text{if } 0 \leq \Delta(x, w, \Gamma_t) \leq S(x, w, \Gamma_t) \\ S(x, \phi_t^1(x), \Gamma_t) & \text{if } \Delta(x, w, \Gamma_t) > S(x, w, \Gamma_t) \\ S(x, \phi_t^0(x), \Gamma_t) & \text{if } \Delta(x, w, \Gamma_t) < 0 \end{cases} \tag{44}$$

B Wage Distribution

This section derives the distribution of wages. It also presents the steady-state distribution of wages.

Wage dynamics

Let $h_t(x, w)$ denote the joint distribution of worker types and wages. At the beginning of period t_+ (after endogenous and exogenous job destruction), the joint distribution of worker types and wages writes:

$$\begin{aligned} h_{t+}(x, w) = (1-\delta)\mathbb{1}\{S_{t+1}(x, \phi_{t+1}^0(x)) \geq 0\} \Big[& \mathbb{1}\{w = \phi_{t+1}^0(x)\}] \int_A h_t(x, e) de \\ & + \mathbb{1}\{w = \phi_{t+1}^1(x)\} \int_B h_t(x, e) de \\ & + \mathbb{1}\{0 \leq \Delta_t(x, w) \leq S_t(x, w)\} h_t(x, w) \Big] \end{aligned} \tag{45}$$

where A is the of wages (inherited from previous periods) that necessitate a renegotiation up to the starting wage $\phi_{t+1}^0(x)$ because the worker would be better-off unemployed rather than working at her current wages. The set B denotes the set of wages that induces a renegotiation down to the promotion wage $\phi_{t+1}^1(x)$ because the firm would be better-off firing the worker rather than keeping the job at the current rate. The former happens when the worker's surplus $\Delta_t(x, w)$ is negative, while the latter happens when the firm's surplus is negative (which happens when the worker's surplus is bigger than the total surplus):

$$\begin{aligned} A &= \{w : \Delta_t(x, w) < 0\} \\ B &= \{w : \Delta_t(x, w) > S_t(x, w)\} \end{aligned} \tag{46}$$

At the end of period t, joint distribution of worker types and wages writes:

$$\begin{aligned} h_{t+1}(x,w) = & [1 - s\lambda_{t+1}]h_{t+}(x,w) \\ & + \mathbb{1}\{w = \phi^0_{t+1}(x)\}\mathbb{1}\{S_{t+1}(x,\phi^0_{t+1}(x)) \geq 0\}\lambda_{t+1}u_{t+}(x) \\ & + \mathbb{1}\{w = \phi^1_{t+1}(x)\}s\lambda_{t+1}\int h_{t+}(x,e)de \end{aligned} \tag{47}$$

The first line of equation (47) takes into account the measure of workers who did not have the chance to meet with an alternative employer. The second line takes into consideration the inflow of new workers hired from unemployed at a wage $\phi^0_{t+1}(x)$. The third line takes into consideration the measure of workers, previously employed, who had the chance to meet with an alternative employer. Meeting with an alternative employer triggered a Bertrand competition, giving the worker the promotion wage $\phi^1_{t+1}(x)$.

Steady-state distribution of wages

The distribution of wages out of the steady-state is a complicated object, because the past distributions of wages matter. As the economy experiences an history of aggregate shocks, different wages accumulate. At the steady-state, the flow equation for the joint distribution of wages and types is considerably simplified. Indeed, for every worker type x only two wages are possible at the steady-state: $\phi^0(x)$ and $\phi^1(x)$. At the stead-state, the joint distribution of worker types and wages in the sub-period $t+$ writes:

$$h_{t+}(x,w) = \begin{cases} (1-\delta)\mathbb{1}\{S_t(x,\phi^0(x)) \geq 0\}h_t(x,\phi^0(x)), & w = \phi^0(x) \\ (1-\delta)\mathbb{1}\{S_t(x,\phi^0(x)) \geq 0\}h_t(x,\phi^1(x)), & w = \phi^1(x) \end{cases} \tag{48}$$

Equation (48) states that, after the occurrence of the productivity shock, the measure of workers of type x at the wages $\phi^0(x)$ or $\phi^1(x)$ must be feasible.

