A manufacturing company producing cheese requires a specific process during its seasoning, as described below.

Let's the desired evolution of the temperature for the next 24 hours of a mass of cheese as the one given below.

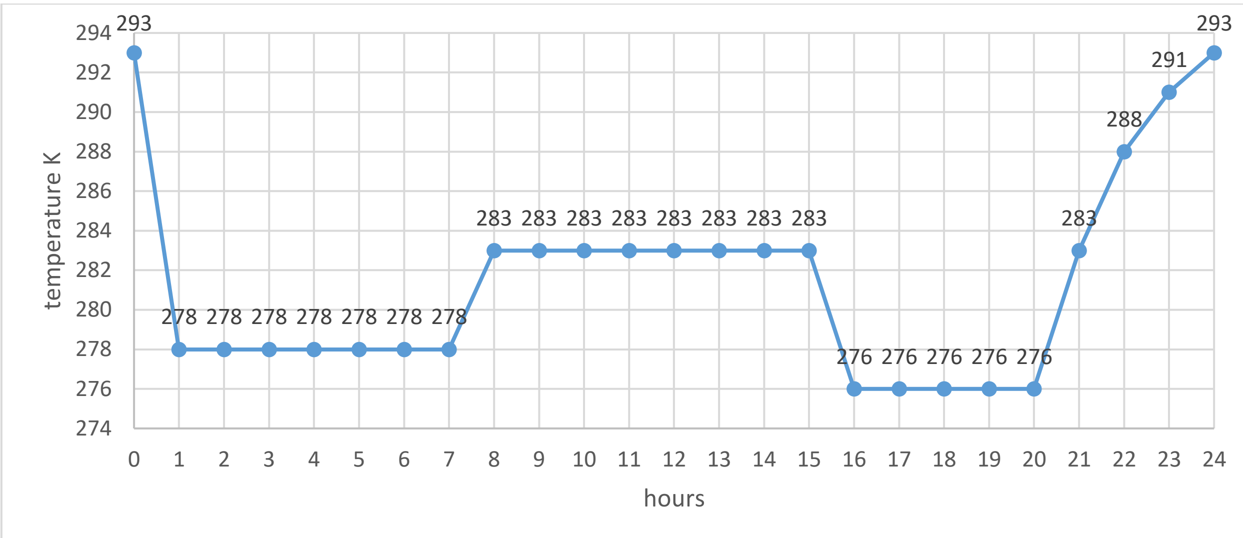

Figure 1. Desired trend of the mass of cheese in the next 24 hours (K)

Let the forecast of the environmental temperature for the next 24 hours be as shown below:

