Robot Dynamics Exercise Session 03

Robotic Systems Lab

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previous weeks: Ex. 1 a – forward kinematics $\{\vec{r}_{IE}, C_{\mathcal{I}\mathcal{E}}\} = \mathrm{FK}(\vec{q})$ unique mapping

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	Ex. 1 b – differential forward kinematics	$\{\vec{v}_{IE},\, \vec{\omega}_{\mathcal{IE}}\} = \mathrm{DFK}(\vec{q},\, \dot{\vec{q}}) = J(\vec{q})\, \dot{\vec{q}}$	unique mapping

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today: Ex. 1 c – inverse kinematics $\vec{q} = \mathrm{IK}\left(\vec{r}_{IE},\, C_{\mathcal{IE}}\right)$ non-unique mapping

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1) IK & DIK

Let $\vec{q} \in \mathbb{R}^n$ be the joint variables, e.g., $q_1,\, q_2,\, \dots$

Let $\vec{\chi} \in \mathbb{R}^m$ be the task variables, e.g., $\vec{r}_{IE},\, C_{\mathcal{I}\mathcal{E}},\, \dots$

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IK problem: find $\vec{q}$ s.t. $\vec{\chi}_d = \vec{f}(\vec{q})$

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DIK problem: find $\dot{\vec{q}}$ s.t. $\dot{\vec{\chi}}_d = J(\vec{q})\,\dot{\vec{q}} \longrightarrow \dot{\vec{q}} = J^{+}(\vec{q})\,\dot{\vec{\chi}}_d$

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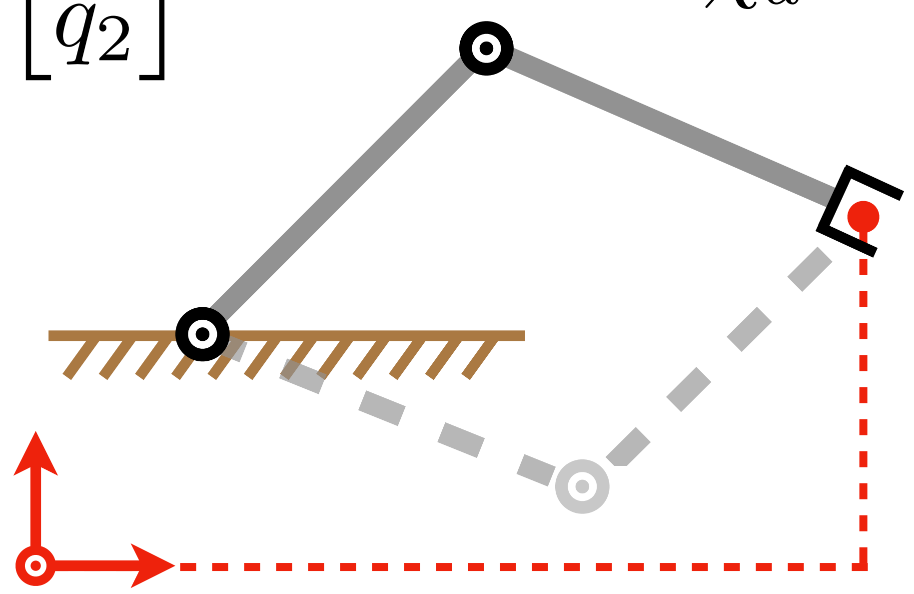

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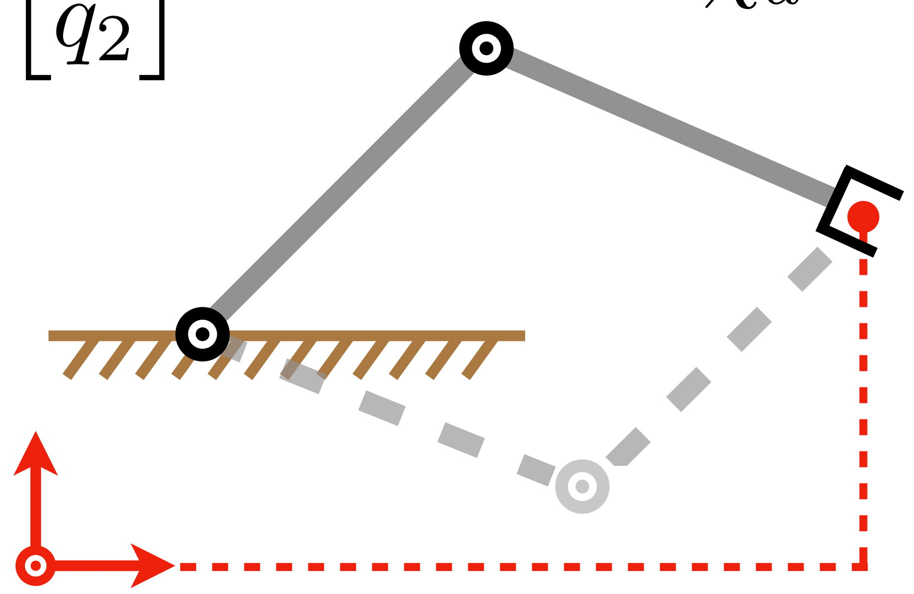

non unique but finitely many solutions for IK

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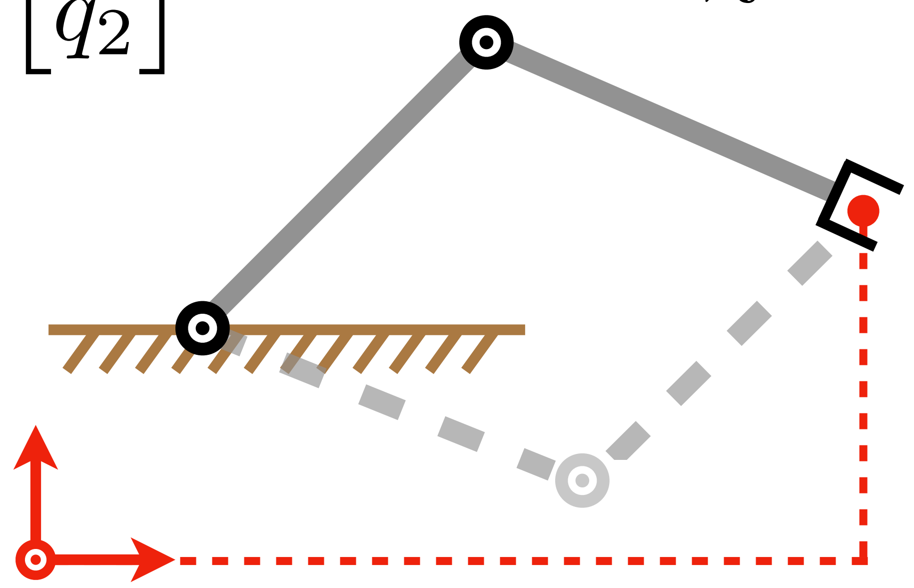

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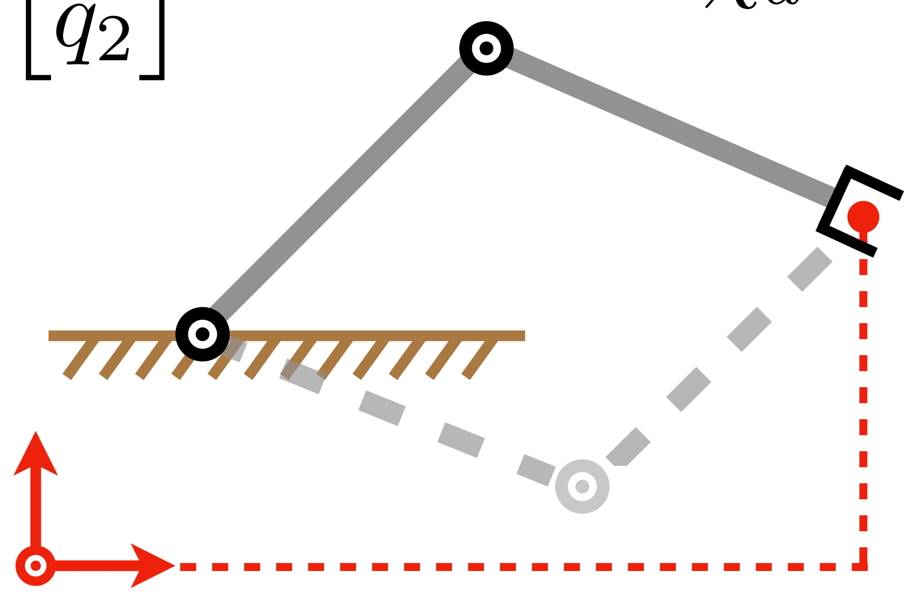

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J^{+} is just J^{-1} as it is square

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<u>Case 2:</u> $m < n$ (redundant)

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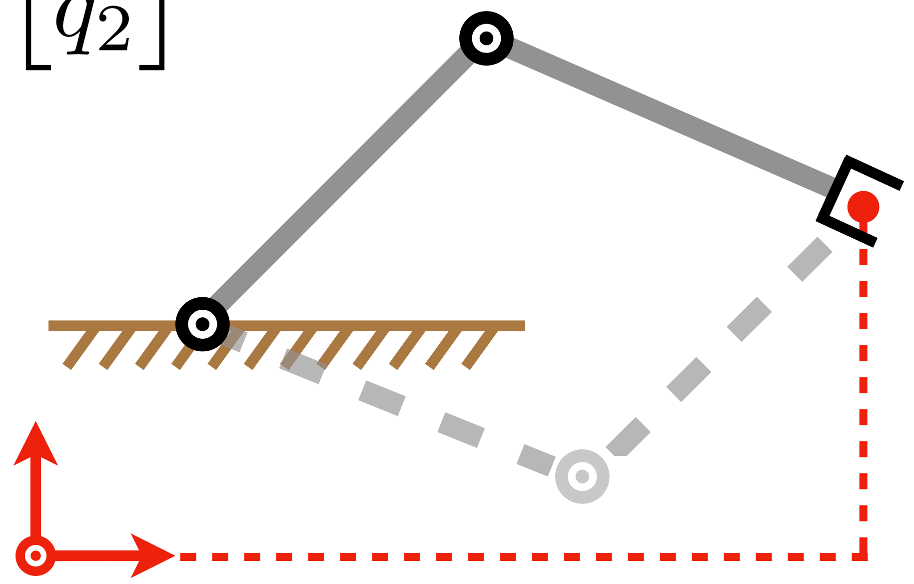

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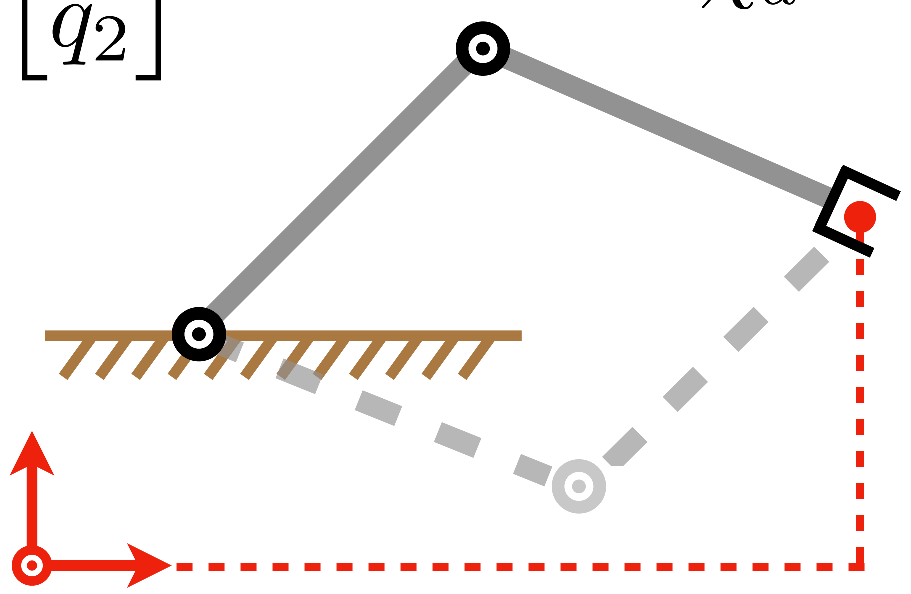

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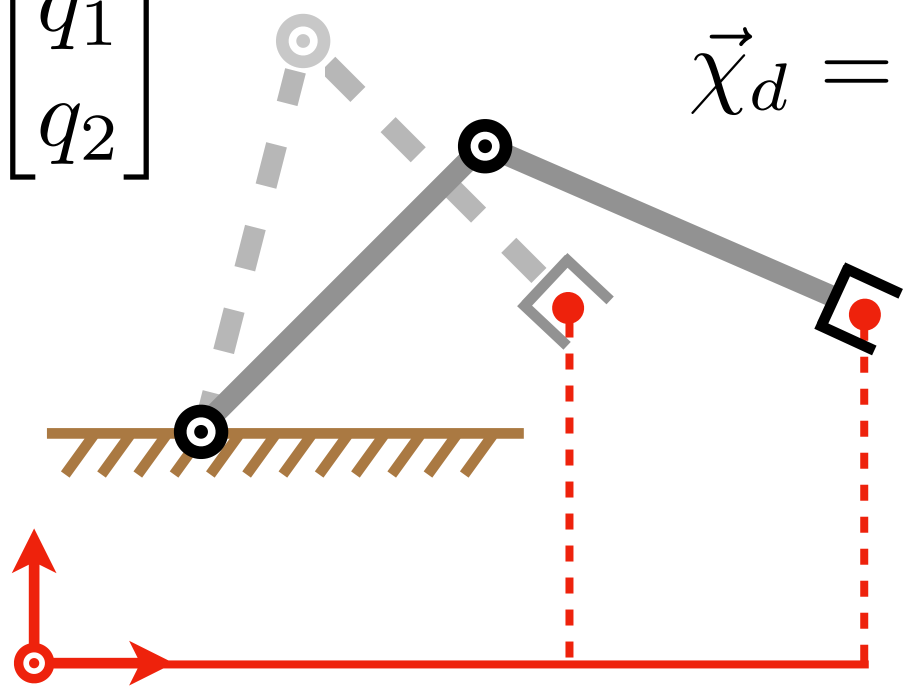

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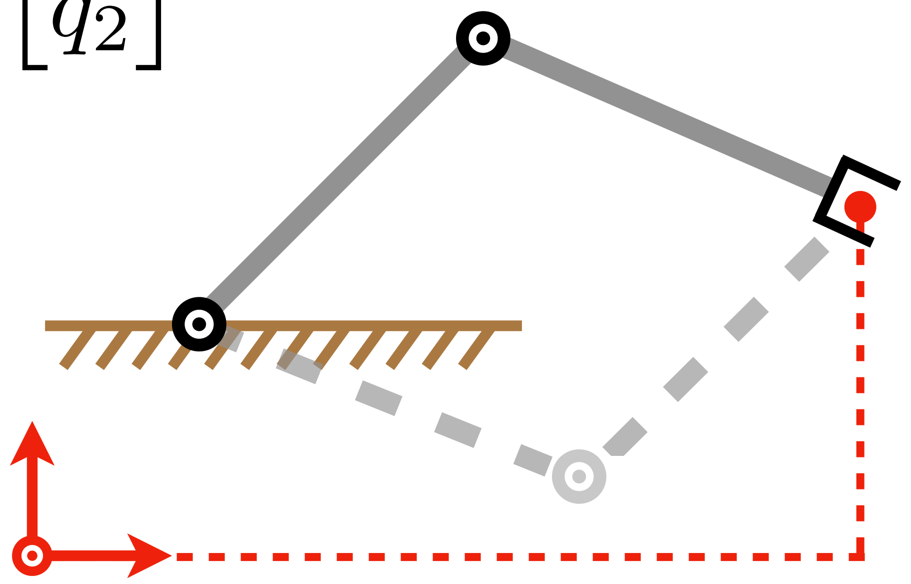

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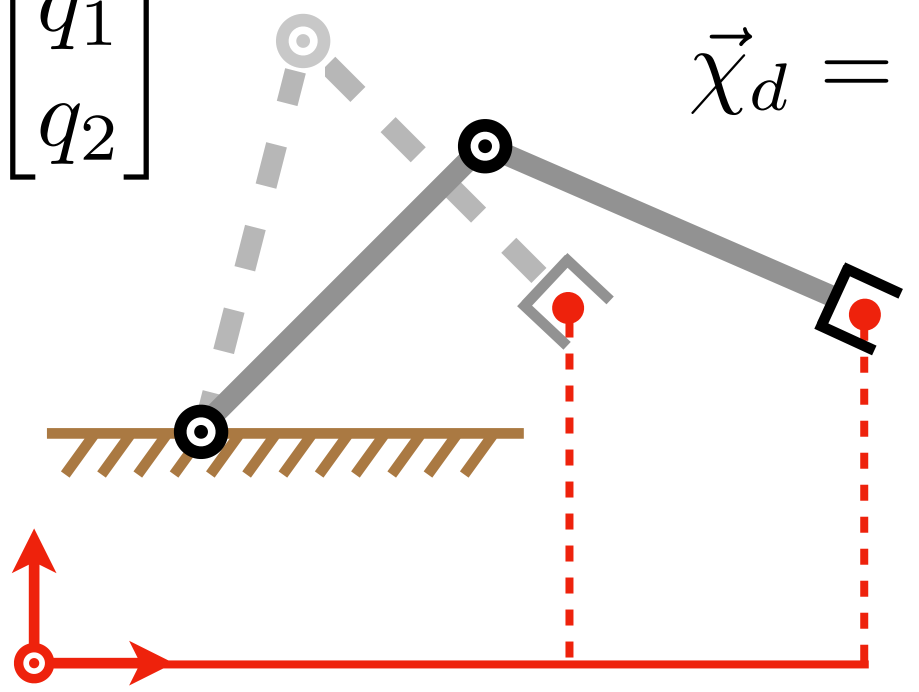

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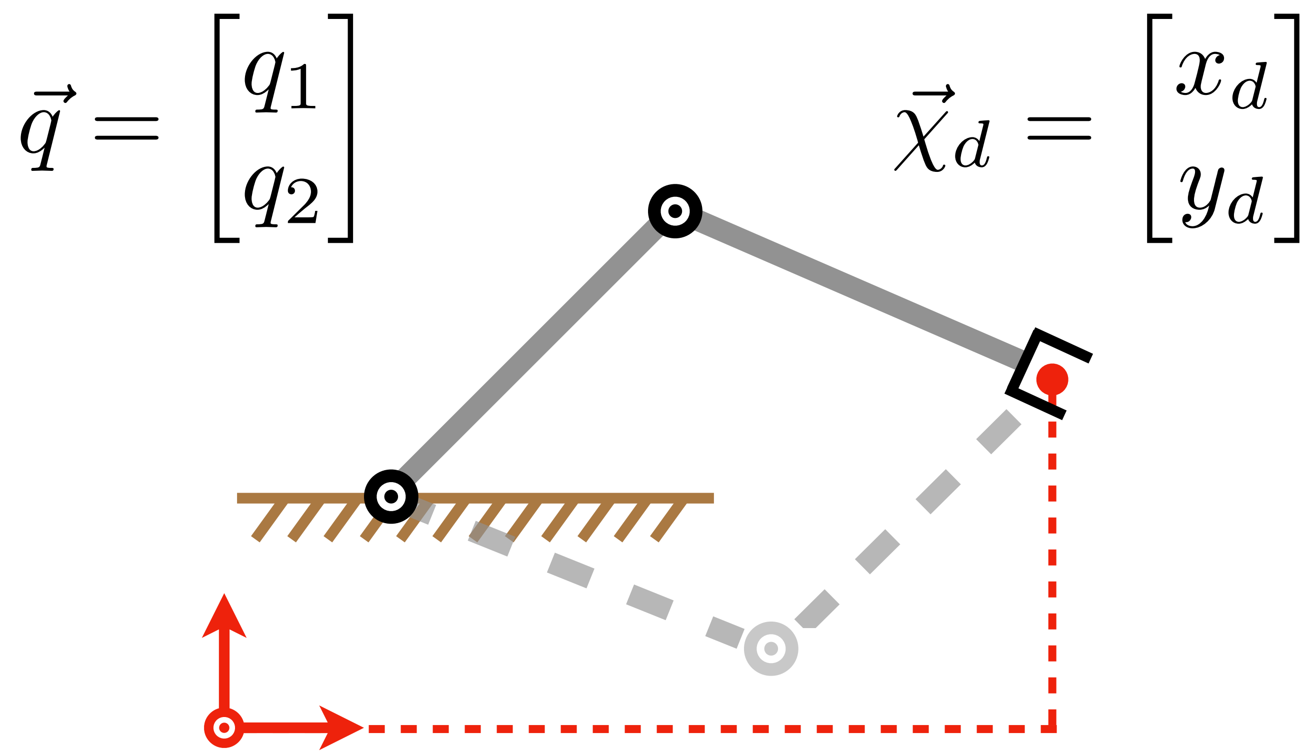

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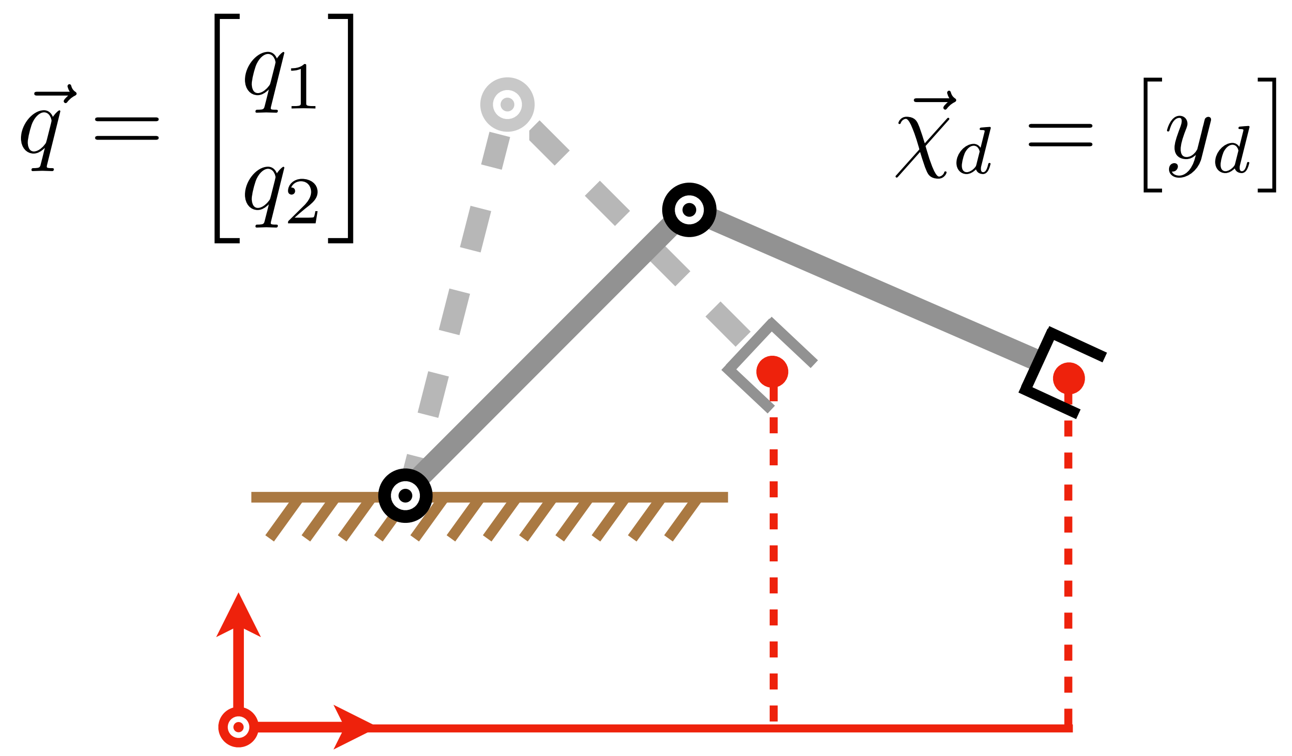

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$$\boxed{J^+ = J^\top \left(JJ^\top\right)^{-1}}$$

right pseudo-inverse

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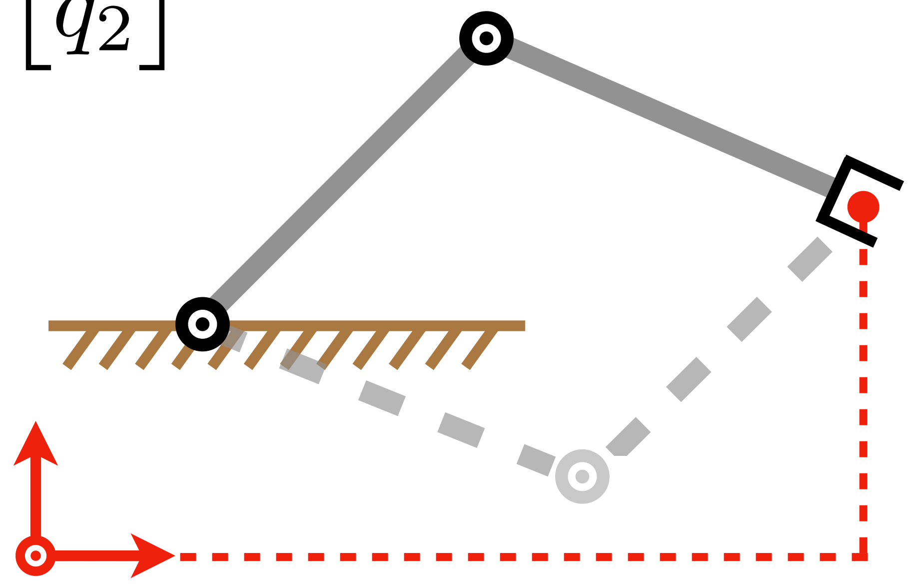

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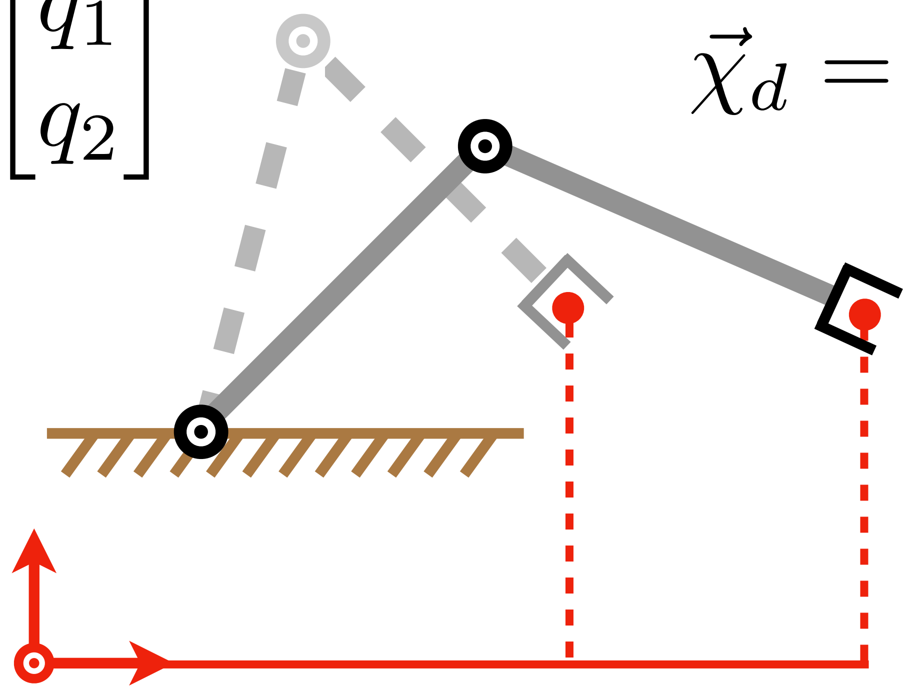

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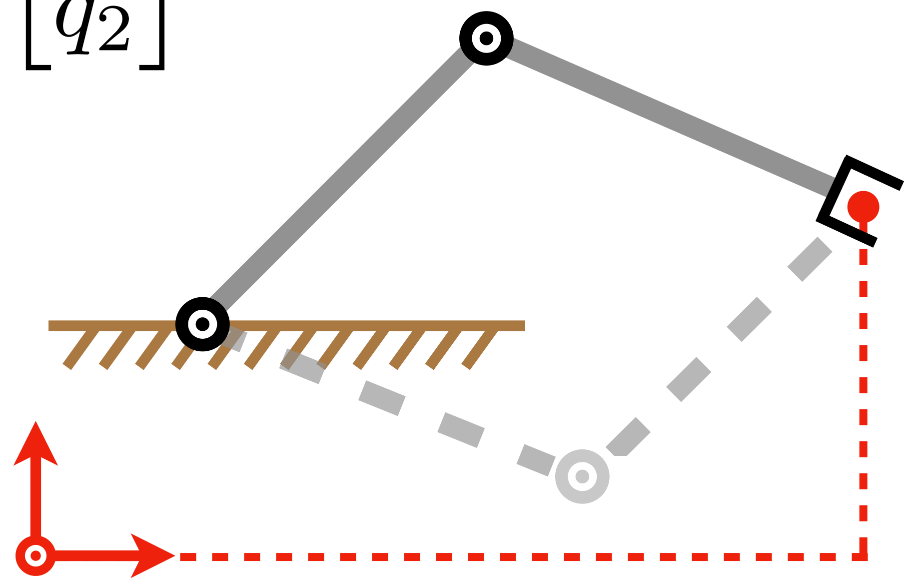

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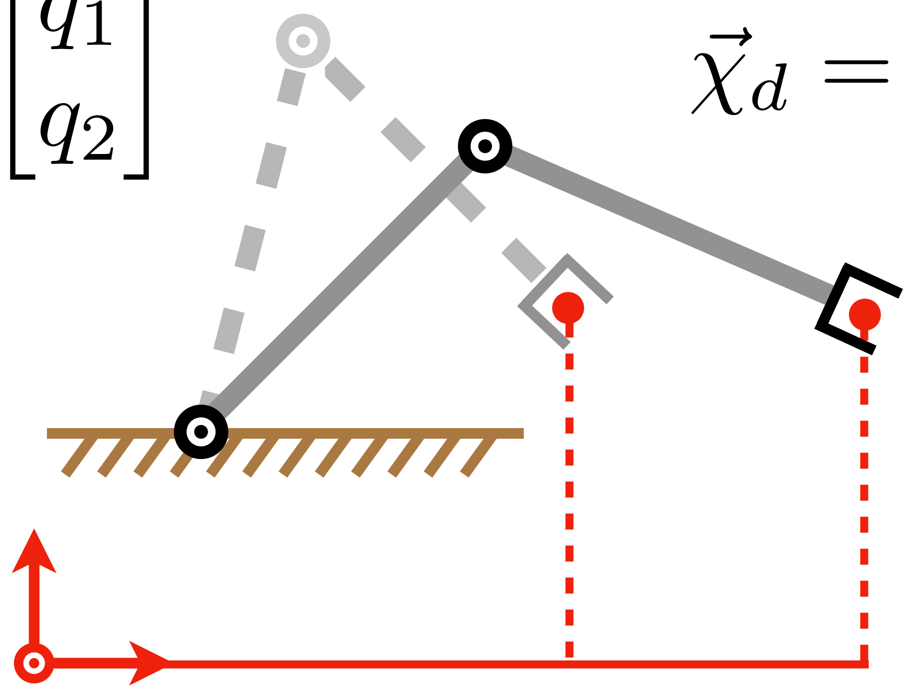

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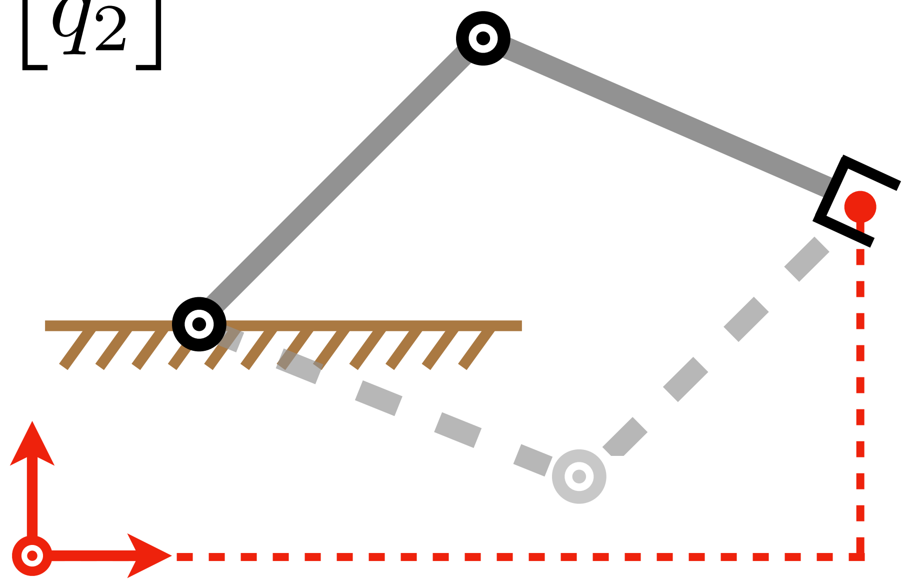

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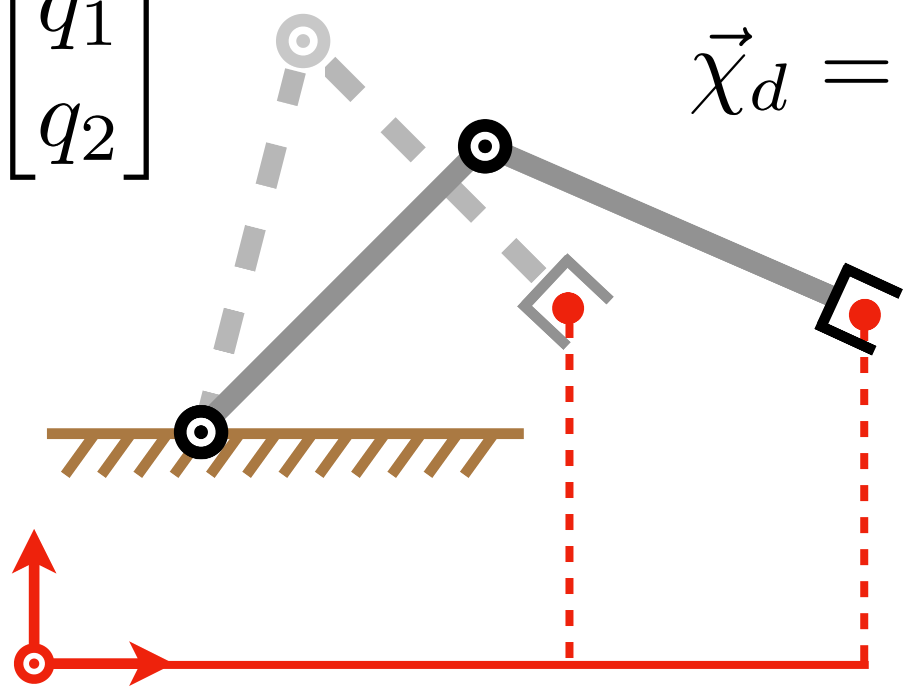

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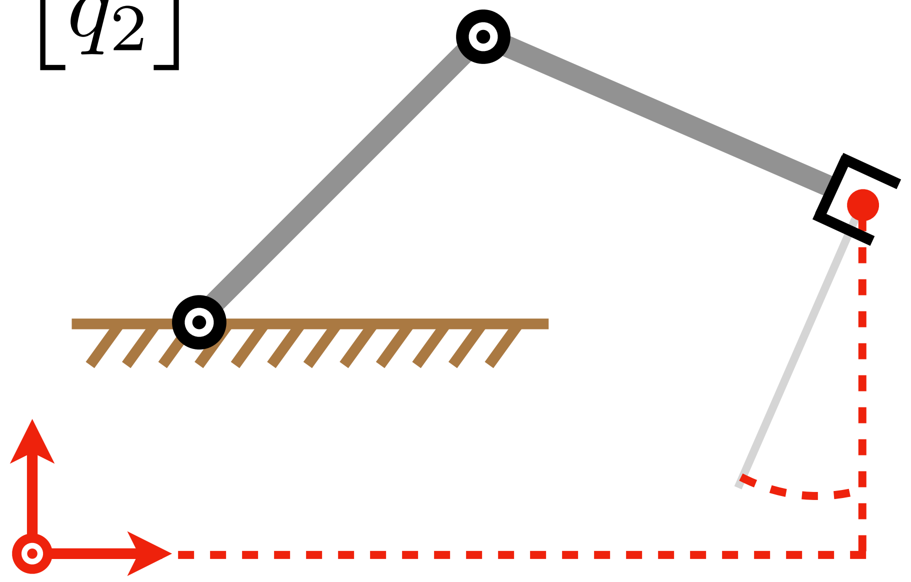

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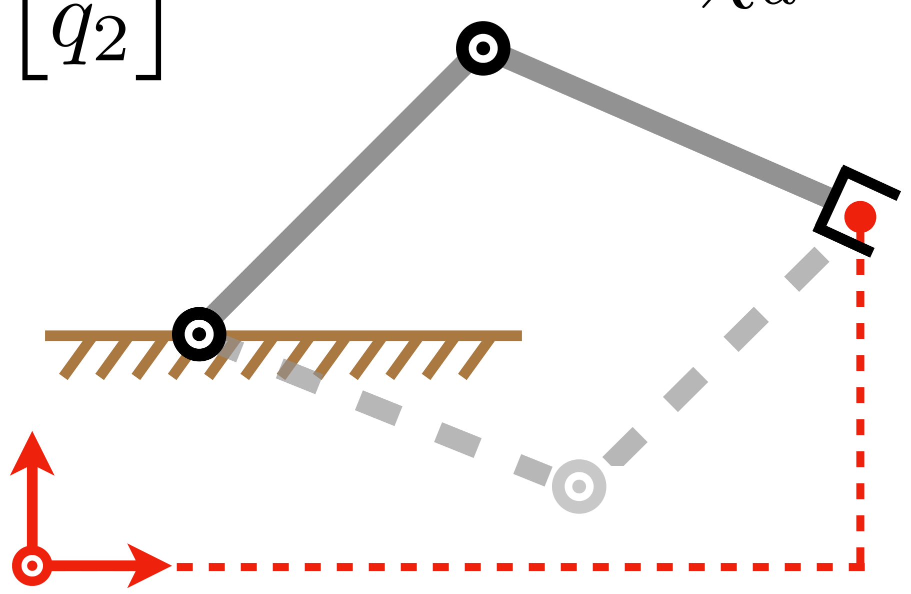

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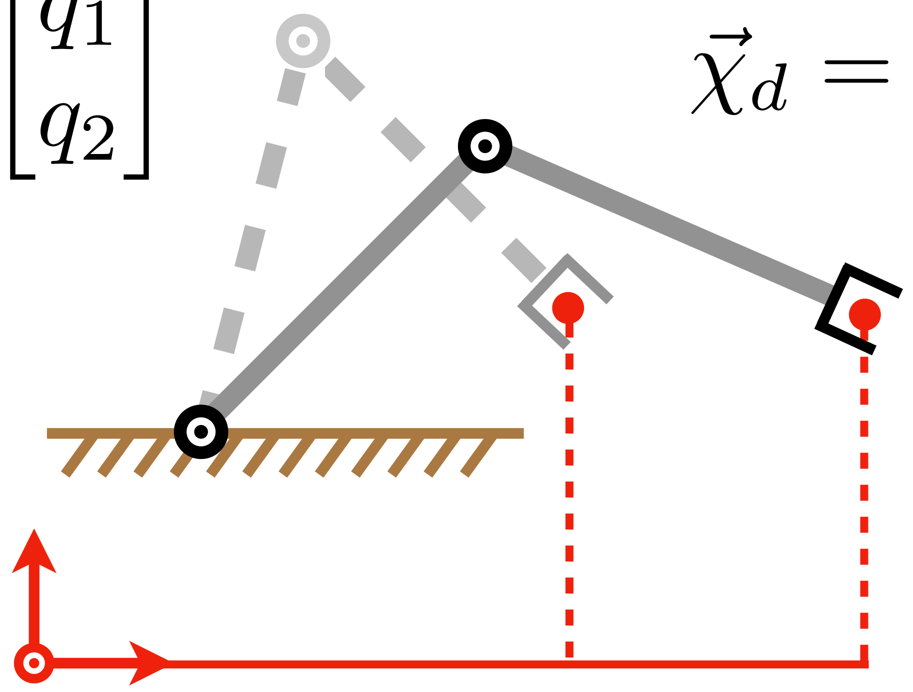

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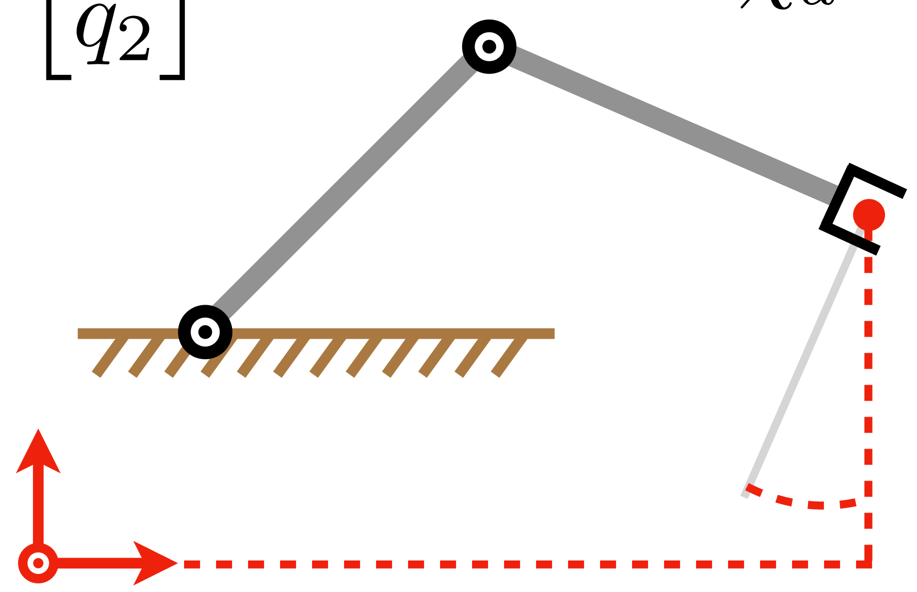

no exact solution for IK

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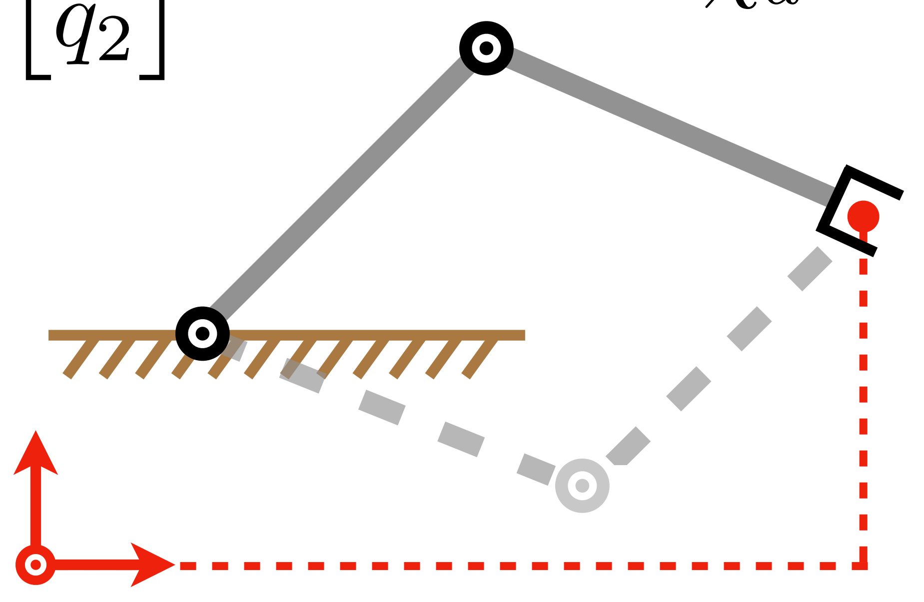

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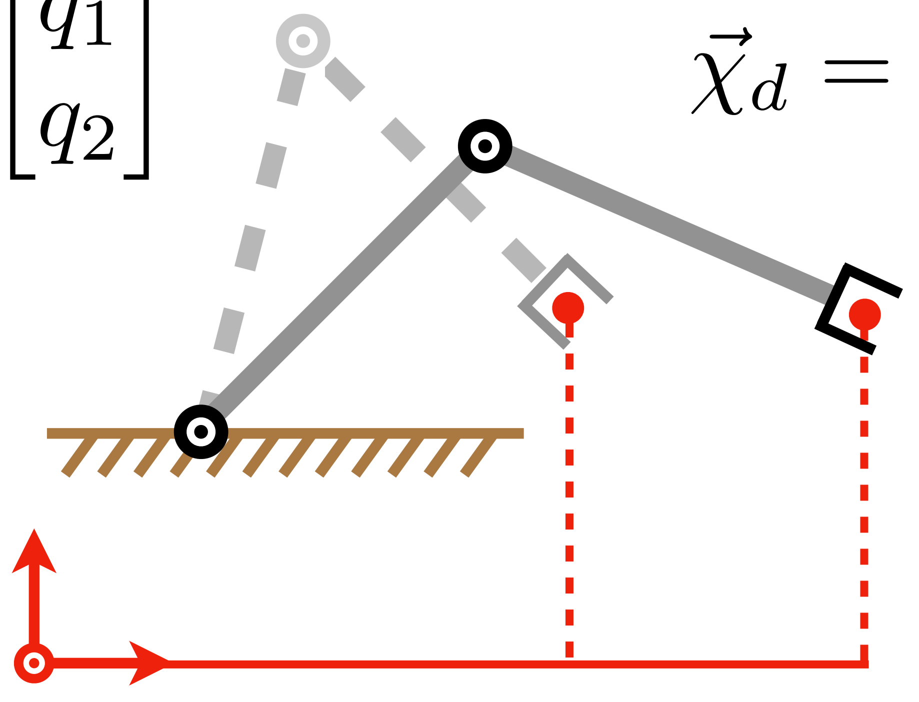

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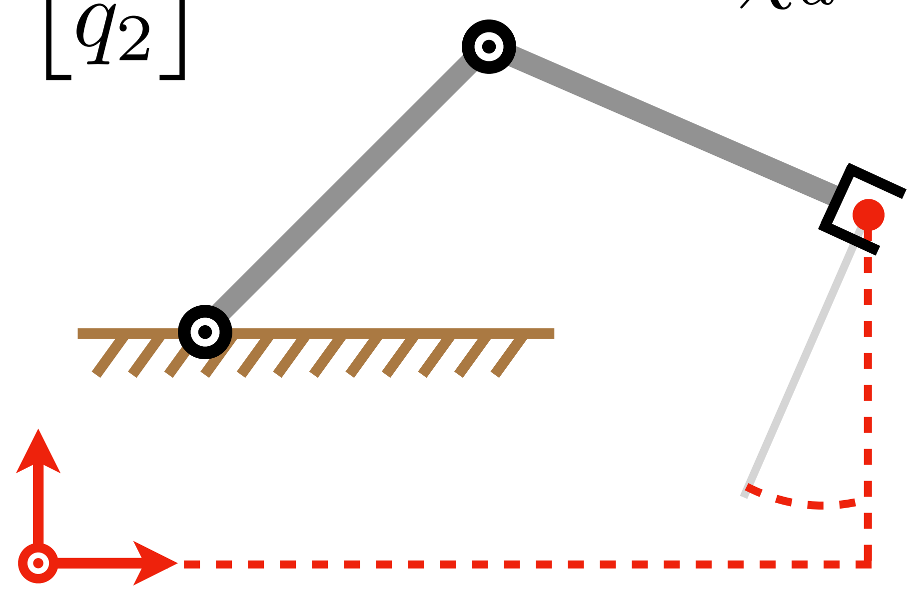

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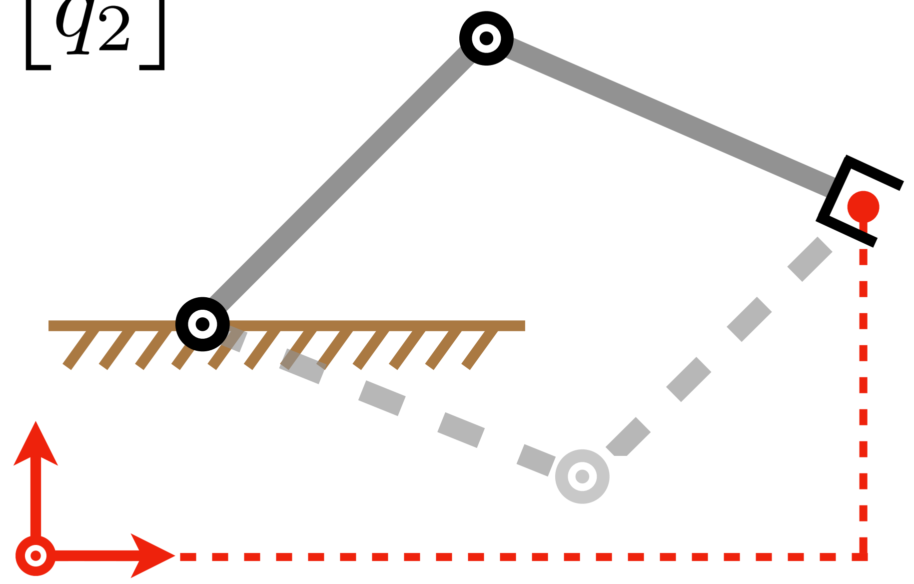

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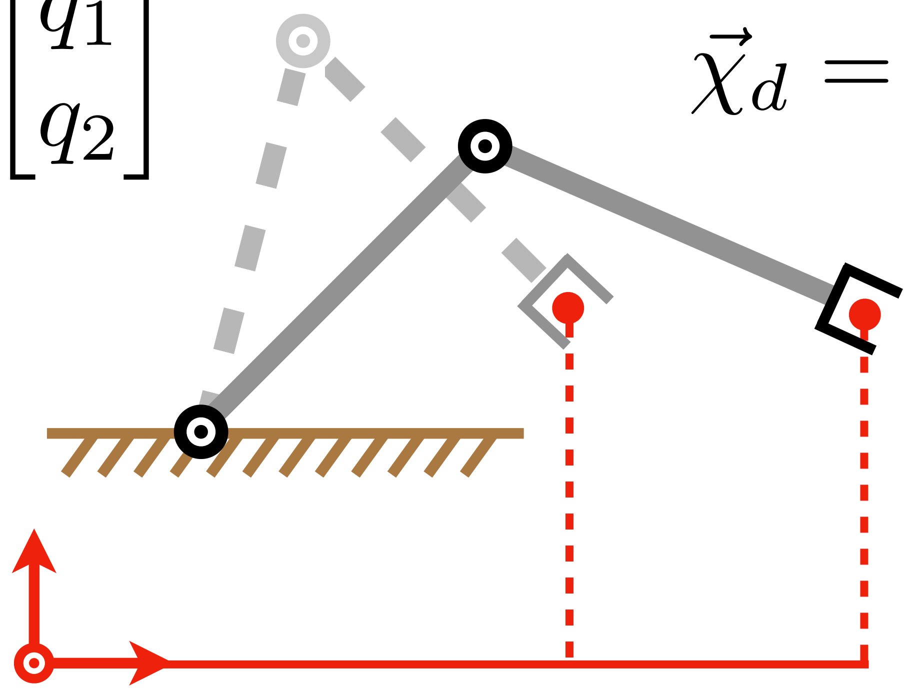

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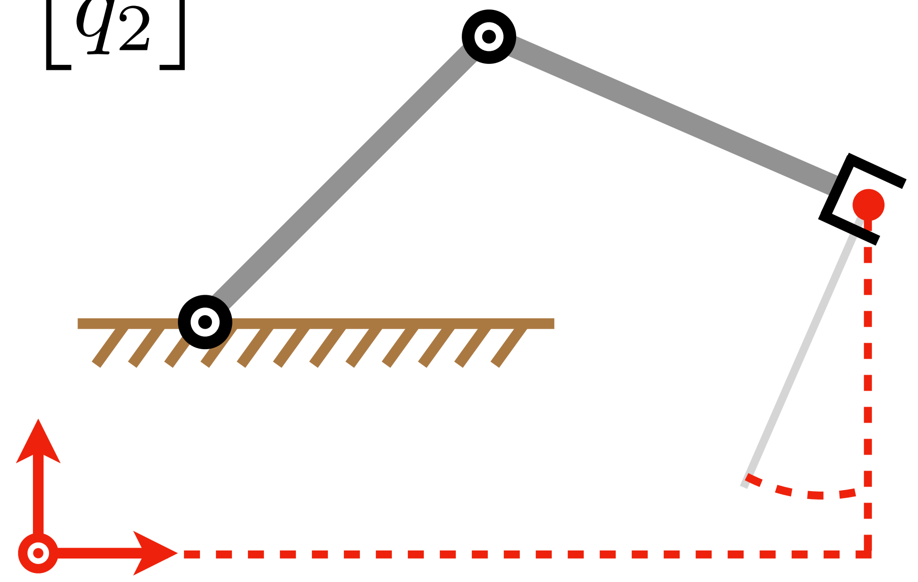
$$\dot{\vec{q}}^* = \begin{cases} \underset{\dot{\vec{q}}}{\operatorname{argmin}} \ \frac{1}{2} \|\dot{\vec{q}}\|_2^2 \\ \text{s.t.} \quad \dot{\vec{\chi}}_d = J(\vec{q})\,\dot{\vec{q}} \end{cases}$$

Case 3: $m > n$

$$\vec{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \qquad \vec{\chi}_d = \begin{bmatrix} x_d \\ y_d \\ \alpha_d \end{bmatrix}$$



no exact solution for IK

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Robot Dynamics Exercise Session 03

Assuming no kinematic singularities, we define the following three cases:

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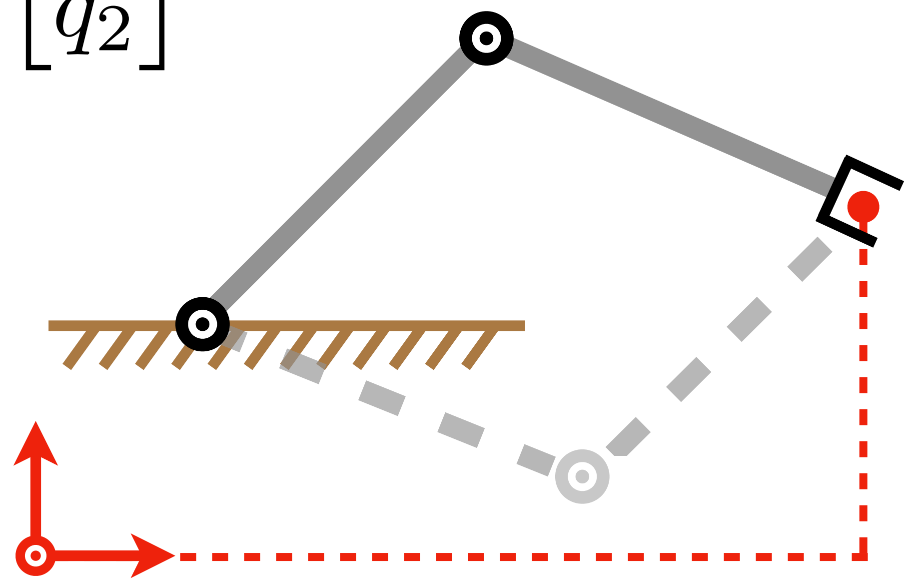

non unique but finitely many solutions for IK

$$\boxed{J^+(\vec{q}) = J^{-1}(\vec{q})}$$

J^+ is just J^{-1} as it is square

Case 2: $m < n$ (redundant)

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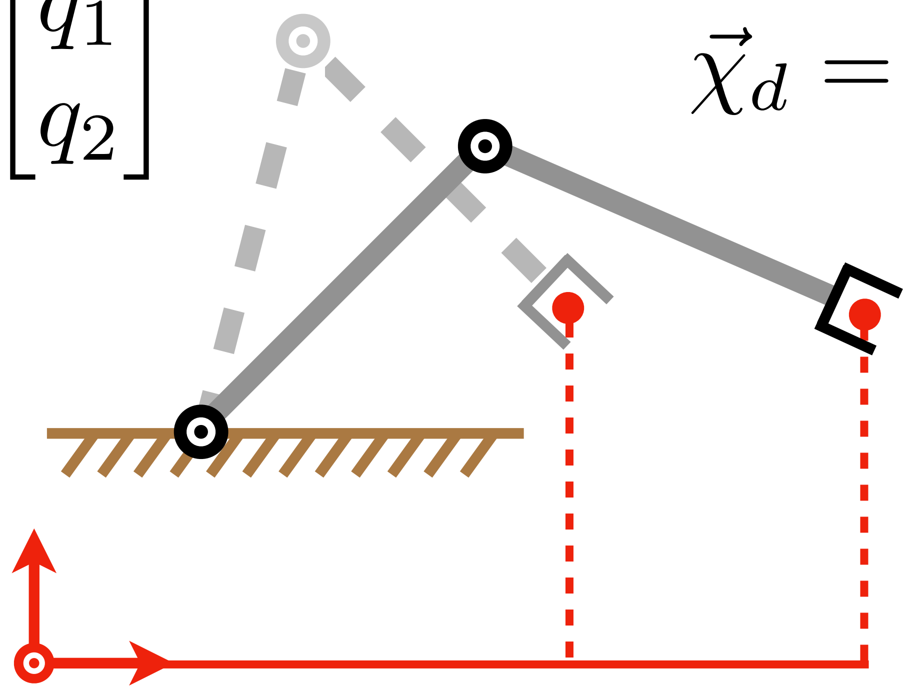

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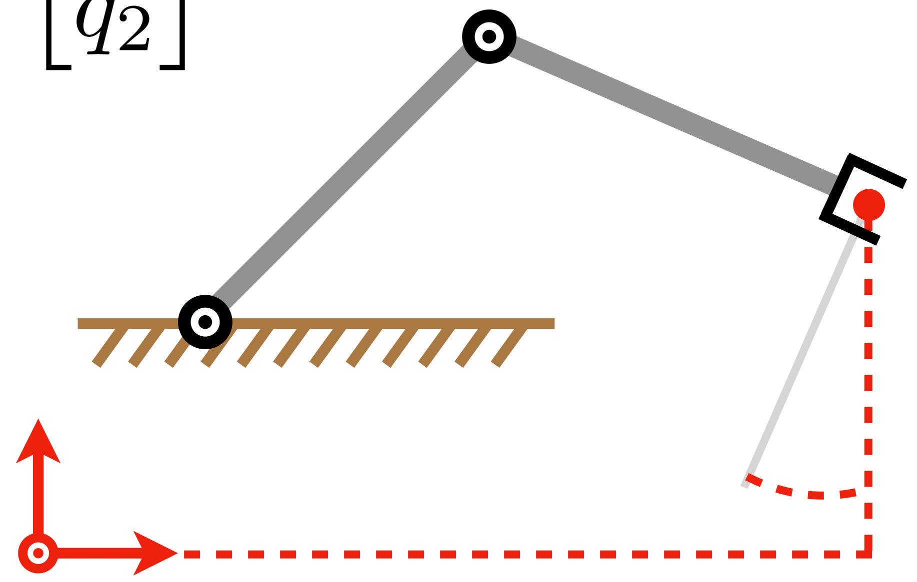

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Robotic Systems Lab

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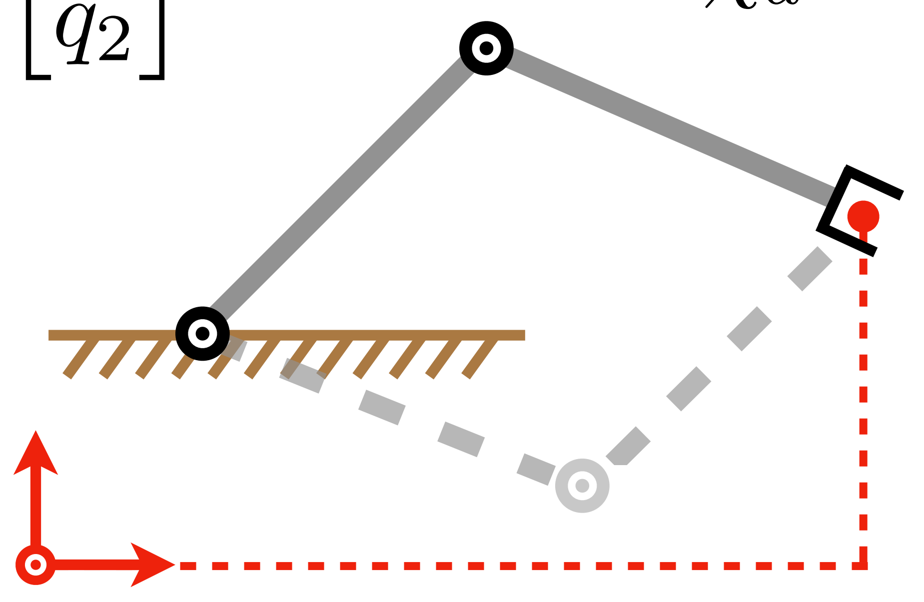

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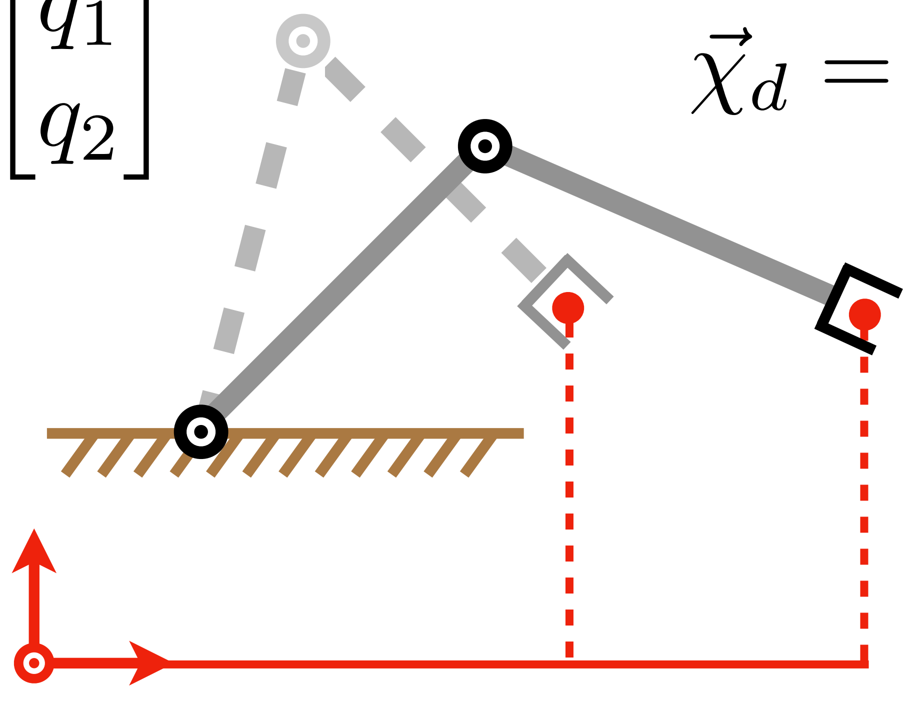

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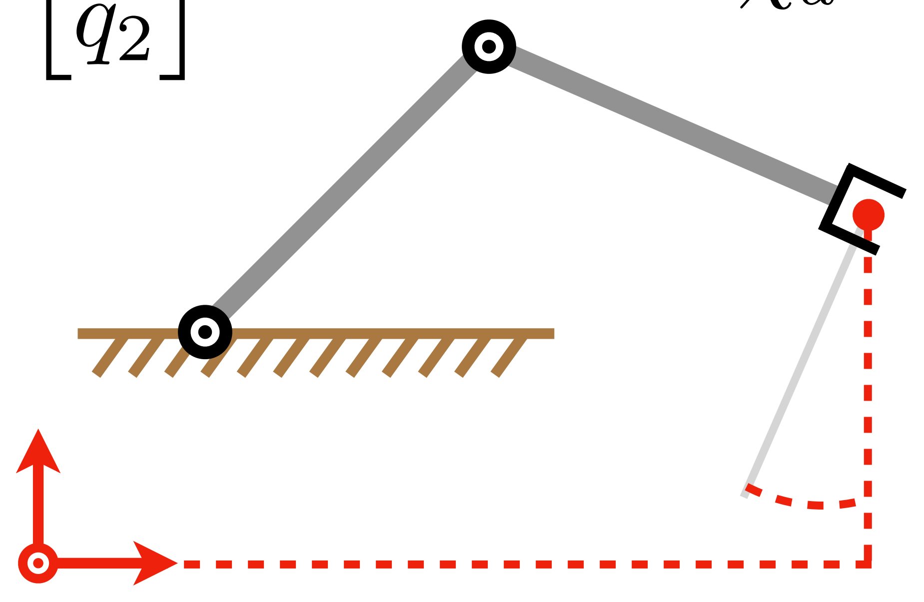

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(Jacobian will have linearly dependent columns in case of singularity)

Robot Dynamics Exercise Session 03

Robot Dynamics Exercise Session 03

2) Iterative Inverse Kinematics

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Robot Dynamics Exercise Session 03

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Solving IK is equivalent to finding the roots of $\vec{g}(\vec{q}) = \vec{\chi}_d - \vec{f}(\vec{q}) = \vec{0}$

Robot Dynamics Exercise Session 03

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Use of numerical technique analogous to Newton's method:

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Robot Dynamics Exercise Session 03

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Robot Dynamics Exercise Session 03

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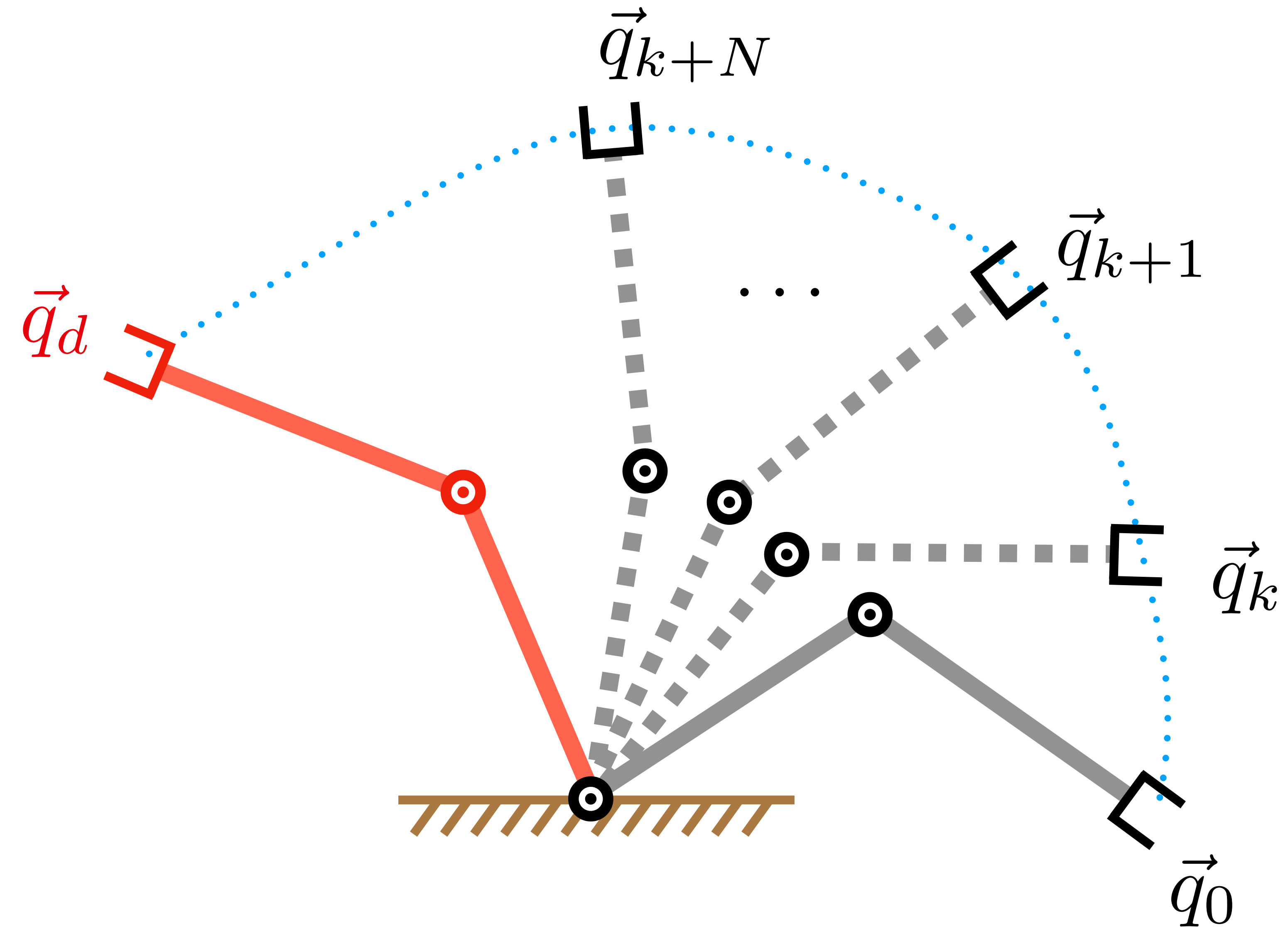

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for which we need an initial guess $\vec{q}_0$
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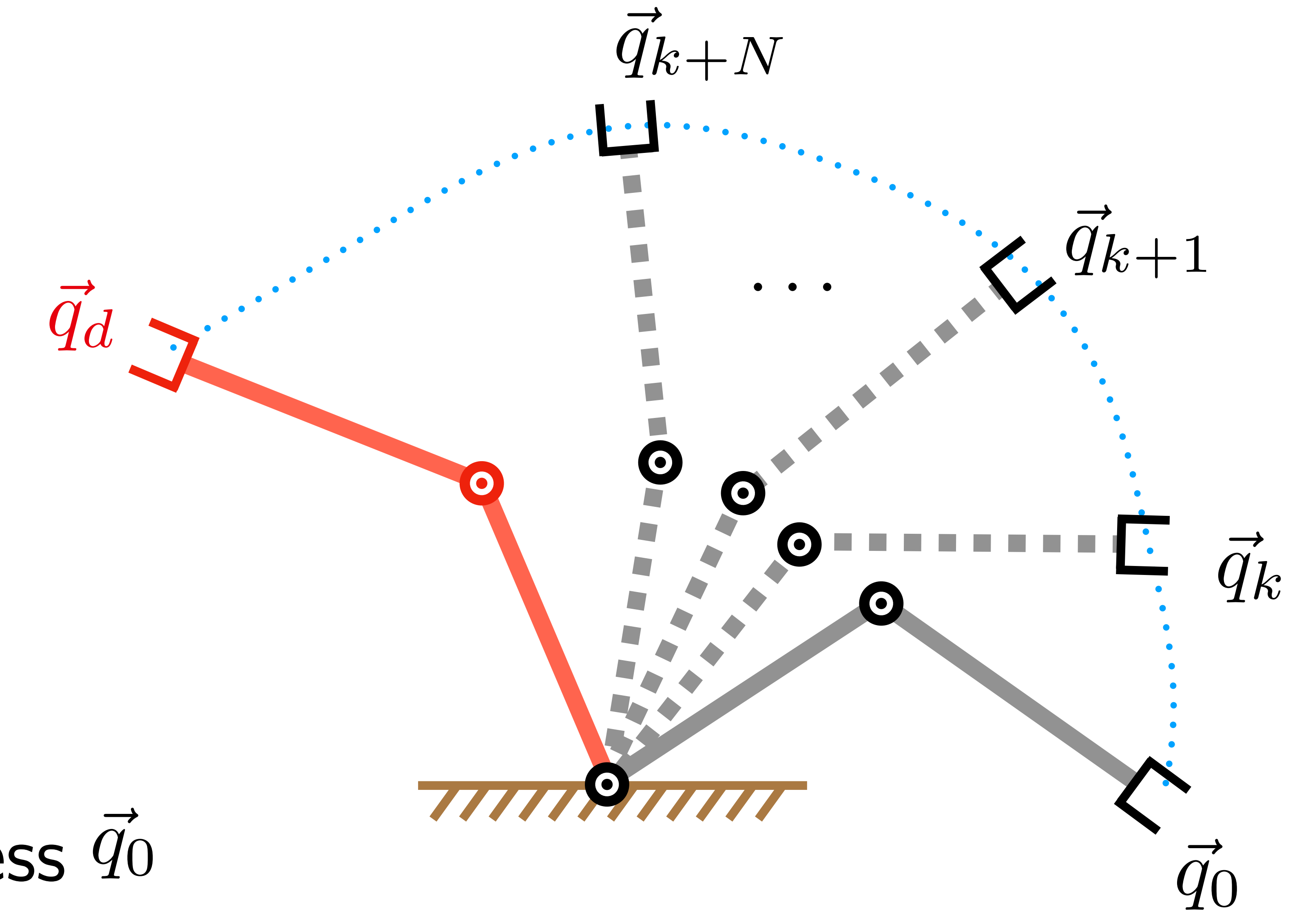

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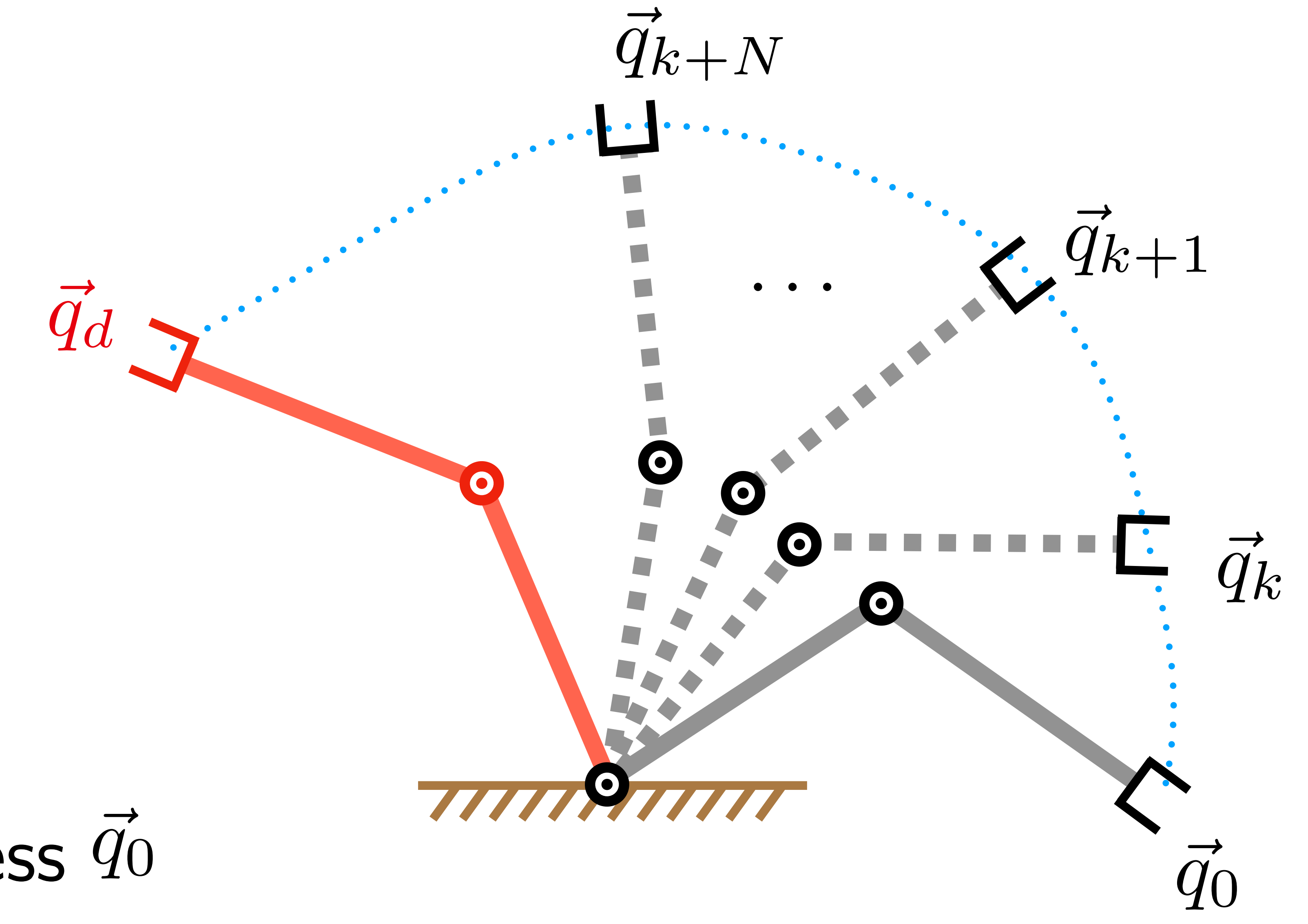

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pose error ⚠

Robot Dynamics Exercise Session 03

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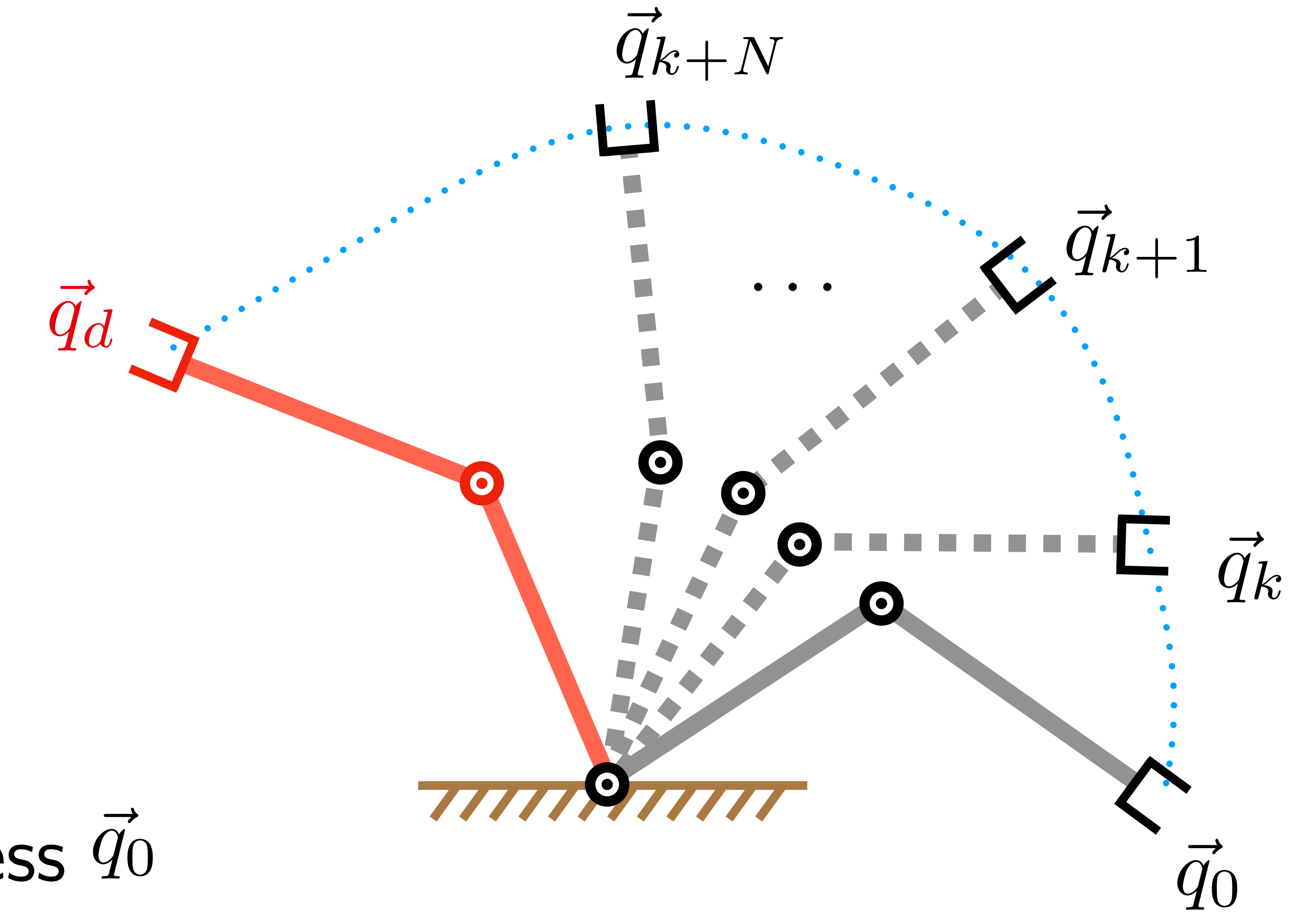

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Robot Dynamics Exercise Session 03

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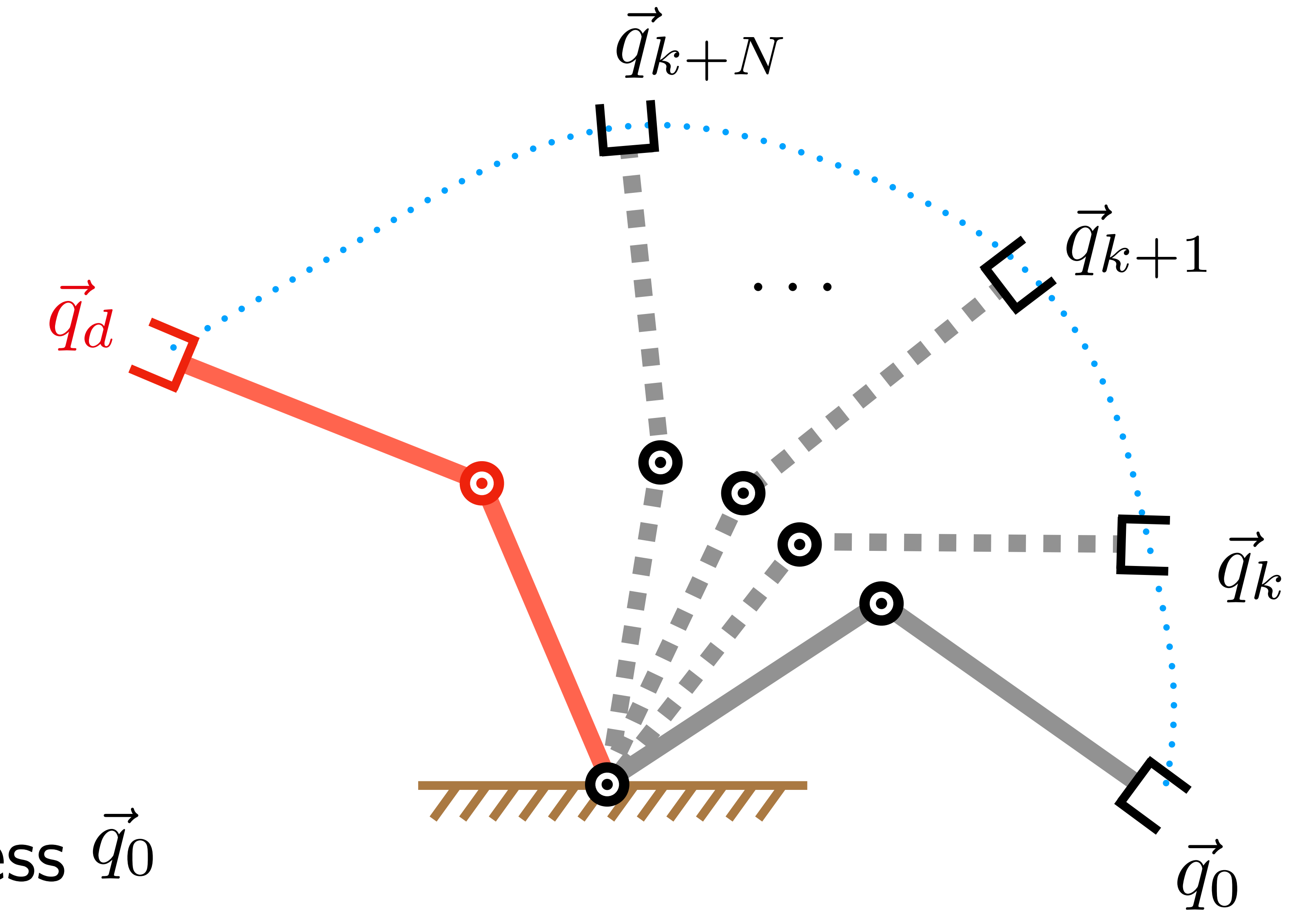

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pose error ⚠

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$$\Delta \vec{\chi}_{\mathcal{I}\mathcal{E}} = \begin{bmatrix} {}_I\vec{r}_{IE^*} - {}_I\vec{r}_{IE}(\vec{q}) \\ \underbrace{\Phi_{\mathcal{I}\mathcal{E}^*} \boxminus \Phi_{\mathcal{I}\mathcal{E}}(\vec{q})}_{{}_{\mathcal{I}}\varphi_{\mathcal{E}\mathcal{E}^*}} \end{bmatrix} \propto \begin{bmatrix} {}_{\mathcal{I}}\vec{v}_{IE} \\ {}_{\mathcal{I}}\vec{\omega}_{\mathcal{I}\mathcal{E}} \end{bmatrix}$$

Robot Dynamics Exercise Session 03

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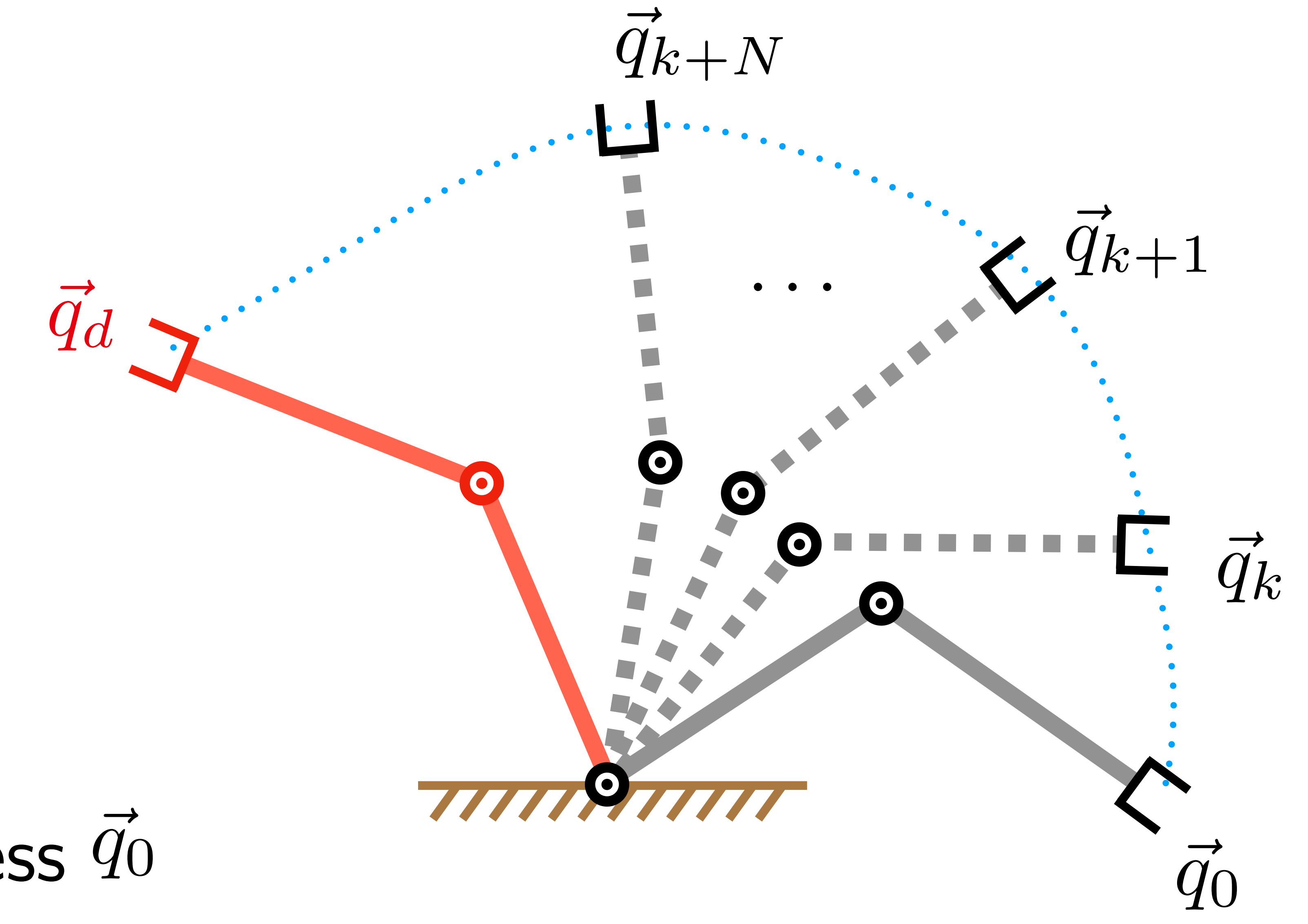

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Robot Dynamics Exercise Session 03

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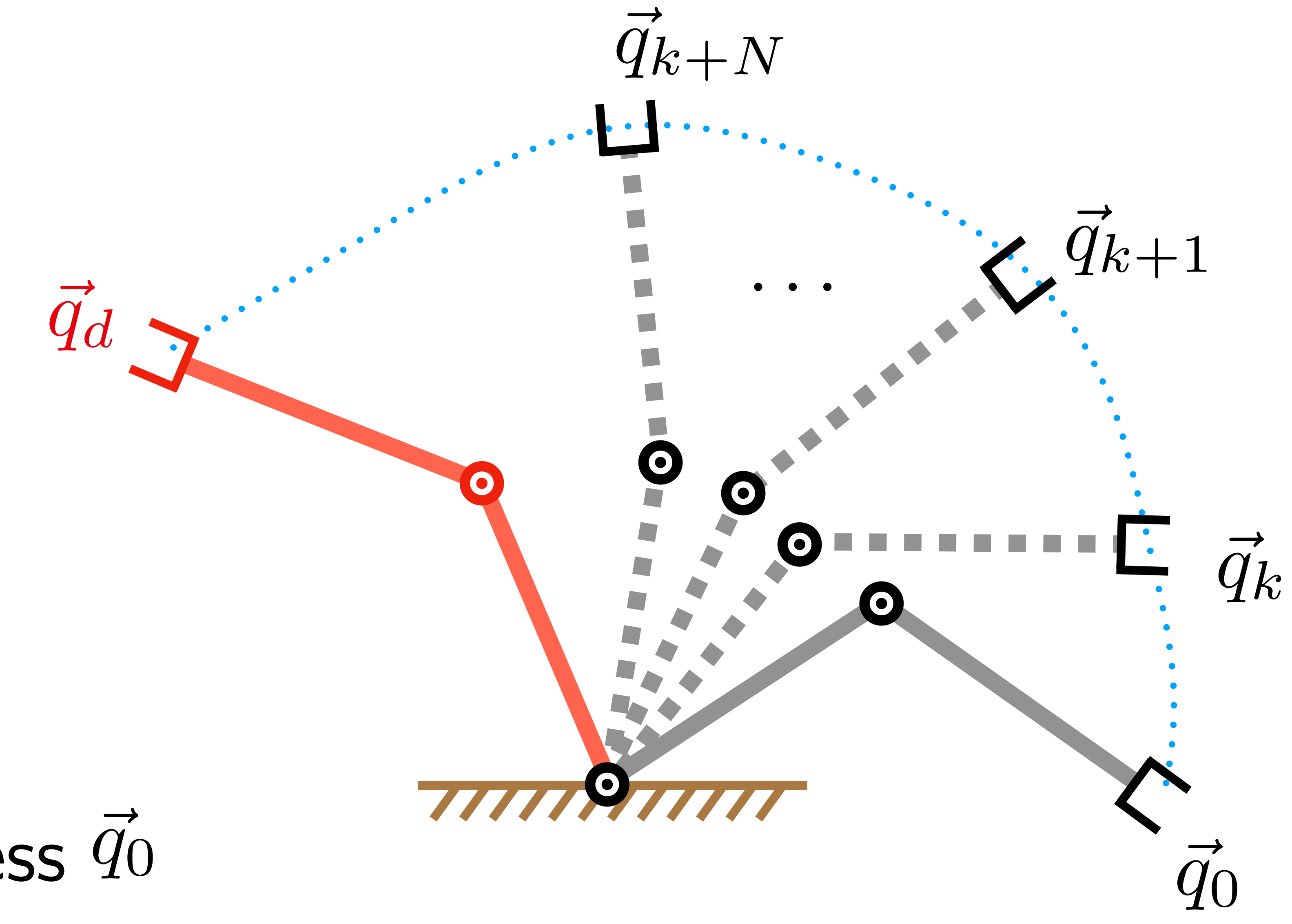

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$\boxminus$: Boxminus: shortest "path" between orientations (minus does not work in orientation space)

Robot Dynamics Exercise Session 03

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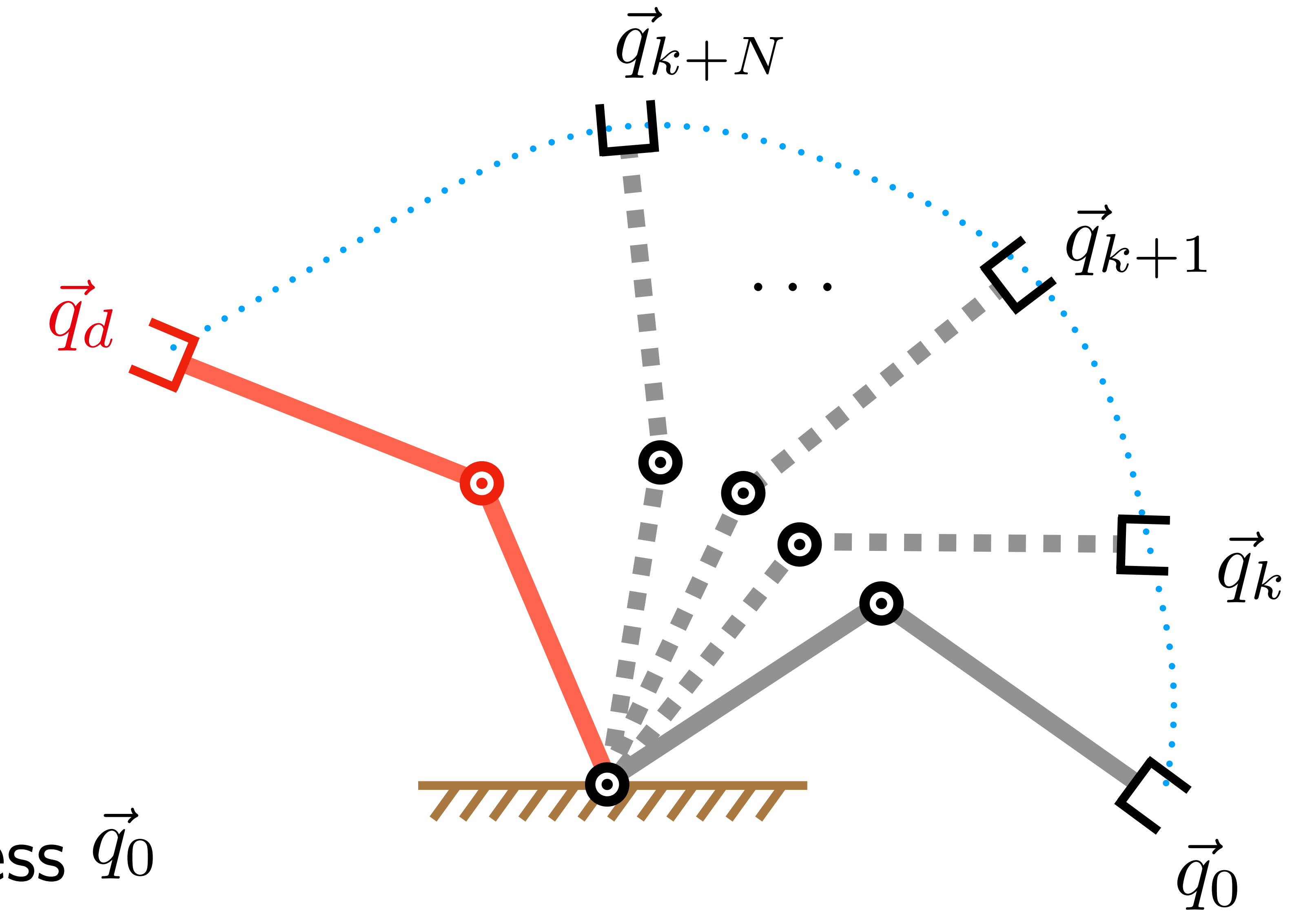

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Given $C_{\mathcal{I}\mathcal{E}}, C_{\mathcal{I}\mathcal{E}^*}$:

$$_{\mathcal{E}}\vec{\varphi}_{\mathcal{E}\mathcal{E}^*} = \mathrm{RotMat2RotVec}(C_{\mathcal{E}\mathcal{E}^*})$$

$$_{\mathcal{I}}\vec{\varphi}_{\mathcal{E}\mathcal{E}^*} = C_{\mathcal{I}\mathcal{E}}\, {}_{\mathcal{E}}\vec{\varphi}_{\mathcal{E}\mathcal{E}^*} = \mathrm{RotMat2RotVec}(C_{\mathcal{I}\mathcal{E}^*}\, C_{\mathcal{I}\mathcal{E}}^{\top}(\vec{q}))$$

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Robot Dynamics Exercise Session 03

Robot Dynamics Exercise Session 03

3) Kinematics Motion Control

Robot Dynamics Exercise Session 03

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Given a reference trajectory defined by $\vec{\chi}_d(t) = \begin{bmatrix} {}_{\mathcal{I}}\vec{r}_{IE^*} \\ \Phi_{\mathcal{I}\mathcal{E}^*} \end{bmatrix}$, $\vec{w}_d(t) = \begin{bmatrix} {}_{\mathcal{I}}\vec{v}_{IE^*} \\ {}_{\mathcal{I}}\vec{\omega}_{\mathcal{I}\mathcal{E}^*} \end{bmatrix}$

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Robot Dynamics Exercise Session 03

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we require feed forward reference if the desired pose is not constant

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relates "changes": $\dot{( )} \sim \Delta$

Robot Dynamics Exercise Session 03

3) Kinematics Motion Control

Given a reference trajectory defined by $\vec{\chi}_d(t) = \begin{bmatrix} {}_{\mathcal{I}}\vec{r}_{IE^*} \\ \Phi_{\mathcal{I}\mathcal{E}^*} \end{bmatrix}$, $\vec{w}_d(t) = \begin{bmatrix} {}_{\mathcal{I}}\vec{v}_{IE^*} \\ {}_{\mathcal{I}}\vec{\omega}_{\mathcal{I}\mathcal{E}^*} \end{bmatrix}$

Control law $\dot{\vec{q}} = J^+ \underbrace{\begin{bmatrix} {}_{\mathcal{I}}\vec{v}_{IE} \\ {}_{\mathcal{I}}\vec{\omega}_{\mathcal{I}\mathcal{E}} \end{bmatrix}}_{\vec{w}(t)}$, where $\vec{w}(t) = \vec{w}_d(t) + \underbrace{K_p}_{>0} \Delta\vec{\chi}_{\mathcal{I}\mathcal{E}}$

we require feed forward reference if the desired pose is not constant

relates "changes": $\dot{(\ )} \sim \Delta$

Reason why this works: error dynamics $\Delta\dot{\vec{\chi}}_{\mathcal{I}\mathcal{E}} + K_p \Delta\vec{\chi}_{\mathcal{I}\mathcal{E}} = \vec{0}$

RSL Robotic Systems Lab

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$\Delta\vec{\chi}_{\mathcal{I}\mathcal{E}} \to \vec{0}$

$\vec{\chi}_{\mathcal{I}\mathcal{E}}(t) \to \vec{\chi}_d(t)$

as $t \to \infty$

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