The joint distribution of x and w at the end of period t is then given by

$$h_{t+1}(x,w) = \begin{cases} (1-s\lambda)h_{t+}(x,\phi^0(x)) + \lambda\mathbb{1}\{S_t(x,\phi^0(x)) \geq 0\}u_{t+}(x) & w = \phi^0(x) \\ h_{t+}(x,\phi^1(x)) + s\lambda h_{t+}(x,\phi^0(x)) & w = \phi^1(x) \end{cases} \tag{49}$$

The flow equations (48) and (49) imply that the steady-state distribution of wages solves:

$$
\begin{aligned}
h(x,\phi^0(x)) &= \mathbb{1}\{S(x,\phi^0(x)) \geq 0\}\frac{\lambda[u(x)+\delta h(x)]}{1-(1-s\lambda)(1-\delta)} \\
h(x,\phi^1(x)) &= \mathbb{1}\{S(x,\phi^0(x)) \geq 0\}\frac{s\lambda(1-\delta)h(x)}{1-(1-s\lambda)(1-\delta)}
\end{aligned}
\tag{50}
$$

Equation (50) show that the measure of workers receiving the starting wage is an increasing function of the measure of unemployed workers $u(x)$. This makes perfect sense because unemployed workers are hired at the starting wages.

C Solving the dynamic model

Our state variable includes the time-varying and infinite-dimensional distribution of employment across types $h_t(x)$. To circumvent this technical difficulty, we solve the dynamic model using a three-step approach based on the methodology first explained in Reiter (2009). Firstly, we provide a discrete representation of the infinite dimensional model, named the discrete model. Secondly, we solve for the non-stochastic steady-state of the discrete model using an iterative algorithm. Thirdly, we linearize the discrete model around its non-stochastic steady-state and we use a rational expectations solver to find approximated aggregated dynamics.

C.1 The discrete model

We provide a discrete representation of the model by replacing the infinite dimensional objects $S(x,w_t,z_t,h_t(x))$, $\Delta(x,w_t,z_t,h_t(x))$ and $h_t(x)$ by finite counterparts.

Distribution of employment. The distribution of employment if approximated by its value on a grid $\Omega_x = \{x_1, ..., x_{n_x}\}$: $h_t = (h_t(x_1)\, h_t(x_2)\, ... h_t(x_n))'$.

Value functions. Let us define a grid for x and w: $\Omega_{xw} = \{x_1, ..., x_{n_x}\} \otimes \{w_1, ..., w_{n_w}\}$. We calculate the value of $S(x,w_t,z_t,h_t)$ and $\Delta(x,w_t,z_t,h_t)$ by value function iteration. Off-grid values are calculated by linear interpolation.

C.2 Solving for the steady state of the discrete model

A non-stochastic steady-state solution of the discrete model can be obtained by solving for two fixed points: one fixed point for the value functions S and Δ, one fixed point for the distribution of employment $h(x)$. The algorithm proceeds as follows:

Step 0: Initialization. Fix an initial distribution of employment $\hat{h}_0$, initialize two value functions $\hat{S}_0$ and $\hat{\Delta}_0$, set an initial set of tax threshold $\{\tilde{\tau}_i\}$, a tolerance level ε, and a maximum number of iterations τ.

For $i = 1, ..., \tau$:

Step 1: Iteration over the policy rules. Holding the set of tax thresholds $\{\tilde{\tau}_i\}$ and the distribution of employment fixed $\hat{h} = \hat{h}_{i-1}$, iterate over the value functions until convergence to $\hat{S}_i$ and $\hat{\Delta}_i$

Step 2: Iteration over the distribution of employment. Holding the set of tax thresholds $\{\tilde{\tau}_i\}$ and value function fixed $\hat{S} = \hat{S}_i$ and $\Delta = \hat{\Delta}_i$, iterate over the distribution of employment until convergence of the distribution $\hat{h}_i$

Step 3: Test for convergence. If $\| \hat{S}_i - \hat{S}_{i-1} \| \leq \varepsilon$, $\| \hat{\Delta}_i - \hat{\Delta}_{i-1} \| \leq \varepsilon$ and $\| \hat{h}_i - \hat{h}_{i-1} \| \leq \varepsilon$ exit the loop; otherwise update the set of tax thresholds $\{\tilde{\tau}_i\}$ using the *c.d.f* of wages implied by $\{S_i, \Delta_i, h_i\}$, increment i and repeat steps 1 to 3.

C.3 Computing aggregate dynamics via perturbation

The discrete model is a rational expectations one, which can be represented by a function F that satisfies the equality condition:

$$F(X_t,\ X_{t-1},\ \eta_t,\ \varepsilon_t) = 0 \tag{51}$$

where η_t is a vector of endogenously determined expectational errors, ε_t is a vector of exogenous random disturbance, and X_t is a vector of a vector of both endogenous and exogenous variables. We numerically differentiate F around the steady-state to obtain a system of n_s equations:

$$F_1(X_t - X_{SS}) + F_2(X_{t-1} - X_{SS}) + F_3\eta_t + F_4\varepsilon_t = 0 \tag{52}$$

with $F_1 = \partial F/\partial X_t$, $F_2 = \partial F/\partial X_{t-1}$,$F_3 = \partial F/\partial \eta_t$, and $F_4 = \partial F/\partial \varepsilon_t$ derivatives matrices evaluated at the steady-state. Equation (52) can be written in Sims (2002) form $\Gamma_0 y_t = \Gamma_1 y_{t-1} + C + \Psi\varepsilon_t + \Pi\eta_t$ by setting $y_t = X_t - X_{SS}$, $\Gamma_0 = F_1$, $\Gamma_2 = F_2$, $\Psi = -F_3$, $\Pi = -F_4$, and $C = 0$. We then use *gensys* solver to determine the solution of the linear(ized) rational expectation equilibrium.

D Data Selection

We use administrative matched employer-employee data from Italy. The LoSai dataset contains information on both any social security contribution paid by around 1/15th of Italian workers to the Italian Social Security Institute (INPS) and any payment obtained by the worker from the INPS from 1977-2012. Several files are available for the user. We use the file "Estratti Conto", which contains precise information on the initial and final period of any social security contribution the worked either paid or received in his life. We select contributions generated by labor market events using the classification provided in the data.

We select our sample as follows. We keep only males aged between 25 and 60. Following the practice adapted in Guvenen et al. (2014), we drop individuals earning less than 1300 euros per year and workers reporting no working weeks in a given year. We also drop jobs with 0 recorded weeks or 0 recorded earnings.

In our estimation we use total annual earnings and average weekly wages net of age and cohort effects. We compute these quantities by regressing earnings and wages on age and cohort fixed effects, obtaining the residuals and re-scaling them to match the 25 years old population-average. The tax schedule (see Appendix E) is calibrated using the distribution of total annual earnings gross of age and cohort effects.

E Personal Income Tax in Italy

Our definition of income tax corresponds to personal income taxes (*IRPEF*). The relevant tax rate for the Italian *IRPEF* tax is computed on the annual income net of social security contributions. For simplicity we consider only income taxes levied by the central government and we do not consider the set of deductions applicable to income tax nor local and regional taxes. Local income taxes (*Addizionale IRPEF*) levied at the city and regional level are relatively small, with city-level taxes that can reach a maximum rate of 0.8% and regional taxes ranging between 0.7% and 3.33%. While considering the set of tax deductions currently in place in Italy would substantially increase the degree of tax progressivity, these type of instruments tend to be widely supported both by proponents of a stepwise system of marginal tax rates and by many of the supporters of a constant flat rate[31]. We therefore focus on the set of marginal tax rates shown in table 23 and study the effects of evening them out.

F Calibration of the Tax Schedule

In order to guarantee that the tax schedule used in our model is in line with the model generated wage distribution, we proceed in two main steps. First, we

1. Obtain taxable annual salary (i.e. "Imponibile Fiscale") as 90.81% of gross wages (i.e. net of mandatory social security contribution).

2. Compute the percentiles of the (not-age-normalized) taxable salary distribution for each year between 1977 and 2012.

3. HP-filter (with smoothing parameter equal to 100) the yearly time series for each percentile and drop its cyclical component.

4. Compute the average percentiles over the period 2007-2012.

5. Interpolate the average percentiles to obtain the estimated empirical taxable salary distribution, $\hat{h}(w)$.

6. For each empirical tax threshold $\hat{\overline{\tau}}_i$ find $\hat{q}_i$, such that

$$\hat{q}_i = \int_{\underline{w}}^{\hat{\overline{\tau}}_i} \hat{h}(w)dw$$

We have so far obtained $\hat{q}_i$: the vector of quantiles of the taxable salary distribution corresponding to the empirical tax thresholds. The next step of the calibration translates these quantities into model-normalized units. We perform this second step at each iteration of our fixed point algorithm (see Appendix C).

1. Compute the distribution of wages implied by the model, $h(w)$.

2. Find the (model-normalized) wage, $\overline{\tau}_i$, corresponding to each quantile in $\hat{\mathbf{q}}$, i.e. find $\overline{\tau}_i$, such that

$$\hat{q}_i = \int_{\underline{w}}^{\overline{\tau}_i} h(w)dw$$

[31] See for example Hall and Rabushka (2013).

We use the vector of model-normalized tax thresholds, $\overline{\boldsymbol{\tau}}$, obtained in this second step to construct the tax schedule in the model.

In our model income tax is levied directly on weekly wages and accordingly the tax thresholds are expressed in terms of wages rather than income. This is a necessary simplifying assumption that we impose in our model. Our calibration of the income/wage tax schedule in the model should therefore be seen as an approximation of the actual Italian income tax schedule. Clearly our approach has no consequence in the case of a flat tax rate as it does not require calibrating any income threshold.