Dynamics of Vehicles Project
Team 2

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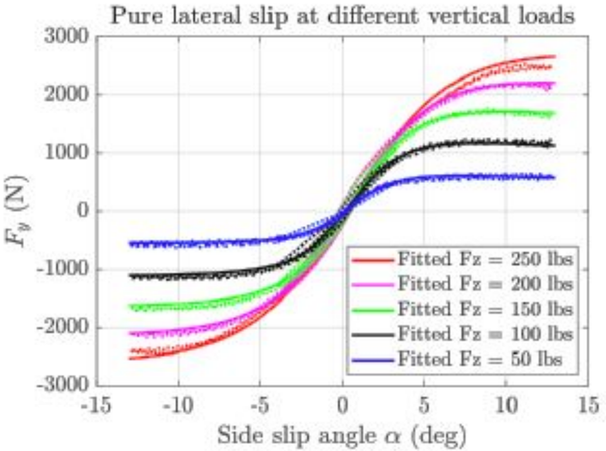
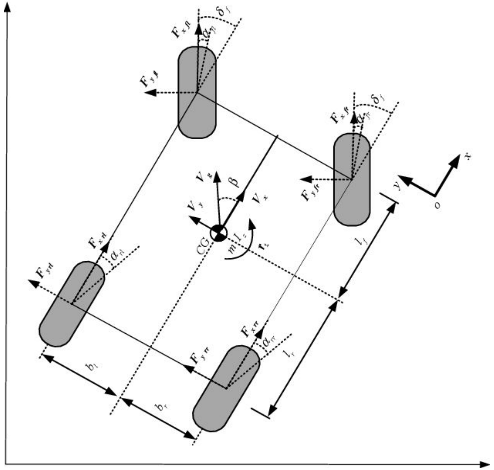
The project was composed by 2 sub-projects

- ➔ ***Tyre data fitting analysis***: fitted the MF96 tyre model coefficients on a dataset of measured tyre forces for a F-SAE tyre
- ➔ ***Steady State Handling analysis:*** analysis of the steady state behaviour of a F-SAE formula type car





Double-track model of the vehicle lateral dynamics

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Tyre data fitting analysis

To study the tyre data fitting of the data provided we used the so called **Magic Formula ‘96 Tyre Model** developed by **Hans B. Pacejka**. In particular this is a widely used semi-empirical tyre model able to calculate steady-state tyre forces and moments characteristics. Furthermore, it is an accurate and numerically efficient model able to fit well experimental data while considering the main features of a physical model like the Brush Model.

In particular this type of model is based on ***6 main factors/parameters*** which are:

- ➢ B (Stiffness Factor): determines the slope at the origin
- ➢ C (Shape Factor)
- ➢ D (Peak Value)
- ➢ E (Curvature Factor): controls the curvature at the peak and its horizontal position
- ➢ Sh (Horizontal Shift)
- ➢ Sv (Vertical Shift)

$$Y(X) = D\sin(C\arctan(Bx - E(Bx - \arctan(Bx)))) + S_V$$

$$x = X + S_H$$

with Y(X) the output variable Fx or Fy (pure lateral or longitudinal forces) and X the input variable α or κ (practical slip)

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Magic Formula Tyre Model ‘96

Magic Formula ‘96 is based on the ***Similarity method*** (Pacejka 1958) which builds upon the ***Reference Conditions*** defined as a tyre running with the following properties:

- ➔ rated nominal load ($Fz0$)
- ➔ null camber ($\gamma = 0$)
- ➔ free rolling ($\kappa = 0$) or null side slip ($\alpha = 0$)
- ➔ given road surface

As can be seen next, the provided experimental data were fitted with the formulas in the next slides with a least-square approach.

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MF96 full equations

Pure Longitudinal force coefficients

$$S_{Hx} = p_{Hx1} + p_{Hx2} \cdot df_z$$
$$\kappa_x = \kappa + S_{Hx}$$
$$C_x = p_{Cx1}$$
$$\mu_x = (p_{Dx1} + p_{Dx2} \cdot df_z)(1 - p_{Dx3}\gamma^2)$$
$$D_x = \mu_x F_z$$
$$K_{xk} = F_z(p_{Kx1} + p_{Kx2} \cdot df_z) \cdot e^{-(p_{Kx3} \cdot df_z)}$$
$$E_x = (p_{Ex1} + p_{Ex2} \cdot df_z + p_{Ex3} \cdot df_z^2)(1 - p_{Ex4}\operatorname{sign}(\kappa_x))$$
$$B_x = K_{xk}/(C_x D_x)$$
$$S_{Vx} = F_z(p_{Vx1} + p_{Vx2} \cdot df_z)$$

Pure Lateral force coefficients

$$S_{Hy} = p_{Hy1} + p_{Hy2} \cdot df_z + p_{Hy3}\gamma$$
$$\alpha_y = \alpha + S_{Hy}$$
$$C_y = p_{Cy1}$$
$$\mu_y = (p_{Dy1} + p_{Dy2} \cdot df_z)(1 - p_{Dy3}\gamma^2)$$
$$D_y = \mu_y F_z$$
$$K_{ya} = F_{z0} \cdot p_{Ky1} \sin\left(2\arctan\left(\frac{F_z}{F_{z0}}\frac{1}{p_{Ky2}}\right)\right)(1 - p_{Ky3}|\gamma|)$$
$$= F_{z0} \cdot p_{Ky1} \sin\left(2\arctan\left(\frac{1 + df_z}{p_{Ky2}}\right)\right)(1 - p_{Ky3}|\gamma|)$$
$$E_y = (p_{Ey1} + p_{Ey2} \cdot df_z)(1 - (p_{Ey3} - p_{Ey4}\gamma)\operatorname{sign}(\alpha_y))$$
$$B_y = K_{ya}/(C_y D_y)$$
$$S_{Vy} = F_z\left(p_{Vy1} + p_{Vy2} df_z + (p_{Vy3} + p_{Vy4} df_z)\gamma\right)$$

Longitudinal force

$$F_{x0} = D_x \sin\left(C_x \arctan\left(B_x\kappa_x - E_x(B_x\kappa_x - \arctan(B_x\kappa_x))\right)\right) + S_{Vx}$$

Lateral force

$$F_{y0} = D_y \sin\left(C_y \arctan\left(B_y\alpha_y - E_y(B_y\alpha_y - \arctan(B_y\alpha_y))\right)\right) + S_{Vy}$$

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MF96 full equations

Combined Longitudinal force coefficients

$$S_{Hxa} = r_{Hx1}$$
$$B_{xa} = r_{Bx1}\cos(\arctan(\kappa \cdot r_{Bx2}))$$
$$C_{xa} = r_{Cx1}$$
$$D_{xa} = 1/\cos(C_{xa}\arctan(B_{xa}S_{Hxa}))$$
$$G_{xa} = D_{xa}\cos(C_{xa}\arctan(B_{xa}(\alpha + S_{Hxa})))$$

Combined Lateral force coefficients

$$D_{Vyk} = \mu_y F_z(r_{Vy1} + r_{Vy2}df_z + r_{Vy3}\gamma)\cos(\arctan(r_{Vy4}\alpha))$$
$$S_{Vyk} = D_{Vyk}\sin(r_{Vy5}\arctan(r_{Vy6}\kappa))$$
$$S_{Hyk} = r_{Hy1}$$
$$B_{yk} = r_{By1}\cos\left(\arctan(r_{By2}(\alpha - r_{By3}))\right)$$
$$C_{yk} = r_{Cy1}$$
$$D_{yk} = 1/\cos(C_{yk}\arctan(B_{yk}S_{Hyk}))$$
$$G_{yk} = D_{yk}\cos(C_{yk}\arctan(B_{yk}(\kappa + S_{Hyk})))$$

Longitudinal force

$$F_x = G_{xa}F_{x0}$$

Lateral force

$$F_y = G_{yk}F_{y0} + S_{Vyk}$$

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Aligning moment

$$S_{Ht} = q_{Hz1} + q_{Hz2} df_z + (q_{Hz3} + q_{Hz4} \cdot df_z)\gamma$$

$$\alpha_t = \alpha + S_{Ht}$$

$$B_t = (q_{Bz1} + q_{Bz2} df_z + q_{Bz3}(df_z)^2)(1 + q_{Bz4}\gamma + q_{Bz5}|\gamma|)$$

$$C_t = q_{Cz1}$$

$$D_t = F_z(q_{Dz1} + q_{Dz2} df_z)(1 + q_{Dz3}\gamma + q_{Dz4}\gamma^2)(R_o/F_{z0})$$

$$E_t = (q_{Ez1} + q_{Ez2} df_z + q_{Ez3} \cdot (df_z)^2)\left(1 + (q_{Ez4} + q_{Ez5}\gamma)\arctan(B_t C_t \alpha_t)\right)$$

$$S_{Hf} = S_{Hy} + S_{Vy}/K_{ya}$$

$$\alpha_r = \alpha + S_{Hf}$$

$$B_r = q_{Bz9} + q_{Bz10} B_y C_y$$

$$D_r = F_z\left(q_{Dz6} + q_{Dz7} df_z + (q_{Dz8} + q_{Dz9} df_z)\gamma\right) R_o$$

$$\boxed{\begin{aligned} M_{zr} &= D_r \cos(\arctan(B_r \alpha_r))\cos(\alpha) \\ t &= D_t \cos\Big(C_t \arctan\big(B_t\alpha_t - E_t(B_t\alpha_t - \arctan(B_t\alpha_t))\big)\Big)\cos(\alpha) \\ M_{z0} &= -tF_y + M_{zr} \end{aligned}}$$

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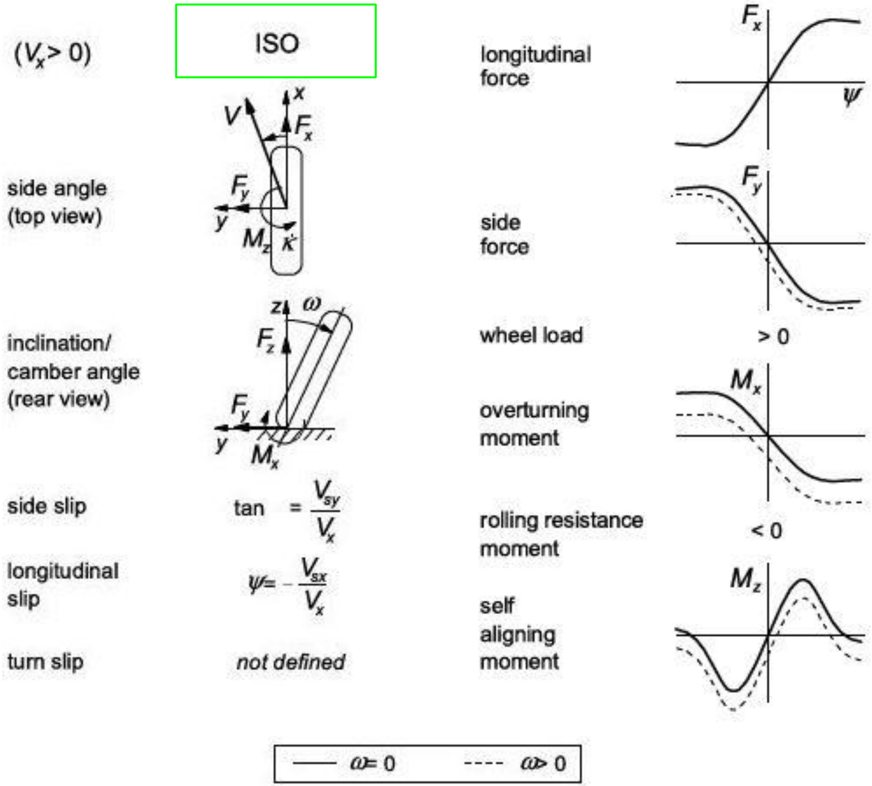
Fitting procedure description



A least-squares approach was used as parameter identification problem to find the optimal parameters that minimize the squares of the residuals between the fitted-predicted function Xi and the real experimental data Xi*

$$\min_{p} \frac{\sum_{i=1}^{N} (X_i - X_i^*)^2}{\sum_{i=1}^{N} (X_i^*)^2}$$

with N being the total number of samples and Xi the longitudinal, lateral force or the aligning moment.

Hoosier 18.0 x 6.0 - 10 R25B

⇩

Selected tyre type

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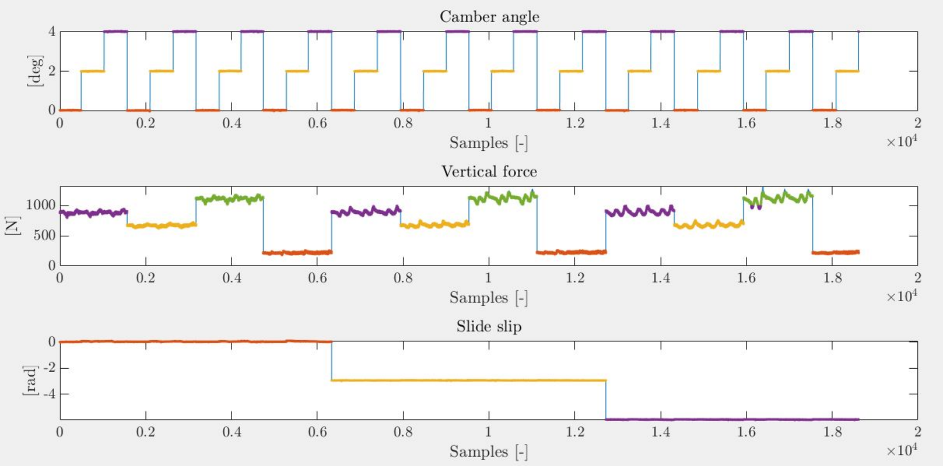
Fitting procedure description

- **Data cropping:** selected data inside `Hoosier_B1464run23.mat` (related to the identification of lateral force and self-aligning moment) and `Hoosier_B1464run30.mat` (related to the identification of combined behaviour and pure longitudinal force)
- **Set of conditions for each analysis:** *pure longitudinal force, pure lateral force, combined pure lateral force, combined pure longitudinal force, self-aligning moment*
- **Optimization of the residuals** using **fmincon** as non-linear Optimization Solver
- **Initial guesses and boundaries vectors** for the parameters to be optimized
- Save tyre data struct with all the ***fitted tyre coefficients for MF96***



⇨ **Data cropping**

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Fitted tyre coefficients for MF96

All the coefficients for each case will be shown in the next slides together with some statistical factor such as *R^2* and *Root Mean Square Error* and the relative plots.

```
%% Save tyre data structure to mat file
%
save(['tyre_' data_set,'.mat'],'tyre_coeffs');
```

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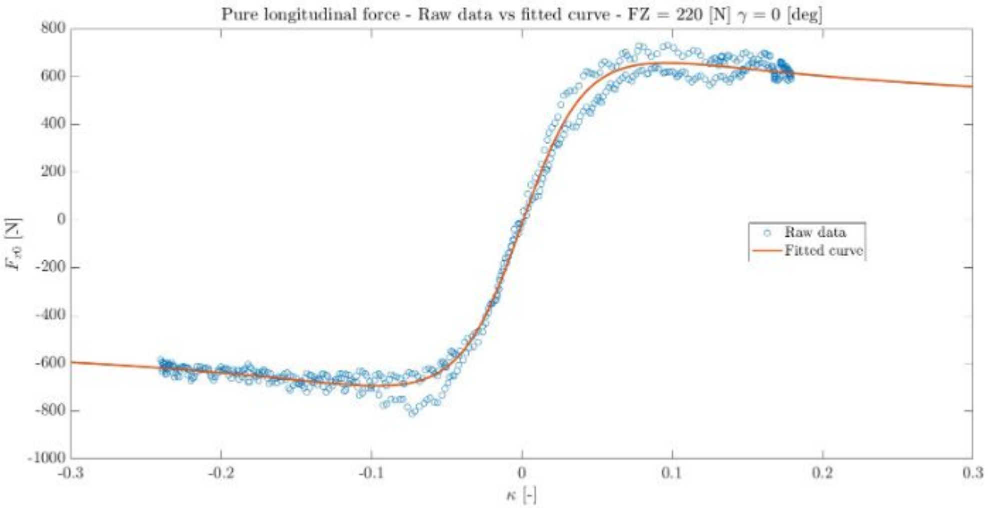
Pure Longitudinal Force

Coefficients	pCx_1	pD_{x_1}	pE_{x_1}	pE_{x_4}	pK_{x_1}	pH_{x_1}	pV_{x_1}
P_0 - initial guess	1	2	1	0	0	1	0
lb - lower bound	1	0.1	0	0	-10	0	-10
ub - upper bound	2	4	1	1	10	100	10
Fitted value	1.5387	3.1469	0.0113	0.0806	82.4171	-2.0920e-05	-0.0858



Fz0 = 220 [N] (nominal vertical load)

Obtained performance indexes:
$R^2 = 99.55\,\%$ and $RMSE = 40.47$ [N]

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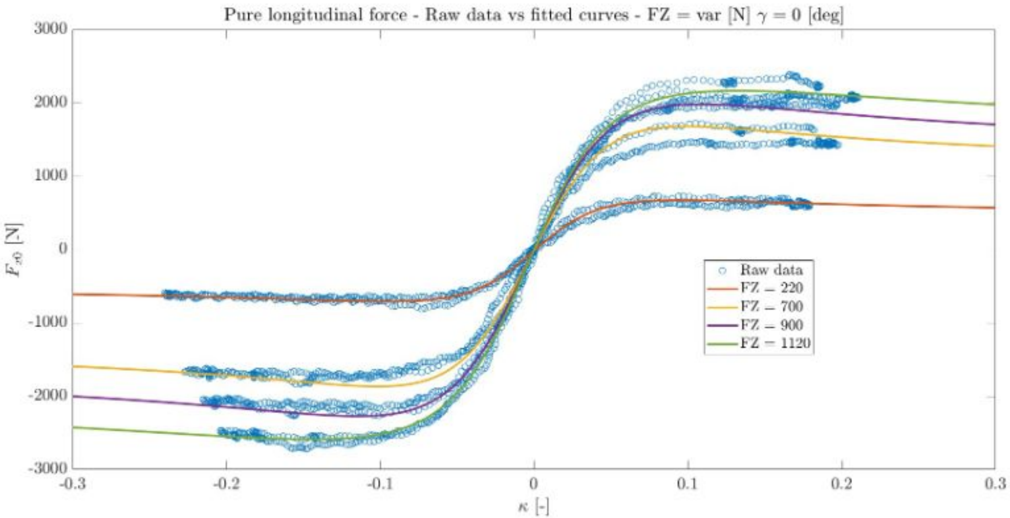
Pure Longitudinal Force (vertical load change)

Coefficients	pD_{x2}	pE_{x2}	pE_{x3}	pH_{x2}	pK_{x2}	pK_{x3}	pV_{x2}
P_0 - initial guess	0	0	0	0	0	0	0
lb - lower bound	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	-
Fitted value	-0.2496	-0.3619	0.1059	0.0011	-0.0019	0.1536	-0.0256



Obtained performance indexes:
$R^2 = 99.71\,\%$ and $RMSE = 85.91$ [N]

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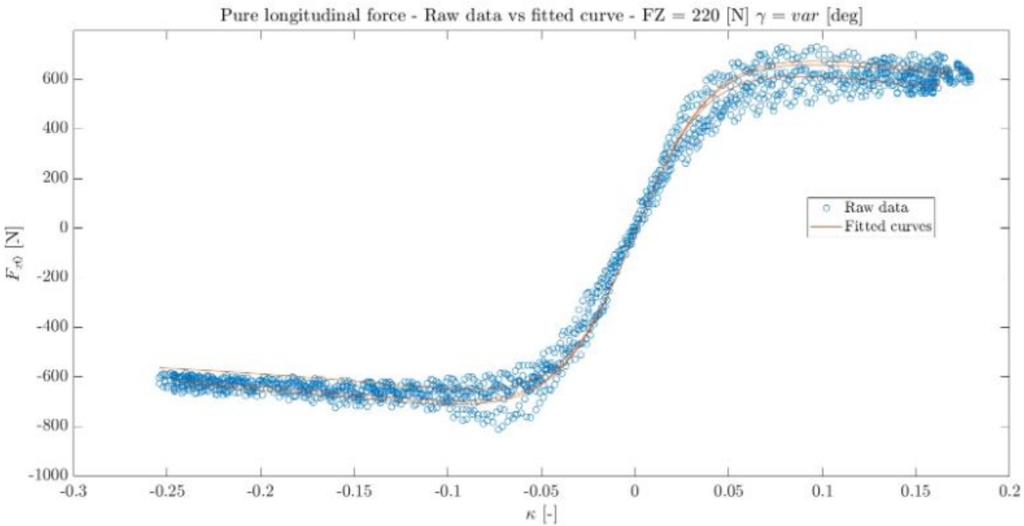
Pure Longitudinal Force (camber angle change)

Coefficients	pD_{x3}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	18.4364



Obtained performance indexes:
$R^2 = 99.03\,\%$ and $RMSE = 57.45$ [N]

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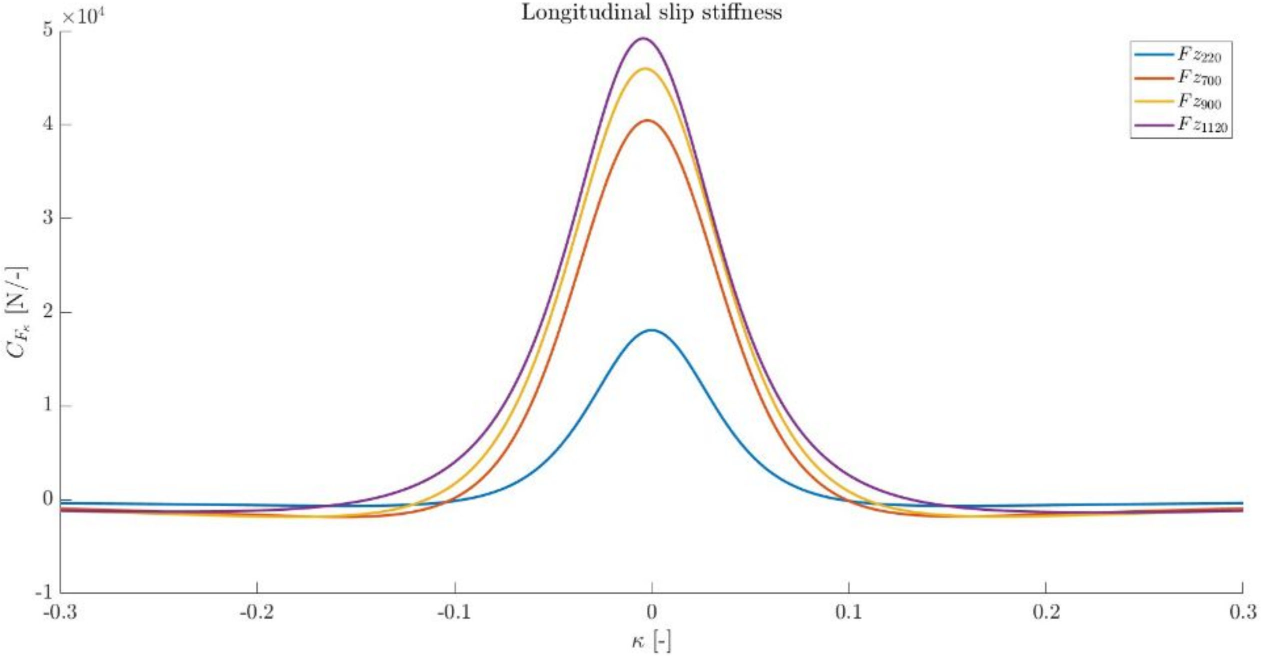
Longitudinal cornering stiffness



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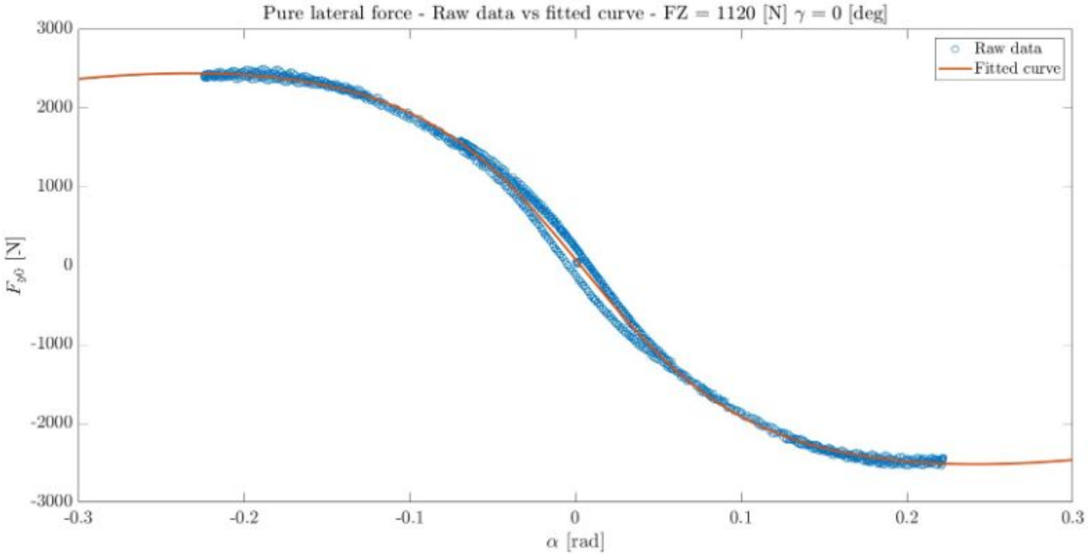
Pure Lateral Force

Coefficients	pC_{y1}	pD_{y1}	pE_{y1}	pH_{y1}	pK_{y1}	pK_{y2}	pV_{y1}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	0.1
lb - lower bound	1	-1000	-1000	-1000	-1000	-1000	-1000
ub - upper bound	2	1000	1000	1000	1000	1000	1000
Fitted value	1.0177	3.7104	1.4733	-0.0046	-23.1279	1.1343	-0.0357



Fz0 = 1120 [N]

Obtained performance indexes:
$R^2 = 99.82\,\%$ and $RMSE = 68.62\,[\mathrm{N}]$

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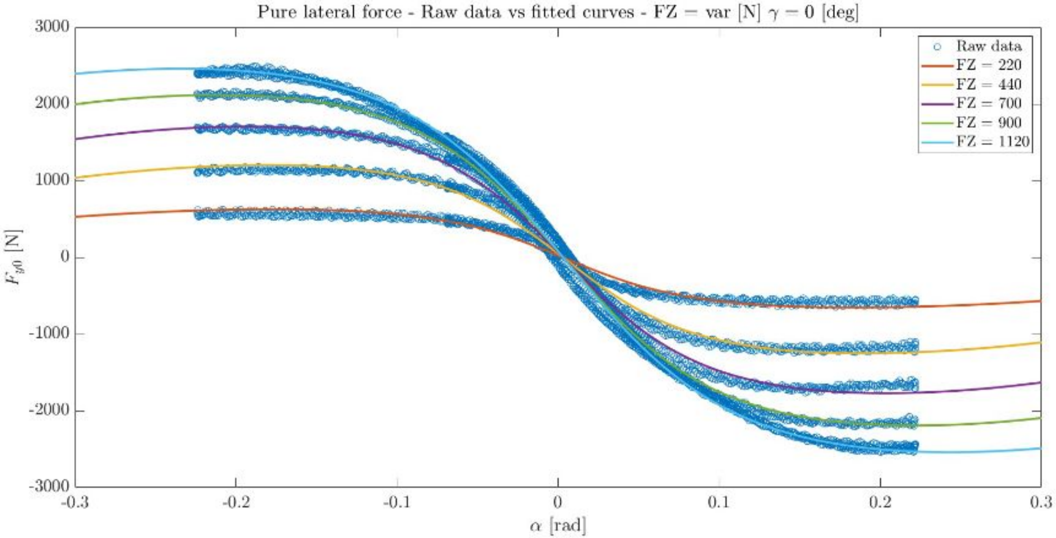
Pure Lateral Force (vertical load change)

Coefficients	pD_{y2}	pE_{y2}	pH_{y2}	pV_{y2}
P_0 - initial guess	0.1	0.1	0.1	0.1
lb - lower bound	-1000	0	-1000	-1000
ub - upper bound	1000	1	1000	1000
Fitted value	-1.3805	8.4470e-05	-2.8188e-04	0.0310



Obtained performance indexes:
$R^2 = 99.68\,\%$ and $RMSE = 75.19$ [N]

