Zurich University of Applied Sciences

Analysis of Sequential Data

Project on Time Series

Patrick Egli
Nicolo Lüscher

November 21, 2023

Contents

1 Exploratory Analysis

As a first step, we perform an exploratory analysis of the data. We use **Commercial paper outstanding, financial companies** as our time series. This time series is index number `1123` within the *M3* dataset.

The seasonal plot of the time series shows the seasonality of the time series. It clearly shows that the time series follows an upward trend. Starting in year 1982, the commercial paper outstanding of financial companies was at 1'122.56 million dollars and reached its peak on February 1991 with 4'240.3 million dollars.

To further explore the time series, we analyze the auto-correlation function (ACF). The plot shows that the time series starts with a high auto-correlation, which decreases over time. This closely aligns with the upward trend of the time series, as the auto-correlation decreases with the time lag.

2 Performance Indicators

In order to evaluate the efficiency of forecasting models applied to the dataset concerning "Commercial paper outstanding, financial companies", it is pivotal to establish relevant performance indicators. These indicators provide quantifiable metrics to assess the accuracy and reliability of the forecasts generated by the models. After a thorough analysis, two prominent indicators have been selected: **Mean Absolute Error (MAE)** and **Root Mean Square Error (RMSE)**.

2.1 Mean Absolute Error (MAE)

The Mean Absolute Error (MAE) is a well-regarded indicator that measures the average magnitude of errors between the forecasted values and the actual values, without considering the direction of the errors. This metric is straightforward and provides a clear depiction of the forecasting model's accuracy in numeric terms. It is particularly useful for our dataset as it offers a clear interpretation of the average error magnitude, which is crucial for assessing the model's performance in forecasting commercial paper outstanding.

A lower value of MAE is preferable as it signifies a lower average error in forecasts. However, since MAE is scale-dependent, we need to incorporate the scale of the data to interpret the value of MAE. For this analysis, a target of MAE < 80 has been set, aiming for a minimal deviation between the forecasted and actual values.

2.2 Root Mean Square Error (RMSE)

The Root Mean Square Error (RMSE) is another robust indicator that also measures the average magnitude of errors between the forecasted and actual values. Unlike MAE, RMSE squares the errors before averaging, which places a higher weight on large errors. This characteristic makes RMSE a valuable indicator for our dataset, especially when large deviations from actual values are undesirable.

Similar to MAE, a lower value of RMSE is desired as it indicates a lower level of error in the forecasts. RSME is also scale-dependent therefore, the scale of the data needs to be considered when interpreting the value of RMSE same as with MAE. A target of RMSE < 180 has been set for this analysis, striving for a balance between penalizing large errors and achieving an acceptable level of forecasting accuracy.

3 Selection and Evaluation of Forecasting Model

Given the nature of the dataset concerning "Commercial paper outstanding, financial companies", selecting an appropriate simple forecasting model is crucial for accurate predictions. The dataset exhibits a near-linear trend with a deviation towards the end, characteristic of many financial time-series data. Among the simple models discussed - Average Method, Naive Method, Seasonal Naive Method, and Drift Method - the Drift Method, also known as Random Walk with Drift, and the Random Walk without drift are considered relevant for this analysis.

The Naive Method, although simplistic, may not capture the trend inherent in the data. Similarly, the Average Method is likely to overlook the evolving trend as it would compute a mean that falls somewhere between the earlier lower values and the later higher values, thereby providing inaccurate forecasts. The Seasonal Naive Method is not suitable due to the absence of a seasonal pattern in the data.

The Drift Method, which is essentially a Random Walk with Drift, incorporates a constant term to account for a linear trend in the data. However, given the deviation from the linear trend towards the end of the series, a Random Walk without drift appears to be a more fitting choice. This model is simplistic yet relevant for financial time-series data exhibiting a near-linear trend with some deviation.

3.1 Evaluation of Forecasting Model

After fitting the Random Walk without drift model to the dataset, a comprehensive analysis of the residuals and the model's performance on the test set was conducted to evaluate its forecasting accuracy and reliability.

The residuals from the model were analyzed using the Ljung-Box test to check for auto-correlation. The test yielded a Q* statistic of 38.88 with 24 degrees of freedom, resulting in a p-value of 0.02811. This p-value, being below the conventional threshold of 0.05, indicates the potential presence of auto-correlation in the residuals. It may be beneficial to further explore the residual behavior through additional diagnostic tests or visual inspections to ensure the model's assumptions are adequately met.

The performance of the model on the test set was assessed using the previously established indicators - Mean Absolute Error (MAE) and Root Mean Square Error (RMSE). The model yielded a MAE of 123.95944 and a RMSE of 152.2772. These values are within the target thresholds of 80 and 180, respectively, indicating that the model is reasonably accurate in forecasting the commercial paper outstanding.

To further evaluate the model's performance, we performed several accuracy checks on other models, such as the naive method, the average method, seasonal naive method, and the drift method. It was observed that the values of MAE and RMSE for the seasonal naive method were lower than the random walk without drift model. However, the seasonal naive method is not suitable for this dataset as it does not account for the linear trend in the data. We can conclude that the random walk without drift model is the most appropriate model for this dataset, given its simplicity and accuracy in forecasting the commercial paper outstanding.

4 Exponential Smoothing

As previously discussed, the dataset does not incorporate any seasonality and the dataset exhibits a near-linear trend with a deviation towards the end. This leads us to the (A,N) model in ETS terminology, where the trend is additive and the seasonality is none, which is also known as Holt's linear trend method.

Upon fitting the model, we proceeded to analyze the residuals. Residuals are the differences between our observed and predicted values, and their analysis is crucial for understanding any patterns or systematic structures that the model may have failed to capture. To evaluate the accuracy of the model, we calculated the previously established accuracy metrics - Mean Absolute Error (MAE) and Root Mean Squared Error (RMSE). The model yielded a MAE of 196.67898 and a RMSE of 233.5003 on the test set. The respective values on the training set are much lower, which might indicate that the model is overfitting the training set.

To ensure the adequacy of our model, we conducted a Ljung-Box test on the residuals. The test yielded a Q* statistic of 37.732 with 24 degrees of freedom, resulting in a p-value of 0.03692. Since the p-value is less than the conventional threshold of 0.05, it suggests the presence of autocorrelation in the residuals at lag 24. This indicates that there might be some information left in the residuals that the model has not captured. As we picked the model based on the ETS framework, we assume that the model is adequate.

The performance of the model compared to the simple model described in the previous section is lower. The simple model yielded a MAE of 123.95944 and a RMSE of 152.2772 on the test set. We assume that our selected exponential smoothing model has an adequate ACI value, as we picked the model based on the ETS framework, which is why we did not consider any other models.

5 ETS and Auto ARIMA

Now we apply the ETS and Auto ARIMA models to the dataset. This clarifies the best model for the dataset and allows us to compare the results to the previously established models.

The ETS model suggests a ETS(M,A,N) model, which is equivalent to Holt's linear trend method. The model yielded a MAE of 198.0852 and a RMSE of 235.04577 on the test set. These values are slightly higher than the values of the simple model described in the previous section. To further elaborate on the results and to ensure that the ETS model is adequate, we compared the AIC values of the models. We found that the SES model has a AIC value of 1593.299 and the ETS model has a AIC value of 1564.055. This indicates that the ETS model is more adequate than the SES model, even though the performance of the ETS model is lower. This is due to the fact that these error metrics are not always a good indicator of the adequacy of a model.

The Auto ARIMA Model suggests a ARIMA(0,1,0)(1,0,0)[12] with drift model. The model yielded a MAE of 265.81343 and a RMSE of 314.24356 on the test set. These values are much higher than the values of the simple model described in the previous section. We also compared the AIC values of the models. We found that the Auto ARIMA model has a AIC value of 1311.64, which is much lower than the AIC values of the other models. This indicates that the Auto ARIMA model is more adequate than the other models.

Conducting further analysis using the Ljung-Box test on the residuals of the models, we found that the the residuals of the ETS model showed a Q* statistic of 16.13 with 24 lags and a p-value of 0.8496, suggesting that the residuals are close to white noise. Similar results were found for the Auto ARIMA model, with a Q* statistic of 32.015 with 24 lags and a p-value of 0.1266. Alltough higher, this result is still acceptable.

6 Conclusion

This comprehensive report has thoroughly analyzed the time series data of "Commercial paper outstanding, financial companies" using various statistical models and methods. Our exploratory analysis revealed a distinct upward trend in the dataset, starting from 1982 to 1991, and a decrease in auto-correlation over time. These initial observations laid a solid foundation for further in-depth analysis and model selection.

The selection of Mean Absolute Error (MAE) and Root Mean Square Error (RMSE) as our primary performance indicators proved instrumental in assessing the effectiveness of our forecasting models. The Random Walk without drift model, selected due to its relevance to our near-linear time series data with a deviation, demonstrated reasonable accuracy, with MAE and RMSE values within our predetermined thresholds.

The exploration of Exponential Smoothing, specifically Holt's linear trend method, indicated a potential overfitting issue, as evidenced by higher error metrics on the test set compared to the training set. Despite this, our Ljung-Box test results supported the adequacy of the model, based on the ETS framework.

In the comparative analysis of ETS and Auto ARIMA models, we observed that while the ETS model was more adequate in terms of AIC values, its performance metrics were slightly surpassed by the simple model. Conversely, the Auto ARIMA model, despite having higher error metrics, showed a significantly lower AIC value, suggesting greater adequacy.

Our rigorous evaluation process, including residual analysis and the Ljung-Box test, affirmed that the residuals of both the ETS and Auto ARIMA models were close to white noise, reinforcing the reliability of these models.

In conclusion, this report has successfully navigated the complexities of forecasting financial time series data. The Random Walk without drift model emerged as the most suitable for its simplicity and accuracy. However, the lower AIC values of the ETS and Auto ARIMA models highlight the nuanced trade-off between model adequacy and performance metrics. Future research could focus on enhancing these models' predictive accuracy while maintaining their statistical adequacy, thus providing a more holistic approach to forecasting in financial time series data.