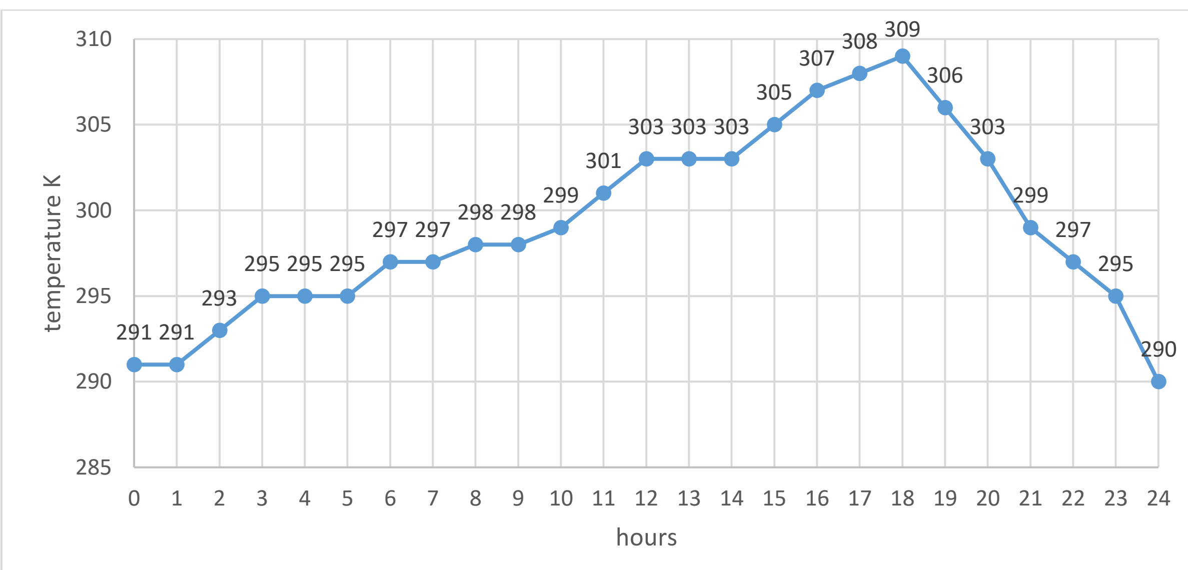

Figure 2. Forecast of the external environmental temperature (K)

Let's a cooling/heating cube box of 1m3 be conditioned by heating air in the power range between -1kW (cooling) and 1 kW (heating).

We wish to insert a cube of 2kg (volume 2dm3) of cheese in the cube box in order to follow such process.

Let the model of the system be described by the following equations:

$$C_c\dot{\theta}_c(t) = k_{fc}\left(\theta_f(t) - \theta_c(t)\right) + \omega_c(t)$$

$$C_f\dot{\theta}_f(t) = k_{fc}\left(\theta_c(t) - \theta_f(t)\right) + k_{af}\left(\theta_a(t) - \theta_f(t)\right) + q(t) + \omega_f(t)$$

Where:

$\theta_c(t)$ temperature of the cheese [K]

$\theta_f(t)$ controlled temperature of the box [K]

$-1000 \leq q(t) \leq 1000$ heating/cooling power [W]

$C_c = c_c * m_c$ [kJ/K] average thermal capacity of the mass of cheese

$m_c = 2kg$ mass of the cheese

$c_c = 2.15$ [kJ/(kg K)]

$C_f = c_c * m_c$ [kJ/K] average thermal capacity of the air in the box

$m_f = 1.3\ kg$ mass of the air in the box

$c_f = 1$ [kJ/(kg K)]

$k_{fc} = \bar{k}_{fc} * s_c$ [W/K] is the overall thermal transmittance between the air in the box and the cheese

s_c surface of the cheese

$\bar{k}_{fc} = 100$ [W/(m^2 K)] average thermal trasmittance x unit surface

$k_{af} = \bar{k}_{af} * s_f$ [W/K] is the overall thermal transmittance between the air in the box and the external environment

$s_c = 6m^2$ surface of the box

$\bar{k}_{fc}$ = 1,22 [W/(m^2 K)] average thermal trasmittance x unit surface

$\omega_c(t) \sim N(0,1)$

$\omega_f(t) \sim N(0,1)$

Question

Imagine to be able to measure the output y given by the following:

$$y(t) = \theta_f(t) + \xi_f(t)$$

$$\xi_f(t) \sim N(0,0.5)$$

1) Implement the system in simulink in continuous time. Sample and hold the output with a sample time of 1 second. Then verify the performance of the controls at point 2 and 3 in terms of:
   a. MSE of the cheese temperature with respect to the desired one
   b. Maximum absolute deviation of the cheese temperature with respect to the desired one
   c. Energy consumption in the 24 hours
   d. Variance of the control in the 24 hours
2) Verify the performance of a LQT / LQG control computed on the discretized system and applied in a zoh to the real continuous system. Please take care to put the problem in a LQ formulation suitable to define the control (see the two annex for any help; specifically the paper by Minciardi and Sacile, page 129, and/or the technical note).
3) Verify the performance with respect to the adoption of a LQT / LQG control using a rolling horizon optimization on different time windows (e.g. 1min, 5min, 9min). Practically the concept of rolling horizon / receding horizon / model predictive control is to optimize on a given window (e.g. solving the LQR on a windows of K instants (K<N where N is the horizon), apply the control just for the first instant, read again the status at the following instant and reapply the control).