

Could the GRB-supernovae GRB 031203 and XRF 060218 be cosmic twins?

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ABSTRACT

The gamma-ray burst (GRB)/X-ray flash (XRF) events GRB 031203, discovered by *INTEGRAL*, and XRF 060218, discovered by *Swift*, represent two of the only five GRB-supernovae (GRB-SNe) with optical spectroscopic confirmation of their SN components. Yet, their observed high-energy properties offer a sharp contrast: while GRB 031203 was detected as a short 40 s burst with a spectrum peaking at $E_{\text{peak}} > 190$ keV, XRF 060218 was a $T_{90} \approx 2100$ s long, smoothly evolving burst with peak energy $E_{\text{peak}} = 4.9$ keV. At the same time, the properties of the two expanding dust-scattered X-ray haloes observed in a fast-response *XMM-Newton* observation of GRB 031203 reveal that this event was accompanied by an ‘X-ray blast’ with fluence comparable to or greater than that of the prompt gamma-ray event. Taking this observation as our starting point, we investigate the likely properties of the X-ray blast from GRB 031203 via detailed modelling of the *XMM* data, discovering a third halo due to scattering off a more distant dust sheet at $d_3 = 9.94 \pm 0.39$ kpc, and constraining the timing of the X-ray blast relative to the GRB trigger time to be $t_0 = 11 \pm 417$ s. Using our constraints, we compare the properties of GRB 031203 to those of other GRB-SNe in order to understand the likely nature of its X-ray blast, concluding that a bright X-ray flare, as in GRB 050502B, or shock breakout event, as in XRF 060218, provides the most likely explanation. In the latter case, we consider the added possibility that XRF 060218 may have manifested an episode of bright gamma-ray emission prior to the burst observed by *Swift*, in which case GRB 031203 and XRF 060218 would be ‘cosmic twin’ explosions with nearly identical high-energy properties.

Key words: supernovae: general – supernovae: individual: SN 2003lw – supernovae: individual: SN 2006aj – dust, extinction – gamma-rays: bursts.

1 INTRODUCTION

The discovery of the connection between long-duration gamma-ray bursts (GRBs) and X-ray flashes (XRFs), on the one hand, and Type Ibc supernovae (SNe) on the other, has confirmed the collapsar model as proposed by Woosley (1993) – for a recent review, see Woosley & Bloom (2006).

Several distinct types of investigation provide support for the GRB- (XRF-) SN connection; however, the most convincing evidence has come from spectroscopic observations of coincident SNe associated with the lowest redshift GRBs and XRFs. As is frequently the case in astronomy, these nearby and best-studied cases have each proven anomalous in their own way; even considered as a group, the properties of GRB-SNe thus resist ready generalization.

In all, five spectroscopically confirmed GRB-SNe – a term we will use to refer to the GRB- and XRF-SNe, as a group – have been observed to date. SN 1998bw offered the first direct evidence for GRB-SN association, as the SN that appeared in the error box of GRB 980425, with an explosion time consistent with the GRB trigger, was shown to be exceptionally luminous and energetic in the optical (Galama et al. 1998), with an associated mildly relativistic ($\Gamma \gtrsim 3$) outflow that made it uniquely radio-bright (Kulkarni et al. 1998). Occurring in a catalogued galaxy at a distance of just 36 Mpc ($z = 0.0085$), however, GRB 980425 was four orders of magnitude less luminous than the typical $z \gtrsim 1$ burst.

XRF 020903, at $z = 0.25$ the first XRF with a determined redshift and observed optical and radio afterglow emission (Soderberg et al. 2004a), also provided the first XRF-associated SN (Soderberg et al. 2005; Bersier et al. 2006); its spectrum was ultimately shown to be similar to that of SN 1998bw (Soderberg et al. 2005), confirming the deep connection between XRFs and GRBs.

GRB 030329/SN 2003dh at $z = 0.17$ provided the first definitive association of a ‘cosmological’ GRB with a coincident Type Ic SN,

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since it had an isotropic-equivalent gamma-ray output of $E_{\gamma, \text{iso}} = 10^{52}$ erg (Vanderspek et al. 2004), within the observed range for $z \gtrsim 1$ events. Broadline SN Ic features were visible within a few days of the GRB (Stanek et al. 2003), and the ultimate evolution of the SN showed a striking similarity to SN 1998bw (Hjorth et al. 2003; Matheson et al. 2003), validating the two earlier associations.

Later that year, GRB 031203 was detected by *INTEGRAL* with a duration of 40 s and a peak energy of >190 keV (Sazonov, Lutovinov & Sunyaev 2004, hereafter SLS04). Six hours after the burst, a fast-response *XMM-Newton* observation of the burst position began; a fading X-ray afterglow was quickly identified (Rodríguez-Pascual et al. 2003), coincident with a $z = 0.105$ galaxy (Prochaska et al. 2004). This was confirmed as the host galaxy once an associated Type Ic SN, SN 2003lw, was discovered (Malesani et al. 2004). Investigation of the radio and X-ray energetics of the event revealed further similarities to SN 1998bw (Soderberg et al. 2004b), although the SN itself was somewhat subluminal by comparison (Gal-Yam et al. 2004).

Contemporaneously, analysis of the *XMM* data revealed two expanding X-ray haloes centred on the fading GRB afterglow (Vaughan et al. 2004, hereafter V04); these haloes were convincingly interpreted as scattered photons from a bright X-ray ‘blast’ that occurred near-simultaneously with GRB 031203. Modelling the properties of the X-ray blast, Watson et al. (2004, 2006), or W04 and W06, argued that the prompt emission of GRB 031203 was, in fact, an XRF, exhibiting greater fluence in X-ray compared to gamma-ray, $\log(S_{2-30 \text{ keV}}/S_{30-400 \text{ keV}}) = 0.6 \pm 0.3 > 0$. Independent analysis by Tiengo & Mereghetti (2006, or TM06) yielded a lower X-ray fluence estimate which none the less also satisfies $S_{2-30 \text{ keV}} > S_{30-400 \text{ keV}}$.

The nature of GRB 031203 is not easily resolved, for two reasons. First, the bright soft X-ray emission cannot be interpreted as the low-energy tail of GRB 031203, as its fluence significantly exceeds the low-energy extrapolation of the *INTEGRAL* spectrum (SLS04; W06; TM06). Secondly, the *INTEGRAL* observations themselves would have detected the X-ray blast if it was emitted over a brief time interval prior to or within 300 s following the burst itself, before 22:06:27 UT, when *INTEGRAL* began to slew to a new pointing. Thus, two distinct explosive episodes are required, with the X-ray blast most likely occurring both physically and temporally apart from the GRB. To satisfy these constraints, W06 consider the possibility that GRB 031203 was followed by a bright X-ray flare such as that observed by *Swift* from GRB 050502B (Burrows et al. 2005); such flares – albeit rarely as bright as that case – are familiar features of the GRB X-ray light curves now routinely gathered by *Swift* (e.g. Chincarini et al. 2007).

XRF 060218, detected by *Swift* (Campana et al. 2006) and coincident with a $z = 0.033$ host galaxy (Mirabal et al. 2006), evolved into a Type Ic SN that was observed from its earliest moments, including extensive spectroscopic observations (Mazzali et al. 2006; Mirabal et al. 2006; Pian et al. 2006), thanks to the prompt alert and extended follow-up observations of *Swift*. Analysis of the X-ray, optical and radio energetics of XRF 060218 demonstrates a deep similarity to SN 1998bw and GRB 031203 (Soderberg et al. 2006); however, XRF 060218 remains an anomaly in that its prompt emission exhibited a uniquely extended evolution, having a 90 per cent containment duration of $T_{90} = 2100$ s and a correspondingly low-peak flux $F_{\text{peak}} = 8.8 \times 10^{-9}$ erg cm $^{-2}$ s $^{-1}$ (15–150 keV). This peak flux is more than an order of magnitude lower than those of all other GRB-SNe except for XRF 020903, which had an even softer spectrum; indeed, it was only detected because of the ‘image trigger’ capability of the *Swift* Burst Alert Telescope (BAT), which

identified XRF 060218 as an uncatalogued source apparent in the first 64 s integration taken at that pointing.

Emission from XRF 060218 is present in the first image taken by *Swift* on that particular orbit; as such, a GRB similar to GRB 031203 could have occurred prior to the start of *Swift* observations. Moreover, a significant fraction of the prompt X-ray emission of XRF 060218 is supplied by a thermal component, unobserved in other GRB-SNe, that is attributed to the shock breakout of the associated SN (Campana et al. 2006) rather than to the relativistic jet usually invoked to explain the prompt emission. If the X-ray blast from GRB 031203 can be explained as a similar shock breakout, then these two events – despite their very different appearances – would in fact be ‘cosmic twins’ exhibiting nearly identical evolution in the X-ray and gamma-ray bands (see also Ghisellini et al. 2006).

In this paper, we undertake a quantitative exploration of this hypothesis by constraining the properties of the X-ray blast in GRB 031203 via detailed modelling of its expanding X-ray haloes; this analysis also allows us to address the alternate theory that the X-ray emission from GRB 031203 was due to a later X-ray flare (W06). Section 2 presents the *XMM* observations of the afterglow and haloes of this burst and the details of our analysis, including derivation of parameter uncertainties. Section 3 discusses our results in light of various hypotheses regarding GRB 031203 and XRF 060218, and places these events in the larger context of the known varieties of cosmic explosion. Section 4 summarizes our conclusions.

2 OBSERVATION AND DATA ANALYSIS

The GRB 031203 data (Observation ID 0158160201), downloaded from the *XMM-Newton* archive, are processed and reduced for the European Photon Imaging Camera (EPIC), Metal-Oxide-Silicon (MOS) and p-n cameras through the standard procedures (tasks EMCHAIN and EPCHAIN), with standard filtering, using the *XMM* Science Analysis System (SAS), version 7.1.0. The observation had start and stop times of 04:10:18 UT and 20:16:20 UT on 2003 December 4, which after standard filtering yields 57.8 ks of good time and 57.3 ks net exposure in CCD 1 of EPIC-MOS1 and MOS2 and 56.2 ks of good time and a CCD-dependent net exposure between 53.4 and 54.0 ks in EPIC-PN. The observation was not affected by significant soft-proton flaring.

2.1 Model construction and fitting

We consider a circular region with radius 311 arcsec centred on the GRB afterglow, and restrict our attention to detected counts in the energy range 0.2 to 3.0 keV (0.2 to 4.0 keV for PN). We generate an exposure map with 1 arcsec pixel resolution, using the EEXPMAP task, to account for PN active regions; exposure map values are treated as binary (on or off) in the subsequent analysis, and no initial exposure map is created for the MOS data as our circular region of interest is fully contained on the central CCD. Bright X-ray sources within the region of interest are identified and all counts within an exclusion radius of 15 arcsec of each source position are excluded from analysis; these exclusion regions are then integrated into exposure maps for the PN and MOS data for analysis purposes. After all exclusions, we accept for analysis 10 754 counts from the MOS-1 detector, 10 535 counts from MOS-2 and 32 085 counts from PN.

We construct a 16-parameter, three-dimensional (x, y, t) maximum-likelihood model including parameters to describe the

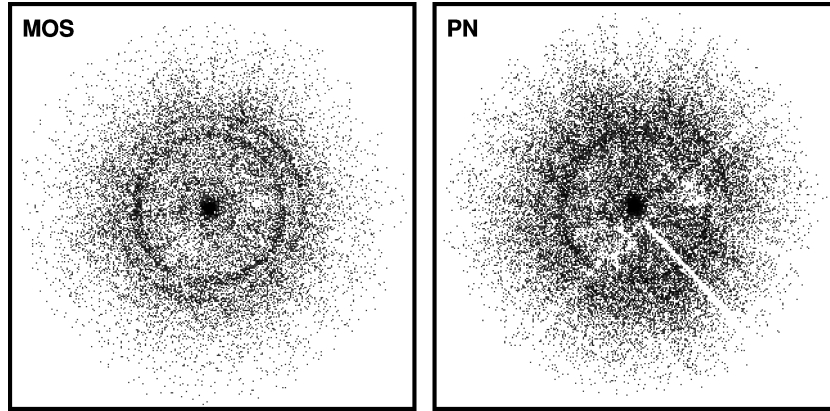


Figure 1. Images of the 2-MOS and PN counts data in normalized coordinates (x/\sqrt{t} , y/\sqrt{t}). The fading GRB afterglow, Halo 1 (outer) and Halo 2 (inner) are readily distinguished; the signature of the more distant and fainter Halo 3 (close to the GRB) is more subtle. White streaks in the images are due to bright source exclusion regions and PN camera chip gaps, which are incorporated into our model via MOS and PN exposure maps.

fading GRB afterglow, the expanding dust-scattered haloes, and the vignettted X-ray (and charged particle-induced) background. The total counts from all model components, for the 2-MOS and PN data sets, are constrained to be equal to the observed count totals given above. The model probability density is evaluated at the position and arrival time of each detected photon, the value of the probability density function is interpreted as being proportional to the likelihood of observing that event, and a global likelihood calculated; we then attempt to maximize the global likelihood using numerical techniques, specifically, the AMOEBA minimization routine (Press et al. 1992) as implemented in the IDL programming environment. For ease of modelling, we convert photon angular positions and arrival times as follows: angular coordinates x and y are measured in arcsec relative to an initial estimate of the GRB position, *XMM* physical coordinates (26 024.8, 23 674.1), with the $+x$ direction corresponding to decreasing RA and $+y$ to increasing Dec., as usual. The temporal coordinate t is measured relative to the burst trigger time ($t = 0$), scaled such that the *XMM* observation begins at $t = 1$, 22.2 ks ($6^{\text{h}}10^{\text{m}}44^{\text{s}}$) after the burst. Using this temporal coordinate, the *XMM* observation extends from $t = 1$ (defined by the earliest arrival time in our data set, from the MOS-2 detector) to $t \approx 3.61$. An illustration of the resulting data set is presented as an image in normalized coordinates (x/\sqrt{t} , y/\sqrt{t}) in Fig. 1.

We represent the fading GRB afterglow using an analytical model for the *XMM* point-spread function (PSF), which approximates the PSF as a King function:

$$s_p(r) = \frac{\alpha - 1}{\pi r_c^2} \left(1 + r^2/r_c^2\right)^{-\alpha}, \quad (1)$$

where α is the King slope, r_c is the King core radius, r is the radial distance from the source (ring centre) and the integrated surface brightness is normalized to unity for $\alpha > 1$. Since we combine the data from the two MOS cameras, we use average MOS PSF parameter values for the MOS data ($r_c = 4.916$ and $\alpha = 1.442$), and PN PSF values for the PN data¹ ($r_c = 6.636$ and $\alpha = 1.525$). The fading of the afterglow is parametrized as a power law with temporal index α_g , referenced to the burst trigger time.

We represent the expanding, fading dust-scattered haloes, centred on the GRB position, using a numerical model for ring-like sources observed with *XMM* that we have developed. The derivation of this model is provided in Appendix A; as realized in our code, this bivariate function is precalculated on a grid and interpolated to the particular values required for each function evaluation. Fading of the haloes is parametrized as a power law with temporal index α_h , referenced to the burst trigger time; this index is assumed to be the same for all rings. In order to avoid complications associated with normalizing the model probability density function for indices $\alpha_h \leq -1$ as well as for $\alpha_h > -1$, we impose a hard limit $\alpha_h \geq -0.95$ on this parameter during fitting. Initial investigations suggested that this limit was encountered in less than 5 per cent of bootstrap trials; however, our final analysis resulted in a somewhat greater proportion of 9 per cent of all trials encountering this limit.

Because the rings expand by a factor of $\sqrt{3.6} \approx 1.9$ over the course of the observation, and because we wish to accurately model the background over the region of interest, we find it necessary to apply an off-axis vignetting function (Lumb et al. 2003) to the rings and background:

$$S(r) = S(0)(1 - mr^e), \quad (2)$$

where $S(0)$ is the surface brightness on-axis, $S(r)$ is the surface brightness at off-axis distance r (measured in arcsec) and m and e are the vignetting parameters for the EPIC cameras. We use $m = 3.43 \times 10^{-5}$ and $e = 1.45$ for the MOS cameras, and $m = 3.51 \times 10^{-5}$ and $e = 1.53$ for the PN, which we find accurately models the vignetting of PN and MOS exposure maps generated in SAS.

The excluded sources and the PN exposure map present discontinuities to the model probability density function, which must be normalized to yield an appropriate fit. We perform this normalization via numerical integration over a three-dimensional rectangular grid (x , y , t), where the grid spacing is 2 arcsec in x and y , and the temporal grid is divided into 40 equal intervals. The same cuts applied to the data, including the circular region of interest, the exclusion regions around identified bright sources, and (for PN only) the PN exposure map, are applied to the grid prior to integration, and are thus (approximately) accounted for.

We maximize the likelihood function using the AMOEBA algorithm, in stages, using multiple fitting rounds at each stage (with fit starting points randomized prior to each round) to ensure a relatively wide-ranging exploration of the parameter space. In sequence, we execute: (i) 10 rounds of fitting the GRB count parameters and

¹ PSF model and parameter values are obtained from the *XMM* calibration document ‘In Flight Calibration of the PSF for the PN Camera’ by Simona Ghizzardi, located at <http://xmm.vilspa.esa.es/docs/documents/CAL-TN-0029-1-0.ps.gz>. See, in particular, equation (1) and table 1 in the document.

fading index α_g ; (ii) 10 rounds of fitting with all ring parameters (counts, radii and fading index α_r) also free; and finally, (3) Five rounds of fitting with all parameters free.

Initial examination of the data suggested the presence of a third dust-scattered halo at small radius (Fig. 1). Preliminary investigation confirmed the reality of this feature, and indicated that it should be interpreted as due to X-ray scattering off a third dust sheet, more distant than the two discussed by previous authors (Section 3.2). We therefore incorporate this third halo into our fits, with the same burst time t_0 and fading index α_h as the others.

Uncertainties in our model parameters are derived with the bootstrap Monte Carlo method, executing multiple trials and accumulating best-fitting parameter values from each trial to realize the posterior distribution of our model parameters. For each trial, a new data set is generated by drawing randomly, with replacement, from the true data set until we have the same number of events as in the actual data. The fitting sequence, described above, is identical for each trial; parameter starting points are randomized based on preliminary estimates (subsequently verified in our final analysis) of the uncertainties in each parameter. The result should approximate the posterior distributions that would result from a full Bayesian analysis using uniform (non-informative) prior distributions for each of the model parameters. For our final analysis, presented below, we executed 1000 trials in all.

The optimization of non-linear functions in many-dimensional spaces is a known hard ('NP') computational problem. We therefore point out that the bootstrap Monte Carlo approach is expected to yield conservative estimates of parameter uncertainties in cases where the model fails to offer a statistically complete representation of the data, or where the underlying fitting procedure fails to identify the global minimum in each and every trial. The reason is that, quite simply, the failure to identify the best possible fit is robustly expected to drive the fitting routine to explore a larger (rather than smaller) region of parameter space. This is true as long as the fitting routine is not trapped in a single local minimum, repeatedly, in trial after trial; in our analysis, we avoid this numerical disorder by randomizing parameter starting values for each trial, as mentioned above.

2.2 Results

The results of our model fitting, including parameter uncertainties quoted both as standard deviations and as 90 per cent confidence intervals, are presented in Table 1. 90 per cent confidence intervals are defined as the minimum-length intervals that encompass 90 per cent of the parameter values from our bootstrap Monte Carlo trials (e.g. 900 of 1000 parameter values). Associated background count totals can be derived by subtracting the counts of the model components given in the table from the total instrument counts of 21 289 for 2-MOS and 32 085 for PN (Section 2.1).

To illustrate the model goodness-of-fit, we present in Fig. 2 a histogram of the data (2-MOS + PN counts) compared to our best-fitting model. In this figure, as well as in Fig. 1, we present the data in terms of the 'normalized radius', which is the radial distance from the GRB afterglow position, divided by the square root of the normalized time (time measured from the GRB trigger, with $t = 1$ at the start of the *XMM* observation). This normalization serves to highlight the expanding haloes as distinct features, while not significantly altering the appearance of the afterglow itself (at small radius); the fall-off in counts at large radius is due to our region of interest cut ($r_{\max} = 311$ arcsec), as smeared out by the conversion to normalized units. We find the overall fit to be satisfactory.

Table 1. Fit parameters for GRB 031203 afterglow and expanding haloes.

Parameter	Median	σ	90 per cent confidence	
GRB x	0.46	0.14	0.23	0.70
y	0.19	0.13	-0.04	0.40
2-MOS counts	2252.7	60.3	2157.3	2355.6
PN counts	2340.0	125.5	2172.4	2559.2
Fading index, α_g	-0.557	0.056	-0.64	-0.46
Halo 1 t_0 (s)	11.0	416.9	-620.6	722.8
Fading index, α_h	-0.77	0.10	(-0.95)	-0.65
Halo 1 radius (arcsec)	145.04	0.70	143.74	146.07
Distance (pc)	871.7	9.7	855.6	886.8
2-MOS Counts	1365.0	67.8	1251.1	1467.4
PN Counts	1686.4	88.8	1568.4	1855.6
Halo 2 Radius (arcsec)	114.93	0.63	113.90	115.94
Distance (pc)	1388.	14.	1364.	1410.
2-MOS Counts	2059.3	65.5	1952.9	2164.1
PN Counts	2229.3	108.9	2041.6	2389.5
Halo 3 Radius (arcsec)	42.97	0.81	41.46	44.06
Distance (kpc)	9.94	0.39	9.32	10.52
2-MOS Counts	268.5	40.7	203.4	334.1
PN Counts	219.7	32.2	177.7	274.8

Note. Median, standard deviation about the median (σ), and 90 per cent confidence intervals are derived from model fits to 1000 bootstrap Monte Carlo realizations of the data set. PN and 2-MOS counts are defined as the total counts gathered from the PN (0.2 to 4.0 keV) and two MOS cameras (0.2 to 3.0 keV), respectively. Halo radii are defined as the radius in arcsec at the start of *XMM* observations. Halo distances are determined by the best-fitting halo radii and t_0 values from each trial and are not independent parameters. GRB coordinates (x, y) are offsets in arcsec from *XMM* physical coordinates (26 024.8, 23 674.1); the median offset of (0.46, 0.19) arcsec corresponds to RA 08^h02^m30^s.20, Dec. -39°51'02".8 (J2000) using the uncorrected *XMM* aspect. t_0 is the time of zero-radius for the expanding haloes relative to the *INTEGRAL* trigger time of 22:01:28 UT. The model enforces a hard limit $\alpha_h \geq -0.95$ which is encountered in 9 per cent of our bootstrap trials.

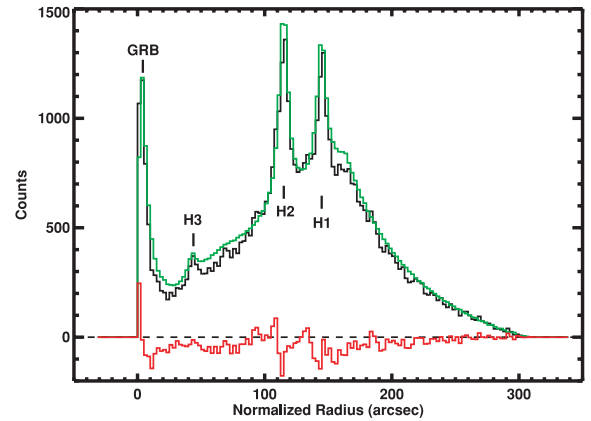


Figure 2. Histogram of 2-MOS + PN counts data (black), best-fitting model (green) and residuals (lower panel), binned according to normalized radius, r/\sqrt{t} . Contributions of the fading GRB afterglow (GRB) and the three expanding haloes (H1, H2, H3) are individually distinguishable; the fall-off in counts at large normalized radius is due to our region of interest cut ($r_{\max} = 311$ arcsec) as smeared out by the normalization procedure.

Specifically, although several systematic features are apparent in the residuals, we have investigated these without identifying any obvious model failings that might account for them. In particular, we have verified that none of the features are due to unaccounted-for bright point sources or instrument artefacts (e.g. hot pixels or

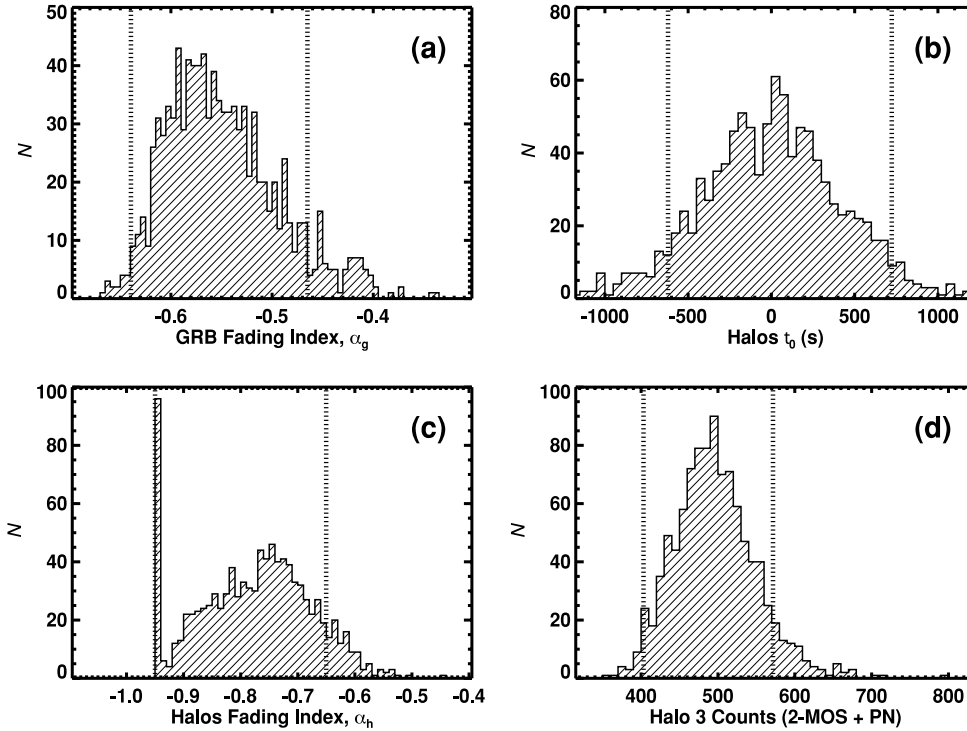


Figure 3. Approximate posterior distributions of selected model parameters, as derived from our bootstrap Monte Carlo analysis. (a) The GRB afterglow temporal fading index, α_g ; (b) t_0 for the expanding haloes, corresponding to the characteristic emission time of the X-ray blast, measured relative to the trigger time for GRB 031203; (c) the haloes temporal fading index, α_h (note this parameter is subject to a hard limit, $\alpha_h \geq -0.95$); (d) The sum of 2-MOS + PN counts for the expanding Halo 3, discovered in our analysis. 90 per cent confidence intervals for each parameter are indicated graphically (dotted lines).

hot columns). Variations in effective exposure within our region of interest, not accounted for by our simple vignetting model, may be responsible for some of these trends. Note that our fitting procedure is executed against the unbinned data, in three dimensions, and that this figure merely represents one possible projection of the model and data on to a one-dimensional space, for visualization purposes.

In Fig. 3, we present the full posterior distribution for four parameters of special interest: α_g , the temporal power-law index for the fading of the GRB afterglow; t_0 , the time of zero-radius for the expanding haloes; α_h , the temporal power-law index for the fading of the expanding haloes; and the sum of the 2-MOS and PN counts for Halo 3, the halo due to the most distant scattering dust sheet, discovered in our analysis. 90 per cent confidence intervals on each of these parameters are indicated.

3 DISCUSSION

In the sections below, we discuss the prior understanding of the GRB 031203 X-ray blast phenomenon and our own results as to the properties and timing of the event. We then discuss the full set of observations of GRB 031203 in the context of the known varieties of cosmic explosion, including XRF 060218.

3.1 Previous findings

V04 originally reported the discovery of two evolving dust-scattered X-ray haloes centred on GRB 031203 and made the first attempt to model the haloes and extract parameters of the X-ray blast and associated dust scattering sheets along the line of sight within the Milky Way. They constrained the time of burst for the inner and outer rings to be $t_0 = 2794^{+2765}_{-3178}$ and 2005^{+2512}_{-2867} s, respectively, relative

to the *INTEGRAL* trigger, and by applying a model of the dust properties, determined that the initial soft X-ray event had a power-law spectrum with photon index $\Gamma \approx 2$.

Subsequently, W04, W06 and TM06, using different analysis approaches, refined the details of this picture. They derived more precise distances to the dust sheets ($d_1 \approx 870$ pc and $d_2 \approx 1390$ pc, respectively), and correspondingly more precise constraints on the timing of the X-ray blast. The time constraint from W06 is $t_0 = 600 \pm 700$ s at 90 per cent confidence; the inferred power-law spectral index for the X-ray blast is reported as either $\Gamma = 2.5 \pm 0.3$ (W04) or $\Gamma = 2.1 \pm 0.2$ (TM06), results which are consistent at the $\approx 1\sigma$ level. The two conclusions as to the fluence of the X-ray blast over 1–2 keV, $F_X = 1.5(3) \times 10^{-6}$ erg cm $^{-2}$ (W06) versus $3.6(2) \times 10^{-7}$ erg cm $^{-2}$ (TM06), are more discrepant, which TM06 attribute to the authors' diverging assumptions on the relation between X-ray scattering optical depth and extinction.

In discussing their model for the dust properties, W06 show a plot of the halo fading (their Fig. 2) which corresponds to a temporal power-law index of $\alpha_h \approx -0.6$. They use this fact (along with the size of the scattering haloes) to derive a maximum size for the dust grains, $a_{\max} = 0.50 \pm 0.03$ μ m; in accounting for the effects of smaller grains they use a power-law size distribution with power-law index equal to -3.5 .

3.2 Discovery of a third halo

Our analysis reveals the presence of a third, faint halo (Figs 1 and 2) in addition to the two identified by previous authors. The presence of this halo is robust in our analysis, with associated total 2-MOS + PN counts between 403 and 572 at 90 per cent confidence (Fig. 3d),

and with roughly equal counts in the two sets of detectors, which are fit independently (Table 1).

Initially, we speculated that the third halo might be the signal of a second X-ray blast or early afterglow emission, with the X-rays scattering off one of the two previously known dust sheets. If this were the case, the third halo would necessarily be associated with a distinct (later) t_0 and would share the same distance d as one of the other haloes.

To investigate this possibility, we performed a distinct bootstrap Monte Carlo analysis of 600 trials, allowing t_0 for the third halo to vary while fixing $t_0 = 0$ for the two other haloes. Given the angular size of the halo at the start of the *XMM* observation, θ , t_0 is directly related to the distance to the dust scattering sheet by $d = 2c(t - t_0)/\theta^2$, where $t - t_0$ is the time from the burst to the start of *XMM* observations. This analysis yields a 90 per cent confidence interval for d_3 of 8597 to 10 503 pc, inconsistent with both $d_1 \approx 870$ pc and $d_2 \approx 1390$ pc. Hence, we conclude that the third halo is due to a third dust sheet and require t_0 to be identical, for all haloes, in our full analysis.

The third dust sheet is located at greater distance and greater Galactic radius ($R \approx 14.5$ kpc) than the two nearby sheets, and nominally at greater distance from the Galactic plane as well ($z \approx -0.83$ kpc), given the Galactic coordinates towards GRB 031203, $l = 255.7^\circ$ and $b = -4.8^\circ$. However, reconstructions of the three-dimensional distribution of neutral hydrogen density throughout the Milky Way (Kalberla et al. 2007; Kalberla & Dedes 2008) reveal a ‘warp’ in the disc which has an amplitude of $\delta z \approx 1$ kpc at $R = 15$ kpc, in precisely the ($-z$) sense needed to place the third dust sheet within or near the densest portion of the H I disc. The existence and location of the third dust sheet is not particularly surprising in this context.

Our final analysis yields 90 per cent confidence intervals on the distances to the dust sheets as follows: $d_1 = 856\text{--}887$ pc, $d_2 = 1364\text{--}1410$ pc and $d_3 = 9321\text{--}10\,523$ pc (Table 1); these uncertainties are inclusive of uncertainties in the value of t_0 itself, which we have not fixed (e.g. Fig. 3b). For Halo 1 and Halo 2, our values agree with those derived by W06 ($d_1 = 868^{+17}_{-16}$ pc and $d_2 = 1395^{+15}_{-30}$ pc) and TM06 ($d_1 = 870 \pm 5$ pc and $d_2 = 1384 \pm 9$ pc); note, however, that TM06 fix $t_0 = 0$ s in their analysis.

The median value of the halo fading index from our analysis is $\alpha_h = -0.77$ with a 90 per cent confidence interval $\alpha_h < -0.65$ that is unconstrained from below because of our hard limit ($\alpha_h \geq -0.95$); Fig. 3(c) presents the full posterior distribution. Our result appears mildly in conflict with the value $\alpha_h \approx -0.6$ from W06, with associated implications for the properties of the scattering dust particles (see also TM06); we have not investigated these implications in detail.

3.3 Properties of the GRB 031203 X-ray blast

Our analysis yields a 90 per cent confidence interval on the timing of the X-ray blast which places it within roughly ten minutes of the GRB 031203 trigger, between -621 and $+723$ s, with median value and standard deviation $t_0 = 11 \pm 417$ s; the full posterior distribution for t_0 is shown in Fig. 3(b).

Our constraint is comparable in precision to the best previous determination, $t_0 = 600 \pm 700$ s at 90 per cent confidence (W06). The shift of our confidence interval to earlier times sharpens the contrast between the implications of the *XMM* and *INTEGRAL* observations: the *INTEGRAL* pointing extends to 300 s after the GRB trigger, and only 22.5 per cent of our trials yield t_0 values consistent with this constraint. If we assume, in accordance with

standard GRB models, that the X-ray blast could not have preceded the gamma-ray trigger, then our upper limit of $t_0 < 545$ s at 90 per cent confidence is also relevant in this context.

An alternative approach to resolving this dilemma would reduce the peak flux of the X-ray blast – making it undetectable in the *INTEGRAL* observation – by extending its fluence over a significant time interval after the GRB. This hypothesis is also constrained by our analysis, since we observe no evidence that the haloes are radially resolved; our analytical model for unresolved ring-like sources (Appendix A) provides a satisfactory fit to the radial profiles of the haloes (Fig. 2).

While we have not constrained the duration of the X-ray blast directly, our results on the timing of the blast demonstrate that our model is sensitive to changes of $\delta t_0 \approx 600$ s, and we expect that, in demonstrating an adequate fit with the unresolved ‘ring’ model, we have constrained the duration of the blast at approximately this magnitude, $\delta t \lesssim 600$ s. Given that the *XMM* observation began $t = 22.2$ ks after the GRB trigger, a duration of 600 s corresponds to $\delta t/t = 3$ per cent or $\delta r_1 = 4$ arcsec for Halo 1, which would be most sensitive to these effects. Since the *XMM* PSF has a width of FWHM = 4 arcsec, it is reasonable to think the data would be sensitive to signs of broadening at this level. Moreover, the dust sheets themselves must be extended to some degree; any such extension would tend to further broaden the radial profile of the haloes, beyond any broadening due to the finite duration of the blast itself.

Finally, we note that W04 call attention to the relatively slow fading of the X-ray afterglow of GRB 031203 during the *XMM* observation, $\alpha_g \approx -0.55$ (Table 1), which was significantly flatter than the canonical $\alpha_x \approx -1.3$ familiar from *Beppo-SAX* observations at the time. In retrospect, we can identify this as one of the first observations of a ‘plateau phase’ in GRB X-ray afterglows, now familiar from *Swift* observations (e.g. Nousek et al. 2006; Racusin et al. 2009). As we have since learned, extrapolation of this slow decay to early times, $t \lesssim 1000$ s, is not particularly revealing of the likely X-ray flux at that time.

3.4 Could GRB 031203 and XRF 060218 be cosmic twins?

As mentioned in Section 1, a GRB similar to GRB 031203, as observed by *INTEGRAL*, could have occurred immediately prior to *Swift* observations of XRF 060218 without being observed by *Swift*. Likewise, a slow-evolving XRF similar to XRF 060218 could have occurred following GRB 031203 without being observed by *INTEGRAL*, if it exhibited either a sufficiently low-peak flux or peaked after $t = 300$ s, when *INTEGRAL* began its slew to a new pointing. In this scenario, then, both events would consist of two separate bursts, one peaking at gamma-ray energies and one peaking in the X-ray. Importantly, while we have evidence for distinct gamma-ray and X-ray emission episodes in GRB 031203, in this picture only the X-ray episode was observed from XRF 060218.

A variation on this scenario, without any separate episode of gamma-ray emission from XRF 060218, was proposed by Ghisellini et al. (2006). These authors were concerned with the question of whether the prompt high-energy emissions of GRB 031203 and GRB 980425 could be reconciled with the high-energy relationships between peak energy and isotropic-equivalent energy output that are observed among the $z \gtrsim 1$ GRB population. They concluded, as we do here, that the observation of XRF 060218 by *Swift* suggests strongly that a similar phenomenon might be responsible for the ‘X-ray blast’ inferred from the expanding X-ray haloes of GRB 031203.

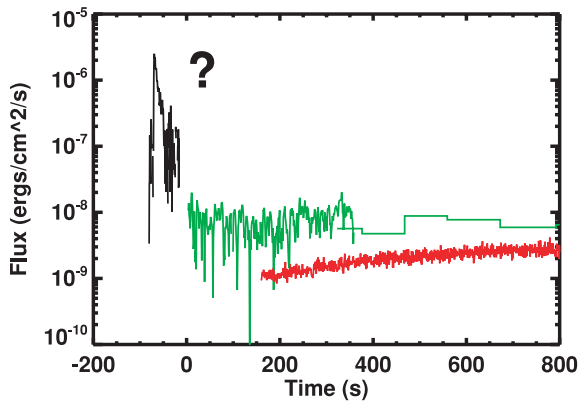


Figure 4. Superposition of XRF 060218 *Swift* BAT (green, 15–350 keV) and XRT (red, 0.3–10 keV) light curves with a hypothetical precursor event having properties identical to GRB 031203 (black, 20–200 keV). The start of *Swift* BAT observations defines $t = 0$; the *INTEGRAL* light curve of GRB 031203 is obtained from SLS04, rebinned, corrected for redshift and time dilation, and offset by -70 s to represent its occurrence prior to the start of *Swift* observations. Negative (background-subtracted) flux measurements are not shown on this logarithmic plot.

To illustrate the nature of our proposal, we have generated a figure that superimposes the prompt-emission light curve of GRB 031203 (SLS04), appropriately rescaled to the redshift and luminosity distance of XRF 060218, on the *Swift* BAT and XRT light curves of the prompt and early emission of XRF 060218 (Fig. 4). With $t = 0$ at the beginning of *Swift* observations, it is necessary to offset the *INTEGRAL* light curve to earlier times in order to avoid detection by the *Swift* BAT; we arbitrarily choose a -70 s offset. The BAT light curve is a combination of the mask-weighted light curve generated by the BAT standard analysis ($t < 350$ s) with data from Campana et al. (2006) (64 s integrations at $t > 350$ s). The XRT light curve (red) for XRF 060218 is obtained from the *Swift*/XRT light curves repository at the University of Leicester² (Evans et al. 2007, 2009).

As Fig. 4 shows, this ‘cosmic twin’ to GRB 031203 would have exhibited a peak flux of $2.7 \times 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1}$ (20–200 keV), roughly 11 times brighter than the original event, which occurred at about three times the distance of XRF 060218. This addresses an important condition on our hypothesis, which is that the burst not be bright enough to trigger detectors of the Interplanetary Network (IPN; e.g. Hurley et al. 2006), and in particular, Konus-WIND (Aptekar et al. 1995), which did not report any trigger associated with XRF 060218. A review of the catalogue of Konus-WIND short burst detections³ reveals that the peak flux distribution peaks at $7.5 \times 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1}$ (15–2000 keV), although bursts are detected down to a minimum peak flux of $2 \times 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1}$. Depending on the high-energy spectrum of GRB 031203, the peak flux of its hypothetical $z = 0.033$ cosmic twin would be from 5.2×10^{-6} to $7.9 \times 10^{-6} \text{ erg cm}^{-2} \text{ s}^{-1}$ (15–2000 keV), likely below the regime of maximum sensitivity for this experiment. With a peak flux falling close to threshold in the brightest case, relatively slight variations in the profile or spectrum of the event would be sufficient to render the burst undetectable.

An associated ‘classical GRB’ like GRB 031203 thus seems plausible; however, our analysis indicates that the X-ray properties of the

two events must differ to some degree. First, we have constrained the timing of the GRB 031203 X-ray blast to fall within $t_0 = 11 \pm 417$ s of the GRB ($t_0 < 545$ s at 90 per cent confidence; Section 3.3), whereas the XRT light curve for XRF 060218 rises slowly until it peaks at $t \approx 1000$ s. Secondly, depending on the assumed relation between extinction and X-ray scattering optical depth (TM06), the GRB 031203 X-ray blast may have released as much as three times the energy of the prompt X-ray emission of XRF 060218 (Table 2).

To illustrate these constraints in detail, and explore other possible explanations for the GRB 031203 X-ray blast, Table 2 presents the gamma-ray and X-ray properties of all five confirmed spectroscopic GRB-SNe, and the properties of two other cosmic explosions that may be interesting in this context, GRB 050502B and SN 2008D.

First, we consider the ‘cosmic twins’ hypothesis, comparing the properties of GRB 031203 and XRF 060218. A distinct, earlier episode of high-energy emission for XRF 060218 is readily accommodated by the total gamma-ray energy budget of GRB 031203; in order to avoid detection by various IPN experiments, the peak luminosity, peak energy or total energy release should be somewhat reduced from GRB 031203 values. The X-ray blast from GRB 031203 released more energy in the X-ray than XRF 060218; however, depending on the assumed relation between extinction and X-ray scattering optical depth, the additional 25 per cent (TM06) to 200 per cent (W06) energy requirement is not necessarily severe. Finally, the time delay from burst trigger to X-ray peak (technically, to the characteristic time of emission) should be at least twice as long for XRF 060218 as for GRB 031203. None of these requirements appear to us prohibitive; collectively, however, they restrict the allowed parameter range of models considerably. The main appeal to satisfying these constraints, in an Occam’s Razor sense, is the requirement for one fewer *sui generis* type of GRB-SN.

Next, we consider the X-ray flare explanation for the GRB 031203 X-ray blast (W06). Even at the maximum inferred energy of $4 \times 10^{49} \text{ erg}$ (2–10 keV), the ratio of X-ray to gamma-ray energies for GRB 031203 would be no greater than observed in GRB 050502B. Moreover, the relative timing of the gamma-ray and X-ray episodes is very close to what is needed to satisfy both our time constraint, derived from the expanding haloes, and the *INTEGRAL* slew time constraint, $t > 300$ s. The latter applies to any relatively short (hence high-peak flux) model for the X-ray blast. At the same time, the X-ray flare from GRB 050502B remains an extreme example of its class (e.g. Chincarini et al. 2007), and in this model the GRB 031203 X-ray blast would be a similarly extreme case; in addition, it would be the only known example of an X-ray flare from a GRB-SN.

None of the other cosmic explosions listed in Table 2 are known to exhibit strong distinct episodes of gamma-ray and X-ray emission. Their properties are presented to illustrate the diversity of the GRB-SN phenomenon, and the gamma-ray and X-ray luminosities that may generally be expected from these events on the basis of past experience. Thus, a prompt high-energy emission episode in XRF 060218 like those seen from SN 1998bw or XRF 020903, and occurring prior to the start of *Swift* observations, would certainly have fallen below IPN threshold, and if present, would help to homogenize the GRB-SNe as a class.

Finally, we note that, given that SN 2008D manifests as an ordinary Type Ibc SN, and that rate estimates for SN 2008D-type X-ray outbursts are consistent with the total rate of Type Ibc SNe (Soderberg et al. 2008), it seems likely that $E_{X,\text{iso}} \approx 2 \times 10^{46} \text{ erg}$ represents a lower bound on the prompt X-ray energy output of any GRB-SN.

² *Swift*/XRT light curves repository: http://www.swift.ac.uk/xrt_curves/.

³ Konus-WIND short burst catalogue: <http://www.ioffe.ru/LEA/shortGRBs/Catalog/>.

Table 2. Properties of GRB 031203 and related cosmic explosions.

