Name: _____ Wisc id: _____

Coin Changing

1. *CLRS 3rd edition (p. 446).* Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer.

   (a) Describe a greedy algorithm to make change consisting of quarters (25¢), dimes (10¢), nickels (5¢), and pennies (1¢). Prove that your algorithm yields an optimal solution.

   Solution:

   (b) Suppose that the available coins are in the denominations that are powers of c, i.e., the denominations are c^0, c^1, ..., c^k for some integers $c > 1$ and $k \geq 1$. Show that the greedy algorithm always yields an optimal solution.

   Solution:

(c) Give a set of coin denominations for which the greedy algorithm does not yield an optimal solution. Your set should include a penny (1¢) so that there is a solution for every value of n.

Solution:

(d) Give an $O(nk)$-time algorithm that makes change for any set of k different coin denomination, assuming one of the coins is a penny.

Solution:

I Love Train Stations

2. There are towns that lies on a straight road, and the government is planning to build a railroad path along this road. You, as the project manager of this construction, needs to decide where to build train stations. Every town must be within distance R of a train station. The goal is to minimize the number of train stations built.

   (a) Consider the following algorithm: repeatedly build train stations where you can maximize the number of towns newly covered. Show that this algorithm is not optimal by giving a counter-example.

   Solution:

   (b) Give an algorithm and prove that it's optimal.

   Solution:

Give Me Classroom

3. Lecture j starts at s_j and finishes at f_j. Find the minimum number of classrooms to schedule all lectures so that no two occur at the same time in the same room.

Solution: