Lab Nr. 3, Probability and Statistics

Continuous Random Variables; CDF and Inverse CDF; Quantiles; Approximations of the Binomial Distribution

1. Let X have one of the following distributions: $X \in N(\mu, \sigma)$ (normal), $X \in T(n)$ (Student), $X \in \chi^2(n)$, or $X \in F(m, n)$ (Fischer). Compute the following:

a) $P(X \leq 0)$ and $P(X \geq 0)$;

b) $P(-1 \leq X \leq 1)$ and $P(X \leq -1$ or $X \geq 1)$;

c) the value x_α such that $P(X < x_\alpha) = \alpha$, for $\alpha \in (0, 1)$ (x_α is called the *quantile* of order α);

d) the value x_β such that $P(X > x_\beta) = \beta$, for $\beta \in (0, 1)$ (x_β is the quantile of order $1 - \beta$).

2. Approximations of the Binomial distribution

- **Normal** approximation of the binomial distribution: For moderate values of p ($0.05 \leq p \leq 0.95$) and large values of n ($n \to \infty$),

$$\text{Bino}(n, p) \approx \text{ Norm}\left(\mu = np, \sigma = \sqrt{np(1-p)}\right).$$

  Write a Matlab code to visualize how the binomial distribution gradually takes the shape of the normal distribution as $n \to \infty$.

- **Poisson** approximation of the binomial distribution: If $n \geq 30$ and $p \leq 0.05$, then

$$\text{Bino}(n, p) \approx \text{ Poisson}(\lambda = np)$$