Name: _____ Wisc id: _____

Coin Change (again)

1. *CLRS 3rd edition (p. 446).* Consider the problem of making change for n cents using the fewest number of coins. Assume that each coin's value is an integer. Give an $O(nk)$-time algorithm that makes change for any set of k different coin denomination, assuming one of the coins is a penny.

Solution:

Coin Collecting

2. Several coins of different value are placed on a $m \times n$ board. Let $c_{i,j} \geq 0$ be the value of the coin placed on grid-(i, j). Note that $c_{i,j} = 0$ implies that there is no coin on grid-(i, j).

   You placed a robot on the board to collect the coins. The robot starts at the top-left corner of the board (grid-$(0, 0)$), but it can only move right or down. What is the maximum total value of coins the robot can collect?

Solution:

Knapsack

3. (0-1 Knapsack) Given n items where i-th item has value v_i and weighs w_i, output the maximum value for a knapsack with capacity of W. Note that each item can be chosen at most once.

Solution:

4. (Unbounded Knapsack) Given n items where i-th item has value v_i and weighs w_i, output the maximum value for a knapsack with capacity of W. Note that each item can be chosen **unlimited times**.

Solution:

5. (Multidimensional Knapsack) Given n items where i-th item has value v_i, weighs w_i, **and has size** d_i , output the maximum value for a knapsack with capacity of W and size of D. Note that each item can be chosen at most once.

Solution:

Farmer

6. A farmer has a row of n trees that grow fruits, and he wants to earn some money by cutting down the trees and selling the fruits. He is old so he only cuts down one tree per day. Also, each day he only cuts down the leftmost tree or the rightmost tree.

   Initially, cutting down the i-th tree allows the farmer to earn $v_i > 0$ dollars. As time goes by, a fruit might rot, so cutting it down earlier allows the farmer to earn more! Cutting down the i-th tree on the d-th day allows the farmer to earn $v_i \times (n - d + 1)$ dollars.

   What is the maximum amount of money the farmer could achieve?

Solution: