

# LinearUCB

Exercise Lecture

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LinearUCB Implementation

# Motivations



When the arms space is huge we can exploit the information gained on observed arms to estimate the reward function of non-observed arms

# Linear Environment

Each arm  $j$  is associated with a feature vector  $x_j = (x_{j1}, x_{j2}, x_{j3}, \dots, x_{jD})$  with  $x_{ji} \in [0,1]$

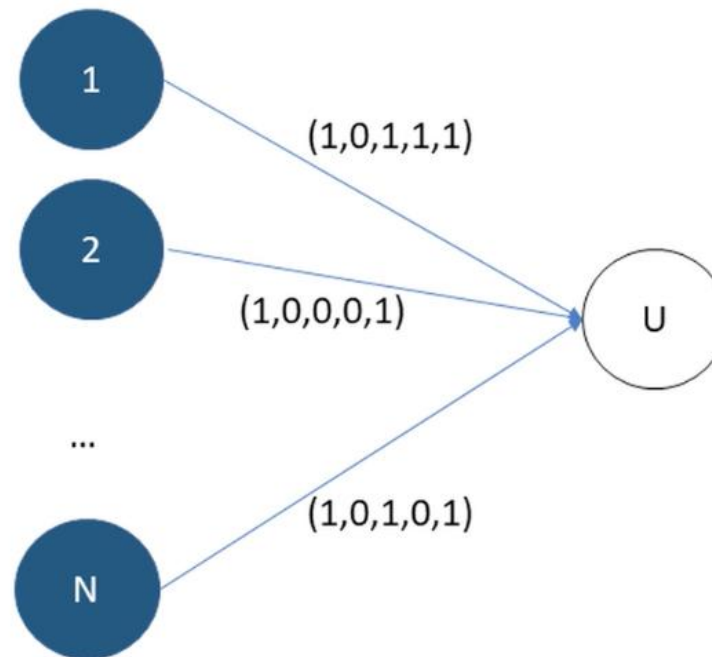
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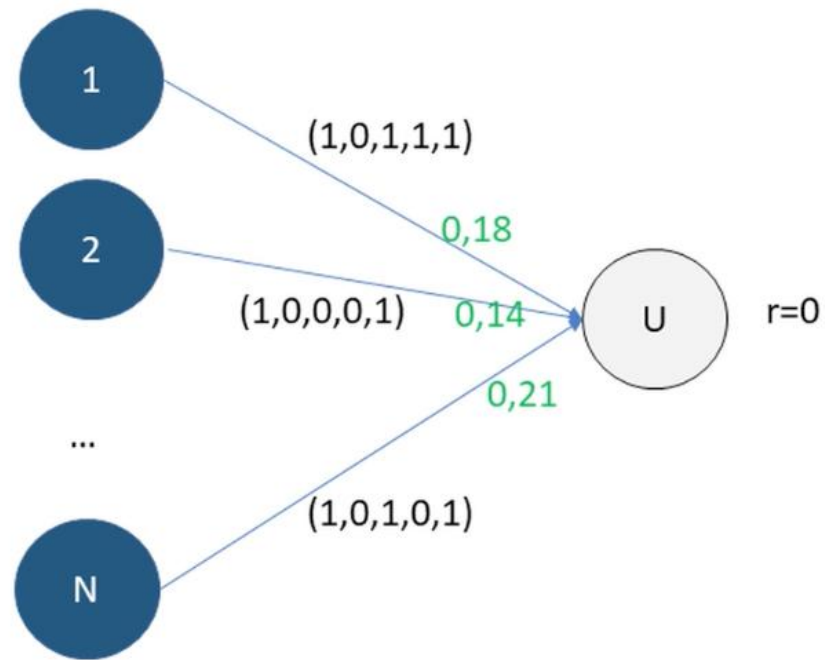
The reward is a linear combination of the arm feature vector and a parameters vector  $\theta$ :

$$r_t = x_t^T \theta$$

# Example: Social Influence



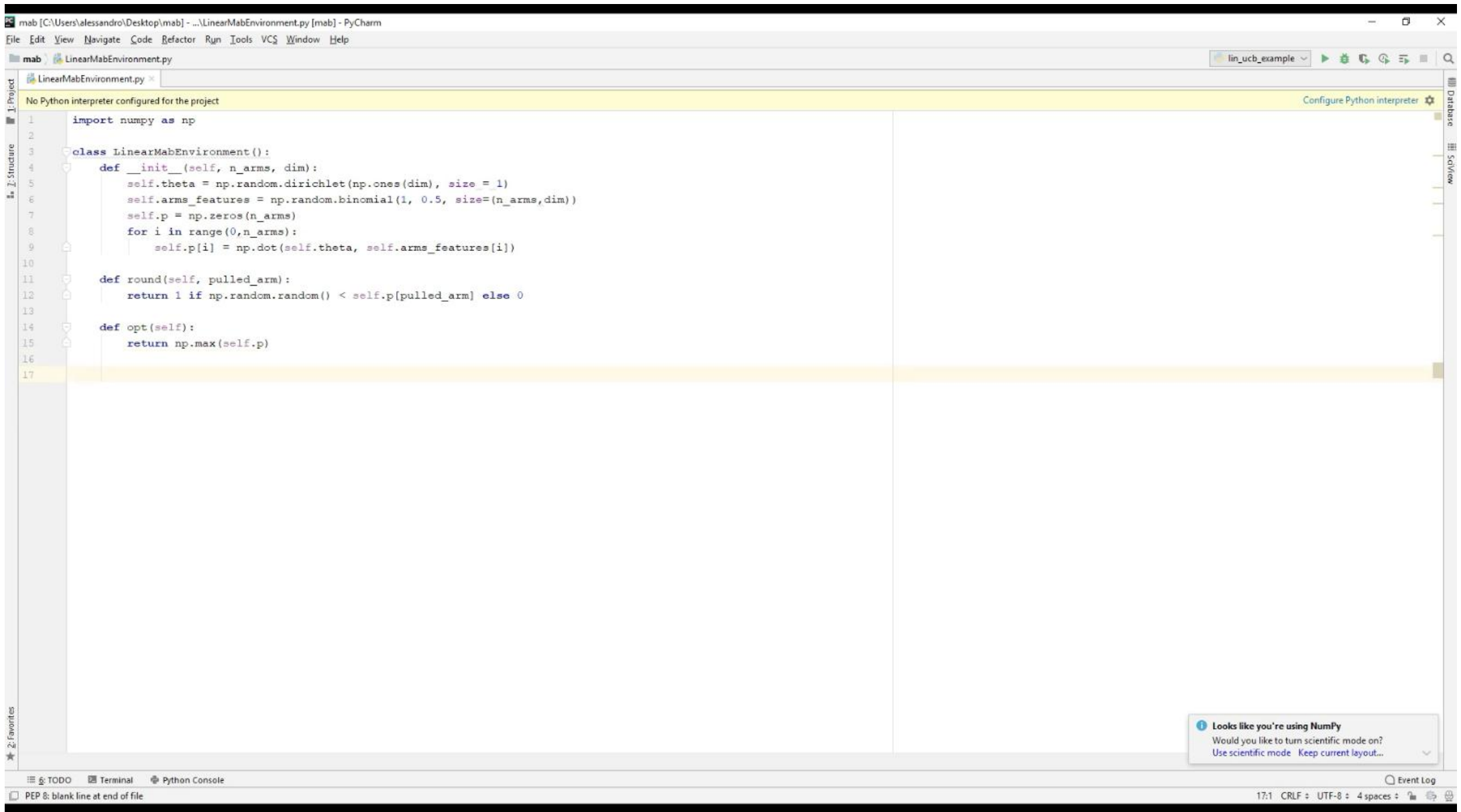
# Example: Social Influence



Let's implement it!







# LinearUCB

Input: arms set  $A$ , parameter  $c > 0$

Initialization:  $B_0 = 0 \in R^d, M_0 = I \in R^{d \times d}$

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For  $t = 1, 2, \dots, n$  do

$$1) \quad \theta_{t-1} = M_{t-1}^{-1} B_{t-1} \quad a$$

$$\text{UCBs} = x^T \theta_{t-1} + c \sqrt{x^T M_{t-1}^{-1} x}$$

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2) Choose the arm with maximum ucb value

3) Update matrices:

a)  $M_t = M_{t-1} \quad B_t = B_{t-1}$

b)  $M_t = M_{t-1} + x_t x_t^T$

and  $B_t = B_{t-1} + x_t r_t$

# IMLinUCB

Wen, Zheng, et al. "**Online influence maximization under independent cascade model with semi-bandit feedback.**" *Advances in neural information processing systems*. 2017.

