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Kinematic design of a double wishbone type front suspension mechanism using multi-objective optimization

J. S. Hwang[1], S. R. Kim[1] and S. Y. Han[2]

[1]Deptment of Mechanical Engineering, Graduate School, Hanyang University, Korea
[2]School of Mechanical Engineering, Hanyang University, Korea

Abstract: A kinematic design of a double wishbone type front suspension mechanism was used to determine the optimal hardpoint positions while considering controllability and stability performances of a vehicle simultaneously. Various performance parameters were classified into two objective functions related to controllability and stability performances. A distance function method was implemented with multi-objective optimization. Multi-objective optimization was performed by using a genetic algorithm. When the multi-objective optimization consisted of the performance parameters related to only controllability or stability performances, variations of each performance parameter were minimized by emphasizing the importance of each performance parameter. It was concluded that multi-objective optimization using the distance function method is very effective for obtaining the optimal hardpoint positions of a suspension mechanism.

Keywords: Controllability Performance, Genetic Algorithm, Performance Parameter, Stability Performance, Suspension Mechanism.

1 Introduction

At the beginning of designing a suspension mechanism, it is essential to determine the hardpoint positions through kinematic analysis in order to satisfy the required controllability and stability performances of a vehicle. The performance parameters such as camber angle, toe angle, king pin angle, and caster angle are related to controllability performance, while the performance parameters of roll center height and percent anti-dive are related to stability performance. All are highly dependent on the hardpoint positions.

Upon kinematic analyses of a suspension mechanism, Suh [1] suggested a nonlinear motion analysis method of a suspension mechanism using the displacement matrix. Kang [2] suggested the constraint equations of a spherical cylinder link and performed motion analysis of a McPherson type suspension mechanism. For the optimum designs of a suspension mechanism, Simionescu [3] performed an optimum design of a multi-link type suspension mechanism to minimize the change of various performance types Kim [4, 5] suggested an approximate composition method of a multi-link type suspension mechanism using an imaginary screw axis when a wheel is in stroke and steering. Also, sensitivity analysis of the suspension mechanism characteristics due to changes in the hardpoint location, kinematic analysis, was necessary. Lee [6] classified the characteristics of wheel alignment into the functions of controllability and straightness performances and performed multi-objective optimization using a genetic algorithm.

In this study, a double wishbone type suspension mechanism was kinematically analyzed. Also, the performance parameters coupled with each other were classified into the performance functions of kinematic controllability and stability. Multi-objective functions consisted of the classified functions. Since the characteristics of the objective functions are nonlinear, a genetic algorithm was used to obtain a global solution. The positions of the hardpoints were established as design variables. The distance function method was implemented with multi-objective optimization in order to make the values of each performance parameter approach the idle values at curb height and to minimize its variation when the wheel is in stroke.

2 Kinematic analysis of a suspension mechanism

2.1 Displacement matrix and constraints

Rigid body motion can be expressed by the displacement matrix $[D_{a,b,g}]$, which consists of rotation angles and displacements. Suh [1] performed a kinematic motion analysis of a rigid body using the displacement matrix and constraints.

Point *q* on a rigid body after rigid motion can be expressed by (1).

$$\begin{Bmatrix} q \\ 1 \end{Bmatrix} = [D_{a,b,g}] \begin{Bmatrix} q_1 \\ 1 \end{Bmatrix} = \begin{bmatrix} [R_{a,b,g}] & (p - [R_{a,b,g}]p_1) \\ 0\ 0\ 0 & 1 \end{bmatrix} \begin{Bmatrix} q_1 \\ 1 \end{Bmatrix} \tag{1}$$

where, *p* and *q* are the points in a rigid body after rigid motion, and p_1 and q_1 are the initial points in a rigid body. $[R_{a,b,g}]$ is the rotation matrix. a, b and g are the rotation angles with respect to z, y, and x axes, respectively.

Constraint equations of a double wishbone type suspension mechanism are shown below. Since the length of a link, which is a component of tie rod, is constant before and after moving in Figure 1, it can be written as (2).

$$\begin{gathered}(H_{1x}-H_{0x})^2+(H_{1y}-H_{0y})^2+(H_{1z}-H_{0z})^2=(H_{ix}-H_{0x})^2+(H_{iy}-H_{0y})^2+(H_{iz}-H_{0z})^2 \\ (i=2,3,\ldots,m)\end{gathered} \tag{2}$$

The length of vector $\overrightarrow{C_iC_0}$ of the upper arm shown in Figure 1 is constant before and after it is moved and can be expressed by (3). The vector $\overrightarrow{C_iC_0}$, which proceeds through point C_0 and is perpendicular to the unit vector $\overrightarrow{QP}$, can be denoted by (4).

$$\begin{gathered}(C_{1x}-C_{0x})^2+(C_{1y}-C_{0y})^2+(C_{1z}-C_{0z})^2=(C_{ix}-C_{0x})^2+(C_{iy}-C_{0y})^2+(C_{iz}-C_{0z})^2 \\ (i=2,3,\ldots,m)\end{gathered} \tag{3}$$

$$\begin{gathered}u_{ux}(C_{ix}-C_{0x})+u_{uy}(C_{iy}-C_{0y})+u_{uz}(C_{iz}-C_{0z})=0, \\ (\mathbf{u_u}\ \text{: unit vector of } \overrightarrow{QA} \quad i=2,3,\ldots,m)\end{gathered} \tag{4}$$

For the lower arm, the same constraints can be expressed by (5) and (6), respectively.

$$\begin{gathered}(B_{1x}-B_{0x})^2+(B_{1y}-B_{0y})^2+(B_{1z}-B_{0z})^2=(B_{ix}-B_{0x})^2+(B_{iy}-B_{0y})^2+(B_{iz}-B_{0z})^2 \\ (i=2,3,\ldots,m)\end{gathered} \tag{5}$$

$$\begin{gathered}u_{lx}(B_{ix}-B_{0x})+u_{ly}(B_{iy}-B_{0y})+u_{lz}(B_{iz}-B_{0z})=0, \\ (\mathbf{u_l}\ \text{: unit vector of } \overrightarrow{GA} \quad i=2,3,\ldots,m)\end{gathered} \tag{6}$$

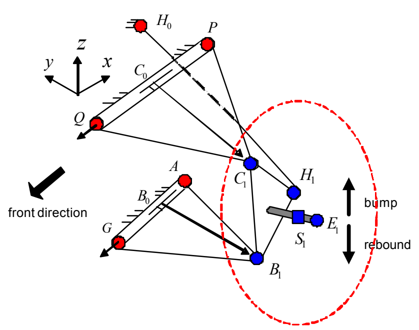


Figure 1 Schematic diagram of a front wheel used with a double wishbon suspension mechanism

3 Optimum design of a suspension mechanism

3.1 Formulation

The multi-objective optimization problem considering kinematic controllability and stability performances of a vehicle in this study can be formulated as shown in (7).

$$\begin{aligned} &\textit{Minimize} && F(x) = w_c \times F_c(x) + w_s \times F_s(x) \\ &\textit{Subject to} && g_i(x) \le 0 \qquad i = 1, 2, \ldots, m \end{aligned} \tag{7}$$

where, $F_c(x)$ is the objective function for controllability performance consisting of camber angle, toe angle, king pin angle, and caster angle. $F_s(x)$ is the objective function for stability performance, which includes roll center height and percent anti-dive. w_c and w_s are the weighting factors for the controllability and stability performance objective functions, respectively.

The distance function method as an optimization technique was implemented with multi-objective optimization in order to make the values of each performance parameter approach the idle values of curb height and minimize their change during wheel stroke. A scale factor was employed to make each performance parameter uniform, and then each performance parameter was evaluated by (8).

$$\begin{aligned} &\text{If } f_{idle}^{kp^*} \le \varepsilon, \text{ where, } \varepsilon \text{ is very closed to zero} \\ &\text{Use } \left[sf^{kp^*} \times \sum_{i=1}^{m} \left| f_i^{kp^*}(x) - f_{idle}^{kp^*} \right|^2 \right]^{1/2}, \qquad m = 13 \\ &\text{Else, use } \left[sf^{kp^*} \times \sum_{i=1}^{m} \left[\frac{\left| f_i^{kp^*}(x) - f_{idle}^{kp^*} \right|}{f_{idle}^{kp^*}} \right]^2 \right]^{1/2} \end{aligned} \tag{8}$$

where, $f_{idle}^{kp^*}$, $f_i^{kp^*}(x)$, and sf^{kp^*} are the idle value, *i* th value, and a scale factor of each performance parameter, respectively. The idle values of each performance parameter at curb height for a double wishbone type suspension mechanism are listed in Table 1 [7].

3.2 Constraints and design variables

Establishment of the acceptable ranges of each performance parameter at curb height is very helpful for controllability and stability performances during the design of a suspension mechanism. From the research results of Lee [6], and Halderman [7], the acceptable ranges of each performance parameter at curb height were referred. Also, since the harmony of camber angle and toe angle when the wheel is in stroke enhances straightness performance and prevention of tire wear, the tendency of toe in and positive camber during rebound, as well as toe out and negative camber during bump, should be required. The positions of the hardpoints attached to the vehicle's body and wheel assembly were established as design variables. The acceptable ranges of the design variables are summarized in Table 2.

4. Optimization results

Optimization results of each performance parameter after adjustment by objective function weighting factors for kinematic controllability and stability performances are shown in Figures 3 and 4. The x-axis denotes the degree of rebound and bump, and the y-axis indicates the variations of each performance parameter during rebound and bump. The weighting factors, $w_i\ (i = 1, 2, 3, 4)$ for kinematic controllability performance were given as 0.25 for $w_i\ (i = 1, 2, 3, 4)$. The weighting factors, $w_j\ (j = 5, 6)$, for kinematic stability performance were given as 0.5.

Figure 3 shows the optimization results for kinematic controllability performance, which consisted of camber angle, toe angle, king pin angle, and caster angle in the case of $w_c = 0.8$, $w_s = 0.2$, and *vice versa*. The acceptable ranges of each performance parameter related to controllability as well as sta-

Table 1 Idle values of each performance parameter

f_{idle}^{kp*}	Cam_{idle}	Toe_{idle}	Kin_{idle}	Cas_{idle}	Rch_{idle}	Fap_{idle}
value	$0°$	$0°$	$3°$	$9.37°$	$45.9\,mm$	27.9%

Table 2 Range of design variables

	Lower	DV*	Upper	Lower	DV*	Upper	Lower	DV*	Upper
Upper arm	1500	Q_x	1600	-500	Q_y	-400	800	Q_z	900
	1750	P_x	1850	-	P_y	-	790	P_z	890
	1550	C_{1x}	1650	-700	C_{1y}	-600	800	C_{1z}	900
Lower arm	1500	G_x	1600	-400	G_y	-300	300	G_z	400
	1800	A_x	1900	-	A_y	-	290	A_z	390
	1550	B_{1x}	1650	-800	B_{1y}	-700	300	B_{1z}	400
Tie Rod	1650	H_{0x}	1750	-350	H_{0y}	-250	350	H_{0z}	450
	1650	H_{1x}	1750	-750	H_{1y}	-650	350	H_{1z}	450

(unit: mm)

Table 3 Optimum solution for $w_c = 0.2$, $w_s = 0.8$, and vice versa

OS*	Q_x	Q_y	Q_z	P_x	P_y	P_z	C_{1x}	C_{1y}	C_{1z}
$w_c = 0.2, w_s = 0.8$	1566.02	-489.00	803.38	1786.00	-489.00	790.00	1649.00	-604.00	821.99
$w_c = 0.8, w_s = 0.2$	156.51	-443.56	804.89	1798.00	-443.56	794.08	1649.38	-617.38	815.77
OS*	G_x	G_y	G_z	A_x	A_y	A_z	B_{1x}	B_{1y}	B_{1z}
$w_c = 0.2, w_s = 0.8$	1597.04	-300.00	301.01	1800.00	-300.00	309.43	1626.00	-716.00	304.05
$w_c = 0.8, w_s = 0.2$	1530.34	-302.00	300.87	1802.63	-302.00	312.44	1265.04	-701.05	300.00
OS*	H_{0x}	H_{0y}	H_{0z}	H_{1x}	H_{1y}	H_{1z}			
$w_c = 0.2, w_s = 0.8$	1650.00	-267.00	351.00	1750.00	-650.00	382.00			
$w_c = 0.8, w_s = 0.2$	1670.02	-324.45	361.00	1730.01	-650.00	363.81			

OS*: optimum solution

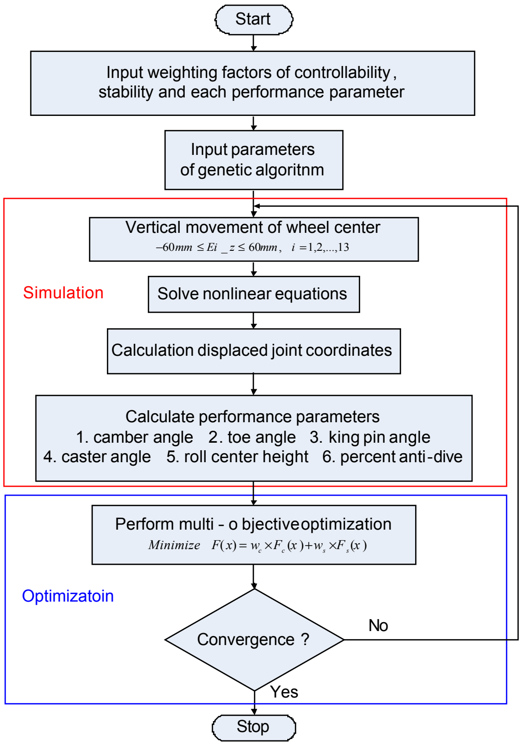
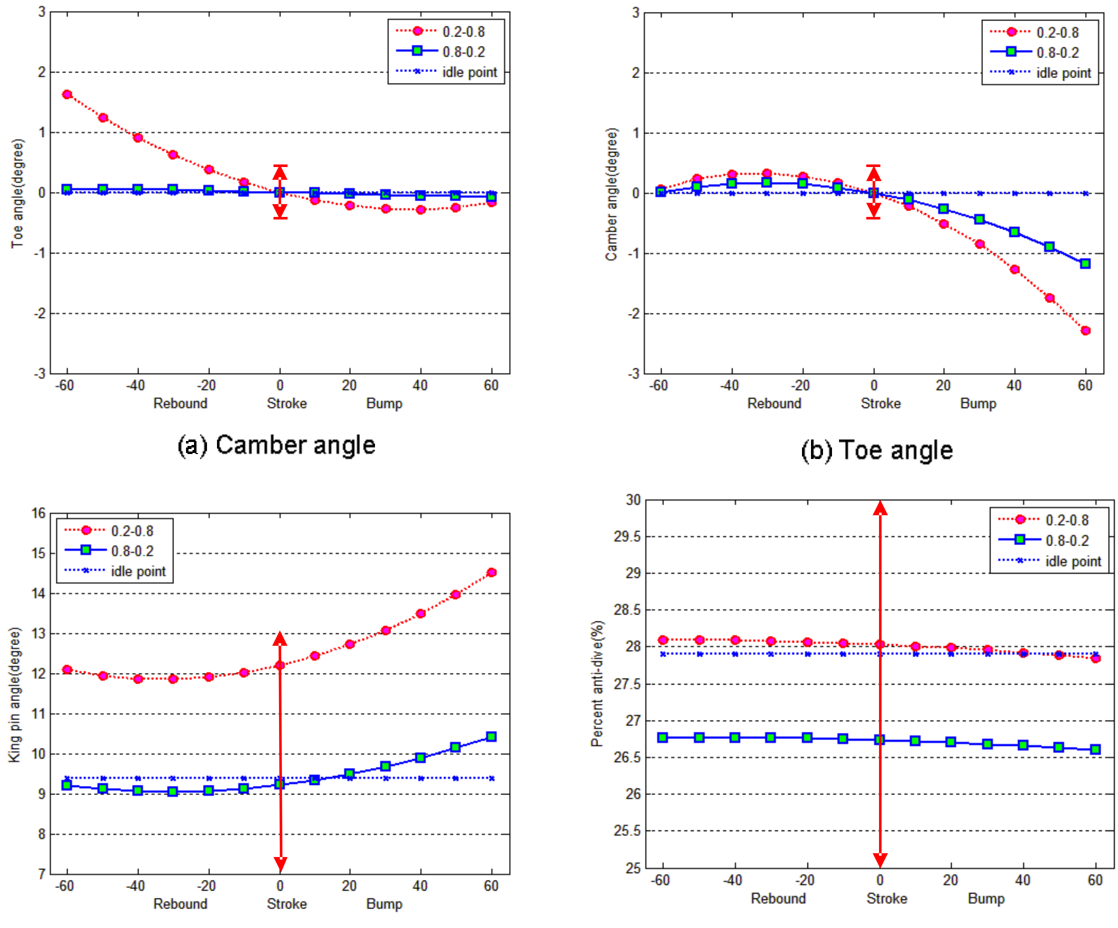


