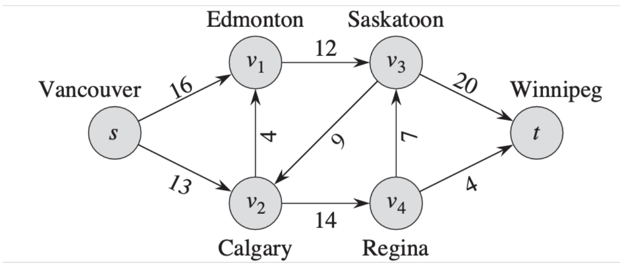
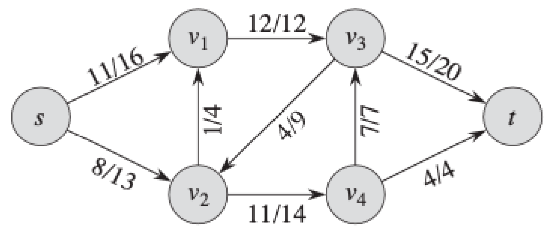
Name: ____________________ Wisc id: ____________________

Basics





1. *CLRS 3rd edition (p. 717).* The figures above are a flow network G and a flow f. What is the flow value $v(f)$? What is the residual network G_f?

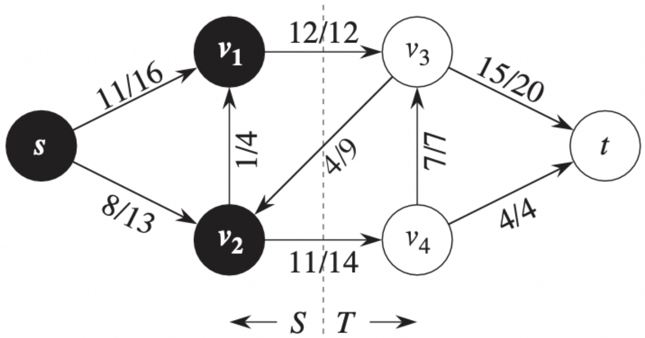
Solution:

2. Consider an augmenting path $p = (s, v_2, v_3, t)$ on G_f. What is `bottleneck(p, f)`? What does the flow network and residual network look like after `augment(f, p)` (increase/augment the flow f along p by `bottleneck(p, f)`)?

Solution:

3. Execute basic Ford-Fulkerson algorithm on the graph.

Solution:



4. Consider a cut (S, T) in G. What is the cut capacity $c(S, T)$? What is the net flow $f(S, T) = \sum_{u \in S} \sum_{v \in T} f(u, v) - \sum_{u \in S} \sum_{v \in T} f(v, u)$?

Solution:

Claim 1. *(CLRS p.721 Lemma 26.4) Let f be any $s - t$ flow and (S, T) be any $s - t$ cut. Then $v(f)$ equals the net flow from S to T. That is, $v(f) = f(S, T)$*

Claim 2. *(CLRS p.723 Lemma 26.5) The value of any flow is bounded above by the capacity of any cut. That is, $v(f) \leq c(S, T)$*

Theorem 1. *(CLRS p.723 Lemma 26.6) The following conditions are equivalent*

1. *f is a max flow of G.*
2. *G_f contains no augmenting path.*
3. *$v(f) = c(S, T)$ for some cut (S, T).*

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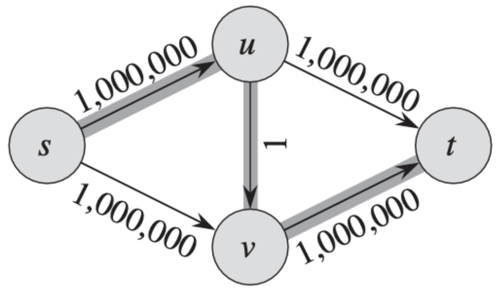
Edmonds-Karp



5. What's wrong with running Ford-Fulkerson on the graph above? How does the Edmonds-Karp algorithm improve it?

Solution: