Fitting Magic Formula '96 coefficients

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CONTENTS

***Abstract*—Tyre models have been developed during the years to satisfy the need of modeling vehicle dynamics. Tyres exhibit non-linear behaviours that require advanced models with many coefficients that shape and fit the curves to data collected from tyre testing machines equipment. The fitted curves are then used to foresee the behaviour of a tyre at any given configuration. This is very important for commercial vehicle dynamics studies, as well as for racing, where the difference between a well modeled tyre and a poor one may lead the team to tune poorly the vehicle. One of the most used models is the Magic Formula '96 (MF96), which can take into account several different effects changing the behaviour of a tyre. The fitting of the MF96 tyre model coefficients on a dataset of measured tyre forces for a F-SAE tyre was required as the first assignment of Dynamics of Vehicles course held by Professor F.Biral. Many different subsequent steps will be presented in the following pages together with comments and plots.**

I. INTRODUCTION

The scope of this first assignment was to fit the MF96 tyre model to Formula SAE tyre data collected by Calspan Tire Research Facility (TIRF) during the Round 5 testing. The provided datasets contained data for the following tyres:

- Continental 205 / 510 R13 (C11/C12)
- Goodyear D2704 20.0x7.0-13
- Hoosier 18.0 x 6.0 - 10 R25B

Each dataset was composed by two `.mat` files:

1) the first for the identification of the pure longitudinal and combined forces;
2) the second for the identification of the pure lateral force and the self aligning moment.

The fitting discussed in these pages is to be referred to Hoosier tyres and the obtained coefficients are for MF96 in ISO reference frame (Fig. 1).

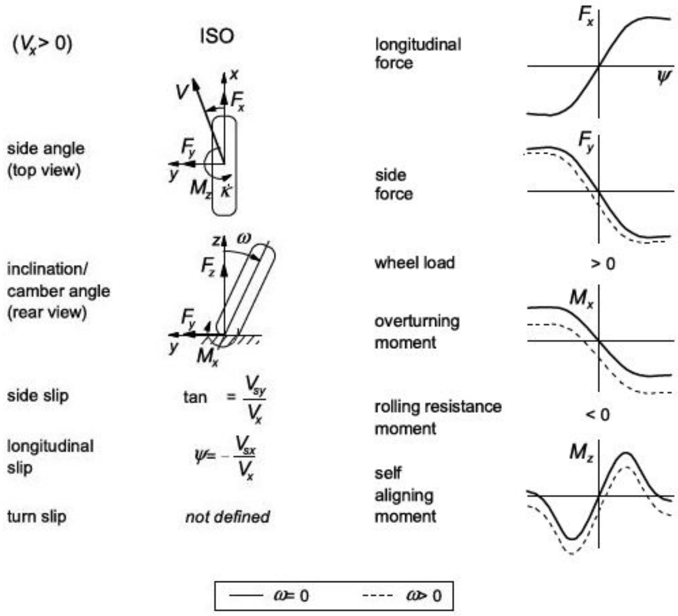


Fig. 1: **ISO reference frame for MF96 formula**
(Credits: Tyre and vehicle dynamics, Hans B. Pacejka)

II. MF96 COEFFICIENTS TYRE DATA FITTING PROCEDURE

Fitting has been carried out in MATLAB® using a least-squares approach, as suggested, and `fmincon` as the non linear optimization solver. Parameters, as initial guesses and upper and lower bounds for `fmincon`, fitted coefficients values, R^2 and $RMSE$ are reported for each fitting procedure carried out in the developed script.

Sample ranges used for fitting were the following:

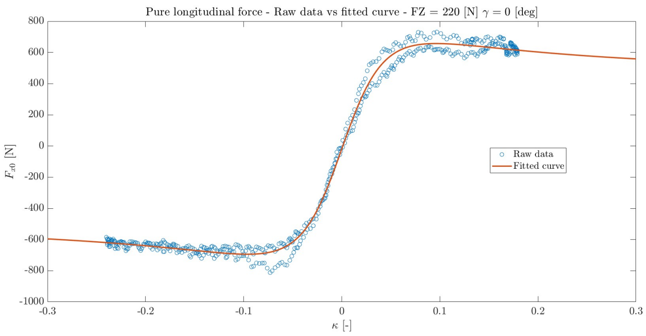
- `Hoosier_B1464run23.mat` $\rightarrow 27760 : 54500$
- `Hoosier_B1464run30.mat` $\rightarrow 19028 : 37643$

A. ***Pure longitudinal force***

Fitting of the pure longitudinal force F_{x0} coefficients with the following 3 steps:

- Fitting of coefficients with **pure conditions: nominal vertical load** $F_{z_0} = 220\,[N]$ **and zero camber angle** $\gamma = 0\,[deg]$

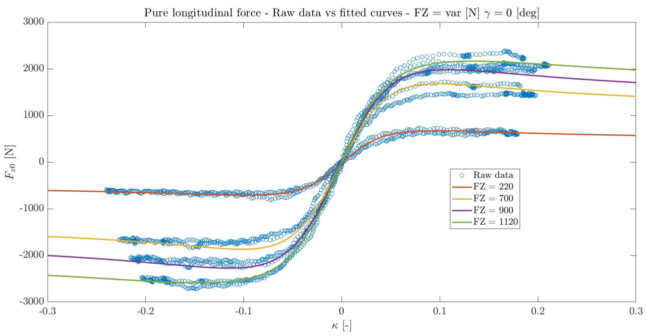
Coefficients	pCx_1	pD_{x_1}	pE_{x_1}	pE_{x_4}	pK_{x_1}	pH_{x_1}	pV_{x_1}
P_0 - initial guess	1	2	1	0	0	1	0
lb - lower bound	1	0.1	0	0	-10	0	-10
ub - upper bound	2	4	1	1	10	100	10
Fitted value	1.5387	3.1469	0.0113	0.0806	82.4171	-2.0920e-05	-0.0858



Obtained performance indexes:
$R^2 = 99.55\,\%$ and $RMSE = 40.47$ [N] .

- Fitting of coefficients depending on $\underline{df_z}$ **load variation with zero camber angle** $\gamma = 0$

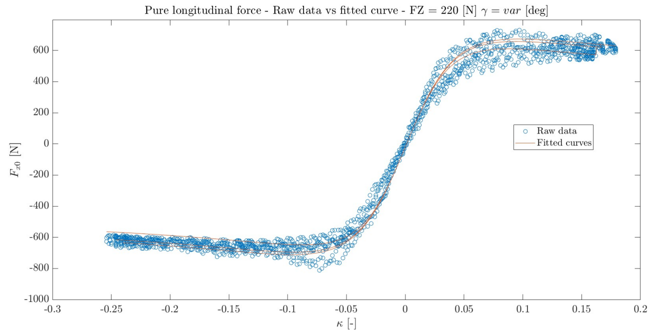
Coefficients	pD_{x_2}	pE_{x_2}	pE_{x_3}	pH_{x_2}	pK_{x_2}	pK_{x_3}	pV_{x_2}
P_0 - initial guess	0	0	0	0	0	0	0
lb - lower bound	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	-
Fitted value	-0.2496	-0.3619	0.1059	0.0011	-0.0019	0.1536	-0.0256



Obtained performance indexes:
$R^2 = 99.71\,\%$ and $RMSE = 85.91$ [N] .

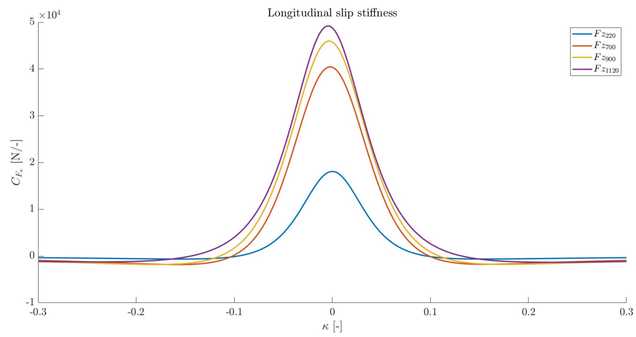
- Fitting of coefficients depending on **variable camber** γ **and nominal vertical load** $F_{z0} = 220$ **[N]**

Coefficients	pD_{x_3}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	18.4364



Obtained performance indexes:
$R^2 = 99.03\,\%$ and $RMSE = 57.45$ [N] .

The longitudinal slip stiffness is plotted as shown below



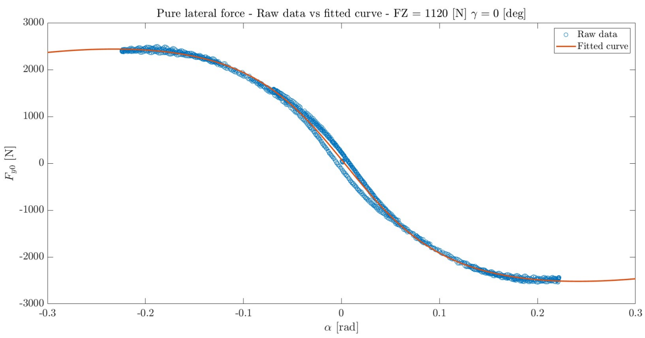
COMMENTS: It can be observed that, for some vertical load cases, the current fitting overestimates or underestimates the longitudinal force for certain values of the slip ratio κ. This may introduce some inaccuracies when using the MF96 curves to predict the behaviour of the tyre. A finer tuning of the coefficients may solve these problems.

B. *Pure lateral force*

Fitting of the pure lateral force F_{y0} coefficients with the following 4 steps:

- Fitting of coefficients with **pure conditions: vertical nominal load** $F_{Z_0} = 1120$ **[N] and zero camber angle** $\gamma = 0$ **[deg]**

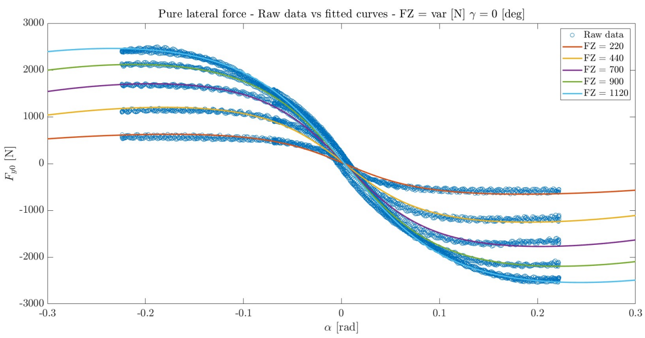
Coefficients	pC_{y1}	pD_{y1}	pE_{y1}	pH_{y1}	pK_{y1}	pK_{y2}	pV_{y1}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	0.1
lb - lower bound	1	-1000	-1000	-1000	-1000	-1000	-1000
ub - upper bound	2	1000	1000	1000	1000	1000	1000
Fitted value	1.0177	3.7104	1.4733	-0.0046	-23.1279	1.1343	-0.0357



Obtained performance indexes:
$R^2 = 99.82\,\%$ and $RMSE = 68.62\,$[N] .

- Fitting of coefficients depending on $\underline{df_z}$ **load variation with zero camber angle** $\gamma = 0$

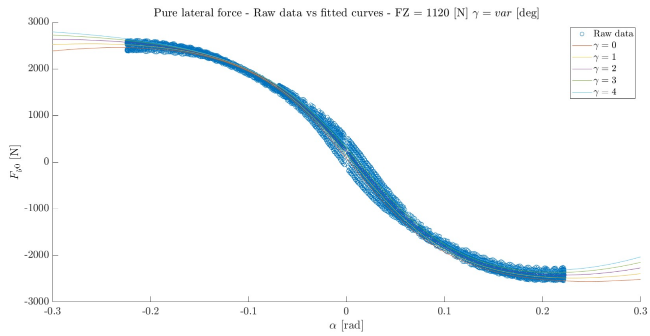
Coefficients	pD_{y2}	pE_{y2}	pH_{y2}	pV_{y2}
P_0 - initial guess	0.1	0.1	0.1	0.1
lb - lower bound	-1000	0	-1000	-1000
ub - upper bound	1000	1	1000	1000
Fitted value	-1.3805	8.4470e-05	-2.8188e-04	0.0310



Obtained performance indexes:
$R^2 = 99.68\,\%$ and $RMSE = 75.19$ [N] .

- Fitting of coefficients depending **on variable camber** γ **and vertical nominal load** $F_{Z_0} = 1120$ **[N]**

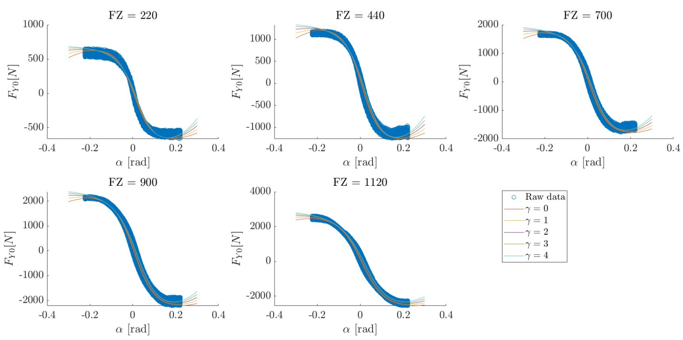
Coefficients	pD_{y3}	pE_{y3}	pE_{y4}	pH_{y3}	pK_{y3}	pV_{y3}
P_0 - initial guess	0.1	0.5	0.1	0.1	0.1	5
lb - lower bound	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-
Fitted value	3.0508	0.0077	-4.0472	-0.2662	0.9497	-1.7757



Obtained performance indexes:
$R^2 = 99.84\,\%$ and $RMSE = 73.49\,$[N] .

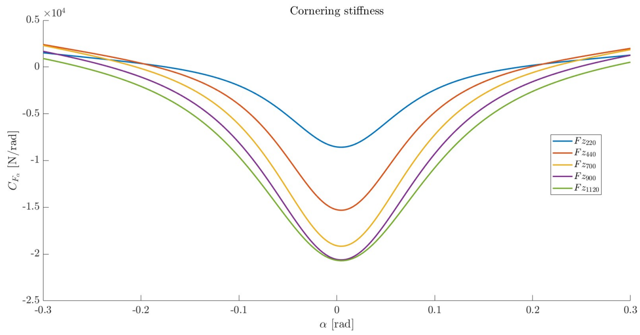
- Fitting of coefficients depending both **on variable camber** γ **and variable load** df_z

Coefficients	pV_{y4}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	4.9444



Obtained performance indexes:
$R^2 = 99.62\,\%$ and $RMSE = 77.57$ [N] .

The cornering stiffness C_{F_α} is shown below



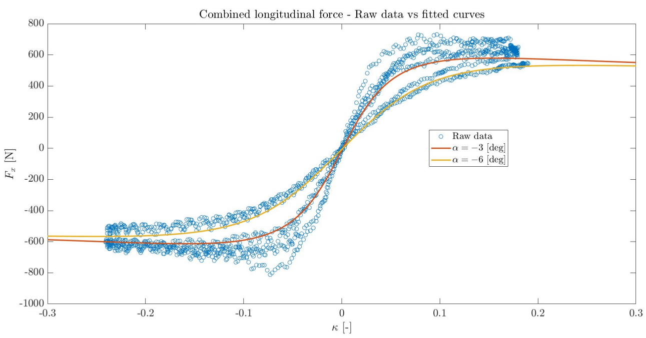
COMMENTS: Differently from the fitting done for F_{X0}, the chosen nominal vertical load for the F_{Y0}'s coefficients is $1120[N]$. This choice leads to more slack `fmincon` conditions and better fitted curves. It is of interest to note that the order of magnitude of the cornering stiffness peaks ranges from about 18 to 38 times the vertical wheel load. Pacejka [2] suggests that the usual range goes from about 6 to 30, with higher values for racing tyres, so the obtained range is compatible with the kind of racing tyre analysed.

C. *Combined Longitudinal Force*

Fitting of the combined longitudinal force F_X coefficients with the following step:

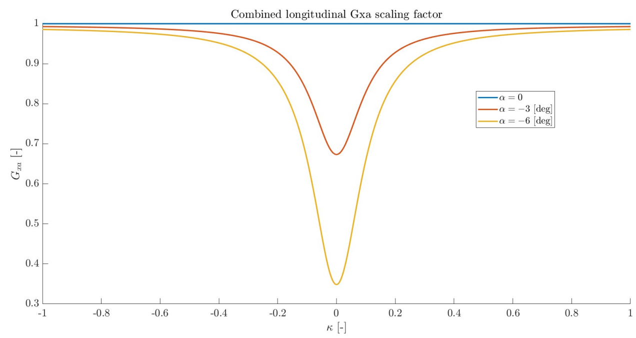
- Fitting of coefficients depending **on pure condition parameters with vertical nominal load** $F_{z_0} = 220\,[N]$ **and zero camber angle** $\gamma = 0\,[deg]$

Coefficients	rB_{x_1}	rB_{x_2}	rC_{x_1}	rH_{x_1}
P_0 - initial guess	8.3	5	0.9	0
lb - lower bound	7	0	0.5	-100
ub - upper bound	20	20	3	1
Fitted value	14.6714	1.2440	1.0022	-19.3506



Obtained performance indexes:
$R^2 = 99.32\,\%$ and $RMSE = 44.21$ [N] .

In the next figure it is shown the scaling factor G_{x_a} as a function of the longitudinal slip κ

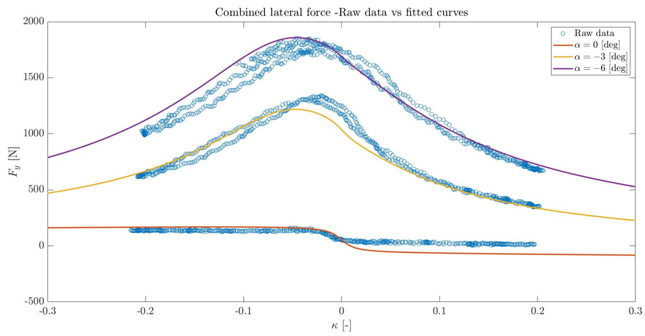


D. *Combined Lateral Force*

Fitting of the combined lateral force F_Y coefficients with the following steps:

- Fitting of coefficients depending on **pure condition parameters with vertical nominal load** $F_{z_0} = 900[N]$ **and zero camber angle** $\gamma = 0\,[deg]$

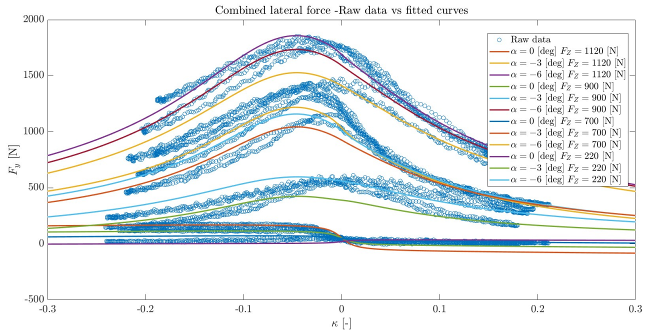
Coefficients	rB_{y_1}	rB_{y_2}	rB_{y_3}	rC_{y_1}	rH_{y_1}	rV_{y_1}	rV_{y_4}	rV_{y_5}	rV_{y_6}
P_0 - initial guess	4.9	2.2	0	1	0.1	0.1	30	0.5	10
lb - lower bound	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000	-1000
ub - upper bound	1000	1000	1000	1000	1000	1000	1000	1000	1000
Fitted value	246.5833	973.0261	-0.0750	0.9995	0.0453	-4.0021	24.4909	0.0032	94.9144



Obtained performance indexes:
$R^2 = 99.57\,\%$ and $RMSE = 63.59$ [N] .

- Fitting of coefficients depending on $\underline{df_z}$ **load variation with zero camber angle** $\gamma = 0$

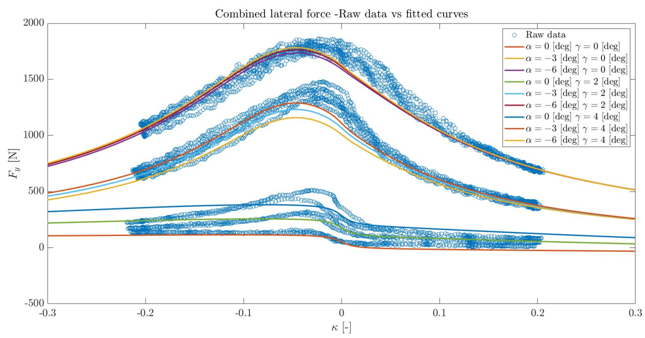
Coefficients	rV_{y_2}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	-9.8945



Obtained performance indexes:
$R^2 = 99.19\,\%$ and $RMSE = 67.91 \quad$ [N] .

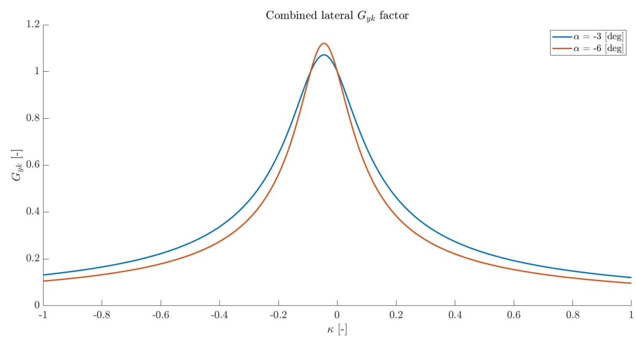
- Fitting of coefficients depending **on variable camber conditions and nominal vertical load $F_{z_0} = 900 \quad$ [N]**

Coefficients	rV_{y3}
P_0 - initial guess	0
lb - lower bound	-
ub - upper bound	-
Fitted value	-31.6715



Obtained performance indexes:
$R^2 = 97.50\,\%$ and $RMSE = 149.69 \quad$ [N] .

In the next figure it is shown the scaling factor G_{y_κ} for the combined lateral force F_y



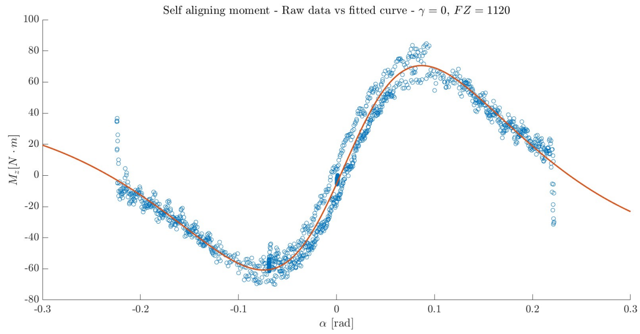
COMMENTS: Combined behaviour fittings were the hardest to achieve, the overall result may be inaccurate for some load/camber configurations. A finer tuning of the coefficients may solve the problem. Similarly to F_{Y_0} fitting, the F_{Z_0} nominal load has been changed to 900 [N] in order to get better results.

E. Pure Self-Aligning Moment

Fitting of the Self-Aligning Moment M_{z_0} coefficients with the following steps:

- Fitting of coefficients with **pure conditions: vertical nominal load $F_{z_0} = 1120\,[N]$ and zero camber angle** $\gamma = 0\,[deg]$

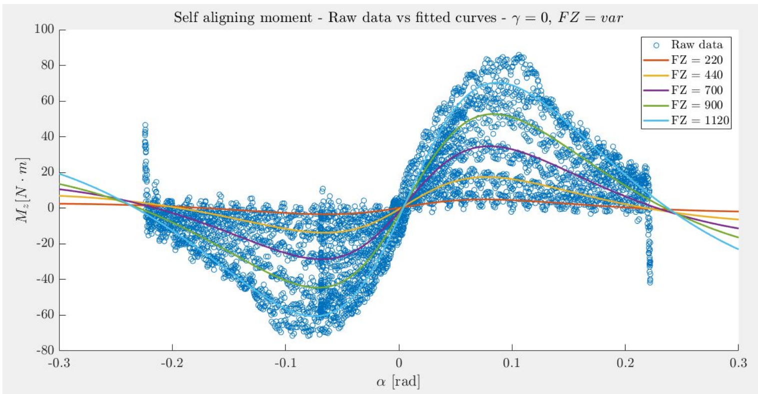
Coefficients	qH_{z_1}	qB_{z_1}	qC_{z_1}	qD_{z_1}	qE_{z_1}	qE_{z_4}	qB_{z_9}	$qB_{z_{10}}$	qD_{z_6}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
lb - lower bound	-	-	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	-	-	-
Fitted value	-0.0132	6.2738	1.7473	0.2400	0.4249	-0.1461	0.2299	-0.5407	-0.0045



Obtained performance indexes:
$R^2 = 97.26\,\%$ and $RMSE = 6.38 \quad$ [N·m] .

- Fitting of coefficients depending on $\underline{df_z}$ **load variation with zero camber angle** $\gamma = 0$

Coefficients	qH_{z_2}	qB_{z_2}	qB_{z_3}	qD_{z_2}	qE_{z_2}	qE_{z_3}	qD_{z_7}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	
lb - lower bound	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	
Fitted value	-0.0024	-0.8874	-0.9187	-0.0254	-0.3624	-1.2557	-0.0156



Obtained performance indexes:
$R^2 = 96.82\,\%$ and $RMSE = 5.3459 \quad$ [N·m] .

- Fitting of coefficients depending **on variable camber γ with nominal vertical load $F_{z_0} = 1120$ [N]**

Coefficients	qH_{z_3}	qB_{z_4}	qB_{z_5}	qD_{z_3}	qD_{z_4}	qE_{z_5}	qD_{z_8}
P_0 - initial guess	0.1	0.1	0.1	0.1	0.1	0.1	0.1
lb - lower bound	-	-	-	-	-	-	-
ub - upper bound	-	-	-	-	-	-	-
Fitted value	0.7460	-7.3893	6.6463	-1.0072	4.7283	-2.5673	1.7837

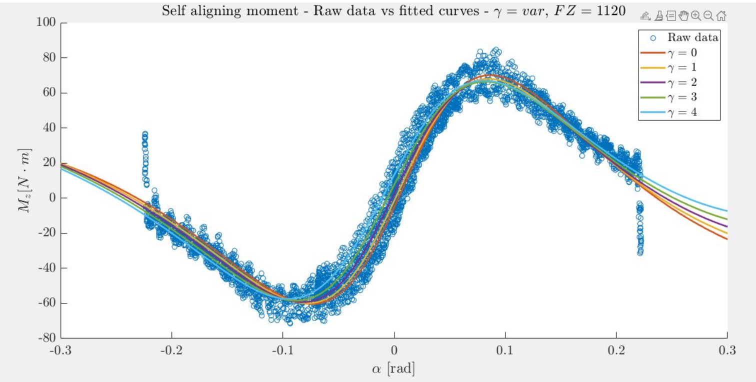
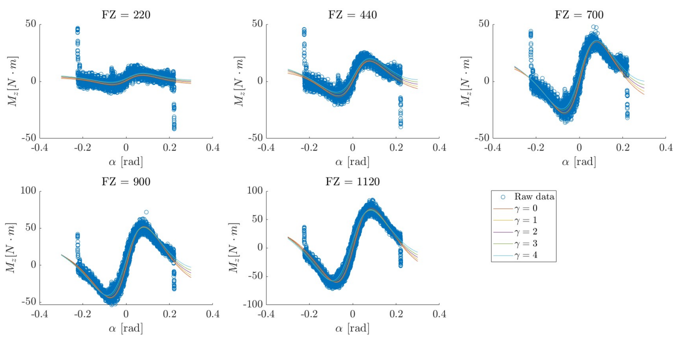


Fig. 2: **Variable camber conditions: plots of raw data and fitting**

Obtained performance indexes:
$R^2 = 97.40\,\%$ and $RMSE = 6.5335$ [N·m] .

- Fitting of coefficients depending on both **variable camber γ and variable load df_z**

Coefficients	qD_{z_9}	qH_{z_4}
P_0 - initial guess	1	0.5
lb - lower bound	0.9	0.4
ub - upper bound	1.1	0.6
Fitted value	0.9000	0.5168



Obtained performance indexes:
$R^2 = 96.17\,\%$ and $RMSE = 5.2640$ [N·m] .

<u>*COMMENTS*</u>: Using $F_{Z0} = 1120\,[N]$, the obtained curves fit well the actual data. The latter two coefficients needed stricter bounds in order to fit better the curves.

REFERENCES

[1] Biral Francesco, Semi empirical tyre models; Magic Formula 96, Trento: Italy, 2023, pp. 34 – 45.
[2] Hans B. Pacejka, Tyre and Vehicle Dynamics, Second edition, Oxford: Butterworth-Heinemann, 2006
[3] Delfttyre TNO, MF-Tyre/MF-Swift 6.2 Help Manual, Document revision: 10/17/2013. The Netherlands, 2013.