Vehicle System Dynamics

ISSN: 0042-3114 (Print) 1744-5159 (Online) Journal homepage: https://www.tandfonline.com/loi/nvsd20

Kinematic design of double-wishbone suspension systems using a multiobjective optimisation approach

Ramon Sancibrian , Pablo Garcia , Fernando Viadero , Alfonso Fernandez & Ana De-Juan

Vehicle System Dynamics
Vol. 48, No. 7, July 2010, 793–813

Taylor & Francis
Taylor & Francis Group

Kinematic design of double-wishbone suspension systems using a multiobjective optimisation approach

Ramon Sancibrian*, Pablo Garcia, Fernando Viadero, Alfonso Fernandez and Ana De-Juan

Department of Structural and Mechanical Engineering, University of Cantabria, ETSIIT, Avda. de los Castros s/n, Santander 39005, Spain

(*Received 29 October 2008; final version received 24 June 2009; first published 9 December 2009*)

This paper is focused on the kinematic design of double-wishbone suspension systems in vehicles, which is tackled using a multiobjective dimensional synthesis technique. The synthesis goal is to optimise an RSSR–SS linkage, subject to some constraints involved in the dynamic behaviour of vehicles. The synthesis method is based on gradient determination using exact differentiation to obtain the elements in the Jacobian matrix. These characteristics make the method adapt well to the optimum design of vehicle suspension systems. The method is capable of handling equality and inequality constraints, thus, the usual ranges of values may be imposed on the functional parameters. The formulation presented is easy to implement and the solutions obtained demonstrate the accuracy and robustness of the method.

Keywords: road vehicles; suspension system design; double-wishbone; kinematic synthesis; computational kinematics

1. Introduction

Nowadays, safety and passenger comfort are unquestionable factors concerning all automobile manufacturers. Independent suspension systems play an important role in reducing the mutual influence between wheels and improving dynamic behaviour during steady and straight-line motion as well as acceleration, cornering and braking. However, the existence of different topologies and configurations confirms that there is no unique solution adapting well to all situations. Indeed, designers try to exploit the advantages of such a system attempting to achieve design requirements (for example, small space, low weight, easier steerability and understeering), which are important in improving road handling, especially in bends or over irregular roads. From a kinematics point of view, a suspension system can be defined as a combination of links and joints, and the changes in orientation and positions undergone by the wheel during jounce and rebound are called the *kinematic characteristics* defined by means of some *functional parameters*, whose values are specified by the design. A higher performance in the design is achieved as the kinematic characteristic is increased. This improves road

*Corresponding author. Email: sancibrr@unican.es

ISSN 0042-3114 print/ISSN 1744-5159 online

DOI: 10.1080/00423110903156574
http://www.informaworld.com

handling and steerability, but the number of links and joints may be increased too much, worsening other important characteristics in the design of suspension systems such as weight and space, and of course, cost. So a solution must be found by compromising after evaluating all factors.

The double-wishbone type follows the aforementioned trends; it being one of the most widely used solutions in both rear and front axles. This type of suspension system in vehicles is also known by the acronym SLA (short-long arm) due to its unequal length of arms. In this kind of system, the wheel is guided by two control links, called upper and lower arms (Figure 1(a)), which are connected to the chassis (or suspension subframe) by means of revolute joints, and to the steering knuckle by means of spherical joints. Furthermore, in the case of the front axle, the steering knuckle is connected to the chassis by means of the tie-rod using spherical joints. Some times the rear suspensions contain a similar link, called toe-link, to allow slight changes of the steering angle. Thus, due to this configuration, the whole three-dimensional kinematic chain is named RSSR–SS and is formed as shown in Figure 1(b).

The main advantage of the double-wishbone configuration is its good ratio of kinematic versatility *vs*. complexity, it being slightly more complex than other systems like the McPherson strut and slightly simpler than others like multi-link suspensions [1]. The large number of design parameters necessary to define a double-wishbone suspension system makes it easy to approach the kinematic characteristic accurately but, at the same time, it is more difficult to synthesise due to the large number of parameters involved in the three-dimensional problem [2]. For instance, the McPherson strut type is relatively easy to design from the kinematic point of view; however, the fulfilment with the design requirements is less than in the double-wishbone type. Another drawback of the McPherson strut is the presence of friction forces which are caused by the sliding motion between two structural links [3]. Indeed, the shock absorber and spring elements are mounted in order to be activated by the relative motion of a prismatic pair. In the double-wishbone suspension system, the shock absorber and spring elements are mounted as independent elements with no kinematic function. For this reason, both the friction forces and the bending moments supported by the mechanism are lower. Furthermore, the McPherson strut requires a high position for the chassis attachment, whereas in double-wishbone systems attachment points remain in lower positions where the reinforcement of the chassis structure is easier.

On the other hand, a multilink suspension system can be considered an evolution of the double-wishbone type. However, it is difficult to describe the kinematic configuration of this kind of system because it is not strictly defined; different car manufacturers have different

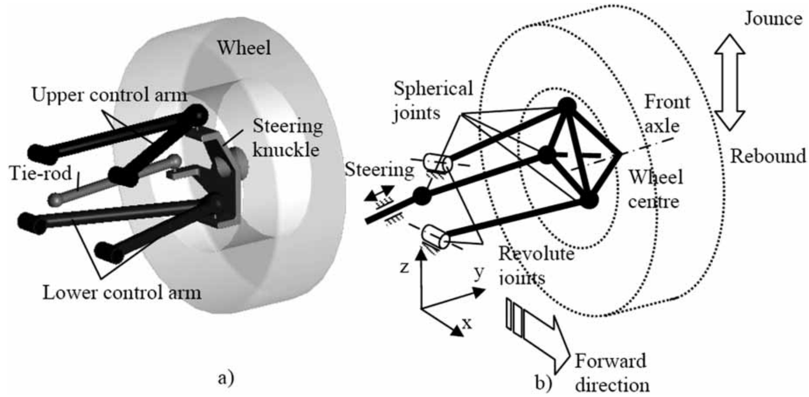

Figure 1. (a) Double-wishbone suspension system and (b) its kinematic scheme.

kinematic solutions for their vehicles, using either four, or sometimes five, links to connect the knuckle to the chassis. Multilink systems are in general costly and complex.

The design of a vehicle suspension system is a typical example of a complex mechanical system. It is an optimisation process whose solution kinematically defines the linkage configuration, including the lengths of the links and the position of ground and floating kinematic pairs. The design parameters define the kinematic configuration of the linkage, and their optimised values are the solution of the problem. At the same time, the formulation of the objective function includes the requirements that must be fulfilled by some functional parameters during the jounce and rebound of the wheel (defining the kinematic characteristic) such as camber, tread width, toe-in and so forth [4]. Two main problems arise in this process. The first is due to the large number of design and functional parameters involved in the objective function. The methodologies developed in suspension design should contain all the variables. However, this fact leads to a complex formulation requiring large computational time to achieve the optimal solution or failing to find it. The second is due to the strong coupling between design and functional parameters. In other words, a small alteration in one design parameter modifies the behaviour of all functional parameters involved in the objective function, or a modification in the required alteration on one functional parameter necessitates the modification of the entire set of design parameters. Furthermore, added to these two difficulties, the problem becomes more pronounced if the number of prescribed positions is increased to obtain greater accuracy. All these characteristics lead to a highly nonlinear system, which is very difficult to solve even with the help of powerful computers.

Over the years, the aforementioned difficulties have been solved using the experience and intuition of expert designers, which are the most important tools in the design of such systems. However, the use of powerful computers and the development of numerical and optimisation methods have made it possible to develop new methodologies and tools, increasingly helping the designer to solve this problem [5]. The design process of suspension systems in vehicles, together with optimal synthesis methodologies, has been studied by several authors. For instance, recently Fallah *et al.* [6] proposed a new nonlinear model of the Macpherson strut suspension in order to improve the ride quality and facilitate the evaluation of the suspension kinematic parameters. Sandor *et al.* [7] developed a rigid-body guidance synthesis procedure with application to RSSR–SS kinematic chains for four exact and four approximate positions. Some authors modelled the double-wishbone linkage as a planar mechanism. In this way, other synthesis procedures developed for planar mechanisms, e.g. Ceccarelli and Vinciguerra [8], can be used to solve this problem. However, accurate solutions require the three-dimensional approach. Displacement matrices were used by Suh [9] with application to the design of some types of suspension systems. Expert systems were used by Liu and Chou [10] to synthesise a planar double-wishbone suspension system. Kovacs [11] developed a procedure for optimal synthesis of three-dimensional mechanisms and applied it to the McPherson strut considering toe, caster and camber angle requirements. The same parameters are considered by Bae *et al.* [12] using an axiomatic approach to the kinematic design of McPherson strut type. Lately, optimal procedures in multi-link suspension systems have achieved great importance due to the wide use of this kind of system by some automobile manufacturers. For instance, Jimenez *et al.*. [13] proposed a method based on the use of fully Cartesian coordinates to synthesise a 5S–5S kinematic chain for camber and tread width alteration (tw). Simiounescu and Beale [14] developed a methodology to optimise a multi-link rear suspension system. Raghavan [15] presented an algorithm for synthesising the location of the tie-rod joints in a suspension system to achieve linear changes in toe-angle during jounce and rebound, which is a valuable tool in suspension design. However, this procedure assumes that the rest of the mechanism has been synthesised previously. Raghavan [16] also presents a dimensional synthesis procedure to control the roll centre height variation during the wheel jounce and rebound.

In spite of all the methodologies developed by all these authors, a general method for the optimal design of double-wishbone suspension systems remains undeveloped. For instance, a general drawback of the aforementioned methods lies in the limitation in the number of design and functional parameters established as requirements. The most complex design must include all the parameters defining the geometry of the double-wishbone system as design parameters. Thus, the design not only includes the lengths of the upper and lower control arms but also all the dimensions of the steering knuckle, the tie-rod and the coordinates of the positions of the joints in the steering knuckle and automobile chassis. At the same time, it is necessary to include the entire kinematic requirements as the limitation in the orientation angles and the necessary offsets. This must be done by means of the accurate definition of their alteration during the jounce and the rebound of the wheel.

In this paper, a procedure for the optimal dimensional synthesis of double-wishbone suspension systems is proposed. Unlike other methods described in the literature, the proposed synthesis procedure is based on obtaining analytical derivatives. This fact reduces the round-off errors, providing advantages such as the use of a greater number of design variables and functional parameters. It also increases the number of precision positions defining the vertical motion of the wheel. For example, there are seven desired functional parameters defined for each prescribed position, four angles: camber, toe, kingpin and caster, two offsets: caster and kingpin, and finally the tw. The kinematic chain is modelled using 14 design parameters including link dimensions, ground and moving joint positions and axle orientation. The synthesis formulation has been completely developed by the authors based on gradient optimisation with specific application to 3D linkages. It is robust, general and efficient. Thus, the method presented in this paper allows engineers to consider a larger number of variables than other methods, reducing the trial and error effort that is necessary in the design of such systems.

2. Kinematic model of the double-wishbone suspension

The front axle double-wishbone suspension system shown in Figure 1(b) is strictly a three-degrees-of-freedom mechanism. However, one of them is provided by the spin motion of the toe-rod (SS link) which is unused in this application. Furthermore, if the steering system remains motionless, the degree of freedom provided by it can be ignored and the mechanism can be analysed as a one-degree-of-freedom linkage.

Figure 2 shows the kinematic scheme used by the authors in this work, where the ground reference frame (*XYZ*) is fixed to the chassis, the frame origin coincides with joint A, and the *XZ* plane is parallel to the vehicle-centre plane in the longitudinal direction. The $\mathbf{R}$ vector gives the tyre radius and the contact position with the road plane (point P). The wheel centre is given by point L shown in the same figure. Vector $\mathbf{u}^{\mathrm{T}} = [\mathbf{u}_1^{\mathrm{T}}\mathbf{u}_2^{\mathrm{T}}, \ldots, \mathbf{u}_{14}^{\mathrm{T}}]$ is the vector of the design parameters whose elements define both lengths of the links and joint positions.

Solid and dashed lines are used for moving links or edges defining moving links, whereas dotted lines are used for describing the position of ground links in the vehicle chassis. Dashed-dotted lines are used as auxiliary vectors in the synthesis procedure; their meaning will be explained later. Each revolute kinematic pair in lower and upper control arms (Figure 1(b)) has been modelled by means of two spherical joints (Figure 2), which permits obtaining an easier formulation in the synthesis method, as will be shown in the forthcoming paragraphs.

Thus, the vector of design parameters contains 14 vectors; all defined using a local reference frame (*xyz*), where the vector origin coincides with the origin of the local frame. The motion of these vectors is defined locally by successive rotations in the following form:

$$\mathbf{u}_i = \mathbf{A}_{zi}\mathbf{A}_{yi}, \quad \tilde{\mathbf{u}}_i = \mathbf{A}_{zyi}\tilde{\mathbf{u}}_i, \tag{1}$$

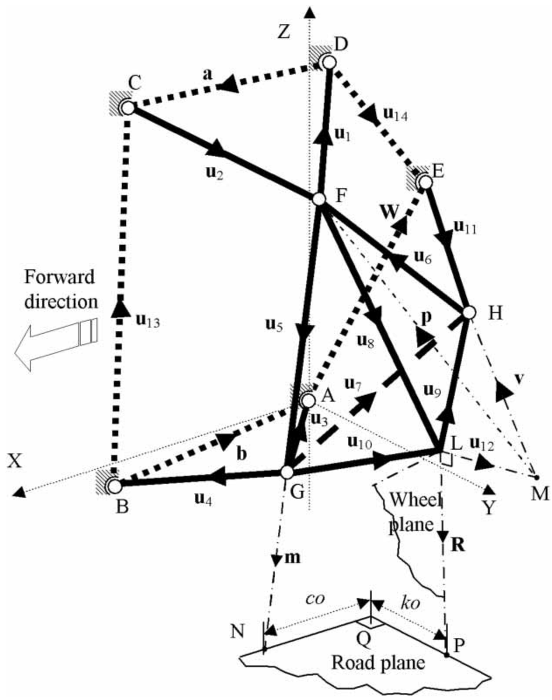

Figure 2. Definition of the kinematic model of a double-wishbone system.

where $\tilde{\mathbf{u}}_i$ are the local coordinates giving the extreme coordinates at the reference position of the vector, and $\mathbf{u}_i$ are the same coordinates after successive rotations. To define the links in the initial position, we consider that they are obtained by means of two successive rotations from position $\tilde{\mathbf{u}}_i = [\mathbf{u}_i\, 0\, 0]$. The matrices $\mathbf{A}_z$ and $\mathbf{A}_y$ are the finite rotations about the z- and y-axis, respectively. Thus, considering rotation θ_i about the z-axis and γ_i about the y-axis the matrix $\mathbf{A}_{zy}$ is defined as

$$A_{zyi} = \begin{bmatrix} \cos\theta_i \cos\gamma_i & -\sin\theta_i & \cos\theta_i \sin\gamma_i \\ \sin\theta_i \cos\gamma_i & \cos\theta_i & \sin\theta_i \sin\gamma_i \\ -\sin\gamma_i & 0 & \cos\gamma_i \end{bmatrix}. \tag{2}$$

Hereafter this matrix will be called simply $\mathbf{A}_i$. As shown in Equation (2), every vector is described by two local coordinates, so it is necessary to formulate a dependent coordinate vector, called $\mathbf{q}$, which contains 28 elements. That is,

$$\mathbf{q}^{\mathrm{T}} = [\theta_1\ \gamma_1\ \theta_2\ \gamma_2\ \ldots\ \theta_{14}\ \gamma_{14}]. \tag{3}$$

The main objective of the dependent coordinates must be to unequivocally describe the orientation of every link in the mechanism. However, some of the parameters within Equation (3) are in the vector of functional requirements that will be defined later, so the $\mathbf{q}$ vector will need to be redefined.

Jounce and rebound motion produce the alteration of functional parameters related with the dependent coordinate vector in the linkage. These parameters must be limited and controlled to provide a desired response in the dynamic behaviour of the vehicle. In the following paragraphs, these parameters will be briefly described and then formulated.

One of the design requirements is given by the alteration undergone by the tread width, normally called tw. This parameter appears in almost all the independent suspension systems during jounce and rebound and has an important influence on the dynamic behaviour of the vehicle [15,17]. *Camber angle* (ϕ) is the angle between the wheel-centre plane and the plane vertical to the road. Typically, it is considered positive when the wheel is inclined outward and negative if the inclination is inwards with respect to the automobile chassis position. The adequate selection of camber angle alteration improves handling and cornering behaviour. Another important angle defined by the kinematic characteristic of the suspension system is the *toe-in alteration* (μ). Toe-in is the angle between the vehicle-centre plane in the longitudinal direction and the line intersecting the centre plane of one wheel with the road plane. It is positive when the front part of the wheel is inwards and negative (toe-out) when it is outwards with respect to the chassis. The correct alteration of this angle during jounce and rebound reduces the tendency to oversteer when the vehicle is cornering. Tread width and camber angle alteration depend mainly on the length of upper and lower control arms, whereas toe-in angle depends mainly on the length of the tie-rod. However, they cannot be considered uncoupled problems because alterations in the dimensions in the control arms or tie-rod influence each other.

Obviously, steering is an important factor in vehicle behaviour during cornering where the suspension and steering systems play a fundamental role. Four kinematic parameters are involved in the suspension system affecting steering, i.e. *kingpin inclination*, *kingpin offset*, *caster angle* and *caster offset* (or *trail*). *Kingpin inclination* (σ) is defined as the angle between steering axis, *FG* in Figure 2, (through the spherical joints in the control arms in the double-wishbone type) projected onto a transversal direction (*YZ* plane), and a plane vertical to the road and parallel to the vehicle-centre plane in the longitudinal direction (*XZ* plane). *Kingpin offset* (ko) is the distance measured on the road plane between the steering axis and the wheel-centre plane in the road (*PQ* distance in Figure 2). On the other hand, *caster angle* (ρ) is the angle between the steering axis projected onto a longitudinal direction (*XZ* plane) and a vertical line drawn through the wheel centre. *Caster offset* (co) is the distance between points N and Q. Both kingpin and caster parameters can have a significant influence on the self-aligning torque during cornering as well as on the force distribution in the suspension system.

All the parameters described above can be formulated using the vectors defined in Figure 2. For instance, the tw is given by the displacement of point P along the *YZ* plane. The wheel plane is defined by a vector $\mathbf{u}_{12}$, normal to its centre plane; thus, the camber angle, ϕ, is given by the γ_{12} coordinate, i.e. $\phi = \gamma_{12}$. The modulus of $\mathbf{u}_{12}$ has no physical meaning so it can be selected as a unitary vector. To obtain an easier formulation using loop closure equations in the synthesis procedure, two additional vectors must be used in the axle definition of $\mathbf{u}_{12}$, and they are $\mathbf{p}$ and $\mathbf{v}$, as shown in Figure 2. Vector $\mathbf{u}_{12}$ again can be used to obtain toe-in angle, μ, in this case defined by means of the θ_{12} coordinate, i.e. $\mu = \theta_{12}\pi/2$ in radians. The kingpin inclination is given by the projection of vector $\mathbf{u}_5$ onto the *YZ* plane and caster angle by projecting the same vector onto the *XZ* plane.

In Figure 2, ko is the *kingpin-offset* parameter measured as the *NP* distance on the Y-axis, where point N is the intersection between the prolongation line of vector $\mathbf{u}_5$ and the road plane. The additional vector necessary to define the distance from G to N is called $\mathbf{m}$. In the same way, co is the *caster-offset* given by the *NP* distance measured on the X-axis.

The vector of functional parameters that must be prescribed by the design and fulfilled by the double-wishbone system can be formulated as

$$\mathbf{r}^{\mathrm{T}} = [\mathrm{tw}\ \phi\ \mu\ \sigma\ \rho\ \mathrm{ko}\ \mathrm{co}]. \tag{4}$$

The functional parameters involved in Equation (4) are not constant in the mobility range of the mechanism and must be defined depending on the position.

The new vector of dependent coordinates is expanded and redefined to be expressed as

$$\mathbf{q}^{\mathrm{T}} = \begin{bmatrix} \mathbf{r}^{\mathrm{T}} & \mathbf{s}^{\mathrm{T}} \end{bmatrix}, \tag{5}$$

where **s** is the vector including the coordinates of the new **p**, and **v** vectors, and the dependent coordinates that are not included in Equation (3). That is,

$$\mathbf{s}^{\mathrm{T}} = [\theta_1\ \gamma_1\ \ldots\ \theta_{11}\ \gamma_{11}\ \theta_{13}\ \gamma_{13}\ \theta_{14}\ \gamma_{14}\ \theta_p\ \gamma_p\ \theta_v\ \gamma_v]. \tag{6}$$

Thus, the total number of dependent coordinates contained in Equation (6) is 37.

3. Kinematic, ground and functional constraints.

Once the kinematic model is defined, it is necessary to formulate the constraint equations. These equations define the kinematic behaviour, the ground joint positions and the functional requirements demanded by the synthesis problem.

The main objective of the kinematic constraint equations is to describe the kinematic behaviour of the mechanism using dependent coordinates, **q**, related to the vector of design parameters, **u**, by means of a set of nonlinear equations. Given a vector of design parameters, **u**, the solution of the constraint equations must define unequivocally the position of everybody during the motion of the system. The system has one degree of freedom and 37 dependent coordinates, so 37 equations have to be imposed. These equations are listed below using matrix notation when necessary, and highlighting their dependence on the functional parameters when this occurs.

$$\mathbf{C}_k(\mathbf{r},\mathbf{s},\mathbf{u}) = \left\{ \begin{matrix} \mathbf{A}_1\mathbf{u}_1 + \mathbf{A}_2\mathbf{u}_2 + \mathbf{A}_a\mathbf{a} \\ \mathbf{A}_3\mathbf{u}_3 - \mathbf{A}_4\mathbf{u}_4 - \mathbf{A}_b\mathbf{b} \\ \mathbf{A}_2\mathbf{u}_2 + \mathbf{A}_5\mathbf{u}_5 + \mathbf{A}_4\mathbf{u}_4 + \mathbf{A}_{13}\mathbf{u}_{13} \\ \mathbf{A}_1\mathbf{u}_1 + \mathbf{A}_6\mathbf{u}_6 + \mathbf{A}_{11}\mathbf{u}_{11} + \mathbf{A}_{14}\mathbf{u}_{14} \\ \mathbf{A}_5\mathbf{u}_5 + \mathbf{A}_6\mathbf{u}_6 + \mathbf{A}_7\mathbf{u}_7 \\ \mathbf{A}_6\mathbf{u}_6 + \mathbf{A}_8\mathbf{u}_8 + \mathbf{A}_9\mathbf{u}_9 \\ \mathbf{A}_7\mathbf{u}_7 - \mathbf{A}_9\mathbf{u}_9 - \mathbf{A}_{10}\mathbf{u}_{10} \\ \mathbf{A}_8\mathbf{u}_8 + \mathbf{A}_{12}(\phi,\mu)\mathbf{u}_{12} + \mathbf{A}_p\mathbf{p} \\ \mathbf{A}_9\mathbf{u}_9 - \mathbf{A}_{12}(\phi,\mu)\mathbf{u}_{12} - \mathbf{A}_v\mathbf{v} \\ u_3\sin\gamma_3 - u_{10}\sin\gamma_{10} + Z_L \end{matrix} \right\} = 0, \tag{7}$$

where $\mathbf{A}_{12}(\phi,\mu)$ contains information about camber and toe angles, and Z_L is the degree of freedom selected as the vertical displacement of the wheel centre.

Kinematic constraints given by Equation (7) are formulated for unalterable position of ground joints. However, in synthesis of suspension systems it may be important to obtain the optimal positions of these fixed joints on the chassis, so their formulation should be introduced in the constraints. We call them ground constraints. Figure 2 shows the vectors involved in this formulation with dotted lines, so we can express them as follows:

$$\mathbf{C}_{\mathbf{g}}(\mathbf{s},\mathbf{u}) = \mathbf{A}_b\mathbf{b} - \mathbf{A}_{13}\mathbf{u}_{13} + \mathbf{A}_a\mathbf{a} - \mathbf{A}_{14}\mathbf{u}_{14} + \mathbf{W} = \mathbf{0}, \tag{8}$$

where **W** is the vector connecting joints A and E. It is important that each new algebraic equation added to the kinematic constraints must introduce a new unknown parameter for every position of the mechanism. In this case, the vectors **a** and **b** may not be altered, because they only have influence in longitudinal dynamic behaviour, so they will not be considered in this study. Since the $\mathbf{u}_{13}$ vector is constrained to be altered within a plane parallel to *ZY*, the only parameters which can be modified in Equation (8) are γ_{13}, θ_{14} and γ_{14}, the three new unknowns.

The functional constraints involve the functional parameters given by the synthesis requirements demanded by the problem. Thus, all the aforementioned parameters must be introduced in their formulation. Some of them (i.e. camber and toe-in) have already been implicitly introduced in kinematic constraint equations so their introduction here is not necessary. Thus, only tw, kingpin and caster angles and both offsets should be introduced as follows:

$$\mathbf{C_f}(\mathbf{r}, \mathbf{s}, \mathbf{u}) = \begin{Bmatrix} -u_3 \sin\theta_3 \cos\gamma_3 + u_{10} \sin\theta_{10} \cos\gamma_{10} + R\cos(\theta_{12} - \pi/2)\sin\gamma_{12} - \mathrm{tw} \\ \mathbf{A}_{10}\mathbf{u}_{10} + \mathbf{R} - \mathbf{A}_{\mathrm{m}}\mathbf{m} - \mathbf{Off}(\mathrm{ko}, \mathrm{co}) \end{Bmatrix} = 0, \tag{9}$$

where tw = *YP* is the tw described by the contact of wheel tread centre and **Off**(ko,co) is the vector containing the offset values

$$\mathbf{Off}(\mathrm{ko}, \mathrm{co})^{\mathrm{T}} = [\mathrm{ko} \quad \mathrm{co} \quad 0]. \tag{10}$$

$\mathbf{A}_{\mathrm{m}}$ in Equation (9) is the same matrix as $\mathbf{A}_5$ with the following angles,

$$\begin{aligned} \theta_m &= \theta_5, \\ \gamma_m &= \gamma_5. \end{aligned} \tag{11}$$

The constraint equations can be formulated altogether as follows:

$$\mathbf{C(r, s, u)} = \begin{Bmatrix} \mathbf{C}_k\mathbf{(r, s, u)} \\ \mathbf{C}_g\mathbf{(s, u)} \\ \mathbf{C}_f\mathbf{(r, s, u)} \end{Bmatrix} = 0 \tag{12}$$

which constitutes a set of 37 equations with 37 unknowns.

4. Multiobjective function and synthesis procedure

Optimal synthesis of linkages can be tackled using global or local approaches. Global methods are based on stochastic approaches, attempting to find the overall best solution without the influence of the starting point. On the other hand, local techniques use deterministic approaches to search for a local minimum, which means that the solution achieved is the nearest to the initial-guess. In this case, the synthesis error is minimised, but there is no guarantee that other solutions exist that could improve the local solution achieved. While global methods seem the ideal solution, they present many problems when the number of variables involved is high, losing characteristics such as robustness, efficiency and accuracy. Furthermore, in global optimisation, complex systems need to dramatically increase the number of constraints to guide the mechanism evolution towards the optimal solution. However, local approaches reduce this effort because the initial guess is orientated to the solution. For instance, when a new vehicle model is designed, the old system to be redesigned can be used as the initial guess mechanism, adapting the design of the suspension system to the new requirements by means of local optimisation. For all these reasons, a deterministic approach has been used in this paper.

Multiobjective or multi-criteria optimisation is the process of simultaneously optimising two or more conflicting objectives. Thus, the first step is to measure the difference between the mechanism generated by the algorithm during the optimisation process and the desired linkage for all functional parameters. Obviously, the desired mechanism remains unknown during the process, so the error measurement must be formulated using the functional parameters. Thus, the difference between both mechanisms can be expressed in the following form:

$$\boldsymbol{\varepsilon}_j = \mathbf{r}_j(\mathbf{u}) - \mathbf{r}_j^{\mathrm{d}}, \tag{13}$$

where d stands for 'desired' and j stands for the mechanism's prescribed position, $\mathbf{r}_j^{\mathrm{d}}$ being the vector of the functional parameters imposed by the problem. Vector $\mathbf{r}_j(\mathbf{u})$ contains the same parameters as $\mathbf{r}_j^{\mathrm{d}}$, but in the mechanism generated by the algorithm. Weighting parameters should be introduced in Equation (13) to give more or less relative importance to functional parameters.

In the proposed formulation, the generated parameters depend on the vector of the design parameters, $\mathbf{u}$, and the appropriate alteration of this vector can reduce the error between the two mechanisms. Thus, rewriting the expression given by Equation (13),

$$\left[\mathbf{r}_j(\mathbf{u} + \boldsymbol{\Delta}\mathbf{u}) - \mathbf{r}_J^{\mathrm{d}}\right] - \boldsymbol{\varepsilon}_{\Delta j} = 0, \tag{14}$$

where $\Delta\mathbf{u}$ is an incremental variation provided to the vector $\mathbf{u}$ to eliminate the error, or at least, to reduce it from $\boldsymbol{\varepsilon}_j$ to $\boldsymbol{\varepsilon}_{\Delta j}$, such that $\boldsymbol{\varepsilon}_{\Delta j} \leq \boldsymbol{\varepsilon}_j$.

Now, applying the Taylor series expansion until the first-order derivative to the term in brackets in Equation (14), and equating it to zero, the following expression is obtained,

$$\left[\mathbf{r}_j(\mathbf{u} + \boldsymbol{\Delta}\mathbf{u}) - \mathbf{r}_J^{\mathrm{d}}\right] = \left[\mathbf{r}_j(\mathbf{u}) - \mathbf{r}_J^{\mathrm{d}}\right] + \mathbf{J}_j\boldsymbol{\Delta}\mathbf{u} = \boldsymbol{\varepsilon}_j + \mathbf{J}_j\boldsymbol{\Delta}\mathbf{u} = 0, \tag{15}$$

which is an important formula that relates the difference between the two mechanisms and the necessary incremental variation of the vector of design parameters to reduce that difference.

The optimisation process proposed consists in minimising the synthesis error function (SEF) defined by the multiobjective function as follows:

$$SEF(\mathbf{u}) = \frac{1}{2}\sum_{j=1}^{n}\left[\mathbf{r}_j(\mathbf{u}) - \mathbf{r}_j^{\mathrm{d}}\right]^{\mathrm{T}}\left[\mathbf{r}_j(\mathbf{u}) - \mathbf{r}_j^{\mathrm{d}}\right], \tag{16}$$

subject to the set of constraint equations defined before. It is assumed that SEF($\mathbf{u}$) is a continuous and differentiable function. Equation (16) is formulated based on the mean square error of the difference given by Equation (13), where n stands for the maximum number of prescribed positions. Differentiating Equation (16) with respect to the vector of design parameters and equating it to zero, the following expression is obtained,

$$\sum_{j=1}^{n}\mathbf{J}_j^{\mathrm{T}}\left[\mathbf{r}_j(\mathbf{u}) - \mathbf{r}_j^{\mathrm{d}}\right] = 0, \tag{17}$$

where $\mathbf{J}_j^{\mathrm{T}}$ is the Jacobian matrix for position j. That is,

$$\mathbf{J}_J^{\mathrm{T}} = \begin{bmatrix} \dfrac{\partial r_{1j}}{\partial u_1} & \cdots & \dfrac{\partial r_{7j}}{\partial u_1} \\ \cdots & \cdots & \cdots \\ \dfrac{\partial r_{1j}}{\partial u_{14}} & \cdots & \dfrac{\partial r_{7j}}{\partial u_{14}} \end{bmatrix}. \tag{18}$$

The Jacobian contains the derivatives of the functional parameters with respect to the vector of the design parameters. This matrix can be regarded as the sensitivity of the requirements with respect to small variations of design parameters, giving valuable information about which design parameters have most influence on these parameters.

Now, using the first-order approximation obtained before and introducing it in Equation (17),

$$\sum_{j=1}^{n} \mathbf{J}_j^{\mathrm{T}} \left(\boldsymbol{\varepsilon}_j + \mathbf{J}_j \Delta \mathbf{u} \right) = 0. \tag{19}$$

In Equation (19) the term $\Delta\mathbf{u}$ is the increment in the vector of the design parameters necessary to reduce the error in all prescribed positions during the motion, so it can be taken out of the expression because its value must be the same in all prescribed positions. Thus, Equation (19) can be rewritten,

$$\boldsymbol{\Delta}\mathbf{u} = -\mathbf{J}^{*}\mathbf{J}^{\mathrm{T}}\boldsymbol{\varepsilon}, \tag{20}$$

where,

$$\begin{aligned} \mathbf{J}^{\mathrm{T}} &= \left[\mathbf{J}_1^{\mathrm{T}} \mathbf{J}_2^{\mathrm{T}} \dots \mathbf{J}_n^{\mathrm{T}} \right], \\ \mathbf{J}^{*} &= \left(\sum_{j=1}^{n} \mathbf{J}_j^{\mathrm{T}} \mathbf{J}_j \right)^{-1} = \left[\mathbf{J}^{\mathrm{T}} \mathbf{J} \right]^{-1}, \\ \boldsymbol{\varepsilon}^{\mathrm{T}} &= \left[\varepsilon_1 \varepsilon_2 \dots \varepsilon_n \right]. \end{aligned} \tag{21}$$

Equation (20) can be regarded as a recursive formula permitting the mechanism to evolve from the initial guess to the optimal solution. However, it is necessary to obtain the Jacobian matrix. The elements of the Jacobian can be obtained by means of exact or numerical differentiation. Numerical differentiation has well-known drawbacks (i.e. higher computational time, less accuracy in the search direction, etc.) compared with exact differentiation; for this reason the latter should always be used when possible. Here, the constraint equations allow exact differentiation to be obtained. Considering the following dependence in Equation (12),

$$\begin{aligned} \mathbf{r} &= \mathbf{r(u)}, \\ \mathbf{s} &= \mathbf{s(u)}, \end{aligned} \tag{22}$$

and differentiating Equation (12) with respect to the vector of design parameters it becomes

$$\left[\frac{\partial \mathbf{C}}{\partial \mathbf{r}} \;\vdots\; \frac{\partial \mathbf{C}}{\partial \mathbf{s}} \right] \begin{bmatrix} \dfrac{\partial \mathbf{r}}{\partial \mathbf{u}} \\ \hline \dfrac{\partial \mathbf{s}}{\partial \mathbf{u}} \end{bmatrix} = \left[\frac{\partial \mathbf{C}}{\partial \mathbf{u}} \right], \tag{23}$$

or in more compact form it may be written as

$$\left[\mathbf{C_r} \;\vdots\; \mathbf{C_s} \right] \begin{bmatrix} \mathbf{r_u} \\ \hline \mathbf{s_u} \end{bmatrix} = [\mathbf{C_u}], \tag{24}$$

where $\mathbf{C}_{\mathrm{r}}$, $\mathbf{C}_{\mathrm{s}}$ and $\mathbf{C}_{\mathrm{u}}$ are the derivatives of the constraint equations with respect to the vectors $\mathbf{r}$, $\mathbf{s}$ and $\mathbf{u}$, respectively. Reordering Equation (24) leads to

$$\begin{bmatrix} \mathbf{r_u} \\ \hline \mathbf{s_u} \end{bmatrix} = \left[\mathbf{C_r} \;\vdots\; \mathbf{C_s} \right]^{-1} [\mathbf{C_u}]. \tag{25}$$

The submatrix $\mathbf{r}_\mathrm{u}$ in Equation (25) is the desired Jacobian matrix expressed in Equation (18), i.e. $\mathbf{J}_\mathrm{j} = \mathbf{r}_\mathrm{u}$. Thus, it is necessary to compute $\mathbf{C}_\mathrm{r}$, $\mathbf{C}_\mathrm{s}$ and $\mathbf{C}_\mathrm{u}$ to obtain the Jacobian, which is gained by differentiating Equation (12) once with respect to vectors $\mathbf{r}$, $\mathbf{s}$ and $\mathbf{u}$, respectively. This is very easy since they can be obtained from direct differentiation of these constraint equations. In the case of $\mathbf{C}_\mathrm{r}$ and $\mathbf{C}_\mathrm{s}$, it will be necessary to compute matrices $\mathbf{A}_{\theta i}$ and $\mathbf{A}_{\gamma i}$ which contain the dependent coordinates. That is,

$$\mathbf{A}_{\theta i} = \frac{\partial \mathbf{A}_i}{\partial \theta_i} = \begin{bmatrix} -\sin\theta_i \cos\gamma_i & -\cos\theta_i & -\sin\theta_i \sin\gamma_i \\ \cos\theta_i \cos\gamma_i & -\sin\theta_i & \cos\theta_i \sin\gamma_i \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{A}_{\gamma i} = \frac{\partial \mathbf{A}_i}{\partial \gamma_i} = \begin{bmatrix} -\cos\theta_i \sin\gamma_i & 0 & \cos\theta_i \cos\gamma_i \\ -\sin\theta_i \sin\gamma_i & 0 & \sin\theta_i \cos\gamma_i \\ -\cos\gamma_i & 0 & -\sin\gamma_i \end{bmatrix}. \tag{26}$$

Once the Jacobian is computed, Equation (20) can be applied to the initial guess mechanism as follows:

$$\mathbf{u}_{k+1} = \mathbf{U}_k - \mathbf{J}_k^{*}\mathbf{J}_k^{\mathrm{T}}\boldsymbol{\varepsilon}_k, \tag{27}$$

and evaluating the error at every step, k, it makes the mechanism, evolve until the optimal solution, $\bar{\mathbf{u}}$, is achieved. Thus, a convergence criterion must be applied as, SEF $(\bar{\mathbf{u}}) \leq \delta$ for a sufficiently small δ.

All the constraint equations considered in the aforementioned formulation are the so-called equality constraints. However, it is quite usual to find inequality constraints in synthesis problems limiting the values adopted by the design parameters within a prescribed range. These can be expressed as follows:

$$\mathbf{g}(\mathbf{u}) \leq \mathbf{0}. \tag{28}$$

One reason for this is related to the impossibility of negative values in the modulus of design parameters because the lengths must always be positive, i.e. $\mathbf{u}_i \geq 0$. Furthermore, in order to avoid singular situations it may be recommended to establish a minimum value for the modulus, i.e. $\mathbf{u}_i^{\min} - \mathbf{u}_i \leq 0$. Focusing on the design of suspension systems, such constraints are used in the limitation of weight and space occupied by the mechanism (e.g. the maximum length in the upper and lower control arms). Normally, they are formulated as a limitation in the length, i.e. $\mathbf{u}_i - \mathbf{u}_i^{\max}$, where $\mathbf{u}_i^{\max}$ is the maximum value that can be adopted by the modulus of design variable $\mathbf{u}_i$. To solve this problem, it is necessary to introduce a new vector, $\boldsymbol{\nu}^\mathrm{T} = [\boldsymbol{\nu}_1^2, \boldsymbol{\nu}_2^2, \ldots, \boldsymbol{\nu}_\mathrm{m}^2]$, which is the vector of the square non-negative slack variables added to the inequality constraints, $\mathbf{g}(\mathbf{u})$, to transform them in equality constraints, $\mathbf{G}(\mathbf{u})$. Thus,

$$\mathbf{G}(\mathbf{u}) = \mathbf{g}(\mathbf{u}) + \nu = \mathbf{0}. \tag{29}$$

Then, it is necessary to add the set of constraint equations given by Equation (29) to the rest and solve them simultaneously.

The whole optimisation procedure is shown in the flowchart in Figure 3 where k stands for the iteration step and p is the number of precision poses. The algorithm begins with the definition of the double-wishbone requirements using the aforementioned vector of desired functional parameters, i.e. vector $\mathbf{r}^\mathrm{d}$. This vector contains the seven desired parameters defined for the entire set of precision poses. The input coordinate is the height of the wheel centre position with respect to the central position and is given by vector Z_L, which contains the p prescribed values. At the same time, in order to define the initial-guess linkage, the vector of

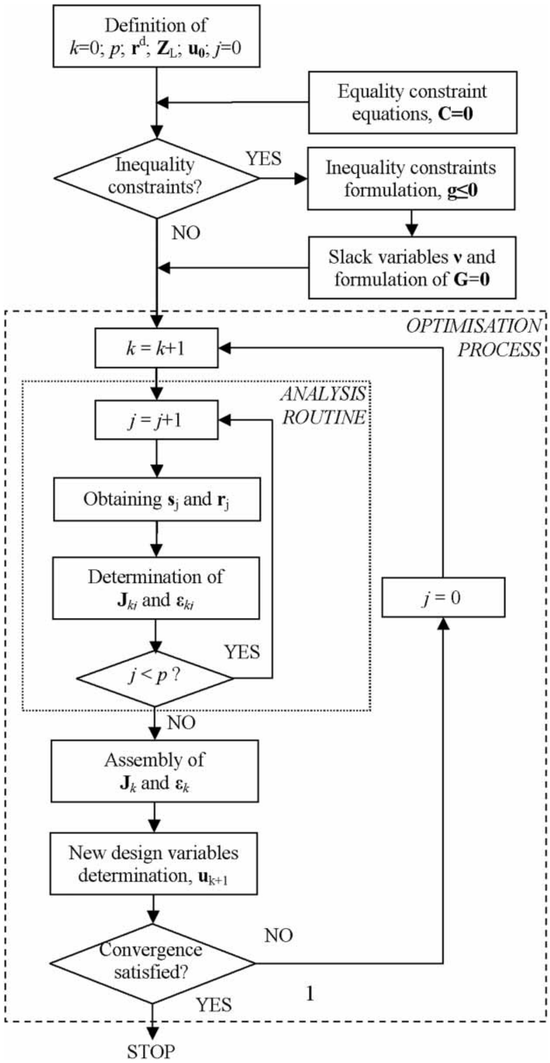

Figure 3. Synthesis process flowchart.

the design parameters $\mathbf{u}_0$ (subscript 0 stands for initial values in the optimisation process) is proposed. This vector contains the dimensions of all links and the coordinates of the points where the linkage is attached to the vehicle chassis. The optimised value of this vector will be the solution of the problem. Here, equality constraints shown in Equation (12) are introduced in the algorithm and, if they exist, inequality constraints are formulated and transformed into equality constraints by means of slack variables, ν. With this step, the problem definition is over and the optimisation process starts. Now, using the analysis routine (Figure 3), the vectors of the dependent coordinates, $\mathbf{s}_j$, and the generated functional parameter, $\mathbf{r}_j$, are obtained by solving the constraint equations. Thus, both vectors are used to obtain the Jacobian, $\mathbf{J}_{kj}$, and the structural error, $\boldsymbol{\varepsilon}_{kj}$, in the jth position. The Jacobian is obtained from Equation (25) and the structural error from Equation (13). The same routine is repeated until the last prescribed pose is achieved. After that, the determination of the new vector of design variables requires the assembly of both the Jacobian matrix and the structural error vector. This is done using the following expressions:

$$\boldsymbol{\varepsilon}_k^{\mathrm{T}} = \left\{ \boldsymbol{\varepsilon}_{k1} \quad \boldsymbol{\varepsilon}_{k2} \quad \dots \quad \boldsymbol{\varepsilon}_{kp} \right\}, \tag{30}$$

$$\mathbf{J}_k^{\mathrm{T}} = \left[\mathbf{J}_{k1}^{\mathrm{T}} \quad \mathbf{J}_{k2}^{\mathrm{T}} \quad \dots \quad \mathbf{J}_{kp}^{\mathrm{T}} \right]. \tag{31}$$

Next, using the recursive formula given by Equation (27), the necessary alteration in the vector of the design variables can be obtained in order to reduce the structural error in the kth iteration. Finally, the synthesis error is evaluated by Equation (16) and the following convergence criterion is applied:

$$|\mathrm{SEF}(\mathbf{u}_{\mathrm{k+1}}) - \mathrm{SEF}(\mathbf{u}_{\mathrm{k}})| \leq \mu_{\mathrm{A}} + \mu_{\mathrm{R}} |\mathrm{SEF}(\mathbf{u}_{\mathrm{k}})| \tag{32}$$

where μ_{A} is the absolute tolerance and μ_{R} is the relative one. Parameter μ_{A} is set to a small value of 10^{-6}, while μ_{R} is set to 1% or 0.01. Only if Equation (32) is satisfied in two consecutive iterations, is the optimisation process stopped. If this criterion is not fulfilled, the same process is repeated for the new vector of design parameters until the required accuracy is achieved.

From the computational point of view, it is very important to highlight that with this formulation the elements of the Jacobian are calculated in an exact form, i.e. no numerical differentiation techniques are required. Furthermore, the SEF is evaluated only once in each iteration. Both the exact differentiation and the reduced number of evaluations of the objective function provide a good convergence ratio.

5. Numerical example

In this section, a numerical example is presented based on the formulation developed above. From the kinematic point of view, the design of a suspension system supposes complying with the performance requirements. The performance requirements provide good handling and comfort characteristics and they are established by means of a set of performance curves giving the variation of the functional parameters (i.e. camber, tread width, etc.) with respect to the vertical motion of the wheel. These curves depend on the type of vehicle where the suspension system will be used. Vehicle manufacturers establish their own curves and these curves can be different, even for similar vehicles. The reason for this discrepancy is because there is not a single best solution. Instead, a trade-off is necessary, and some vehicle characteristics must prevail over others. A discussion on these issues is beyond the scope of this article, for this reason, in the example, a set of performance curves is suggested. It should be pointed out that the application of the proposed procedure to another vehicle is similar to the case presented here, it being only necessary to change the set of curves to fit the new conditions of the vehicle.

The goal in the example presented here is to improve the design of a double-wishbone suspension system in a bus front axle. With this aim, we use a double-wishbone suspension system used in low-floor buses as a reference. This system has been proved and provides good dynamic characteristics, so its kinematic parameters are established as the objective ones. However, the dimensions of such a system in the transversal plane to the vehicle are too long, reducing the passenger space inside the bus. In this case, due to the necessity of the bus kneeling to enable access to disabled people, the suspension system requires special jounce displacement together with the requirements of the tw, camber, toe-in, and so on. For this reason, instead of considering the normal interval of the wheel carrier vertical travel, e.g. $-45\,\mathrm{mm} \leq Z_L \leq +55\,\mathrm{mm}$, a larger interval has been selected as $-80\,\mathrm{mm} \leq Z_L \leq +80\,\mathrm{mm}$. Obviously, with the aim of kneeling, such a displacement is not necessary during the rebound, but has been considered for other purposes, for instance, driving through irregular roads.

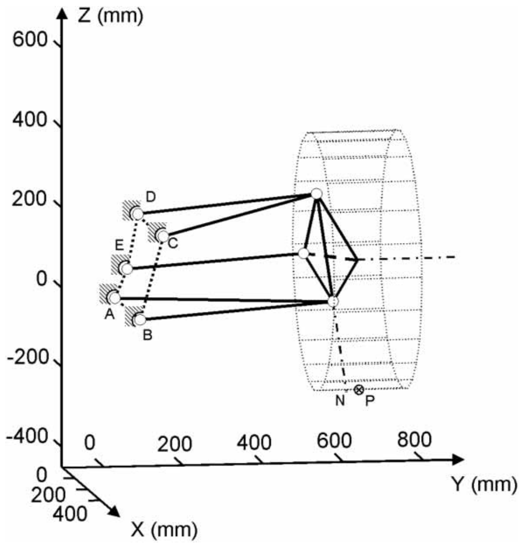

Figure 4. Original design of a double-wishbone suspension system in buses.

Figure 4 shows the scheme of the reference suspension system where the maximum dimension from the inner attachment point to the wheel centre is 518 mm. This means, in a bus with a tread width of 1.8 m, the inside free space between wheel houses is less than 764 mm, which supposes an important reduction of space. Thus, in this type of design, it is crucial to improve the passenger space available inside the bus cabin by means of reducing the transversal dimensions occupied by both wheel houses.

Special attention must be paid to the upper and lower control arms to reduce the global dimensions of this kind of system. These two elements are the main contributors to the transversal dimension, but their alteration substantially modifies the dynamic behaviour of the vehicle. The reduction of the control arms must be in keeping with the tie-rod dimension controlling the alteration of toe-in angle.

Table 1 shows the dimensions of the actual solution used in mid-sized buses, together with the initial guess mechanism proposed to be synthesised. A global reduction (between 16 and 28%) in the upper and lower control arm is suggested as a starting point of iteration. This reduction is shown in Figure 5 and may be better adjusted by means of trial and error, but here, it is suggested more or less randomly by the authors to demonstrate the capacity of the method to search for the optimal solution.

Different positions have been considered for chassis attachment points, i.e. points A, B, C, D and E. In the initial guess mechanism, only spherical joints C and D may be altered by the algorithm, the rest will remain unaffected during the optimisation process.

The difference between wheel centre position in the original mechanism and the initial guess is 77 mm. To avoid singularities and ensure space reduction, some inequality constraints are applied as shown in Table 1.

Thirteen prescribed positions along the wheel carrier travel have been used to define the synthesis problem. Table 2 shows the Z_L coordinates together with the seven associated desired functional parameters. This table gives the multiobjective optimisation criteria for the double-wishbone kinematic design. Thus, considering all prescribed poses, the total number of requirements is 91. The tw is measured in the Y direction (Y-axis in Figure 2) with respect to the longitudinal central plane in the vehicle parallel to the XZ plane.

Table 1. Original and initial guess values of the design parameters together with the feasible regions.

Design variable	Original	Initial guess	Reduction (%)	Inequality constraints
$\mathbf{u}_1$	461.74	330	28.53	$10 \leq \mathbf{u}_1 \leq 330$
$\mathbf{u}_2$	472.02	394	16.53	$10 \leq \mathbf{u}_2 \leq 394$
$\mathbf{u}_3$	554.14	443	20.06	$10 \leq \mathbf{u}_3 \leq 443$
$\mathbf{u}_4$	549.06	407	25.87	$10 \leq \mathbf{u}_4 \leq 407$
$\mathbf{u}_5$	273.18	250	8.49	$\mathbf{u}_5 \geq 10$
$\mathbf{u}_6$	217.21	185	14.83	$\mathbf{u}_6 \geq 10$
$\mathbf{u}_7$	191.13	274	−43.36	$\mathbf{u}_7 \geq 10$
$\mathbf{u}_8$	196.74	225	−14.37	$\mathbf{u}_8 \geq 10$
$\mathbf{u}_9$	185.00	260	−40.54	$\mathbf{u}_9 \geq 10$
$\mathbf{u}_{10}$	119.24	165	−38.37	$\mathbf{u}_{10} \geq 10$
$\mathbf{u}_{11}$	440.65	350	20.57	$\mathbf{u}_{11} \geq 10$
$\mathbf{u}_{13}$	155.34	153	1.50	$\mathbf{u}_{13} \geq 10$
$\mathbf{u}_{14}$	217.17	260	−19.72	$\mathbf{u}_{14} \geq 10$

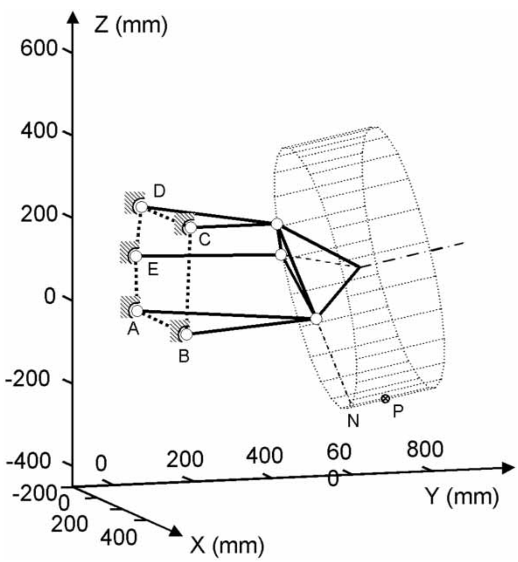

Figure 5. Initial guess mechanism.

Normally, this point requires special attention in other synthesis methods with limitation in the maximum number of prescribed poses and functional parameters. However, since the proposed method is not limited, the designer should select freely those positions giving the necessary accuracy in the parameter definitions. Nevertheless, it is necessary to bear in mind that a larger number of prescribed positions requires more computational time to achieve the optimal solution.

The algorithm achieves the convergence after 28 iterations spending 8.32 CPU seconds in a PC Pentium DC at 2.2 GHz. The configuration of the optimal solution mechanism is shown in Figure 6 and the numerical results are shown in Table 3. In Table 3, together with the dimensions of the final mechanism, the reduction with respect to the initial guess mechanism is shown. Here, it is important to highlight that the wheel centre position in the final mechanism is reduced by 182 mm with respect to the original one, which means that there is 364 mm more

Table 2. Prescribed poses and desired values of functional parameters.

Poses	1	2	3	4	5	6	7	8	9	10	11	12	13
Z_L (mm)	80.0	66.7	53.3	40.0	26.7	13.3	0.00	−13.3	−26.6	−40.0	−53.3	−66.6	−80.0
tw (mm)	567.26	569.78	571.94	573.74	575.18	576.26	577.00	577.38	577.40	577.08	576.40	575.37	573.97
ϕ (deg)	2.51	2.05	1.61	1.18	0.77	0.38	0.00	−0.36	−0.72	−1.07	−1.40	−1.73	−2.04
μ (deg)	1.11	0.90	0.70	0.51	0.33	0.16	0.00	−0.15	−0.29	−0.43	−0.55	−0.66	−0.77
σ (deg)	10.39	9.92	9.46	9.02	8.60	8.19	7.80	7.42	7.05	6.70	6.35	6.02	5.70
ρ (deg)	4.05	4.04	4.04	4.03	4.03	4.02	4.02	4.02	4.01	4.01	4.01	4.01	4.00
ko (mm)	−32.32	−32.37	−32.42	−32.46	−32.50	−32.54	−32.57	−32.61	−32.64	−32.66	−32.69	−32.71	−32.74
co (mm)	24.29	23.94	23.61	23.32	23.06	22.831	22.62	22.43	22.27	22.13	22.01	21.90	21.82

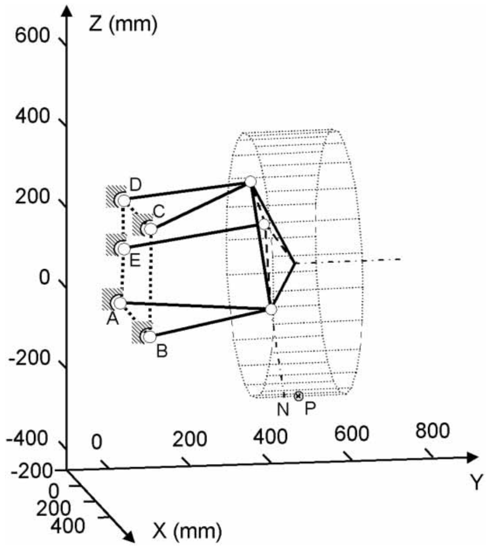

Figure 6. Optimal solution.

Table 3. Optimal values of the design parameters and variations in the lengths.

		Alteration with respect to	
Design variables	Optimal solution	Original mechanism%	Initial guess
$\mathbf{u}_1$	328.71	28.81	0.39
$\mathbf{u}_2$	393.11	16.72	0.23
$\mathbf{u}_3$	376.50	32.06	15.01
$\mathbf{u}_4$	405.36	26.17	0.40
$\mathbf{u}_5$	315.14	−15.36	−26.06
$\mathbf{u}_6$	204.69	5.76	10.64
$\mathbf{u}_7$	243.57	−27.44	11.11
$\mathbf{u}_8$	229.38	−16.59	−1.95
$\mathbf{u}_9$	177.99	3.79	31.54
$\mathbf{u}_{10}$	124.69	−4.57	24.43
$\mathbf{u}_{11}$	350.15	20.54	−0.04
$\mathbf{u}_{13}$	158.54	−2.06	−3.62
$\mathbf{u}_{14}$	263.94	−21.54	−1.52

for the bus platform. Furthermore, the largest reduction in the lengths of the links with respect to the initial guess is 31.54%, and with respect to the original system, it is 32.06%.

To avoid the effect of the different units present in the objective function all terms in this function have been normalised using weighting coefficients and the SEF is transformed into an adimensional function. For the initial guess mechanism the value of SEF is 109.6212. The optimisation results are very good since the optimised mechanism is found with a structural error of 0.1740. Figure 7 shows the evolution of the SEF during the optimisation process for every functional parameter. Each parameter shows a different speed of convergence; some of them achieve the convergence in a few iterations and others suffer more difficulties, but the trend is almost the same in all cases, reducing the synthesis error in each iteration. Finally, Table 4 shows the alteration in the coordinate positions of the fixed pairs (chassis attachments).

In Figure 8 the diagrams of the functional parameters during the jounce and rebound are given in three cases: original mechanism, initial guess and final solution. Thus, this figure

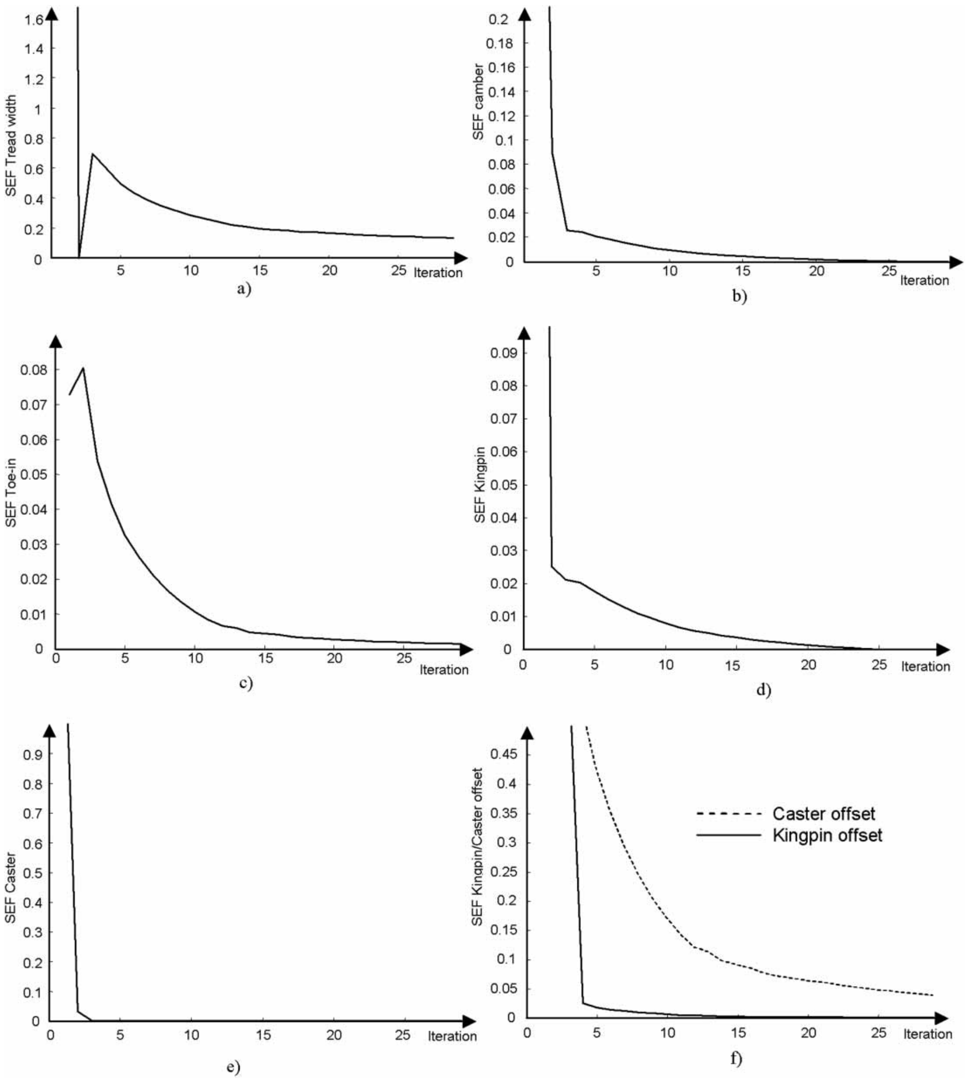

Figure 7. Synthesis error evolution in the functional parameters.

Table 4. Attachment points coordinates.

	Original	Initial guess	Optimal solution
x_C	400.00	400.00	400.00
y_C	59.00	10.00	3.84
z_C	209.00	254.00	255.91
x_D	0.00	0.00	0.00
y_D	59.00	10.00	3.84
z_D	209.00	250.00	255.91

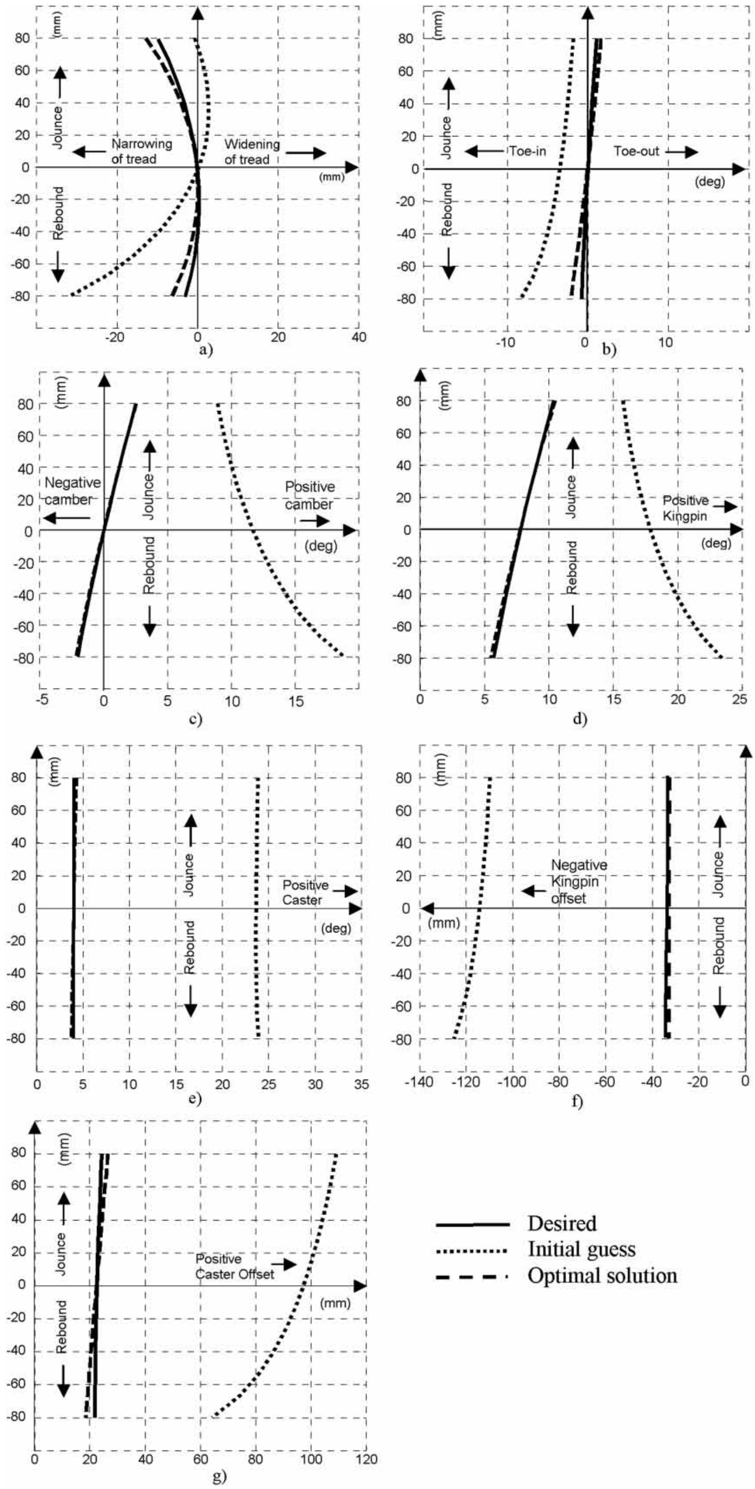

Figure 8. Alteration of the functional parameters in the desired, initial guess, and optimal solution.

represents the deviation of the generated functional parameters with respect to the desired ones. Figure 8(a) shows the reduction of the synthesis error in the tw. As can be seen, the solution shows differences with respect to the desired curve only in extreme values (upper and lower) required for special tasks (e.g. kneeling), but in the central values where the dynamic response is necessary, it can be considered adequate. The rest of the diagrams (Figures 8(b)–(g)) show high accuracy in the results of all functional parameters. In some cases, curves fit so well that it is difficult to differentiate the solution and desired curves.

6. Concluding remarks

This paper has tackled the kinematic design of a double-wishbone suspension system using a dimensional synthesis procedure. The mechanism is modelled as an RSSR–SS spatial linkage and formulated by means of constraint equations. The main advantage of the method, besides the conceptual simplicity, is its capability to consider a large number of design parameters, functional parameters and prescribed positions.

Design parameters are the lengths of the links and, indirectly, the position of the ground joints. In this way, it is possible to have control over the general linkage dimensions and the attachment points to the chassis. Functional parameters represent the mechanism requirements demanded by the problem and must be fulfilled as well as possible since there is no exact solution. Seven functional parameters have been considered during the up-and-down motion of the wheel: tread width, camber, toe-in, kingpin and caster angles, and kingpin and caster offsets.

The method does not have a limited number of prescribed positions, so high accuracy can be achieved in the definition of the functional parameters during the motion of the wheel carrier. However, it is worth pointing out that when a large number of precision poses are defined, a large computational time is spent to achieve the convergence and it may limit the strengths of the method.

The proposed method formulates the objective function as a measurement of the synthesis error between the generated and desired mechanisms. This function is minimised subject to a set of constraints which can be formulated as equality or inequality equations. These kinds of constraints are very common in kinematic design to obtain a valid range of the design parameters.

The method is based on gradient determination and uses exact derivatives to search for the optimal solution. In this way, the coupled effects appearing in these sorts of problems can be considered simultaneously, obtaining a general solution for all variables. An effective recursive formula has been obtained to make the system evolve from the initial guess mechanism to the optimal solution. The capacity of the method to improve the design of double-wishbone suspension systems with efficiency and robustness is shown in the example presented.

Acknowledgements

This paper has been developed in the framework of the Project DPI2006-13945 funded by the Spanish Ministry of Education and Science.

References

[1] K. Chen and D.G. Beale, *Base dynamic parameter estimation of a MacPherson suspension mechanism*, Vehicle System Dynamics 39(3) (2003), pp 227–244.

[2] D.J. Cole, *Fundamental issues in suspension design for heavy road vehicles*, Vehicle System Dynamics 35 (2001), pp. 319–360.
[3] J. Reimpell and H. Stoll, *The Automotive Chassis: Engineering Principles*, Vogel-Buchverlag, Würzburg, 1998.
[4] K. Strandemar and B. Thorvald, *Ride diagram: A tool for analysis of vehicle suspension settings*, Vehicle System Dynamics 44 (2006), pp. 913–920.
[5] M.M. Chatillon, L. Jezequel, P. Coutant, and P. Baggio, *Hierarchical optimisation of the design parameters of a vehicle suspension system*, Vehicle System Dynamics 44 (2006), pp. 817–839.
[6] M.S. Fallah, R. Xie, and W.F. Xie, *New model and simulation of Macpherson suspension system for ride control applications*, Vehicle System Dynamics 47(2) (2009), pp 195–220.
[7] G.N. Sandor, L.J. Xu, and S.P. Yang, *Computer aided synthesis of two-closed loop RSSR–SS spatial motion generators with branching and sequence constraints*, Mech. Mach. Theory 21 (1986), pp. 345–350.
[8] M. Ceccarelli and A. Vinciguerra, *Approximate four-bar circle-tracing mechanisms: Classical and new synthesis*, Mech. Mach. Theory 35 (2000), pp. 1579–1599.
[9] C.H. Suh, *Synthesis and analysis of suspension mechanisms with use of displacement matrices*, SAE 890098 (1989), pp. 189–200.
[10] T.S. Liu and C.C. Chou, *Type synthesis of vehicle planar suspension mechanism using graph theory*, J. Mech. Des. 115 (1993), pp. 652–657.
[11] A.P. Kovacs, *Geometric event constraints: Design synthesis by multibody Lagrangian type I formalism*, Int. J. Vehicle Des. 29 (2002), pp. 160–177.
[12] S. Bae, J.M. Lee, W.J. Choi, J.R. Jun, and T.O. Tak, *Axiomatic approach to the kinematic design of an automotive suspension system with the McPherson strut type*, Int. J. Vehicle Des. 31 (2003), pp. 58–71.
[13] J.M. Jimenez, G. Alvarez, J. Cardenal, and J. Cuadrado, *A simple and general method for kinematic synthesis of spatial mechanisms*, Mech. Mach. Theory 32 (1997), pp. 323–341.
[14] P.A. Simiounescu and D. Beale, *Synthesis and analysis of the five-link rear suspension system used in automobiles*, Mech. Mach. Theory 37 (2002), pp. 815–832.
[15] M. Raghavan, *Suspension design for linear toe curves: A case study in mechanism synthesis*, J. Mech. Des. 126 (2004), pp. 278–282.
[16] M. Raghavan, *Suspension synthesis for N: 1 roll center motion*, J. Mech. Des. 127 (2005), pp 673–678.
[17] W.F. Milliken and D.L. Milliken, *Race Car Vehicle Dynamics*, Society of Automotive Engineers, Warrendale, PA, 1995.