Figure 2 Flow chart of the simulation and optimization process



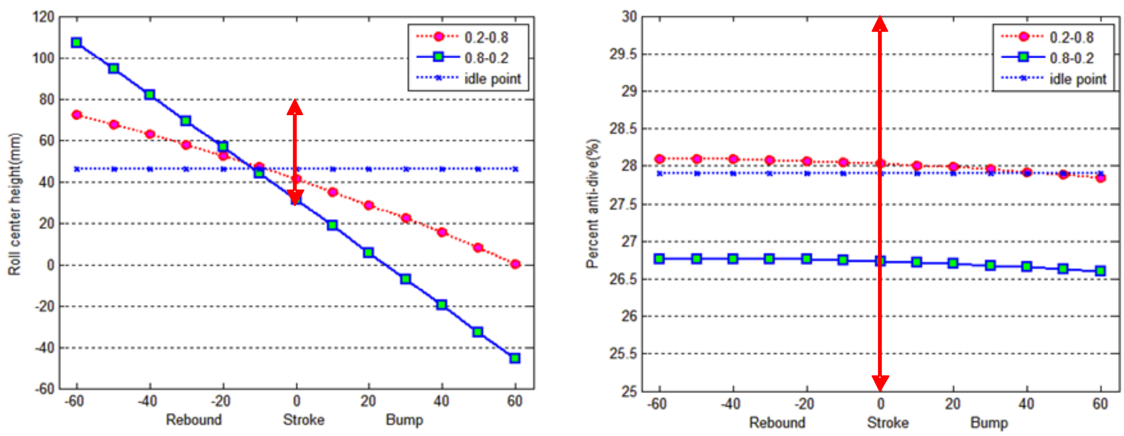
(a) Camber angle

(b) Toe angle

(c) King pin angle

(d) Caster angle

Figure 3 Variations of each performance parameter related to controllability performance



(a) Roll center height

(b) Percent anti-dive

Figure 4 Variations of each performance parameter related to stability performance bility were satisfied at curb height. In the case of the larger weight factor for controllability performance, the variations of each performance parameter were minimized, and approached each idle point compared with the larger weight factor for stability performance. Furthermore it was shown that the variations of toe angle were very small and almost the same value as the idle point.

Figure 4 shows the optimization results for kinematic stability performance that consisted of roll center height and percent anti-dive in the case of $w_c = 0.2$, $w_s = 0.8$ and *vice versa*. The acceptable ranges of each performance parameter related to controllability as well as stability performances were satisfied at curb height. In the case of the larger weighting factor for stability performance, the variation of each performance parameter related to it was minimized, and approached each idle point compared with the larger weight factor for controllability performance. The optimal solutions for the two cases are summarized in Table 3. The acceptable ranges of all performance parameters at curb height were satisfied for all cases. In the case of the larger weighting factor for each performance parameter related to stability performance, its variations were minimized, and approached each idle point compared to the smaller weighting factors for the other performance parameters. In particular, it is shown that the variations of percent anti-dive were very small and remained close to the idle point.

5 Conclusions

In this study, a double wishbone type suspension mechanism was kinematically analyzed, and the distance function method was implemented with multi-objective optimization. Optimal positions of the hardpoints were determined by a genetic algorithm through multi-objective optimization. The conclusions derived from this study are as follows:

(1) It was verified that multi-objective optimization was effectively performed using the distance method in order to make the value of each characteristic factor approach the idle values at curb height and minimize their variations throughout the wheel stroke.

(2) In the case of the larger weighting factor for controllability performance, the variations of camber angle, toe angle, king pin angle, and caster angle were minimized. Conversely, in the case of the larger weighting factor for stability performance, the variations of roll center height and percent anti-dive were minimized. Moreover, it was shown that the variations of toe angle were very small, producing a similar value to that of the idle point.

(3) In the case of the larger weighting factor for each performance parameter related to controllability or stability performances, its variations were minimized and approached each idle point in comparison to the smaller weighting factors for the other performance parameters. Moreover, it was shown that the variation of the toe angle was very small and remained near the idle point.

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