Name: _____ Wisc id: _____

Basics

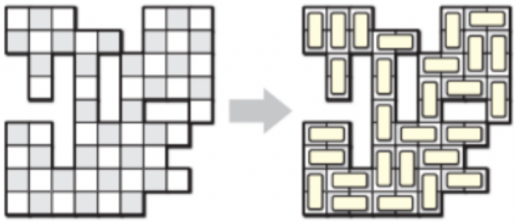
1. A spammer is located at one node q in an undirected communication network G and peaceful email users are located at nodes denoted by the set S. Let $c(u, v)$ denote the effort required to install a spam filter for the network edge (u, v). The problem is to determine the minimal effort required to isolate the spammer from the peaceful email users using the spam filters.

Solution:

2. Several families go out to dinner together. To increase their social interaction, they would like to sit at tables so that no two members of the same family are at the same table. Assume that there are m families and that the i-th family has f_i members. Assume also that there are n tables, and that the j-th table has a seating capacity of c_j. Show how to find a satisfying assignment of people to tables in polynomial time.

Solution:

3. Suppose you are given an $n \times n$ checkerboard with some of the squares deleted. You have a lot of dominos, each of which just the right size to cover two squares of the checkerboard.Describe and analyze an algorithm to determine whether you can cover the board with dominos – each domino must cover exactly two squares, and each square must be covered by exactly one domino.



Solution:

4. Suppose we are given an array $A[1 \cdots m][1 \cdots n]$ of non-negative real numbers. We want to round A to an integer matrix, by replacing each entry x in A with either $\lfloor x \rfloor$ or $\lceil x \rceil$, without changing the sum of entries in any row or column of A. For example:

$$\begin{bmatrix} 1.2 & 3.4 & 2.4 \\ 3.9 & 4.0 & 2.1 \\ 7.9 & 1.6 & 0.5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 2 \\ 4 & 4 & 2 \\ 8 & 1 & 1 \end{bmatrix}$$

Describe and analyze an algorithm that either rounds A in this fashion, or reports correctly that no such rounding is possible.

Solution: