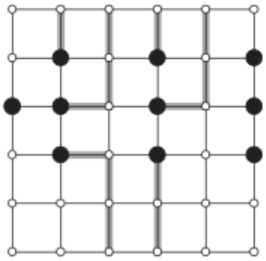
Name: _____ Wisc id: _____

1. Suppose we have capacities on the vertices as well as the edges. That is, in addition to edge capacity $c(u, v)$, we require that for any vertex v other than source s and sink t, the total flow into v (and therefore the total flow out of v) is at most some non-negative value $c(v)$. How can we compute a maximum flow with these new constraints?

Solution:

More Reductions

2. *CLRS p.760* An $n \times n$ grid is an undirected graph consisting of n rows and n columns of vertices. We denote the vertex in the i-th row and the j-th column by (i, j). All vertices in a grid have exactly four neighbors, except for the boundary vertices,which are the points (i, j) for which $i = 1, i = n, j = 1$ or $j = n$. Given $m \leq n^2$ starting points $(x_1, y_1), ..., (x_m, y_m)$ in the grid, the **escape problem** is to determine whether or not there are m edge-disjoint paths from the starting points to any m different points on the boundary. For example, the grid in the following figure has an escape. Give an algorithm to solve the escape problem. What if we're looking for vertex-disjoint paths?



Solution:

3. You're organizing the First Annual UW-Madison Computer Sciences 72-Hour Dance Exchange, to be held all day Friday, Saturday, and Sunday. Several 30-minute sets of music will be played during the event, and a large number of DJs have applied to perform. You need to hire DJs according to the following constraints.

   - Exactly k sets of music must be played each day, and thus 3k sets altogether.
   - Each set must be played by a single DJ in a consistent music genre (ambient, bubblegum, dubstep, horrorcore, hyphy, trip-hop, Nitzhonot, Kwaito, J-pop, Nashville country, . . .).
   - Each genre must be played at most once per day.
   - Each candidate DJ has given you a list of genres they are willing to play.
   - Each DJ can play at most three sets during the entire event.

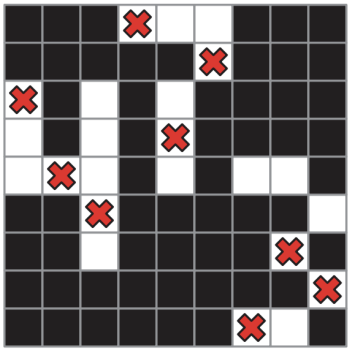
   Suppose there are n candidate DJs and g different musical genres available. Describe and analyze an efficient algorithm that either assigns a DJ and a genre to each of the 3k sets, or correctly reports that no such assignment is possible.

Solution:

4. Suppose we are given an $n \times n$ square grid, some of whose squares are colored black and the rest white. Describe and analyze an algorithm to determine whether tokens can be placed on the grid so that

   - every token is on a white square
   - every row of the grid contains exactly one token
   - every column of the grid contains exactly one token

   Your input is a two dimensional array $IsWhite[1..n, 1..n]$ of booleans, indicating which squares are white. Your output is a single boolean. For example, given the grid above as input, your algorithm should return True.



Solution: