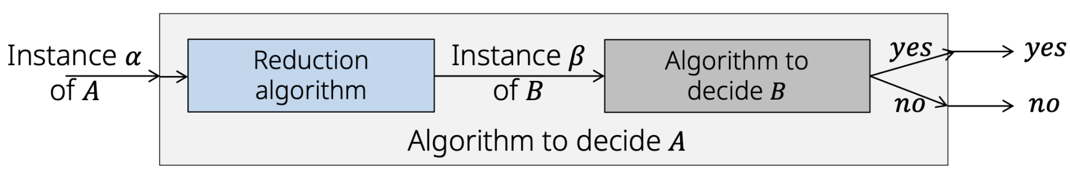
Name: ______________________ Wisc id: ______________________

True/False

1.
- All problems in NP can be reduced to NP-Hard problems in polynomial time.
- All problems in NP can be reduced to NP-Complete problems in polynomial time.
- All NP-Complete problems can be reduced to each other in polynomial time.
- If $A \leq_p B$ and B is in NPC, then A is in NPC, too.
- If $A \leq_p B$ and $B \leq_p C$, then $A \leq_p C$.
- Decision problems are no harder than their respective optimization problems.
- Assume $P! = NP$. Given an NPC problem A, no instance of A can be solved in polynomial time.

Solution:

Proving NPC



To prove B is NP-Complete:

- Prove $B \in NP$
- Prove $B \in NP\text{-}hard$
  - Select a known NPC problem A
  - Construct a reduction f transforming every instance of A to an instance of B
  - Prove that B outputs 1 if and only if A outputs 1
    That is, for all α, $Sol\text{-}A(\alpha) = 1 \iff Sol\text{-}B(\beta) = 1$
  - Prove that f is a polynomial time transformation

3-conjunctive normal form (3-CNF):

- AND of clauses, each is the OR of exactly 3 distinct literals
- A literal is an occurrence of a variable or its negation, e.g. x_1 or $\neg x_1$

e.g. $\phi = (x_1 \vee \neg x_1 \vee \neg x_2) \wedge (x_3 \vee \neg x_2 \vee x_4)$
One solution: [1, 1, 1, 0]

3-CNF-SAT: Given a boolean formula ϕ in 3-CNF, is it satisfiable?

A clique in $G = (V, E)$ is a subset of V

- Each pair of vertices in a clique is connected by an edge in E
- Size of a clique = Number of vertices it contains

CLIQUE: Is there a clique of size k in G?

2. Prove that CLIQUE is NP-complete.

Solution:

A vertex cover of $G = (V, E)$ is a subset $V' \subseteq V$ such that if $(w, v) \in E$, then $w \in V'$ or $v \in V'$ or both.

VERTEX-COVER: Is there a vertex-cover of size k in G?

3. Show that VERTEX-COVER is NP-Complete.

Solution: