Aggregate Uncertainty and the Micro-Dynamics of Firms

Nicolò Dalvit *

November 1, 2020

Click for Most Recent Version

Abstract

Using firm level micro-data, I find evidence that firms with lower growth prospects are more sensitive to aggregate shocks. I interpret these findings using a model of demand accumulation and endogenous entry and exit decisions, which I then estimate on French data. The resulting cyclical dynamics of firms provide an explanation for the observed counter-cyclical dispersion in firms' growth rates. They suggest that cyclical dispersion is the result of a pre-existing and persistent characteristic of the firm and caution against its use as a proxy for time-varying uncertainty. The estimated negative correlation between a firm's sensitivity to aggregate shocks and its expected future growth rate is shown to have important consequences for the cyclical characteristics of entering and exiting firms. The quantitative model suggests that this compositional effect is sizeable and equivalent to around 10.5% of the drop in aggregate employment between 2008 and 2009.

*Nicolò Dalvit: World Bank, Science Po, nicolo.dalvit@gmail.com. I would like to thank Jean-Marc Robin for his support and advice. I would also like to thank Pierre Cahuc, Oliver Cassagneau-Francis, Quoc-Anh Do, Vincent Sterk and Mirko Wiederholt as well as seminar participants at Sciences Po for their comments and feedback. My work was supported by a public grant overseen by the French National Research Agency (ANR) as part of the Investissements d'avenir program (reference : ANR-10-EQPX-17 Centre d'accès sècurisé aux données CASD).

1 Introduction

The dispersion in firm growth rates is counter-cyclical, its time-series showing spikes during recessions. This is a widely accepted business cycle fact. The origins of this cyclical pattern on the other hand remain debated. Some authors suggest that the cyclical fluctuation in dispersion is the realisation of changes in the volatility of the business environment[1]. Accordingly, they should be interpreted as a sign of time-varying uncertainty. Other authors have instead argued that recessions are times in which markets and agents become more responsive to underlying differences in economic fundamentals. As differences between agents become more salient, the dispersion in micro-economic variables changes, generating cyclical dispersion even without cyclical volatility[2].

These views bear fundamentally different implications for the way we should think about the cyclical dynamics of firms and the degree of uncertainty they face. More evidence on the sources of these cyclical fluctuations can help us to better understand the decisions that firms take during times of crisis and that ultimately determine their role in creating jobs. In this article I provide new evidence on the origins of the observed counter-cyclical dispersion in firms' growth rates, which I rationalise using a model of demand accumulation where low growth firms are relatively more exposed to aggregate shocks. I then study the implication of this mechanism for the relation between dispersion and uncertainty and for the dynamics of aggregate employment.

The contributions of this paper are threefold. First, I document two new facts on the cyclical dynamics of firm-level log output, pointing to its pro-cyclical persistence. I show that these pro-cyclical fluctuations in persistence comove negatively with the dispersion in firms' growth rates. Second, I develop a simple general equilibrium model of firm growth that can *jointly* rationalise these cyclical movements. The intuition is simple: firms that are on a downward trend of demand accumulation are weaker and more sensitive to aggregate shocks. I then estimate the model on firm-level data and show that, by matching the observed movements in persistence, the quantitative model is able to reproduce the empirical fluctuations in dispersion without requiring a second moment shock. It suggests that cyclical dispersion is the result of a pre-existing and persistent characteristic of the firm rather than the direct consequence of increased volatility in its business environment. It therefore caution against the use of realised dispersion in firm growth rates as a proxy for time-varying uncertainty.

[1]See for example Bloom (2009) and Bloom et al. (2018).

[2]Recent works considering cyclical dispersion in micro variables as a result of cyclical responsiveness include Berger and Vavra (2017), Ilut et al. (2018), Munro (2018) and Kuhn and George (2019).

Third, I show that the relation between a firm's growth potential and its sensitivity to the aggregate state has important consequences for the cyclical composition of firms entering and exiting the market and can magnify the cyclical dynamics of aggregate employment.

Using French firm-level data I document three facts. First, the dispersion in firm growth rates is strongly counter cyclical. This is a well-known and robust business cycle fact. Second, the persistence in firm-level output is pro-cyclical, showing a strong negative correlation with the dispersion in growth rates. Third, inspecting the autocovariance function of output shows that changes in persistence at longer lags co-move because of variation happening at the lead rather than lag year. I show that these facts hold for other measures of productivity and cannot be explained by a cyclical selection effect. To the best of my knowledge the second and third facts are novel in the literature[3]. Importantly, I show that the second moment shock argument often used to interpret the first fact cannot rationalise the other two.

Borrowing from frictional models of demand accumulation, I develop a simple model of firm growth that can provide a common explanation to the three empirical facts. I model the growth process as a state-space system where the importance of the unobserved dynamic type determining a firm's future expected growth depends on the size of aggregate consumption. The intuition behind this representation is simple: firms that are losing market shares are being outperformed by their high growth competitors and lose relatively more when the market contracts and becomes tighter. The process for firm growth is embedded in a standard general equilibrium model with endogenous entry and exit decisions, which I estimate using firm-level data. The estimated model implies large heterogeneity in the sensitivity to the aggregate state. Importantly, the estimation suggests that the endogenous exit bias, while present, can explain only a small fraction of the cyclical fluctuations in persistence.

Targeting the cyclical fluctuations in the persistence of log firm-level output, the model is able to quantitatively match the cyclical movements in the dispersion of growth rates, without requiring a second moment shock. The model is also able to reproduce the co-movements in the autocovariance function of log output. Importantly, this is achieved without exploiting information on the cyclical properties of dispersion, which are left un-targeted in the estimation. I then consider a set of alternative models based on a direct size-dependent sensitivity to the aggregate state, which can match the pro-cyclical persistence, but are shown to fail at

[3]Indirect evidence of the second fact is found in Moscarini and Postel-Vinay (2012) who use French, Danish and US data to show that the relative contribution of small versus big firms to job creation negatively co-moves with aggregate employment

generating enough dispersion in growth rate. The analysis of these alternatives suggests that the key ingredient needed to jointly match the three empirical facts is a strong covariance between sensitivity to the aggregate state and firms' idiosyncratic growth rates, thus lending support to my interpretation.

The mechanism highlighted in this paper implies that the observed cyclical dispersion in growth rates is not the realization of a cyclical volatility in the business environment but is rather the result of underlying characteristics of the firm. This argument breaks down the direct link between fluctuations in dispersion and cyclical volatility and suggests that the use of realized dispersion to calibrate models with time-varying micro uncertainty might result in mis-measurement. It also suggests that firms' exposure to aggregate risk is heterogeneous, with struggling low growth firms facing higher aggregate uncertainty than their performing high growth competitors.

Finally, I consider firms entry and exit decisions and their contribution to net employment creation. I show that the positive correlation between a firm's sensitivity to the aggregate state and its future growth has important consequences for the cyclical characteristics of entering and exiting firms. Compared to a counter-factual economy where an aggregate shock has a proportional effect on all firms, firms exiting during a recession tend to be larger while firms entering the market tend to be smaller on average. At the same time this heterogeneous sensitivity channel has a pro-cyclical effect on the average growth rate of exiting firms and a counter-cyclical effect on the average growth rate of start-ups. The estimated model suggests that the compositional changes induced by the sensitivity channel can have significant consequences on the dynamics of aggregate employment: the total compositional effect on entering and exiting firms due to the heterogeneous sensitivity channel is equivalent to around 10.5% of the total change in aggregate employment between 2008 and 2009 seen in the data.

Related Literature Several authors have investigated the cyclical movements in the dispersion of firm-level variables. While this interest is not new (see for example Abraham and Katz 1986), thanks to the increasing availability of firm-level data, it has been the object of renewed interest[4]. An increasing number of works have provided evidence of counter-cyclical dispersion in several firm-level variables. Significant attention has been given to the counter-cyclical dispersion in the growth rates of output and TFP (Bloom et al. 2018).

[4] In their work Abraham and Katz (1986) use sector-level data to show that the dispersion of sector level employment growth rates is positively correlated with the aggregate unemployment rate.

Recent work has shown evidence of similar cyclical properties for other firm-level variables including prices (Vavra 2014), TFP levels (Kehrig 2015) and business forecasts (Bachmann et al. 2013). A notable exception is investment, which has been found by Bachmann and Bayer (2014) to show a pro-cyclical dispersion, a fact that the authors suggest is the result of fixed costs in capital adjustment. Investigating the origins of the cyclical dispersion in firm growth rates, using physical output data Carlsson et al. (2019) provide evidence that most of its cyclical variation comes from demand rather than productivity shocks. On the other hand, the cyclical persistence of firm-level output and productivity measures has been given limited attention. Indirect evidence of a pro-cyclical persistence in firm sizes, as measured by employment, can be found in Moscarini and Postel-Vinay (2012) who use French, Danish and US data to show that the relative contribution of small versus big firms to job creation negatively co-moves with aggregate employment.

The counter-cyclical dispersion in firm growth-rates has been interpreted as a sign of counter-cyclical uncertainty in the business environment (Bloom et al. 2018). In this view the increase in dispersion observed during recessions is simply the realization of a more volatile and thus more uncertain process of firm growth. In line with this interpretation several authors have modelled time-varying uncertainty as a second moment shock and used realized cyclical dispersion to calibrate its process. Recent works using second moment shocks include Bloom (2009), Schaal (2017), Bachmann et al. (2018), Senga (2018). While counter-cyclical dispersion has been frequently used as a proxy for uncertainty, some authors have cautioned against this practice. Jurado et al. (2015) show that while uncertainty and dispersion are positively correlated, they show important independent variation and their properties can differ substantially.

A growing number of works have supported a different explanation for this cyclical pattern arguing that cyclical dispersion can be the result of a time-varying responsiveness to idiosyncratic differences. Recent work has shown evidence of such time-varying responsiveness. Berger and Vavra (2017) show that periods in which firms respond more to changes in exchange rates are also periods of high dispersion in price adjustments, inline with a model of time-varying responsiveness. Ilut et al. (2018) find that firms hiring rules are concave suggesting that periods of low growth are also periods of higher responsiveness and higher dispersion in employment adjustments. Kuhn and George (2019) shows that a pro-cyclical responsiveness can arise when firms are capacity constrained and capital adjustment is frictional. On the consumer side Munro (2018) shows theoretically that the product demand elasticity can rise in recessions if consumers devote more time to product search, a fact that has been documented in Aguiar et al. (2013). This on the other hand can result in a pro-cyclical demand elasticity, which magnifies differences in productivity among firms and can result in a counter-cyclical dispersion of firm-growth rates.

The intuition behind my model of demand accumulation borrows from a growing number of works modelling the role of demand factors and frictional demand accumulation in determining firms' growth dynamics. The role of frictional demand accumulation through marketing/search effort is receiving increasing attention in the literature. Recent examples include Gourio and Rudanko (2014), Perla (2016) and Kaas and Kimasa (2018). More generally, Foster et al. (2016) provide evidence of the importance of demand accumulation in explaining firms' size differences and Foster et al. (2008) discuss the importance of demand factors for firms' selection. Demand factors have also been identified as the primary source of counter-cyclical dispersion in firm growth rates by Carlsson et al. (2019).

My work is also related to the literature studying the role of cyclical entry and exit decisions in determining aggregate employment fluctuations and analysing the cyclical composition of start-ups. Most closely related to the analysis carried out in this paper is Sedláček and Sterk (2017). The authors show that cyclical differences in the return to starting-up firms producing mass rather than niche goods have important consequences for the cyclical composition of start-ups. While the focus on firms' growth potential is similar, the interpretation of this term is different. In my analysis growth potential refers to the dynamic evolution of a firm's demand and captures its ability to retain its customers and acquire new ones. In this sense, it is a measure of its performance, relative to other firms operating in the market. In Sedláček and Sterk (2017) the growth potential of start-ups is related to the type of good that the firm produces (mass or niche), which is chosen at entry and remains fixed over time. It is therefore not related to the performance of a given firm but rather to its specialisation in the production of mass and niche goods. Other recent works studying the cyclical role of entry and exit in determining aggregate employment include Clementi and Palazzo (2016), who show that entry and exit tend to increase the persistence and unconditional volatility of aggregate macroeconomic variables.

Finally, my analysis of the cyclical persistence in firm-level output is linked to works studying the autocovariance function of firm variables to learn about the process of firm growth and their underlying types. Most recently, Pugsley et al. (2019) study the (long-term) autocovariance function of employment to investigate the sources of observed differences in firm sizes. They find substantial heterogeneity in the growth profile and long-run size of firms, which are found to be key determinants of firms' growth dynamics.

Outline The remainder of the paper is organized as follows. Section 2 presents empirical facts on the cyclical properties of firm level log output. Section 3 discusses a model of demand accumulation and endogeneous entry and exit that can rationalize these facts. Section 4 describes the estimation and calibration of the quantitative model. Section 5 discusses the results of the estimation and their implications for the dynamics of aggregate employment. Section 6 concludes.

2 Empirical Evidence

The cyclical dynamics of firm level output, productivity and demand measures has received increasing attention in the literature. This section presents three empirical facts on the cyclical dynamics of firm-level log output, as measured by the log of total sales.

Fact 1. The dispersion of firms' growth rates is counter-cyclical.

Fact 2. The persistence of firm-level log-output is pro-cyclical and negatively comoves with the dispersion in firms' growth rates.

Fact 3. The autocovariance function of firm-level log output between a lag year t and a lead year $s = t + i$ for $i > 0$ comoves at the lead year s.

The first empirical regularity is well-known in the literature and has been documented for a wide set of variables and countries, including TFP, output and prices[5]. To the best of my knowledge the second and third regularities are novel in the literature[6].

2.1 Data

I focus on the manufacturing sector and use two sources of French firm-level data. The first, FICUS/FARE, is an annual administrative balance sheet dataset covering the universe of French private firms over the period 1994-2016. The second source, EAE, covers a sub sample of private firms over the period 1984-2007. The EAE and the FICUS/FARE contain similar information, but the EAE oversamples big firms. I HP-filter all statistics presented in

[5] See the review of the literature in section 1.

[6] The closest evidence to the one shown in this section is given in Moscarini and Postel-Vinay (2012) for France, the US and Denmark. The authors show that the relative contribution of small versus big firms to job creation negatively co-moves with aggregate employment, which is inline with Fact 2.

this section using an annual smoothing parameter of 100 and plot their cyclical component[7]. Appendix A explains how the sample is selected and gives additional information on the dataset as well as descriptive statistics on the sample. My measure of aggregate output is taken from the INSEE series on annual output from the manufacturing sector, also HP-filtered with smoothing parameter 100. Reported correlations across series are computed as weighted correlations calculated on both datasets, with FICUS/FARE being used in years where the two datasets overlap.

Due to differences in the characteristics of firms in the EAE and FICUS/FARE samples, quantitative comparisons between aggregate statistics computed on the two datasets should be done with caution. In order to guarantee consistency, I carry out the estimation in section 4 using information from the FICUS/FARE sample, the highest quality one, only. Statistics computed from the EAE sample are only shown in this section to provide descriptive evidence on the pre-1994 period.

2.2 Counter-Cyclical Dispersion in Growth Rates

Let $y_{j,t}$ be the log of total sales for a firm j at time t. The mid panel in Figure 1 presents the cyclical component of the HP-filtered inter-quartile range of $\Delta y_{j,t} = y_{j,t} - y_{j,t-1}$ conditional on every year t in the sample. The top panel in Figure 1 displays the growth rate in aggregate manufacturing output. The inter-quartile range shows a clear counter-cyclical behaviour, displaying a correlation with the growth rate in manufacturing output of -0.47. This is a well-known business cycle fact. Counter-cyclical dispersion has been documented for other firm-level variables including TFP growth, prices and business forecast[8].

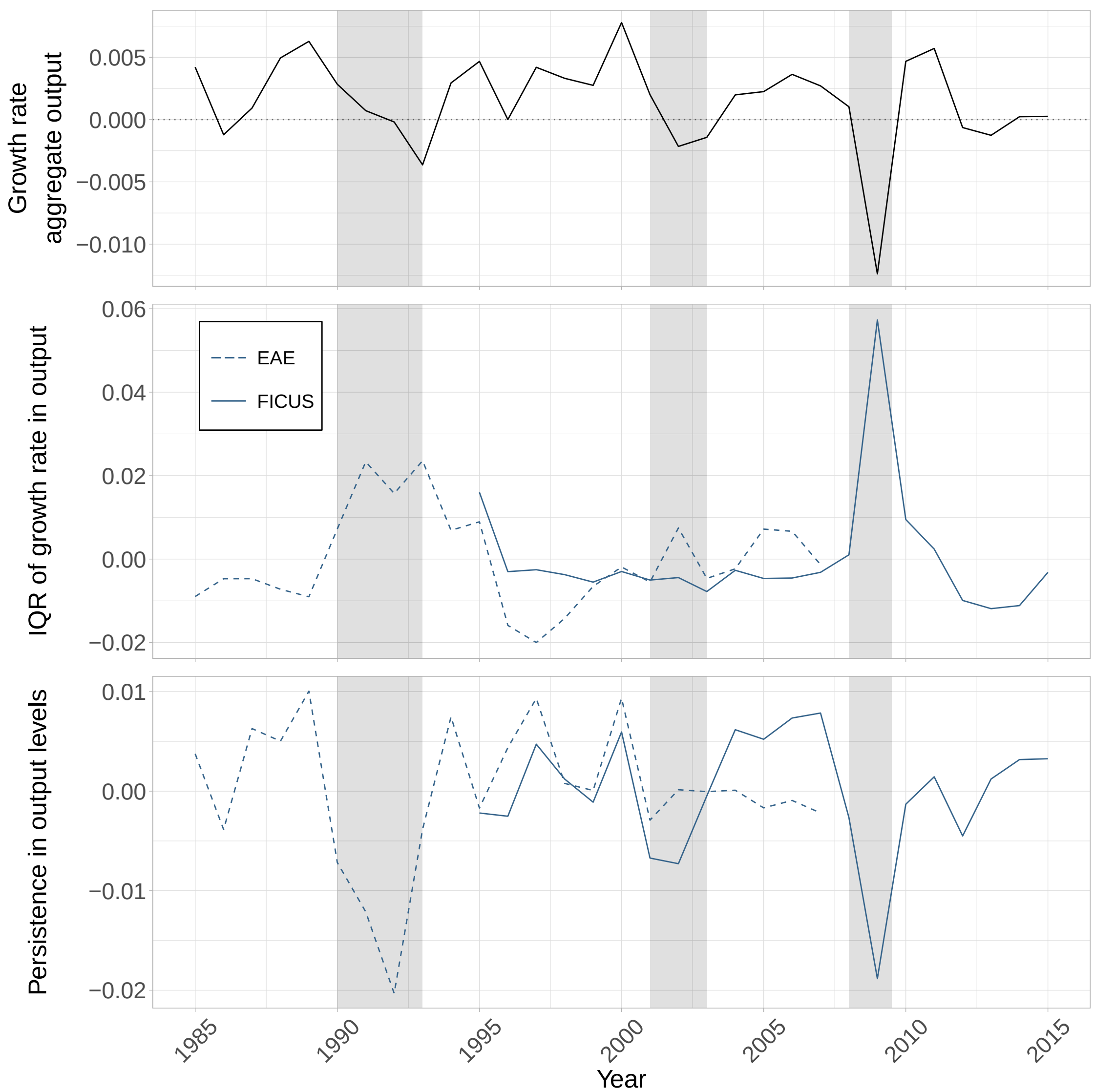
Starting with Bloom (2009), several authors have interpreted this counter-cyclical dispersion as evidence of counter-cyclical uncertainty, modelling it as the result of a second moment shock to the innovation in the process of firm growth[9]. In line with this interpretation, the observed counter-cyclical dispersion in firms' growth rates has frequently been used to calibrate models with time-varying micro uncertainty.

[7] Trends are present in EAE data, but negligible in FICUS/FARE data. Raw series for the mid and bottom panel in Figure 1 are reported in Appendix A.

[8] A notable exception is investment, which is found in Bachmann and Bayer (2014) to have a pro-cyclical dispersion.

[9] Usually a shock to the variance of the innovation in an autoregressive model with fixed effects as in Bloom (2009) and Bloom et al. (2018).

Figure 1: Cyclical Dispersion and Persistence



Notes: The top panel plots the log-change in aggregate manufacturing output. The mid panel plots the cyclical component of the HP-filtered inter-quartile range of yearly log-changes in output for manufacturing firms for every year between 1984-2015. The bottom panel plots cyclical component of the HP-filtered ratio between $\text{Cov}_t(y_{j,t}, y_{j,t-1})$ and $\text{Var}_t(y_{j,t-1})$ for manufacturing firms for every year between 1984-2015. The dotted line is based on EAE data, while the solid line uses FICUS/FARE data. Data are taken from INSEE time series. Highlighted intervals correspond to periods of recession. Raw series for the statistics in the mid and bottom panels are reported in Appendix A.

2.3 Cyclical Persistence

The second piece of evidence relates to the cyclical persistence of firms' log output. I first compute the ratio between $\text{Cov}_t(y_{j,t}, y_{j,t-1})$ and $\text{Var}_t(y_{j,t-1})$, a measure of persistence, condi-

Table 1: Correlation Between Ratios of Autocovariances

At:	Lag Year (t)				Lead Year (s)			
	i=1	i=2	i=3	i=4	i=1	i=2	i=3	i=4
i=1	1	-	-	-	1	-	-	-
i=2	0.28	1	-	-	0.89	1	-	-
i=3	-0.2	0.08	1	-	0.79	0.95	1	-
i=4	-0.39	-0.47	0.06	1	0.67	0.85	0.95	1

Note: The table reports the correlations between measures of persistence (as defined in the main text) computed at different lags i. The first four columns reports correlations taken with respect to the lag year t. The last four columns reports correlations taken with respect to the lead year s.

tional on calendar year t for every year in the sample. I use the subscript t to indicate that variances and covariances are computed conditional on calendar year. I then HP-filter the resulting time-series for this measure and plot its cyclical component in the bottom panel of Figure 1.

The plot shows strong pro-cyclical behaviour of the persistence measure. The series displays a correlation with the growth rate in aggregate output of 0.61. It also shows a strong negative correlation with the measure of dispersion in growth rates, with a correlation coefficient between the two series equal to -0.66. The comovements between the two series and aggregate output is particularly evident during the 1993-1994 and 2008-2009 crisis and to a lesser extent during the 2003-2004 crisis. The evidence in Figure 1 suggests that recessions tend to be periods both of high dispersion in firms' growth rates and where firms' log output is less persistent.

The pro-cyclical persistence documented here suggests that the growth rate of big firms comoves more strongly with the aggregate state of the economy than the growth rate of small firms. This is inline with the evidence provided in Moscarini and Postel-Vinay (2012) who use data from France, the US and Denmark to show that the relative contribution to job creation of small versus big firms negatively co-moves with aggregate employment.

2.4 Cyclical Co-movements in the Autocovariance Function

The last piece of evidence relates to the cyclical comovements in the autocovariance function of firm-level log-output. I first compute the autocovariance of log-output for every year t (lag year) and $s = t + i$ (lead year), for $i \geq 0$, for every year in the sample, $\text{Cov}_t(y_{j,t}, y_{j,s})$. I then compute the ratio between $\text{Cov}_t(y_{j,t}, y_{j,s})$ and $\text{Cov}_t(y_{j,t}, y_{j,s-1})$ for $s > t$. This is a measure of the evolution in the persistence of $y_{j,t}$ between year $s-1$ and s. The comovements between these statistics contain important information on the nature of the counter-cyclical persistence shown in Figure 1.

To understand these comovements I compute the time-series correlation between these ratios. I first compute correlations based on the lag year t. They capture the common variation coming from year t when the lead year s is allowed to vary. The left half of Table 1 reports the correlations for the first four lags ($0 < i \leq 4$). The results show no clear pattern, with coefficients of different sign and distributed around zero, suggesting that no strong comovement is taking place because of events happening at the lag year t. I then compute correlations based on the lead year s. These correlations capture the common variation coming from the lead years s and $s-1$ when the lag year t is allowed to vary. The right half of Table 1 reports the correlations for the first four lags. This time the results show a clear pattern, with very large and positive correlation coefficients, suggesting that changes in the covariance ratios are linked to events happening at the lead periods s and $s-1$.

2.5 Interpreting the Evidence

How does this evidence relate to the standard interpretation of the counter-cyclical dispersion in growth rates as resulting from a pro-cyclical second moment shock? Second moment shocks are usually modelled assuming an AR(1) model with fixed unobserved heterogeneity

$$y_{j,t} = \tilde{\eta}_j + \tilde{\alpha}_{j,t}$$

$$\tilde{\alpha}_{j,t} = \rho \tilde{\alpha}_{j,t-1} + \tilde{\sigma}_t \tilde{u}_{j,t}$$

where $\tilde{u}_{j,t}$ is a zero mean, variance 1 innovation, $\tilde{\eta}_j$ is a fixed type uncorrelated to $\tilde{\alpha}_{j,t}$ and $\tilde{\sigma}_t$ is a time varying dispersion parameter. In this framework the autocovariance ratios used in section 2.4 are equal to

$$\frac{\mathrm{Cov}_t(y_{j,t}, y_{j,s})}{\mathrm{Cov}_t(y_{j,t}, y_{j,s-1})} = \frac{\mathrm{Var}(\tilde{\eta}_j) + \rho^i \mathrm{Var}(\tilde{\alpha}_{j,t})}{\mathrm{Var}(\tilde{\eta}_j) + \rho^{i-1} \mathrm{Var}(\tilde{\alpha}_{j,t})} \tag{1}$$

Through its effect on $\mathrm{Var}(\tilde{\alpha}_{j,t})$, a pro-cyclical $\tilde{\sigma}_t$ does affect the persistence measures used in section 2.4. However, note that equation 1 is at odds with the correlation pattern from Fact 3. As the effect of a second moment shock comes solely from its effect on $\mathrm{Var}(\tilde{\alpha}_{j,t})$ at the lag year t, this argument would imply comovements between the autocovariance ratios based on the lag year t rather than on the lead year s. The empirical autocovariance function on the other hands shows that these movements are linked to events happening at the lead years s and $s-1$, rejecting a second moment shock as a potential explanation for Fact 2.

At this point one possibility is to fit the first fact with a second moment shock and introduce a new mechanism to match the other two. Alternatively, one can read these facts as being linked and look for a mechanism that can jointly rationalize them. In the next section I take the latter option and present a simple model that provides a single explanation for the three empirical facts.

Before moving to the next section, I briefly mention two potential alternative explanations. The first is a size dependent sensitivity to the aggregate state of the economy. In Section 5 I show that this explanation is unable to quantitatively match Fact 1 and Fact 2, due to a weak link between a firm's sensitivity to the aggregate state and its idiosyncratic growth. The second explanation is the presence of an endogenous exit bias. The estimation in Section 5 takes into account endogenous selection and finds the cyclical selection bias to be small compared to the cyclical movements in Figure 1.

3 Model

In this section I present a simple model of frictional demand accumulation and endogenous entry and exit, which I use to interpret the stylized facts from Section 2.

Households and Preferences The economy is populated by an infinitely lived representative household which provides labor N_t on a perfectly competitive market, consumes a composite good C_t and owns all firms. Time is discrete and indexed with t. The composite basket C_t is given by the Dixit-Stiglitz aggregator

$$C_t = \Big(\int_{j \in \Upsilon_t} \psi_{j,t}^{\frac{1}{\phi}} c_{j,t}^{\frac{\phi-1}{\phi}} dj \Big)^{\frac{\phi}{\phi-1}}$$

where Υ_t is the set of available goods, $\psi_{j,t}$ is the j good specific demand shifter, $c_{j,t}$ is the quantity of good j consumed by the household and $\phi > 1$ is the elasticity of substitution between goods.

The utility of the household depends on the quantity of the composite good consumed and on the amount of labor supplied. The household choose consumption, C_t, and labor, N_t, in order to maximize the expected present discounted value of the future utility flows

$$\sum_{i=t}^{\infty} \beta^i U(C_t, N_t) = \sum_{i=t}^{\infty} \beta^i (\log(C_t) - \chi N_t)$$

subject to the aggregate budget constraint

$$\int_{j\in\Omega_t} p_{j,t}c_{j,t}dj = P_t w_t N_t + \Pi_t$$

where β is the household discount rate, χ is the disutility of labor parameter, $w_t = \frac{W}{P_t}$ is the real wage, W is the nominal wage which is assumed to be fixed, Π_t are nominal aggregate firm profits, $p_{j,t}$ is the price of good j and $P_t = \Big(\int_{i\in\Omega_t} \psi_{j,t} p_{j,t}^{1-\phi} dj\Big)^{\frac{1}{1-\phi}}$ is the aggregate price index.

Households' preferences imply that the good j specific demand at time t is given by

$$c_{j,t} = \psi_{j,t}\Big(\frac{p_{j,t}}{P_t}\Big)^{-\phi} C_t$$

which depends on the relative price of good j, aggregate consumption C_t and the good's specific demand shifter.

Accumulation of Demand I assume that firms compete for demand[10]. Whenever a firm j enters the market it draws a value η_j, which determines the potential demand (quality) of its product[11]. The product market is frictional and firms slowly accumulate intangible customer capital $\omega_{j,t}$, which allows them to exploit their demand potential. In every period firms invest an exogenous amount of effort in demand accumulation, with the returns on their effort being inversely related to the distance between their customer base and their potential demand according to the following law of motion

$$\omega_{j,t} = \omega_{j,t-1} + \gamma(\eta_j - \omega_{j,t-1}) + u_{j,t} \tag{2}$$

where $\gamma \geq 0$ governs the return on demand accumulation and $u_{j,t}$ is an exogenous innovation to the demand accumulation process with standard deviation σ. Equation 2 implies that firms with a customer base, $\omega_{j,t}$, below their potential demand tend, on average, to grow their customer base over time. Vice versa, firms with a customer base above their potential demand tend, on average, to lose customers. The gap between potential demand and current customer base thus determines a firm's potential to grow its customer base. I define $z_{j,t} = \eta_j - \omega_{j,t}$ as the *growth potential* of a firm j at time t. In Appendix B I show how this model relates to the standard AR(1) process with fixed heterogeneity commonly used in the literature.

[10]Recent evidence in Carlsson et al. (2019) suggests that most of the cyclical dispersion in growth rates comes from demand shocks.

[11]In this version of the model η_j is modelled as being fixed over time. Extending the model to allow for a time-varying η_j is feasible, but more demanding in terms of identification.

I then assume that a firm's current demand, $v_{j,t} = \log(\psi_{j,t})$, is a function of its current customer base, $\omega_{j,t}$, its potential demand, η_j, and the current market conditions, as summarized by the term λ_t. In practice I model λ_t as governing the relative importance of customer capital and potential demand in determining a firm's current demand

$$\begin{aligned} v_{j,t} &= \omega_{j,t-1} + \lambda_t \gamma (\eta_j - \omega_{j,t-1}) + u_{j,t} \\ &= \omega_{j,t-1} + \lambda_t \gamma z_{j,t-1} + u_{j,t} \end{aligned} \tag{3}$$

inline with the representation of firm growth as resulting from a frictional process of intangible capital accumulation, the term λ_t can be interpreted as governing the importance of frictions in determining a firm's current demand[12]. When $\lambda_t\gamma$ tends to one frictions play no role and the frictional product market approaches perfect competition where current demand reflects the underlying differences in quality, η_j, among firms. When $\lambda_t\gamma$ tends to zero on the other hand, current demand equals $\omega_{j,t}$ and reflects only differences in intangible/customer capital. In line with this interpretation, I define $\omega_{j,t}$ as the frictional demand of a firm j at time t, as opposed to its potential demand η_j. The set of demand fundamentals and the current period idiosyncratic shock $u_{j,t}$ then define the good j's specific state at time t, $\kappa_{j,t} = (\omega_{j,t-1}, \eta_j, u_{j,t})$.

The intuition for this mechanism is the following. Consider firms that are above their potential demand. These firms do not immediately lose their excess demand because frictions in the product market prevent competitors from seizing their excess market shares. The frictional process of demand accumulation limits the scope of competition, generating persistence in excess demand. In periods where frictions lose importance, firms above their demand potential find it more difficult to defend their excess market share as competition becomes fiercer. As a result firms that, being unable to increase or retain their demand, are showing a poor competitive performance would suffer relatively more in periods where frictions are reduced and fundamental differences in quality become more salient.

Incumbent Firms There is an endogenous measure of incumbent firms active in the market at time t. Each firm produces one of the available goods in the economy using labor as the only input in a constant return to scale production function $q_{j,t} = A_t N_{j,t}$ where A_t is an aggregate productivity term, which evolves according to

$$\log(A_t) = (1 - \rho_A)\log(\bar{A}) + \rho_A \log(A_{t-1}) + \epsilon_t^A$$

[12]While I do not micro-found this mechanism, one simple way to think about λ_t is as a measure of competition in the market. In a model of frictional demand accumulation, competition is linked to the tightness of the product market. As the market tightness tends to infinity, frictional markets tend to behave as competitive ones and outcomes reflect more closely the agents' fundamentals (η_j in this case).

where ρ_A is a persistence parameter and ϵ_t^A is an i.i.d. innovation term with mean 0 and standard deviation σ_A. Prices freely adjust to clear the market implying that $q_{j,t} = c_{j,t}$. At the beginning of every period incumbent firms face the risk of losing their product with probability δ and exiting exogenously the market. Incumbent firms can also decide to exit, lose their product and avoid paying the fixed over-head production cost f_c[13]. The continuation value of a firm at the time the exit decision is taken is then a function of the aggregate state of the economy Ψ_t and of the firm/good's state $\kappa_{j,t}$

$$V(\Psi_t, \kappa_{j,t}) = \pi(\Psi_t, \kappa_{j,t}) - f_c + \Lambda_t(1-\delta)E_t[\max(V(\Psi_{t+1}, \kappa_{t+1}), 0)]$$

where $\pi(\Psi_t, \kappa_{j,t}) = \frac{p_{j,t}q_{j,t}}{P_t} - w_t N_t$ is the firm j's real profits at time t and $\Lambda_t = \beta \frac{C_t}{C_{t+1}}$ is the stochastic discount factor. A firm optimally decides to exit at the beginning of period t if $V(\Psi_t, \kappa_{j,t}) < 0$[14].

Firm Entry and Available Business Opportunities There is a measure 1 of business opportunities in the economy, each associated with a good j and defined by their demand fundamentals $\kappa_{j,t}$. In every period there is a large number of homogeneous potential entrants that compete to seize the set of opportunities not operated by incumbent firms. At the beginning of each period each opportunity is randomly assigned to a potential entrant who, after having observed $\kappa_{j,t}$, can either pay a fix entry cost $f_e = \xi f_c$ and become an incumbent or decide to lose the good and leave the market. A potential entrant decides to enter at time t if $V(\Psi_t, \kappa_{j,t}) > f_e$.

Equilibrium and Market Clearing I assume that the market condition term is a function of aggregate consumption C_t, $\lambda_t = \lambda(C_t)$, and thus model the evolution in the market conditions as arising from a market size effect[15]. Denoting as μ_t the distribution of $\kappa_{j,t} = (\omega_{j,t}, \eta_j, u_{j,t})$ over active firms in the economy at time t, labor market-clearing implies that

$$N_t = \int N(\kappa, \Psi_t) d\mu_t$$

[13]The fact that exiting firms lose their variety upon exit means that incumbent firms are not allowed to temporarily exit the market.

[14]Note that firms observe the realisation of all exogenous variables (η and z) before paying the over-head cost.

[15]Several works, most notably in the trade literature, have considered the importance of market size effects on firms. Recent examples include Zhelobodko et al. (2012), Mayer et al. (2014) and Edmond et al. (2015).

The goods market-clearing condition is then

$$C_t = A_t \Big(\int \psi^{\frac{1}{\phi}} N(\kappa, \Psi_t)^{\frac{\phi-1}{\phi}} d\mu_t \Big)^{\frac{\phi}{\phi-1}}$$

The evolution of μ_t is consistent with the steady state dynamics of the distribution of $\kappa_{j,t}$ among business opportunities, the evolution of A_t, household maximization and firms' optimal choices of entry and exit. Finally, the economy's aggregate state at time t is given by $\Psi_t = (A_t, \mu_t)$. Note that as long as the entry cost f_e is different from zero the distribution of active firm types is an aggregate state of the economy.

Market Condition, Dispersion and Persistence Recent evidence has shown that recessions tend to be periods where the product market becomes more sensitive to fundamental differences in quality[16]. Accordingly, one could expect λ_t to behave counter-cyclically, with frictions playing a larger role during expansions and demand more closely reflecting quality during recessions.

Would a counter-cyclical λ_t be able to rationalise the empirical facts presented in section 2? Intuitively, λ_t governs the importance of $z_{j,t-1}$ in determining $v_{j,t}$. In line with the second empirical fact, as long as $Cov(v_{j,t-1}, z_{j,t-1}) < 0$, a counter-cyclical λ_t would have a pro-cyclical effect on the persistence of $v_{j,t}$. As $z_{j,t-1}$ is positively correlated with $\Delta v_{j,t} = v_{j,t} - v_{j,t-1}$, a counter-cyclical λ_t would also have a counter-cyclical effect on $\text{Var}(\Delta v_{j,t})$, inline with the first empirical fact. In Appendix B I discuss the cyclical properties of $v_{j,t}$ more formally.

It is worth noting two important differences between the mechanism highlighted here and the second moment argument often used in the literature. First, similarly to a positive second moment shock, the effect of increasing λ_t ,as measured by the difference $v_{j,t} - \omega_{j,t}$, is positively correlated with $\Delta\omega_{j,t}$: on average firms that would have grown between time $t-1$ and time t gain from an increase in λ_t. On the other hand and contrary to the case of a positive second moment shock, the effect of increasing λ_t is positively correlated with the expected idiosyncratic future growth in $\omega_{j,t}$: firms that expect to be growing more in the future gain from an increase in λ_t. Second, the effect of λ_t works though the amplification of a pre-existing dynamic type of the firm, $z_{j,t}$. A cyclical second moment shock instead amplifies the effect of an unknown innovation to the dynamic type. In Section 5 I discuss how these differences have implications for firms' choices and the degree of uncertainty that

[16]Several works using households' time use surveys have shown that households tend to increase their shopping time during recessions, a fact that has been interpreted as evidence of increased search for products. Recent works in this direction include Aguiar et al. (2013) and Munro (2018). Other works on the cyclical responsivness of markets and firms include Berger and Vavra (2017), Ilut et al. (2018) and Kuhn and George (2019).

they face.

4 Quantitative implementation

4.1 Parametrization

The dependence of the market condition on aggregate demand is parametrized as a simple linear function of C_t

$$\lambda(C_t) = 1 + \overline{\lambda} \log(C_t)$$

where the term $\tilde{\lambda}$ governs the dependence of λ_t on aggregate consumption and is estimated as explained below[17]. The distribution of η_j is parametrised as a normal distribution with mean μ^η and standard deviation σ^η.

As long as $f_e > 0$ the distribution of active firm types in the economy is an aggregate state variable affecting the equilibrium price, P_t, independently of the aggregate productivity state A_t. To deal with the problem of having an infinite dimensional state space I follow Krusell and Smith (1998) and approximate a firm's forecast of P_{t+1} at time t with a linear forecasting rule

$$\log(P_{t+1}) = \tau_1 + \tau_2 \log(P_t) + \tau_3 \log(A_{t+1}) + \epsilon^f$$

where τ_1, τ_2, τ_3 are parameters that are estimated by OLS on a simulated economy as discussed in Appendix D.

4.2 Estimation and Calibration

A first group of parameters is set a priori using values that are standard in the literature. I set the annual discount factor at $\beta = 0.96$ inline with an average interest rate of 4%. The disutility of work parameter is set to $\chi = 2$ and the elasticity of substitution between goods to $\phi = 4$ giving a constant mark-up of 33%. The relative size of fixed entry and over-head cost, ξ, is set to 0.44 inline with Barseghyan and DiCecio (2011). I set $\rho^A = 0.65$, which corresponds to a quarterly persistence parameter equal to 0.9 and I normalize $\overline{A} = 1$. Finally I normalize $W = \chi$, so that the household maximization problem gives $C_t = \frac{1}{P_t}$[18].

The rest of the parameters are recovered from the data. The details of the estimation procedure are given in Appendix D. I recover a first set of parameters $\theta^s = (f_c, \delta, \sigma^\eta, \mu^\eta, \gamma, \sigma)$ by simulated method of moments (SMM), matching a set of six steady state moments, ap-

[17] I have also tested an exponential specification where $\lambda(C_t) = \overline{\lambda} C_t$ and the results in Section 5 remain very similar.

[18] See section C in the Appendix for more details on this derivation.

Table 2: Parameter Values

Estimated			Set A Priori		
Parameter	Value	Target	Parameter	Value	Source
At the steady state			β	0.96	4% Average interest rate
γ	0.235	Autocovariance function	ϕ	4	33% mark-up
σ	0.271	Variance first-difference	$\overline{A}$	1	Normalization
f_c	0.223	Exit probability by size	ρ_A	0.65	0.9 quarterly (standard)
δ	0.43	Average exit rate	χ	2	Standard
σ^η	0.56	Observed dispersion in size	f_e	$0.82{*}\mathrm{f}_c$	Barseghyan and DiCecio (2011)
μ^η	2.86	$C_t = 1$ at the steady state	W	2	Normalization
Out of the steady state					
σ_A	0.0368	Volatility of $\log(A_t)$			
$\overline{\lambda}$	-5.4	Autocovariance			
τ_1	-0.008				
τ_1	0.032				
τ_2	-1.093				

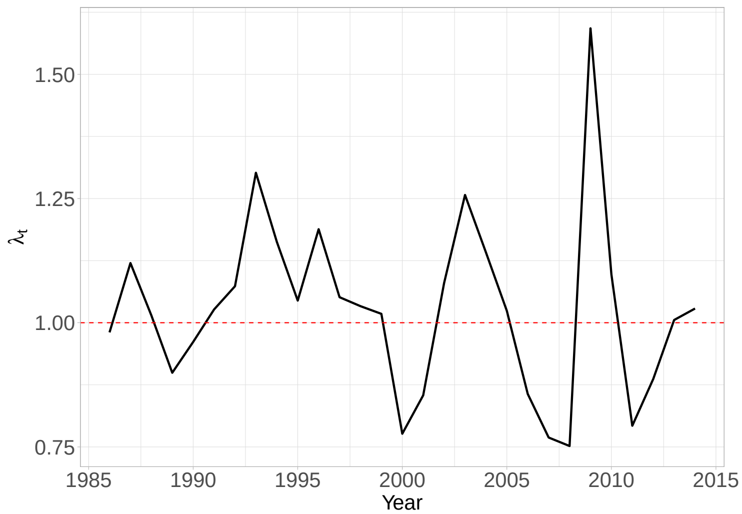
Note: The table presents the parameters values used in the quantitative exercises. Values are derived as described in section 4 and appendix D.

proximated by their trend averages. While the estimates are recovered jointly, each moment is chosen to inform the estimation of one of the six parameters. The fixed operational cost relates to the size dependency of exit rates, while the exogenous exit rate δ relates to the residual exit probability. The mean of the distribution of η_j, μ^η, is chosen so as to normalize $C_t = 1$ at the steady state, while the choice of σ^η is informed by the empirical dispersion in the size of incumbent firms. Finally, the persistence parameter γ is recovered from information on the autocovariance function of revenue productivity and the parameter σ relates to the long-run dispersion in growth rates.

The volatility of $\log(A_t)$ is recovered from the volatility of the series for aggregate TFP as defined in Appendix A. This leaves the market condition parameter $\overline{\lambda}$ and the parameters of the forecasting rule to be estimated. I do this with an iterative GMM algorithm that: (i) updates the τ on model simulated data and (ii) (re)estimates $\overline{\lambda}$ using moments on the cyclical persistence of revenue productivity adjusted for selection using simulated selection correction terms based on the previous iteration. The details are given in Appendix D. Importantly, the estimation of λ_t relies on the cyclical variation in the covariance function of firm-level revenue productivity. Moments on the variance in growth rates conditional on calendar year are not used in the estimation, and its cyclical properties are therefore not targeted[19].

[19] Only the average of its trend component is used in the estimation.

Figure 2: Estimated Market Condition Parameter



Notes: Estimated level of the market condition parameter λ_t for every year between 1986 and 2015. The horizontal dotted line shows the steady state level of λ_t, which is normalized to 1.

5 Discussion and Results

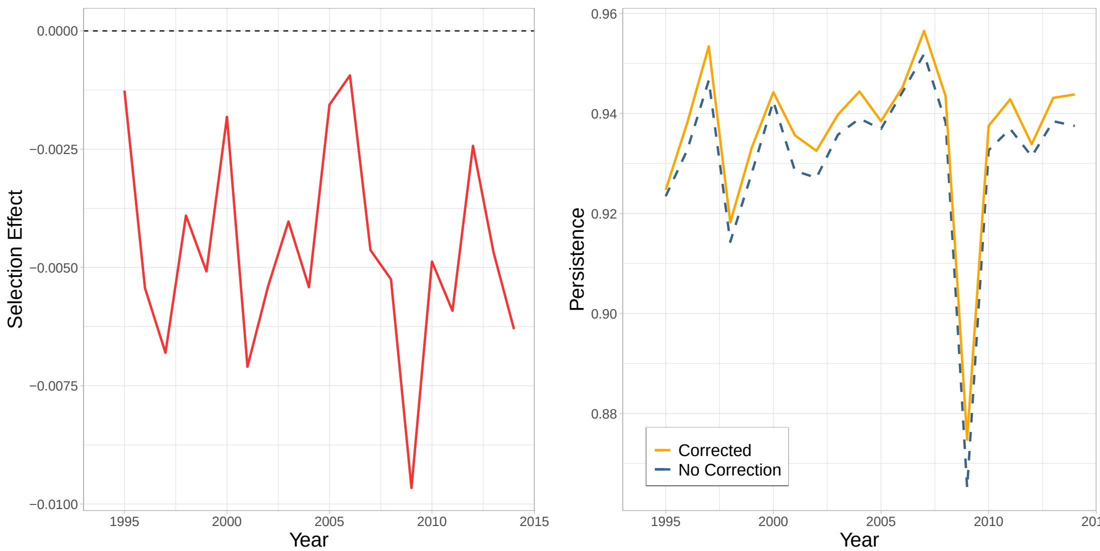
5.1 Estimated Model

5.1.1 Expected Growth and Aggregate Shock

The estimated value for $\overline{\lambda}$ reported in table 2 strongly supports the presence of a countercyclical λ_t. Seen through the lenses of the model, this evidence suggests that recessions are periods where a firm's relative demand aligns more closely with its potential value. This in turn implies that a firm's sensitivity to the aggregate state of the economy is negatively related to its expected future growth rate: firms expecting to lose market share in the future are hit the hardest during economic downturns. Figure 2 plots the estimated value of λ_t for every year in the sample. The cyclical fluctuations in λ_t are large, with the parameter in 2009 being around 60% higher than its steady state value and 110% higher than its value in 2008.

The economic magnitude of these fluctuations is also important. In order to quantify the extent to which market conditions affect firms differently, I focus on profitability and consider the effect of a shock to A_t on the log of real profits (net of the fixed overhead cost),

Figure 3: Selection Bias



Notes: The left panel plots the estimated endogenous exit bias in the covariance-variance ratio (persistence coefficient) for $v_{j,t}$. The right panel plots the ratio estimated from the data (dotted blue line) and the simulated ratio computed on the unselected sample (solid orange line). The difference between the two lines corresponds to the solid red line in the left panel.

a quantity closely related to firms' entry and exit decisions[20]

$$\log(\pi_{j,t}) = \underbrace{v_{j,t}}_{\text{idiosyncratic term}} + \underbrace{\log(A_t^{\phi-1} C_t P_t^{\phi-1} B)}_{\text{common term}} \tag{4}$$

where $B = W^{1-\phi}(\phi-1)^{\phi-1}\phi^{-\phi}$ is a constant term[21]. The estimate for $\tilde{\lambda}$ implies that, starting from the steady state, a one standard deviation, σ_A, drop in aggregate productivity leads to a $-0.81\sigma_A$ standard deviations drop in the common term and a $-0.72\sigma_A$ differential effect on $\log(\pi_{j,t})$ between incumbent firms at the 25th and 75th percentile of the distribution of $z_{j,t}$[22]: firms at the 25th percentile of the distribution of $z_{j,t}$ experience a drop in $\pi_{j,t}$ of $-1.22\sigma_A$ while firms at the 75th percentile see a drop of $-0.5\sigma_A$, suggesting a sizeable difference in their exposure to the aggregate shocks.

5.1.2 Selection Bias

A potential concerns is that endogenous selection could generate a cyclical persistence even if the parameter λ_t was constant. Intuitively, if recessions are periods of increased endogenous exit due to worsened profitability, the endogenous selection bias can show a pro-cyclical pattern[23]. The left panel in Figure 4 plots the estimated selection bias for every year in the sample in a model with λ_t fixed and set at its steady state value of 1. Appendix D explains how the selection bias is estimated. The result confirms the intuition and shows a pro-cyclical behaviour of the selection bias. Even in the absence of a time-varying λ_t the persistence coefficient would show some degree of pro-cyclicality. Note however that the magnitude of the effect is small. The right panel in Figure 3 plots the empirical persistence coefficient for $v_{j,t}$ and its residual fluctuations once the selection bias is taken into account. This is equivalent to the persistence parameter that one would estimate if an unselected sample was observed. The residual variation in persistence is considerable with the cyclical selection effect being able to account only for around 7% of the difference between the persistence coefficient in 2009 and its long run trend, suggesting that most of the observed cyclical persistence is not due to a selection effect.

5.1.3 Fitting the Empirical Facts

Having shown that the endogenous selection bias is unable to explain the cyclical fluctuations in persistence, the next question to ask is whether the mechanism highlighted in the model is able to quantitatively match the empirical facts presented in the descriptive section.

I first assess the ability of the model to fit the cyclical behaviour of the persistence measure, a moment that is targeted in the estimation. The left panel in Figure 4 plots the empirical autocovariance-variance ratio for $v_{j,t}$ (solid blue line) and its predicted value from the model (dashed orange line). The overall fit is good, with the model being able to match around 90% of the 2009 drop in the persistence measure compared to its trend. While the overall fit is good, some unexplained variation remains. This is notably the case during the recovery from the 2009 recession, suggesting that market conditions as summarized by λ_t are likely to be more persistent than what is implied by the parametrization adopted in section 4.

Next, I consider whether the model is able to fit the counter-cyclical dispersion in the

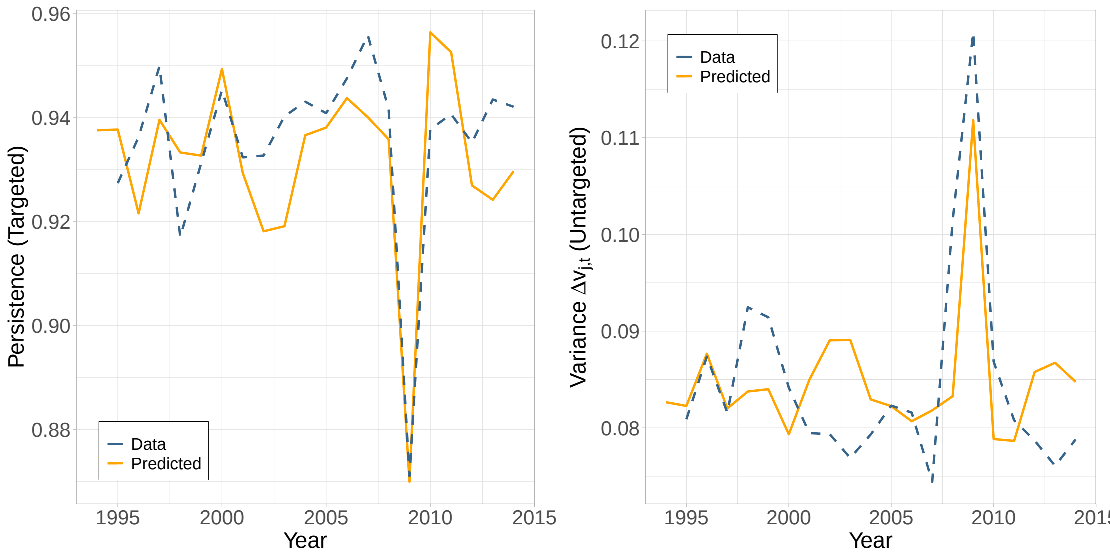
[20] See Foster et al. (2008) for a discussion on the role of profitability for firms survival.

[21] See appendix C for the derivation.

[22] I use firms forecasting rule as estimated in Section 4 to obtain these numbers. The percentiles are computed on the distribution of $z_{j,t}$ among incumbent firms at the steady-state.

[23] If the correlation between size and exit is negative, an increase in the magnitude of the selection bias would generate pro-cyclical fluctuations in its value.

Figure 4: Model Fit



Notes: The left panel plots the selection corrected empirical persistence coefficient for $v_{j,t}$ corrected for selection (dotted blue line) and its predicted value from the estimated parametric model (solid orange line). The right panel plots the empirical variance of $\Delta v_{j,t}$ corrected for selection (dotted blue line) and its predicted value from the estimated parametric model (solid orange line). The left panel is targeted in the estimation while the right panel is untargeted. Empirical series are HP-filtered and then scaled up using the mean trend component. Raw empirical series are reported in Appendix A

growth rates of $v_{j,t}$ as seen in the data. Importantly, the cyclical fluctuations in dispersion are not targeted in the estimation. The ability of the model to fit this stylized fact therefore rests only on the correct specification of the relationship between persistence and dispersion. The right panel of Figure 4 plots the empirical and model predicted variance in revenue productivity growth rates. By matching the pro-cyclical movements in the persistence measure, the model is able to reproduce most of the counter-cyclical dispersion seen in the data: matching the drop in persistence from 2008 to 2009, the model generates 3/4 of the spike in dispersion seen in 2009 compared to the long-run average. Figure 4 therefore lends support to the mechanism highlighted in this paper as a common explanation for these two empirical facts, showing that it is quantitatively able to jointly generates both a pro-cyclical persistence in output levels and a counter-cyclical dispersion in growth rates as seen in the data. While some unexplained variation is still evident in the right panel of Figure 4 it is interesting to notice that it is contemporaneous to the unexplained variation in persistence, suggesting that a more flexible parametrization of λ_t could explain most of these differences.

Finally, Table 3 reports the simulated correlations between persistence measures at different lags. While the simulated correlations tend to be higher than their empirical counterparts

Table 3: Simulated Correlation At Different Lags

At:	Lag Year (t)				Lead Year (s)			
	i=1	i=2	i=3	i=4	i=1	i=2	i=3	i=4
i=1	1	-	-	-	1	-	-	-
i=2	-0.27	1	-	-	0.96	1	-	-
i=3	-0.34	-0.13	1	-	0.95	0.99	1	-
i=4	0.03	-0.35	-0.16	1	0.93	0.97	0.98	1

Note: The table reports the simulated counter-parts of the empirical correlations presented in Table 1.

as reported in Table 1, the model is able to reproduce the comovements patterns from the third empirical fact presented in Section 2[24]. In appendix B I discuss more formally how the model is able to generate an autocovariance function that is inline with the empirical data.

5.1.4 Alternative Models

At this point one might wonder whether any model that can match the drop in persistence seen in Figure 4 is able to match the empirical increase in dispersion. The most straightforward alternative explanation for the drop in persistence is that sensitivity to the aggregate shock is directly correlated with size, rather than indirectly via the correlation between size and growth potential. To show why this alternative explanation fails to match the cyclical fluctuation in the dispersion of firms' growth rates I consider two alternative models of size dependent sensitivity. First (alternative 1),

$$v_{j,t} = \lambda_t^a \omega_{j,t}$$

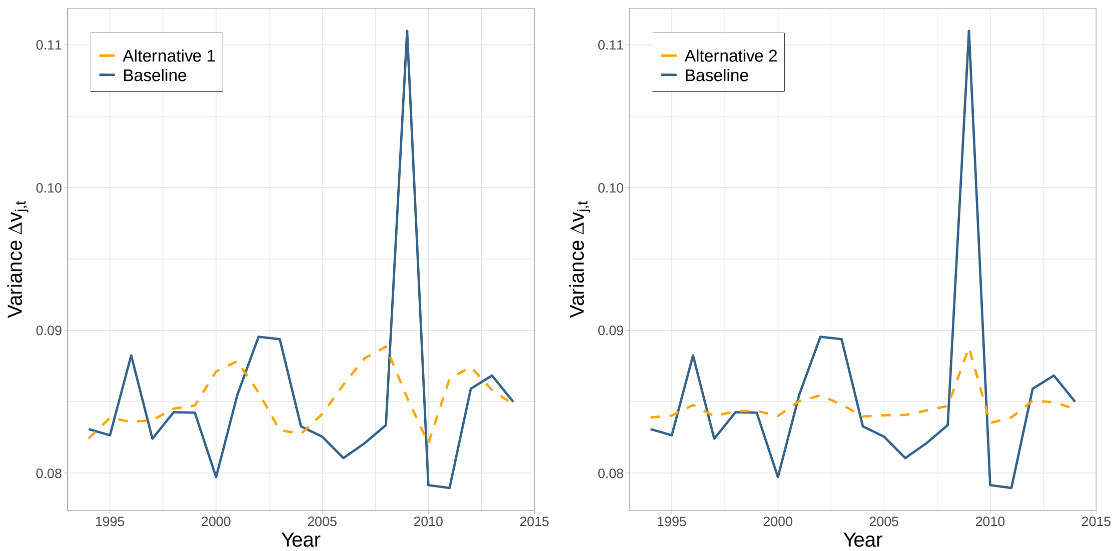
and second (alternative 2)

$$v_{j,t} = \lambda_t^a(\omega_{j,t-1} + \gamma z_{j,t-1}) + u_{j,t}$$

where $z_{j,t}$, $\omega_{j,t}$ and $u_{j,t}$ follow the processes described in Section 3 and λ_t^a is a time-varying parameter capturing the effect of size-dependent sensitivity. Note that the only difference between the two cases comes from whether the innovation $u_{j,t}$ is affected by the contemporaneous sensitivity parameter. I then estimate λ_t^a in a similar manner to what is done for the baseline case, matching the simulated moments from the baseline model. In Appendix

[24] The fact that the simulated correlations are higher compared to their empirical counter-parts is to be expected if the actual data are noisier than the simulated ones.

Figure 5: Predicted Variance: Alternative Models



Notes: Predicted dispersion from models with size dependent sensitivity. Left panel refers to a model where sensitivity depends on the size at time t. Right panel refers a model where sensitivity depends on the size at time t excluding the shock $u_{j,t}$.

E I plot the estimated persistence in the two alternative models, which almost perfectly match the one from the baseline model. Figure 5 shows the predicted time-series for the dispersion in firm growth rates in the two alternative cases. From the graphs it is clear that the two models cannot replicate the empirical variation in dispersion. Alternative 2 can explain only around 15% of the 2009 increase in the baseline model while alternative 1 produces pro-cyclical rather than counter-cyclical dispersion. Figure 5 therefore rejects pure size dependent sensitivity as an alternative joint explanation for the two facts.

To understand the origins of the differences between the three models, I then perform a decomposition of $Var(\Delta v_{j,t})$. I consider the effect of a shock of intensity equal to the 2009 crisis, hitting the economy at the non-stochastic steady state and decompose $\Delta v_{j,t}$ into its steady state value and its deviation from the steady state

$$\Delta v_{j,t} = \underbrace{\Delta\omega_{j,t}}_{\text{steady state growth}} + \underbrace{(\Delta v_{j,t} - \Delta\omega_{j,t})}_{\text{deviation from steady state growth}}$$

Defining $\Delta\tilde{v}_{j,t} = (\Delta v_{j,t} - \Delta\omega_{j,t})$, I then decompose the variance of $\Delta v_{j,t}$ as

$$\text{Var}(\Delta v_{j,t}) = \text{Var}(\Delta\omega_{j,t}) + \text{Var}(\Delta\tilde{v}_{j,t}) + 2\text{Cov}(\Delta\omega_{j,t}, \Delta\tilde{v}_{j,t})$$

where the first term is simply the steady state variance in growth rates, which is by construc-

Table 4: Variance Decomposition

	Baseline	Alternative 1	Alternative 2
Total Change	0.0193	-0.0027	0.003
Variance Term	0.0047	0.0007	0.0008
Covariance Term	0.0146	-0.0034	0.0022

tion equal in the three models, the second term is the variance of the cyclical component and the third term captures the effect of the comovement between steady state growth and cyclical growth. When the sensitivity to the aggregate shock is unrelated to the steady state growth rate of firms, the third term is then equal to zero. Table 4 reports the results of the decomposition for each of the three models. The covariance term explains much of the variation in all three cases, explaining 76% percent of the variation in the baseline model and driving the pro-cyclical behaviour of the variance in alternative 1. It also explains the bulk of the variation across specifications. Shutting down the covariance channel the difference between the baseline model and alternative 2 would be 76% smaller and the difference with alternative 1 82% smaller. The covariance term also explains 98% of the difference between alternative 1 and 2. This decomposition provides some insights on the mechanism that allows the baseline model to quantitatively match the counter-cyclical variance in growth rates. Other things equal, if the firm type driving the heterogeneous response to the aggregate state was completely unrelated to a firm steady state growth rate (growth in $\omega_{j,t}$), the magnitude of the cyclical fluctuations in dispersion would be around four times smaller. In order to match the data, the heterogeneous effect of the aggregate state needs to work through a characteristic of the firm that is both negatively correlated to its size and positively correlated to its steady state growth rate. This decomposition exercise therefore lends further support to the mechanism highlighted in this paper.

5.2 Dispersion and Uncertainty

5.2.1 Volatility and Realized Dispersion

The dispersion in firm level growth rates has often been used as a proxy for uncertainty. This approach rests on the assumption that the cyclical fluctuations in dispersion are a direct result of the underlying volatility in business conditions[25]. However, uncertainty is related to the volatility in future growth rates *conditional* on a firm's current information

[25]Under this assumption, realized dispersion would can be used to proxy the degree of micro-uncertainty that firms face. Micro-uncertainty is related to the volatility in the idiosyncratic component of a firm's profits as opposed to macro-uncertainty which pertains to the volatility in the aggregate state of the economy.

set, rather than to their *unconditional* realized dispersion. As long as realized dispersion fluctuates for reasons unrelated to the underlying degree of business volatility, it does not constitute a good proxy for cyclical uncertainty.

Seen through the lenses of the model, the observed fluctuations in the dispersion of $\Delta v_{j,t}$ are driven by firms' underlying characteristics and their interaction with the aggregate state, rather than being the result of an unpredictable random process. To the extent that firms are aware of their type, this cyclical dispersion is predictable and does not constitute uncertainty.

To see how dispersion relates to business volatility in my model, let us again take profitability as the quantity of interest for firms. Let us first consider the volatility in the profits at period $t+1$. The variance of $\log(\pi_{j,t+1})$ conditional on the information at time t, $I_{j,t} = (\omega_{j,t}, z_{j,t}, u_{j,t}, A_t, P_t)$, is then[26]

$$\mathrm{Var}(\log(\pi_{j,t+1})|I_{j,t}) = \mathrm{Var}(\lambda_{t+1}|I_{j,t})\gamma^2 z_{j,t}^2 + \sigma^2 + \Sigma + \mathrm{Cov}(\lambda_{t+1}, \log(A_{t+1})|I_{j,t})\Gamma\gamma z_{j,t} \tag{5}$$

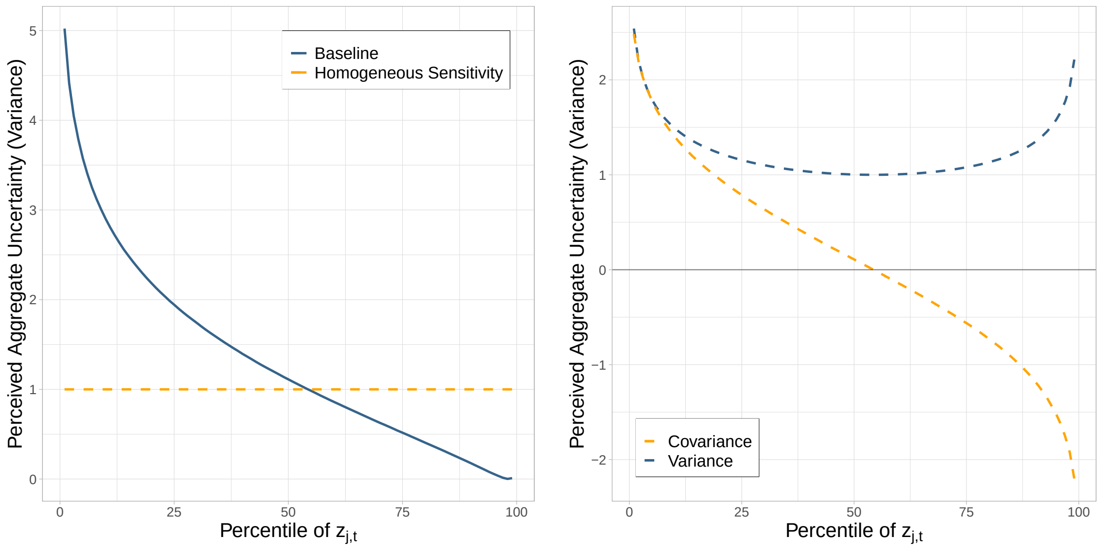
where the first two terms represent the conditional variance of the idiosyncratic term in equation 4, $\Sigma = [(\phi-1) + (\phi-2)\tau_2]^2\sigma_A^2$ is the variance of the common aggregate term and the last term captures the covariance between the idiosyncratic and aggregate terms, with $\Gamma = [(\phi-1) + \tau_2(\phi-2)]$. As Σ is constant, any cyclical fluctuation must come from the idiosyncratic component and its comovement with the aggregate term. Its dependence on time on the other hand depends on the functional form of $\lambda(C_{t+1})$ and on firms' forecast of P_{t+1} given $I_{j,t}$. In the simple linear parametrization chosen in Section 4 the conditional variance of λ_{t+1} and its conditional covariance with $\log(A_{t+1})$ are time-invariant and equal to $\mathrm{Var}(\lambda_{t+1}|I_{j,t}) = \overline{\lambda}^2\tau_2^2\sigma_A^2$ and $\mathrm{Cov}(\lambda_{t+1}, \log(A_{t+1})|I_{j,t}) = -\overline{\lambda}\tau_2\sigma_A^2$. Thus, the model shows that the empirical counter-cyclical dispersion can be generated even when the degree of uncertainty is fixed over time.

What about uncertainty over periods beyond $t+1$? In appendix E I show that the negative linear dependence of λ_t on C_t causes a reduction in the volatility of $v_{j,t+2}$ conditional on $\omega_{j,t}, z_{j,t}, u_{j,t}$ (micro-uncertainty) during periods of low aggregate productivity. Micro-volatility, defined as the dispersion in the future idiosyncratic demand $v_{j,t+2}$, is the result of the random process $u_{j,t+1}$, which on average increases the gap between a firm's frictional demand, $\omega_{j,t+1}$, and its potential value, η_j. As a higher λ_t reduces the importance of frictional demand, it limits the impact of $u_{j,t+1}$ on $v_{j,t+2}$, reducing the degree of micro-volatility in $v_{j,t+2}$[27].

[26] This is the perceived variance in $\log(\pi_{j,t+1})$ as I use the estimated forecasting rule in its derivation. This is consistent with the idea of uncertainty in the model.

[27] This result is due to the fact that the random component $u_{j,t}$ does not affect η_j. If this was the case the direction of the effect of λ_t on micro-volatility would be less clear.

Figure 6: Perceived Aggregate Uncertainty by Firm Type



Notes: The left panel plots the level of aggregate uncertainty perceived by each incumbent firm type. Value are for a situation where $A_t = 1$ and $P_t = 1$ (steady state). The solid blue line plots values from the baseline model. The dashed orange line plots values from a model where sensitivity to the aggregate state is homogeneous.

While the exact relation between uncertainty and realized dispersion depends on the specific functional form of λ_t and on firm's forecast of future economic conditions, the model proposed in this article suggests that the observed cyclical dispersion in growth rates might well be unrelated (or negatively related) to micro-level uncertainty. The results in this article should not be interpreted as evidence against counter-cyclical uncertainty, especially with respect to aggregate variables[28]. However, the model shows that if cyclical dispersion is the result of a pre-existing and persistent characteristic of the firm, it is unlikely to be a good proxy for time-varying micro uncertainty. This result is inline with recent evidence in Jurado et al. (2015), who show that while uncertainty and realized dispersion are positively correlated, their cyclical properties differ considerably with their time-series displaying substantial independent variation.

5.2.2 Heterogeneous Uncertainty

As already discussed, the model suggests that the observed counter-cyclical dispersion in growth rates is the result of some pre-existing characteristics of the firm, which become more salient during periods of low growth. If different types of firms are exposed differently

[28] A counter-cyclical level of σ_A can easily be added to the model in section 3 and would have no effect on fugure 4.

to the aggregate shock, the degree to which aggregate uncertainty is transmitted to and perceived by these firms will differ. This is evident from equation 5 where the measure of uncertainty, $\text{Var}(\pi_{j,t+1}|I_{j,t})$, depends on a firm's growth potential $z_{j,t}$. To see the extent of these differences, I consider the level of $\text{Var}(\pi_{j,t+1}|I_{j,t})$ net of the common idiosyncratic term σ^2, $\widetilde{\text{Var}}(\pi_{j,t+1}|I_{j,t}) = \text{Var}(\pi_{j,t+1}|I_{j,t}) - \sigma^2$, that is faced by firms when the economy is at the (stochastic) steady state[29]. This residual volatility captures the contribution of the volatility in aggregate TFP to the level of uncertainty faced by firms. The left panel in Figure 6 plots the level of $\widetilde{\text{Var}}(\pi_{j,t+1}|I_{j,t})$ by percentile of the distribution of $z_{j,t}$ among incumbent firms. Values are normalized by the level of $\widetilde{\text{Var}}(\pi_{j,t+1}|I_{j,t})$ when $\lambda_t = 1$ and uncertainty is homogeneous across firms. The graph shows that the heterogeneity is substantial. Firms at the 10th percentile face a variance that is three and a half times higher than in the homogeneous case. Firms at the 90th percentile, on the other hand, are almost entirely insured against aggregate shocks.

The right panel in Figure 6 decomposes $\widetilde{\text{Var}}(\pi_{j,t+1}|I_{j,t})$, the solid line in the right panel, into two components. The first (dashed blue line) is the sum of the variance of the common aggregate component and the variance of the idiosyncratic component net of the fixed idiosyncratic term σ^2, $\text{Var}(\lambda_{t+1}|I_{j,t})\gamma^2 z_{j,t}^2 + \Sigma$. This term captures the dircet effect of λ_t. The second (dashed orange line) is the covariance between the idiosyncratic and aggregate term, $\text{Cov}(\lambda_{t+1}, \log(A_{t+1})|I_{j,t})\Gamma\gamma z_{j,t}$. This term captures the effect in the co-movements between λ_t and aggregate TFP A_t. While the first term affects firms above and below their potential demand symmetrically, the second term decreases with $z_{j,t}$, increasing the aggregate uncertainty perceived by low growth firms and insuring high growth firms against fluctuations in the aggregate state[30]. Taken together, these two effects result in the positive relation between a firm's growth potential and its perceived aggregate uncertainty[31].

5.3 Entry, Exit and the Dynamics of Employment

As highlighted in the previous section, one of the main implications of the estimated model is that a firm's sensitivity to the aggregate condition of the economy is negatively correlated

[29] This is simply obtained by setting $A_t = 1$ and $P_t = 1$.

[30] The negative correlation between growth potential and size implies that bigger firms face, on average, a higher degree of aggregate uncertainty. This also implies that, should time-varying aggregate uncertainty be introduced in the model, big firms would be, on average, more sensitive to its cyclical fluctuations. This is inline with the evidence from US firm-level expectations data presented in Senga (2018), who finds that the cyclical dynamics of uncertainty are driven by low uncertainty firms, which the author finds to be bigger on average. The fact that bigger firms face on average lower uncertainty on the other hand is not at odds with my explanation as long as bigger firms face, on average, lower idiosyncratic uncertainty.

[31] I use the term "perceived aggregate uncertainty" to define the extent to which aggregate uncertainty is transmitted to a firm's profitability given its type.

to its expected future growth. The natural question to ask at this point is whether this negative correlation has any effect on firms' decisions and on the economy's aggregates. In this section I consider firms' entry and exit decisions and focus on how a negative correlation between sensitivity and expected future growth can affect the composition of entering and exiting firms and their contribution to aggregate employment.

Consider first incumbent firms that are indifferent between exiting and staying. These firms have operational value equal to zero, $V(\Psi_t, \kappa_{j,t}) = 0$. Now consider two firms with different levels of $z_{j,t-1}$ but with the same operational value $V(\Psi_t, \kappa_{j,t}) = 0$. Other things equal, $V(\Psi_t, \kappa_{j,t})$ is an increasing function of $z_{j,t-1}$, meaning that the firm with higher $z_{j,t-1}$ will have a lower value of $\omega_{j,t}$: marginal firms with higher growth potential tend to have smaller customer bases. This is intuitive as expected future growth compensates for current lower demand[32].

Consider now a shock of fixed size that hits one of the two firms. Allocating the shock to the low $z_{j,t-1}$ rather than to the high $z_{j,t-1}$ firm would cause the low $z_{j,t-1}$ firm to exit the market and the high $z_{j,t-1}$ one to stay. Compared to the situation where the high $z_{j,t-1}$ firm is hit and the low $z_{j,t-1}$ is not, the exiting firm is larger, while its future expected growth rate is lower. Extending this argument to the entire economy, going from an economy where λ_t is fixed and equal to 1 to the heterogenous sensitivity case presented in this article is equivalent to a reallocation of the burden away from high growth firms and towards low growth ones. This reallocation would cause the average size of entering firms to be more pro-cyclical and their average growth rate more counter-cyclical[33]. At the same time it would have a counter-cyclical effect on the average size of exiting firms and a pro-cyclical effect on their average growth rates. Combining these compositional effects, the sensitivity channel highlighted in this paper has a pro-cyclical effect on the average size of active firms and a counter-cyclical effect on their growth rates. In the rest of this section I quantify this compositional effect within the framework of my model.

The compositional effect works through the decisions made by firms with operational values that are close to the entry and exit thresholds. For expositional purposes I first focus on a subset of firms that closely relates to these *marginal* firms. I then extend the result to the entire economy and derive its aggregate implications.

[32] A higher $z_{j,t-1}$ must be compensated by a change in $\omega_{j,t-1}$ and $u_{j,t}$. It can be shown that any combination of $\omega_{j,t-1}$ and $u_{j,t}$ that compensate for a higher $z_{j,t-1}$ leads to a lower $\omega_{j,t}$.

[33] Recent work by Sedláček and Sterk (2017) has pointed out that the growth potential of start-ups created during recessionary periods is lower than that of those created during booms. This fact is not at odds with the mechanism outlined here. The compositional effect discussed in this paper is relative to a counter-factual economy where sensitivity to the aggregate state is homogeneous. If the baseline model features other cyclical composition effect the total effect of those effect and the heterogeneous sensitivity effect highlighted in this paper.

I start by considering the set of incumbent firms who endogeneously decide to exit the market at time t and those potential entrants that decide to enter at time t with a business opportunity that was not exogenously destroyed at period $t-1$[34]. In the rest of this section I refer to this subset of entering firms as those endogenously entering the economy[35]. To quantify the compositional effect induced by entry and exit decisions only I follow Sedláček and Sterk (2017) and set firms' employment level to their type-specific steady state value $\hat{N}(\kappa_{j,t})$ and compute changes coming solely from variations in the distribution of types induced by entry and exit decisions. I compute the average size of firms choosing to exit (EX) and of endogenous entrants (EN) as

$$N_t^{EX} = \frac{\int_{\Upsilon_t^{EX}} f(\kappa)\hat{N}(\kappa)d\kappa}{\int_{\Upsilon_t^{EX}} f(\kappa)d\kappa}$$

$$N_t^{EN} = \frac{\int_{\Upsilon_t^{EN}} f(\kappa)\hat{N}(\kappa)d\kappa}{\int_{\Upsilon_t^{EN}} f(\kappa)d\kappa}$$

where $f(\kappa)$ is the distribution of types among business opportunities (active and idle) and Υ_t^{EX} and Υ_t^{EN} is the set of endogenously exiting firms and endogenous entrants at time t, respectively[36]. I then shut down the heterogeneous sensitivity channel by setting $\lambda_t = 1$ and simulate the counter-factual economy. I define this alternative model as the *homogeneous sensitivity* case. Given this counter-factual simulation I record the counter-factual sets $\widetilde{\Upsilon}_t^{EX}$ and $\widetilde{\Upsilon}_t^{EN}$ and compute

$$\widetilde{N}_t^{EX} = \frac{\int_{\widetilde{\Upsilon}_t^{EX}} f(\kappa)\hat{N}(\kappa)d\kappa}{\int_{\widetilde{\Upsilon}_t^{EX}} f(\kappa)d\kappa}$$

$$\widetilde{N}_t^{EN} = \frac{\int_{\widetilde{\Upsilon}_t^{EN}} f(\kappa)\hat{N}(\kappa)d\kappa}{\int_{\widetilde{\Upsilon}_t^{EN}} f(\kappa)d\kappa}$$

the counter-factual average size of endogenously entering and exiting firms. The left panel in Figure 7 plots the difference in average sizes between the baseline and counter-factual models, normalized by the average size in the counter-factual economy. The solid blue line plots the difference for endogenously exiting firms, $(N_t^{EX} - \widetilde{N}_t^{EX})/\widetilde{N}^{EX}$, and the dashed

[34]Most of the opportunities that exogenously exit the market at time $t-1$ and are exploited by a new entrepreneur at time t have values that are well above the entry threshold and are therefore far from the definition of a marginal entrant.

[35]I use this term for expositional conciseness. While all entering firms choose to enter, most of the firms the enter with a product that was exogenously destroyed in the previous period have values that are considerably above the entry cost.

[36]The distribution $f(\kappa)$ is fixed over time and given by equation 2 under stationary. The sets Υ_t^{EX} and Υ_t^{EX} on the other hand evolve over time inline with firms' entry and exit decisions.

Figure 7: Composition Effect on Endogenously Entering and Exiting Firms

Notes: The left panel plots the percent difference in average employment size of endogenously entering (dashed orange line) and exiting (blue solid line) between the baseline and counter-factual model. The right panel plots the difference in the growth rate in the average employment in firms endogenously entering (dashed orange line) and exiting (blue solid line) between the baseline and counter-factual model. The growth rate at t refers to firms entering and exiting in period $t-1$.

orange line does the same for endogenously entering firms, plotting $(N_t^{EN} - \widetilde{N}_t^{EN})/\widetilde{N}^{EN}$. The result shows that allowing for a positive correlation between sensitivity to the aggregate state and growth potential has a counter-cyclical effect on the average size of endogenous entrants. Compared to the homogeneous sensitivity case, firms that decide to exit the market in 2009 are more than 12% larger. Conversely, the counter-cyclical fluctuations of λ_t have a pro-cyclical effect on the average size of endogenously entering firms. Compared to the homogeneous sensitivity case, firms that endogenously enter the market in 2009 are around 5.5% smaller.

The heterogeneous sensitivity channel has a second compositional effect, affecting the expected future growth rate of entering and exiting firms. To quantify this second effect I compute the period t to period $t+1$ growth rate in the total employment created by firms endogenously entering and exiting at time t, $(N_{t+1}^{EN} - N_t^{EN})/N_t^{EN}$ and $(N_{t+1}^{EX} - N_t^{EX})/N_t^{EX}$. I compute the same quantity for the counter-factual economy where $\lambda_t = 1$, $(\widetilde{N}_{t+1}^{EN} - \widetilde{N}_t^{EN})/\widetilde{N}_t^{EN}$ and $(\widetilde{N}_{t+1}^{EX} - \widetilde{N}_t^{EX})/\widetilde{N}_t^{EX}$. The dashed line in the right panel of Figure 7 plots the difference in the average growth rate of entering firms in the baseline and counter-factual economies, $(N_{t+1}^{EN} - N_t^{EN})/N_t^{EN} - (\widetilde{N}_{t+1}^{EN} - \widetilde{N}_t^{EN})/\widetilde{N}_t^{EN}$. The composition effect induced by the sensitivity channel increases the growth in the net employment created by firms entering during a recession. The size of the effect is important. Total employment in firms endogenously created in 2009 would have grown by 2.4 percentage points less should the sensitivity mechanism have been shut down. The compositional effect has the opposite sign on the average (potential) growth rate of endogenously exiting firms. The solid line plots the effect on the employment growth rate for firms endogenously exiting at time t, $(N_{t+1}^{EX} - N_t^{EN})/N_t^{EX} - (\widetilde{N}_{t+1}^{EX} - \widetilde{N}_t^{EX})/\widetilde{N}_t^{EX}$. The compositional effect is large and pro-cyclical: shutting down the heterogeneous sensitivity channel, the average growth potential of firms that exited in 2009 would have been 5.2 percentage higher[37].

Depending on the local properties of the distribution of $\kappa_{j,t}$ around the entry and exit thresholds, the $z_{j,t}$-dependent sensitivity can affect the number of endogenously entering and exiting firms and not just their composition. I discuss this effect in Appendix E.

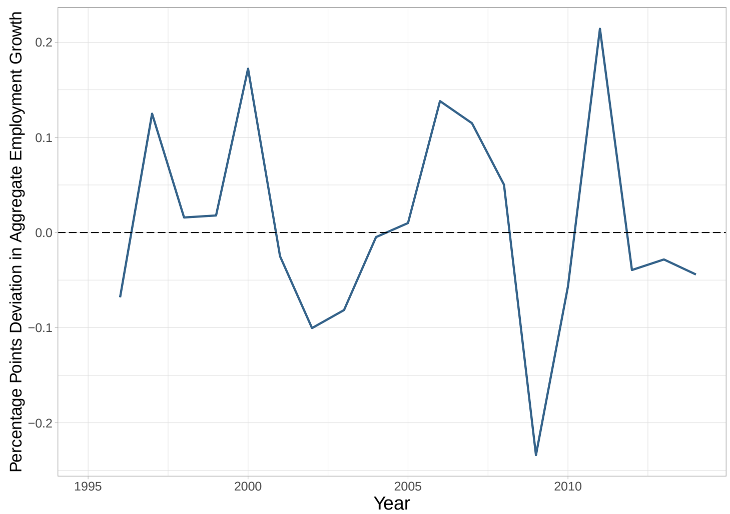
Up until now I have focused on the subset of firms endogenously entering and exiting the market. The next natural question to ask is whether the mechanism highlighted in this section is quantitatively relevant for macroeconomic aggregates. I therefore quantify the sensitivity effect on the total net job creation from entering and exiting firms, pooling the two composition effects and the effect on the number of firms together. To isolate the total effect of entry and exit I once again follow Sedláček and Sterk (2017) and decompose aggregate employment as

$$N_t = \int_{\Upsilon_t} f_t(\kappa) N_t(\kappa) d\kappa = \underbrace{\int_{\Upsilon_t} f_t(\kappa) \hat{N}(\kappa) d\kappa}_{\text{extensive margin}} + \underbrace{\int_{\Upsilon_t} f_t(\kappa)[N_t(\kappa) - \hat{N}(\kappa)] d\kappa}_{\text{intensive margin}} \tag{6}$$

where Υ_t represents the set of active firms in the economy and $\hat{N}(\kappa)$ is the steady state level of employment for a firm of type κ. The first term in equation 6 then captures the effect of entry and exit decisions only (the extensive margin) on aggregate employment. Changes in the first term are only due to changes in the distribution (number and composition) of active firms. The second term captures changes at the intensive margin (employment change within firms). Similarly, I decompose $\widetilde{N}_t$, the counter-factual level of employment, using equation 6 where $N_t(\kappa)$ is substituted by $\widetilde{N}_t(\kappa)$ and Υ_t is substituted by $\widetilde{\Upsilon}_t$: the set of active firms in the counter-factual economy.

[37] In their work Sedláček and Sterk (2017) show evidence that the growth potential of start-ups is pro-cyclical. This is evidence is not in contrast with the argument made in this section. The counter-factual exercise isolate the effect of the heterogeneous sensitivity channel on the composition of start-ups and exiting firms, other things held fixed. If other mechanisms at play generate a pro-cyclical average growth potential of start-ups, the counter-factual exercise simply suggests that absent the sensitivity effect this pro-cyclical behaviour would be even stronger.

Figure 8: Effect on Aggregate Employment at the Extensive Margin



Notes: Percentage points difference in the growth rate of aggregate employment coming from the sensitivity effect on the entry and exit margin.

Starting from the counter-factual economy, figure 8 plots the additional percentage point growth in total employment that would be obtained by substituting the intensive margin term from the counter-factual economy with the one in the baseline economy, everything else fixed. This is a measure of the total effect of the heterogeneous sensitivity channel on aggregate employment growth via entry and exit decisions[38]. The graph shows that the negative correlation between sensitivity to the aggregate state and future growth potential amplifies the effect of TFP shocks on aggregate employment: baseline employment growth tends to be higher during periods of expansion and lower during periods of recession compared its counter-factual level. The estimated model suggests that between 2008 and 2009 the sensitivity effect is responsible for a 0.23 percentage point drop in employment or about 10.5% of the total change in the data[39]. Considering that the effect reported in Figure 8 works only through entry and exit decisions, its magnitude is sizeable. The model suggests that the dependence of a firm's sensitivity to an aggregate shock on its expected future performance can have important consequences for the dynamics of macroeconomic aggregates.

Before concluding, it is worth stressing two points. First, the compositional effect works through the entry and exit decisions of marginal firms: firms with present discounted values

[38] This is a partial equilibrium exercise. By construction, if the economy was allowed to fully adjust in the simple general equilibrium model presented in section 3, prices would adjust to guarantee that employment remains constant. The only effect would then be on the relative importance of the first and second term in equation 6 in explaining N_t.

[39] Manufacturing employed growth for France is shown in Figure 13 in section E of the Appendix. As shown in the graph employment continues to drop in 2010 and starts recovering in 2011.

that are close to the entry and exit thresholds. In models where all entering firms are marginal, the compositional effect is likely to have larger aggregate consequences. This would notably be the case in models where entering firms choose their type and free entry drives the value of all opportunities to zero[40]. Second, in the model considered in this paper the sensitivity effect works solely via the entry and exit channel. In models where frictions influence the decisions of incumbent firms, including models with capital adjustment costs, price rigidities and frictional labor markets, a broader set of decisions would be affected by the mechanism highlighted in this paper. Investigating the effect on the frictional adjustment of incumbent firms is an interesting avenue for future research.

6 Conclusion

In this paper I exploit French firm-level data to provide new evidence on the cyclical dynamics of firm growth. The analysis shows that the persistence of firm-level log output is pro-cyclical and negatively co-moves with the dispersion in firm growth rates. A simple model of firm growth based on the idea that firms slowly gain access to their full demand potential provides a common explanation to these empirical regularities.

Seen through the lenses of the model these cyclical facts suggest that firms expecting to lose market share in the future are hit the hardest during economic downturns. The mechanism challenges the second moment shock interpretation for the observed counter-cyclical dispersion in growth rates. It suggests that cyclical dispersion is the result of a pre-existing and persistent characteristic of the firm rather than the direct consequence of increased volatility in its business environment. The results in this paper therefore caution against the use of realized dispersion in growth rates to proxy for time-varying micro uncertainty.

The correlation between a firm's future growth prospects and its sensitivity to the aggregate state is shown to have consequences for the cyclical characteristics of entering and exiting firms. The magnitude of this composition effect is important: the change in the size, number and expected growth of exiting firms and start-ups that is induced by this sensitivity channel is found to be equivalent to 10.5% of the total 2008-2009 drop in employment.

Much of this paper has focused on highlighting and providing evidence of a new mechanism that can explain well-known counter-cyclical behaviour of the dispersion in firms' growth rates. Many questions remain open for future research. I conclude by mentioning two. First, improving our understanding of the cyclical dynamics of firms would require a micro-foundation of the *market size* effect estimated in this paper. In line with the intuition

[40] See for example the model in Sedláček and Sterk (2017). The same would be true for exiting firms in a model that can explain the entire pattern of exit without requiring an exogenous exit shock.

provided here, a natural way of proceeding is within the framework of a frictional model of demand accumulation and competition for consumers. This is a something that I am currently exploring in other work. Second, in a model where incumbents' input and pricing decisions are affected by frictions and adjustment costs, the positive correlation between sensitivity to the aggregate state and growth potential can have broader consequences for the economy. Investigating these channels is an interesting avenue for future research.

References

Abraham, K. G. and Katz, L. F. (1986). Cyclical unemployment: sectoral shifts or aggregate disturbances? *Journal of political Economy*, 94(3, Part 1):507–522.

Aguiar, M., Hurst, E., and Karabarbounis, L. (2013). Time use during the great recession. *American Economic Review*, 103(5):1664–96.

Bachmann, R. and Bayer, C. (2014). Investment dispersion and the business cycle. *American Economic Review*, 104(4):1392–1416.

Bachmann, R., Carstensen, K., Lautenbacher, S., and Schneider, M. (2018). Uncertainty and change: Survey evidence of firms? subjective beliefs.

Bachmann, R., Elstner, S., and Sims, E. R. (2013). Uncertainty and economic activity: Evidence from business survey data. *American Economic Journal: Macroeconomics*, 5(2):217–49.

Barseghyan, L. and DiCecio, R. (2011). Entry costs, industry structure, and cross-country income and tfp differences. *Journal of Economic Theory*, 146(5):1828–1851.

Berger, D. and Vavra, J. (2017). Shocks vs responsiveness: What drives time-varying dispersion?

Bloom, N. (2009). The impact of uncertainty shocks. *econometrica*, 77(3):623–685.

Bloom, N., Floetotto, M., Jaimovich, N., Saporta-Eksten, I., and Terry, S. J. (2018). Really uncertain business cycles. *Econometrica*, 86(3):1031–1065.

Carlsson, M., Clymo, A., and Joslin, K.-E. (2019). Dispersion over the business cycle: Productivity versus demand.

Clementi, G. L. and Palazzo, B. (2016). Entry, exit, firm dynamics, and aggregate fluctuations. *American Economic Journal: Macroeconomics*, 8(3):1–41.

Edmond, C., Midrigan, V., and Xu, D. Y. (2015). Competition, markups, and the gains from international trade. *American Economic Review*, 105(10):3183–3221.

Foster, L., Haltiwanger, J., and Syverson, C. (2008). Reallocation, firm turnover, and efficiency: selection on productivity or profitability? *American Economic Review*, 98(1):394–425.

Foster, L., Haltiwanger, J., and Syverson, C. (2016). The slow growth of new plants: learning about demand? *Economica*, 83(329):91–129.

Gourio, F. and Rudanko, L. (2014). Customer capital. *Review of Economic Studies*, 81(3):1102–1136.

Ilut, C., Kehrig, M., and Schneider, M. (2018). Slow to hire, quick to fire: Employment dynamics with asymmetric responses to news. *Journal of Political Economy*, 126(5):2011–2071.

Jurado, K., Ludvigson, S. C., and Ng, S. (2015). Measuring uncertainty. *American Economic Review*, 105(3):1177–1216.

Kaas, L. and Kimasa, B. (2018). Firm dynamics with frictional product and labor markets.

Kehrig, M. (2015). The cyclical nature of the productivity distribution. *Earlier version: US Census Bureau Center for Economic Studies Paper No. CES-WP-11-15*.

Krusell, P. and Smith, Jr, A. A. (1998). Income and wealth heterogeneity in the macroeconomy. *Journal of political Economy*, 106(5):867–896.

Kuhn, F. and George, C. (2019). Business cycle implications of capacity constraints under demand shocks. *Review of Economic Dynamics*, 32:94–121.

Mayer, T., Melitz, M. J., and Ottaviano, G. I. P. (2014). Market size, competition, and the product mix of exporters. *American Economic Review*, 104(2):495–536.

Moscarini, G. and Postel-Vinay, F. (2012). The contribution of large and small employers to job creation in times of high and low unemployment. *American Economic Review*, 102(6):2509–39.

Munro, D. R. (2018). Consumer behavior and firm volatility. Technical report, Working paper, Middlebury Coll.

Perla, J. (2016). Product awareness, industry life cycles, and aggregate profits.

Pugsley, B. W., Sedlacek, P., and Sterk, V. (2019). The Nature of Firm Growth. Discussion Papers 1737, Centre for Macroeconomics (CFM).

Schaal, E. (2017). Uncertainty and unemployment. *Econometrica*, 85(6):1675–1721.

Sedláček, P. and Sterk, V. (2017). The growth potential of startups over the business cycle. *American Economic Review*, 107(10):3182–3210.

Senga, T. (2018). A new look at uncertainty shocks: Imperfect information and misallocation. Technical report, working paper.

Vavra, J. (2014). Inflation dynamics and time-varying volatility: New evidence and an ss interpretation. *The Quarterly Journal of Economics*, 129(1):215–258.

Zhelobodko, E., Kokovin, S., Parenti, M., and Thisse, J.-F. (2012). Monopolistic competition: Beyond the constant elasticity of substitution. *Econometrica*, 80(6):2765–2784.

Appendix

A Data Appendix

Datasets I use two sources of French firm-level data. The first dataset is FICUS/FARE, an administrative record of the universe of French firms containing standard firm-level balance sheet data, which include information on sales, value added, employment, capital, intermediate input and total wage bill. The dataset cover the period 1994-2016 and is based on firms' annual tax filling documents. The second source is a survey-based dataset used to complement tax based information. It covers all firms that either employ at least 20 employees or have total annual sales above ? and a sample of smaller firms, covering the period 1984-2007. Descriptive statistics on the samples are given in Table 6.

To recover the series of aggregate TFP I use two aggregate series from the INSEE. The first is a series on total output, Q_t, for the manufacturing sector (series 001689779). The second series reports total manufacturing employment, N_t, (series 001577235). Given the definition of aggregate TFP, A_t, in my model as

$$Q_t = \sum_j A_t N_{j,t} = A_t N_t \tag{7}$$

the series for aggregate TFP is obtained by dividing total output by total employment, which is then HP-filtered with a smoothing parameter equal to 100 to obtain its cyclical component.

Table 5: Sample Descriptives

	FICUS/FARE	EAE
Period	1994-2016	1984-2007
Observations	2,480,359	571,152
Unique firms	226,959	76,046
Average employees	37	120
Average sales (thousands)	10,046	26,550

Note: Descriptives are for the sample used in the estimation and empirical analysis. Sales are reported in millions of 2010 Euro. Unique firms count is based on the administrative unique identifier SIREN.

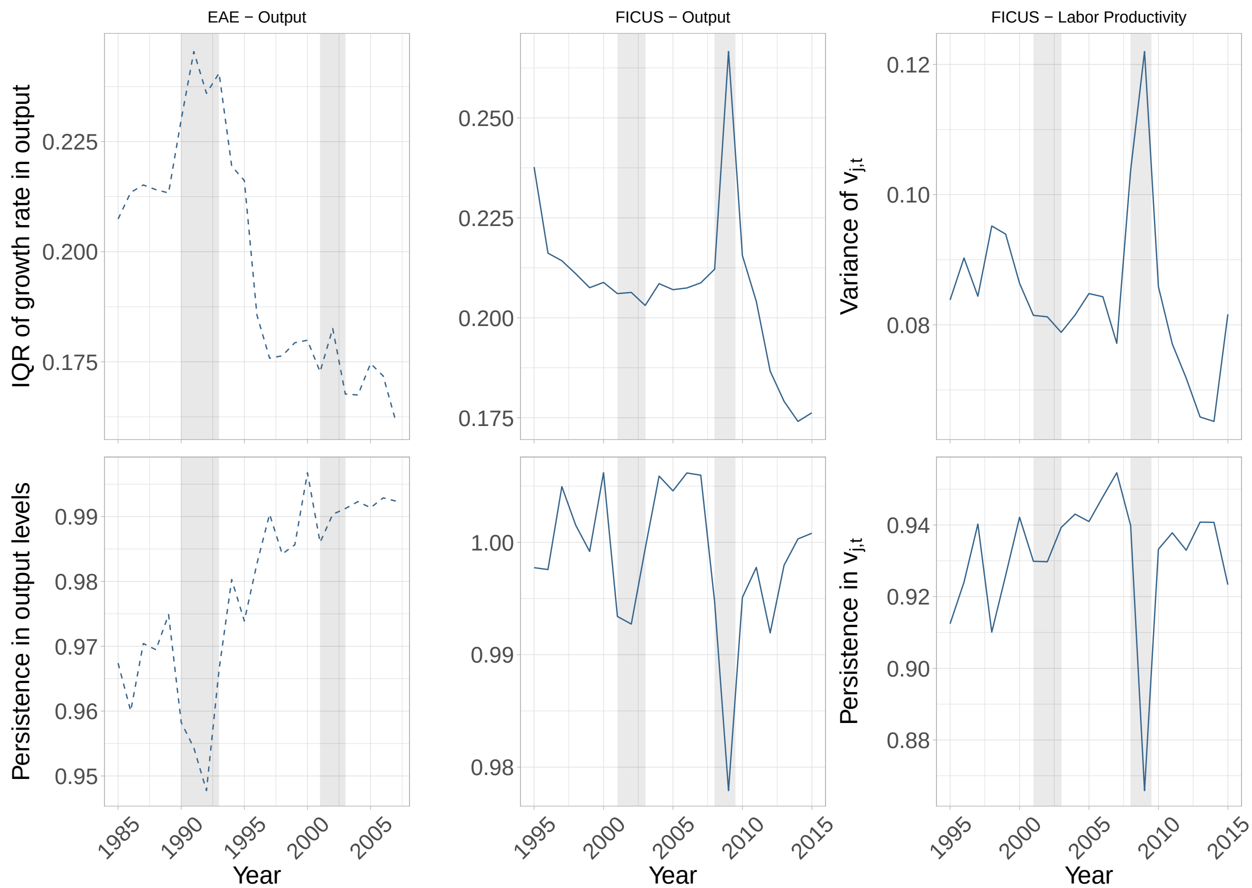
Comparability of EAE and FICUS/FARE Samples The discrepancy in the sample covered by the two sources challenges the comparability of aggregate statistics computed in the two samples. While qualitative patterns are similar in the datasets, quantitative comparison across sources should be cautious. To guarantee consistency, the estimation and counter-factual exercises carried out in the paper are based solely on the FICUS/FARE dataset, with the EAE dataset used only to show that the empirical facts presented in Section 2 hold before 1994. In order to plot Figure 1 I first HP-filter each series separately on each dataset with smoothing parameter 100. I then take the two series of cyclical components (one for each dataset) and add the average of the trend component from the FICUS/FARE dataset. In interpreting the results from Figure **??** one should therefore apply caution when comparing the magnitude of the cyclical fluctuations across datasets.

Sample Selection In the empirical analysis I focus on the manufacturing sector. In line with the guidelines issues by the FICUS/FARE data provider (INSEE) I select only firms subject to the tax filling scheme BRN and drop those subject to the simplified scheme RSI. This selection guarantees consistency in the way information is reported overtime allowing the data to be used as panel. Firms subject to BRN are on average bigger than those subject to the RSI. To limit the impact of outliers on the estimated series used in the paper, these are calculated by dropping firms in the top and bottom 0.5% of the distribution of log-changes in $x_{j,t}$, where $x_{j,t}$ is the variable used to compute the series. The qualitative results in the paper remain unchanged when the data are not trimmed.

Variables I use total sales (variable CATOTAL) as a measure of output, $Y_{j,t}$, and the average size of the workforce over the year (variable EFFSALM in EAE and FICUS and variable redi$_0$0 in FARE) as a measure of employment, $N_{j,t}$. Total sales are deflated by the sector-specific output deflators from the EU KLEMS dataset. The empirical value for a firm j labor productivity at time t, $v_{j,t}$, is then calculated as

$$v_{j,t} = \phi\big[\log Y_{j,t} - \Big(1 - \frac{1}{\phi}\Big)\log N_{j,t}\big]$$

Figure 9: Raw Empirical Series



Notes: The graph plots the raw, non HP-filtered series corresponding to mid and bottom panels in Figure 1 (first two columns) and to the empirical series in Figure 4 (last column).

Raw Series The first two columns in Figure 9 reports the raw, non HP-filtered series corresponding to mid and bottom panels in Figure 1. Trends are visible in the EAE dataset, but absent in the series for the persistence coefficient computed on FICUS/FARE data. The inter-quartile range on FICUS/FARE data shows a downward trend. The last column in Figure 9 reports the raw, non HP-filtered series for the empirical values of $Var_t(\Delta v_{j,t})$

and for the persistence coefficient on $v_{j,t}$. These are the the raw series from which the HP-filtered empirical series in Figure 4 are derived. Similarly to the case considering output, in the FICUS/FARE there is a small downward trend in the $Var_t(\Delta v_{j,t})$ and no trend in the persistence coefficient. Overall, the stylized facts presented in Figure 1 remain evident in the raw series.

B Demand

Steady State Properties I derive here the properties of the frictional demand, $\omega_{j,t}$, and growth potential $z_{j,t}$ for the set of business opportunities in the economy (active and inactive). First, note that $\omega_{j,t}$ has the following infinite representation

$$\omega_{j,t} = \lim_{i \to \infty} [(1-\gamma)^i \omega_{j,t-i} + \gamma \eta_j \sum_{l=0}^{i-1} (1-\gamma)^l + \sum_{l=0}^{i-1} (1-\gamma)^l u_{j,t-l}]$$

the covariance between $\omega_{j,t}$ and η_j is then obtained as

$$\text{Cov}(\omega_{j,t}, \eta_j) = \lim_{i \to \infty} (1-\gamma)^i \text{Cov}(\omega_{j,t-1}, \eta_j) + \gamma \text{Var}(\eta_j) \lim_{i \to \infty} \sum_{l=0}^{i-1} (1-\gamma)^l = \text{Var}(\eta_j)$$

where the first term tends to 0 and the sum tends to $\frac{1}{\gamma}$ as i tends to infinity. Assuming stationarity and given $Cov(\omega, \eta) = Var(\eta_j)$, the variance of ω can then be expressed as

$$\text{Var}(\omega) = \text{Var}(\eta) + \frac{(\text{Var}(u)}{1-(1-\gamma)^2)}$$

using the fact that $z_{j,t} = \eta_j - \omega_{j,t}$, the variance of $z_{j,t}$ is given by

$$\text{Var}(z) = \text{Var}(\eta) + \text{Var}(\omega) - 2\text{Cov}(\omega, \eta) = \text{Var}(\omega) - \text{Var}(\eta)$$

Finally, the covariance between $z_{j,t}$ and $\omega_{j,t}$ can be derived as

$$\text{Cov}(\omega, z) = \text{Cov}(\omega, \eta) - \text{Var}(\omega) = -\text{Var}(z)$$

First, note that as long as frictions are present and the system is stationary the variance of realized (frictional) demand $\omega_{j,t}$ at the steady state is higher than the variance of fundamental (potential) demand η_j. This also implies that growth potential is negatively correlated to realized demand. Second, these properties are derived on the entire set of business opportunities. In the presence of endogenous exit and entry the properties of $\omega_{j,t}$, η_j and $z_{j,t}$

on the sample of active firms can differ substantially.

Relation to AR(1) + Fixed Effect Model To see how the model in 2 relates to a standard AR(1) plus fixed effect model first note that the model can be rewritten as a system of two equations

$$\omega_{j,t} = \omega_{j,t-1} + \gamma z_{j,t-1} + u_{j,t} \tag{8}$$

$$z_{j,t} = (1-\gamma) z_{j,t-1} - u_{j,t} \tag{9}$$

Let us then rewrite 2 as

$$\begin{aligned} \omega_{j,t} &= \omega_{j,t-1} + \gamma[\eta_j - \omega_{j,t-1}] + u_{j,t} \\ &= \eta_j + (1-\gamma)[\omega_{j,t-1} - \eta_j] + u_{j,t} \end{aligned}$$

where $\omega_{j,t} - \eta_j$ being equal to $-z_{j,t}$ has an autoregressive law of motion

$$\begin{aligned} \omega_{j,t} - \eta_j = -z_{j,t} &= (\gamma - 1) z_{j,t-1} + u_{j,t} \\ &= (1-\gamma)[\omega_{j,t-1} - \eta_j] + u_{j,t} \end{aligned}$$

Further notice that given that $\mathrm{Cov}(\omega_{j,t}, \eta_j) = \mathrm{Var}(\eta_j)$ as shown above, the covariance between η_j and $[\omega_{j,t-1} - \eta_j]$ is zero

$$\mathrm{Cov}(\eta_j, \omega_{j,t-1} - \eta_j) = \mathrm{Cov}(\omega_{j,t}, \eta_j) - \mathrm{Var}(\eta_j) = 0$$

The system in 8 and 9 therefore has a familiar representation and can be rewritten as an AR(1) model with fixed heterogeneity. The autocovariance function of $\omega_{j,t}$ in 8 and 9 in particular is equivalent to that of $y_{j,t}$ in the model

$$y_{j,t} = \tilde{\eta}_j + \tilde{\alpha}_{j,t}$$

$$\tilde{\alpha}_{j,t} = \rho \tilde{\alpha}_{j,t-1} + \tilde{u}_{j,t}$$

where $\rho = 1 - \gamma$, $\mathrm{Var}(\tilde{u}_{j,t}) = \mathrm{Var}(u_{j,t})$, $\mathrm{Var}(\tilde{\eta}_j) = \mathrm{Var}(\eta_j)$, $\mathrm{Var}(\tilde{\alpha}_{j,t}) = \mathrm{Var}(\omega_{j,t}) - \mathrm{Var}(\tilde{\eta}_j)$ and $\mathrm{Cov}(\tilde{\eta}_j, \tilde{\alpha}_{j,t}) = 0$.

Cyclical Properties I first consider persistence. To derive the cyclical properties of persistence I use a slightly different, but closely related, measure that is easier to handle compared to the variance covariance ratio use in Section 2. In particular I use the difference $\mathrm{Cov}(v_{j,t}, v_{j,t-1}) - \mathrm{Var}(v_{j,t-1})$. As will become clear in a few lines, this statistics has the advantage of differencing out the variance of $\omega_{j,t}$ and for this reason all persistence moments used in the estimation are based on this measure[41]. The expression for this persistence statistics is then

$$
\begin{aligned}
&\mathrm{Cov}(v_{j,t}, v_{j,t-1}) - \mathrm{Var}(v_{j,t-1}) = \\
&\qquad (1+\lambda_t(1-\gamma)-\lambda_{t-1})\gamma \underbrace{[\mathrm{Cov}(\omega_{j,t-2}, z_{j,t-2}) + \lambda_{t-1}\gamma \mathrm{Var}(z_{j,t-2})]}_{\mathrm{Cov}(v_{j,t-1}, z_{j,t-2})} - \lambda_t \gamma \mathrm{Var}(u_{j,t-1})
\end{aligned} \tag{10}
$$

First, note that for $\gamma > 0$ the effect of an increase of λ_t on the second term is always negative. The direction of the effect on the first term on the other hand depends on the term in square brackets, which is simply the covariance between $v_{j,t-1}$ and $z_{j,t-2}$. The interpretation is simple, as long as past growth potential $z_{j,t-2}$ is negatively correlated to past demand, increasing λ_t results in a drop of the persistence measure, other things equal. Note that past market conditions also play a role in determining persistence. The intuition is simple. As the characteristics that are affected by the market condition parameters are pre-existing and persistent, both the evolution of λ_t and its level play a role in determining the persistence in $v_{,t}$. In particular, looking at equation 10 the first term captures the effect through the pre-exiting persistent level of z, while the second term capture the effect through the innovation in z.

Consider now the expression for $\mathrm{Var}_t(\Delta v_{j,t})$, where the subscript t means that the variance is computed conditional on calendar year

$$
\mathrm{Var}_t(\Delta v_{j,t}) = (1+\lambda_t(1-\gamma)-\lambda_{t-1})^2\gamma^2 \mathrm{Var}(z_{j,t-2}) + \lambda_t^2\gamma^2\mathrm{Var}(u_{j,t-1}) + \mathrm{Var}(u_{j,t}) \tag{11}
$$

As long as the term in brackets is positive, increasing λ_t has a clear contemporaneous positive positive effect on the dispersion in growth rates between $t-1$ and t, meaning that a counter-cyclical λ_t would result in a counter-cyclical $\mathrm{Var}_t(\Delta v_{j,t})$. Note that the magnitude of these fluctuations in $\mathrm{Var}_t(\Delta v_{j,t})$ depends on the difference between $\lambda_t(1-\gamma)-\lambda_{t-1}$. In particular, a high past value λ_{t-1} reduces the effect of increasing λ_t. The intuition is the same given for the persistence statistics. As the characteristics that are affected by the market condition parameters are pre-existing and persistent, part of the change in $\mathrm{Var}_t(\Delta v_{j,t})$ is due to the change in the relevance of these persistence characteristics. This in turn implies that both the evolution of λ_t and its level play a role in the cyclical behaviour of $\mathrm{Var}_t(\Delta v_{j,t})$. Looking at equation 11 the first term then captures the effect through the pre-exiting persistent level of z, while the second term capture the effect through the innovation in z.

[41] The properties of the two measures are very similar. Using the ratio of covariances rather than their difference in the estimation has no significant impact on the results. The similarity between the two measures in my sample is also proved in Figure 4 where the estimate obtained using the difference is shown to be able to match the cyclical fluctuations in the ratios.

C Model Appendix

The first order conditions with respect to C_t and N from the household maximization problem

$$\begin{aligned} \max_{C_t, N_t} \quad & \sum_{i=t}^{\infty} \beta^i (\log(C_t) - \chi N_t) \\ \text{s.t.} \quad & \int_{j \in \Omega_t} p_{j,t} c_{j,t} dj = P_t w_t N_t + \Pi_t \end{aligned} \tag{12}$$

give the set of equations

$$\frac{1}{C_t} + \iota_t P_t = 0$$

$$\chi + \iota_t W = 0$$

where ι_t is the Lagrange multiplier on the budget constraint. Putting them together gives the condition $\frac{W}{P_t} = \chi C_t$.

Inverting the expression for the good's specific demand $c_{j,t}$ we get $p_{j,t} = \psi^{\frac{1}{\phi}} c_{j,t}^{-\frac{1}{\phi}} C_t^{\frac{1}{\phi}} P_t$. Using the production function equation $q_{j,t} = A_t N_{j,t}$ the real profit function can then be rewritten in terms of $N_{j,t}$ as

$$\pi_{j,t} = \psi^{\frac{1}{\phi}} A_t^{(1-\frac{1}{\phi})} N_{j,t}^{(1-\frac{1}{\phi})} C_t^{\frac{1}{\phi}} - N_{j,t} w_t$$

The firm's optimal choice of $N_{j,t}$ maximizes $\pi_{j,t}$ and gives

$$N_{j,t} = \psi A_t^{\phi-1} W^{-\phi} (1 - \frac{1}{\phi})^{\phi} C_t P_t^{\phi}$$

substituting in the expression for $\pi_{j,t}$ we get

$$\pi_{j,t} = \psi A_t^{\phi-1} C_t P_t^{\phi-1} W^{1-\phi} (\phi - 1)^{\phi-1} \phi^{-\phi}$$

Using the expression for the production function and the optimal choice of $N_{j,t}$, the equation for $p_{j,t}$ can be rewritten in terms of the markup over the nominal marginal cost of production

$$p_{j,t} = \frac{\phi}{\phi - 1} \frac{w_t P_t}{A_t}$$

implying that all firms set the same price. Using this expression and the fact that $P_t C_t =$

$\int_{j\in\Upsilon_t} p_{j,t}c_{j,t}$ one can show that

$$\begin{aligned}\frac{W}{\chi} = P_tC_t &= \int_{j\in\Upsilon_t} p_{j,t}c_{j,t} \\ &= \int_{j\in\Upsilon_t} \frac{\phi}{\phi-1}\frac{w_tP_t}{A_t}A_tN_{j,t} \\ &= \frac{\phi}{\phi-1}WN_t\end{aligned}$$

which implies a constant level of N_t.

D Estimation

Conditional on the parameters defined a priori, I recover estimates for the remaining parameters in several steps. First, I calibrate the set of parameters $\theta^s = (f_c, \delta, \sigma^\eta, \mu^\eta, \gamma, \sigma)$ to match a set of six steady state moments by simulated method of moments (SMM). In practice the estimation is carried out by solving and simulating the economy at the non-stochastic steady state. To approximate the steady state values of the six moments I compute their value for each year in the sample and compute the mean value from their HP-100 trend component. The estimated values are then chosen to minimize the distance between the model generated moments and these approximated steady state empirical values. The advantage of this approach is that it requires solving the model only at the non-stochastic steady state, substantially speeding up the estimation procedure[42].

While the estimates are recovered jointly, each moment has a clear relation to one of the six parameters. In particular, to identify f_c I match the slope coefficient of the linear probability model

$$d_{j,t} = \beta_0 + \beta_1 v_{j,t-1} + \epsilon_{j,t}$$

where $v_{j,t-1}$ is a firm revenue productivity at period $t-1$ and $d_{j,t}$ is equal to 1 if a firm exit the market at period t and 0 otherwise. The residual exit rate then identifies the exogenous exit probability δ. The observed dispersion in firm sizes among incumbent firms identifies σ^η, while μ^η is set such that aggregate consumption C_t is normalized to 1 at the non-stochastic steady state. The average of the ratio

$$\zeta_{t,i} = \frac{\text{Cov}(v_{j,t+i}, v_{j,t}) - \text{Cov}(v_{j,t+i-1}, v_{j,t})}{\text{Cov}(v_{j,t+i-1}, v_{j,t}) - \text{Cov}(v_{j,t+i-2}, v_{j,t})}$$

[42] At the non-stochastic steady state the state-space is only three dimensional, while it is five-dimensional when aggregate dynamics are taken into considerations.

Table 6: (Long-Run) Empirical Moments

Moment	Value	Identified Parameter
Exit probability: slope	-0.0381	f_c
Exit rate	0.063	δ
Var($N_{j,t}$)	0.31	σ^{η}
E[$\zeta_{t,i}$]	0.774	γ
Var($v_{j,t}$)	0.0846	σ

Note: The table shows the moments used in the estimation of the six parameters recovered from the steady-state of the model. Moments are computed from FICUS/FARE data for the period 1994-2016. The six moment used in the estimation is the normalization $C_t = 1$ at the steady-state.

for $1 < i \leq 4$ contains information on the persistence parameter γ[43]. Finally, the dispersion in the growth rates of revenue productivity is used to identify σ. The value for these moments are given in Table .

Conditional on ρ_A, the parameter σ_A is recovered by matching the dispersion in aggregate TFP as defined in equation 7. This leaves the parameter, $\overline{\lambda}$, and the parameters of the forecasting rule, $\tau = (\tau_1, \tau_2, \tau_3)$, to be estimated. Conditional on the values of γ and σ, in the absence of selection an estimate for $\overline{\lambda}$ can be easily recovered using the series of moments from equation 10. In order to correct for selection, I estimate $\overline{\lambda}$ using a simple iterative GMM algorithm. I first solve and simulate the economy out of the steady state for a given set of parameters $\theta = (\overline{\lambda}, \tau)$. Simulations are set to reproduce the empirical economy between 1994 and 2016 as described in below. From the simulated economy I recover a set of year-specific selection correction terms for the set of quantities $Cov(v_{j,t}, v_{j,t-1}) - Var(v_{j,t-1})$, $Var(u_{j,t-1})$, $Cov(\omega_{j,t-2}, z_{j,t-2})$ and $Var(z_{j,t-2})$. These selection correction terms show the model-implied size of the unobserved unselected quantity relative to the observed selected one for every year t. They can therefore be used to correct the observed empirical moments to account for the effect of selection. In practice I iterate between three steps

1. Solve and simulate the economy out-of-the steady state for a given value $\theta_{i-1} = (\overline{\lambda}_{i-1}, \tau_{i-1})$.

2. Derive the selection correction term from the simulated economy and update the estimate for $\overline{\lambda}_i$.

3. Use θ_{i-1} to simulate the economy out of the steady state for T periods and update the

[43] In absence of selection the process for the accumulation of the demand in section 3 implies that this ratio is equal to $(1 - \gamma)$ for every i. I exclude $i = 1$ in order to limit the impact of measurement error and transitory shocks on the estimated γ.

estimate τ_i.

I initiate the iteration using the estimate for $\overline{\lambda}$ obtained ignoring selection and setting $\tau = (0, 0, -1)$ and iterate until convergence. The estimated forecasting rule precisely tracks the dynamics of P_t, with an R-squared equal to 0.97.

Simulations In order to obtain the steady state of the economy I first simulate for 1.000 periods the process for (ω, z, u) and obtain its steady state distribution. I then simulate (ω, z, u) together with the exit and entry decisions of firms for other 1.000 periods. The parameters calibrated at the steady state are recovered from simulated steady state based on a cross-section of 10.000 simulated business opportunities[44].

Selection correction terms are estimated using 100.000 business opportunities setting 1979 as the steady state and letting the economy adjust to the estimated series of aggregate shocks for the period 1980-2015. Note that taking a prior calendar year as the steady state has little impact on the dynamics over the period 1994-2015. The forecasting rule is updated using 10.000 simulated business opportunities over 5.000 periods, with period 1 being the steady state. The first 1.000 periods are then dropped, and the forecasting rule is updated by OLS on the remaining 4.000 periods.

Partial Identification Figure 10 plots the distance between each of the empirical moments used in the SMM estimation and its empirical counter-part when the parameter they are chosen to identify changes and the other parameters are held fixed at their estimated value. The distance is computed as the sqaured percent deviation between simulated and empirical moments.

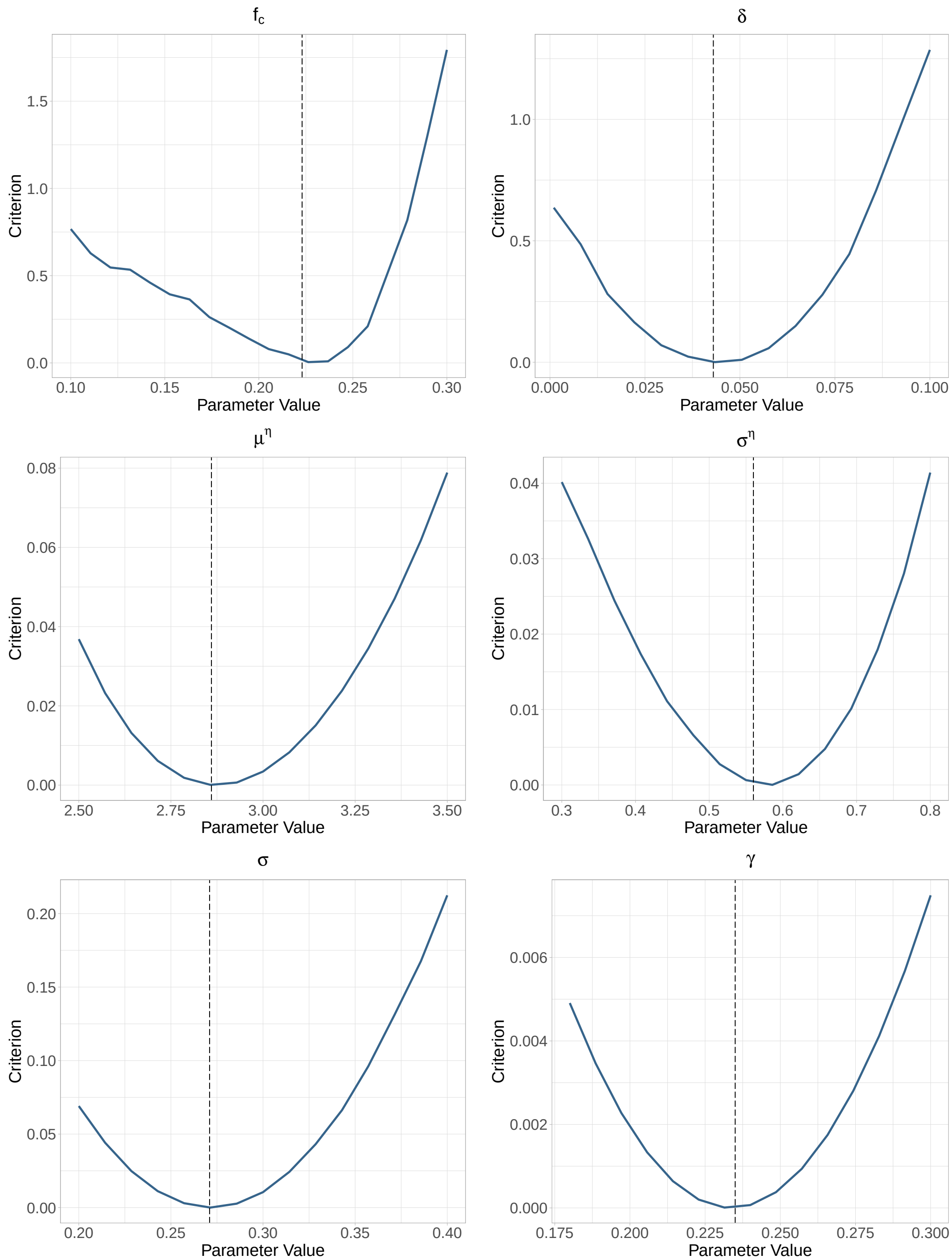
E Discussion

E.1 Alternative Models

Figure 11 shows the persistence coefficient for two alternative models and the baseline model. The alternative models are closely match the cyclical behaviour of the persistence coefficient from the baseline model, while generating substantially less dispersion in growth rates as seen in Figure 5.

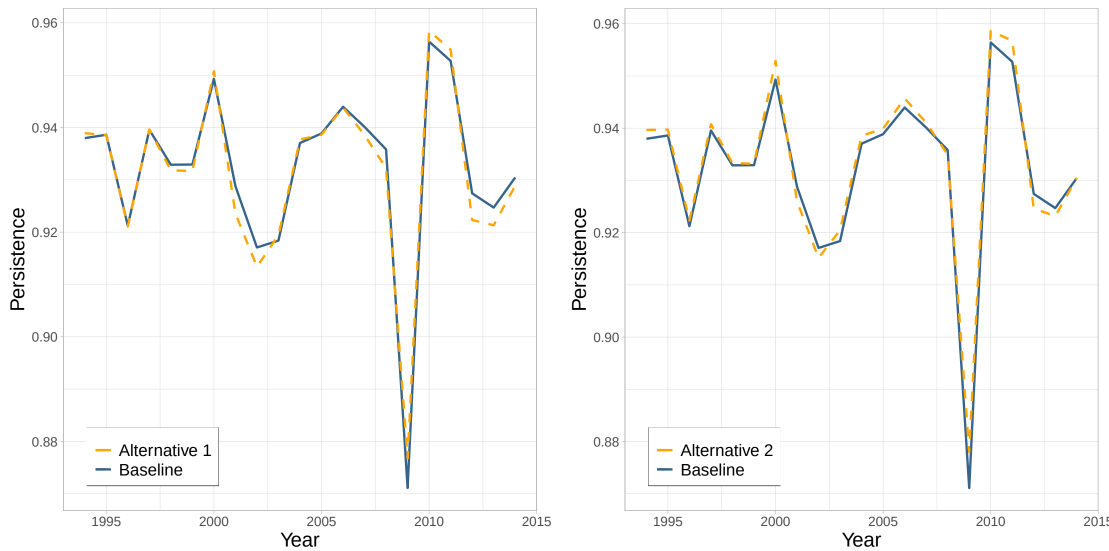
[44]The realizations of the random processes are held constant across simulations

Figure 10: Partial Identification



Notes: The graph plots the distance between the empirical and simulated moment that is most closely related to each parameter $\theta_p \in \theta^s$, as a function of θ_p when the other parameters are held constant at their estimated values. The blu solid line plots the squared percent deviation between the simulated and empirical moment. The dotted vertical line shows the value of the estimated parameter θ_p. The moment chosen for each parameter is described in the main body of section D.

Figure 11: Predicted Persistence: Alternative Models



Notes: The graphs plot the persistence statistics in the baseline model (solid blue line), in the alternative 1 model (dashed orange line, left panel) and in the alternative 2 model (dashed orange line, right panel).

E.2 Dispersion and Uncertainty

To see how uncertainty over $t+2$ is affected by the level of aggregate productivity let us consider the variance of $v_{j,t+2}$, the idiosyncratic component of $\log(\pi_{j,t+2})$, conditional on the information set at time t, I_t. Using the law of total covariance and the law of total variance it can be shown that

$$\mathrm{Var}(v_{j,t+2}|I_t) = [1-\gamma+\overline{\lambda}\tau_2\rho_A^2\log(A_t)]^2\sigma^2+\widetilde{\Sigma} \tag{13}$$

where $\widetilde{\Sigma} = [\sigma^2+(1-\gamma)^2 z_{j,t}]\overline{\lambda}\gamma^2\tau_2^2(1+\rho_A^2)\sigma_A^2+\sigma^2$ is a time-invariant term. The conditional volatility in $v_{j,t+2}$ is not constant over time. The level of $\log(A_t)$ in fact mediates the effect of $u_{j,t+1}$ on $v_{j,t+2}$, with direct consequences on the volatility of $v_{j,t+2}$. Taking the derivative of the first term in equation 13 with respect to $\log(A_t)$ it is clear that the conditional volatility of $v_{j,t+2}$ is increasing in the level of $\log(A_t)$ as long as $\tau_2 < 0$, $\overline{\lambda} < 0$ and

$$[1-\gamma+\overline{\lambda}\tau_2\rho_A^2\log(A_t)] > 0$$

E.3 Entry, Exit and the Dynamics of Employment

Effect on the Number of Entering and Exiting Firms As discussed in the main text the fact that $z_{j,t}$ is not evenly spread across firms values, means that the correlation between

Figure 12: Effect on the Number of Entering and Exiting Firms

Notes: The left panel plots the percent difference in the number of endogenously entering (dashed orange line) and exiting (blue solid line) firms between the baseline and counter-factual model.

growth potential and sensitivity to the aggregate state can have an effect not just on the composition of firms exiting and entering the market, but also on their number. I therefore compute the total mass of endogenously entering and exiting firms in the baseline model

$$n_t^{EN} = \int_{\Upsilon_t^{EN}} f_t(\kappa) d\kappa$$

$$n_t^{EX} = \int_{\Upsilon_t^{EX}} f_t(\kappa) d\kappa$$

I do the same for the counter-factual model

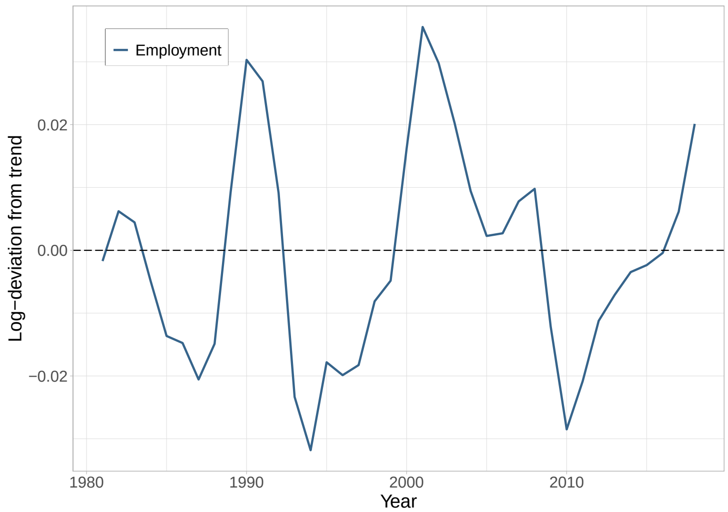
$$\tilde{n}_t^{EN} = \int_{\tilde{\Upsilon}_t^{EN}} f_t(\kappa) d\kappa$$

$$\tilde{n}_t^{EX} = \int_{\tilde{\Upsilon}_t^{EX}} f_t(\kappa) d\kappa$$

and I compute the percent difference in the number of endogenously entering and exiting firms between the two economies as $(n_t^{EN} - \tilde{n}_t^{EN})/\tilde{n}_t^{EN}$ and $(n_t^{EX} - \tilde{n}_t^{EX})/\tilde{n}_t^{EX}$. Figure 12 plots the two deviations. The results show a pro-cyclical effect on the number of exiting firms and a counter-cyclical effect on the number of exiting firms. This additional effect therefore goes in the same direction as the effect shown in 7 and strengthen the total effect on employment.

Manufacturing Employemnt in France Fugure 13 shows the HP-filtered series for manufacturing employment in France. The series is filtered using a smoothing parameter equal to 100.

Figure 13: Manufacturing Employment (Data)



Notes: Cyclical component of the HP-filtered series of log-employment in the manufacturing sector. The series is filtered using a smoothing parameter equal to 100. Data are from INSEE (series 001577235).