

# Revealing the Landscape of Globally Color-Dual Multi-loop Integrands

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**ABSTRACT:** We report on progress in understanding how to construct color-dual multi-loop amplitudes. First we identify a cubic theory, semi-abelian Yang-Mills, that unifies many of the color-dual theories studied in the literature, and provides a prescriptive approach for constructing  $D$ -dimensional color-dual numerators through one-loop directly from Feynman rules. By a simple weight counting argument, this approach does not further generalize to two-loops. As a first step in understanding the two-loop challenge, we use a  $D$ -dimensional color-dual bootstrap to successfully construct globally color-dual *local* two-loop four-point nonlinear sigma model (NLSM) numerators. The double-copy of these NLSM numerators with themselves, pure Yang-Mills, and  $\mathcal{N} = 4$  super-Yang-Mills correctly reproduce the known unitarity constructed integrands of special Gallileons, Born-Infeld theory, and Dirac-Born-Infeld-Volkov-Akulov theory, respectively. Applying our bootstrap to two-loop four-point pure Yang-Mills, we exhaustively search the space of local numerators and find that it fails to satisfy global color-kinematics duality, completing a search previously initiated in the literature. We pinpoint the failure to the bowtie unitarity cut, and discuss a path forward towards *non-local* construction of color-dual integrands at generic loop order.

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### 1 Chern-Simons Theory

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### 1 Chern-Simons Theory

The final theory that is unified by semi-abelian Yang-Mills is that Chern-Simons theory. As we will show below, plugging in appropriate on-shell states to the integrands constructed in the text will produce precisely the kinematic numerators for this theory.

$$\begin{aligned}
 & \begin{array}{c} 3 \\ \downarrow \\ \text{---} \bullet \text{---} \\ \uparrow \quad \uparrow \\ 1 \quad 2 \end{array} = \epsilon^{\mu\nu\rho} \varepsilon_1^\mu \varepsilon_2^\nu \varepsilon_3^\rho \quad \Rightarrow \quad \begin{array}{c} 3 \\ \downarrow \\ \text{---} \bullet \text{---} \\ \uparrow \quad \uparrow \\ 1 \quad 2 \end{array} = (\varepsilon_1 \bar{\varepsilon}_3)(\varepsilon_2 k_3) - (\varepsilon_2 \bar{\varepsilon}_3)(\varepsilon_1 k_3) \quad (1.1) \\
 & A^\nu \xrightarrow{k \rightarrow} A^\mu = \frac{i}{k^2} \epsilon^{\mu\nu\rho} k^\rho \quad \Rightarrow \quad A^\nu \xrightarrow{k \rightarrow} \bar{A}^\mu = \frac{i}{k^2} \eta^{\mu\nu} \quad (1.2)
 \end{aligned}$$