Response to Referee: JHEP_031P_1223

(Dated: February 7, 2024)

We would like to begin by sincerely thanking the referee for their careful reading of the manuscript. We believe their thoughtful comments and suggestions have produced a better paper.

The referee pointed out that the manuscript contained insufficient context for the two-loop results. Specifically, the referee noted that a comparison with $\mathcal{N} = 4$ super-Yang Mills (sYM) was absent. We could not agree more and we apologize for the oversight. Below is summary of the modifications to the manuscript.

- A small comment has been added to the introduction at the top of page 2 reminding readers that color-dual representations of sYM have been found to high loop order.
- At the bottom of page 2, when stating that there is no local 4pt 2-loop numerator for pure YM we remind readers that such a numerator exists for sYM.
- At the end of the introduction at the top of page 3 we clarify that the bowtie cut failure is specific to pure YM. We go on to speculate that this is because sYM and NLSM are simpler theories, at least as far as color-kinematics is concerned.
- Although the referee suggested adding comments to the introduction and section 4, we have included one additional modification. Section 2.3 deals with ansatz locality so this seemed like a natural place to mention that the sYM ansätze incorporate locality as a key simplifying assumption. At the bottom of page 7 and the top of page 8, we emphasize that the sYM numerators through four loops are local. Even though the double copy has been performed for sYM at five loops, local numerators have not been found. We comment in footnote 4 that the full set of local numerators has never been fully explored for sYM at 4pt 5-loop.
- The first paragraph in section 4 closes by putting our results in the context of sYM, specifically by reminding readers of the state of the art in that theory.
- Footnote 10 at the beginning of section 4.3 reminds readers again of the state of the art in sYM.
- After stating the bowtie failure condition for pure YM in (4.9) we state that NLSM and sYM do not suffer from this failure, likely because they are simpler theories in some sense.

We would like to thank the referees again for their carefully considered feedback. Their suggestions have improved the readability of the manuscript. We believe all of their comments have been addressed and the the paper should be ready for publication.

Best regards,
Alex Edison, James Mangan, and Nic Pavao