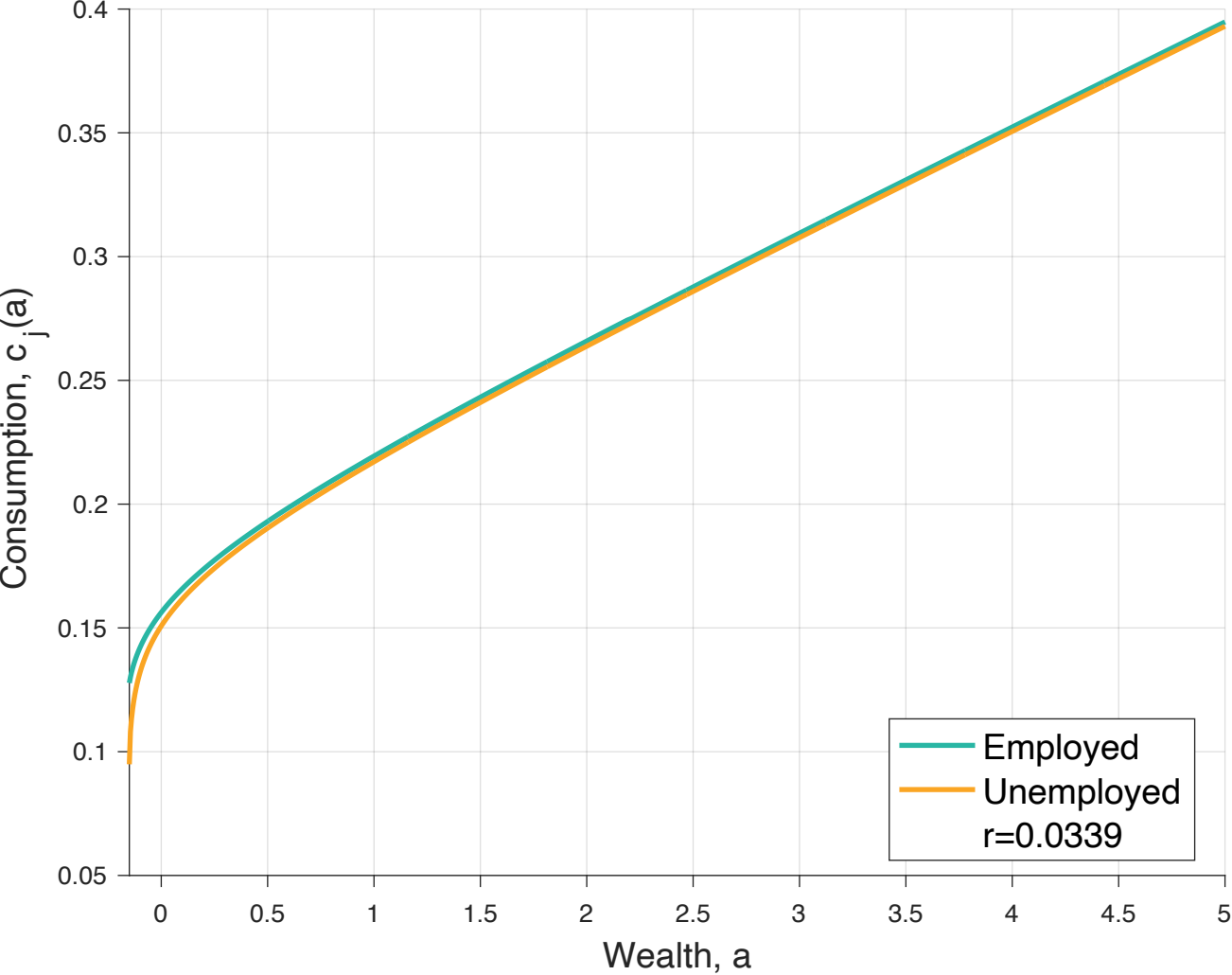
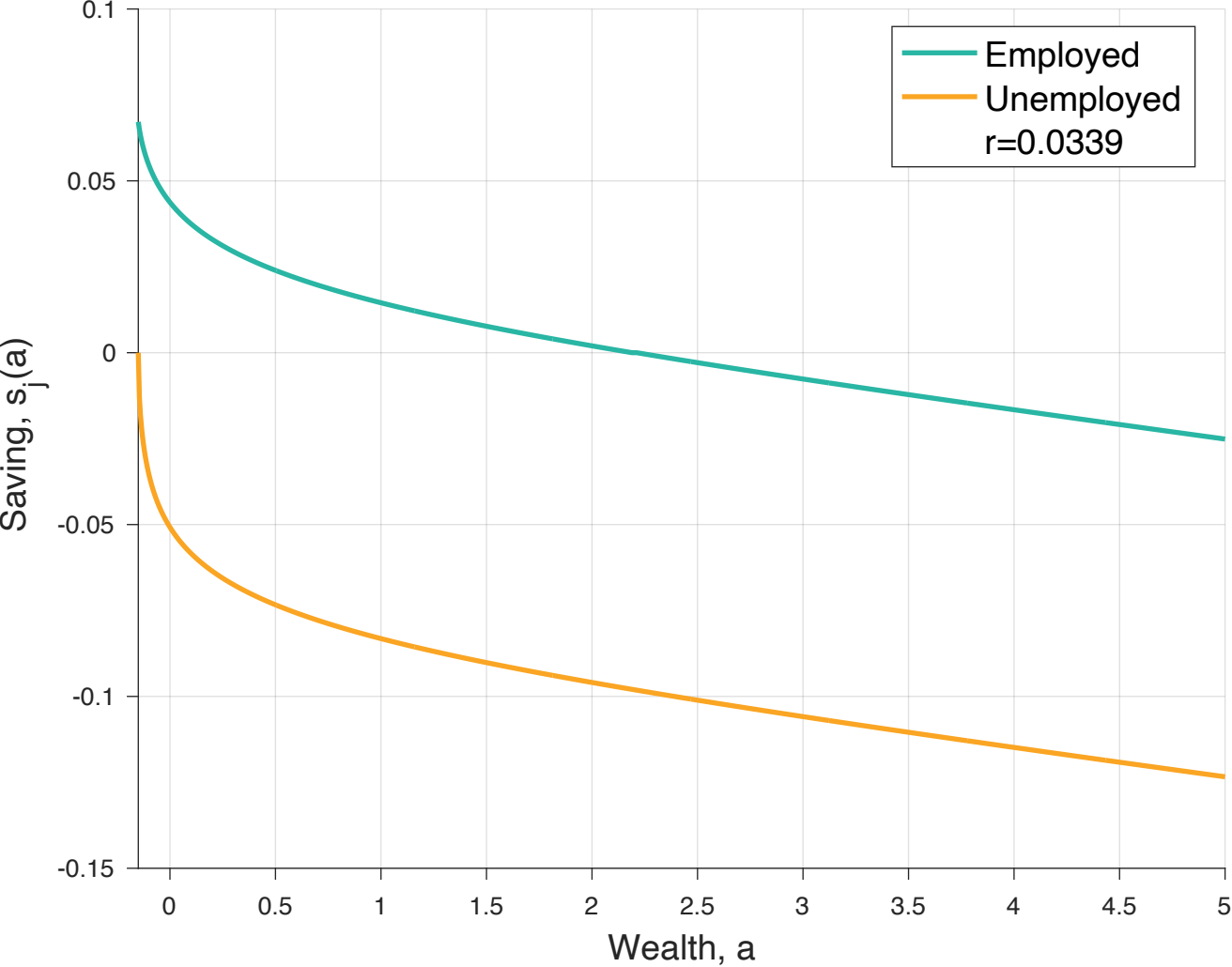
econ 202A; problem set 4 codings

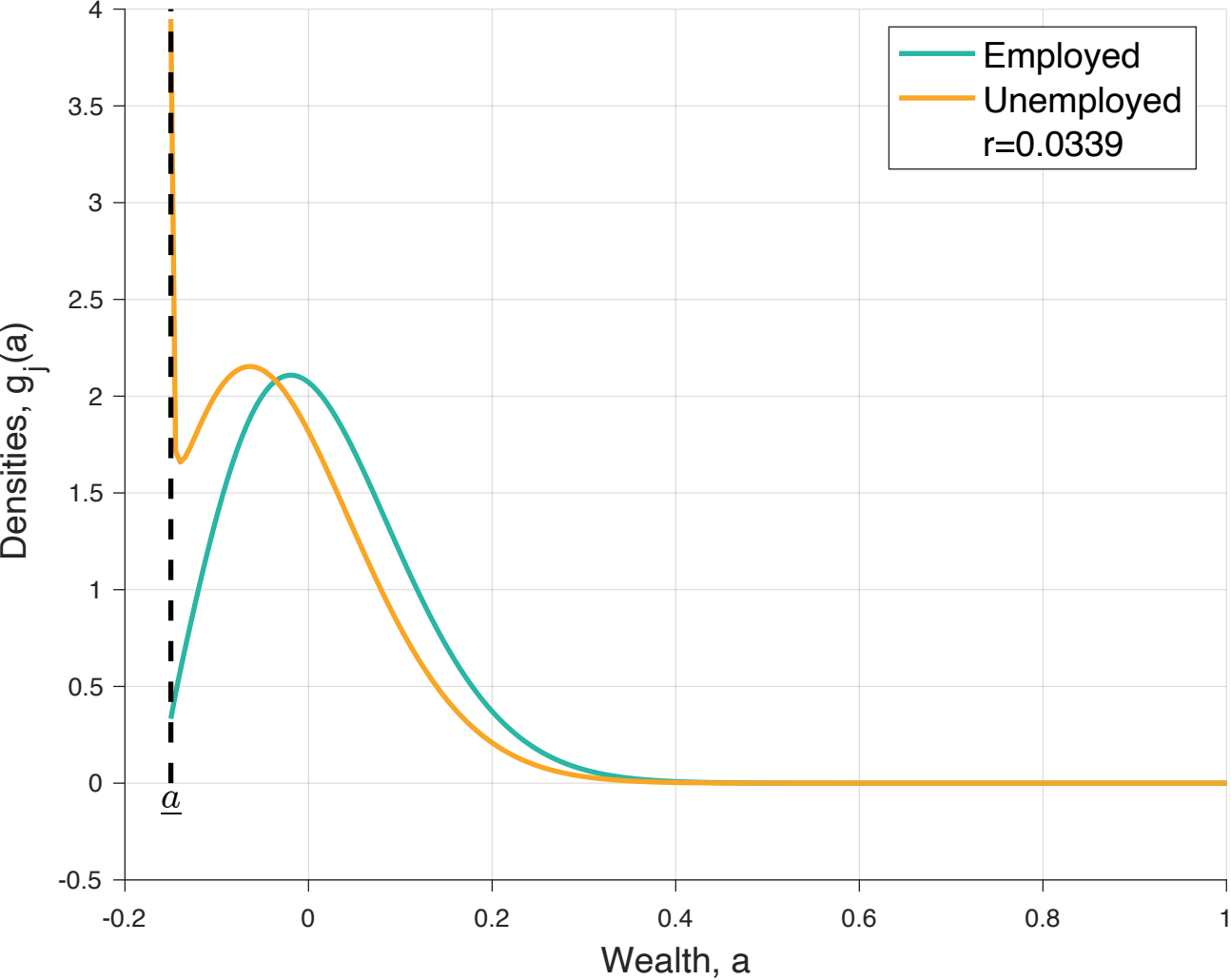
1. Similar to problem set 3, the gap in consumption favors the employed group at every asset level. The difference is largest at the borrowing constraint, $a = -.15$, and rapidly decreases as consumption increases. This difference reflects the larger MPC of unemployed individuals, as they obtain higher marginal utility from increased assets due to their income constraint.

2. Similar to problem set 3, the gap in savings favors the employed group at every asset level. Additionally, this gap increases as assets increase. This difference reflects agents' desire to consumption smooth: the unemployed group factors in their future employment states, where they can save more, & the converse for the employed group. Additionally, those who are employed but have low assets ($a \leq 2.5$), save a positive amount, reflecting precautionary saving behavior.

3. The distribution of assets for the employed group is centered around 0, with a long right tail. In contrast, the distribution of assets for the unemployed group spikes at the borrowing constraint, & then rapidly decreases in wealth. This suggests that the unemployed group is constrained in their ability to accumulate wealth. These observations correspond with the saving behavior noted in 2 - unemployed individuals do not save at any asset level. Comparatively, the unemployed distribution is left-shifted from the employed distribution, reflecting that the former is concentrated at lower wealth levels.


Consumption, c_j(a)
Wealth, a
Employed
Unemployed
r=0.0339
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Employed
Unemployed
r=0.0339
Saving, s_j(a)
Wealth, a
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5


Employed
Unemployed
r=0.0339
Densities, $g_j(a)$
Wealth, a
$\underline{a}$
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-0.2
0
0.2
0.4
0.6
0.8
1

problem_sets/problem_set_4/problem_set_4_code.m

```matlab
%{
------------------------------------------------------------------------------------------------------------------------
Purpose: Coding part of problem set 4
Created: Nico Rotundo 2024-11-26 (Adapted from Kiyea's section code)
------------------------------------------------------------------------------------------------------------------------
%}

%% Define parameters and initialize grids

%{
------------------------------------------------------------------------------------------------------------------------
Define parameters
------------------------------------------------------------------------------------------------------------------------
%}

clear all;

% Relative risk aversion coefficient
sigma = 2;

% Income
z_e = 0.2;
z_u = 0.1;
z = [z_e, z_u];

% Transition rates
lambda_e = 1.2;
lambda_u = 1.2;
lambda = [lambda_e, lambda_u];

% Discount rate
rho = 0.05;

%{
------------------------------------------------------------------------------------------------------------------------
Define asset grid
------------------------------------------------------------------------------------------------------------------------
%}

% Borrowing constraint minimum
a_min = -0.15;

% Borrowing constraint maximum
a_max = 5;

% Number of grid points
num_points = 1000;

%{
------------------------------------------------------------------------------------------------------------------------
Define utility function and its derivative
------------------------------------------------------------------------------------------------------------------------
%}

% Utility function (CRRA), handling vector inputs
U = @(c) (c.^(1 - sigma)) / (1 - sigma);

% Derivative of utility, handling vector inputs
U_prime = @(c) c.^(-sigma);

% Inverse of derivative of utility
U_prime_inv = @(Vp) (Vp).^(-1 / sigma);

%{
------------------------------------------------------------------------------------------------------------------------
Tuning parameters (general)
------------------------------------------------------------------------------------------------------------------------
%}

% Step size: can be arbitrarily large in implicit method
Delta = 1000;

% The maximum number of value function iterations
max_iterations_vf = 100;

% Tolerance for value function iterations
tolerance = 10^(-6);

 %{
------------------------------------------------------------------------------------------------------------------------
Tuning parameters (interest rate iteration)
------------------------------------------------------------------------------------------------------------------------
%}

% The maximum number of interest rate iterations
num_iterations_r = 1000;

% Tolerance for interest rate iterations
tolerance_S = 10^(-5);

a_grid = linspace(a_min, a_max, num_points)';
da = (a_max-a_min)/(num_points-1);

aa = [a_grid, a_grid]; % I*2 matrix

%% Initialize matrices before iterating

%{
------------------------------------------------------------------------------------------------------------------------
Define differential operator
------------------------------------------------------------------------------------------------------------------------
%}


% Forward; satisfies Df*V=dVf
Df = zeros(num_points, num_points);
for i = 1:num_points-1
    Df(i,i) = -1/da; Df(i,i+1) = 1/da;
end
Df = sparse(Df);

% Backward; satisfies Db*V=dV
Db = zeros(num_points, num_points);
for i = 2:num_points
    Db(i,i-1) = -1/da; Db(i,i) = 1/da;
end
Db = sparse(Db);

%{
------------------------------------------------------------------------------------------------------------------------
Define A-switch matrix
------------------------------------------------------------------------------------------------------------------------
%}

A_switch = [speye(num_points).*(-lambda(1)), speye(num_points).*lambda(1);
            speye(num_points).*lambda(2), speye(num_points).*(-lambda(2))];

%{
------------------------------------------------------------------------------------------------------------------------
Define initial guesses
------------------------------------------------------------------------------------------------------------------------
%}

% Guess for initial value of the interest rate
r_0_guess = 0.03;

% Set bounds on interest rate
r_min = 0.01;
r_max = 0.04;

% Define the number of points * 2 matrix
z = ones(num_points, 1).*z;

% The value function of "staying put"
r = r_0_guess;

% Initial guess for value function
V_0 = U(z + r.*aa)./rho;
V = V_0;

%% Iterate over interest rates and value function

% Loop over number of iterations for the interest rate
for nr=1:num_iterations_r

    % Set interest rate equal to current
    r_r(nr) = r;

    % Set bounds on the interest rate
    rmin_r(nr) = r_min;
    rmax_r(nr) = r_max;

    % Use the value function solution from the previous interest rate iteration
    % as the initial guess for the next iteration
    if nr>1
        V_0 = V_r(:,:,nr-1);
        V = V_0;
    end

    %% Value function iteration

    % Loop over number of interations for the value function
    for n=1:max_iterations_vf

        % Derivative of the forward value function
        dVf = Df*V;

        % Derivative of the backward value function
        dVb = Db*V;

        % Boundary condition on backwards value function i.e., the borrowing constraint; a>=a_min
        dVb(1,:) = U_prime(z(1,:) + r.*aa(1,:));

        % Boundary condition on forward value function; a<=a_max
        dVf(end,:) = U_prime(z(end,:) + r.*aa(end,:));

        % Indicator whether value function is concave; for stability purposes
        I_concave = dVb > dVf;

        % Compute optimal consumption using forward derivative
        cf = U_prime_inv(dVf);

        % Compute optimal consumption using backward derivative
        cb = U_prime_inv(dVb);

        % Compute optimal savings using forward derivative
        sf = z + r.*aa - cf;

        % Compute optimal savings using backward derivative
        sb = z + r.*aa - cb;

        % Upwind scheme
        If = sf>0;
        Ib = sb<0;
        I0 = 1-If-Ib;
        dV0 = U_prime(z + r.*aa); % If sf<=0<=sb, set s=0

        dV_upwind = If.*dVf + Ib.*dVb + I0.*dV0;

        c = U_prime_inv(dV_upwind);

        % Update value function
        V_stacked = V(:);

        % Update consumption function
        c_stacked = c(:);

        % A = SD
        SD_u = spdiags(If(:,1).*sf(:,1), 0, num_points, num_points)*Df + spdiags(Ib(:,1).*sb(:,1), 0, num_points, num_points)*Db;
        SD_e = spdiags(If(:,2).*sf(:,2), 0, num_points, num_points)*Df + spdiags(Ib(:,2).*sb(:,2), 0, num_points, num_points)*Db;
        SD = [SD_u, sparse(num_points, num_points);
            sparse(num_points, num_points), SD_e];

        % P = A + A_switch
        P = SD + A_switch;

        % B = [(rho + 1/Delta)*I - P]
        B = (rho + 1/Delta)*speye(2*num_points) - P;

        % b = u(c) + 1/Delta*V
        b = U(c_stacked) + (1/Delta)*V_stacked;

        % V = B\b;
        V_update = B\b;
        V_change = V_update - V_stacked;
        V = reshape(V_update, num_points, 2);

        % Convergence criterion
        dist(n) = max(abs(V_change));

        if dist(n)<tolerance
            disp('Value function converged. Iteration = ')
            disp(n)
        break

        end
    end

    %% KF Equation

    % Solve for 0=gdot=P'*g
    PT = P';
    gdot_stacked = zeros(2*num_points,1);

    % Fix one value to obtain a non-singular matrix, otherwise matrix is singular
    i_fix = 1;
    gdot_stacked(i_fix)=.1;

    row_fix = [zeros(1,i_fix-1),1,zeros(1,2*num_points-i_fix)];
    AT(i_fix,:) = row_fix;

    g_stacked = PT\gdot_stacked;

    % Normalization
    g_sum = g_stacked'*ones(2*num_points,1)*da;
    g_stacked = g_stacked./g_sum;

    % Reshape
    gg = reshape(g_stacked, num_points, 2);

    % Compute interest-rate dependent variables for this iteration
    g_r(:,:,nr) = gg;
    adot(:,:,nr) = z + r.*aa - c;
    V_r(:,:,nr) = V;
    dV_r(:,:,nr) = dV_upwind;
    c_r(:,:,nr) = c;

    S(nr) = gg(:,1)'*a_grid*da + gg(:,2)'*a_grid*da;

    %% Update interest rate using Newton's method

    % Only if iteration is past 1
    if nr > 1

        % Change in S
        dS = S(nr) - S(nr-1);

        % Change in r
        dr = r - r_old;

        % Derivative of S matrix
        S_prime = dS/dr;

        % Save current iteration interest rate
        r_old = r;

        % Ppdate interest rate
        r = r - S(nr)/S_prime;

    else
            r_old = r;
            r = r + 1e-4;
    end

    % Check that convergence
    if sqrt(sum((r-r_old).^2))<tolerance
        disp(r)
        break
    end

end

%% Plots

%{
------------------------------------------------------------------------------------------------------------------------
Question 1: Optimal consumption
------------------------------------------------------------------------------------------------------------------------
%}

% Initialize plot
set(gca, 'FontSize', 18)

    % Employed
    plot(a_grid, c_r(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', [41/255, 182/255, 164/255])
    hold on

    % Unemployed
    plot(a_grid, c_r(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', [250/255, 165/255, 35/255])

    % Blank line for legend
    plot(NaN, NaN, 'LineStyle', 'none')

    hold off

    grid
        xlabel('Wealth, a','FontSize', 14)

        set(gca, 'TickDir', 'out');
            box off;

        ylabel('Consumption, c_j(a)','FontSize', 14)

        xlim([a_min a_max])

        legend(sprintf('Employed'), ...
            sprintf('Unemployed'), ...
            sprintf('r=%.4f', r), 'Location', 'best', 'FontSize', 14)

% Export graph
exportgraphics(gcf, '/Users/nicorotundo/Documents/GitHub/DynamicProgramming2024/problem_sets/problem_set_4/output/question_1.pdf', 'ContentType', 'vector');

%{
------------------------------------------------------------------------------------------------------------------------
Question 2: Optimal savings
------------------------------------------------------------------------------------------------------------------------
%}

% Initialize plot
set(gca, 'FontSize', 18)

    % Employed
    plot(a_grid, adot(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', [41/255, 182/255, 164/255])
    hold on

    % Unemployed
    plot(a_grid, adot(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', [250/255, 165/255, 35/255])

    % Blank line for legend
    plot(NaN, NaN, 'LineStyle', 'none')
    hold off

    grid
        xlabel('Wealth, a', 'FontSize', 14)

        ylabel('Saving, s_j(a)', 'FontSize', 14)

        set(gca, 'TickDir', 'out');
            box off;

        xlim([a_min a_max])

        legend(sprintf('Employed'), ...
            sprintf('Unemployed'), ...
            sprintf('r=%.4f', r), 'Location', 'best', 'FontSize', 14)

% Export graph
exportgraphics(gcf, '/Users/nicorotundo/Documents/GitHub/DynamicProgramming2024/problem_sets/problem_set_4/output/question_2.pdf', 'ContentType', 'vector');

%{
------------------------------------------------------------------------------------------------------------------------
Question 3: Wealth distribution
------------------------------------------------------------------------------------------------------------------------
%}

% Initialize plot
set(gca, 'FontSize', 14)

    % Employed
    plot(a_grid, g_r(:,1,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', [41/255, 182/255, 164/255])
    hold on

    % Unemployed
    plot(a_grid, g_r(:,2,nr), 'LineWidth', 2, 'LineStyle', '-', 'Color', [250/255, 165/255, 35/255])

    % Blank line for legend
    plot(NaN, NaN, 'LineStyle', 'none')

    hold off

    grid
        xlabel('Wealth, a', 'FontSize', 14)

        set(gca, 'TickDir', 'out');
            box off;

        ylabel('Densities, g_j(a)', 'FontSize', 14)

        yy = get(gca, 'yLim');

        hold on

        plot([a_min, a_min], yy, '--k', 'LineWidth', 2)

        hold off

        text(-0.15, yy(1)-0.02*(yy(2) - yy(1)), '$\underline{a}$', 'HorizontalAlignment', 'center', 'FontSize', 15, 'Interpreter', 'latex')

        xlim([-0.2 1])

        legend(sprintf('Employed'), ...
            sprintf('Unemployed'), ...
            sprintf('r=%.4f', r), 'Location', 'best', 'FontSize', 14)

% Export graph
exportgraphics(gcf, '/Users/nicorotundo/Documents/GitHub/DynamicProgramming2024/problem_sets/problem_set_4/output/question_3.pdf', 'ContentType', 'vector');
```