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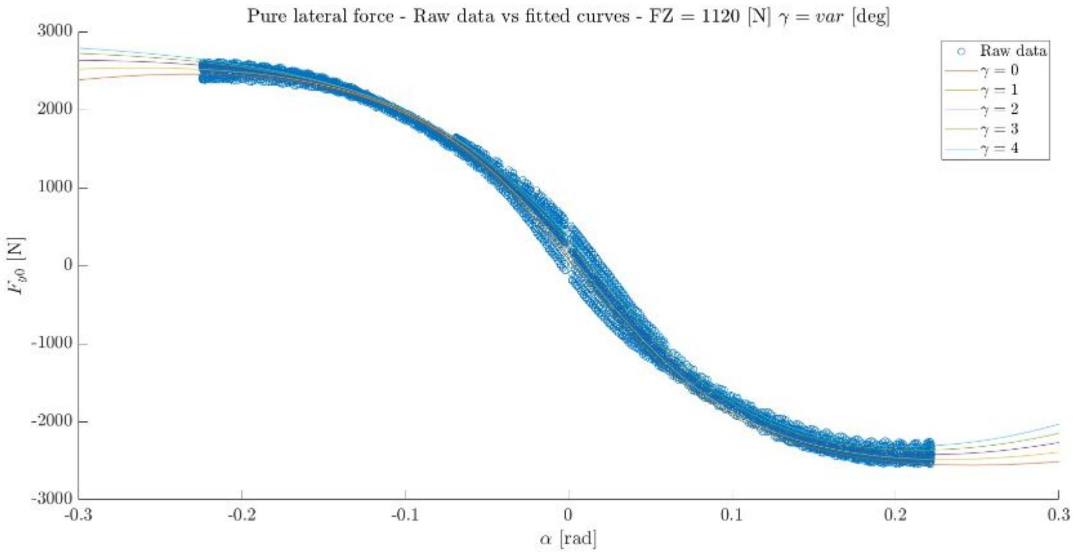
Pure Lateral Force (camber angle change)

Coefficients	pD_{y3}	pE_{y3}	pE_{y4}	pH_{y3}	pK_{y3}	pV_{y3}
P_0 - initial guess	0.1	0.5	0.1	0.1	0.1	5
lb - lower bound	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-
Fitted value	3.0508	0.0077	-4.0472	-0.2662	0.9497	-1.7757



Obtained performance indexes:
$R^2 = 99.84\,\%$ and $RMSE = 73.49\,$[N]

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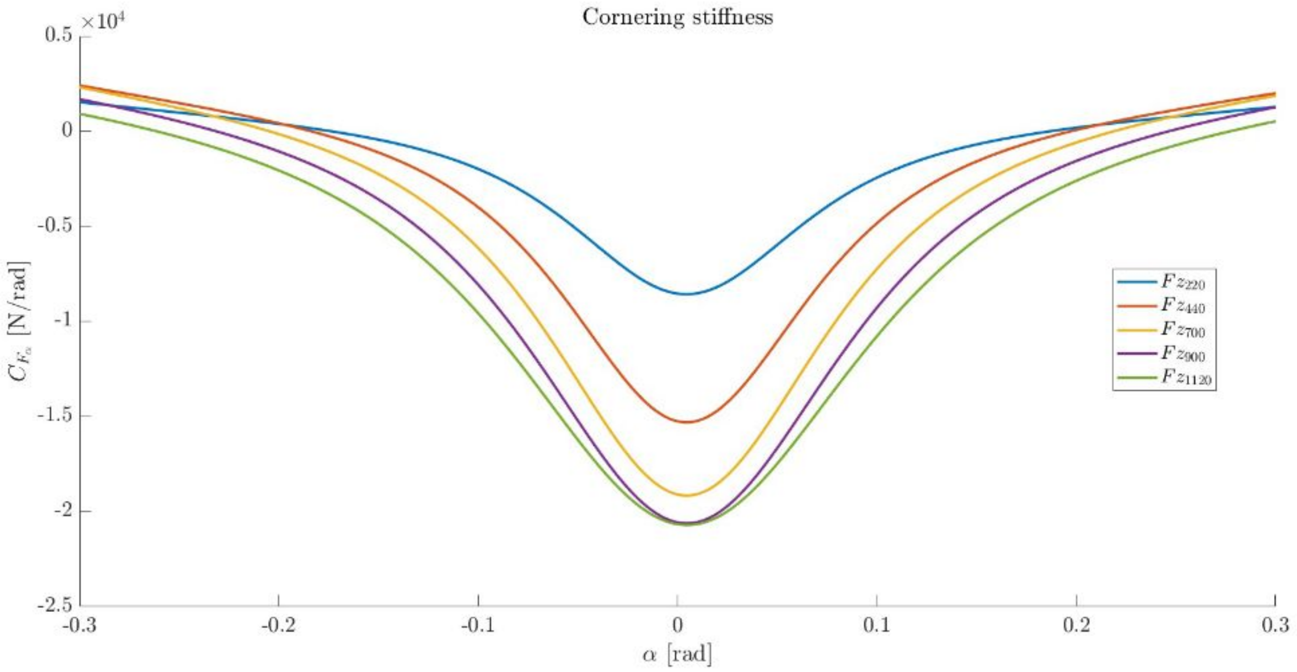
Lateral cornering stiffness



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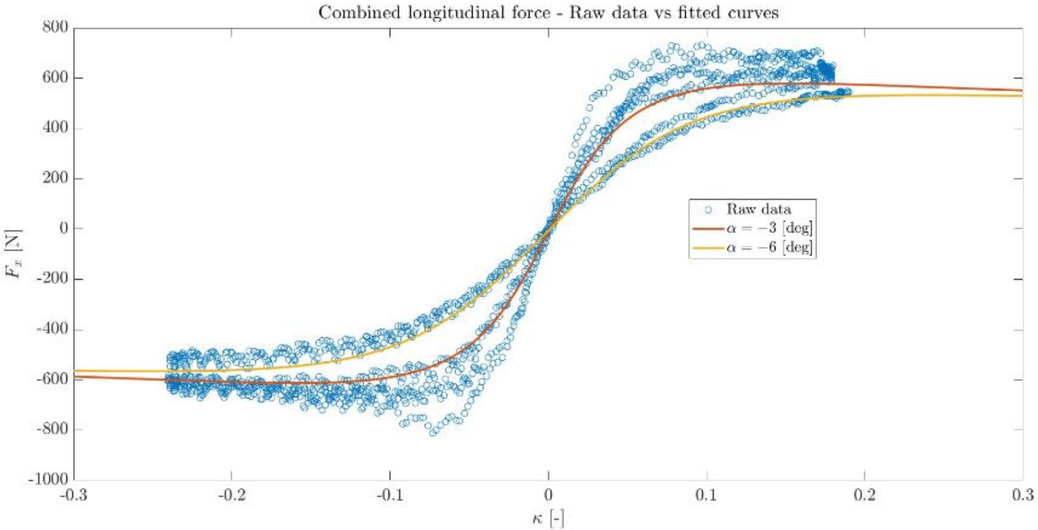
Combined Longitudinal Force

Coefficients	rB_{x_1}	rB_{x_2}	rC_{x_1}	rH_{x_1}
P_0 - initial guess	8.3	5	0.9	0
lb - lower bound	7	0	0.5	-100
ub - upper bound	20	20	3	1
Fitted value	14.6714	1.2440	1.0022	-19.3506



Fz0 = 220 [N]

Obtained performance indexes:
$R^2 = 99.32\,\%$ and $RMSE = 44.21$ [N]

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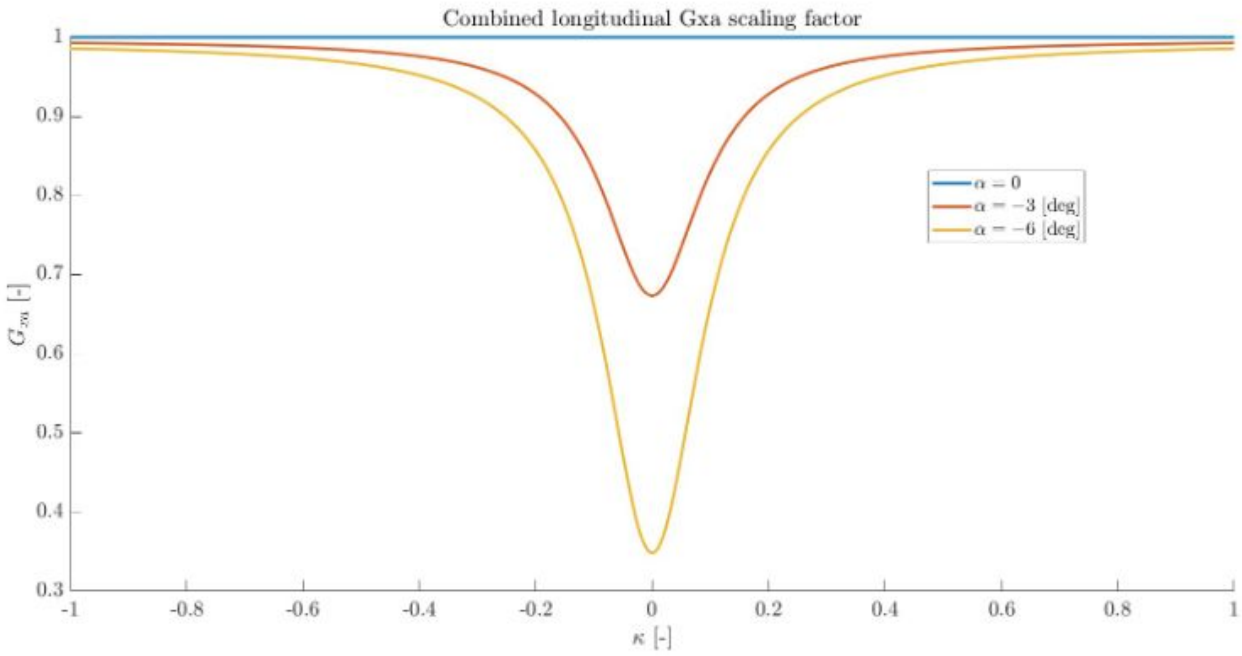
Gxa weighting function of the longitudinal force



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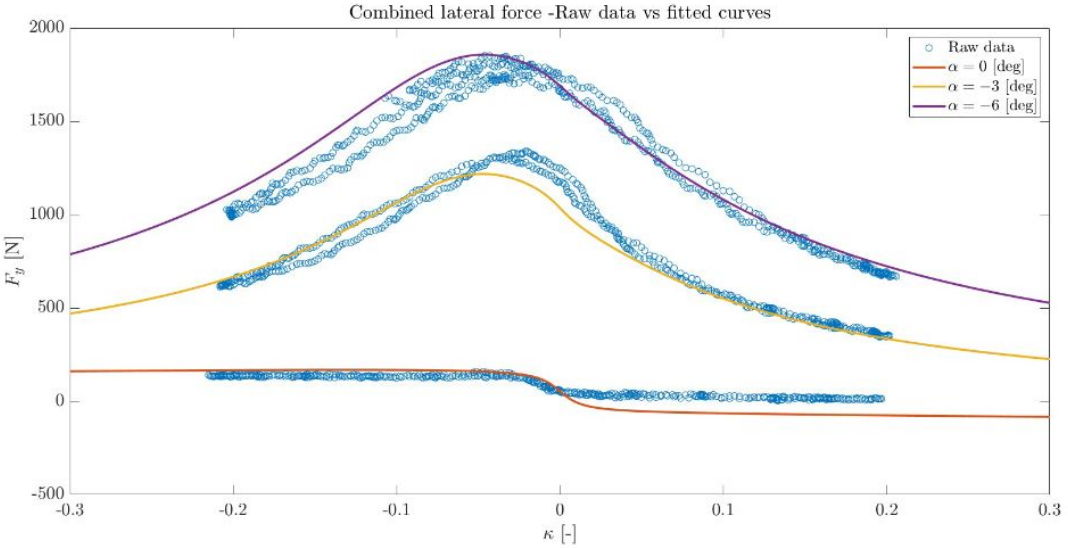
Combined Lateral Force

Coefficients	rB_{y1}	rB_{y2}	rB_{y3}	rC_{y1}	rH_{y1}	rV_{y1}	rV_{y4}	rV_{y5}	rV_{y6}
P_0 - initial guess	4.9	2.2	0	1	0.1	0.1	30	0.5	10
lb - lower bound	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000
ub - upper bound	1000	1000	1000	1000	1000	1000	1000	1000	1000
Fitted value	246.5833	973.0261	-0.0750	0.9995	0.0453	-4.0021	24.4909	0.0032	94.9144



Fz0 = 900 [N]

Obtained performance indexes:
$R^2 = 99.57\,\%$ and $RMSE = 63.59 \quad$ [N]

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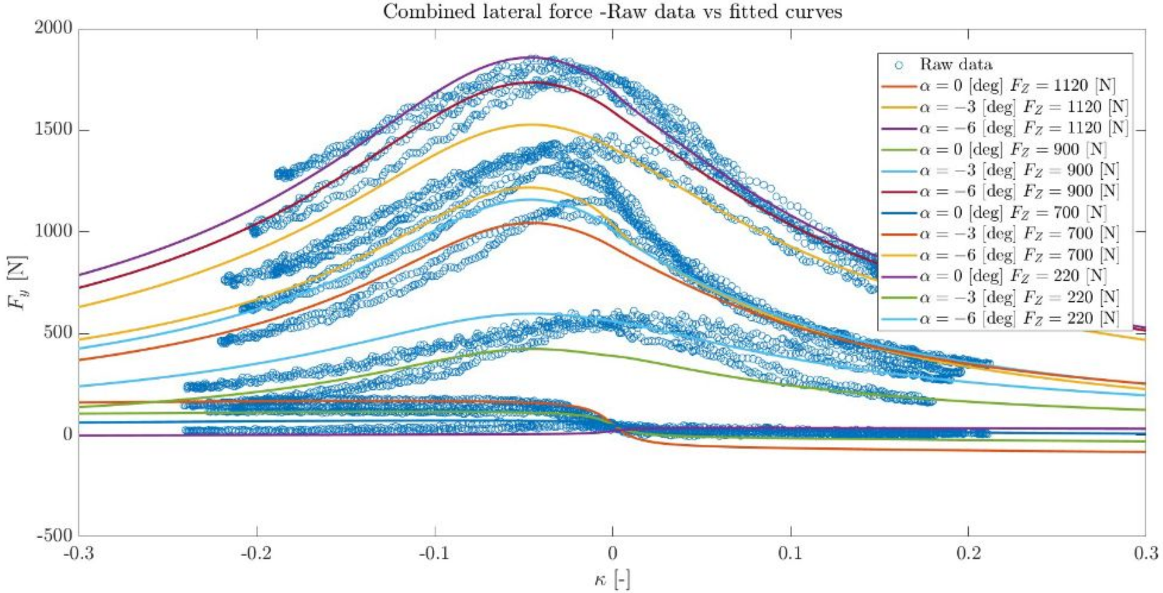
Combined Lateral Force (vertical load change)

Coefficients	rV_{y2}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	-9.8945



Obtained performance indexes:

$R^2 = 99.19\,\%$ and $RMSE = 67.91$ [N]

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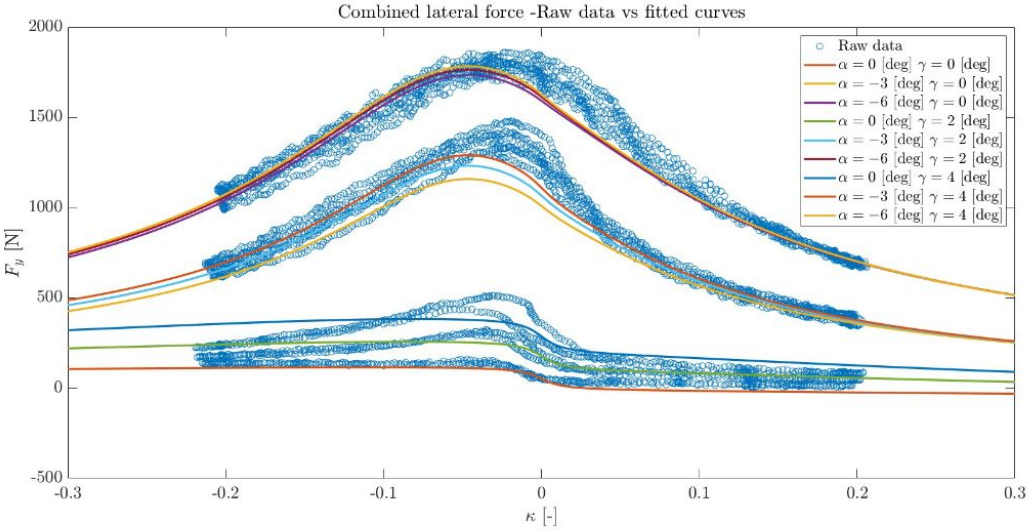
Combined Lateral Force (camber angle change)

Coefficients	rV_{y3}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	-31.6715



Obtained performance indexes:
$R^2 = 97.50\,\%$ and $RMSE = 149.69$ [N]

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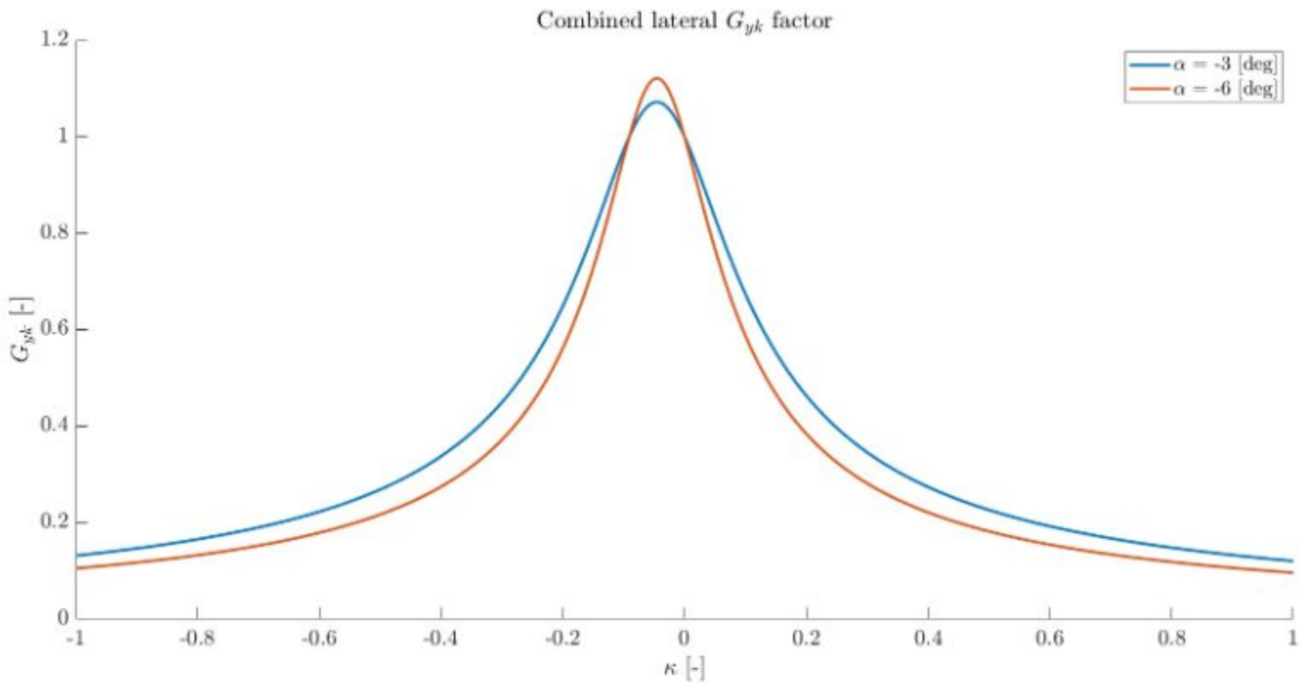
Gyk weighting function of the lateral force



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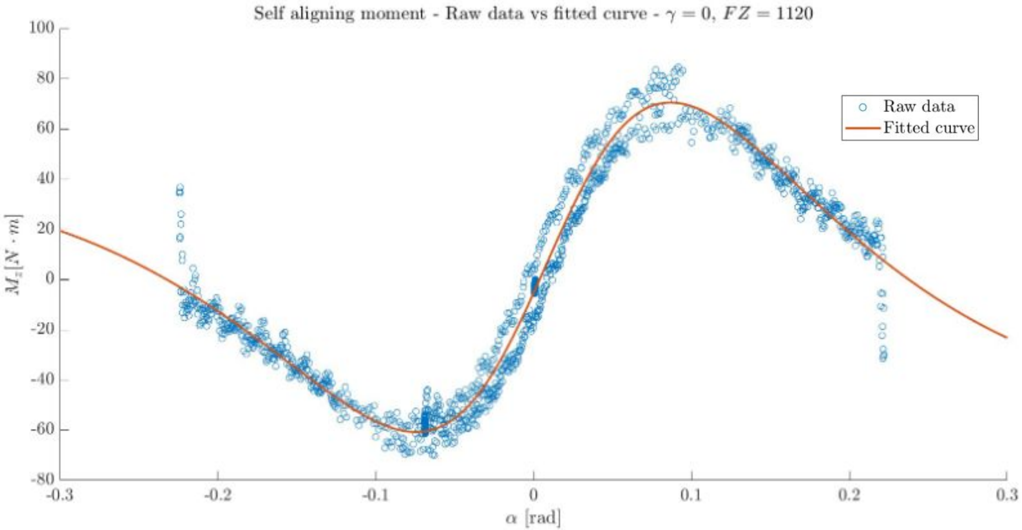
Self-Aligning Moment

Coefficients	qH_{z1}	qB_{z1}	qC_{z1}	qD_{z1}	qE_{z1}	qE_{z4}	qB_{z9}	qB_{z10}	qD_{z6}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
lb - lower bound	-	-	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	-	-	-
Fitted value	-0.0132	6.2738	1.7473	0.2400	0.4249	-0.1461	0.2299	-0.5407	-0.0045



Fz0 = 1120 [N]

Obtained performance indexes:
$R^2 = 97.26\,\%$ and $RMSE = 6.38$ [N·m]

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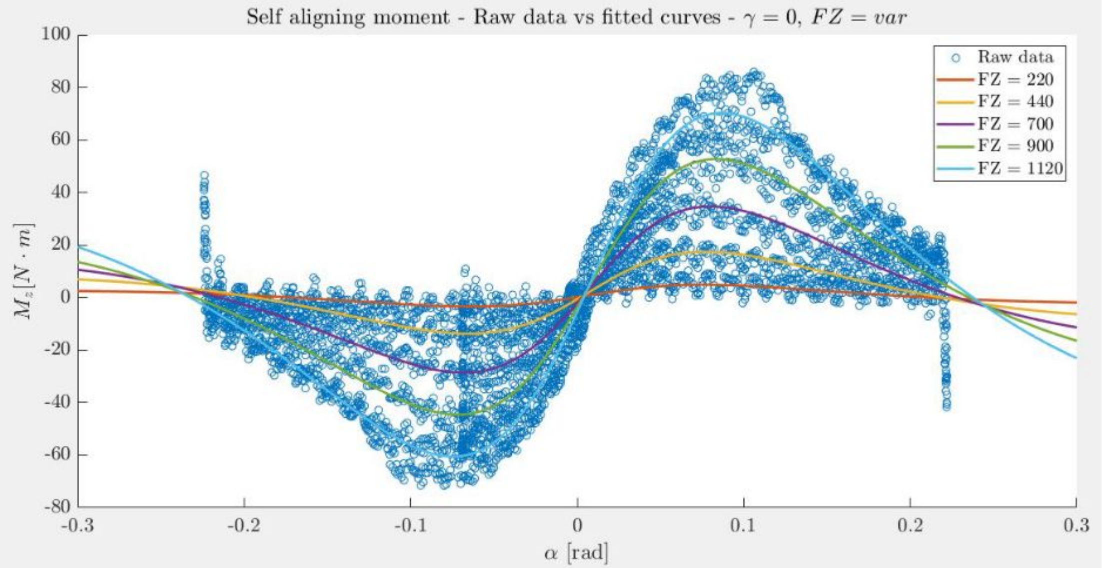
Self-Aligning Moment (vertical load change)

Coefficients	qH_{z_2}	qB_{z_2}	qB_{z_3}	qD_{z_2}	qE_{z_2}	qE_{z_3}	qD_{z_7}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	
lb - lower bound	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	
Fitted value	-0.0024	-0.8874	-0.9187	-0.0254	-0.3624	-1.2557	-0.0156



Obtained performance indexes:

$R^2 = 96.82\,\%$ and $RMSE = 5.3459$ [N·m]

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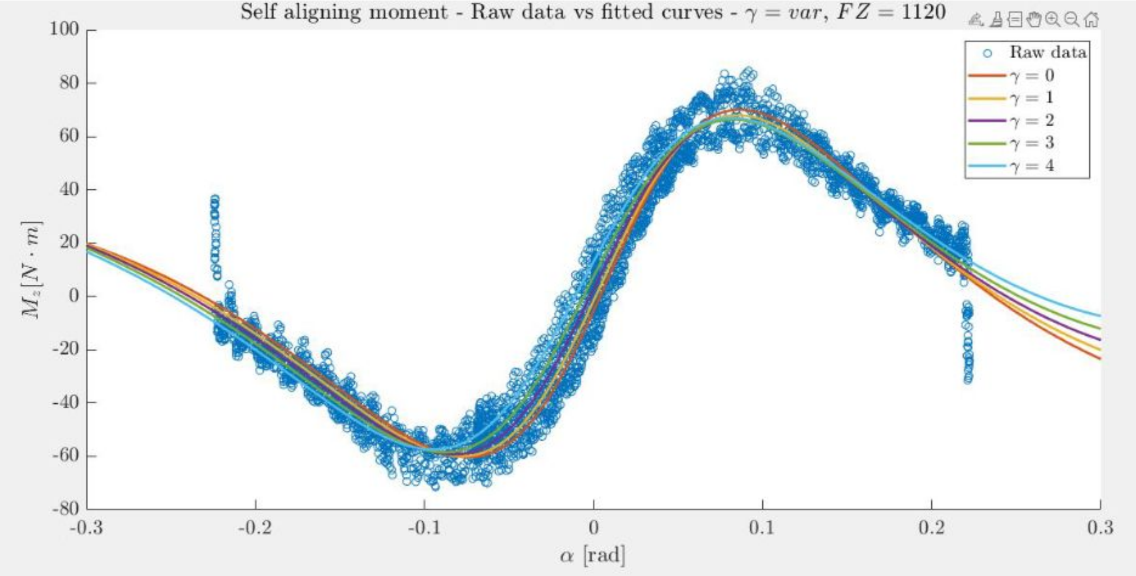
Self-Aligning Moment (camber angle change)

Coefficients	qH_{z3}	qB_{z4}	qB_{z5}	qD_{z3}	qD_{z4}	qE_{z5}	qD_{z8}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	0.1
lb - lower bound	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	-
Fitted value	0.7460	-7.3893	6.6463	-1.0072	4.7283	-2.5673	1.7837



Obtained performance indexes:
$R^2 = 97.40\,\%$ and $RMSE = 6.5335$ [N·m]

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Steady-State Handling Analysis

For the study of the Steady-state behaviour of the F-SAE car provided to us we used the so called ***Double Track Model or 8 DoFs model***, a vehicle model for performance analysis.

This model allows to study the lateral and longitudinal vehicle response together with the combined behaviour of the vehicle. In particular, it is very suited to analyze how the changes of the main system components or parameters affect the vehicle steering and traction/braking characteristics

3 dofs for vehicle chassis: $[x, y, \psi]^T$ or $[u, v, \Omega]^T$

1 roll motion: ϕ

4 wheel spin: $[\omega_{rr}, \omega_{rl}, \omega_{fr}, \omega_{fl}]$

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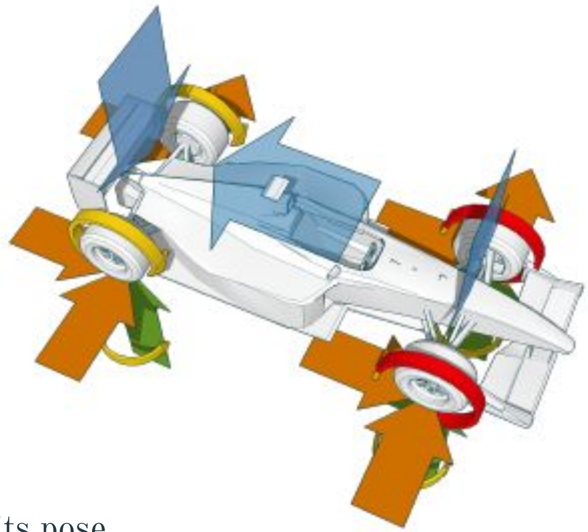
Double Track Model

The model is sufficiently accurate including the major effects of all sub-systems of the vehicle.

In particular the **model requirements** are as follows:

- ➢ Tyre characteristics
- ➢ Suspension characteristics
- ➢ Inertia properties
- ➢ Driveline and braking system
- ➢ Steering system



Most importantly the **model assumptions** are:

- ➔ Flat road, 4 wheels, 2 axles
- ➔ Negligible effects of the suspensions on the contact points positions
- ➔ Roll motion (no heave and pitch motion) with roll axis not changing its pose
- ➔ Front Steering system only and vertical steering axis
- ➔ Small steering angle (easier equations)
- ➔ Open differential (torque split equally on right and left wheel)

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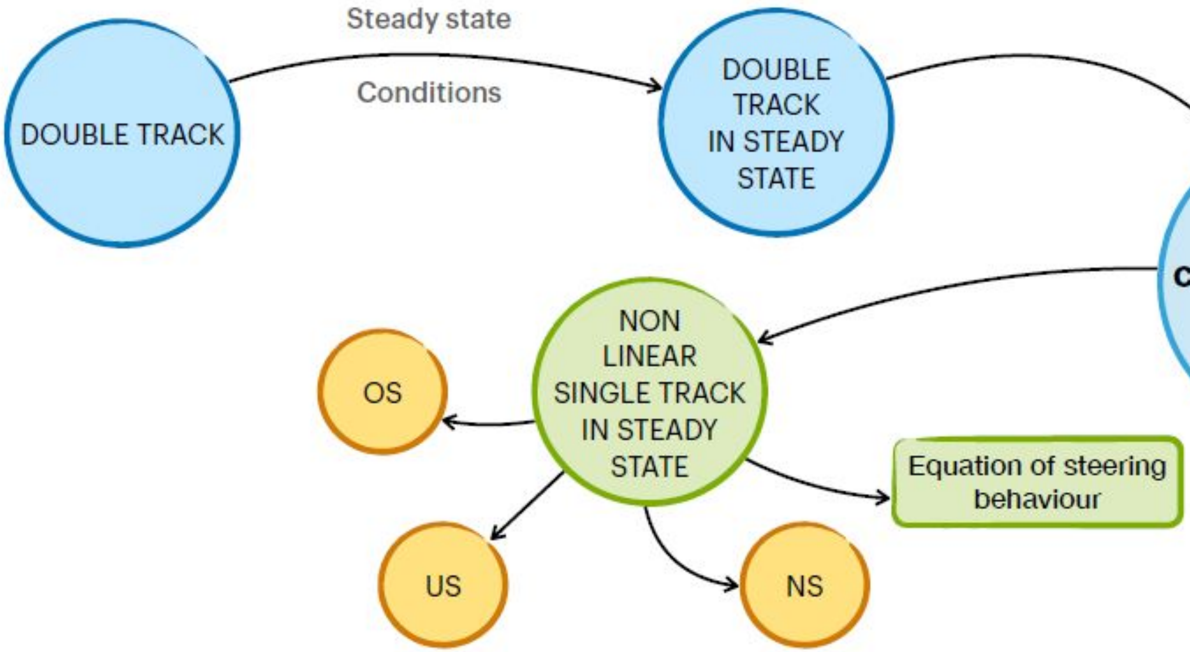
Process to study handling

Analytical approach to study handling steady state



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Goals of Steady State Handling Analysis

- Modify data according the provided vehicle_data.m file and use the tyre data fitted in the *first assignment*

- Use the vehicle model (in Simulink) to simulate the requested steady state tests such as ***Steer ramp test*** at constant speed and ***Speed ramp test*** at constant steering angle

- Collect the corresponding telemetry data from the aforementioned tests

- Compute main parameters and indices to evaluate the model such as:
  Lateral load transfer
  Normalised axle characteristics
  Handling diagram
  Understeering gradient (theoretical and fitted)
  Yaw rate gain, Beta gain

- Evaluate the effect of suspension roll stiffness, static toe angle and camber angle on steering characteristic/handling (optionally investigate the chassis roll stiffness)

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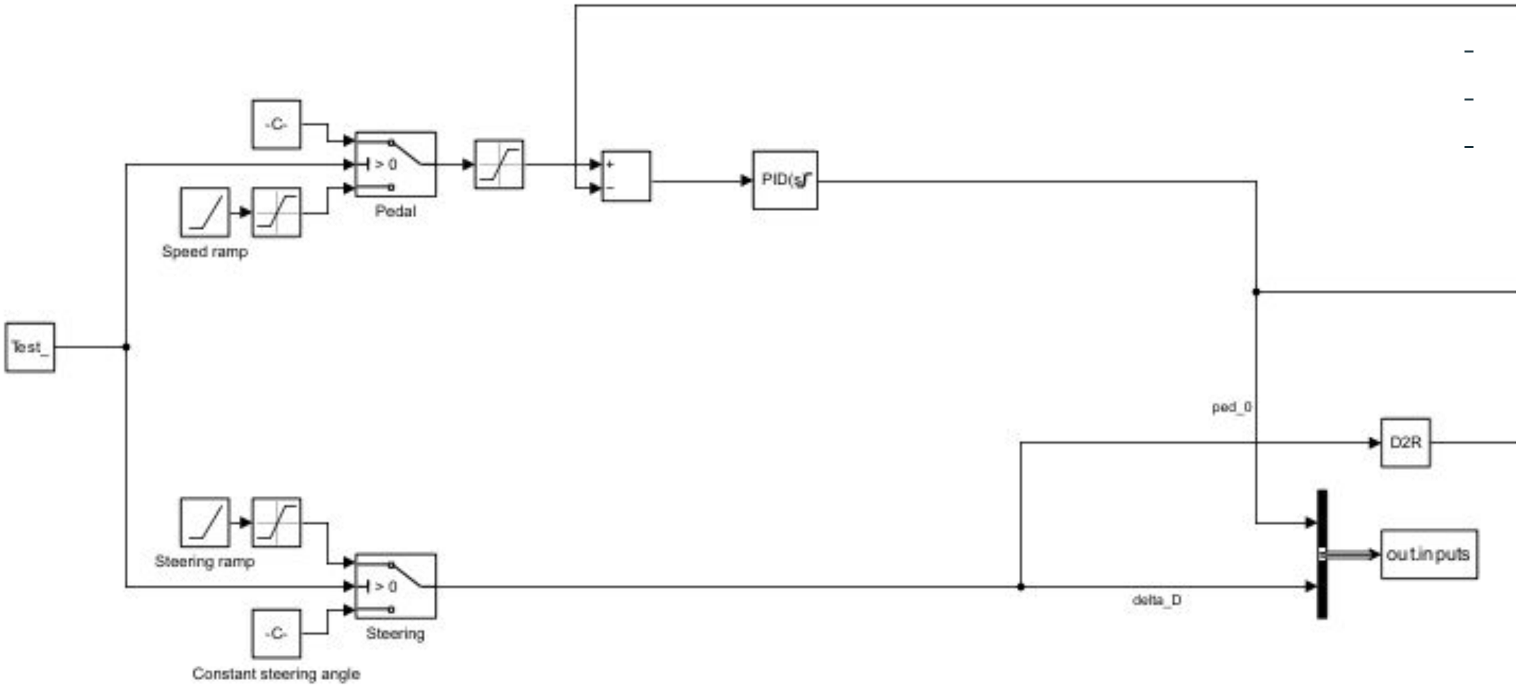
Simulink Model implementation



- Ramp blocks
- Saturation blocks
- Switches

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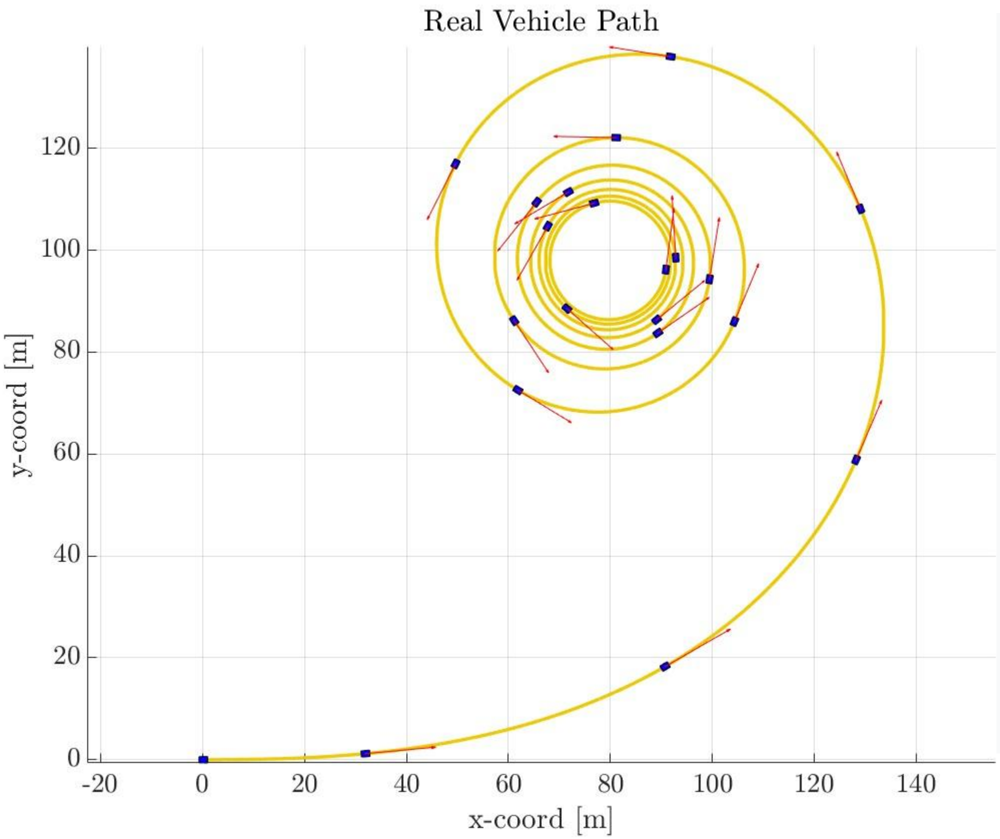
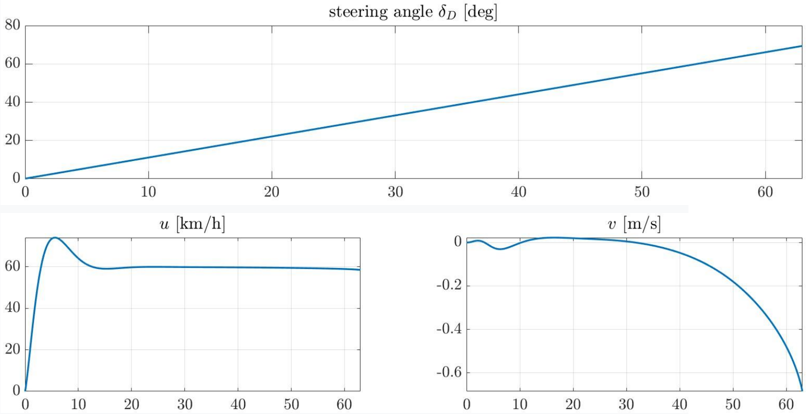
Test #1 (Steer Ramp Test)



- Constant speed: 60 Km/h
- Linear steer angle ramp
- Simulation ends when car starts losing control (~ 63 [s])



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Lateral Load Transfer

We consider the case of **Suspension deformation** only (tyres assumed infinitely rigid) and the fact that we are in steady-state so we have neither yaw moment effect nor an angular acceleration.

$$\Delta F_{z_r} = m\, a_y \left(\boxed{\frac{L_f}{L} \frac{h_{r_r}}{W_r}} + \boxed{\frac{h_s}{W_r} \frac{K^s_{\phi_r}}{(K^s_{\phi_r} + K^s_{\phi_f})}} \right)$$

$$\Delta F_{z_f} = m\, a_y \left(\boxed{\frac{L_r}{L} \frac{h_{r_f}}{W_f}} + \boxed{\frac{h_s}{W_f} \frac{K^s_{\phi_f}}{(K^s_{\phi_r} + K^s_{\phi_f})}} \right)$$

Instantaneous load transfer

Elastic or transient load transfer

with

$$\epsilon_s = \frac{K^s_{\phi_f}}{(K^s_{\phi_r} + K^s_{\phi_f})} = \frac{K^s_{\phi_f}}{K^s_{\phi_{tot}}} \in [0,\, 1]$$

and the assumption that $m\, g\, h_s \ll K^s_{\phi_r} + K^s_{\phi_f} = K^s_{\phi_{tot}}$

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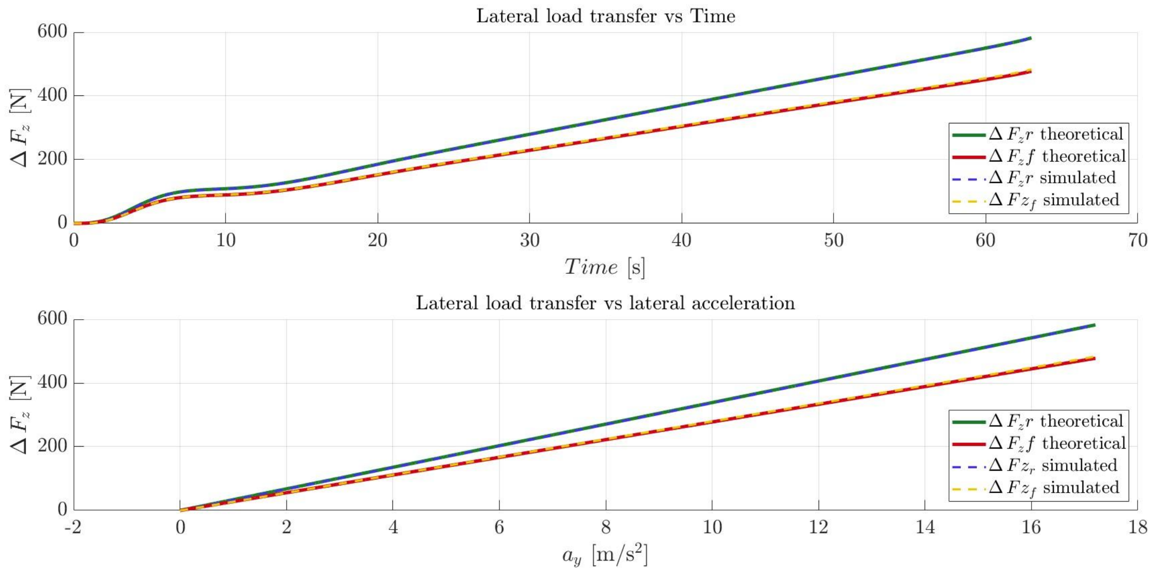
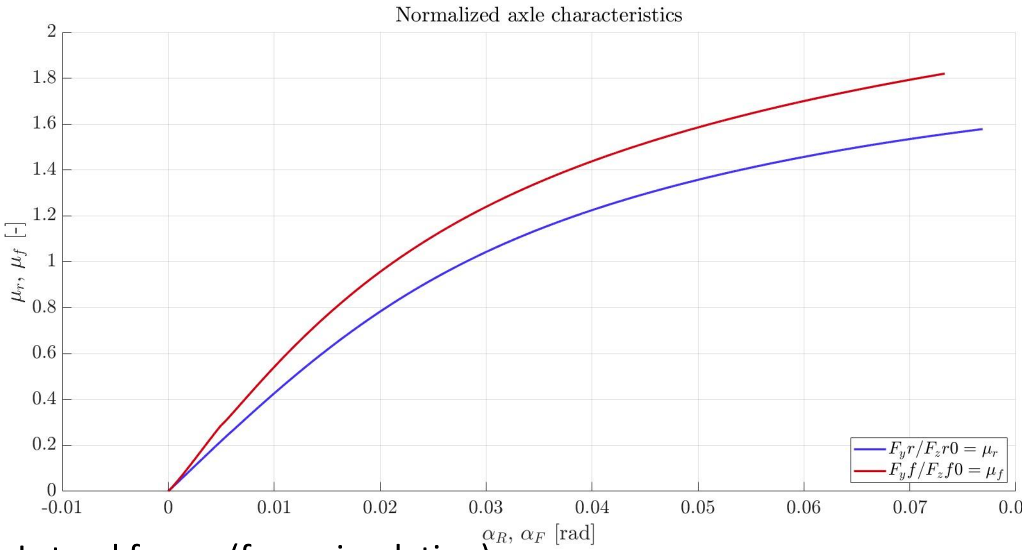
Lateral load transfer



$$\Delta Fz_f = \frac{ma_y}{W_f}\left(\frac{L_r}{L}h_{rf} + \epsilon_\phi h_s\right) \qquad \Delta Fz_r = \frac{ma_y}{W_r}\left(\frac{L_f}{L}h_{rr} + (1-\epsilon_\phi)h_s\right)$$

Normalized axle characteristics



- Front axle is able to develop higher lateral forces than the rear axle
- This means that the vehicle will oversteer

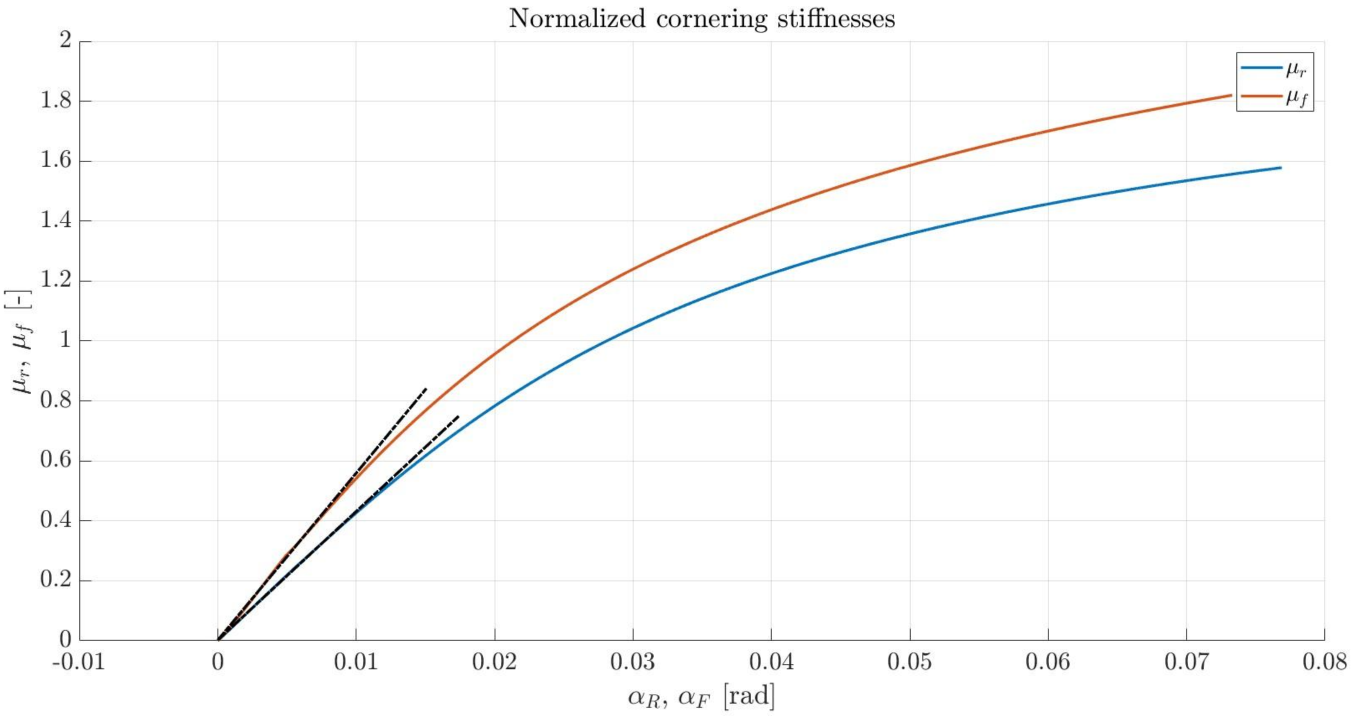
Lateral forces (from simulation):

$$Fyf = Fy_{fl} + Fy_{fr} \qquad Fyr = Fy_{rl} + Fy_{rr}$$

Vertical forces distribution:

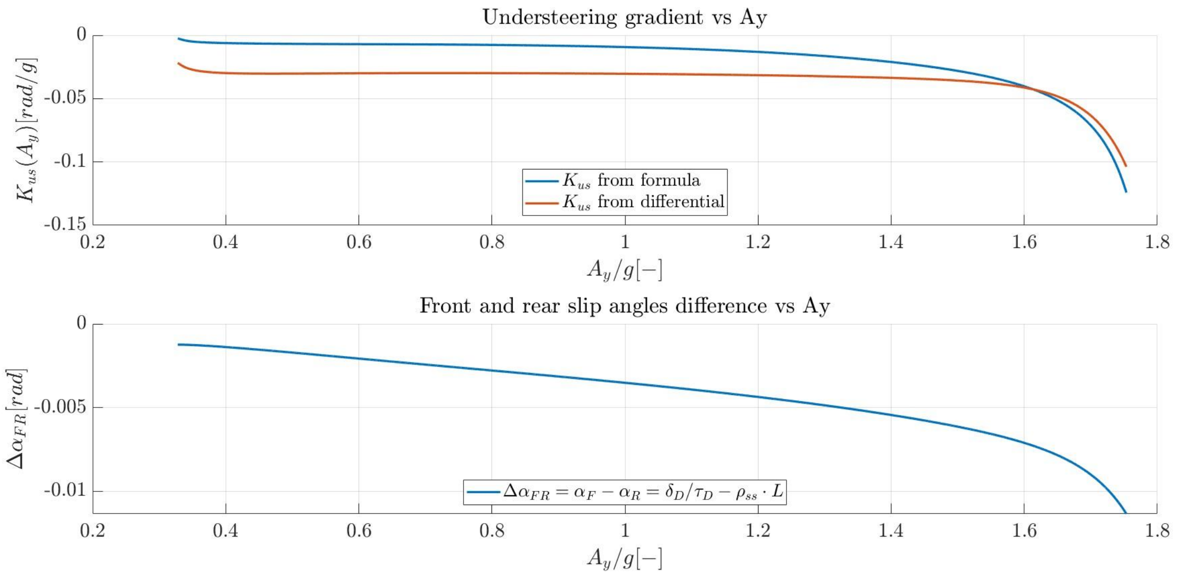
$$Fzf_0 = m \cdot g \cdot \frac{Lr}{L} \qquad Fzr_0 = m \cdot g \cdot \frac{Lf}{L}$$

Cornering stiffnesses from axle characteristics



$$C_{\alpha F} = \delta\mu_f/\delta\alpha_f = 55.72 \qquad C_{\alpha R} = \delta\mu_r/\delta\alpha_r = 42.92$$

Understeering gradient comparison



$$K_{US}^{diff} = \frac{d\delta_H}{da_y} = \frac{L}{u^2\tau_H} - \frac{\frac{d\alpha_r}{da_y} - \frac{d\alpha_f}{da_y}}{\tau_H}$$

$$K_{us} = -\frac{m}{\tau_H L}\left(\frac{L_F}{C_{\alpha R}} - \frac{L_R}{C_{\alpha F}}\right)$$

Handling diagram

From the lateral balance in steady state and the axle characteristics we get the axle side slips. Once we have the axle slips we are interested to find the relationship with vehicle state and in particular the trajectory curvature, the body slip and the driver steering angle.

The relationship can be found from

$$\alpha_r(u^0, v^0, \Omega^0) = -\beta + L_r \rho_{ss}$$

$$\alpha_f(u^0, v^0, \Omega^0) = \delta_f - \beta - L_f \rho_{ss}$$

By making the difference between the 2 equations above we get the **Steering Characteristic equation**

$$\rho_0 L - \delta_H \tau_H = \alpha_r(\frac{a_y}{g}) - \alpha_f(\frac{a_y}{g})$$

Reshuffling the last equations we get the Handling curve equation

$$\boxed{\delta_H \, \tau_H - \rho_0 \, L = K_{US} \cdot a_y + \overline{K_{US}}(a_y) \cdot a_y}$$

As we can see it is made by a linear term in the lateral acceleration and by a Non linear term which is fitted with a polynomial function.

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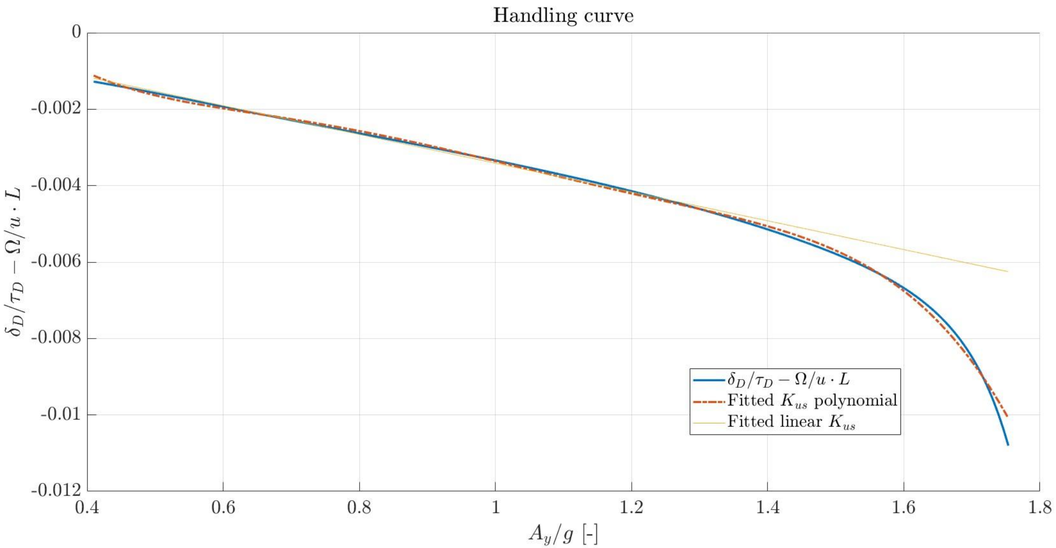
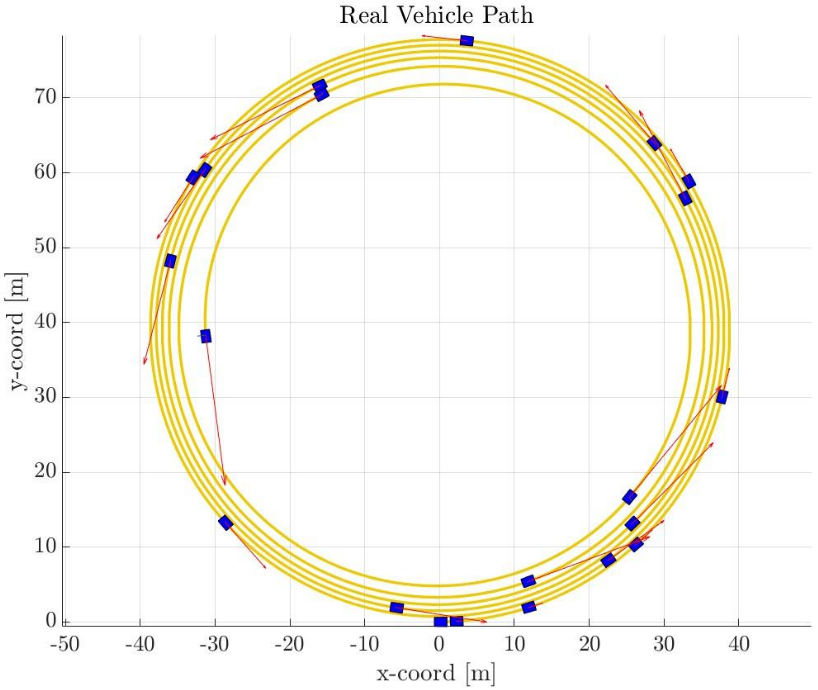
Handling diagram



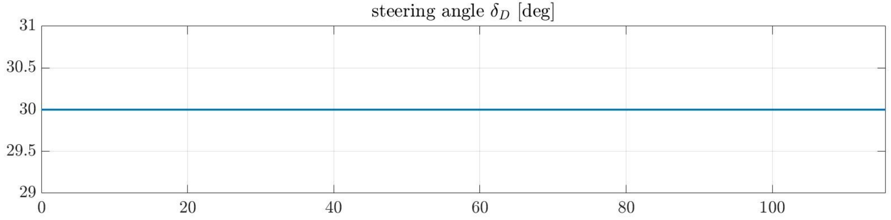
Handling curves give us the steering behaviour of the vehicle as function of the normalized lateral acceleration.

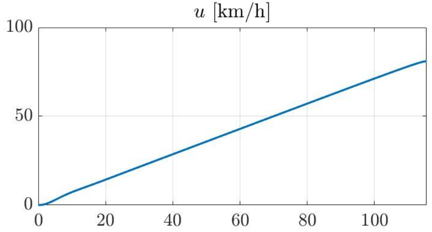
Oversteering

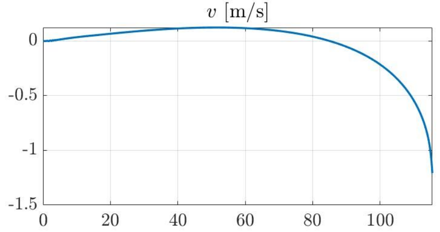
Test #2 (Speed Ramp Test)



- Linear speed ramp
- Constant steering angle: 30°
- Simulation ends when car starts losing control (~115.5 [s])







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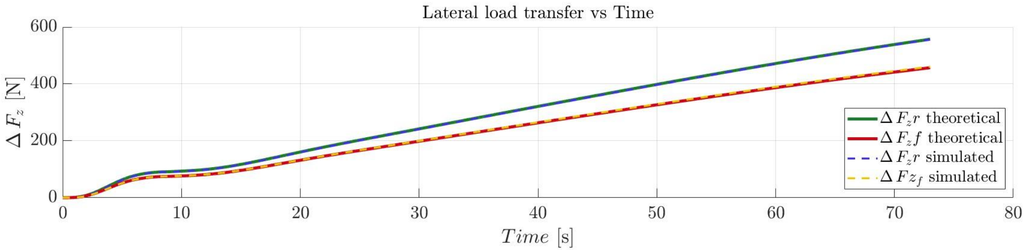
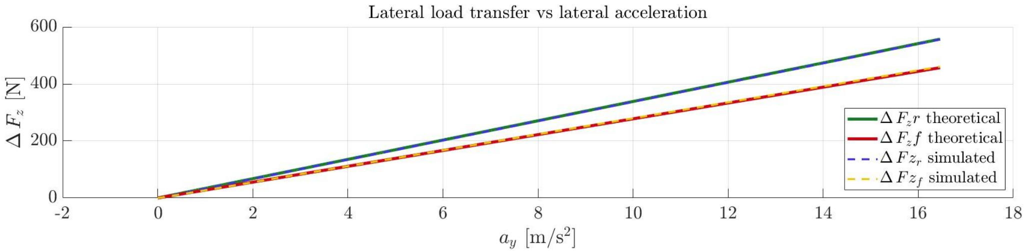
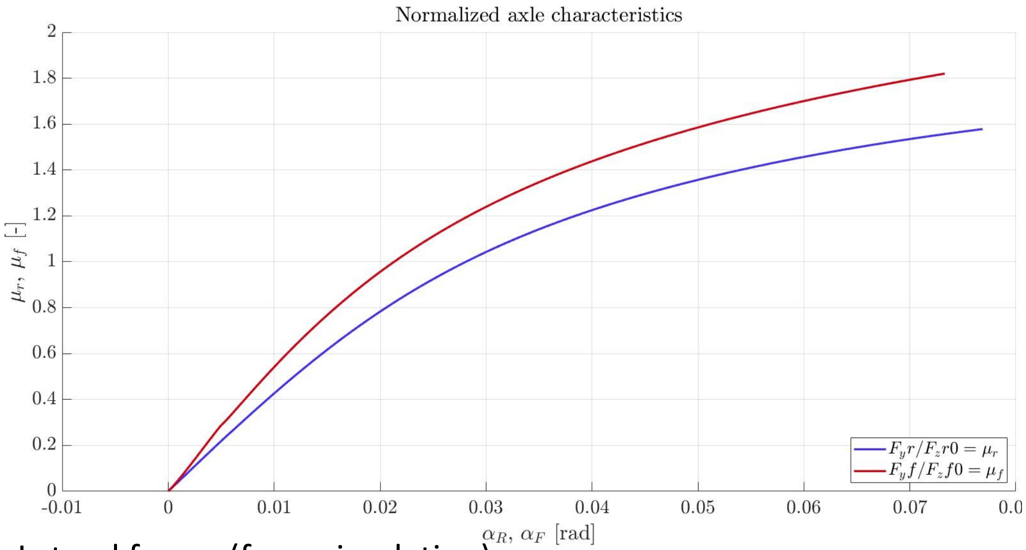
Lateral load transfer





$$\Delta F z_f = \frac{m a_y}{W_f}\left(\frac{L_r}{L} h_{rf} + \epsilon_\phi h_s\right) \qquad \Delta F z_r = \frac{m a_y}{W_r}\left(\frac{L_f}{L} h_{rr} + (1-\epsilon_\phi) h_s\right)$$

Normalized axle characteristics



Same results as for steer ramp test

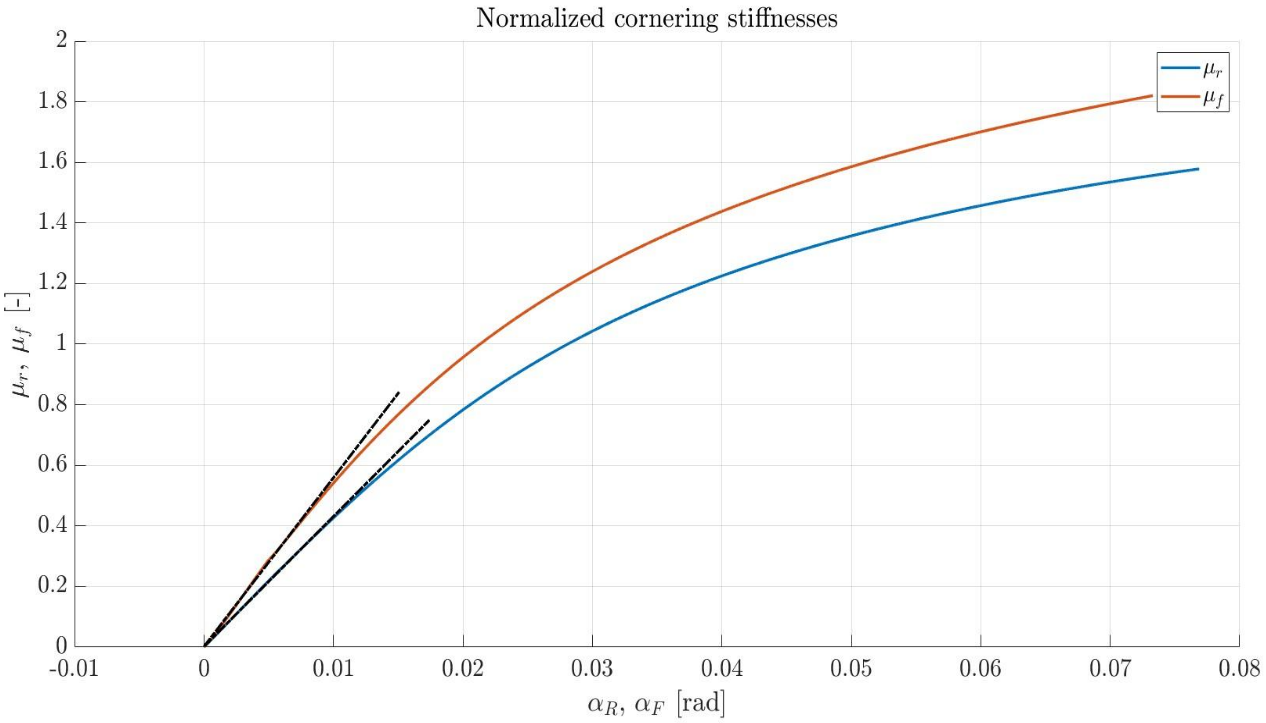
Lateral forces (from simulation):

$$Fyf = Fy_{fl} + Fy_{fr} \qquad Fyr = Fy_{rl} + Fy_{rr}$$

Vertical forces distribution:

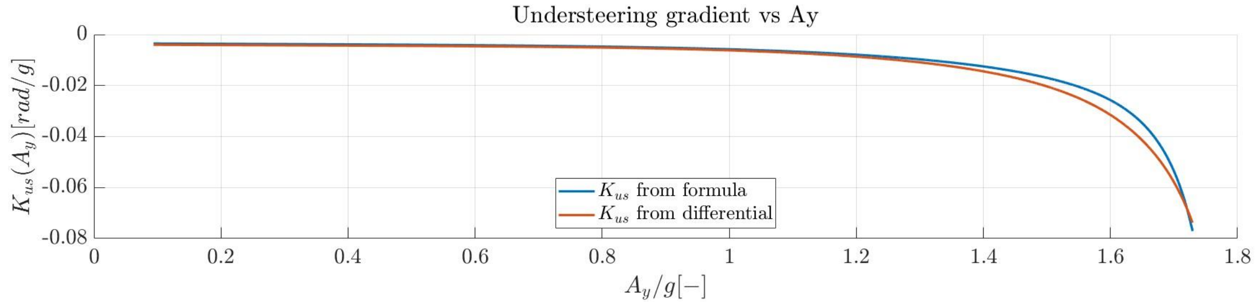
$$Fzf_0 = m \cdot g \cdot \frac{Lr}{L} \qquad Fzr_0 = m \cdot g \cdot \frac{Lf}{L}$$

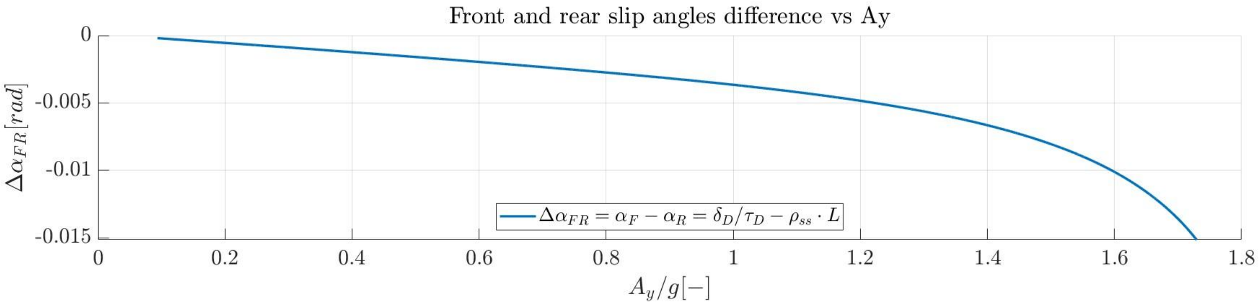
Cornering stiffnesses from axle characteristics



$$C_{\alpha F} = \delta\mu_f/\delta\alpha_f = 55.72 \qquad C_{\alpha R} = \delta\mu_r/\delta\alpha_r = 42.92$$

Understeering gradient comparison

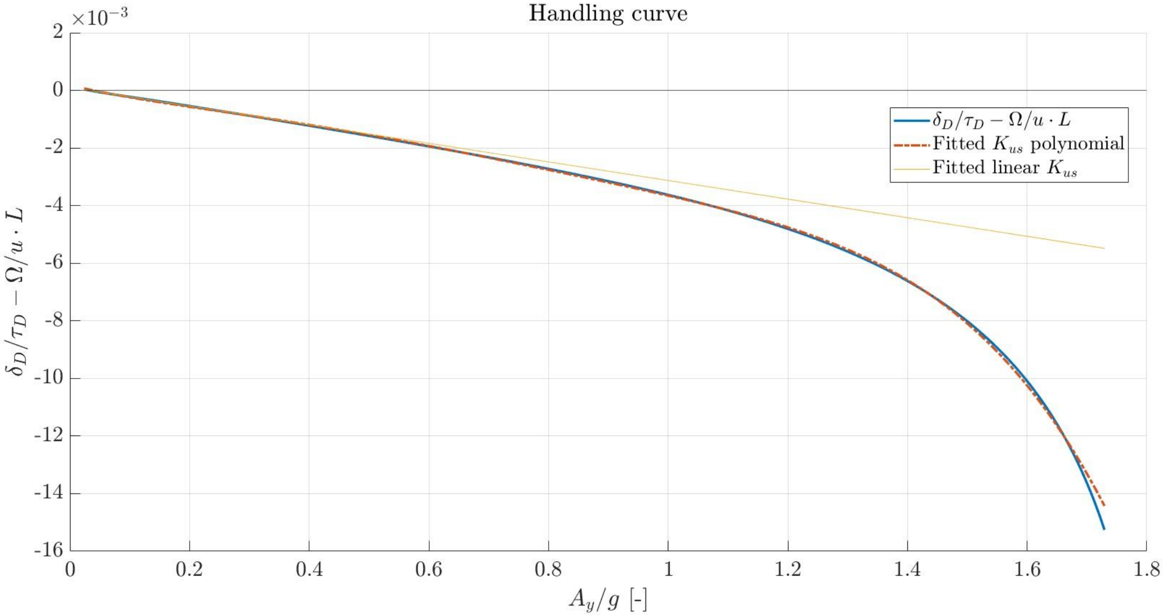




$$\boxed{K_{US}^{diff} = -\frac{\frac{d\alpha_r}{da_y} - \frac{d\alpha_f}{da_y}}{\tau_H}}$$

$$\boxed{K_{us} = -\frac{m}{\tau_H L^2}\left(\frac{L_F}{C_{\alpha R}} - \frac{L_R}{C_{\alpha F}}\right)}$$

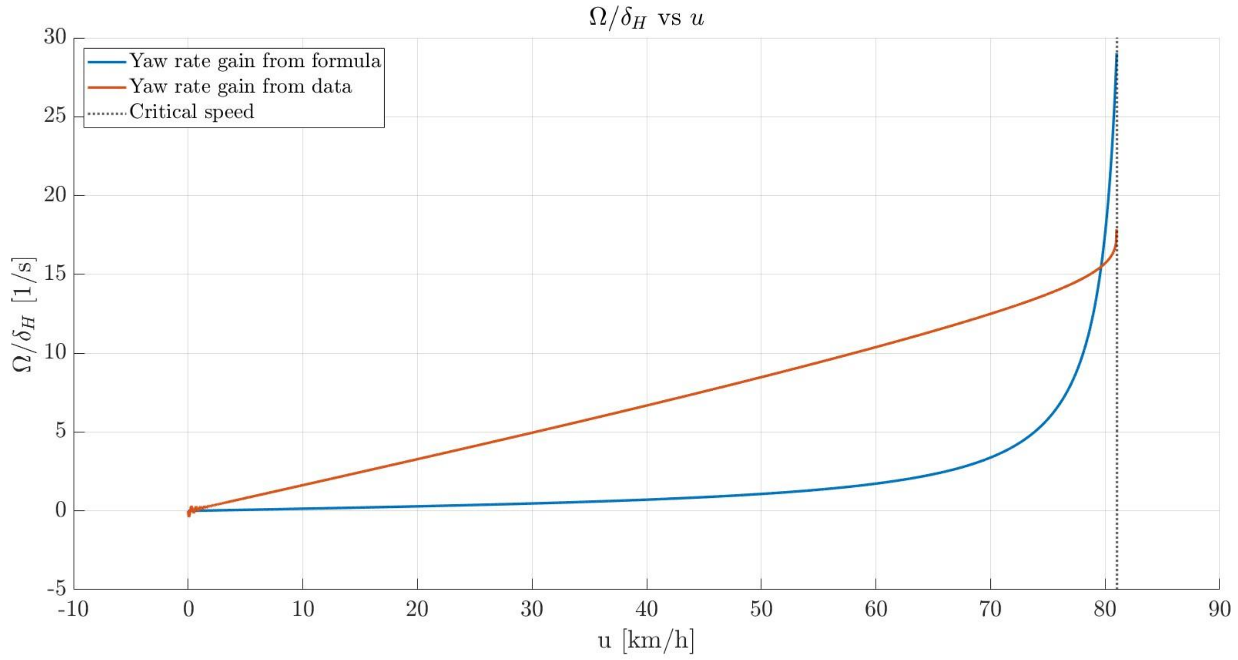
Handling diagram



Handling curves give us the steering behaviour of the vehicle as function of the lateral acceleration.

Oversteering

Yaw rate gain



$$\frac{\Omega}{\delta_H} = \frac{u\,\tau_H}{L(1+\frac{K_{US}}{L}u^2)}$$

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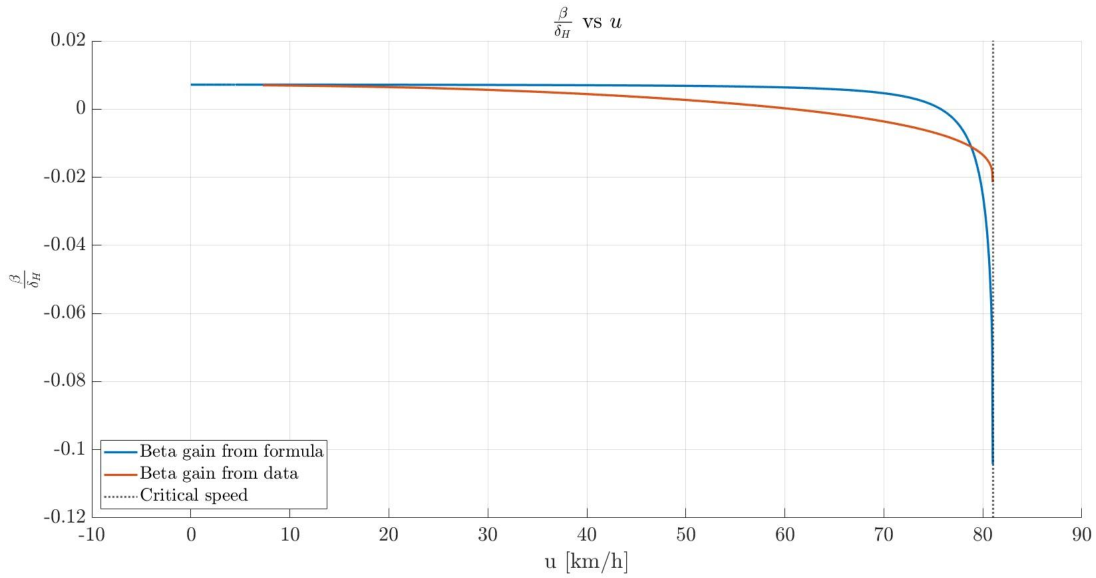
Beta gain

$$\frac{\beta}{\delta_H} = \tau_H \frac{L_r}{L} - \frac{m}{L^3}\left(\frac{L_f^2}{K_{y_r}} + \frac{L_r^2}{K_{y_f}}\right)\frac{\tau_H\, u^2}{(1 + \frac{K_{US}}{L} u^2)}$$



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Additional effects analysis

1. *Static toe angle*
2. *Camber angle*
3. *Suspension roll stiffness*

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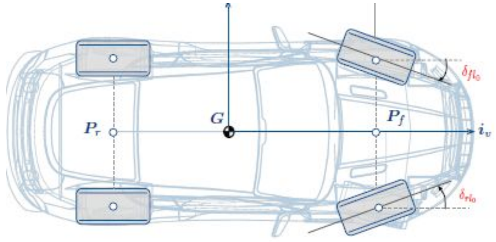
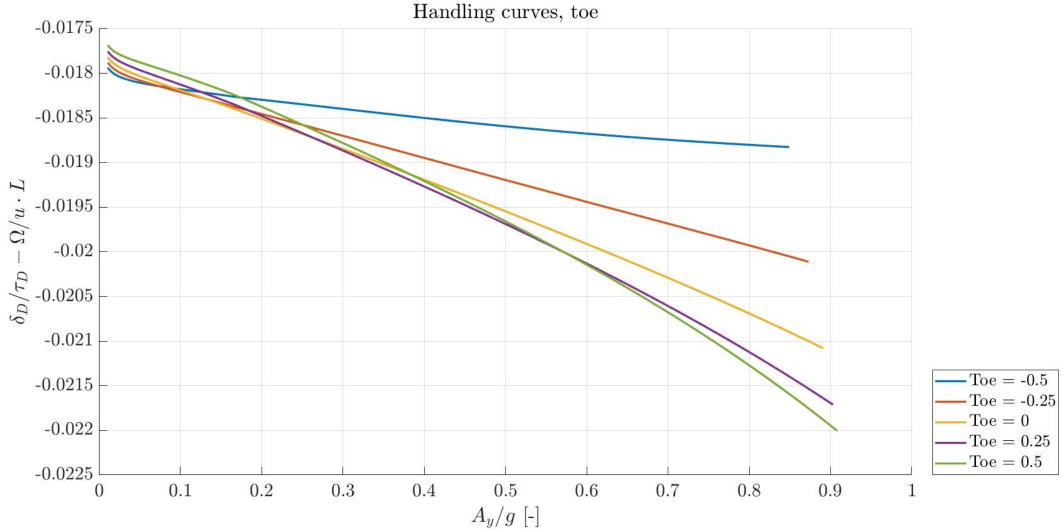
Static toe angle effect



Positive toe



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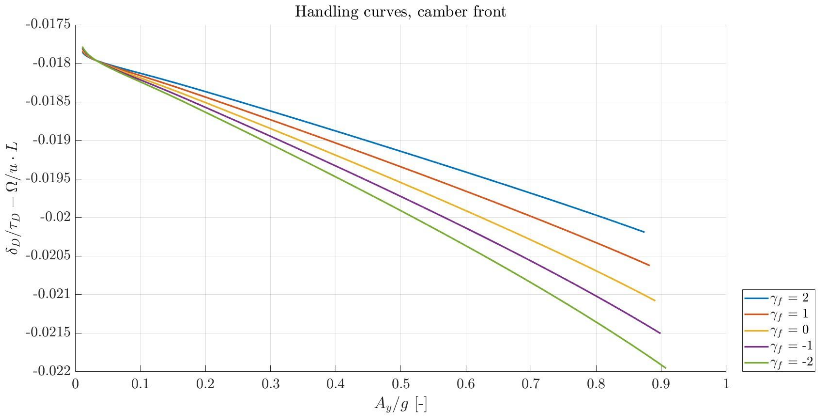
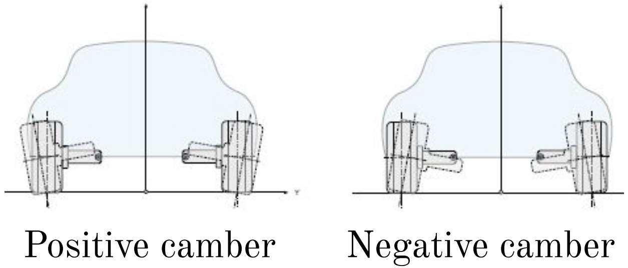
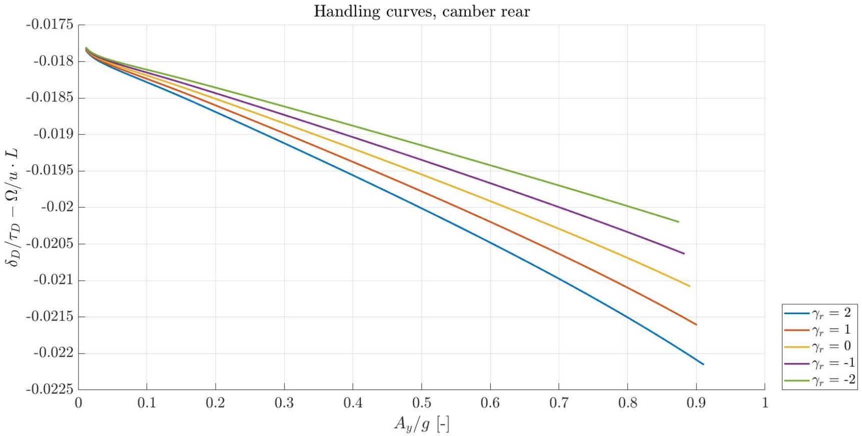
Camber angle effect







Suspension roll stiffness effect

The suspension stiffness (usually via Anti-Roll bar) affects lateral load transfer on each axle so consequently it changes the axles cornering stiffness

$$\Delta F_z^{TRANS} = m a_y \frac{h_{G_s}}{W_f} \frac{K_\phi^f}{K_\phi^f + K_\phi^r} = m a_y \frac{h_{G_s}}{W_f} \epsilon_\phi$$

The elastic term or transient lateral load transfer is the part of the lateral load transfer going through the spring of the suspension deforming it

In particular we have as follows

$$\uparrow K_{\phi_f} \Rightarrow \uparrow \Delta F_{z_f} \Rightarrow \uparrow \Delta \alpha_f \Rightarrow \uparrow US$$
$$\uparrow K_{\phi_r} \Rightarrow \uparrow \Delta F_{z_r} \Rightarrow \uparrow \Delta \alpha_r \Rightarrow \uparrow OS$$

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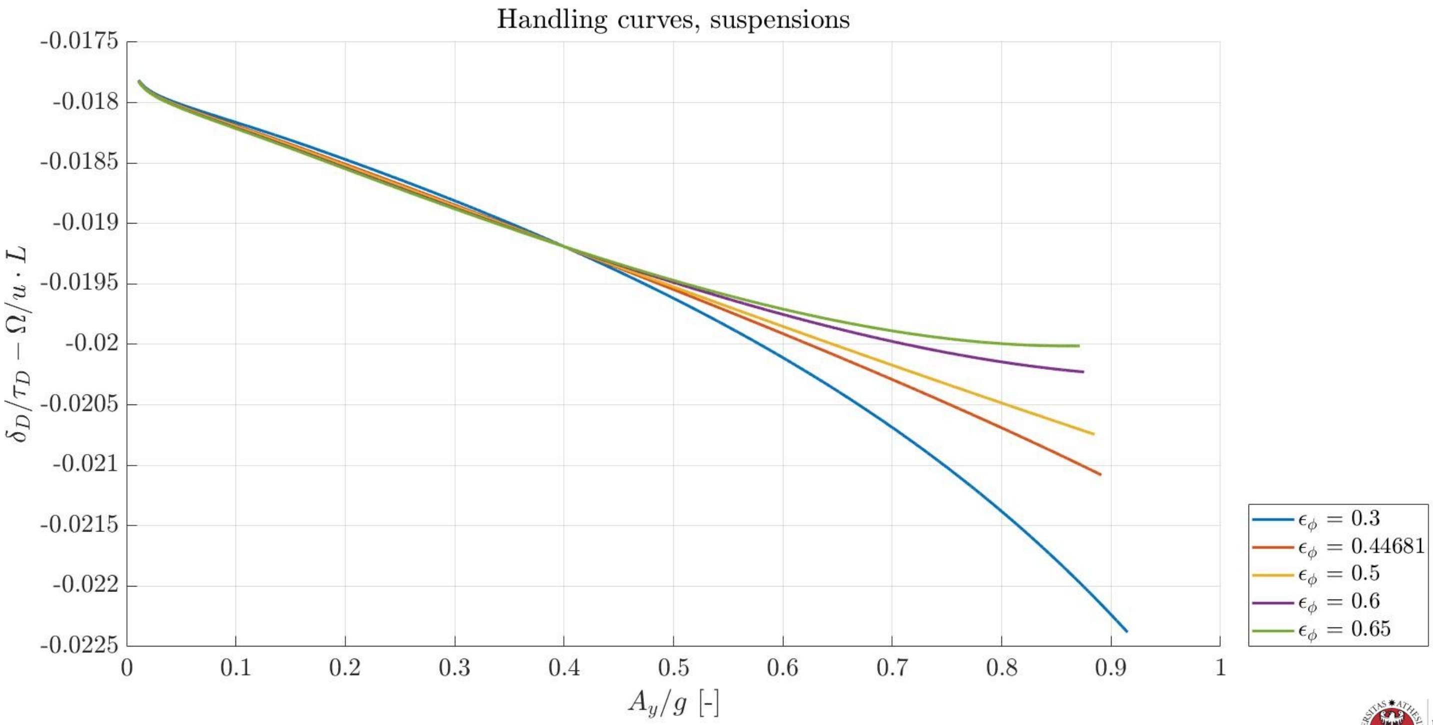
Suspension roll stiffness effect



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Thank you for the attention!!!

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