

# Optimal Control in a Cooperative Network of Smart Power Grids

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**Abstract**—The possibility to store energy, to exchange power and information on demand and production among grids allows us to achieve an active distribution which is of major interest for cooperative smart power grids, that are grids which can forecast demand and production and are able to exchange power in order to enhance the quality of the service. In this paper, a model to support optimal decisions in a network of cooperative grids is formalized as an original discrete and centralized problem here defined as cooperative network of smart power grids (CNSPG) problem. In the CNSPG problem, the control variables are the instantaneous flows of power in the network of grids, which can be obtained from the solution of a linear quadratic Gaussian problem on a fixed time horizon. A simple case study showing the enhancement which may be obtained from the introduction of direct connections among microgrids according to a lattice network is shown and finally discussed.

**Index Terms**—Energy storage, linear quadratic Gaussian (LQG) control, optimal control, power microgrids, renewable energy, smart grids.

## I. INTRODUCTION

THE USE OF renewable energy sources (RESs) is one of the fundamental strategies to fight against climate change and to reduce the dependence on fossil fuels. Wind and solar sources seem to be particularly promising and several countries are investing in technologies to exploit these RESs. However, due to the fact that the wind and solar energy sources cannot be controlled and are dependent on meteorological conditions, an intermittent, stochastic behavior characterizes such sources.

On the other hand, the possibility to use modern smart meters provides the knowledge of the current electric consumption of each user in real-time. Besides, it is also possible to predict forthcoming user demand, although also in this case, due to unexpected user behaviors, the prediction is affected by an error which can be modeled as a stochastic process. These possibilities lead to the concept of active distribution [1]–[3].

Among RESs, wind and solar sources are not directly exploitable for active distribution. Technological and methodological approaches are necessary to make smart power grids more flexible and agile to react promptly to the forecasted user demand.

In European Union (EU), new system concepts have been adopted to implement such an active distribution [4]. Two

examples are: microgrids, and virtual utilities. According to [4], microgrids are defined as low voltage networks with distributed generation sources, together with local storage devices and controllable loads. They have a total installed capacity in the range from a few hundred kilowatts to a couple of megawatts. Microgrids mostly operate connected to the distribution network. Within the main grid, a microgrid can be regarded as a controlled entity which can be operated as a single aggregated load or generator and as a small source of power supporting the main grid. In [4], virtual utilities have an Internet-like structure with information exchange and trading capability. Power is purchased and delivered to nodes and it is determined by the supplier, e.g., using a conventional generator, RES, or an energy storage device. EU microgrid and virtual utilities research go in the same direction of United States Gridwise initiative (see [5], for more details).

Among the possible approaches to allow a RES base active distribution, the following options seem to be particularly relevant to enhance the flexibility and the agility of a smart power microgrid: to integrate different RESs into a so-called hybrid system, with the intent to provide a more stable energy supply; to provide a microgrid or a renewable energy system with effective local storage devices [6], in order to store energy in periods when demand is lower than produced power, and, vice versa, to provide power when demand is higher than the produced power; to cooperate either with other microgrids or with the National grid itself to exchange information on predicted demand and production, as well as power in order to enhance the overall service provision.

In regards to the three options quoted above, several recent contributions are available in the literature. Some of them are quoted hereinafter.

In regards to hybrid systems, Anagnostopoulos and Papan-tonis [7] presented a numerical study for the optimum sizing and design of a station unit in a hybrid wind-hydro plant (i.e., wind turbines and hydroelectric plant). Dagdougui *et al.* [8] introduced a dynamic decision model for the real time control of hybrid renewable energy production systems, which can be particularly suitable for autonomous systems, such as islands or isolated villages. In their paper, the demand of energy is coupled with the demand of water and hydrogen, where the hydrogen and water reservoirs also work as storage energy systems. Korpas and Holen [9] presented a methodology for the definition of control strategies for a hybrid plant with wind power and hydrogen storage. Their objective is to maximize the expected profit from power exchange in a day-ahead

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market. The generation scheduling is based on forecasts of electricity price, loads, and wind generation. During online operation, a receding horizon strategy is applied to determine the set points for the electrolyzer and the fuel cell power. The market model is defined both for isolated and grid-connected systems.

