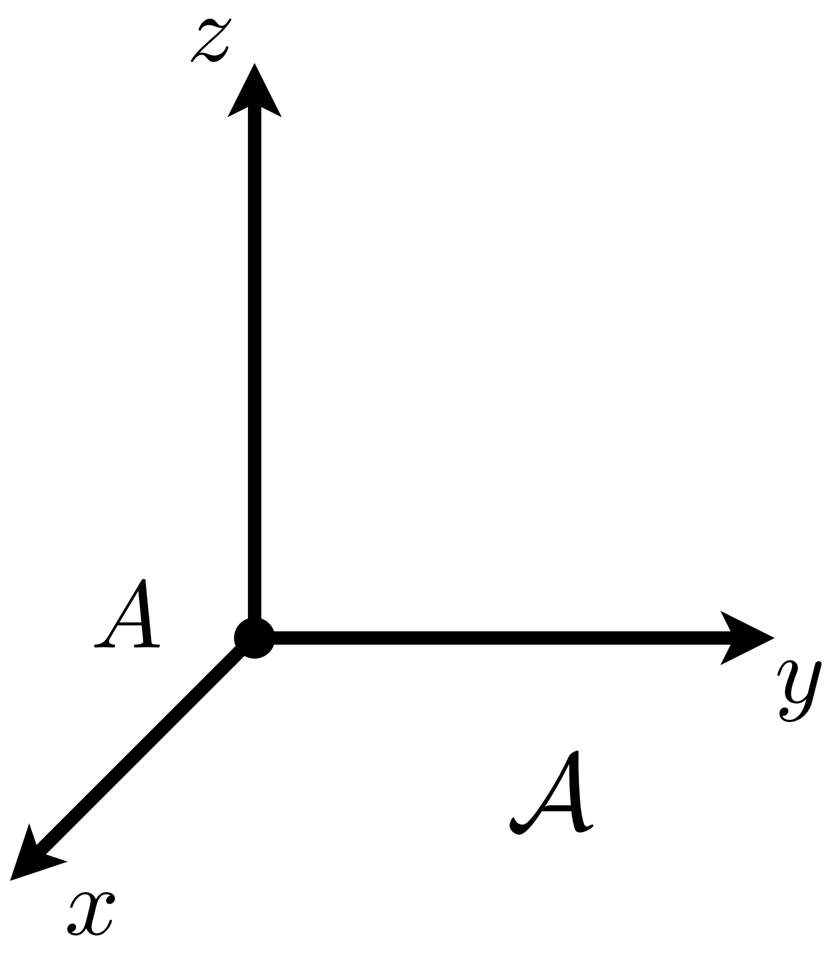
Robot Dynamics Exercise Session 01

Robot Dynamics Exercise Session 01

position vectors

Robot Dynamics Exercise Session 01

position vectors



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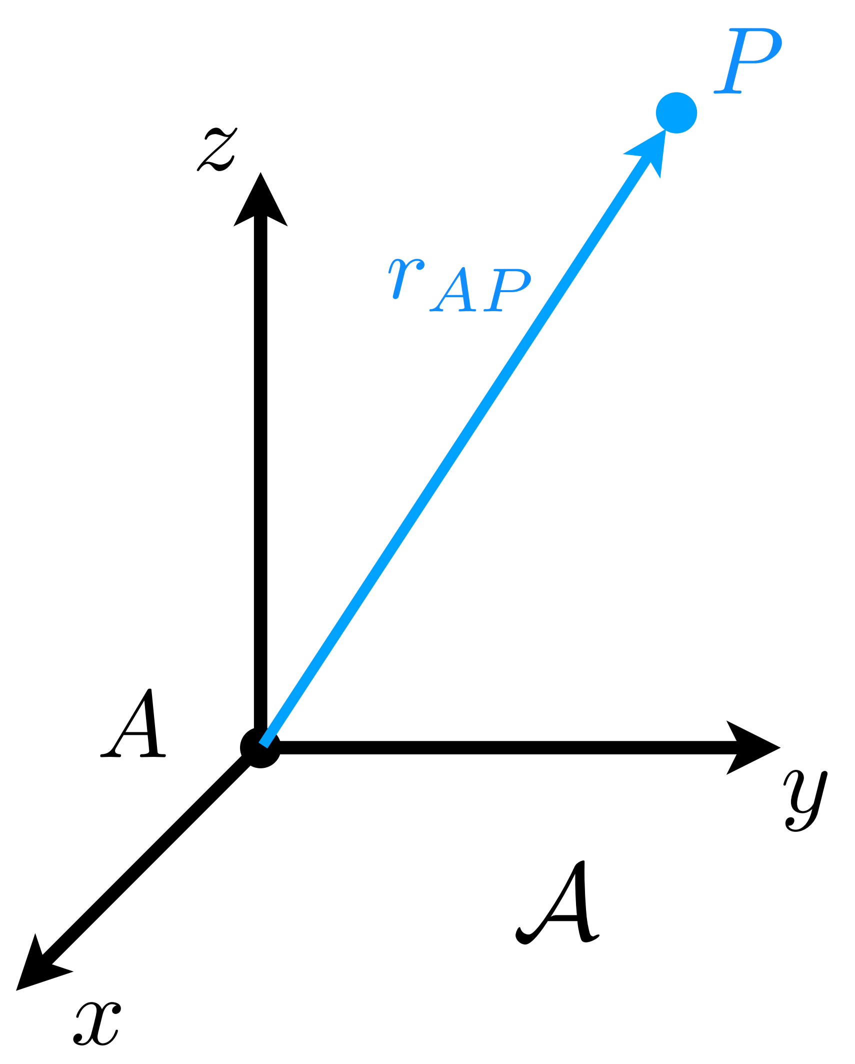
position vectors



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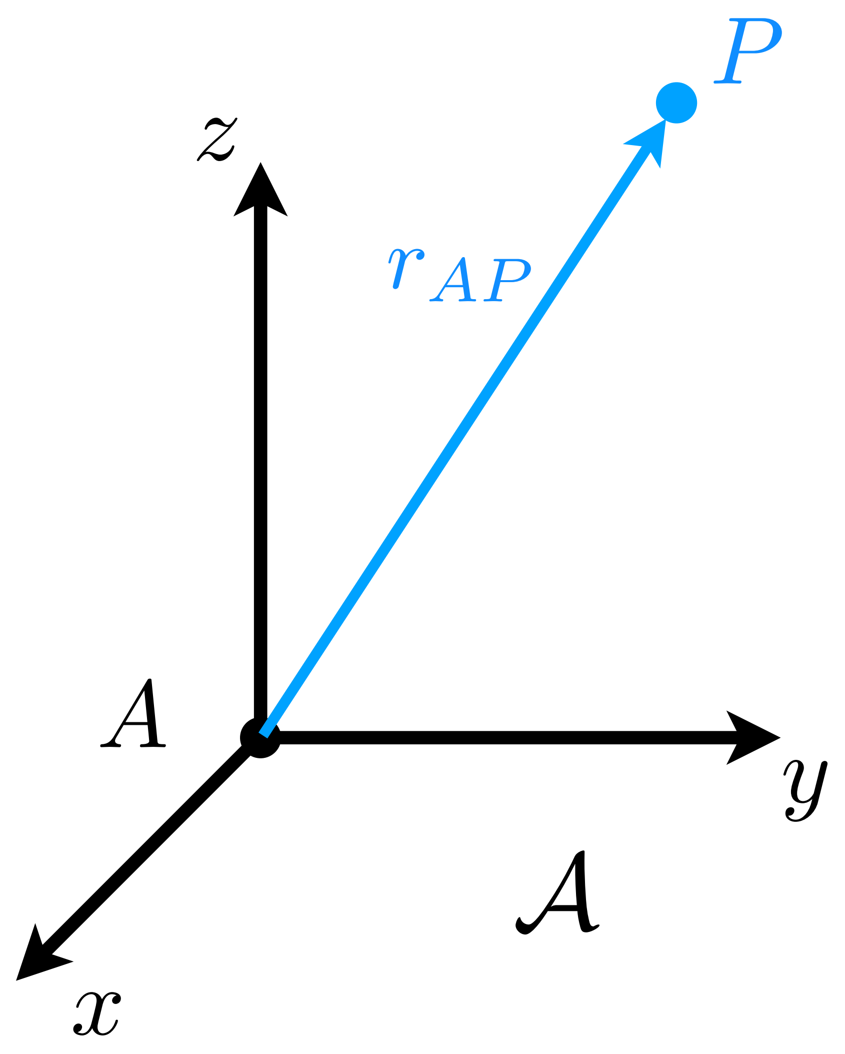
position vectors



$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

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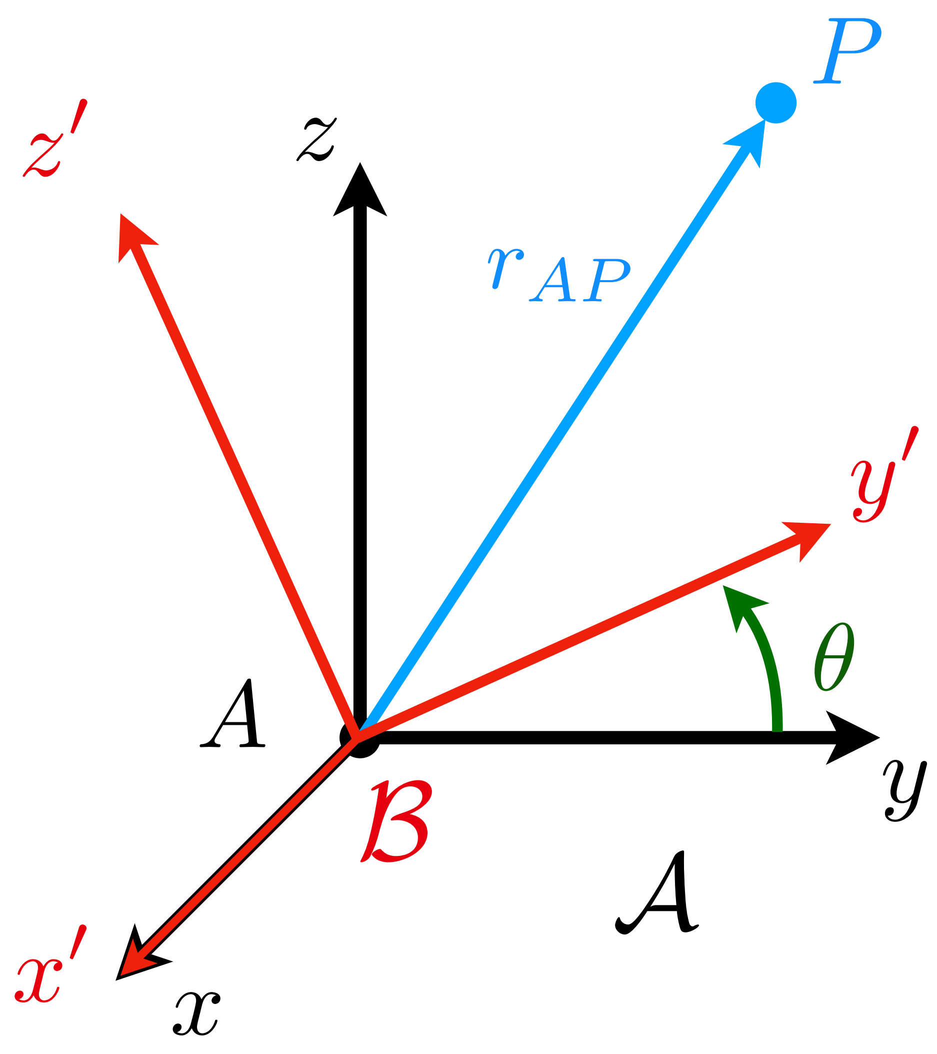
position vectors



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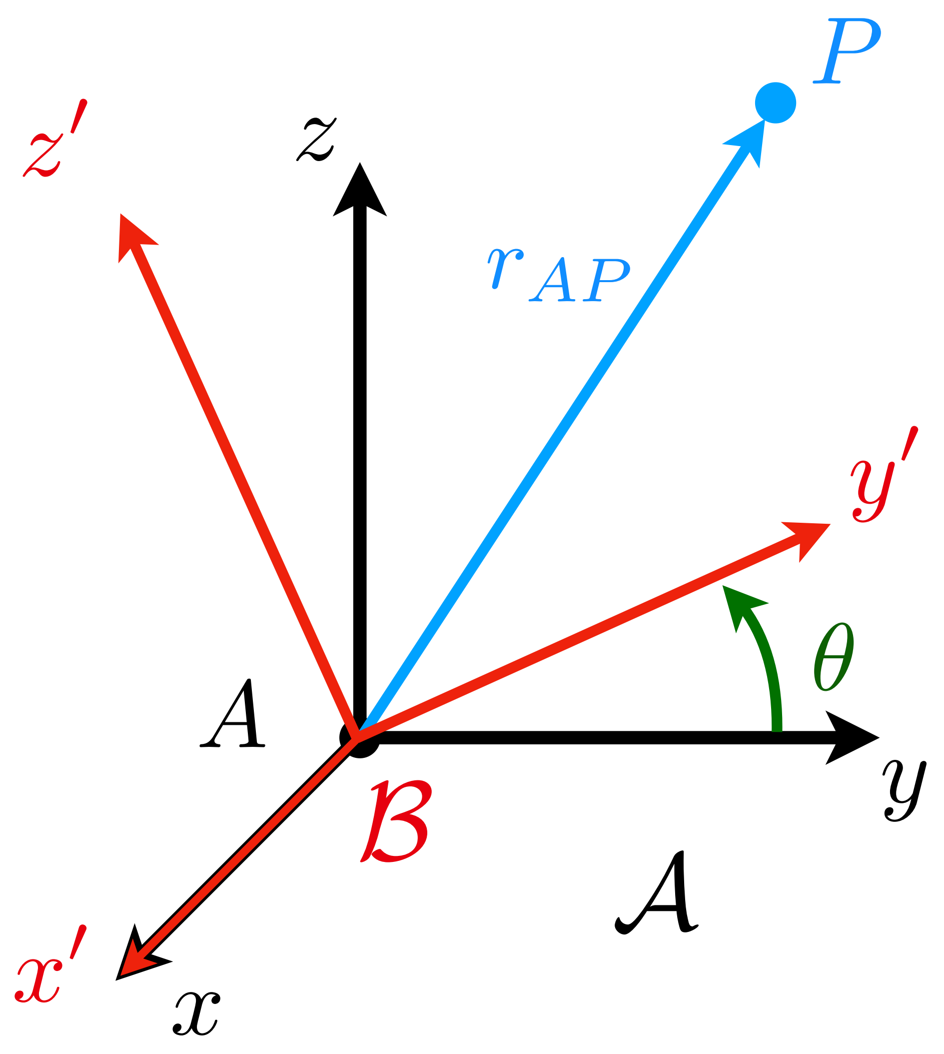

$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

$$_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\mathcal{A}}\ _{\mathcal{A}}\vec{r}_{AP}$$

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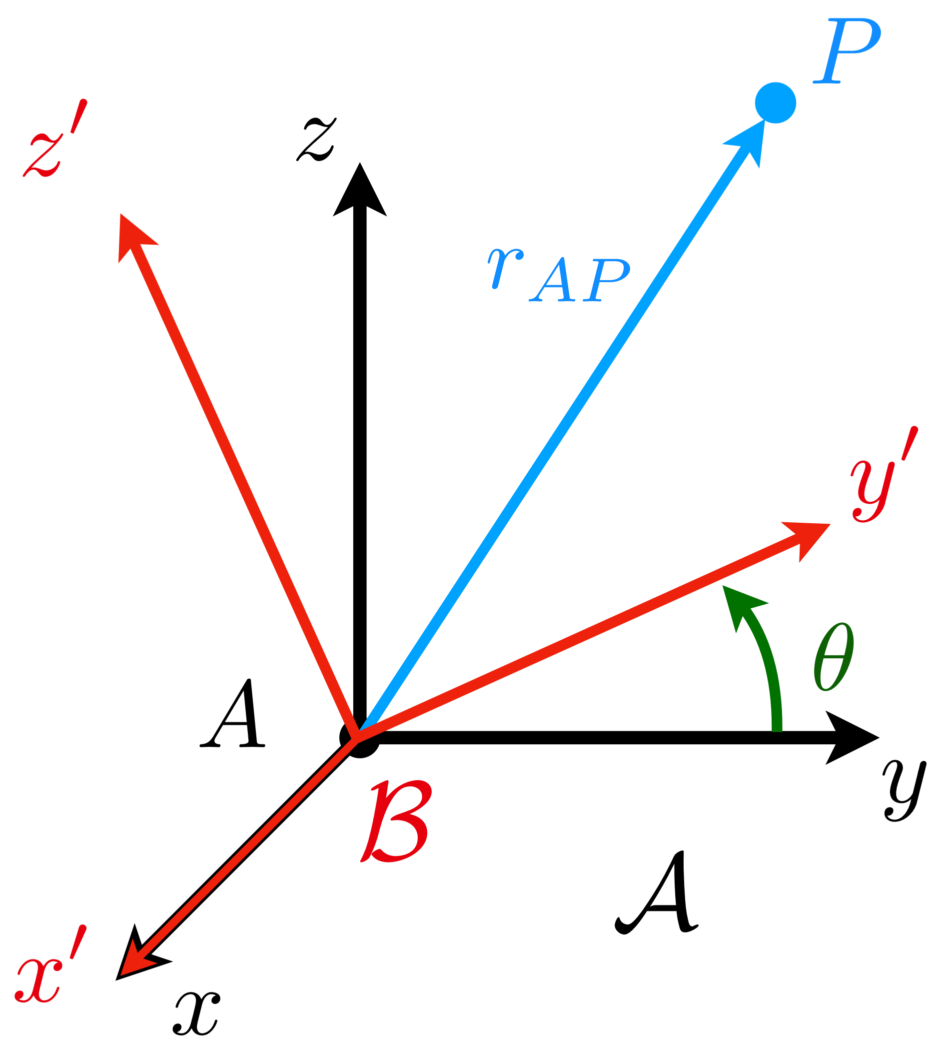
position vectors



$$ {}_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} {}_{\mathcal{A}}r_{AP,x} \\ {}_{\mathcal{A}}r_{AP,y} \\ {}_{\mathcal{A}}r_{AP,z} \end{bmatrix} $$

$$ {}_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}}\boxed{{}_{\mathcal{A}}\;{}_{\mathcal{A}}}\vec{r}_{AP} $$

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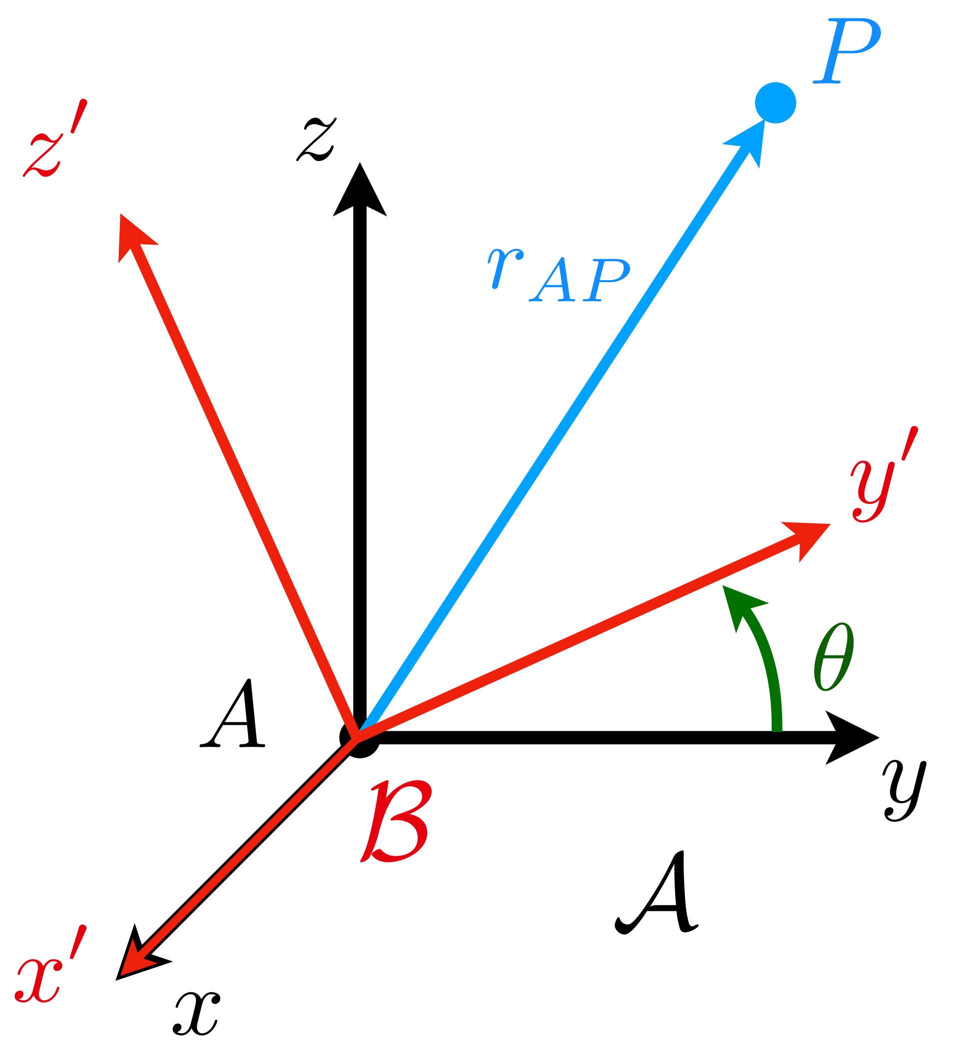
position vectors



$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

$$_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}}} \, \boxed{_{\mathcal{A}}}\vec{r}_{AP}$$

$C_{\mathcal{A}\mathcal{B}}$ is a change of representation (or basis)

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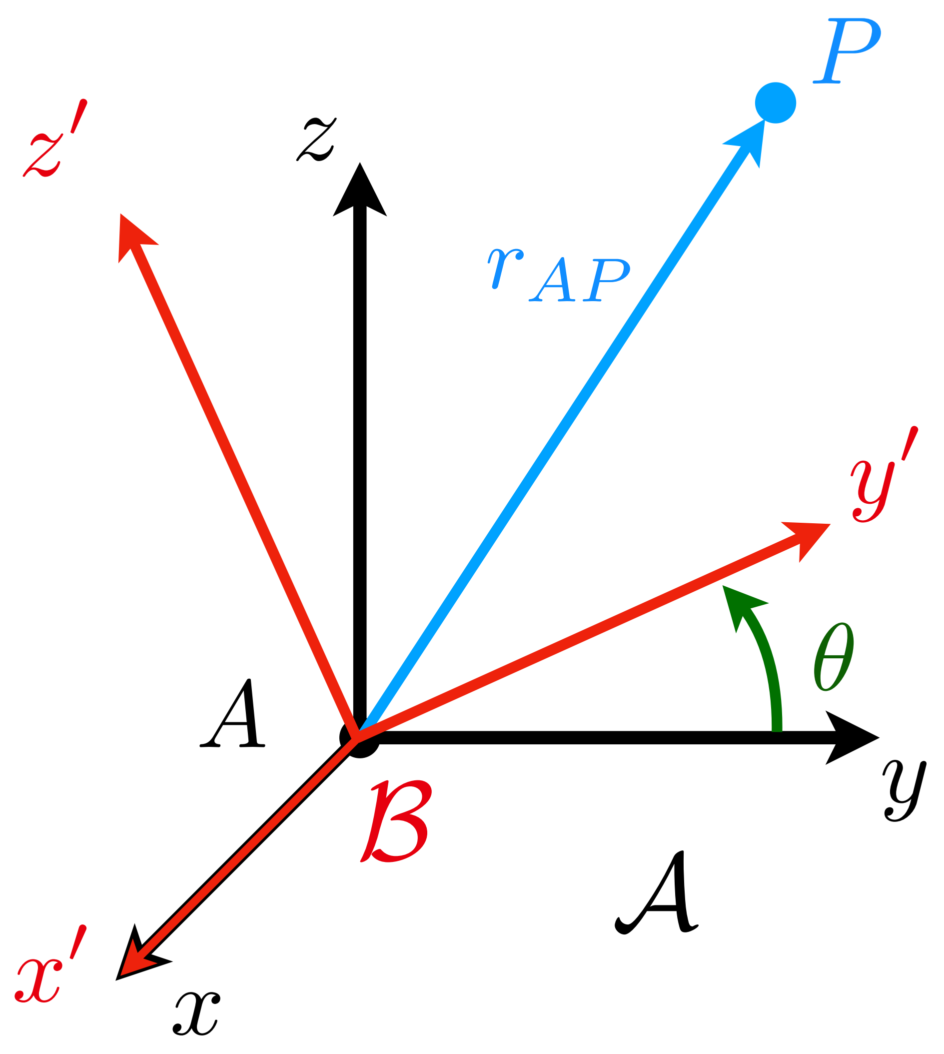
position vectors



$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

$$C_{\mathcal{A}\mathcal{B}} = \begin{bmatrix} _{\mathcal{A}}\vec{e}_x^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_y^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_z^{\mathcal{B}} \end{bmatrix}$$

$$_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}\ _{\mathcal{A}}}}\vec{r}_{AP}$$

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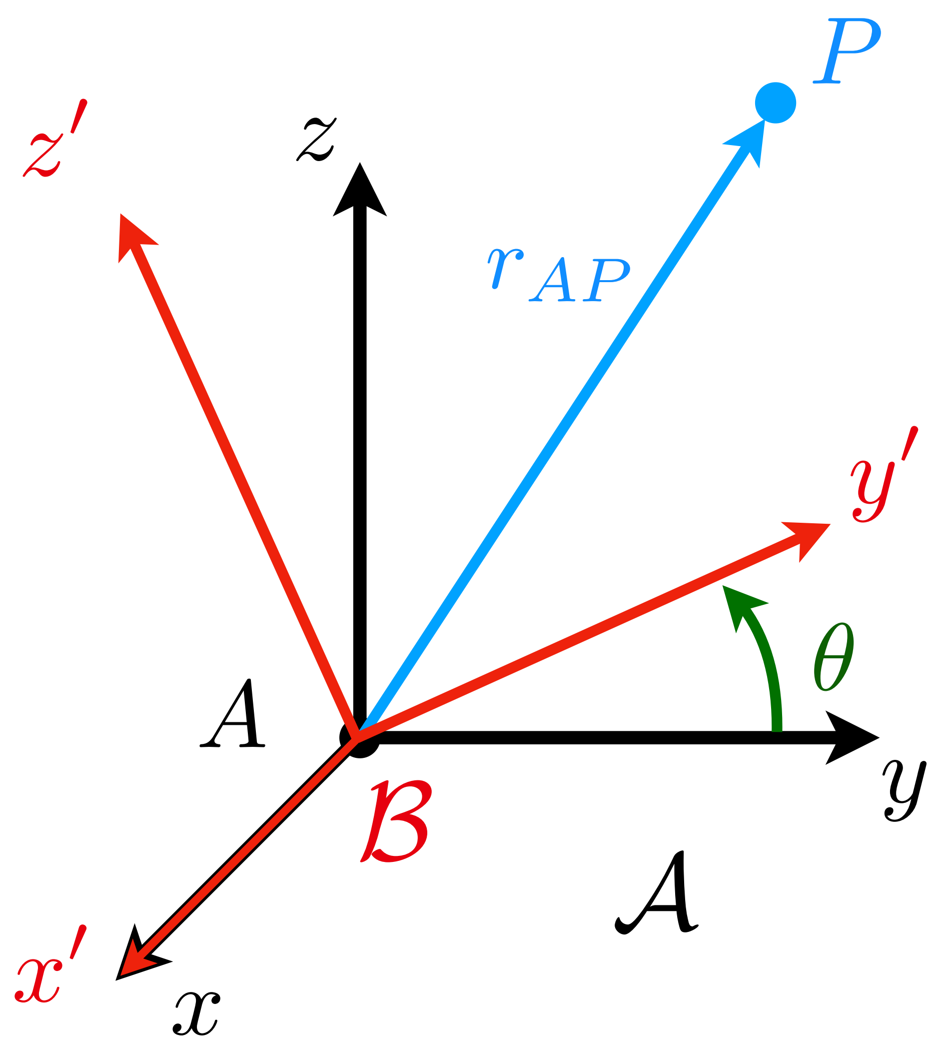
position vectors



$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

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$C_{\mathcal{A}\mathcal{B}}$ is a change of representation (or basis)

$$C_{\mathcal{A}\mathcal{B}} = \begin{bmatrix} _{\mathcal{A}}\vec{e}_x^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_y^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_z^{\mathcal{B}} \end{bmatrix}$$

$$C_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

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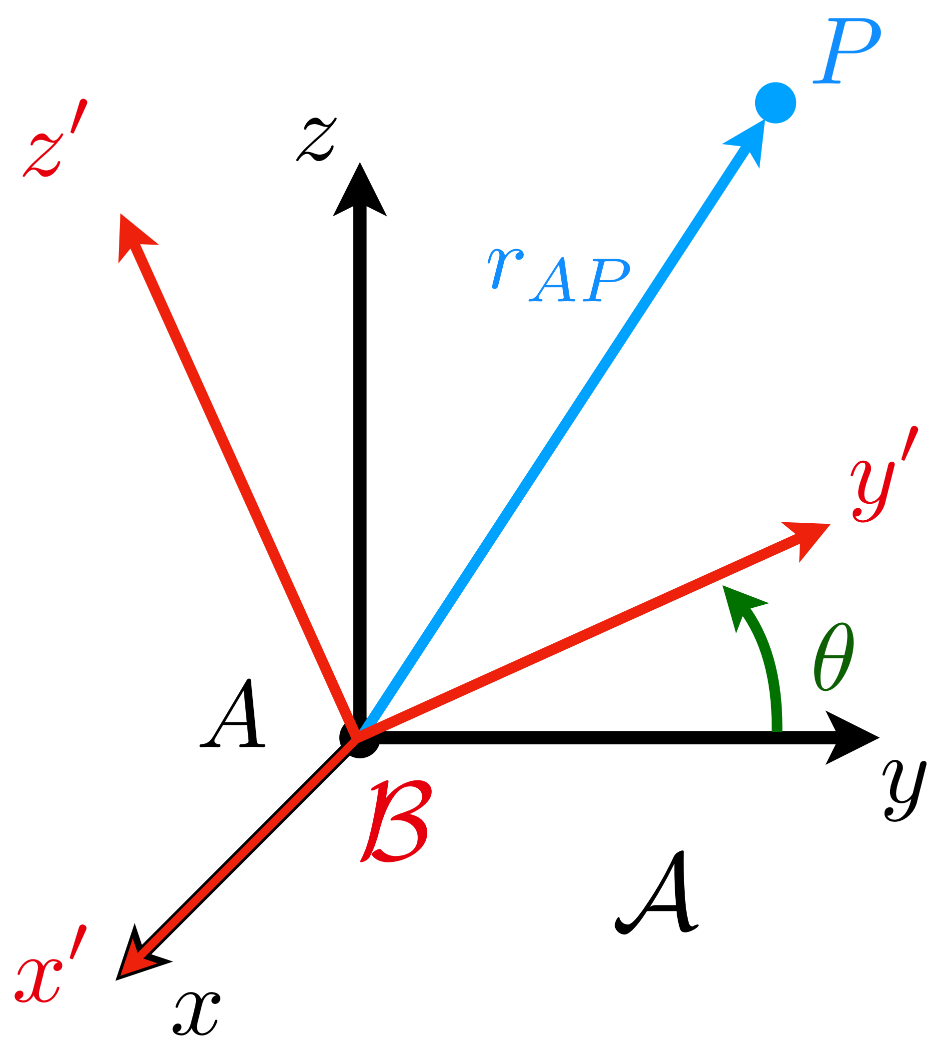
position vectors



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$$ {}_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}\ {}_{\mathcal{A}}}}\vec{r}_{AP} $$

$C_{\mathcal{AB}}$ is a change of representation (or basis)

$$ C_{\mathcal{AB}} = \begin{bmatrix} {}_{\mathcal{A}}\vec{e}_x^{\mathcal{B}} & {}_{\mathcal{A}}\vec{e}_y^{\mathcal{B}} & {}_{\mathcal{A}}\vec{e}_z^{\mathcal{B}} \end{bmatrix} $$

$$ C_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} $$

inverse: $C_{\mathcal{BA}} = C_{\mathcal{AB}}^{-1} = C_{\mathcal{AB}}^{\top} \quad \leftrightarrow \quad C_{\mathcal{AB}}\underbrace{C_{\mathcal{AB}}^{\top}}_{C_{\mathcal{BA}}} = \mathbb{I}_{3\times 3}$

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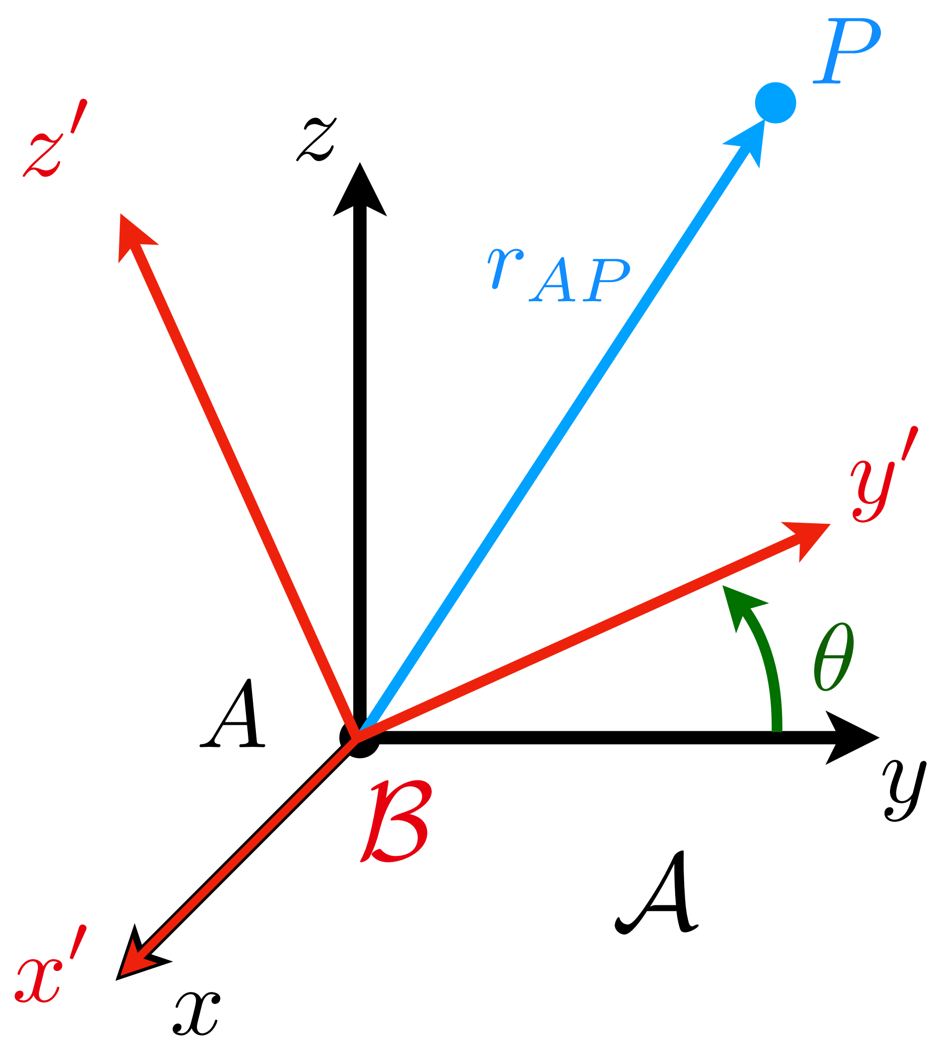
position vectors



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$${}_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}\ \mathcal{A}}}\vec{r}_{AP}$$

$C_{\mathcal{AB}}$ is a change of representation (or basis)

$$C_{\mathcal{AB}} = \begin{bmatrix} {}_{\mathcal{A}}\vec{e}_x^{\,\mathcal{B}} & {}_{\mathcal{A}}\vec{e}_y^{\,\mathcal{B}} & {}_{\mathcal{A}}\vec{e}_z^{\,\mathcal{B}} \end{bmatrix}$$

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$C_{\mathcal{AB}}$ is an orthogonal matrix: $\det\left(C_{BA}\right) = 1$ (length preserving)

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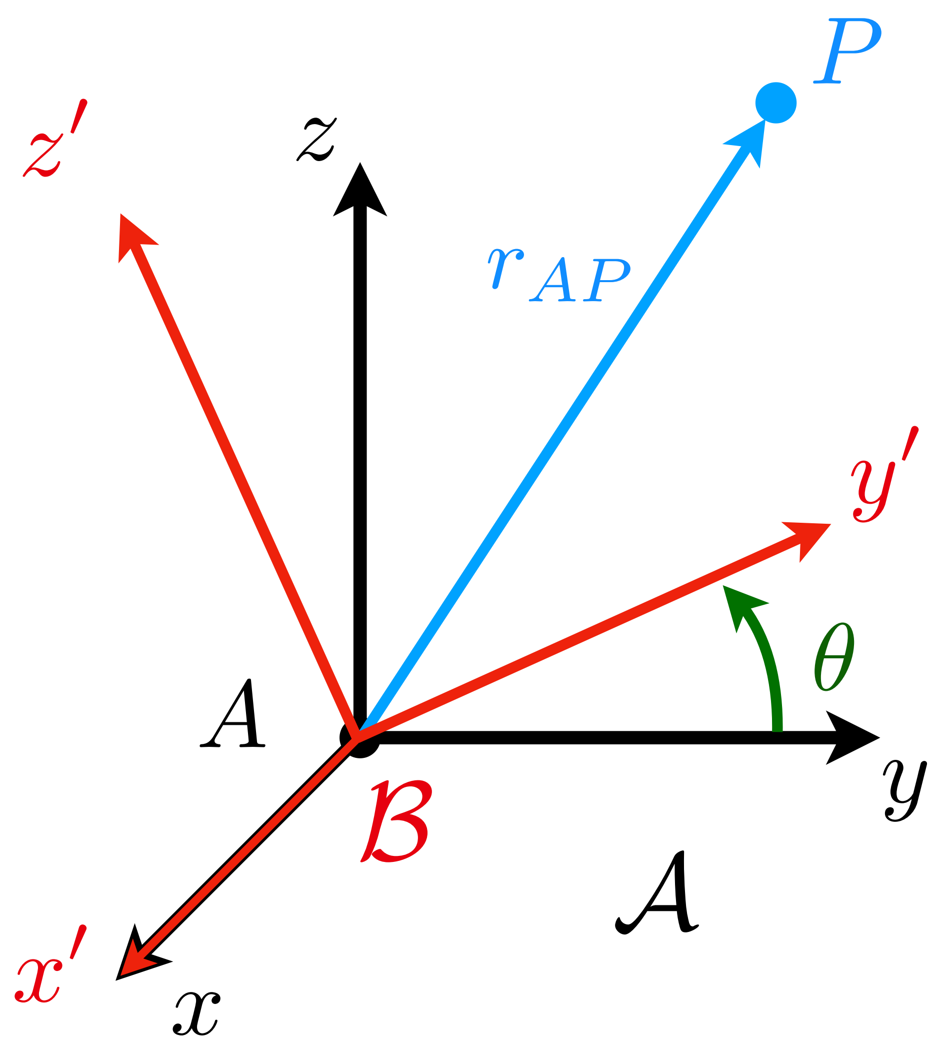
position vectors



$$ {}_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} {}_{\mathcal{A}}r_{AP,x} \\ {}_{\mathcal{A}}r_{AP,y} \\ {}_{\mathcal{A}}r_{AP,z} \end{bmatrix} $$

$$ {}_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}\ {}_{\mathcal{A}}}}\vec{r}_{AP} $$

$C_{\mathcal{AB}}$ is a change of representation (or basis)

$$ C_{\mathcal{AB}} = \begin{bmatrix} {}_{\mathcal{A}}\vec{e}_x^{\,\mathcal{B}} & {}_{\mathcal{A}}\vec{e}_y^{\,\mathcal{B}} & {}_{\mathcal{A}}\vec{e}_z^{\,\mathcal{B}} \end{bmatrix} $$

$$ C_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix} $$

inverse: $C_{\mathcal{BA}} = C_{\mathcal{AB}}^{-1} = C_{\mathcal{AB}}^{\top} \quad \leftrightarrow \quad C_{\mathcal{AB}}\,\underbrace{C_{\mathcal{AB}}^{\top}}_{C_{\mathcal{BA}}} = \mathbb{I}_{3\times 3}$

$C_{\mathcal{AB}}$ is an orthogonal matrix: $\det\left(C_{BA}\right) = 1$ (length preserving)

composition: $C_{\mathcal{AC}} = C_{\mathcal{AB}}\,C_{\mathcal{BC}}$

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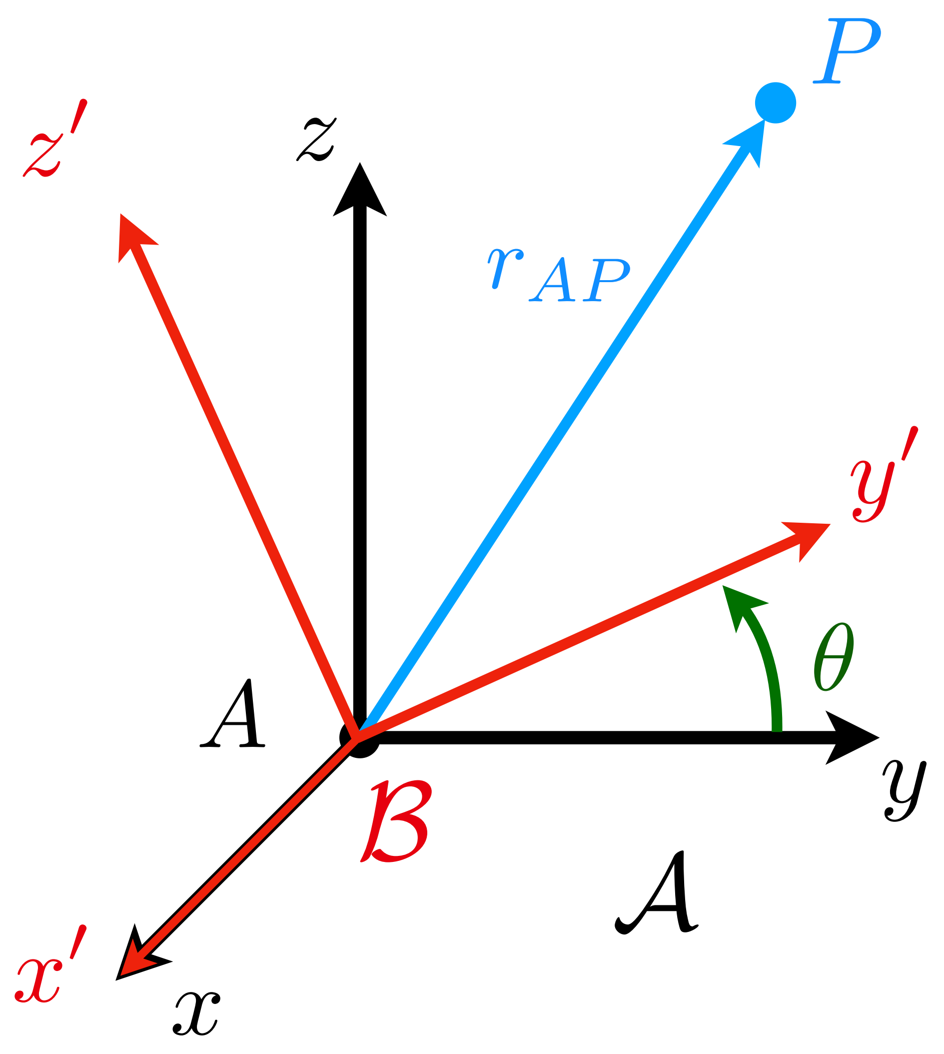
position vectors



$$_{\mathcal{A}}\vec{r}_{AP} = \begin{bmatrix} _{\mathcal{A}}r_{AP,x} \\ _{\mathcal{A}}r_{AP,y} \\ _{\mathcal{A}}r_{AP,z} \end{bmatrix}$$

$$_{\mathcal{B}}\vec{r}_{AP} = C_{\mathcal{B}\boxed{\mathcal{A}\ _{\mathcal{A}}}}\vec{r}_{AP}$$

$C_{\mathcal{A}\mathcal{B}}$ is a change of representation (or basis)

$$C_{\mathcal{A}\mathcal{B}} = \begin{bmatrix} _{\mathcal{A}}\vec{e}_x^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_y^{\mathcal{B}} & _{\mathcal{A}}\vec{e}_z^{\mathcal{B}} \end{bmatrix}$$

$$C_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta) & -\sin(\theta) \\ 0 & \sin(\theta) & \cos(\theta) \end{bmatrix}$$

inverse: $C_{\mathcal{B}\mathcal{A}} = C_{\mathcal{A}\mathcal{B}}^{-1} = C_{\mathcal{A}\mathcal{B}}^{\top} \quad \leftrightarrow \quad C_{\mathcal{A}\mathcal{B}}\underbrace{C_{\mathcal{A}\mathcal{B}}^{\top}}_{C_{\mathcal{B}\mathcal{A}}} = \mathbb{I}_{3\times 3}$

$C_{\mathcal{A}\mathcal{B}}$ is an orthogonal matrix: $\det(C_{BA}) = 1$ (length preserving)

composition: $C_{\mathcal{A}\mathcal{C}} = C_{\mathcal{A}\mathcal{B}}\, C_{\mathcal{B}\mathcal{C}}$

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Robot Dynamics Exercise Session 01

homogeneous transformations

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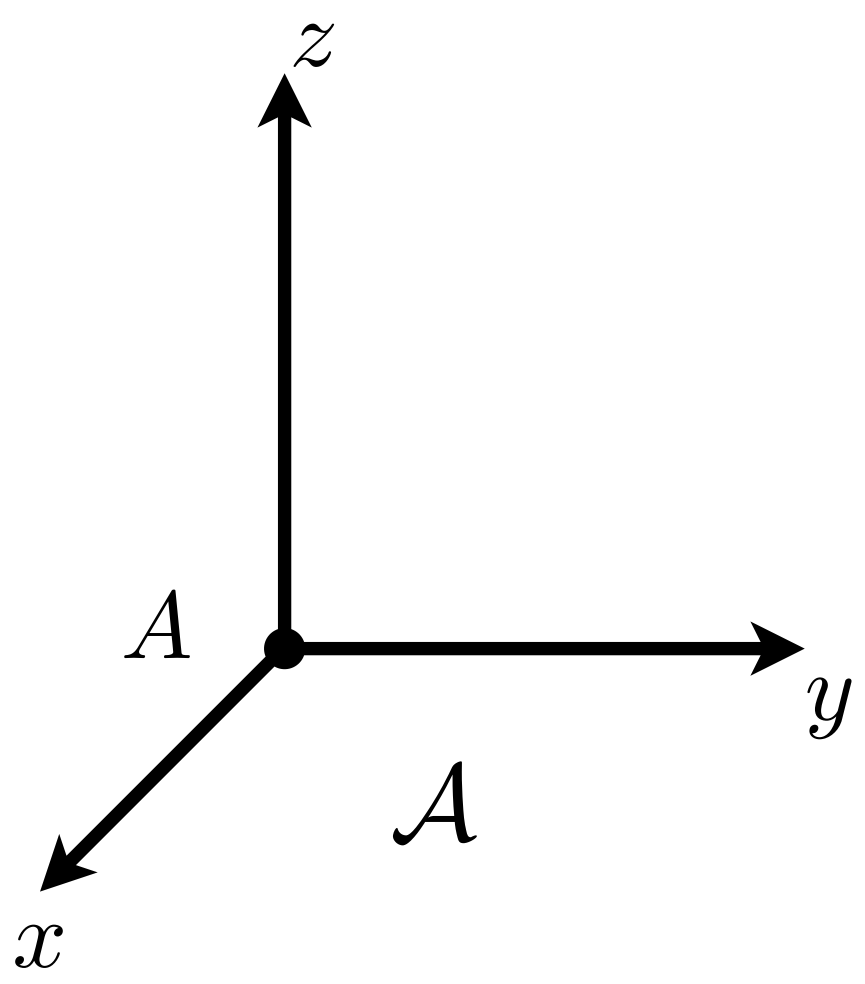
Robot Dynamics Exercise Session 01

homogeneous transformations



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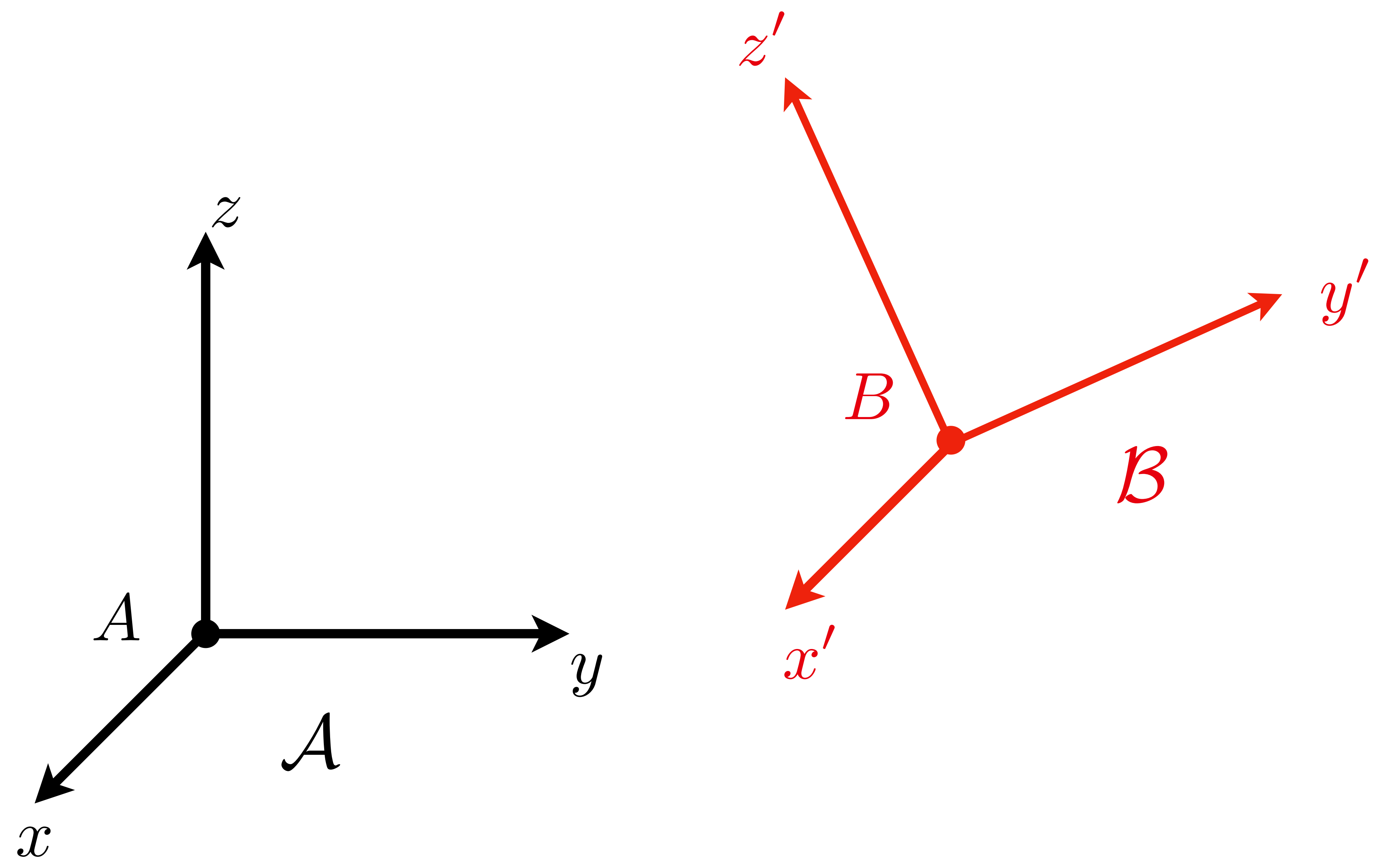
homogeneous transformations



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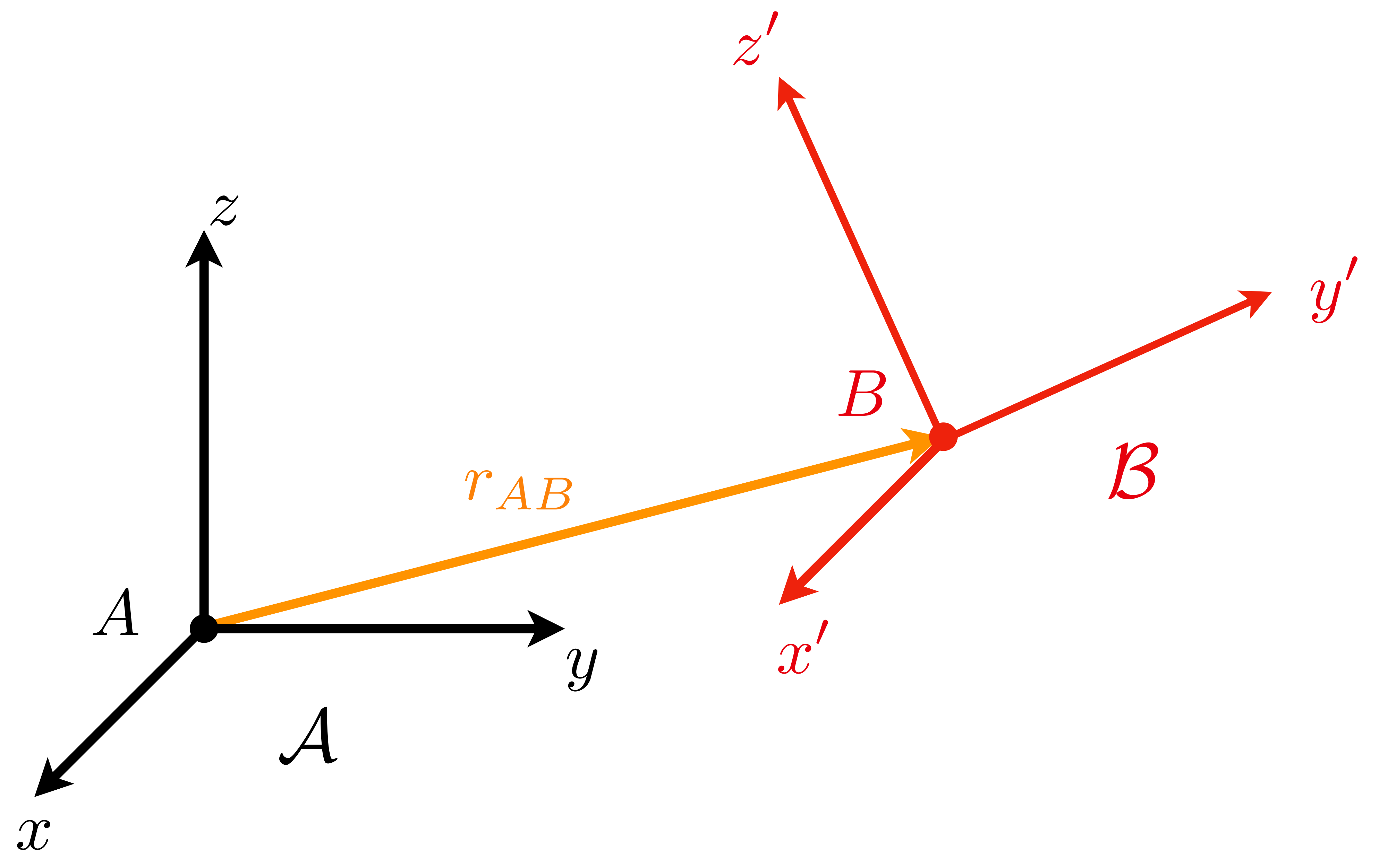
homogeneous transformations



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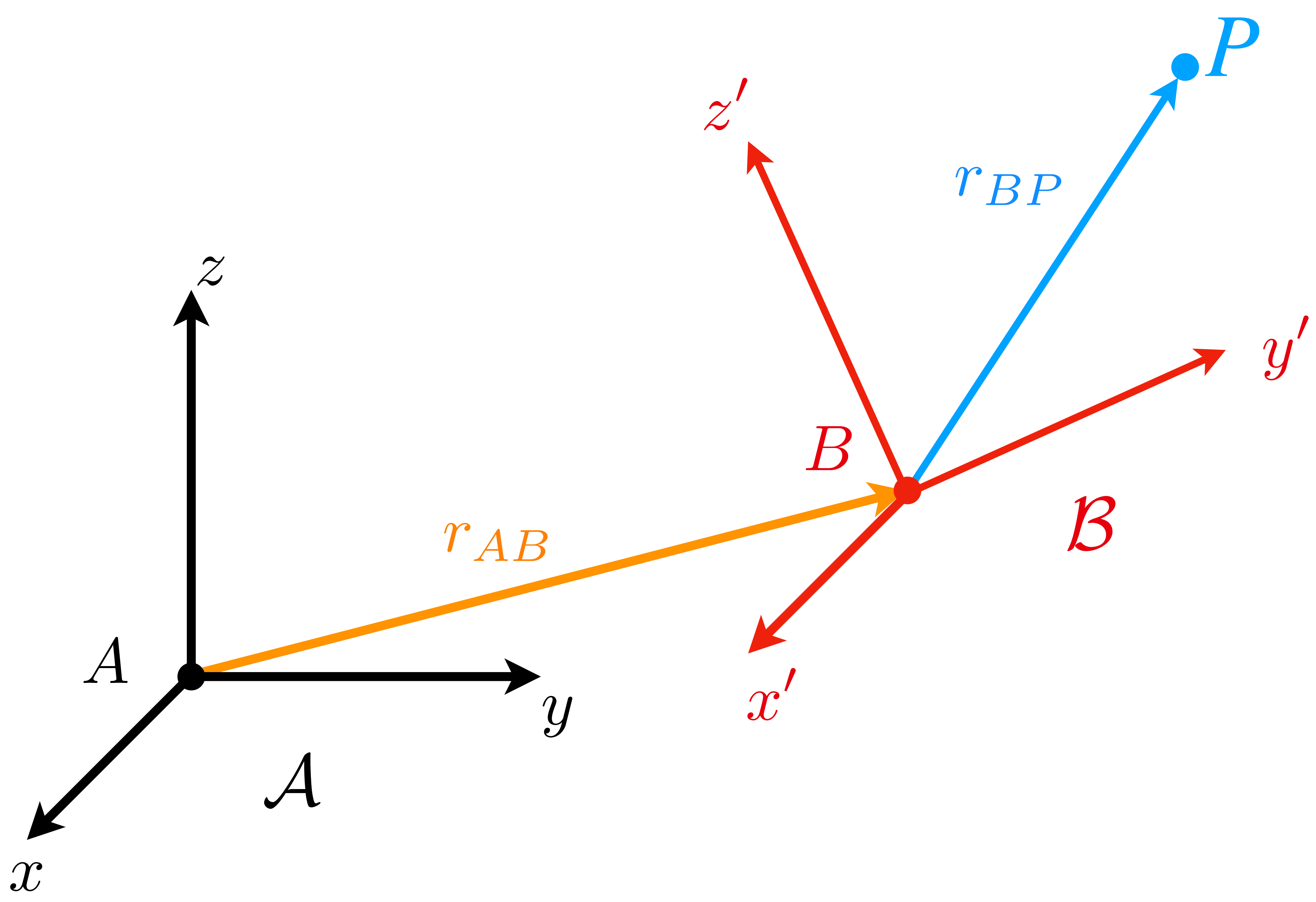
homogeneous transformations



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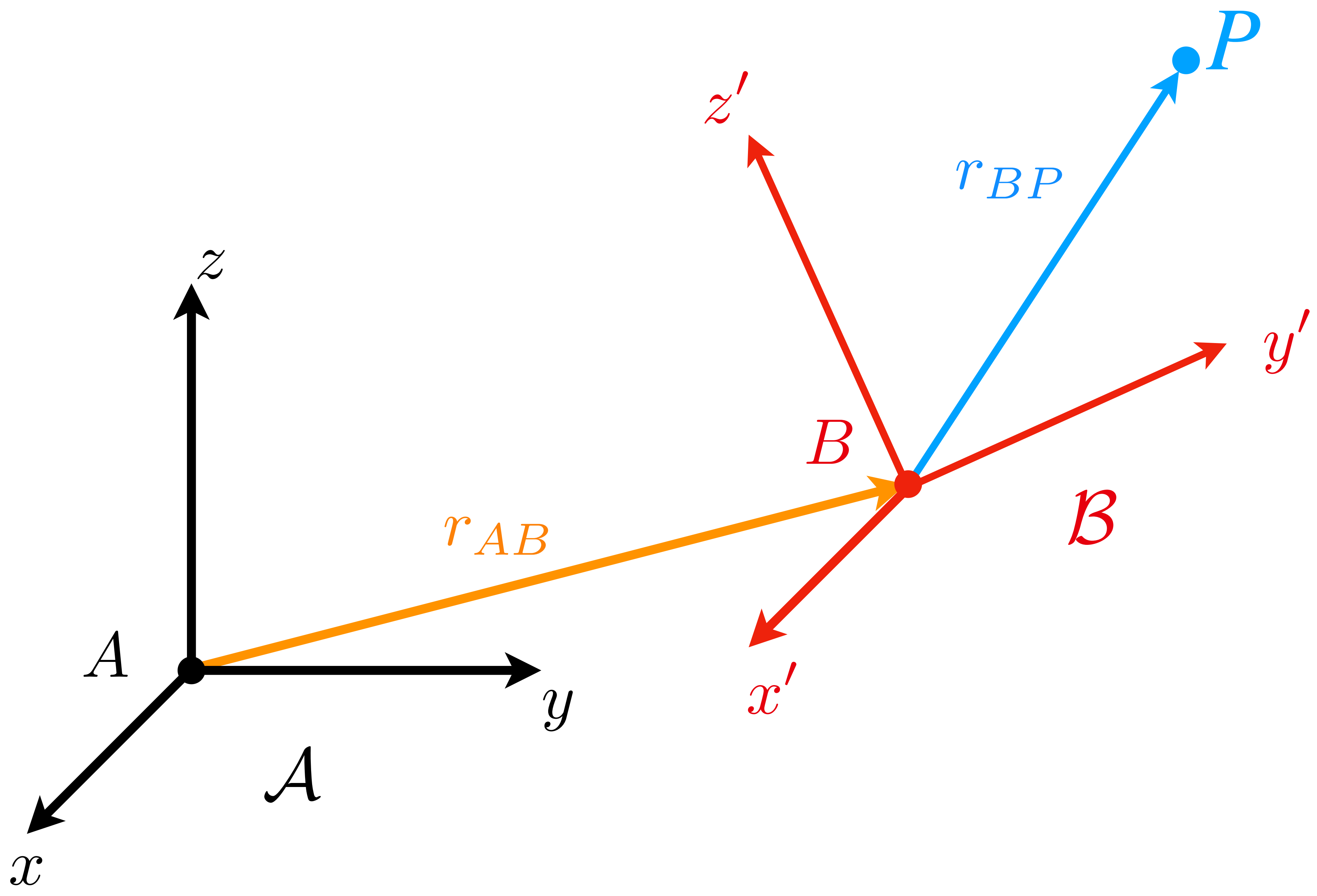
homogeneous transformations



given: $\quad {}_\mathcal{A}\vec{r}_{AB},\ C_{\mathcal{AB}},\ {}_\mathcal{B}\vec{r}_{BP}$

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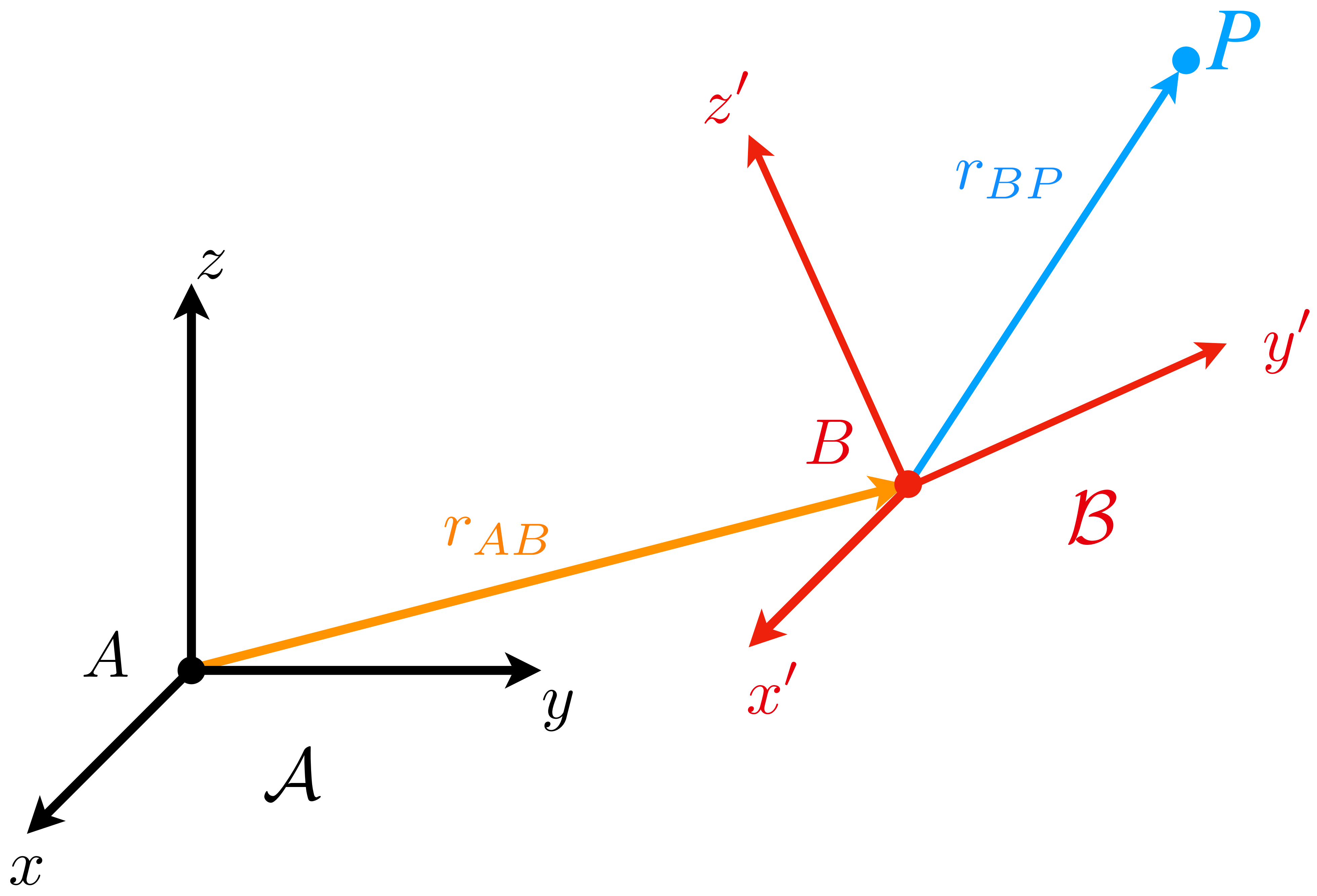
homogeneous transformations

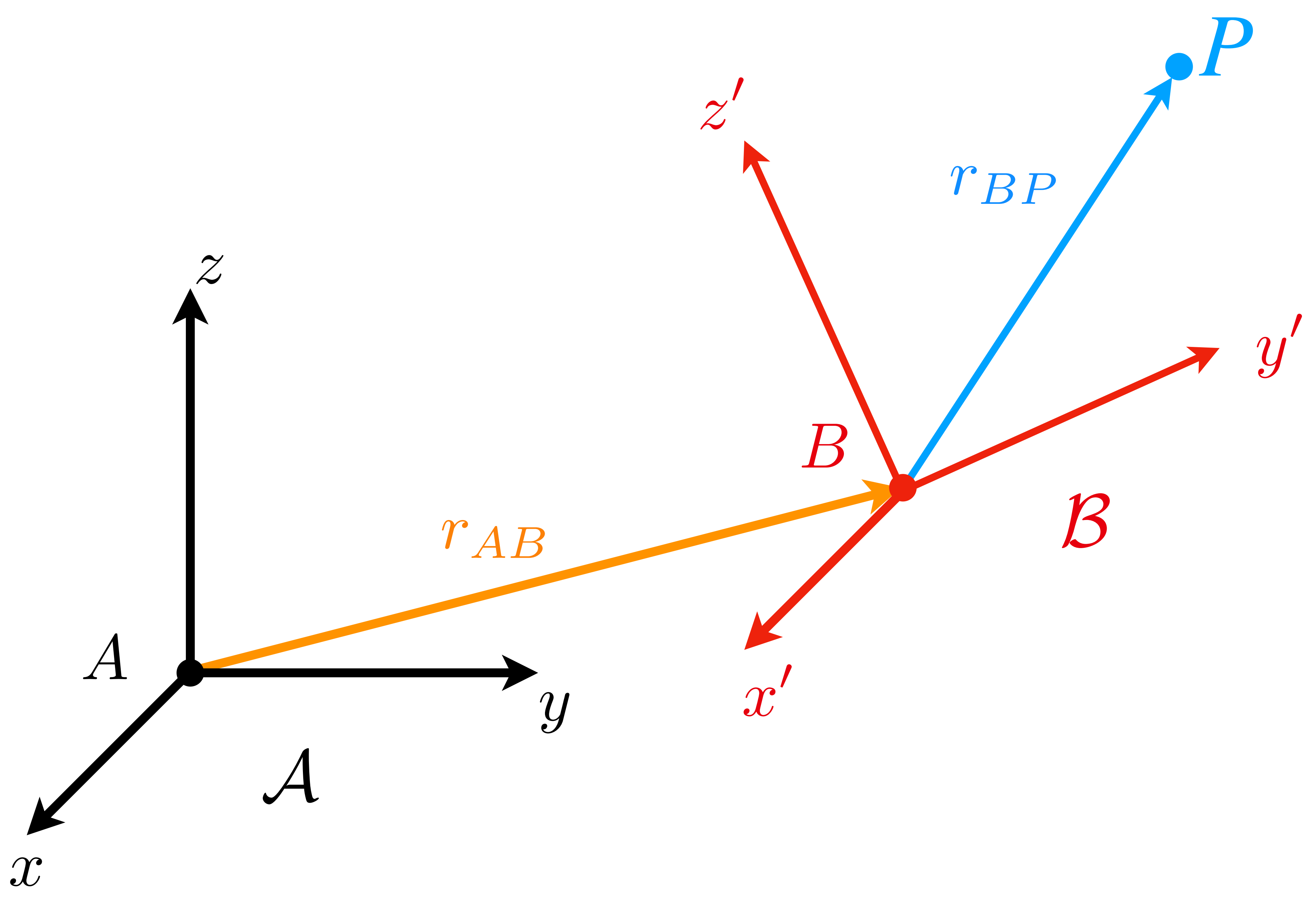


given: $_{\mathcal{A}}\vec{r}_{AB}$, $C_{\mathcal{AB}}$, $_{\mathcal{B}}\vec{r}_{BP}$

find: $_{\mathcal{A}}\vec{r}_{AP}$

Robot Dynamics Exercise Session 01

homogeneous transformations



given: ${}_{\mathcal{A}}\vec{r}_{AB}$, $C_{\mathcal{AB}}$, ${}_{\mathcal{B}}\vec{r}_{BP}$

find: ${}_{\mathcal{A}}\vec{r}_{AP}$

$$\vec{r}_{AP} = \vec{r}_{AB} + \vec{r}_{BP}$$
$${}_{\mathcal{A}}\vec{r}_{AP} = {}_{\mathcal{A}}\vec{r}_{AB} + {}_{\mathcal{A}}\vec{r}_{BP}$$
$${}_{\mathcal{A}}\vec{r}_{AP} = {}_{\mathcal{A}}\vec{r}_{AB} + C_{\mathcal{AB}}\,{}_{\mathcal{B}}\vec{r}_{BP}$$

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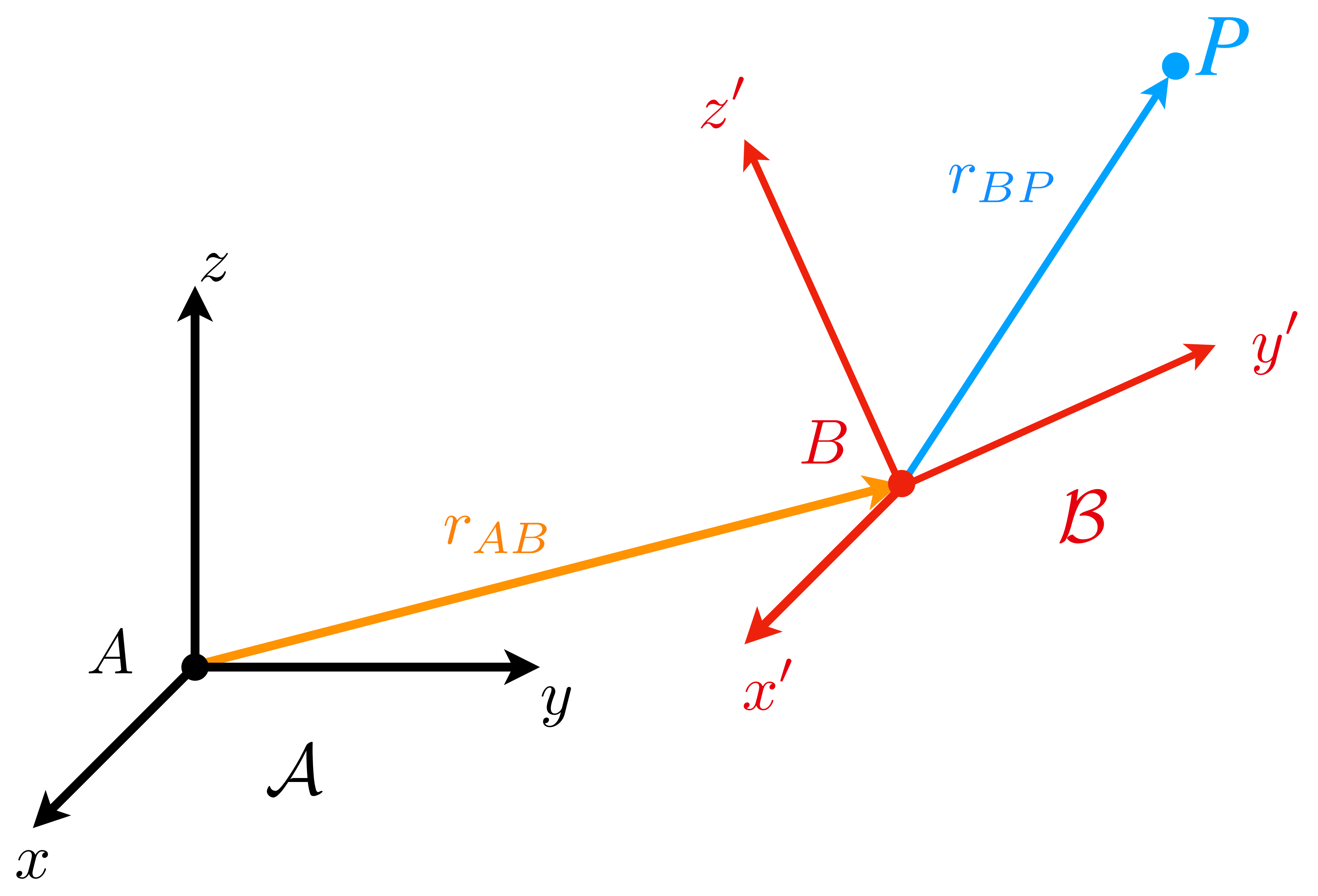
homogeneous transformations



given: ${}_{\mathcal{A}}\vec{r}_{AB}$, $C_{\mathcal{AB}}$, ${}_{\mathcal{B}}\vec{r}_{BP}$

find: ${}_{\mathcal{A}}\vec{r}_{AP}$

$$\vec{r}_{AP} = \vec{r}_{AB} + \vec{r}_{BP}$$
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$${}_{\mathcal{A}}\vec{r}_{AP} = {}_{\mathcal{A}}\vec{r}_{AB} + C_{\mathcal{AB}}\,{}_{\mathcal{B}}\vec{r}_{BP}$$

homogeneous coordinates:

$$\begin{bmatrix} {}_{\mathcal{A}}\vec{r}_{AP} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} C_{\mathcal{AB}} & {}_{\mathcal{A}}\vec{r}_{AB} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{T_{\mathcal{AB}}\ (4\times 4)} \begin{bmatrix} {}_{\mathcal{B}}\vec{r}_{BP} \\ 1 \end{bmatrix}$$

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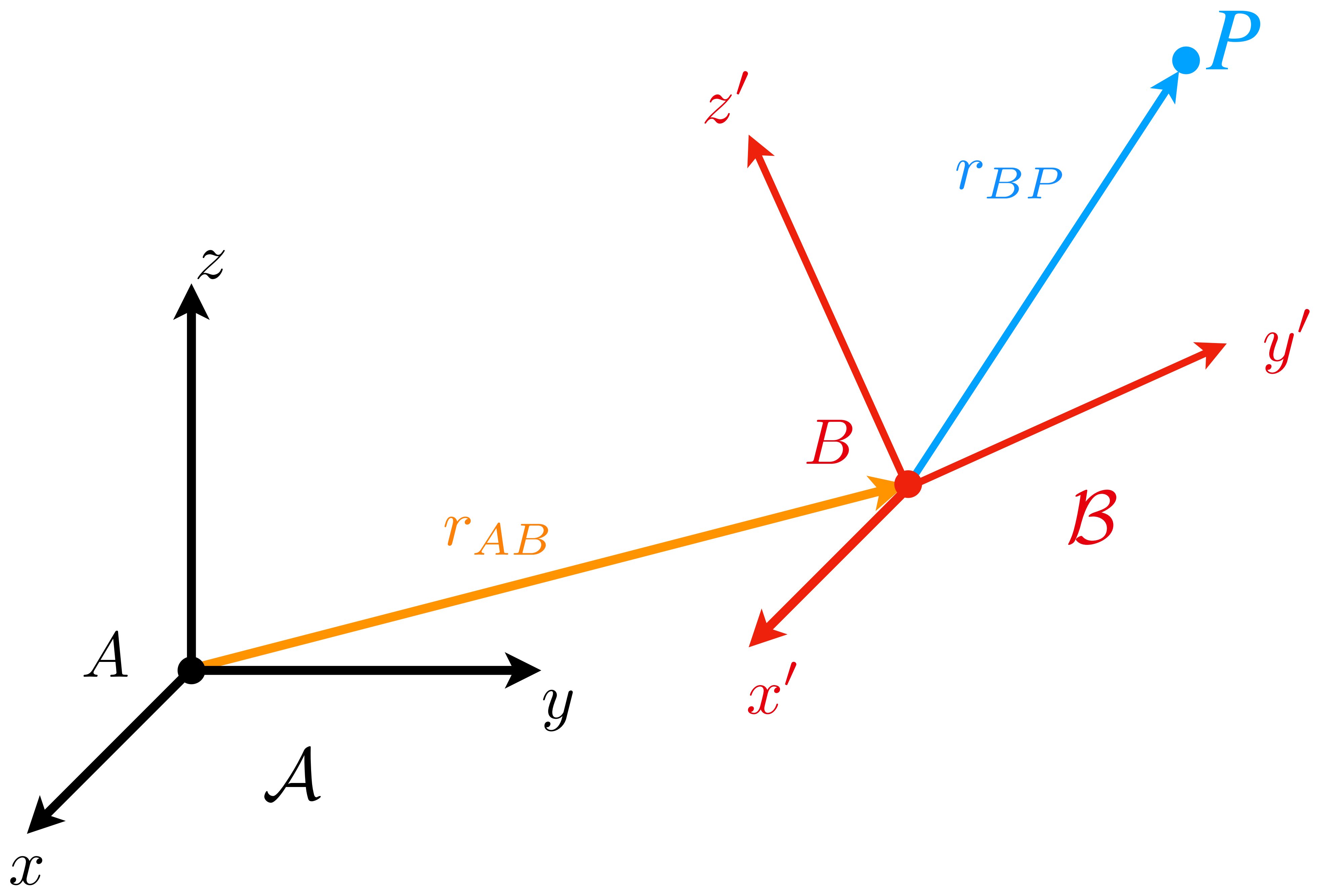
homogeneous transformations



given: $_{\mathcal{A}}\vec{r}_{AB}$, $C_{\mathcal{AB}}$, $_{\mathcal{B}}\vec{r}_{BP}$

find: $_{\mathcal{A}}\vec{r}_{AP}$

$$\vec{r}_{AP} = \vec{r}_{AB} + \vec{r}_{BP}$$
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homogeneous coordinates:

$$\begin{bmatrix} _{\mathcal{A}}\vec{r}_{AP} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} C_{\mathcal{AB}} & _{\mathcal{A}}\vec{r}_{AB} \\ 0_{1\times 3} & 1 \end{bmatrix}}_{T_{\mathcal{AB}}\ (4\times 4)} \begin{bmatrix} _{\mathcal{B}}\vec{r}_{BP} \\ 1 \end{bmatrix}$$

change of representation and origin

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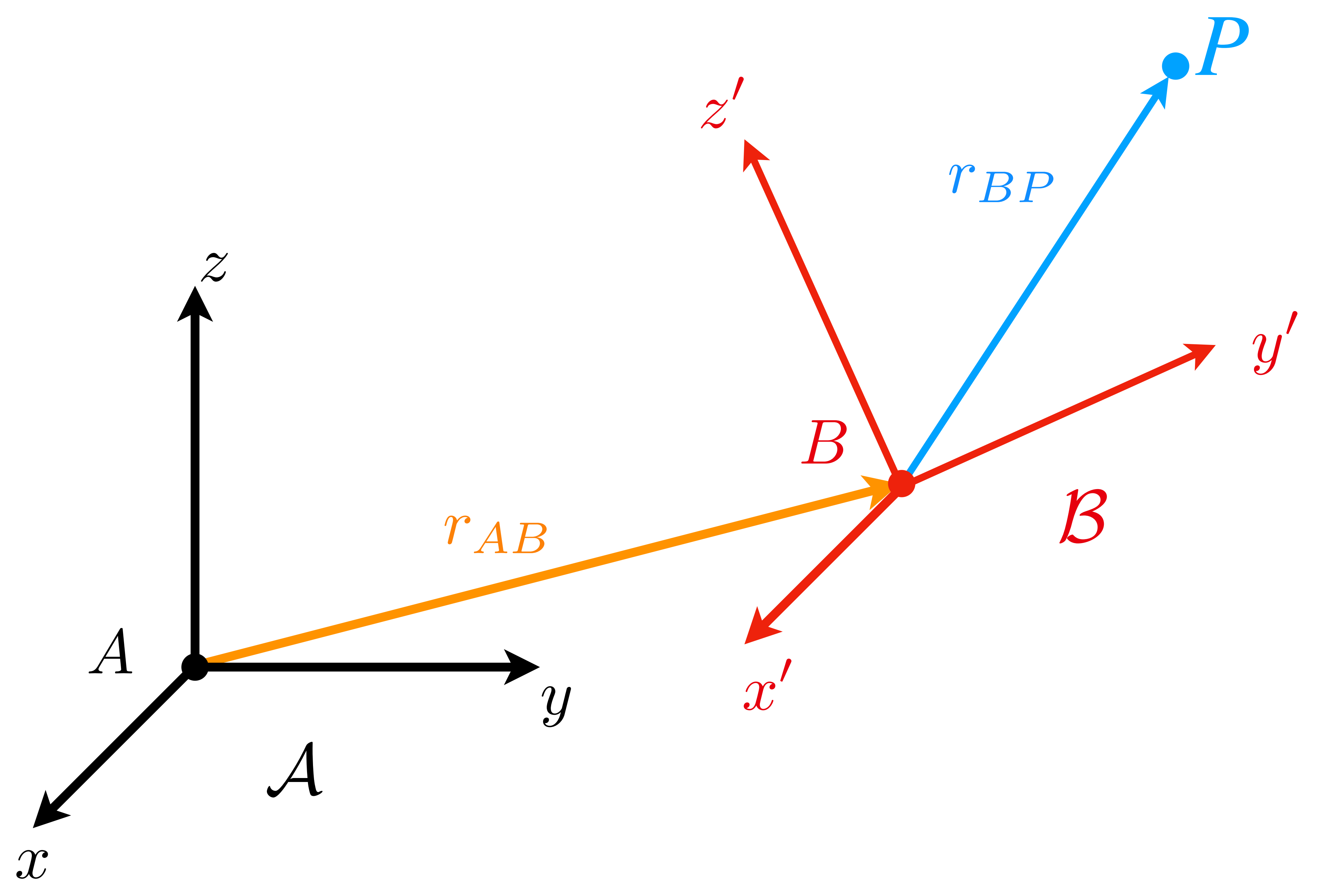
homogeneous transformations



given: ${}_{\mathcal{A}}\vec{r}_{AB}$, $C_{\mathcal{AB}}$, ${}_{\mathcal{B}}\vec{r}_{BP}$

find: ${}_{\mathcal{A}}\vec{r}_{AP}$

$$\vec{r}_{AP} = \vec{r}_{AB} + \vec{r}_{BP}$$
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homogeneous coordinates:

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change of representation and origin

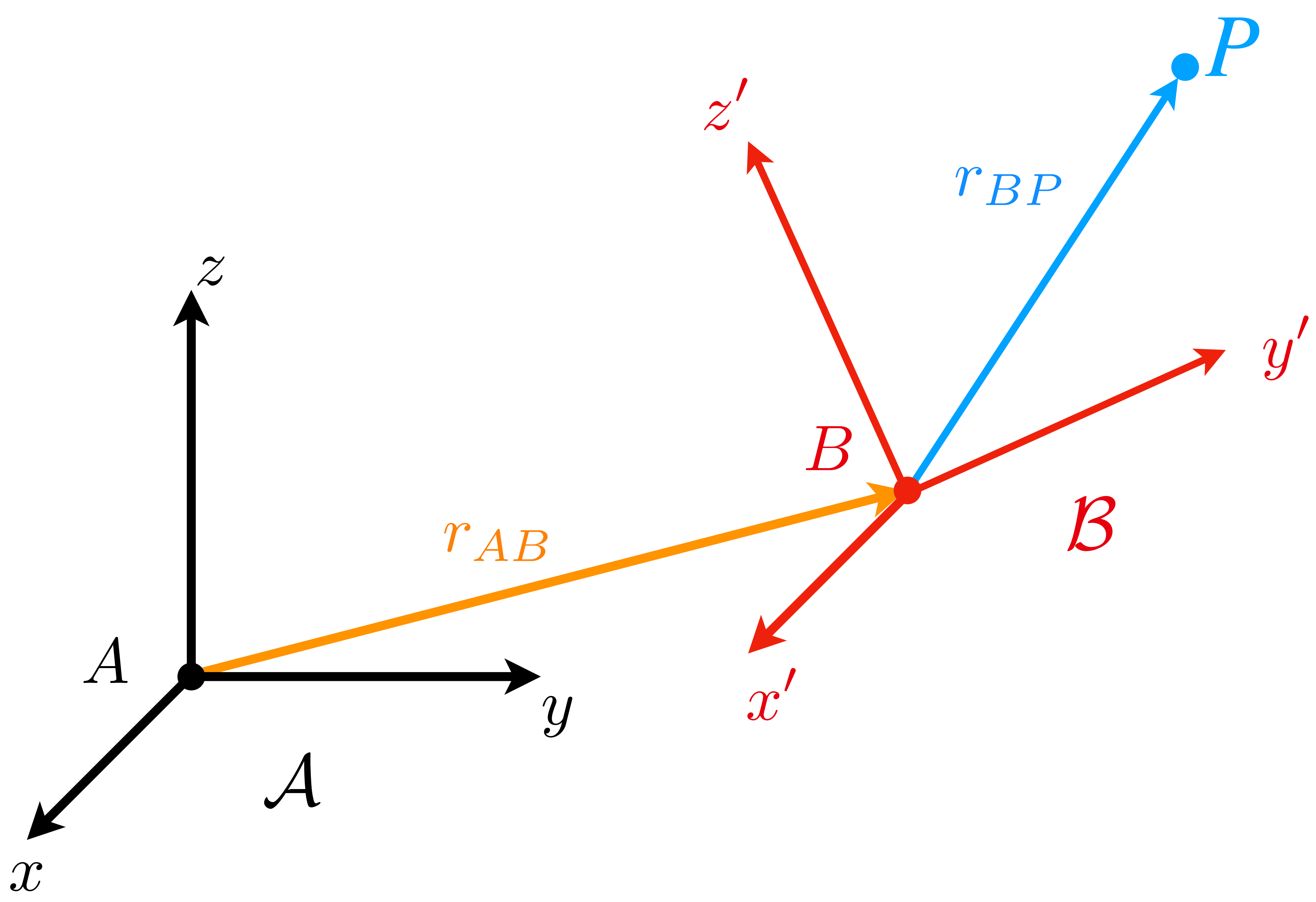
inverse: $$T_{\mathcal{AB}}^{-1} = T_{\mathcal{BA}} = \begin{bmatrix} C_{\mathcal{BA}} & {}_{\mathcal{B}}\vec{r}_{BA} \\ 0_{1\times 3} & 1 \end{bmatrix}$$
$$= \begin{bmatrix} C_{\mathcal{AB}}^{\top} & C_{\mathcal{BA}}\, {}_{\mathcal{A}}\vec{r}_{BA} \\ 0_{1\times 3} & 1 \end{bmatrix}$$
$$= \begin{bmatrix} C_{\mathcal{AB}}^{\top} & -C_{\mathcal{AB}}^{\top}\, {}_{\mathcal{A}}\vec{r}_{AB} \\ 0_{1\times 3} & 1 \end{bmatrix}$$

Robot Dynamics Exercise Session 01

homogeneous transformations

given: ${}_\mathcal{A}\vec{r}_{AB},\ C_{\mathcal{AB}},\ {}_\mathcal{B}\vec{r}_{BP}$

find: ${}_\mathcal{A}\vec{r}_{AP}$



$$\vec{r}_{AP} = \vec{r}_{AB} + \vec{r}_{BP}$$
$${}_\mathcal{A}\vec{r}_{AP} = {}_\mathcal{A}\vec{r}_{AB} + {}_\mathcal{A}\vec{r}_{BP}$$
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homogeneous coordinates:

$$\begin{bmatrix} {}_\mathcal{A}\vec{r}_{AP} \\ 1 \end{bmatrix} = \underbrace{\begin{bmatrix} C_{\mathcal{AB}} & {}_\mathcal{A}\vec{r}_{AB} \\ 0_{1\times3} & 1 \end{bmatrix}}_{T_{\mathcal{AB}}\ (4\times4)} \begin{bmatrix} {}_\mathcal{B}\vec{r}_{BP} \\ 1 \end{bmatrix}$$

change of representation and origin

inverse: $$T_{\mathcal{AB}}^{-1} = T_{\mathcal{BA}} = \begin{bmatrix} C_{\mathcal{BA}} & {}_\mathcal{B}\vec{r}_{BA} \\ 0_{1\times3} & 1 \end{bmatrix}$$
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$$= \begin{bmatrix} C_{\mathcal{AB}}^\top & -C_{\mathcal{AB}}^\top\,{}_\mathcal{A}\vec{r}_{AB} \\ 0_{1\times3} & 1 \end{bmatrix}$$

$$\boxed{\vec{r}_{AB} = -\vec{r}_{BA}}$$

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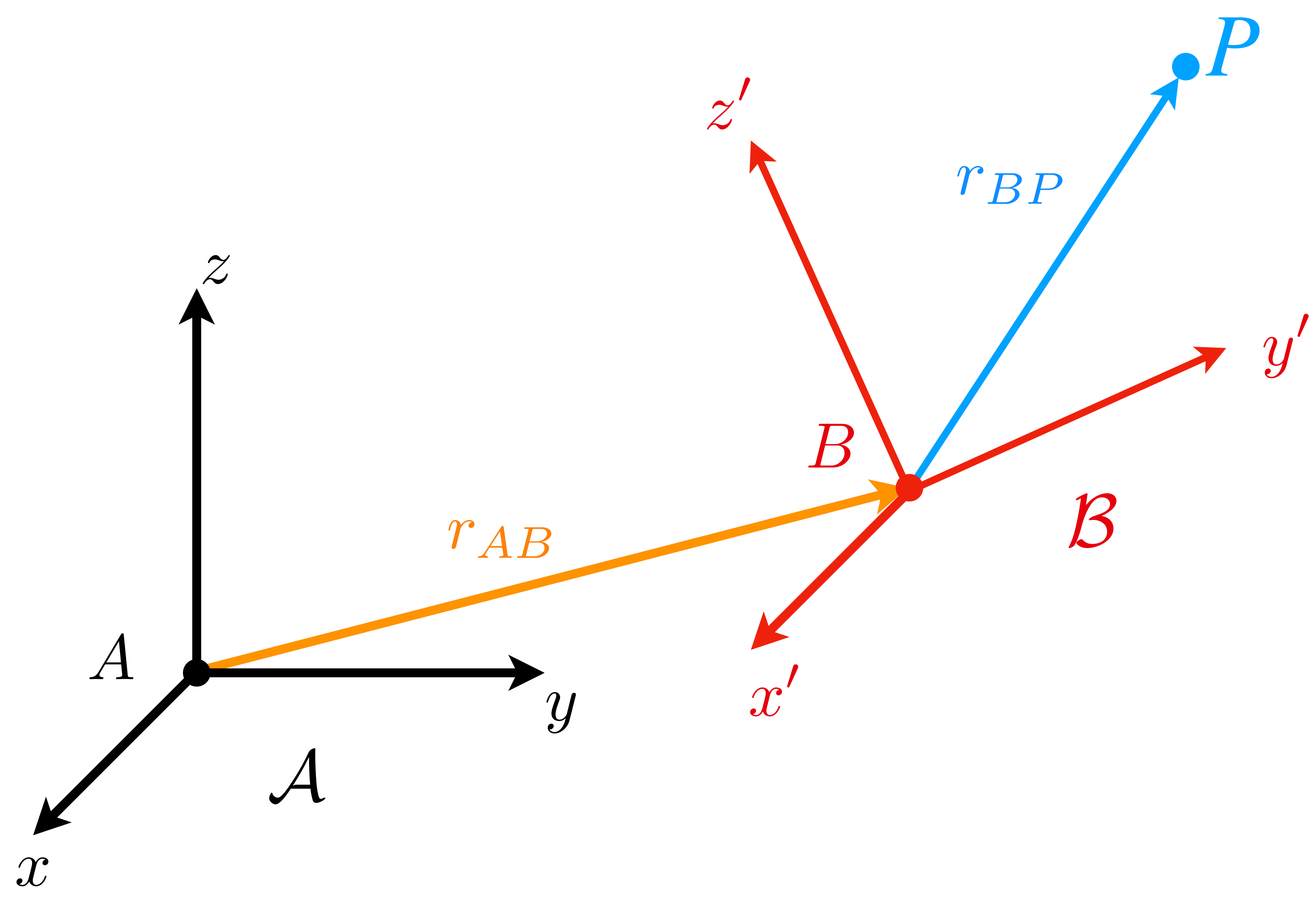
homogeneous transformations



given: $_{\mathcal{A}}\vec{r}_{AB},\ C_{\mathcal{AB}},\ _{\mathcal{B}}\vec{r}_{BP}$

find: $_{\mathcal{A}}\vec{r}_{AP}$

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$$_{\mathcal{A}}\vec{r}_{AP} = {_{\mathcal{A}}\vec{r}_{AB}} + {_{\mathcal{A}}\vec{r}_{BP}}$$
$$_{\mathcal{A}}\vec{r}_{AP} = {_{\mathcal{A}}\vec{r}_{AB}} + C_{\mathcal{AB}}\, {_{\mathcal{B}}\vec{r}_{BP}}$$

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change of representation and origin

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$$\boxed{\vec{r}_{AB} = -\vec{r}_{BA}}$$

composition:

$$T_{\mathcal{AC}} = T_{\mathcal{AB}}\, T_{\mathcal{BC}}$$

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example: kinematics of a planar manipulator

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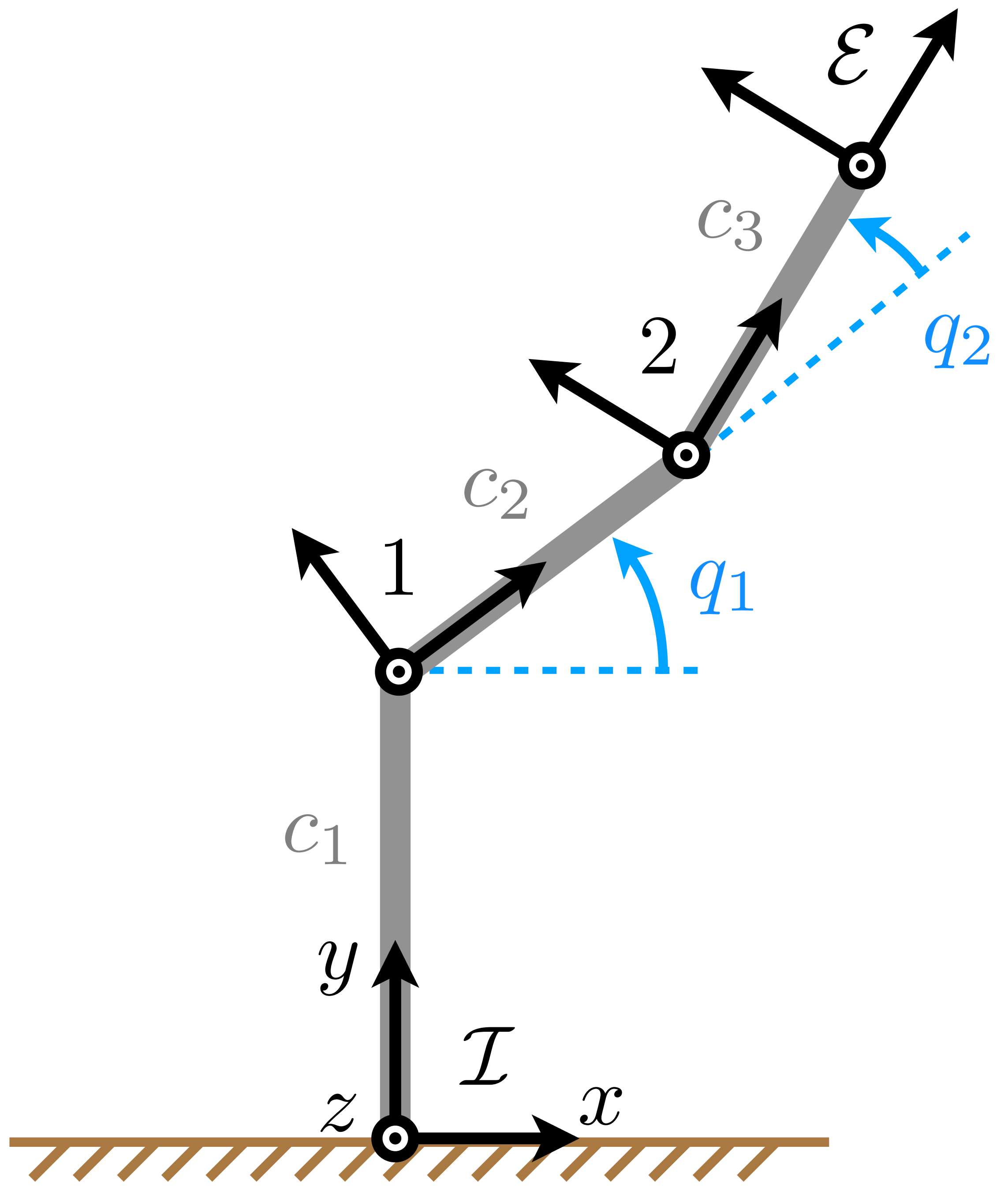
example: kinematics of a planar manipulator



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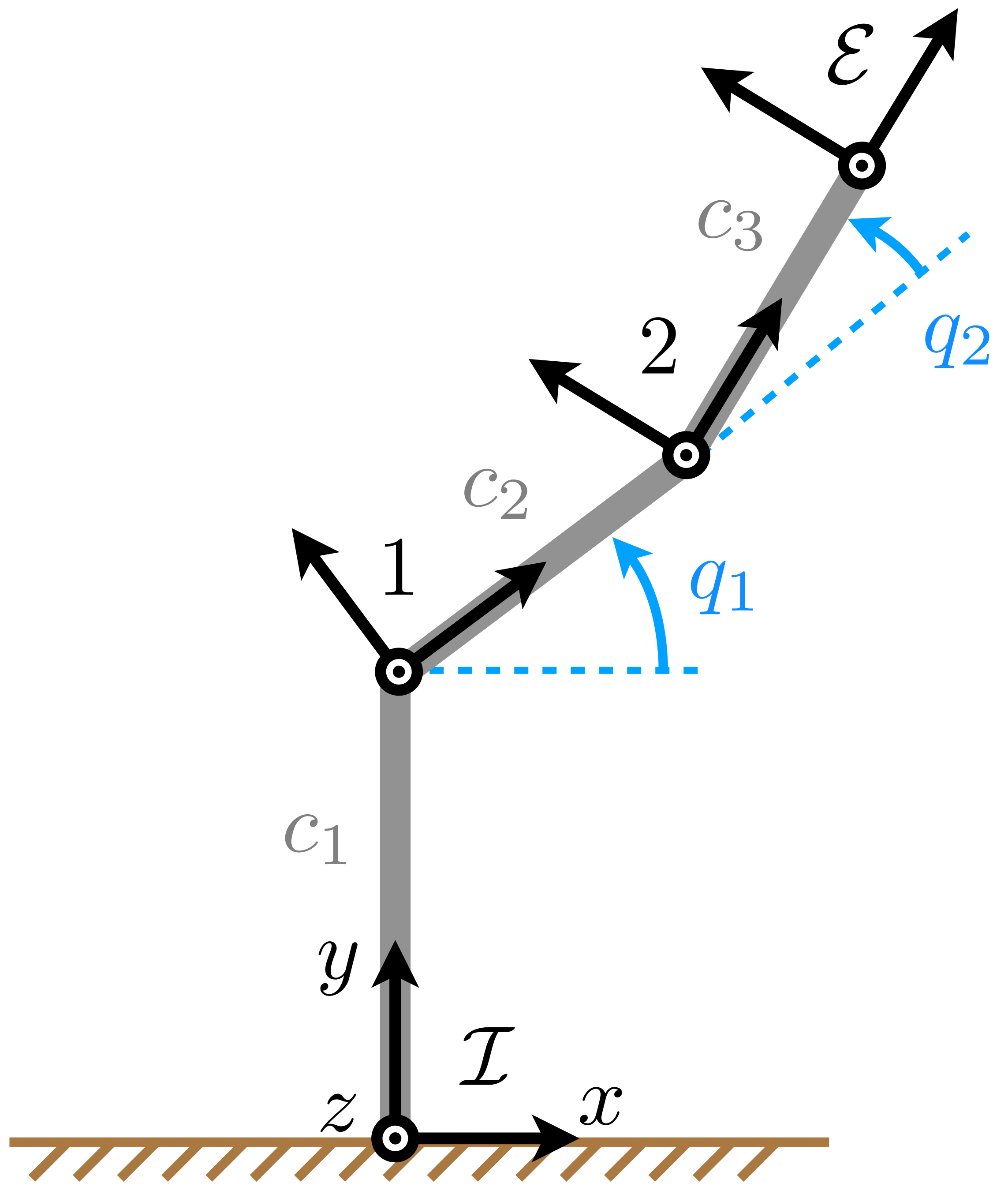
example: kinematics of a planar manipulator

find: $C_{\mathcal{I}\mathcal{E}}$, ${}_{\mathcal{I}}\vec{r}_{IE}$



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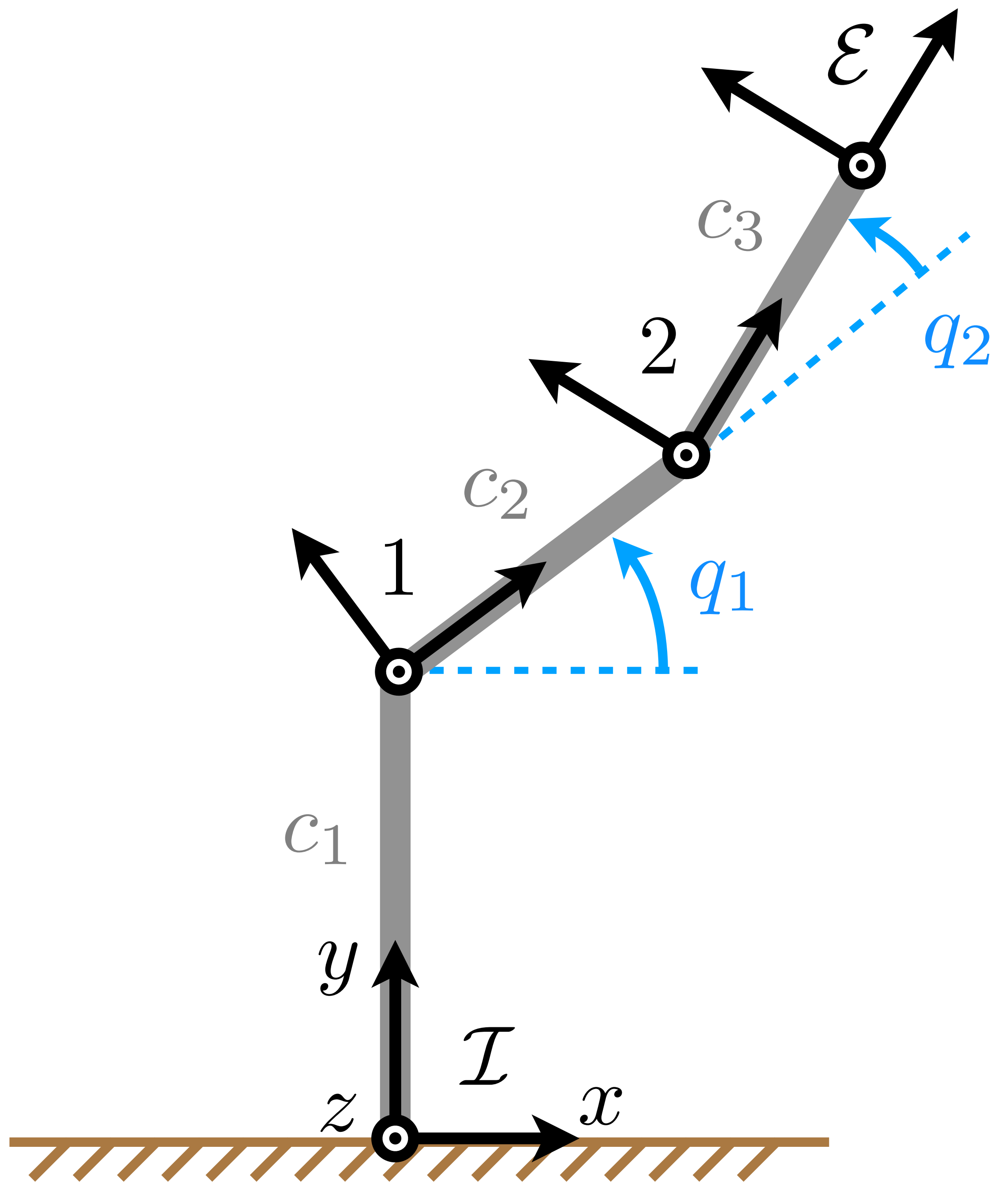
example: kinematics of a planar manipulator

$\boxed{\text{find:} \quad C_{\mathcal{I}\mathcal{E}},\ {}_{\mathcal{I}}\vec{r}_{IE}}$



Step 1: find $T_{\mathcal{I}1},\ T_{12},\ T_{2\mathcal{E}}$

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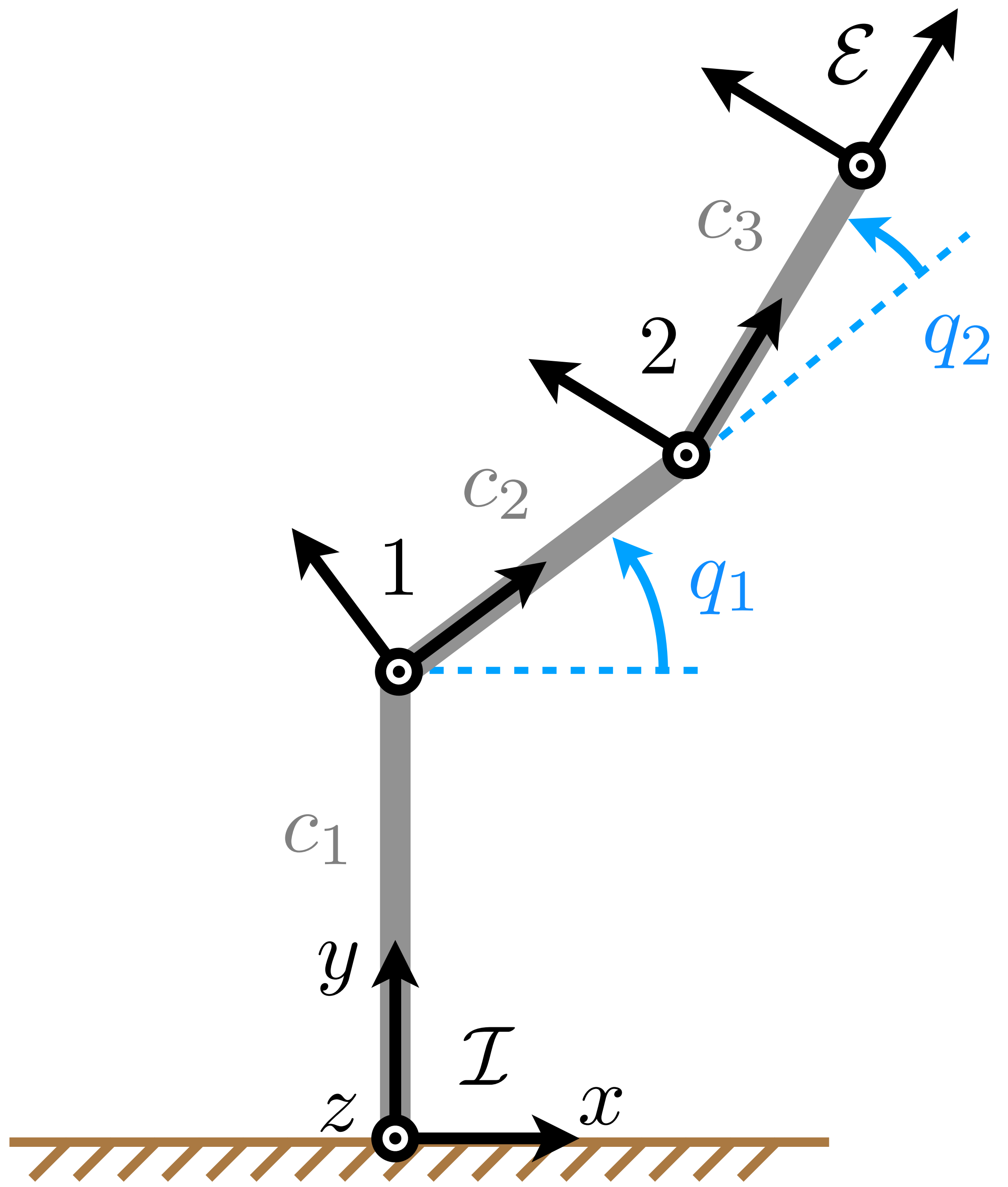
example: kinematics of a planar manipulator

find: $C_{\mathcal{I}\mathcal{E}}, \; _{\mathcal{I}}\vec{r}_{IE}$



Step 1: find $T_{\mathcal{I}1}, T_{12}, T_{2\mathcal{E}}$

$$T_{\mathcal{I}1} = \begin{bmatrix} C_z(q_1) & \begin{pmatrix} 0 \\ c_1 \\ 0 \end{pmatrix} \\ 0_{1\times3} & 1 \end{bmatrix}$$

Robot Dynamics Exercise Session 01

example: kinematics of a planar manipulator

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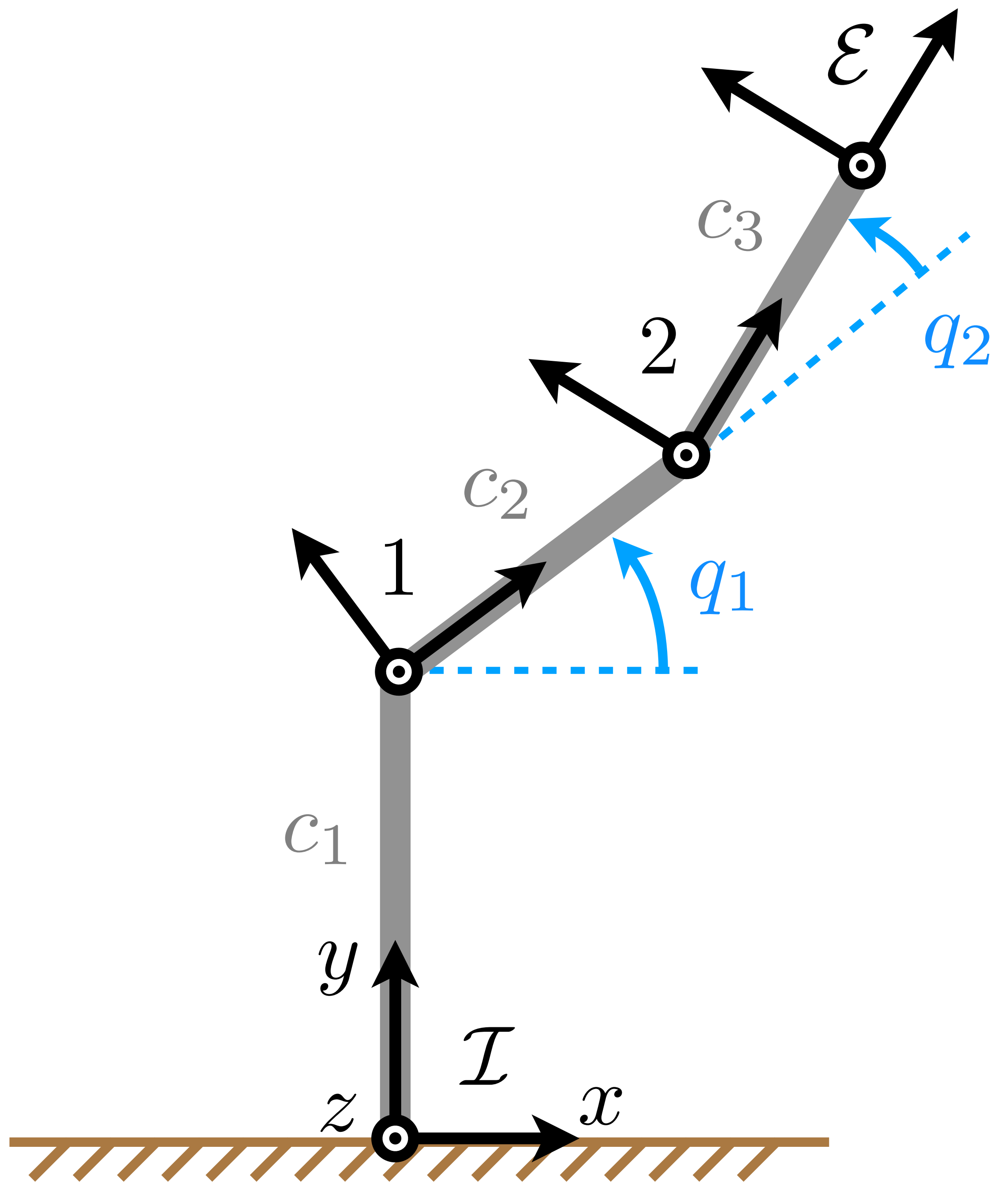

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Robot Dynamics Exercise Session 01

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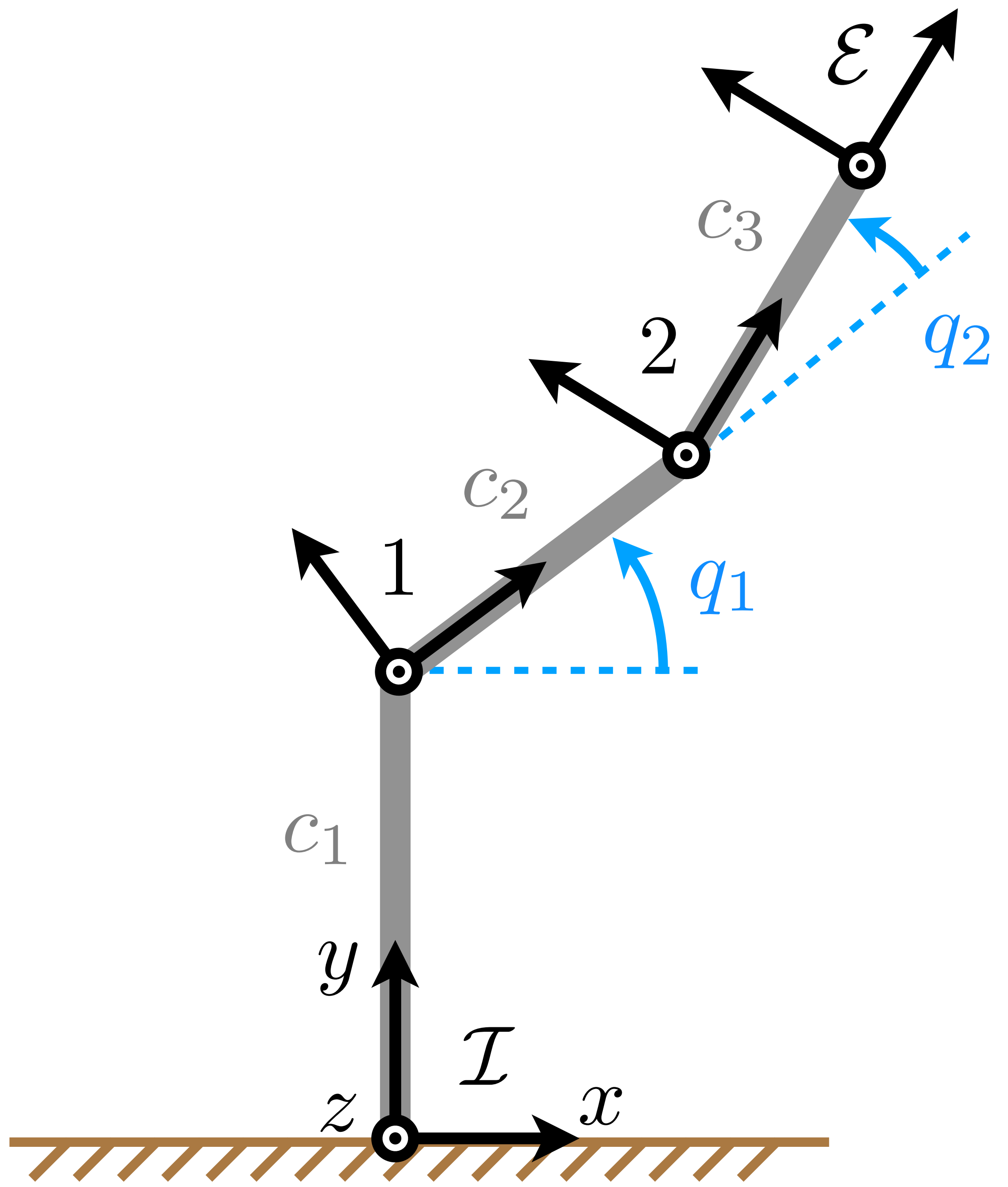

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Robot Dynamics Exercise Session 01

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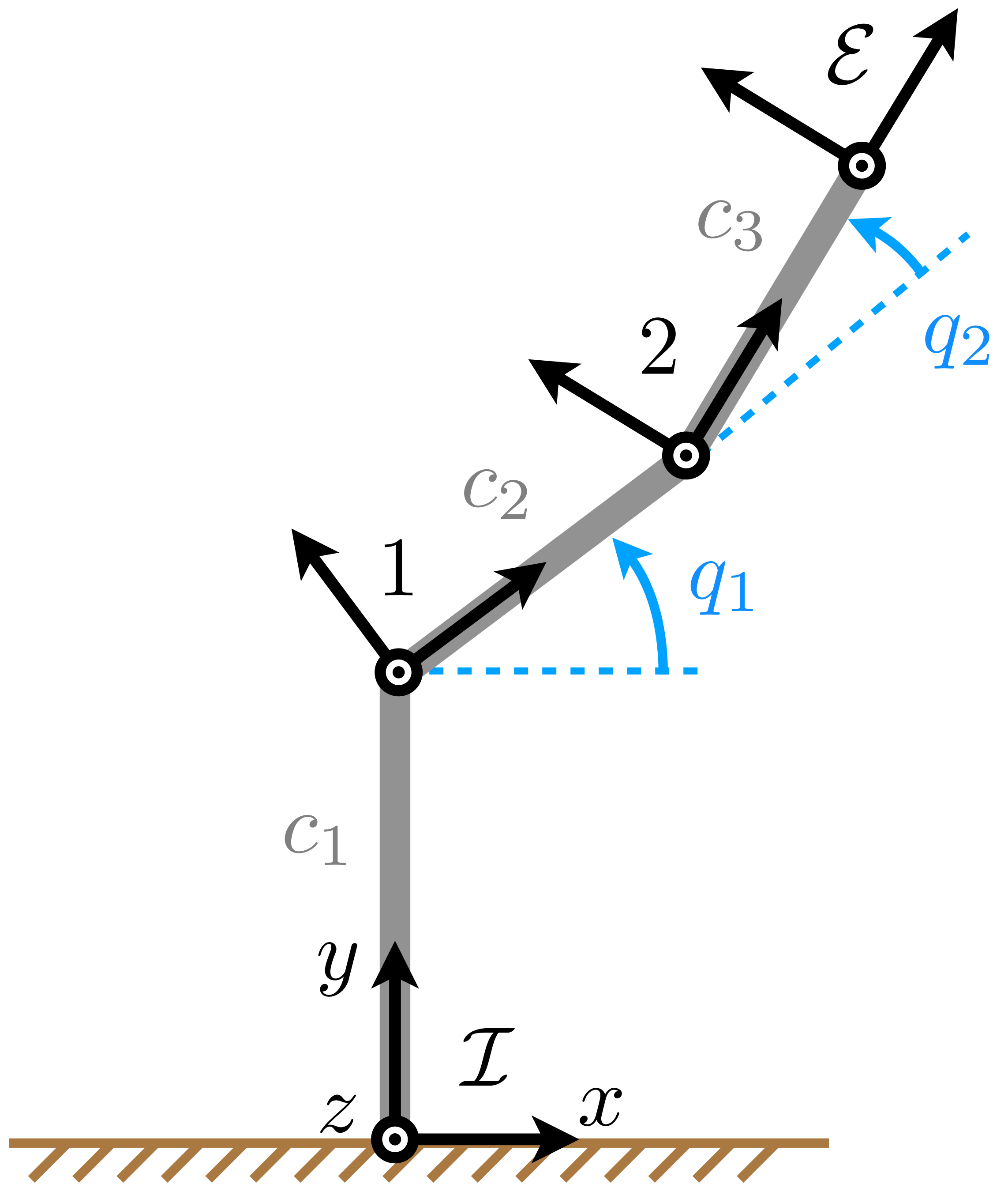

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Step 2: composition $T_{\mathcal{I}\mathcal{E}} = T_{\mathcal{I}1}\, T_{12}\, T_{2\mathcal{E}}$

Robot Dynamics Exercise Session 01

example: kinematics of a planar manipulator

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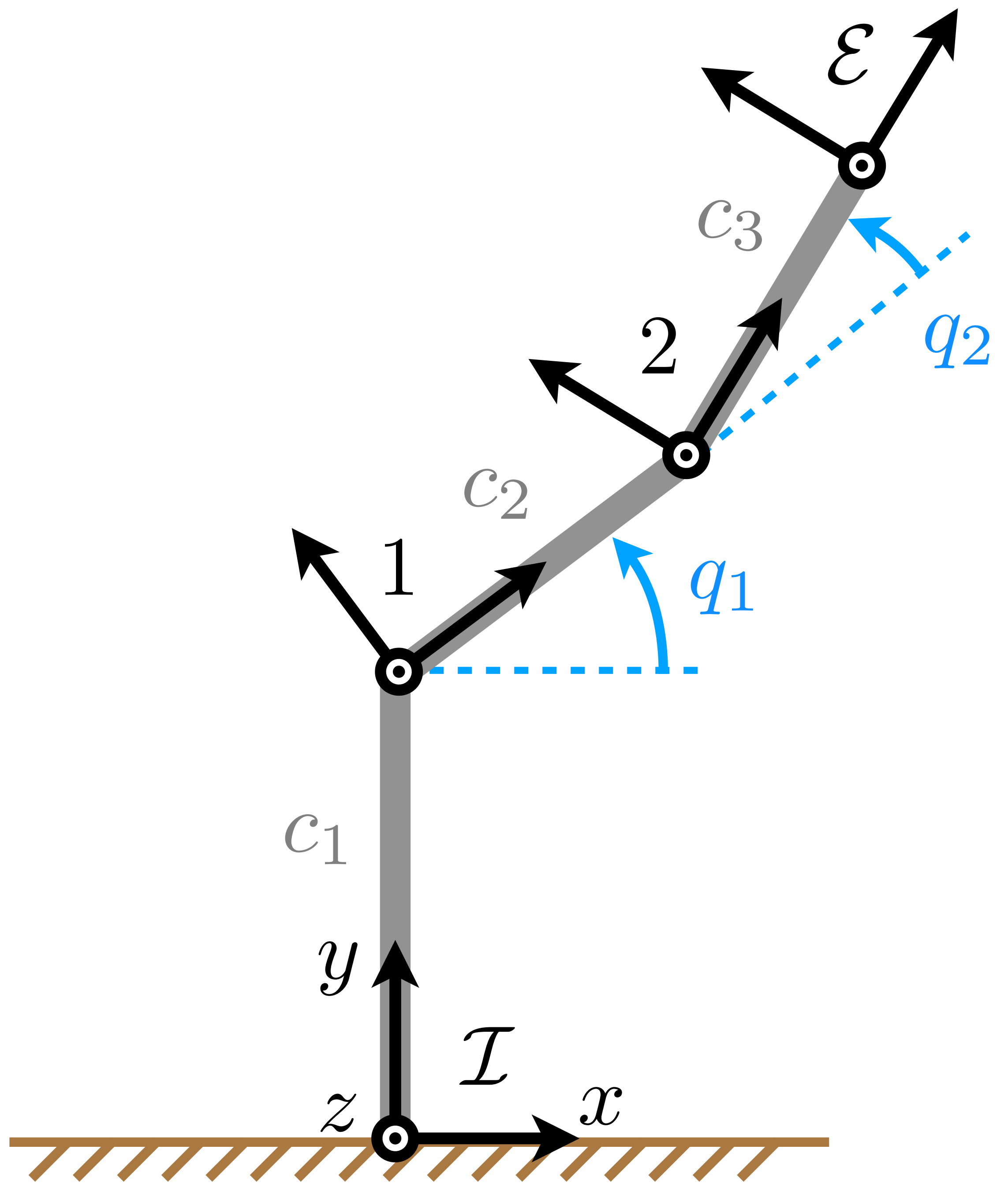

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Robot Dynamics Exercise Session 01

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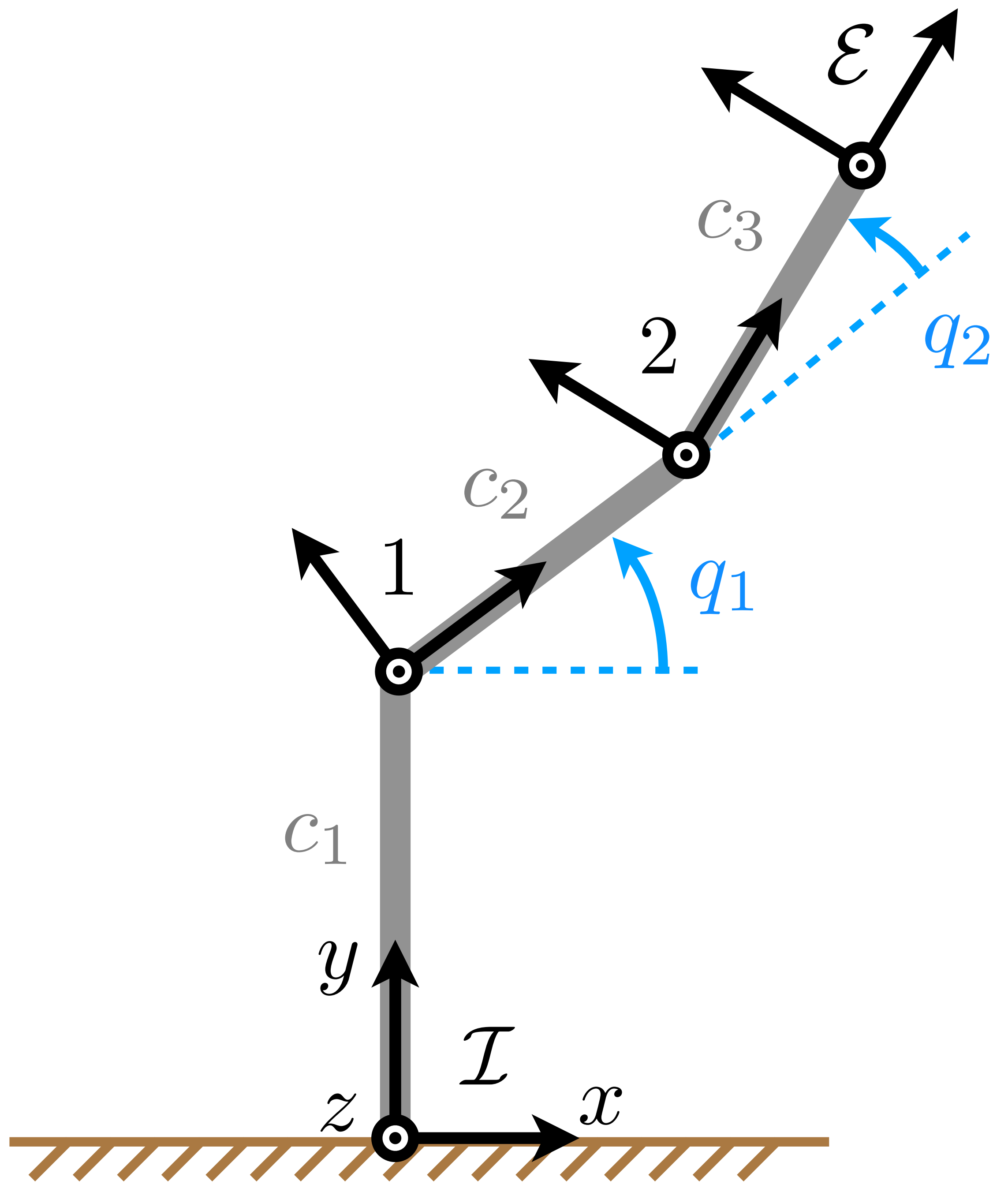

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Robotic Systems Lab