Let's implement it!



mab [C:\Users\alejandro\Desktop\mab] - ...LinUcbLearner.py [mab] - PyCharm

File Edit View Navigate Code Refactor Run Tools VCS Window Help

mab LinUcbLearner.py

lin\_ucb\_example

Project

mab C:\Users\alejandro\Desktop\mab

BiddingEnvironment.py  
comparison\_example.py  
Environment.py  
GaussianEnvironment.py  
gpts\_experiment.py  
GPTS\_Learner.py  
Greedy\_Learner.py  
GTS\_Learner.py  
Learner.py  
lin\_ucb\_example.py  
LinearMabEnvironment.py  
LinUcbLearner.py  
Non\_Stationary\_Environment.py  
non\_stationary\_example.py  
prova.py  
SWTS\_Learner.py  
TS\_Learner.py  
External Libraries  
Scratches and Consoles

No Python interpreter configured for the project

Configure Python interpreter

```
4 class LinUcbLearner():
5     def __init__(self, arms_features):
6         self.arms = arms_features
7         self.dim = arms_features.shape[1]
8         self.collected_rewards = []
9         self.pulled_arms = []
10        self.c = 2.0
11        self.M = np.identity(self.dim)
12        self.b = np.atleast_2d(np.zeros(self.dim)).T
13        self.theta = np.dot(np.linalg.inv(self.M), self.b)
14
15
16    def compute_ucbs(self):
17        self.theta = np.dot(np.linalg.inv(self.M), self.b)
18        ucbs = []
19        for arm in self.arms:
20            arm = np.atleast_2d(arm).T
21            ucb = np.dot(self.theta.T, arm) + self.c * np.sqrt(np.dot(arm.T, np.dot(np.linalg.inv(self.M), arm)))
22            ucbs.append(ucb[0][0])
23        return ucbs
24
25    def pull_arm(self):
26        ucbs = self.compute_ucbs()
27        return np.argmax(ucbs)
28
29    def update_estimation(self, arm_idx, reward):
30        arm = np.atleast_2d(self.arms[arm_idx]).T
31        self.M += np.dot(arm, arm.T)
32        self.b += reward * arm
33
34
35    def update(self, arm_idx, reward):
36        self.pulled_arms.append(arm_idx)
37        self.collected_rewards.append(reward)
38
```

Event Log

Statement seems to have no effect

38:10 CRLF UTF-8 4 spaces

Looks like you're using NumPy  
Would you like to turn scientific mode on?  
Use scientific mode Keep current layout...



mab [C:\Users\alejandro\Desktop\mab] - ...LinUcbLearner.py [mab] - PyCharm

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mab LinUcbLearner.py lin\_ucb\_example

Project C:\Users\alejandro\Desktop\mab

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External Libraries

Scratches and Consoles

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12 self.b = np.atleast_2d(np.zeros(self.dim)).T
13 self.theta_ = np.dot(np.linalg.inv(self.M), self.b)
14
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16 def compute_ucbs(self):
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18     ucbs = []
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20         arm = np.atleast_2d(arm).T
21         ucb = np.dot(self.theta.T, arm) + self.c * np.sqrt(np.dot(arm.T, np.dot(np.linalg.inv(self.M), arm)))
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36     self.pulled_arms.append(arm_idx)
37     self.collected_rewards.append(reward)
38     self.update_estimation(arm_idx, reward)
39
40
```

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Event Log

PEP 8: blank line at end of file

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mab [C:\Users\alejandro\Desktop\mab] - ...lin\_uch\_example.py [mab] - PyCharm

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mab lin\_uch\_example.py

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lin\_uch\_example.py

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```
1 import numpy as np
2 import matplotlib.pyplot as plt
3 from LinearMabEnvironment import *
4 from LinUcbLearner import *
5
6
7
8 n_arms = 10
9 T = 1000
10 n_experiments = 100
11 lin_uch_rewards_per_experiment = []
12
13 env = LinearMabEnvironment(n_arms=n_arms, dim=10)
14
15 for e in range(0, n_experiments):
16     lin_uch_learner = LinUcbLearner(arms_features=env.arms_features)
17     for t in range(0, T):
18         pulled_arm = lin_uch_learner.pull_arm()
19         reward = env.round(pulled_arm)
20         lin_uch_learner.update(pulled_arm, reward)
21     lin_uch_rewards_per_experiment.append(lin_uch_learner.collected_rewards)
22
23
24 opt = env.opt()
25 plt.figure(0)
26 plt.ylabel("Regret")
27 plt.xlabel("t")
28 plt.plot(np.cumsum(np.mean(opt - lin_uch_rewards_per_experiment, axis=0)), 'r')
29 plt.legend(["LinUCB"])
30 plt.show()
31
32
```

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Event Log

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