In regards to storage devices, there are various possibilities for the storage of the produced electrical energy, such as water pumping reversible hydro plants, batteries, compressed air storage, and hydrogen storage [6]. This possibility of energy storage can significantly reduce the drawbacks related to the fluctuating behavior of wind and solar energy resources and can provide cost-effective means to satisfy peak energy demand. Maity and Rao [10] described a framework for an electrical power microgrid, modeled as a demand and supply problem in a multiagent system. The resulting microgrid system model is able to determine the optimum operation of a solar-powered microgrid with respect to load demand, environmental requirements, photovoltaic panel, and battery capacity. Among the energy storage devices, hydrogen based systems have been the subject of several studies. Christopher *et al.* [11] presented a method for the evaluation of a wind-hydrogen energy system. The method includes simulations and economic computations to define the size of the plants. Bernal-Agustin and Dufo-Lopez [12] proposed a complete technical-economic analysis of the hourly energy management in wind-hydrogen systems. In particular, the authors proposed a method to adjust the generation curve to the demand curve. Their method consists of the generation of hydrogen and storing it in a hydrogen tank during off-peak (low demand) hours, while during the rest of the hours (peak hours, high demand) the stored hydrogen can be used to generate electricity. In Korpas *et al.* [13], an operation strategy for a general energy storage device connected to a wind farm is presented. A dynamic algorithm is applied for daily scheduling in a power market. The objective of the online operation strategy is to follow a given generation schedule as closely as possible. A recent comprehensive review to control local energy storage devices in microgrids can be found in [14].

In regards to cooperation with other grids, the current research trend is to provide agent based frameworks where decisions can be taken in a distributed way, in which microgrids can be inserted in a sort of “plug & play” mechanism. The work by Hommelberg *et al.* [15] follows this research approach. Mohod and Aware [16] described the problems due to the injection of several wind power systems into an electric grid and they presented an approach to mitigate these effects.

More recently, Ender *et al.* [17] developed a design tool that enables tradeoffs between various energy systems. Their tool is based on a neural network surrogate model of a publicly available power system modeling tool. It is important to underline that, in this respect, power systems will be required to interact with other quite different energy systems, such as transportation systems. In this respect, Aber and Venayagamoorthy [18] focused on RES integration and grid-able vehicles to maximize emissions reduction: among the three proposed models, the results obtained by the smart grid model show the highest potential for sustainability.

This paper focuses on the problem to evaluate quantitatively the advantages of a cooperative network of power grids, fully exchanging real-time information on energy demand and production. This problem is defined hereinafter as co-operative network of smart power grids (CNSPG). CNSPG is specifically interesting for a network of microgrids where direct connections might be used to exchange power and to decrease the variability of the load of the power exchanged with the connection to the main grid. Section II describes the proposed model and shows how to achieve an optimal control of the CNSPG, while Section III shows an example with two microgrids and a related evaluation on the advantages of exchanging power by a direct connection.

## II. OPTIMAL CONTROL OF THE ENERGY STORAGE IN A COOPERATIVE NETWORK OF SMART POWER GRIDS

The simplified energy storage model proposed hereinafter is based on the assumption that, in the power grid, there is always the technology to store energy locally adopting a device with a given efficiency. The grid has the possibility either to give out or to acquire power from one or more connections to other grids. The local power production of the grid is RES based, specifically coming from wind and solar sources, whose power production is supposed to be fully exploited. It is also supposed that the user demand has to be fully satisfied. The main decisions are whether to store exceeding instantaneous power production or to send it to some of the grid connections. Alternatively, in case of energy shortage, to decide whether it is convenient to acquire energy from some other grids or to utilize (if any) the energy stored in the local energy reservoir. These decisional aspects, specifically under a collaborative framework which is of interest for example in a regional network of microgrids, lead to the formalization of an optimal control problem described in this section. In this framework, the following terminology will be adopted. Each grid will be referred to as a “smart power microgrid” (or simply as a “microgrid”), the connection to the local energy provider as a connection to the “main grid,” and the overall set of microgrids connected among them and to the main grid will be defined as the “network.”

Specifically, each smart power microgrid is supposed to be connected to a regional network of similar grids, and, at least for one microgrid, to one main grid. This network is modeled as a directed graph  $G = (V, E)$ , where  $V$  is the set of vertex with cardinality  $S$ , representing either microgrids or the main grid, and  $E$  is the set of directed links with cardinality  $W$ , representing the power connections existing among the vertexes. As a convention, the  $S$ th node is associated to the main grid.

The evolution over time of the energy stored in the network of microgrids is supposed to be described by the following discrete time state equation:

$$x_{t+1} = Ax_t + (in_t - out_t + Bu_t)\Delta t, \quad t = 0, \dots, T-1 \quad (1)$$

$x_0, \quad \text{given}$

which can be rewritten introducing the following change of state variable:

$$z_t = x_t - \hat{z} \quad (2)$$

as

$$\begin{aligned} z_{t+1} &= Az_t + (A - I)\hat{z} + (in_t - out_t + Bu_t)\Delta t, \\ t &= 0, \dots, T - 1 \\ z_0 &= x_0 - \hat{z} \end{aligned} \quad (3)$$

where the following holds true.