	Event	z	$\log E_{\gamma, \text{iso}}$	$\log E_{X, \text{iso}}$	$\Delta t_{X-\gamma}$	Interpretation	Refs
SN	1998bw	0.0085	47.8	47.1	+10	GRB-SN Ic	1,2
XRF	020903	0.251	48.5	48.9	0	XRF-SN Ic	3,4
GRB	030329	0.167	52.0	51.1	+3	GRB-SN Ic	5
GRB	031203	0.105	50.0	49.2–49.6	± 600	GRB-SN Ic + ?	6,7,8,9
GRB	050502B	(2.0)	51.4	51.0	+250	GRB + X-ray flare	10,11
XRF	060218	0.033	49.4	49.1	+1050	XRF-SN Ic + shock	12
SN	2008D	0.0065	<46.5	46.3	–	SN Ic shock	13

Note. $\log E_{\gamma, \text{iso}}$ is the isotropic-equivalent gamma-ray energy over 20 keV to 2 MeV, rest frame (ergs); $\log E_{X, \text{iso}}$ is the isotropic-equivalent prompt X-ray energy over 2 to 10 keV, rest frame (ergs); and $\Delta t_{X-\gamma}$ is the mean or estimated delay between gamma-ray and X-ray emission, rest frame (s). A redshift of $z = 2$ is assumed for GRB 050502B; no redshift-related corrections are applied for SN 1998bw and SN 2008D. The two values of $\log E_{X, \text{iso}}$ for GRB 031203 are from TM06 (49.2) and W04 (49.6), respectively. The X-ray energy and time delay for XRF 060218 are derived by analysis of the XRT light curve from the *Swift* XRT light curves repository (Evans et al. 2007, 2009). The upper limit on $E_{\gamma, \text{iso}}$ for SN 2008D is extrapolated from the *Swift* BAT upper limit using an $\alpha = 1.5$ power-law spectrum. References: (1) Galama et al. (1998); (2) Frontera et al. (2000); (3) Soderberg et al. (2004a); (4) Sakamoto et al. (2004); (5) Vanderspek et al. (2004); (6) SLS04; (7) W04; (8) TM06; (9) this work; (10) Burrows et al. (2005); (11) Sakamoto et al. (2008); (12) Campana et al. (2006); (13) Soderberg et al. (2008).

4 CONCLUSIONS

The expanding X-ray haloes observed by *XMM-Newton* in association with GRB 031203 provide a rare opportunity to study the prompt soft X-ray properties of a nearby GRB-SN (V04). We have undertaken a detailed investigation of these haloes in hopes of clarifying the nature of the ‘X-ray blast’ (W06) whose dust-scattered X-rays are responsible for the haloes.

In agreement with previous authors (V04; W04; W06; TM06), we find that the properties of the haloes are consistent with an X-ray blast occurring simultaneously with the GRB and subsequently scattering off dust sheets at distances $d_1 = 871.7 \pm 9.7$ pc and $d_2 = 1388 \pm 14$ pc away in the plane of the Milky Way. In addition, we discover a third expanding halo that reveals scattering by a third dust sheet at distance $d_3 = 9.94 \pm 0.39$ kpc; the presence of this third halo is a robust feature of our model as applied to the independent EPIC-MOS and EPIC-PN data sets, and due to the warping of the Milky Way disc, its location is well within the densest H I regions of the outer disc.

Our constraints on the timing of the X-ray blast, $t_0 = 11 \pm 417$ s by comparison to the GRB 031203 trigger time ($t_0 < 545$ s at 90 per cent confidence), are comparable in precision to the previous best constraint (W06) while suggesting a greater degree of synchronization between the GRB and the X-ray blast. Combined with upper limits from *INTEGRAL* observations of GRB 031203, which extend to $t \approx 300$ s after trigger, our findings significantly restrict the parameter space of allowed models for the X-ray blast.

We explore the implications of our findings, in reference to the properties of all other spectroscopically confirmed GRB-SNe, and conclude that two alternative interpretations seem possible. On the one hand (Ghisellini et al. 2006), the X-ray blast may be the signature of a high-energy ‘shock breakout’ event, as observed from XRF 060218; in this case, the event should exhibit roughly $\times 2$ faster evolution and 25 to 200 per cent greater X-ray energy than the XRF 060218 event. Alternatively (W06), the X-ray blast may be the signature of an X-ray flare that occurred during the interval $300 < t_0 < 545$ s; if so, the ratio of X-ray flare to GRB energies for GRB 031203 would be high but not unprecedented.

We point out that observations of XRF 060218 by *Swift* and IPN experiments allow for the presence of a ‘precursor’ GRB as lumi-

nous as (or more likely, somewhat less luminous than) GRB 031203. In this case, a shock-breakout origin for the GRB 031203 X-ray blast would make GRB 031203 and XRF 060218 ‘cosmic twin’ explosions with nearly identical high-energy properties. If GRB 031203 and XRF 060218 thus represent two facets of a single type of GRB-SN, detection and observation of future nearby GRB-SNe by *Swift* can be expected to yield additional examples of these events.

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APPENDIX A: RING SURFACE BRIGHTNESS

In this appendix, we develop a semi-analytical treatment of the surface brightness of ring-type sources as observed with *XMM*. The results are incorporated into the numerical model of expanding dust-scattered haloes which is used in the main text to analyse the soft X-ray properties of GRB 031203.

We begin with the angularly symmetric expression for the *XMM* PSF (equation 1 in the main text), expressed as a King function:

$$s_p(r) = \frac{\alpha - 1}{\pi r_c^2} (1 + r^2/r_c^2)^{-\alpha}, \quad (\text{A1})$$

where r is the radial coordinate and the PSF parameters, core radius r_c and scaling index α are functions of the incident photon energy and exhibit slightly different functional forms for the MOS-1, MOS-2 and PN detectors; representative values for the parameters are $r_c = 4.9$ arcsec and $\alpha = 1.44$. As expressed here, the surface brightness distribution is normalized to unit integral over $0 < r < \infty$ as long as $\alpha > 1$. We note that the *XMM* PSF is known not to be angularly symmetric and that this form is adopted strictly as a useful approximation to the actual PSF form.

Fig. A1 serves to define the coordinate system we use in our analysis. We seek to describe the surface brightness at the point x , due to a ring of radius R , as an integral over contributions from various angles, $0 \leq \phi < 2\pi$, where the angular coordinate is defined from the perspective of the ring centre. Owing to the angularly symmetric nature of the PSF, we may take the point x to lie on the x -axis, as shown, without loss of generality.

The integral we wish to solve is thus expressed as

$$s_r(x) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\alpha - 1}{\pi r_c^2} (1 + r^2/r_c^2)^{-\alpha} d\phi, \quad (\text{A2})$$

where the squared distance r^2 from the point x to the ring element at angle ϕ can be derived by simple trigonometry as $r^2 = R^2 + x^2 - 2Rx \cos \phi$. Inserting this definition into the expression for the surface brightness, and factoring out all coefficients on the $\cos \phi$ term, we find

$$s_r(x) = \frac{\alpha - 1}{2\pi^2} r_c^{2(\alpha-1)} (2Rx)^{-\alpha} \int_0^{2\pi} \left(\frac{r_c^2 + R^2 + x^2}{2Rx} - \cos \phi \right)^{-\alpha} d\phi, \quad (\text{A3})$$

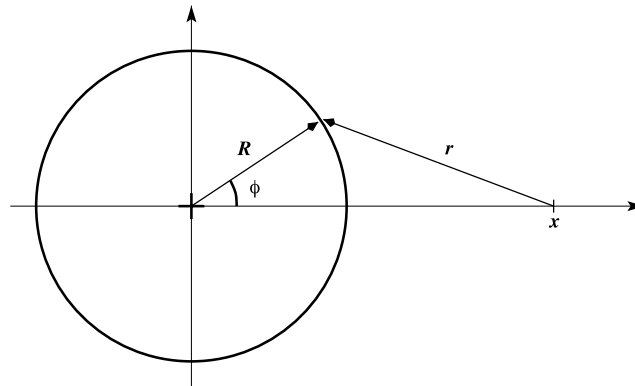


Figure A1. Coordinate definitions for analytical treatment of the surface brightness of ring-like sources, as observed with *XMM*. The ring has angular radius R , and we seek an expression for the surface brightness an angular distance x from the ring centre, which we can choose to lie on the x -axis, as here, without loss of generality in an angularly symmetric treatment. In analysing the contribution from a particular differential segment of the ring, at angular coordinate ϕ , we define the distance from the segment position to the point x to be r .

where the integral is now in a form that can be solved by use of one of the hypergeometric functions, ${}_2F_1$. In particular, if we define $A \equiv (r_c^2 + R^2 + x^2)/2Rx$, then

$$\int_0^{2\pi} (A - \cos \phi)^{-\alpha} d\phi = \pi(A-1)^{-\alpha} {}_2F_1\left(\frac{1}{2}, \alpha, 1, \frac{-2}{A-1}\right) + \pi(A+1)^{-\alpha} {}_2F_1\left(\frac{1}{2}, \alpha, 1, \frac{2}{A+1}\right), \quad (\text{A4})$$

where we make use of the fact that $A > 1$ in the present case (since r_c , R and x are all positive real numbers). The hypergeometric function ${}_2F_1$ has a series expansion that is valid and convergent over the full domain of interest; however, for the sake of computational speed we take two distinct approaches in our numerical calculations.

In the first case, near the ring centre where $x \ll R$, we have $A \gg 1$, and the fourth argument of the ${}_2F_1$ function in each term of the integral solution is small. In this case, we expand each term to second order in the fourth argument:

$${}_2F_1\left(\frac{1}{2}, \alpha, 1, z\right) = 1 + \frac{\alpha z}{2} + \frac{3}{16} \alpha(\alpha+1)z^2 + \dots, \quad (\text{A5})$$

so that, keeping terms to second order in $1/A$ throughout, the overall expression for the ring surface brightness becomes

$$s_{r2}(x) = \frac{\alpha-1}{\pi} r_c^{2(\alpha-1)} (r_c^2 + R^2 + x^2)^{-\alpha} \left[1 + \frac{\alpha(\alpha+1)}{4A^2} \right]. \quad (\text{A6})$$

This is our expression for the ring surface brightness in the second-order approximation.

For x values near or beyond the ring, $x \gtrsim R$, with $R \gg r_c$ still holding, we take a different approach. Writing $x = R + \Delta$, we expand the integral expression for the ring surface brightness to second order in Δ/R :

$$s_{rn}(x) = \frac{\alpha-1}{2\pi^2} r_c^{2(\alpha-1)} (2Rx)^{-\alpha} \int_0^{2\pi} \left(1 + \frac{\Delta^2}{2R^2} + \frac{r_c^2}{2R^2} - \cos \phi \right)^{-\alpha} d\phi. \quad (\text{A7})$$

The integrand in this expression consists of a single, strictly positive, parenthetical term that is taken to the $(-\alpha)$ power. In the case of the *XMM* PSF model, $\alpha \approx 1.5 > 1$. Thus, the integral will be dominated by contributions from the regime where the parenthetical term is near its minimum. Or, to take a more practical perspective, the bulk of the surface brightness will be contributed by the relatively nearby portions of the ring that have $\phi \ll 1$. Separately, we note that the integral limits may be changed from $0 < \phi < 2\pi$ to $-\pi < \phi < \pi$ without effect.

We then make two further approximations, expanding to second order in ϕ , $\cos \phi \approx 1 - \frac{1}{2}\phi^2$, and increasing the now-symmetrical limits of integration from $\pm\pi$ to $\pm\infty$. The expression for the near-ring surface brightness then becomes

$$s_{rn}(x) = \frac{\alpha-1}{2\pi^2} r_c^{2(\alpha-1)} (2Rx)^{-\alpha} \int_{-\infty}^{\infty} \left(\frac{\Delta^2 + r_c^2}{2R^2} + \frac{1}{2}\phi^2 \right)^{-\alpha} d\phi, \quad (\text{A8})$$

where we have avoided expanding x to $R + \Delta$ in the $(2Rx)$ term only. Making a change of variables to $\psi = \phi \sqrt{R^2/(\Delta^2 + r_c^2)}$ leads to the simplified form:

$$s_{rn}(x) = \frac{\alpha-1}{2\pi^2} r_c^{2(\alpha-1)} (2Rx)^{-\alpha} \sqrt{2} \left(\frac{\Delta^2 + r_c^2}{2R^2} \right)^{-\alpha+1/2} \int_{-\infty}^{\infty} (1 + \psi^2)^{-\alpha} d\psi, \quad (\text{A9})$$

where the integral is now in a form that can be solved via the Euler gamma function:

$$\int_{-\infty}^{\infty} (1 + \psi^2)^{-\alpha} d\psi = \frac{\sqrt{\pi} \Gamma(\alpha - \frac{1}{2})}{\Gamma(\alpha)}. \quad (\text{A10})$$

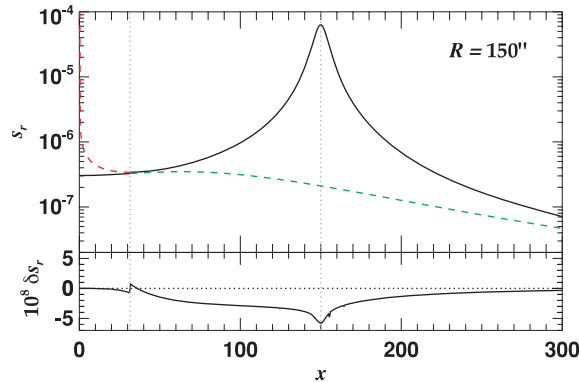


Figure A2. Example surface brightness profile for a ring-like source, and comparison of our approximate approach to a full numerical integration. Upper panel: surface brightness profile for a ring of radius $R = 150$ arcsec convolved with the azimuthally symmetric approximation of the *XMM* PSF (equation A1). Two distinct approximations discussed in the text, $s_{r2}(x)$ and $s_{rm}(x)$, are employed. The s_{r2} approximation is used close to the ring centre ($x \leq x_{\text{sep}}$), and is plotted as a dashed green line beyond this domain. The s_{rm} approximation is used for $x > x_{\text{sep}}$, and is plotted as a dashed red line near the ring centre; in the case shown, $x_{\text{sep}} \approx 31.5$ arcsec. Lower panel: residuals of the approximate approach, as compared to a full numerical integration of equation (A3).

As a result, the final expression the ring surface brightness in our ‘near-ring’ approximation is

$$s_{rn}(x) = \frac{(\alpha - 1)}{\pi^{3/2}\sqrt{2}} r_c^{2(\alpha-1)} (2Rx)^{-\alpha} \left[\frac{(x - R)^2 + r_c^2}{2R^2} \right]^{-\alpha+1/2} \left[\frac{\Gamma(\alpha - 1/2)}{\Gamma(\alpha)} \right]. \quad (\text{A11})$$

It remains for us to determine when to use each approximation. In our code, we use one or the other approximation depending on the comparison of the value of x to the ‘separating value’, x_{sep} :

$$x_{\text{sep}} = \frac{2\alpha r}{4\alpha - 1} \left[1 - \frac{\sqrt{1 - (4\alpha - 1)(1 + r_c^2/r^2)}}{2\alpha} \right], \quad (\text{A12})$$

with the test applying as follows:

$$s_r(x) = \begin{cases} s_{r2}(x), & x \leq x_{\text{sep}}; \\ s_{rn}(x), & x > x_{\text{sep}}. \end{cases} \quad (\text{A13})$$

An illustrative comparison of our numerical approach to a full numerical integration of equation (A3) is shown in Fig. A2. The systematic underprediction of the true surface brightness in the s_{rn} approximation is remedied when we renormalize the ring surface brightness over the active area of the detector, via a numerical integration. Overall, we find this level of accuracy to be sufficient to our purposes; moreover, as discussed in the main text, our use of a bootstrap Monte Carlo approach means that any inadequacies in our modelling should result in conservative estimates of parameter uncertainties.

This paper has been typeset from a \LaTeX file prepared by the author.