- 1)  $x_t \in R^{S-1}$  (kWh) is the state vector, whose  $i$ th entry is the energy stored at microgrid  $i$  at instant  $t$ .
- 2)  $z_t \in R^{S-1}$  (kWh) is the state vector, whose  $i$ th element is the energy stored at microgrid  $i$  at time instant  $t$ , with respect to an optimal working level  $\hat{z}^i$ . It is supposed that each element of  $z_t$  may assume positive as well as negative values.
- 3)  $u_t \in R^W$  (kW) is the vector of power flows (that are the decision variables) in time interval  $(t, t+1)$ . Specifically, the  $j$ th element of vector  $\mu_t$  is the power flow in the directed link  $j$  in time interval  $(t, t+1)$ . Such a flow may be either positive (i.e., in accordance with the conventional direction of that link) or negative.
- 4)  $in_t \in R^{S-1}$  (kW) is the vector of stochastic processes corresponding to the power flow in input to each microgrid, in time interval  $(t, t+1)$ , as given by the RES exploited in each microgrid.
- 5)  $out_t \in R^{S-1}$  (kW) is the vector of the stochastic power demand processes of each microgrid in time interval  $(t, t+1)$ .
- 6)  $A$  is a  $(S-1) \times (S-1)$  diagonal matrix whose generic diagonal element  $\alpha_{ii}$  is the efficiency of the energy storage technology in the  $i$ th microgrid ( $0 \leq \alpha_{ii} \leq 1$ ).
- 7)  $B$  is the  $(S-1) \times W$  incidence matrix, representing the network topology, where  $b_{i,j} = -1$  if link  $j$  exits the  $i$ th microgrid,  $b_{i,j} = 1$  if link  $j$  enters the  $i$ th microgrid, and 0 otherwise. The direction of the link is purely conventional, as the energy in each link is supposed to be allowed to flow in both directions. It is also worthwhile underlining that real world power grids are sparsely connected, and  $B$  will generally be associated to small-world networks [19].
- 8)  $\Delta t$  is the time discretization interval.

Under the hypothesis that  $in_t$  and  $out_t$  can be forecasted on a given interval  $[0, T]$ , with a certain degree of uncertainty, both of them can be split into their deterministic (respectively  $in_t^d$  and  $out_t^d$ ) and stochastic (respectively  $w_t^{in}$  and  $w_t^{out}$ ) vector components giving rise to

$$in_t = in_t^d + w_t^{in} \quad (4)$$

$$out_t = out_t^d + w_t^{out}. \quad (5)$$

Let  $e_t$  be the vector of energy balance in each microgrid given by

$$e_t = in_t - out_t. \quad (6)$$

Under a simplifying hypothesis, let  $e_t$  be a vector whose elements are represented by a Gaussian white noise, where

$$E\{e_t\} = \eta_t = in_t^d - out_t^d. \quad (7)$$

Thus,  $\{\eta_t, t = 0, 1, \dots, T\}$ , with  $\eta_t \in R^{S-1}$  is a known sequence of values over the interval  $[0, T]$ , while  $\{\omega_t, t = 0, 1, \dots, T\}$ , with  $\omega_t \in R^{S-1}$  being  $\omega_t$  defined as

$$\omega_t = e_t - \eta_t \quad (8)$$

is a sequence of independent identically distributed zero-mean Gaussian random vectors. It is also assumed that the sequences  $\{\omega_t\}$  and  $\{\eta_t\}$  are independent.

Under the assumption quoted above, and to simplify the notation assuming  $\Delta t = 1h$ , the following is the state equation of the energy storage in the microgrid network:

$$\begin{aligned} z_{t+1} &= Az_t + Bu_t + \mu_t + \omega_t \quad t = 0, \dots, T - 1 \\ z_0 &\text{ given} \\ \mu_t &= \eta_t + (A - I)\hat{z} \end{aligned} \quad (9)$$

where:

- 1)  $\mu_t \in R^{S-1}$  is a known sequence of values over the interval  $[0, T]$ ;
- 2)  $\omega_t \in R^{S-1}$  is a vector whose  $i$ th element is the stochastic error affecting the prediction of the energy balance  $\eta_t$  of microgrid  $i$  in time interval  $(t, t+1)$ , modeled as above specified.

It is supposed that there is a perfect knowledge of the state of each local storage. In addition, a cooperative strategy among the grids is followed, whose aim is to keep the level of energy in each local storage system as close as possible to the optimal working level, as well as to reduce, as far as possible, the power flow among the grids. For these reasons, it seems reasonable to choose the following objective function for the definition of the optimal control problem:

$$\min J(z, u) = E \left\{ \sum_{t=0}^{T-1} c(z_t, u_t) + z_T' M_T z_T \right\} \quad (10)$$

$$c(z_t, u_t) = z_t' M z_t + u_t' N u_t \quad (11)$$

where the following holds true.

- 1)  $M$  is a  $(S-1) \times (S-1)$  diagonal matrix, related to the cost of an exceeding/lacking quantity of energy stored in each energy storage device. This matrix is supposed to be positive definite, i.e.,  $M > 0$  and constant for each instant  $t \neq T$ .
- 2)  $M_T$ , with  $M_T > 0$ , has the same definition of  $M$ , but it is only referred to instant  $t = T$ .
- 3)  $N$  is a  $W \times W$  matrix,  $N > 0$ , related to the cost of the power sent on each edge of the network, whose elements are constant for each time interval  $t$ .

The CNSPG problem is so completely expressed in (10) and (11), subject to the state equation (9) with known initial state  $z_0$ . The problem is a linear quadratic Gaussian (LQG) problem, that is “non-standard,” due to the presence of the known input sequence  $\{\mu_t\}$  in the state equation. However, the optimal control strategy can be found in closed form as demonstrated below.

*Result:* For the discrete time LQG problem with non-zero mean input noise, defined by (9), (10) and (11), the optimal control strategy is given by

$$u_t^* = K_t (z_t - z_t^{d2}) + K_t^g g_{t+1} \quad t = 0, \dots, T-1 \quad (12)$$

where the following holds true.

1)  $K_t$  is a  $W \times (S-1)$  matrix given by

$$K_t = -(N + B' P_{t+1} B)^{-1} (B' P_{t+1} A) \quad (13)$$

being  $P_{t+1}$  a  $(S-1) \times (S-1)$  matrix given by the discrete time Riccati equation

$$P_t = M + A' P_{t+1} (I + B N^{-1} B' P_{t+1})^{-1} A \quad t = T-1, \dots, 0 \\ P_T = M_T. \quad (14)$$

2)  $K_t^g$  is a  $W \times (S-1)$  matrix given by

$$K_t^g = (N + B' P_{t+1} B)^{-1} B'. \quad (15)$$

3) The vector  $z_t^{d2}$  is given by

$$z_{t+1}^{d2} = A z_t^{d2} + \mu_t \quad t = 0, \dots, T-1 \\ z_0^{d2} = z_0. \quad (16)$$

4) The vector  $g_t$  is given by

$$g_t = (A' - P_{t+1} (I + B N^{-1} B' P_{t+1})^{-1} B N^{-1} B') g_{t+1} - M z_t^{d2} \\ g_T = M_T z_T^{d2}. \quad (17)$$

*Proof:* The proof follows the theorem proof relevant to an analogous problem in the continuous domain (see [20, problem 1]). Due to its linearity, the system state equation (9) can be written separating the stochastic and deterministic components

$$z_{t+1}^s + z_{t+1}^d = A(z_t^s + z_t^d) + B(u_t^s + u_t^d) + \mu_t + \omega_t \quad t = 0, \dots, T-1. \quad (9^i)$$

System (9) can be so decomposed into a stochastic subsystem

$$z_{t+1}^s = A z_t^s + B u_t^s + \omega_t \quad t = 0, \dots, T-1 \quad z_0^s = 0 \quad (9^{ii})$$

and into a deterministic subsystem

$$z_{t+1}^d = A z_t^d + B u_t^d + \mu_t \quad t = 0, \dots, T-1 \quad z_0^d = z_0. \quad (9^{iii})$$

Similarly, the cost function can be decomposed into its stochastic and deterministic components represented by the functions  $J(z^s, u^s)$  and  $J(z^d, u^d)$ , defined in a straightforward way. This means that for each admissible solution  $(z, u)$ , and for the corresponding stochastic  $(z^s, u^s)$  and deterministic  $(z^d, u^d)$  components, the following identity holds:

$$J(z, u) = J(z^s, u^s) + J(z^d, u^d). \quad (18)$$

This immediately follows from the assumptions on the stochastic process  $\omega_t$ , that imply  $E\{z_t^s\} = E\{u_t^s\} = 0 \quad \forall t$ .

From (18), it follows that the LQG problem with non-zero input noise has a unique optimal solution given by

$$z^* = z^{*,s} + z^{*,d} \quad (19)$$

$$u^* = u^{*,s} + u^{*,d} \quad (20)$$

being, of course,  $(z^{*,d}, u^{*,d})$  and  $(z^{*,s}, u^{*,s})$  the optimal solutions of the problem corresponding to the optimization of the deterministic and stochastic costs, respectively. In fact, for every admissible solution  $(z, u)$ , the following relationship can be written:

$$J(z, u) = J(z^s, u^s) + J(z^d, u^d) \geq J(z^{*,s}, u^{*,s}) \\ + J(z^{*,d}, u^{*,d}) = J(z^*, u^*). \quad (21)$$

The original problem can be so decomposed into a LQG regulation and a deterministic discrete linear quadratic (LQ) tracking problem. For the LQG regulation problem, the optimal control is [21]

$$u_t^{*,s} = k_t z_t^s \quad (22)$$

$$K_t = -(N + B' P_{t+1} B)^{-1} (B' P_{t+1} A) \quad t = 0, \dots, T-1 \quad (23)$$

where  $P_{t+1}$  is given by

$$P_t = M + A' P_{t+1} (I + B N^{-1} B' P_{t+1})^{-1} A \quad t = T-1, \dots, 0 \quad P_T = M_T. \quad (24)$$

The LQ tracking problem can be introduced as follows. Observe that  $z_t^d$  may be thought as composed by the two components  $z_t^{d1}$  and  $z_t^{d2}$ , such that  $z_t^d = z_t^{d1} + z_t^{d2}$ , and

$$z_{t+1}^{d1} = A z_t^{d1} + B u_t^d \quad t = 0, \dots, T-1$$

$$z_0^{d1} = 0 \quad (9^{iv})$$

$$z_{t+1}^{d2} = A z_t^{d2} + \mu_t \quad t = 0, \dots, T-1$$

$$z_0^{d2} = z_0. \quad (9^v)$$

Obviously, the vector  $z_t^{d2}$  may be exactly determined for each instant  $t$ . This means that the deterministic component of the cost function can only be influenced by the values of  $z_t^{d1}$ . Then, putting  $z_t^{d2} = -r_t$ , the optimization of  $J(z^d, u^d)$  is equivalent to the optimization of

$$J(z^{d1}, u^d) = \sum_{t=0}^{T-1} \left( (-r_t + z_t^{d1})' M (-r_t + z_t^{d1}) + u_t^{d'} M u_t^d \right) \\ + (-r_T + z_T^{d1})' M_T (-r_T + z_T^{d1}) \quad (25)$$

which actually gives rise to a deterministic LQ tracking problem, whose solution is [22]

$$u_t^{*,d} = K_t z_t^{d1} + K_t^g g_{t+1} \quad t = 0, \dots, T-1 \quad (26)$$

where the feedback gain  $K_t$  is defined in (19), and the feed forward gain  $K_t^g$  is given by

$$K_t^g = (N + B' P_{t+1} B)^{-1} B' \quad t = 0, \dots, T-1 \quad (27)$$

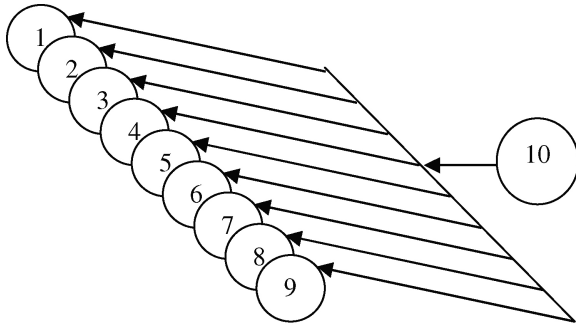


Fig. 1. Logical structure of the network of independent microgrids: circles 1–9 are representative of the microgrids, while circle 10 represents the main grid. The arrows represent the independent power connections with the same main grid.

and the vector  $g_t$  is given by the following recursion:

$$\begin{aligned} g_t &= A - A'P_{t+1} (I + BN^{-1}B'P_{t+1})^{-1} BN^{-1}B'g_{t+1} \\ &\quad + Mr_t \quad t = T-1, \dots, 0 \\ g_T &= M_T r_T. \end{aligned} \quad (17')$$

So, as the end of the proof, it has been demonstrated that

$$\begin{aligned} u_t^* &= u_t^{*,s} + u_t^{*,d} = K_t z_t^s + K_t z_t^{d1} + K_t^g g_{t+1} \\ &= k_t (z_t - z_t^{d2}) + k_t^g g_{t+1} \quad t = 0, \dots, T-1. \end{aligned} \quad (12')$$

■

### III. SIMPLE EXAMPLE ON A LATTICE NETWORK

A simple fictional example follows illustrating one possible application of the control scheme introduced in the previous section. In a region, nine microgrids can produce energy for their user community but their current energy balance between demand and production is often subject to intermittences mainly due to their wind/solar RESs as well as to a user demand significantly varying according to the hours of the day. To mitigate this effect, all of them have a local energy storage possibility, whose size is adequate to the microgrid needs so that it can be taken into account as an energy storage of infinite capacity. The microgrids are also already connected to the same main grid, with which they exchange power. The decision maker (who has the responsibility of managing all the microgrids) needs to evaluate how far the introduction of the possibility of directly exchanging power among the microgrids according to a lattice network can help to improve the overall performance of the system (i.e., of the nine microgrids), for example by reducing the overall power load on the main grid as well as limiting the variations of the energy stocked in the local storages.

Two case studies are compared, the first one taking into account each microgrid as independent, and the second one taking into account the microgrid connected among them by a lattice network. In the latter case, it is supposed that a unique decision maker (DM) can decide the optimal strategy to control the storage level and the power flows in all microgrids, following a cooperative approach as described by the CNSPG problem.

TABLE I

$W_i$ ,  $\phi_i$ ,  $D_i$  PARAMETERS DEFINED FOR EACH MICROGRID ACCORDING TO (28)

Microgrid	$W_i$	$\phi_i$	$D_i$
1	5	0	3
2	5	1	-2
3	4	2	-1
4	3	3	3
5	6	4	-5
6	2	5	1
7	5	6	1
8	6	7	-1
9	5	8	1

#### A. Optimal Management with no Direct Connection Among the Microgrids (Independent Microgrids)

Here the nine microgrids (i.e., from 1 to 9) are supposed to be just connected to the main grid (i.e., the tenth grid), while no connection is present among them (Fig. 1). In this case, nine separate and independent simple control problems are dealt with, each one with one state variable and one control variable, representing the power flow from the main grid. The optimal strategy is also given by (12), taking into account that in this case all variables and parameters are scalars.

It is supposed that the storage devices in the nine microgrids have the same efficiency, that is  $A = 85\%$ , while  $B = 1$ . The evaluation is performed on a period of 168 h (7 days). The predictions of the local energy balance are available. For this fictional case study, the prediction  $\eta_t^i$  for each microgrid  $i = 1, \dots, 9$ ,  $t = 0, \dots, T-1$ , is represented by a square wave function defined as follows:

$$\eta_t^i = W_i \text{sgn}(\sin(\frac{\pi}{12}(t + \phi_i))) + D_i \quad (28)$$

where  $\text{sgn}$  is the sign function and  $W_i$ ,  $\phi_i$ ,  $D_i$  are parameters defined as reported in Table I.

The true energy balance for each microgrid is supposed to result from the sum of the predicted energy balance and an additional Gaussian white noise  $\omega_t^i$ , which for the case study is  $\omega_t^i \sim N(0, 1)$  for all the microgrids. The same realization of such a stochastic stream has been used in all trials described hereinafter.

The scalar parameters in the cost function are set to the following values:  $M = M_T = 1$ , and  $N = 1$  in trial 1 and  $N = 100$  in trial 2. The reason for considering the two trials is to show results where the cost of the state is comparable to the cost of the control (trial 1), and when the latter cost is higher (trial 2). For all the microgrids the initial state value is set to  $z_0 = 0$ . The main statistical characteristics of the resulting optimal values  $u^*$  and  $z^*$  obtained in trials 1 and 2 on the whole set of microgrids are shown in Tables II and III.

#### B. Optimal Management with Direct Connections Among the Microgrids According to a Lattice Network Model (Cooperative Microgrids)

Here the network of nine microgrids is connected according to a lattice graph, as shown in Fig. 2, where each vertex, numbered (highlighted), represents a microgrid. With respect

TABLE II  
MAIN STATISTICAL CHARACTERISTICS (MAXIMUM, MINIMUM,  
AVERAGE, AND STANDARD DEVIATION) OF THE RESULTING OPTIMAL  
VARIABLES FOR CASE A, TRIAL 1,  $M = M_T = 1$ ,  $N = 1$

Microgrid Variable	<i>max</i>	<i>min</i>	<i>avg</i>	<i>sd</i>
$u^{*,1}$	3.1	-9.1	-2.9	4.5
$u^{*,2}$	7.8	-3.8	2.0	4.3
$u^{*,3}$	6.7	-4.2	1.0	3.6
$u^{*,4}$	1.1	-6.8	-2.9	2.7
$u^{*,5}$	11.7	-2.2	4.9	5.2
$u^{*,6}$	1.8	-4.3	-0.9	1.8
$u^{*,7}$	5.1	-6.7	-0.9	4.4
$u^{*,8}$	8.4	-6.1	1.1	5.2
$U^{*,9}$	5.1	-6.6	-1.0	4.3
$z^{*,1}$	5.7	-6.2	0.5	2.0
$z^{*,2}$	4.9	-6.3	-0.3	2.0
$z^{*,3}$	4.3	-5.8	-0.2	1.8
$z^{*,4}$	4.0	-4.7	0.4	1.4
$z^{*,5}$	6.8	-7.2	-0.8	2.2
$z^{*,6}$	5.0	-3.2	0.0	1.2
$z^{*,7}$	4.8	-5.7	-0.2	1.9
$z^{*,8}$	7.5	-6.2	-0.2	2.1
$z^{*,9}$	7.8	-5.4	0.2	1.9

TABLE III  
MAIN STATISTICAL CHARACTERISTICS (MAXIMUM, MINIMUM,  
AVERAGE, AND STANDARD DEVIATION) OF THE RESULTING OPTIMAL  
VARIABLES FOR CASE A, TRIAL 2,  $M = M_T = 1$ ,  $N = 100$

Microgrid Variable	<i>max</i>	<i>min</i>	<i>avg</i>	<i>sd</i>
$u^{*,1}$	0.0	-1.7	-0.9	0.5
$u^{*,2}$	1.4	-0.2	0.6	0.5
$u^{*,3}$	0.9	-0.3	0.3	0.4
$u^{*,4}$	-0.1	-1.4	-0.9	0.3
$u^{*,5}$	2.4	0.3	1.4	0.6
$u^{*,6}$	0.1	-0.7	-0.3	0.2
$u^{*,7}$	0.4	-1.1	-0.3	0.5
$u^{*,8}$	1.2	-0.6	0.3	0.6
$u^{*,9}$	0.48	-1.1	-0.3	0.5
$z^{*,1}$	37.6	-11.8	13.5	14.7
$z^{*,2}$	15.9	-33.2	-9.1	14.7
$z^{*,3}$	14.3	-25.9	-5.4	11.6
$z^{*,4}$	29.1	-2.9	13.1	9.2
$z^{*,5}$	7.5	-53.1	-22.1	17.1
$z^{*,6}$	17.2	-6.7	4.6	6.0
$z^{*,7}$	29.7	-20.3	4.3	14.2
$z^{*,8}$	24.7	-33.9	-4.5	17.4
$z^{*,9}$	29.9	-20.8	4.2	14.0

to case A, the state variables have the same physical meaning, while the control variables have different meanings due to the different network structure of case B.

The nine microgrids have the same storage efficiency of case A, that is 85%. The 12 links, numbered in *italics* and conventionally directed (for positive power flow values) from left to right and from top to bottom, represent the power lines connecting the microgrids among them. It is supposed that the microgrid 9 is connected to the main grid, also modeled as a vertex numbered as 10, by an additional power line modeled

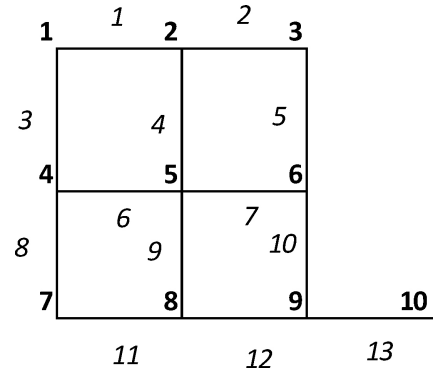


Fig. 2. Lattice network model.

as a link numbered as 13. In this case, the  $9 \times 13$  incidence matrix  $B$  is as follows:

$$B = \begin{bmatrix} -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -1 \end{bmatrix} \quad (29)$$

and the  $9 \times 9$  matrix  $A$  is assumed to be diagonal with  $a_{i,i} = 0.85$ . It is assumed that the DM prefers to stress the minimization of the power load on the connection  $u_{13}$ , that is he/she wishes to minimize the exchanges with the external main grid. So, the  $13 \times 13$   $N$  matrix is a diagonal matrix whose elements  $n_{i,i} = 1 \quad \forall i \neq 13$ , and  $n_{13,13} = 100$ . Finally, it is assumed  $M = M_T$ , and to be  $9 \times 9$  diagonal matrixes with  $m_{i,i} = 1$ .

The evaluation of the quality of the results obtained in connection with case study A and case study B cannot be assessed according to the evaluation of the values of the respective objective functions since the configuration of the regional network of microgrids is different in the two case studies. On the other hand, some indications can be given to the DM according to the trends and the related statistical features of the control and state variables in the different case studies. In addition, the trends of the overall power exchanges with the main grid may also represent an important factor for the DM, whose main goal was to reduce the variations of power loads on the main grid due to the intermittency of the RES based microgrids.

The main statistical characteristics of the resulting optimal values  $u^*$  and  $z^*$  obtained are shown in Table IV. The lower values of the maximum and minimum of  $u^{*,13}$  with respect to the other control variables are due to the characteristics of the  $N$  matrix which has been adopted. Fig. 3 shows the pattern of the power exchanges with the external main grid, as resulting for case A (both trials 1 and 2) from the overall algebraic sum of the power exchanges on the links of each microgrid, and for case B from the power flow on link 13. It is not the aim of this example to assess whether it

TABLE IV  
MAIN STATISTICAL CHARACTERISTICS (MAXIMUM, MINIMUM,  
AVERAGE, AND STANDARD DEVIATION) OF THE RESULTING  
OPTIMAL VARIABLES FOR CASE B

Microgrid Variable	max	min	avg	sd
$u^{*,1}$	6.9	-1.4	2.7	2.3
$u^{*,2}$	5.3	-4.6	0.4	2.8
$u^{*,3}$	3.4	-3.1	0.2	2.2
$u^{*,4}$	3.9	-3.3	0.3	2.4
$u^{*,5}$	4.9	-6.6	-0.6	3.3
$u^{*,6}$	5.7	0.2	2.9	1.5
$u^{*,7}$	4.2	-5.2	-0.6	2.7
$u^{*,8}$	4.6	-3.9	0.4	2.7
$u^{*,9}$	5.4	-8.5	-1.1	4.8
$u^{*,10}$	7.1	-7.1	-0.2	4.4
$u^{*,11}$	6.5	-3.8	1.3	2.8
$u^{*,12}$	5.5	-6.9	-0.9	3.9
$u^{*,13}$	0.2	-0.2	0.0	0.1
$z^{*,1}$	6.0	-6.0	0.6	3.3
$z^{*,2}$	5.7	-6.9	0.1	2.9
$z^{*,3}$	4.9	-5.9	0.0	2.3
$z^{*,4}$	5.8	-4.4	0.6	2.6
$z^{*,5}$	5.8	-5.1	0.1	2.5
$z^{*,6}$	4.6	-4.0	0.1	1.6
$z^{*,7}$	6.8	-5.7	0.3	3.3
$z^{*,8}$	6.9	-7.8	0.1	3.8
$z^{*,9}$	10.2	-9.8	0.3	4.0

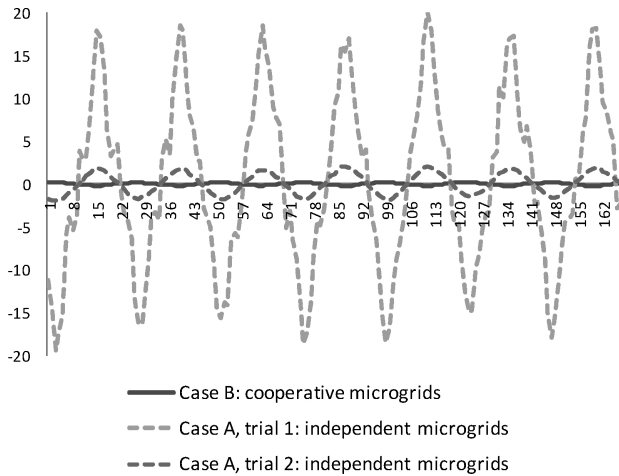


Fig. 3. Trend of the overall power exchanges with the main grid. In the case B, this is represented by the power flow on link 13, while in the cases A trial 1 and 2, this is represented by the overall sum of the power exchanges of all microgrids with the main grid. The x-axis is related to the temporal instants (in hours) and the y-axis to the power flows (expressed in kW).

is worthwhile or not connecting the microgrids in a lattice network, since this fictional case has the aim to provide an example of the methodology described in the previous session which may be subject of additional investigations, for example by a proper tuning of the  $N$ ,  $M$ , and  $M_T$  parameters. Besides, another issue to be taken into account is the one related to the evolution of the cost for the setting and the management of the physical network for the power exchange.

#### IV. CONCLUSION

The proposed CNSPG problem is an original formulation which may be viewed as a preliminary attempt to model and control the exchange of power in a network of microgrids and grids considered as a system of systems [24], [25]. Its main originality is the use and the exchange of information and forecast of energy production and consumption on the whole set of microgrids, to improve the overall quality of the power management and energy stocking.

The main limitation of the approach is the hypothesis to know in real time the status of all storage devices in a centralized way. In the future, it is reasonable to assume that power networks will become more similar to an Internet network where grids may be on-line or not, and will be controlled by decentralized autonomous devices. In addition, even with the presence of a centralized controller, the communication system can operate at a much slower speed: routine status information to the central controller will most likely be updated only once every several minutes [23]. Another limitation of the current work is the fact that no lower and upper bounds are introduced for the control and state variables, as it may be required by the infrastructural elements. The adequacy of their values, as defined by the control law, could be obtained by an appropriate tuning of the parameters in the cost function.

Thus, new research efforts should be devoted to model the information flow in the network, and the related effects of delays in data acquisition, both in a centralized and in a decentralized decision making approach.

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