CHAPTER 1
What is Physics?

Figure 1.1 Galaxies, such as the Andromeda galaxy pictured here, are immense in size. The small blue spots in this photo are also galaxies. The same physical laws apply to objects as large as galaxies or objects as small as atoms. The laws of physics are, therefore, surprisingly few in number. (NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics).

Chapter Outline

INTRODUCTION Take a look at the image above of the Andromeda Galaxy (Figure 1.1), which contains billions of stars. This galaxy is the nearest one to our own galaxy (the Milky Way) but is still a staggering 2.5 million light years from Earth. (A light year is a measurement of the distance light travels in a year.) Yet, the primary force that affects the movement of stars within Andromeda is the same force that we contend with here on Earth—namely, gravity.

You may soon realize that physics plays a much larger role in your life than you thought. This section introduces you to the realm of physics, and discusses applications of physics in other disciplines of study. It also describes the methods by which science is done, and how scientists communicate their results to each other.

1.1 Physics: Definitions and Applications

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the definition, aims, and branches of physics
- Describe and distinguish classical physics from modern physics and describe the importance of relativity, quantum mechanics, and relativistic quantum mechanics in modern physics
- Describe how aspects of physics are used in other sciences (e.g., biology, chemistry, geology, etc.) as well as in everyday technology

Section Key Terms

atom	classical physics	modern physics
physics	quantum mechanics	theory of relativity

What Physics Is

Think about all of the technological devices that you use on a regular basis. Computers, wireless internet, smart phones, tablets, global positioning system (GPS), MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above their tracks, *invisibility cloaks* that bend light around them, and microscopic robots that fight diseased cells in our bodies. All of these groundbreaking advancements rely on the principles of **physics**.

Physics is a branch of science. The word *science* comes from a Latin word that means *having knowledge*, and refers the knowledge of how the physical world operates, based on objective evidence determined through observation and experimentation. A key requirement of any scientific explanation of a natural phenomenon is that it must be testable; one must be able to devise and conduct an experimental investigation that either supports or refutes the explanation. It is important to note that some questions fall outside the realm of science precisely because they deal with phenomena that are not scientifically testable. This need for objective evidence helps define the investigative process scientists follow, which will be described later in this chapter.

Physics is the science aimed at describing the fundamental aspects of our universe. This includes what things are in it, what properties of those things are noticeable, and what processes those things or their properties undergo. In simpler terms, physics attempts to describe the basic mechanisms that make our universe behave the way it does. For example, consider a smart phone (Figure 1.2). Physics describes how electric current interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics relationships to determine the travel time from one location to another.

Figure 1.2 Physics describes the way that electric charge flows through the circuits of this device. Engineers use their knowledge of physics to construct a smartphone with features that consumers will enjoy, such as a GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (@gletham GIS, Social, Mobile Tech Images)

As our technology evolved over the centuries, physics expanded into many branches. Ancient peoples could only study things that they could see with the naked eye or otherwise experience without the aid of scientific equipment. This included the study of kinematics, which is the study of moving objects. For example, ancient people often studied the apparent motion of objects in the sky, such as the sun, moon, and stars. This is evident in the construction of prehistoric astronomical observatories, such as Stonehenge in England (shown in Figure 1.3).

Figure 1.3 Stonehenge is a monument located in England that was built between 3000 and 1000 B.C. It functions as an ancient astronomical observatory, with certain rocks in the monument aligning with the position of the sun during the summer and winter solstices. Other rocks align with the rising and setting of the moon during certain days of the year. (Citypeek, Wikimedia Commons)

Ancient people also studied statics and dynamics, which focus on how objects start moving, stop moving, and change speed and direction in response to forces that push or pull on the objects. This early interest in kinematics and dynamics allowed humans to invent simple machines, such as the lever, the pulley, the ramp, and the wheel. These simple machines were gradually combined and integrated to produce more complicated machines, such as wagons and cranes. Machines allowed humans to gradually do more work more effectively in less time, allowing them to create larger and more complicated buildings and structures, many of which still exist today from ancient times.

As technology advanced, the branches of physics diversified even more. These include branches such as acoustics, the study of sound, and optics, the study of the light. In 1608, the invention of the telescope by a Germany spectacle maker, Hans Lippershey, led to huge breakthroughs in astronomy—the study of objects or phenomena in space. One year later, in 1609, Galileo Galilei began the first studies of the solar system and the universe using a telescope. During the Renaissance era, Isaac Newton used observations made by Galileo to construct his three laws of motion. These laws were the standard for studying kinematics and dynamics even today.

Another major branch of physics is thermodynamics, which includes the study of thermal energy and the transfer of heat. James Prescott Joule, an English physicist, studied the nature of heat and its relationship to work. Joule's work helped lay the foundation for the first of three laws of thermodynamics that describe how energy in our universe is transferred from one object to another or transformed from one form to another. Studies in thermodynamics were motivated by the need to make engines more efficient, keep people safe from the elements, and preserve food.

The 18th and 19th centuries also saw great strides in the study of electricity and magnetism. Electricity involves the study of electric charges and their movements. Magnetism had long ago been noticed as an attractive force between a magnetized object and a metal like iron, or between the opposite poles (North and South) of two magnetized objects. In 1820, Danish physicist Hans Christian Oersted showed that electric currents create magnetic fields. In 1831, English inventor Michael Faraday showed that moving a wire through a magnetic field could induce an electric current. These studies led to the inventions of the electric motor and electric generator, which revolutionized human life by bringing electricity and magnetism into our machines.

The end of the 19th century saw the discovery of radioactive substances by the French scientists Marie and Pierre Curie. Nuclear physics involves studying the nuclei of **atoms**, the source of nuclear radiation. In the 20th century, the study of nuclear physics eventually led to the ability to split the nucleus of an atom, a process called nuclear fission. This process is the basis for nuclear power plants and nuclear weapons. Also, the field of **quantum mechanics**, which involves the mechanics of atoms and molecules, saw great strides during the 20th century as our understanding of atoms and subatomic particles increased (see below).

Early in the 20th century, Albert Einstein revolutionized several branches of physics, especially relativity. Relativity revolutionized our understanding of motion and the universe in general as described further in this chapter. Now, in the 21st century, physicists continue to study these and many other branches of physics.

By studying the most important topics in physics, you will gain analytical abilities that will enable you to apply physics far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any career you choose to pursue.

Physics: Past and Present

The word physics is thought to come from the Greek word *phusis*, meaning nature. The study of nature later came to be called *natural philosophy*. From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, mathematics, and medicine. Over the last few centuries, the growth of scientific knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. Physics, as it developed from the Renaissance to the end of the 19th century, is called **classical physics**. Revolutionary discoveries starting at the beginning of the 20th century transformed physics from classical physics to **modern physics**.

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: (1) matter must be moving at speeds less than about 1 percent of the speed of light, (2) the objects dealt with must be large enough to be seen with the naked eye, and (3) only weak gravity, such as that generated by Earth, can be involved. Very small objects, such as atoms and molecules, cannot be adequately explained by classical physics. These three conditions apply to almost all of everyday experience. As a result, most aspects of classical physics should make sense on an intuitive level.

Many laws of classical physics have been modified during the 20th century, resulting in revolutionary changes in technology, society, and our view of the universe. As a result, many aspects of modern physics, which occur outside of the range of our everyday experience, may seem bizarre or unbelievable. So why is most of this textbook devoted to classical physics? There are two main reasons. The first is that knowledge of classical physics is necessary to understand modern physics. The second reason is that classical physics still gives an accurate description of the universe under a wide range of everyday circumstances.

Modern physics includes two revolutionary theories: relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. The **theory of relativity** was developed by Albert Einstein in 1905. By examining how two observers moving relative to each other would see the same phenomena, Einstein devised radical new ideas about time and space. He came to the startling conclusion that the measured length of an object travelling at high speeds (greater than about one percent of the speed of light) is shorter than the same object measured at rest. Perhaps even more bizarre is the idea the time for the same process to occur is different depending on the motion of the observer. Time passes more slowly for an object travelling at high speeds. A trip to the nearest star system, Alpha Centauri, might take an astronaut 4.5 Earth years if the ship travels near the speed of light. However, because time is slowed at higher speeds, the astronaut would age only 0.5 years during the trip. Einstein's ideas of relativity were accepted after they were confirmed by numerous experiments.

Gravity, the force that holds us to Earth, can also affect time and space. For example, time passes more slowly on Earth's surface than for objects farther from the surface, such as a satellite in orbit. The very accurate clocks on global positioning satellites have to correct for this. They slowly keep getting ahead of clocks at Earth's surface. This is called time dilation, and it occurs because gravity, in essence, slows down time.

Large objects, like Earth, have strong enough gravity to distort space. To visualize this idea, think about a bowling ball placed on a trampoline. The bowling ball depresses or curves the surface of the trampoline. If you rolled a marble across the trampoline, it would follow the surface of the trampoline, roll into the depression caused by the bowling ball, and hit the ball. Similarly, the Earth curves space around it in the shape of a funnel. These curves in space due to the Earth cause objects to be attracted to Earth (i.e., gravity).

Because of the way gravity affects space and time, Einstein stated that gravity affects the space-time continuum, as illustrated in Figure 1.4. This is why time proceeds more slowly at Earth's surface than in orbit. In black holes, whose gravity is hundreds of times that of Earth, time passes so slowly that it would appear to a far-away observer to have stopped!

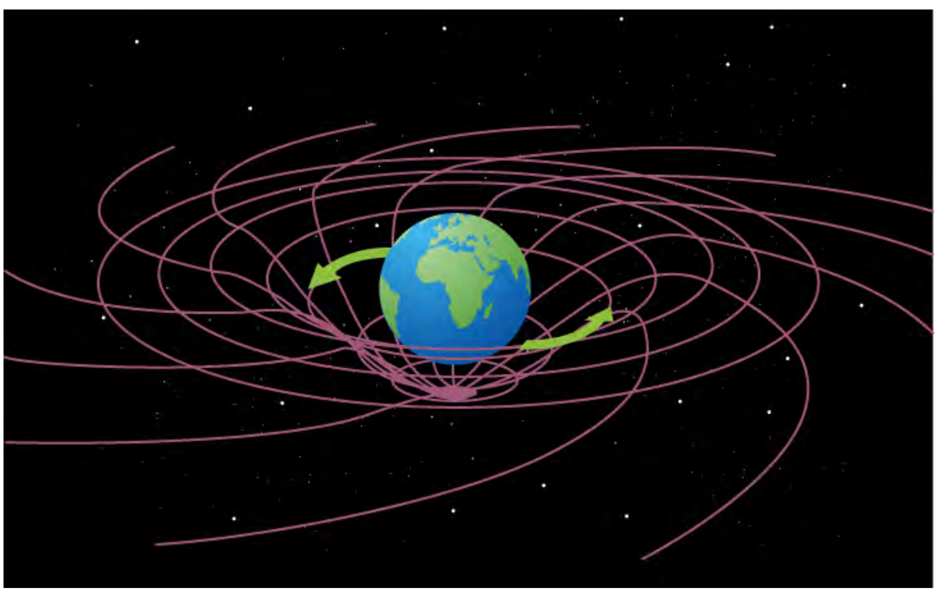


Figure 1.4 Einstein's theory of relativity describes space and time as an interweaved mesh. Large objects, such as a planet, distort space, causing objects to fall in toward the planet due to the action of gravity. Large objects also distort time, causing time to proceed at a slower rate near the surface of Earth compared with the area outside of the distorted region of space-time.

In summary, relativity says that in describing the universe, it is important to realize that time, space and speed are not absolute. Instead, they can appear different to different observers. Einstein's ability to reason out relativity is even more amazing because we cannot see the effects of relativity in our everyday lives.

Quantum mechanics is the second major theory of modern physics. Quantum mechanics deals with the very small, namely, the subatomic particles that make up atoms. Atoms (Figure 1.5) are the smallest units of elements. However, atoms themselves are constructed of even smaller subatomic particles, such as protons, neutrons and electrons. Quantum mechanics strives to describe the properties and behavior of these and other subatomic particles. Often, these particles do not behave in the ways expected by classical physics. One reason for this is that they are small enough to travel at great speeds, near the speed of light.

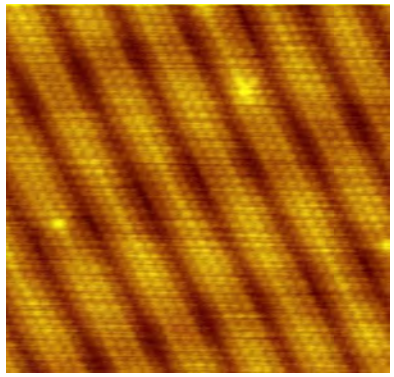


Figure 1.5 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (Erwinrossen)

At particle colliders (Figure 1.6), such as the Large Hadron Collider on the France-Swiss border, particle physicists can make subatomic particles travel at very high speeds within a 27 kilometers (17 miles) long superconducting tunnel. They can then study the properties of the particles at high speeds, as well as collide them with each other to see how they exchange energy. This has led to many intriguing discoveries such as the Higgs-Boson particle, which gives matter the property of mass, and antimatter, which causes a huge energy release when it comes in contact with matter.

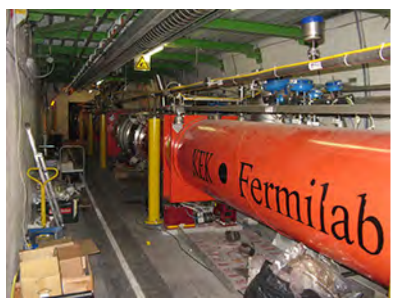


Figure 1.6 Particle colliders such as the Large Hadron Collider in Switzerland or Fermilab in the United States (pictured here), have long tunnels that allows subatomic particles to be accelerated to near light speed. (Andrius.v)

Physicists are currently trying to unify the two theories of modern physics, relativity and quantum mechanics, into a single, comprehensive theory called relativistic quantum mechanics. Relating the behavior of subatomic particles to gravity, time, and space will allow us to explain how the universe works in a much more comprehensive way.

Application of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. For example, physics can help you understand why you shouldn't put metal in the microwave (Figure 1.7), why a black car radiator helps remove heat in a car engine, and why a white roof helps keep the inside of a house cool. The operation of a car's ignition system, as well as the transmission of electrical signals through our nervous system, are much easier to understand when you think about them in terms of the basic physics of electricity.

Figure 1.7 Why can't you put metal in the microwave? Microwaves are high-energy radiation that increases the movement of electrons in

metal. These moving electrons can create an electrical current, causing sparking that can lead to a fire. (= MoneyBlogNewz)

Physics is the foundation of many important scientific disciplines. For example, chemistry deals with the interactions of atoms and molecules. Not surprisingly, chemistry is rooted in atomic and molecular physics. Most branches of engineering are also applied physics. In architecture, physics is at the heart of determining structural stability, acoustics, heating, lighting, and cooling for buildings. Parts of geology, the study of nonliving parts of Earth, rely heavily on physics; including radioactive dating, earthquake analysis, and heat transfer across Earth's surface. Indeed, some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics also describes the chemical processes that power the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements (Figure 1.8). Medical therapy Physics also has many applications in biology, the study of life. For example, physics describes how cells can protect themselves using their cell walls and cell membranes (Figure 1.9). Medical therapy sometimes directly involves physics, such as in using X-rays to diagnose health conditions. Physics can also explain what we perceive with our senses, such as how the ears detect sound or the eye detects color.

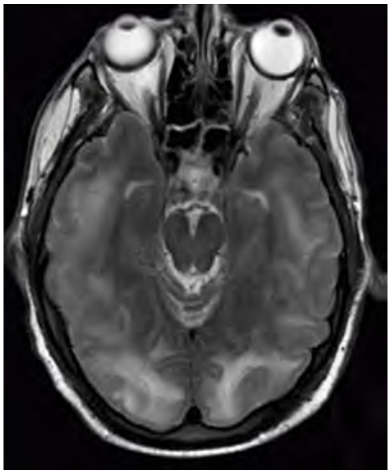


Figure 1.8 Magnetic resonance imaging (MRI) uses electromagnetic waves to yield an image of the brain, which doctors can use to find diseased regions. (Rashmi Chawla, Daniel Smith, and Paul E. Marik)

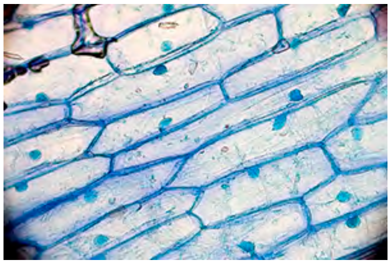


Figure 1.9 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (Umberto Salvagnin)

BOUNDLESS PHYSICS

The Physics of Landing on a Comet

On November 12, 2014, the European Space Agency's Rosetta spacecraft (shown in Figure 1.10) became the first ever to reach and orbit a comet. Shortly after, Rosetta's rover, Philae, landed on the comet, representing the first time humans have ever landed a space probe on a comet.

Figure 1.10 The Rosetta spacecraft, with its large and revolutionary solar panels, carried the Philae lander to a comet. The lander then detached and landed on the comet's surface. (European Space Agency)

After traveling 6.4 billion kilometers starting from its launch on Earth, Rosetta landed on the comet 67P/Churyumov-Gerasimenko, which is only 4 kilometers wide. Physics was needed to successfully plot the course to reach such a small, distant, and rapidly moving target. Rosetta's path to the comet was not straight forward. The probe first had to travel to Mars so that Mars's gravity could accelerate it and divert it in the exact direction of the comet.

This was not the first time humans used gravity to power our spaceships. Voyager 2, a space probe launched in 1977, used the gravity of Saturn to *slingshot* over to Uranus and Neptune (illustrated in Figure 1.11), providing the first pictures ever taken of these planets. Now, almost 40 years after its launch, Voyager 2 is at the very edge of our solar system and is about to enter interstellar space. Its sister ship, Voyager 1 (illustrated in Figure 1.11), which was also launched in 1977, is already there.

To listen to the sounds of interstellar space or see images that have been transmitted back from the Voyager I or to learn more about the Voyager mission, visit the Voyager's Mission website (https://openstax.org/l/28voyager) .

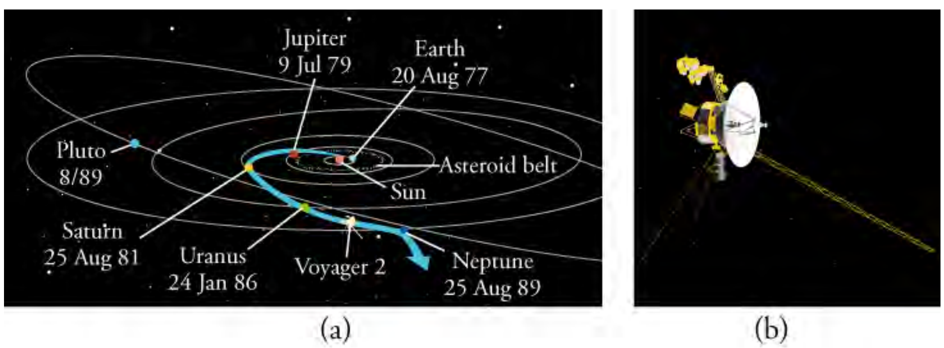


Figure 1.11 a) Voyager 2, launched in 1977, used the gravity of Saturn to slingshot over to Uranus and Neptune. NASA b) A rendering of Voyager 1, the first space probe to ever leave our solar system and enter interstellar space. NASA

Both Voyagers have electrical power generators based on the decay of radioisotopes. These generators have served them for almost 40 years. Rosetta, on the other hand, is solar-powered. In fact, Rosetta became the first space probe to travel beyond the asteroid belt by relying only on solar cells for power generation.

At 800 million kilometers from the sun, Rosetta receives sunlight that is only 4 percent as strong as on Earth. In addition, it is very cold in space. Therefore, a lot of physics went into developing Rosetta's low-intensity low-temperature solar cells.

In this sense, the Rosetta project nicely shows the huge range of topics encompassed by physics: from modeling the movement of gigantic planets over huge distances within our solar systems, to learning how to generate electric power from low-intensity light. Physics is, by far, the broadest field of science.

GRASP CHECK

What characteristics of the solar system would have to be known or calculated in order to send a probe to a distant planet, such as Jupiter?

a. the effects due to the light from the distant stars
b. the effects due to the air in the solar system
c. the effects due to the gravity from the other planets
d. the effects due to the cosmic microwave background radiation

In summary, physics studies many of the most basic aspects of science. A knowledge of physics is, therefore, necessary to understand all other sciences. This is because physics explains the most basic ways in which our universe works. However, it is not necessary to formally study all applications of physics. A knowledge of the basic laws of physics will be most useful to you, so that you can use them to solve some everyday problems. In this way, the study of physics can improve your problem-solving skills.

Check Your Understanding

1. Which of the following is *not* an essential feature of scientific explanations?
   a. They must be subject to testing.
   b. They strictly pertain to the physical world.
   c. Their validity is judged based on objective observations.
   d. Once supported by observation, they can be viewed as a fact.

2. Which of the following does *not* represent a question that can be answered by science?
   a. How much energy is released in a given nuclear chain reaction?
   b. Can a nuclear chain reaction be controlled?
   c. Should uncontrolled nuclear reactions be used for military applications?
   d. What is the half-life of a waste product of a nuclear reaction?

3. What are the three conditions under which classical physics provides an excellent description of our universe?
   a. 1. Matter is moving at speeds less than about 1 percent of the speed of light
      2. Objects dealt with must be large enough to be seen with the naked eye.
      3. Strong electromagnetic fields are involved.

   b. 1. Matter is moving at speeds less than about 1 percent of the speed of light.
      2. Objects dealt with must be large enough to be seen with the naked eye.
      3. Only weak gravitational fields are involved.

   c. 1. Matter is moving at great speeds, comparable to the speed of light.
      2. Objects dealt with are large enough to be seen with the naked eye.
      3. Strong gravitational fields are involved.

   d. 1. Matter is moving at great speeds, comparable to the speed of light.
      2. Objects are just large enough to be visible through the most powerful telescope.
      3. Only weak gravitational fields are involved.

4. Why is the Greek word for nature appropriate in describing the field of physics?
   a. Physics is a natural science that studies life and living organism on habitable planets like Earth.
   b. Physics is a natural science that studies the laws and principles of our universe.
   c. Physics is a physical science that studies the composition, structure, and changes of matter in our universe.
   d. Physics is a social science that studies the social behavior of living beings on habitable planets like Earth.

5. Which aspect of the universe is studied by quantum mechanics?
   a. objects at the galactic level
   b. objects at the classical level
   c. objects at the subatomic level
   d. objects at all levels, from subatomic to galactic

1.2 The Scientific Methods

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how the methods of science are used to make scientific discoveries
- Define a scientific model and describe examples of physical and mathematical models used in physics
- Compare and contrast hypothesis, theory, and law

Section Key Terms

experiment	hypothesis	model	observation	principle
scientific law	scientific methods	theory	universal	

Scientific Methods

Scientists often plan and carry out investigations to answer questions about the universe around us. Such laws are intrinsic to the universe, meaning that humans did not create them and cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. The cornerstone of discovering natural laws is observation. Science must describe the universe as it is, not as we imagine or wish it to be.

We all are curious to some extent. We look around, make generalizations, and try to understand what we see. For example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how data may be organized. We then formulate models, theories, and laws based on the data we have collected, and communicate those results with others. This, in a nutshell, describes the **scientific method** that scientists employ to decide scientific issues on the basis of evidence from observation and experiment.

An investigation often begins with a scientist making an **observation**. The scientist observes a pattern or trend within the natural world. Observation may generate questions that the scientist wishes to answer. Next, the scientist may perform some research about the topic and devise a **hypothesis**. A hypothesis is a testable statement that describes how something in the natural world works. In essence, a hypothesis is an educated guess that explains something about an observation.

Scientists may test the hypothesis by performing an **experiment**. During an experiment, the scientist collects data that will help them learn about the phenomenon they are studying. Then the scientists analyze the results of the experiment (that is, the data), often using statistical, mathematical, and/or graphical methods. From the data analysis, they draw conclusions. They may conclude that their experiment either supports or rejects their hypothesis. If the hypothesis is supported, the scientist usually goes on to test another hypothesis related to the first. If their hypothesis is rejected, they will often then test a new and different hypothesis in their effort to learn more about whatever they are studying.

Scientific processes can be applied to many situations. Let's say that you try to turn on your car, but it will not start. You have just made an observation! You ask yourself, "Why won't my car start?" You can now use scientific processes to answer this question. First, you generate a hypothesis such as, "The car won't start because it has no gasoline in the gas tank." To test this hypothesis, you put gasoline in the car and try to start it again. If the car starts, then your hypothesis is supported by the experiment. If the car does not start, then your hypothesis is rejected. You will then need to think up a new hypothesis to test such as, "My car won't start because the fuel pump is broken." Hopefully, your investigations lead you to discover why the car won't start and enable you to fix it.

Modeling

A **model** is a representation of something that is often too difficult (or impossible) to study directly. Models can take the form of physical models, equations, computer programs, or simulations—computer graphics/animations. Models are tools that are especially useful in modern physics because they let us visualize phenomena that we normally cannot observe with our senses, such as very small objects or objects that move at high speeds. For example, we can understand the structure of an atom using models, despite the fact that no one has ever seen an atom with their own eyes. Models are always approximate, so they are simpler to consider than the real situation; the more complete a model is, the more complicated it must be. Models put the intangible or the extremely complex into human terms that we can visualize, discuss, and hypothesize about.

Scientific models are constructed based on the results of previous experiments. Even still, models often only describe a phenomenon partially or in a few limited situations. Some phenomena are so complex that they may be impossible to model them in their entirety, even using computers. An example is the electron cloud model of the atom in which electrons are moving around the atom's center in distinct clouds (Figure 1.12), that represent the likelihood of finding an electron in different places. This model helps us to visualize the structure of an atom. However, it does not show us exactly where an electron will be within its cloud at any one particular time.

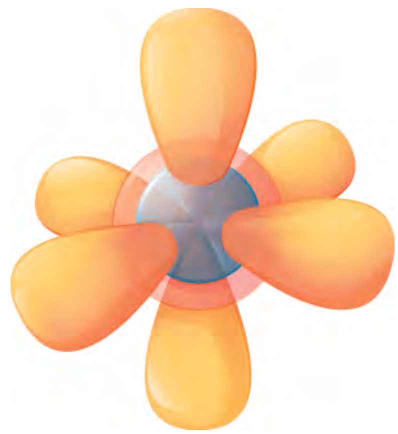


Figure 1.12 The electron cloud model of the atom predicts the geometry and shape of areas where different electrons may be found in an atom. However, it cannot indicate exactly where an electron will be at any one time.

As mentioned previously, physicists use a variety of models including equations, physical models, computer simulations, etc. For example, three-dimensional models are often commonly used in chemistry and physics to model molecules. Properties other than appearance or location are usually modelled using mathematics, where functions are used to show how these properties relate to one another. Processes such as the formation of a star or the planets, can also be modelled using computer simulations. Once a simulation is correctly programmed based on actual experimental data, the simulation can allow us to view processes that happened in the past or happen too quickly or slowly for us to observe directly. In addition, scientists can also run virtual experiments using computer-based models. In a model of planet formation, for example, the scientist could alter the amount or type of rocks present in space and see how it affects planet formation.

Scientists use models and experimental results to construct explanations of observations or design solutions to problems. For example, one way to make a car more fuel efficient is to reduce the friction or drag caused by air flowing around the moving car. This can be done by designing the body shape of the car to be more aerodynamic, such as by using rounded corners instead of sharp ones. Engineers can then construct physical models of the car body, place them in a wind tunnel, and examine the flow of air around the model. This can also be done mathematically in a computer simulation. The air flow pattern can be analyzed for regions smooth air flow and for eddies that indicate drag. The model of the car body may have to be altered slightly to produce the smoothest pattern of air flow (i.e., the least drag). The pattern with the least drag may be the solution to increasing fuel efficiency of the car. This solution might then be incorporated into the car design.

Snap Lab

Using Models and the Scientific Processes

Be sure to secure loose items before opening the window or door.

In this activity, you will learn about scientific models by making a model of how air flows through your classroom or a room in your house.

- One room with at least one window or door that can be opened
- Piece of single-ply tissue paper
  1. Work with a group of four, as directed by your teacher. Close all of the windows and doors in the room you are working in. Your teacher may assign you a specific window or door to study.

2. Before opening any windows or doors, draw a to-scale diagram of your room. First, measure the length and width of your room using the tape measure. Then, transform the measurement using a scale that could fit on your paper, such as 5 centimeters = 1 meter.
3. Your teacher will assign you a specific window or door to study air flow. On your diagram, add arrows showing your hypothesis (before opening any windows or doors) of how air will flow through the room when your assigned window or door is opened. Use pencil so that you can easily make changes to your diagram.
4. On your diagram, mark four locations where you would like to test air flow in your room. To test for airflow, hold a strip of single ply tissue paper between the thumb and index finger. Note the direction that the paper moves when exposed to the airflow. Then, for each location, predict which way the paper will move if your air flow diagram is correct.
5. Now, each member of your group will stand in one of the four selected areas. Each member will test the airflow Agree upon an approximate height at which everyone will hold their papers.
6. When you teacher tells you to, open your assigned window and/or door. Each person should note the direction that their paper points immediately after the window or door was opened. Record your results on your diagram.
7. Did the airflow test data support or refute the hypothetical model of air flow shown in your diagram? Why or why not? Correct your model based on your experimental evidence.
8. With your group, discuss how accurate your model is. What limitations did it have? Write down the limitations that your group agreed upon.

GRASP CHECK

Your diagram is a model, based on experimental evidence, of how air flows through the room. Could you use your model to predict how air would flow through a new window or door placed in a different location in the classroom? Make a new diagram that predicts the room's airflow with the addition of a new window or door. Add a short explanation that describes how.

a. Yes, you could use your model to predict air flow through a new window. The earlier experiment of air flow would help you model the system more accurately.
b. Yes, you could use your model to predict air flow through a new window. The earlier experiment of air flow is not useful for modeling the new system.
c. No, you cannot model a system to predict the air flow through a new window. The earlier experiment of air flow would help you model the system more accurately.
d. No, you cannot model a system to predict the air flow through a new window. The earlier experiment of air flow is not useful for modeling the new system.

Scientific Laws and Theories

A **scientific law** is a description of a pattern in nature that is true in all circumstances that have been studied. That is, physical laws are meant to be **universal**, meaning that they apply throughout the known universe. Laws are often also concise, whereas theories are more complicated. A law can be expressed in the form of a single sentence or mathematical equation. For example, Newton's second law of motion, which relates the motion of an object to the force applied (F), the mass of the object (m), and the object's acceleration (a), is simply stated using the equation

$$F = ma.$$

Scientific ideas and explanations that are true in many, but not all situations in the universe are usually called **principles**. An example is Pascal's principle, which explains properties of liquids, but not solids or gases. However, the distinction between laws and principles is sometimes not carefully made in science.

A **theory** is an explanation for patterns in nature that is supported by much scientific evidence and verified multiple times by multiple researchers. While many people confuse theories with educated guesses or hypotheses, theories have withstood more rigorous testing and verification than hypotheses.

As a closing idea about scientific processes, we want to point out that scientific laws and theories, even those that have been supported by experiments for centuries, can still be changed by new discoveries. This is especially true when new technologies emerge that allow us to observe things that were formerly unobservable. Imagine how viewing previously invisible objects with a microscope or viewing Earth for the first time from space may have instantly changed our scientific theories and laws! What discoveries still await us in the future? The constant retesting and perfecting of our scientific laws and theories allows our knowledge of nature to progress. For this reason, many scientists are reluctant to say that their studies *prove* anything. By saying *support* instead of *prove*, it keeps the door open for future discoveries, even if they won't occur for centuries or even millennia.

Check Your Understanding

6. Explain why scientists sometimes use a model rather than trying to analyze the behavior of the real system.
   a. Models are simpler to analyze.
   b. Models give more accurate results.
   c. Models provide more reliable predictions.
   d. Models do not require any computer calculations.

7. Describe the difference between a question, generated through observation, and a hypothesis.
   a. They are the same.
   b. A hypothesis has been thoroughly tested and found to be true.
   c. A hypothesis is a tentative assumption based on what is already known.
   d. A hypothesis is a broad explanation firmly supported by evidence.

8. What is a scientific model and how is it useful?
   a. A scientific model is a representation of something that can be easily studied directly. It is useful for studying things that can be easily analyzed by humans.
   b. A scientific model is a representation of something that is often too difficult to study directly. It is useful for studying a complex system or systems that humans cannot observe directly.
   c. A scientific model is a representation of scientific equipment. It is useful for studying working principles of scientific equipment.
   d. A scientific model is a representation of a laboratory where experiments are performed. It is useful for studying requirements needed inside the laboratory.

9. Which of the following statements is correct about the hypothesis?
   a. The hypothesis must be validated by scientific experiments.
   b. The hypothesis must not include any physical quantity.
   c. The hypothesis must be a short and concise statement.
   d. The hypothesis must apply to all the situations in the universe.

10. What is a scientific theory?
   a. A scientific theory is an explanation of natural phenomena that is supported by evidence.
   b. A scientific theory is an explanation of natural phenomena without the support of evidence.
   c. A scientific theory is an educated guess about the natural phenomena occurring in nature.
   d. A scientific theory is an uneducated guess about natural phenomena occurring in nature.

11. Compare and contrast a hypothesis and a scientific theory.
   a. A hypothesis is an explanation of the natural world with experimental support, while a scientific theory is an educated guess about a natural phenomenon.
   b. A hypothesis is an educated guess about natural phenomenon, while a scientific theory is an explanation of natural world with experimental support.
   c. A hypothesis is experimental evidence of a natural phenomenon, while a scientific theory is an explanation of the natural world with experimental support.
   d. A hypothesis is an explanation of the natural world with experimental support, while a scientific theory is experimental evidence of a natural phenomenon.

1.3 The Language of Physics: Physical Quantities and Units

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Associate physical quantities with their International System of Units (SI)and perform conversions among SI units using scientific notation
- Relate measurement uncertainty to significant figures and apply the rules for using significant figures in calculations
- Correctly create, label, and identify relationships in graphs using mathematical relationships (e.g., slope, *y*-intercept, inverse, quadratic and logarithmic)

Section Key Terms

accuracy	ampere	constant	conversion factor	dependent variable
derived units	English units	exponential relationship	fundamental physical units	independent variable
inverse relationship	inversely proportional	kilogram	linear relationship	logarithmic (log) scale
log-log plot	meter	method of adding percents	order of magnitude	precision
quadratic relationship	scientific notation	second	semi-log plot	SI units
significant figures	slope	uncertainty	variable	*y*-intercept

The Role of Units

Physicists, like other scientists, make observations and ask basic questions. For example, how big is an object? How much mass does it have? How far did it travel? To answer these questions, they make measurements with various instruments (e.g., meter stick, balance, stopwatch, etc.).

The measurements of physical quantities are expressed in terms of units, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in meters (for sprinters) or kilometers (for long distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way (Figure 1.13).

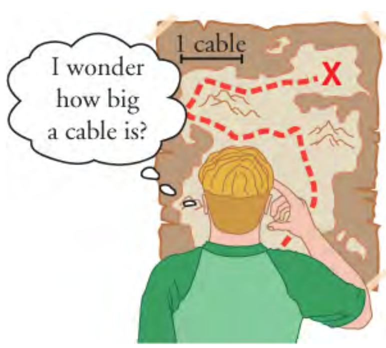


Figure 1.13 Distances given in unknown units are maddeningly useless.

All physical quantities in the International System of Units (SI) are expressed in terms of combinations of seven **fundamental**

physical units, which are units for: length, mass, time, electric current, temperature, amount of a substance, and luminous intensity.

SI Units: Fundamental and Derived Units

There are two major systems of units used in the world: **SI units** (acronym for the French Le Système International d'Unités, also known as the metric system), and **English units** (also known as the imperial system). English units were historically used in nations once ruled by the British Empire. Today, the United States is the only country that still uses English units extensively. Virtually every other country in the world now uses the metric system, which is the standard system agreed upon by scientists and mathematicians.

Some physical quantities are more fundamental than others. In physics, there are seven fundamental physical quantities that are measured in base or physical fundamental units: length, mass, time, electric current temperature, amount of substance, and luminous intensity. Units for other physical quantities (such as force, speed, and electric charge) described by mathematically combining these seven base units. In this course, we will mainly use five of these: length, mass, time, electric current and temperature. The units in which they are measured are the meter, kilogram, second, ampere, kelvin, mole, and candela (Table 1.1). All other units are made by mathematically combining the fundamental units. These are called **derived units**.

Quantity	Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	a
Temperature	Kelvin	k
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

Table 1.1 SI Base Units

The Meter

The SI unit for length is the **meter** (m). The definition of the meter has changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar. (The bar is now housed at the International Bureau of Weights and Meaures, near Paris). By 1960, some distances could be measured more precisely by comparing them to wavelengths of light. The meter was redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition as the distance light travels in a vacuum in 1/299,792,458 of a second (Figure 1.14).

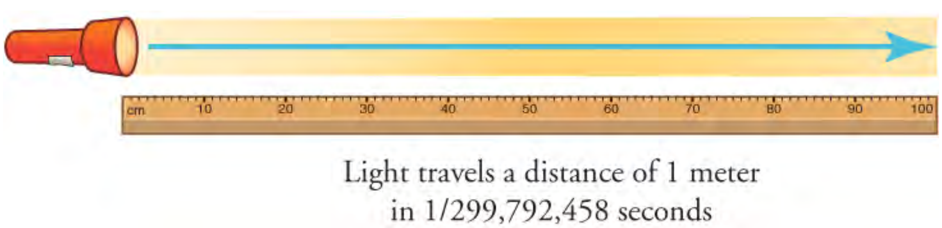


Figure 1.14 The meter is defined to be the distance light travels in 1/299,792,458 of a second through a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (kg). It is defined to be the mass of a platinum-iridium cylinder, housed at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram cylinder are kept in numerous locations throughout the world, such as the National Institute of Standards and Technology in Gaithersburg, Maryland. The determination of all other masses can be done by comparing them with one of these standard kilograms.

The Second

The SI unit for time, the **second** (s) also has a long history. For many years it was defined as 1/86,400 of an average solar day. However, the average solar day is actually very gradually getting longer due to gradual slowing of Earth's rotation. Accuracy in the fundamental units is essential, since all other measurements are derived from them. Therefore, a new standard was adopted to define the second in terms of a non-varying, or constant, physical phenomenon. One constant phenomenon is the very steady vibration of Cesium atoms, which can be observed and counted. This vibration forms the basis of the cesium atomic clock. In 1967, the second was redefined as the time required for 9,192,631,770 Cesium atom vibrations (Figure 1.15).

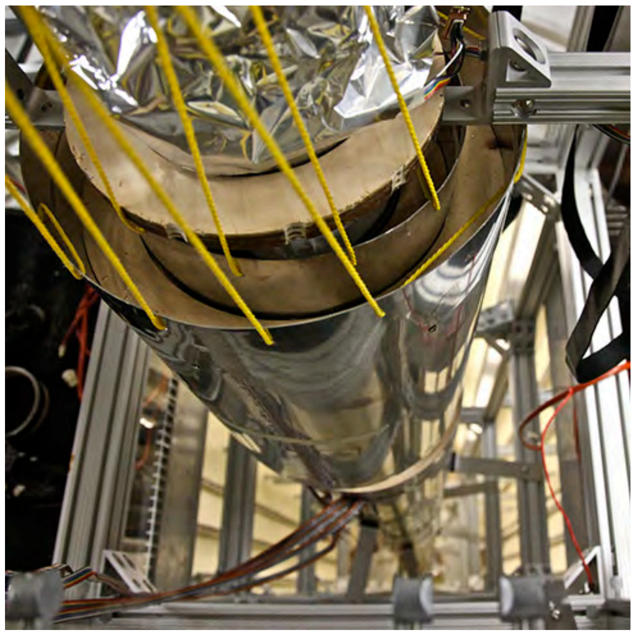


Figure 1.15 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of one microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic clock. (Steve Jurvetson/Flickr)

The Ampere

Electric current is measured in the **ampere** (A), named after Andre Ampere. You have probably heard of amperes, or *amps*, when people discuss electrical currents or electrical devices. Understanding an ampere requires a basic understanding of electricity and magnetism, something that will be explored in depth in later chapters of this book. Basically, two parallel wires with an electric current running through them will produce an attractive force on each other. One ampere is defined as the amount of electric current that will produce an attractive force of 2.7×10^{-7} newton per meter of separation between the two wires (the newton is the derived unit of force).

Kelvins

The SI unit of temperature is the **kelvin** (or kelvins, but not degrees kelvin). This scale is named after physicist William Thomson, Lord Kelvin, who was the first to call for an absolute temperature scale. The Kelvin scale is based on absolute zero. This is the point at which all thermal energy has been removed from all atoms or molecules in a system. This temperature, 0 K, is equal to –273.15 °C and –459.67 °F. Conveniently, the Kelvin scale actually changes in the same way as the Celsius scale. For example, the freezing point (0 °C) and boiling points of water (100 °C) are 100 degrees apart on the Celsius scale. These two temperatures are also 100 kelvins apart (freezing point = 273.15 K; boiling point = 373.15 K).

Metric Prefixes

Physical objects or phenomena may vary widely. For example, the size of objects varies from something very small (like an atom) to something very large (like a star). Yet the standard metric unit of length is the meter. So, the metric system includes many prefixes that can be attached to a unit. Each prefix is based on factors of 10 (10, 100, 1,000, etc., as well as 0.1, 0.01, 0.001, etc.). Table 1.2 gives the metric prefixes and symbols used to denote the different various factors of 10 in the metric system.

Prefix	Symbol	Value[1]	Example Name	Example Symbol	Example Value	Example Description
exa	E	10^{18}	Exameter	Em	10^{18} m	Distance light travels in a century
peta	P	10^{15}	Petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	Terawatt	TW	10^{12} W	Powerful laser output
giga	G	10^{9}	Gigahertz	GHz	10^{9} Hz	A microwave frequency
mega	M	10^{6}	Megacurie	MCi	10^{6} Ci	High radioactivity
kilo	k	10^{3}	Kilometer	km	10^{3} m	About 6/10 mile
hector	h	10^{2}	Hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	Dekagram	dag	10^{1} g	Teaspoon of butter
____	____	10^{0} (=1)				
deci	d	10^{-1}	Deciliter	dL	10^{-1} L	Less than half a soda
centi	c	10^{-2}	Centimeter	Cm	10^{-2} m	Fingertip thickness
milli	m	10^{-3}	Millimeter	Mm	10^{-3} m	Flea at its shoulder
micro	µ	10^{-6}	Micrometer	µm	10^{-6} m	Detail in microscope
nano	n	10^{-9}	Nanogram	Ng	10^{-9} g	Small speck of dust
pico	p	10^{-12}	Picofarad	pF	10^{-12} F	Small capacitor in radio
femto	f	10^{-15}	Femtometer	Fm	10^{-15} m	Size of a proton
atto	a	10^{-18}	Attosecond	as	10^{-18} s	Time light takes to cross an atom

Table 1.2 Metric Prefixes for Powers of 10 and Their Symbols [1]See Appendix A for a discussion of powers of 10. Note—Some examples are approximate.

The metric system is convenient because conversions between metric units can be done simply by moving the decimal place of a number. This is because the metric prefixes are sequential powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as U.S. customary units, the relationships are less simple—there are 12 inches in a foot, 5,280 feet in a mile, 4 quarts in a gallon, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by switching to the most-appropriate metric prefix. For example, distances in meters are suitable for building construction, but kilometers are used to describe road construction. Therefore, with the metric system, there is no need to invent new units when measuring very small or very large objects—you just have to move the decimal point (and use the appropriate prefix).

Known Ranges of Length, Mass, and Time

Table 1.3 lists known lengths, masses, and time measurements. You can see that scientists use a range of measurement units. This wide range demonstrates the vastness and complexity of the universe, as well as the breadth of phenomena physicists study. As you examine this table, note how the metric system allows us to discuss and compare an enormous range of phenomena, using one system of measurement (Figure 1.16 and Figure 1.17).

Length (m)	Phenomenon Measured	Mass (Kg)	Phenomenon Measured[1]	Time (s)	Phenomenon Measured[1]
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron (9.11×10^{-31} kg)	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom (1.67×10^{-27} kg)	10^{-22}	Mean life of an extremely unstable nucleus
10^{14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of a visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cell of living organism	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day (8.64×10^{4} s)
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (3.16×10^{7} s)
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from Earth to the sun	10^{23}	Mass of the moon (7.35×10^{22} kg)	10^{17}	Age of Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of Earth (5.97×10^{24} kg)	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way Galaxy	10^{30}	Mass of the Sun (1.99×10^{24} kg)		

Table 1.3 Approximate Values of Length, Mass, and Time [1] More precise values are in parentheses.

Length (m)	Phenomenon Measured	Mass (Kg)	Phenomenon Measured[1]	Time (s)	Phenomenon Measured[1]
10^{22}	Distance from Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Table 1.3 Approximate Values of Length, Mass, and Time [1] More precise values are in parentheses.

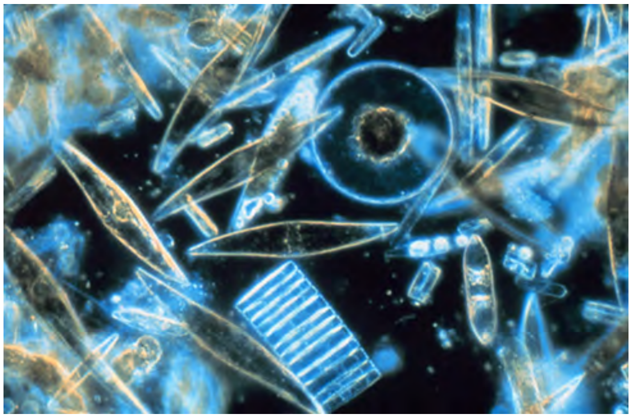


Figure 1.16 Tiny phytoplankton float among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

Figure 1.17 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Using Scientific Notation with Physical Measurements

Scientific notation is a way of writing numbers that are too large or small to be conveniently written as a decimal. For example, consider the number 840,000,000,000,000. It's a rather large number to write out. The scientific notation for this number is 8.40×10^{14}. Scientific notation follows this general format

$$x \times 10^y.$$

In this format x is the value of the measurement with all placeholder zeros removed. In the example above, x is 8.4. The x is multiplied by a factor, 10^y, which indicates the number of placeholder zeros in the measurement. Placeholder zeros are those at the end of a number that is 10 or greater, and at the beginning of a decimal number that is less than 1. In the example above, the factor is 10^{14}. This tells you that you should move the decimal point 14 positions to the right, filling in placeholder zeros as you go. In this case, moving the decimal point 14 places creates only 13 placeholder zeros, indicating that the actual measurement value is 840,000,000,000,000.

Numbers that are fractions can be indicated by scientific notation as well. Consider the number 0.0000045. Its scientific notation is 4.5×10^{-6}. Its scientific notation has the same format

$$x \times 10^y.$$

Here, x is 4.5. However, the value of y in the 10^y factor is negative, which indicates that the measurement is a fraction of 1. Therefore, we move the decimal place to the left, for a negative y. In our example of 4.5×10^{-6}, the decimal point would be moved to the left six times to yield the original number, which would be 0.0000045.

The term **order of magnitude** refers to the power of 10 when numbers are expressed in scientific notation. Quantities that have the same power of 10 when expressed in scientific notation, or come close to it, are said to be of the same order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Both numbers have the same value for y. Therefore, 800 and 450 are of the same order of magnitude. Similarly, 101 and 99 would be regarded as the same order of magnitude, 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9} m, while the diameter of the sun is on the order of 10^9 m. These two values are 18 orders of magnitude apart.

Scientists make frequent use of scientific notation because of the vast range of physical measurements possible in the universe, such as the distance from Earth to the moon (Figure 1.18), or to the nearest star.

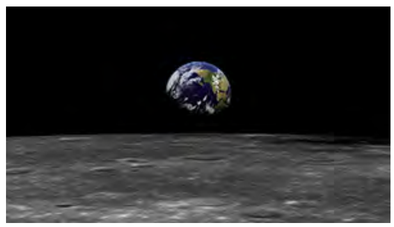


Figure 1.18 The distance from Earth to the moon may seem immense, but it is just a tiny fraction of the distance from Earth to our closest neighboring star. (NASA)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook in the United States, some quantities may be expressed in liters and you need to convert them to cups. A Canadian tourist driving through the United States might want to convert miles to kilometers, to have a sense of how far away his next destination is. A doctor in the United States might convert a patient's weight in pounds to kilograms.

Let's consider a simple example of how to convert units within the metric system. How can we want to convert 1 hour to seconds?

Next, we need to determine a **conversion factor** relating meters to kilometers. A **conversion factor** is a ratio expressing how many of one unit are equal to another unit. A conversion factor is simply a fraction which equals 1. You can multiply any number by 1 and get the same value. When you multiply a number by a conversion factor, you are simply multiplying it by one. For example, the following are conversion factors: (1 foot)/(12 inches) = 1 to convert inches to feet, (1 meter)/(100 centimeters) = 1 to convert centimeters to meters, (1 minute)/(60 seconds) = 1 to convert seconds to minutes. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor (1 km/1,000m) = 1, so we are simply multiplying 80m by 1:

$$1\ \cancel{h} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{h}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} = 3600\ \text{s} = 3.6 \times 10^2\ \text{s} \qquad (1.1)$$

When there is a unit in the original number, and a unit in the denominator (bottom) of the conversion factor, the units cancel. In this case, hours and minutes cancel and the value in seconds remains.

You can use this method to convert between any types of unit, including between the U.S. customary system and metric system. Notice also that, although you can multiply and divide units algebraically, you cannot add or subtract different units. An expression like *10 km + 5 kg* makes no sense. Even adding two lengths in different units, such as *10 km + 20 m* does not make sense. You express both lengths in the same unit. See Appendix C for a more complete list of conversion factors.

WORKED EXAMPLE

Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note—Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

2. Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0\ \text{km}}{20.0\ \text{min}} = 0.500\ \frac{\text{km}}{\text{min}}$$

3. Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is $60\ \text{min}/1\text{h}$. Thus,

$$\text{average speed} = 0.500\ \frac{\text{km}}{\text{min}} \times \frac{60\ \text{min}}{1\ \text{h}} = 30.0\ \frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

1. Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows

$$\frac{\text{km}}{\text{min}} \times \frac{1\ \text{hr}}{60\ \text{min}} = \frac{1}{60}\,\frac{\text{km}\cdot\text{h}}{\text{min}^2},$$

   which are obviously not the desired units of km/h.

2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.
3. Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/h does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 min, so the precision of the conversion factor is perfect.
4. Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution (b)

There are several ways to convert the average speed into meters per second.

1. Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.
2. Multiplying by these yields

$$\text{Averagespeed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\ \text{h}}{3{,}600\ \text{s}} \times \frac{1{,}000\ \text{m}}{1\ \text{km}}$$

$$\text{Averagespeed} = 8.33\,\frac{\text{m}}{\text{s}}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces?

WORKED EXAMPLE

Using Physics to Evaluate Promotional Materials

A commemorative coin that is 2" in diameter is advertised to be plated with 15 mg of gold. If the density of gold is 19.3 g/cc, and the amount of gold around the edge of the coin can be ignored, what is the thickness of the gold on the top and bottom faces of the coin?

Strategy

To solve this problem, the volume of the gold needs to be determined using the gold's mass and density. Half of that volume is distributed on each face of the coin, and, for each face, the gold can be represented as a cylinder that is 2" in diameter with a height equal to the thickness. Use the volume formula for a cylinder to determine the thickness.

Solution

The mass of the gold is given by the formula $m = \rho V = 15 \times 10^{-3}\ \text{g}$, where $\rho = 19.3\ \text{g/cc}$ and V is the volume. Solving for the volume gives $V = \frac{m}{\rho} = \frac{15 \times 10^{-3}\ \text{g}}{19.3\ \text{g/cc}} \cong 7.8 \times 10^{-4}\ \text{cc}$.

If t is the thickness, the volume corresponding to half the gold is $\frac{1}{2}\left(7.8 \times 10^{-4}\right) = \pi r^2 t = \pi (2.54)^2 \text{t}$, where the 1" radius has been converted to cm. Solving for the thickness gives $t = \frac{\left(3.9 \times 10^{-4}\right)}{\pi (2.54)^2} \cong 1.9 \times 10^{-5}\ \text{cm} = 0.00019\ \text{mm}$.

Discussion

The amount of gold used is stated to be 15 mg, which is equivalent to a thickness of about 0.00019 mm. The mass figure may make the amount of gold sound larger, both because the number is much bigger (15 versus 0.00019), and because people may have a more intuitive feel for how much a millimeter is than for how much a milligram is. A simple analysis of this sort can clarify the significance of claims made by advertisers.

Accuracy, Precision and Significant Figures

Science is based on experimentation that requires good measurements. The validity of a measurement can be described in terms of its accuracy and its precision (see Figure 1.19 and Figure 1.20). **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard piece of printer paper. The packaging in which you purchased the paper states that it is 11 inches long, and suppose this stated value is correct. You measure the length of the paper three times and obtain the following measurements: 11.1 inches, 11.2 inches, and 10.9 inches. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate. This is why measuring instruments are calibrated based on a known measurement. If the instrument consistently returns the correct value of the known measurement, it is safe for use in finding unknown values.

Figure 1.19 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The known masses are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (Serge Melki)

Figure 1.20 Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, some digital scales can measure the mass of an object up to the nearest thousandth of a gram. As in other measuring devices, the precision of a scale is limited to the last measured figures. This is the hundredths place in the scale pictured here. (Splarka, Wikimedia Commons)

Precision states how well repeated measurements of something generate the same or similar results. Therefore, the precision of measurements refers to how close together the measurements are when you measure the same thing several times. One way to analyze the precision of measurements would be to determine the range, or difference between the lowest and the highest measured values. In the case of the printer paper measurements, the lowest value was 10.9 inches and the highest value was 11.2 inches. Thus, the measured values deviated from each other by, at most, 0.3 inches. These measurements were reasonably precise because they varied by only a fraction of an inch. However, if the measured values had been 10.9 inches, 11.1 inches, and 11.9 inches, then the measurements would not be very precise because there is a lot of variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target. Then think of each GPS attempt to locate the restaurant as a black dot on the bull's eye.

In Figure 1.21, you can see that the GPS measurements are spread far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 1.22, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system. Finally, in Figure 1.23, the GPS is both precise and accurate, allowing the restaurant to be located.

Figure 1.21 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (Dark Evil)

Figure 1.22 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Figure 1.23 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Uncertainty

The accuracy and precision of a measuring system determine the **uncertainty** of its measurements. Uncertainty is a way to describe how much your measured value deviates from the actual value that the object has. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 inches plus or minus 0.2 inches or 11.0 ± 0.2 inches. The uncertainty in a measurement, A, is often denoted as δA ("delta A"),

The factors contributing to uncertainty in a measurement include the following:

1. Limitations of the measuring device
2. The skill of the person making the measurement
3. Irregularities in the object being measured
4. Any other factors that affect the outcome (highly dependent on the situation)

In the printer paper example uncertainty could be caused by: the fact that the smallest division on the ruler is 0.1 inches, the person using the ruler has bad eyesight, or uncertainty caused by the paper cutting machine (e.g., one side of the paper is slightly longer than the other.) It is good practice to carefully consider all possible sources of uncertainty in a measurement and reduce or eliminate them,

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement, A, is expressed with uncertainty, δA, the percent uncertainty is

$$\%\text{ uncertainty} = \frac{\delta \mathrm{A}}{\mathrm{A}} \times 100\%. \tag{1.2}$$

WORKED EXAMPLE

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.\ 8\text{lb}$
- Week 2 weight: 5.3 lb
- Week 3 weight: $4.\ 9\text{lb}$
- Week 4 weight: 5.4 lb

You determine that the weight of the 5 lb bag has an uncertainty of ±0.4 lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight

$$\%\text{ uncertainty} \quad = \quad \frac{\delta \mathrm{A}}{\mathrm{A}} \times 100\%.$$

Solution

Plug the known values into the equation

$$\%\text{ uncertainty} = \frac{0.4\ \text{ lb}}{5\ \text{ lb}} \times 100\% = 8\%.$$

Discussion

We can conclude that the weight of the apple bag is 5 lb ± 8 percent. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100 percent. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainty in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the both the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements in the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used. This method says that the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2 percent and 1 percent, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3 percent (expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter).

For a quick demonstration of the accuracy, precision, and uncertainty of measurements based upon the units of measurement, try this simulation (http://openstax.org/l/28precision) . You will have the opportunity to measure the length and weight of a desk, using milli- versus centi- units. Which do you think will provide greater accuracy, precision and uncertainty when measuring the desk and the notepad in the simulation? Consider how the nature of the hypothesis or research question might influence how precise of a measuring tool you need to collect data.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements is the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, consider measuring the thickness of a coin. A standard ruler can measure thickness to the nearest millimeter, while a micrometer can measure the thickness to the nearest 0.005 millimeter. The micrometer is a more precise measuring tool because it can measure extremely small differences in thickness. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool (such as the rulers shown in Figure 1.24). For example, if you use a standard ruler to measure the length of a stick, you may measure it with a decimeter ruler as 3.6 cm. You could not express this value as 3.65 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36 mm and 37 mm. He or she must estimate the value of the last digit. The rule is that the last digit written down in a measurement is the first digit with some uncertainty. For example, the last measured value 36.5 mm has three digits, or three significant figures. The number of **significant figures** in a measurement indicates the precision of the measuring tool. The more precise a measuring tool is, the greater the number of significant figures it can report.

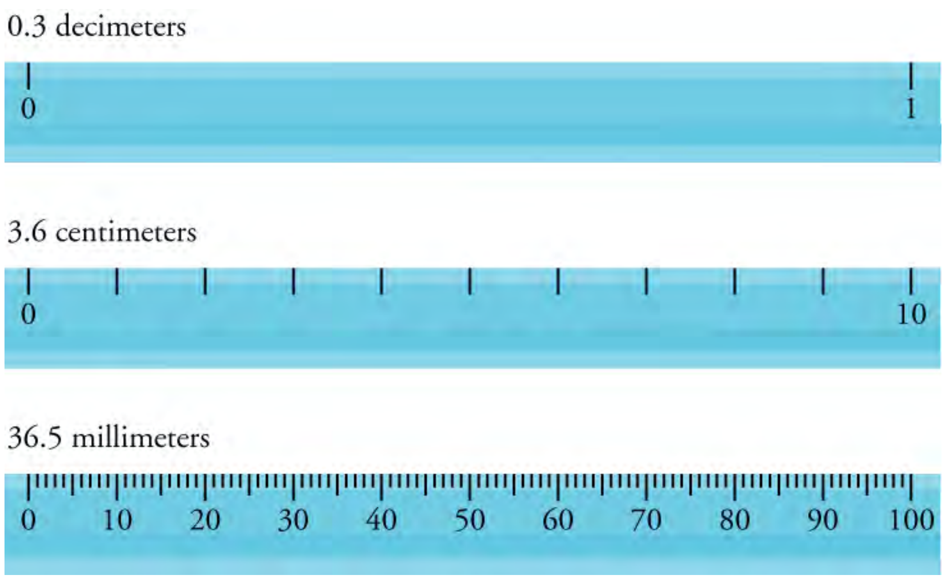


Figure 1.24 Three metric rulers are shown. The first ruler is in decimeters and can measure point three decimeters. The second ruler is in centimeters long and can measure three point six centimeters. The last ruler is in millimeters and can measure thirty-six point five millimeters.

Zeros

Special consideration is given to zeros when counting significant figures. For example, the zeros in 0.053 are not significant because they are only placeholders that locate the decimal point. There are two significant figures in 0.053—the 5 and the 3. However, if the zero occurs between other significant figures, the zeros are significant. For example, both zeros in 10.053 are significant, as these zeros were actually measured. Therefore, the 10.053 placeholder has five significant figures. The zeros in 1300 may or may not be significant, depending on the style of writing numbers. They could mean the number is known to the last zero, or the zeros could be placeholders. So 1300 could have two, three, or four significant figures. To avoid this ambiguity, write 1300 in scientific notation as 1.3 × 10^3. Only significant figures are given in the *x* factor for a number in scientific notation (in the form $x \times 10^y$). Therefore, we know that 1 and 3 are the only significant digits in this number. In summary, zeros are significant except when they serve only as placeholders. Table 1.4 provides examples of the number of significant figures in various numbers.

Number	Significant Figures	Rationale
1.657	4	There are no zeros and all non-zero numbers are always significant.
0.4578	4	The first zero is only a placeholder for the decimal point.
0.000458	3	The first four zeros are placeholders needed to report the data to the ten-thoudsandths place.
2000.56	6	The three zeros are significant here because they occur between other significant figures.
45,600	3	With no underlines or scientific notation, we assume that the last two zeros are placeholders and are not significant.
15895$\underline{00}$0	7	The two underlined zeros are significant, while the last zero is not, as it is not underlined.
5.457×10^{13}	4	In scientific notation, all numbers reported in front of the multiplication sign are significant
6.520×10^{-23}	4	In scientific notation, all numbers reported in front of the multiplication sign are significant, including zeros.

Table 1.4

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value. There are two different rules, one for multiplication and division and another rule for addition and subtraction, as discussed below.

1. **For multiplication and division:** The answer should have the same number of significant figures as the starting value with the fewest significant figures. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area will have if the radius has only two significant figures, for example, $r = 2.0$ m. Then, using a calculator that keeps eight significant figures, you would get

$$A = \pi r^2 = (3.1415927...) \times (2.0 \text{ m})^2 = 4.5238934 \text{ m}^2.$$

But because the radius has only two significant figures, the area calculated is meaningful only to two significant figures or

$$A = 4.5 \text{ m}^2$$

even though the value of π is meaningful to at least eight digits.

2. **For addition and subtraction**: The answer should have the same number places (e.g. tens place, ones place, tenths place, etc.) as the least-precise starting value. Suppose that you buy 7.56 kg of potatoes in a grocery store as measured with a scale having a precision of 0.01 kg. Then you drop off 6.052 kg of potatoes at your laboratory as measured by a scale with a precision of 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with a precision of 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{rl} 7.56 & \text{kg} \\ -6.052 & \text{kg} \\ +13.7 & \text{kg} \\ \hline 15.208 & \text{kg} \end{array}$$

The least precise measurement is 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer should be rounded to the tenths place, giving 15.2 kg. The same is true for non-decimal numbers. For example,

$$6527.23 + 2 = 6528.23 = 6528\ .$$

We cannot report the decimal places in the answer because *2* has no decimal places that would be significant. Therefore, we can only report to the ones place.

It is a good idea to keep extra significant figures while calculating, and to round off to the correct number of significant figures only in the final answers. The reason is that small errors from rounding while calculating can sometimes produce significant errors in the final answer. As an example, try calculating $5{,}098 - (5.000) \times (1{,}010)$ to obtain a final answer to only two significant figures. Keeping all significant during the calculation gives 48. Rounding to two significant figures in the middle of the calculation changes it to $5{,}100 - (5.000) \times (1{,}000) = 100$, which is way off. You would similarly avoid rounding in the middle of the calculation in counting and in doing accounting, where many small numbers need to be added and subtracted accurately to give possibly much larger final numbers.

Significant Figures in this Text

In this textbook, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, such as optics, more than three significant figures will be used. Finally, if a number is exact, such as the *2* in the formula, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

WORKED EXAMPLE

Approximating Vast Numbers: a Trillion Dollars

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks, like that shown in Figure 1.25, and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Figure 1.25 A bank stack contains one hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (Andrew Magill)

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

1. Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is

$$\begin{aligned}\text{volume of stack} &= \text{length} \times \text{width} \times \text{height},\\ \text{volume of stack} &= 6\text{ in.} \times 3\text{ in.} \times 0.5\text{ in.},\\ \text{volume of stack} &= 9\text{ in.}^3.\end{aligned}$$

2. Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $\$100$ bills is equal to $\$10,000$, or $\$1 \times 10^4$. The number of stacks you will have is

$$\$1 \times 10^{12}\text{ (a trillion dollars) / }\$1 \times 10^4\text{ per stack} = 1 \times 10^8\text{ stacks.} \tag{1.3}$$

3. Calculate the area of a football field in square inches. The area of a football field is $100\ \text{yd} \times 50\text{yd}$, which gives $5,000\ \text{yd}^2$. Because we are working in inches, we need to convert square yards to square inches

$$\begin{aligned}\text{Area} &= 5{,}000\text{ yd}^2 \times \frac{3\text{ ft}}{1\text{yd}} \times \frac{3\text{ ft}}{1\text{yd}} \times \frac{12\text{ in.}}{1\text{ foot}} \times \frac{12\text{ in.}}{1\text{ foot}} = 6{,}480{,}000\text{ in.}^2,\\ \text{Area} &\approx 6 \times 10^6\text{ in.}^2.\end{aligned}$$

This conversion gives us $6 \times 10^6\ \text{in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

4. Calculate the total volume of the bills. The volume of all the $100-bill stacks is
$9\text{ in.}^3\text{ / stack} \times 10^8\text{ stacks} = 9 \times 10^8\text{ in.}^3$
5. Calculate the height. To determine the height of the bills, use the following equation

$$\begin{aligned}\text{volume of bills} &= \text{area of field} \times \text{height of money} \\ \text{Height of money} &= \frac{\text{volume of bills}}{\text{area of field}} \\ \text{Height of money} &= \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.} \\ \text{Height of money} &= 1{\times}10^2 \text{ in.} = 100 \text{ in.}\end{aligned}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100 \text{ in.} \times \frac{1\text{ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft.}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough *guesstimates* versus carefully calculated approximations?

In the example above, the final approximate value is much higher than the first friend's early estimate of 3 in. However, the other friend's early estimate of 10 ft. (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise suggest about the value of rough *guesstimates* versus carefully calculated approximations?

Graphing in Physics

Most results in science are presented in scientific journal articles using graphs. Graphs present data in a way that is easy to visualize for humans in general, especially someone unfamiliar with what is being studied. They are also useful for presenting large amounts of data or data with complicated trends in an easily-readable way.

One commonly-used graph in physics and other sciences is the line graph, probably because it is the best graph for showing how one quantity changes in response to the other. Let's build a line graph based on the data in Table 1.5, which shows the measured distance that a train travels from its station versus time. Our two **variables**, or things that change along the graph, are time in minutes, and distance from the station, in kilometers. Remember that measured data may not have perfect accuracy.

Time (min)	Distance from Station (km)
0	0
10	24
20	36
30	60
40	84
50	97
60	116
70	140

Table 1.5

1. Draw the two axes. The horizontal axis, or *x*-axis, shows the **independent variable**, which is the variable that is controlled or manipulated. The vertical axis, or *y*-axis, shows the **dependent variable**, the non-manipulated variable that changes with (or is dependent on) the value of the independent variable. In the data above, time is the independent variable and should be plotted on the *x*-axis. Distance from the station is the dependent variable and should be plotted on the *y*-axis.

2. Label each axes on the graph with the name of each variable, followed by the symbol for its units in parentheses. Be sure to leave room so that you can number each axis. In this example, use *Time (min)* as the label for the x-axis.
3. Next, you must determine the best scale to use for numbering each axis. Because the time values on the x-axis are taken every 10 minutes, we could easily number the x-axis from 0 to 70 minutes with a tick mark every 10 minutes. Likewise, the y-axis scale should start low enough and continue high enough to include all of the *distance from station* values. A scale from 0 km to 160 km should suffice, perhaps with a tick mark every 10 km.

   In general, you want to pick a scale for both axes that 1) shows all of your data, and 2) makes it easy to identify trends in your data. If you make your scale too large, it will be harder to see how your data change. Likewise, the smaller and more fine you make your scale, the more space you will need to make the graph. The number of significant figures in the axis values should be coarser than the number of significant figures in the measurements.

4. Now that your axes are ready, you can begin plotting your data. For the first data point, count along the x-axis until you find the 10 min tick mark. Then, count up from that point to the 10 km tick mark on the y-axis, and approximate where 22 km is along the y-axis. Place a dot at this location. Repeat for the other six data points (Figure 1.26).

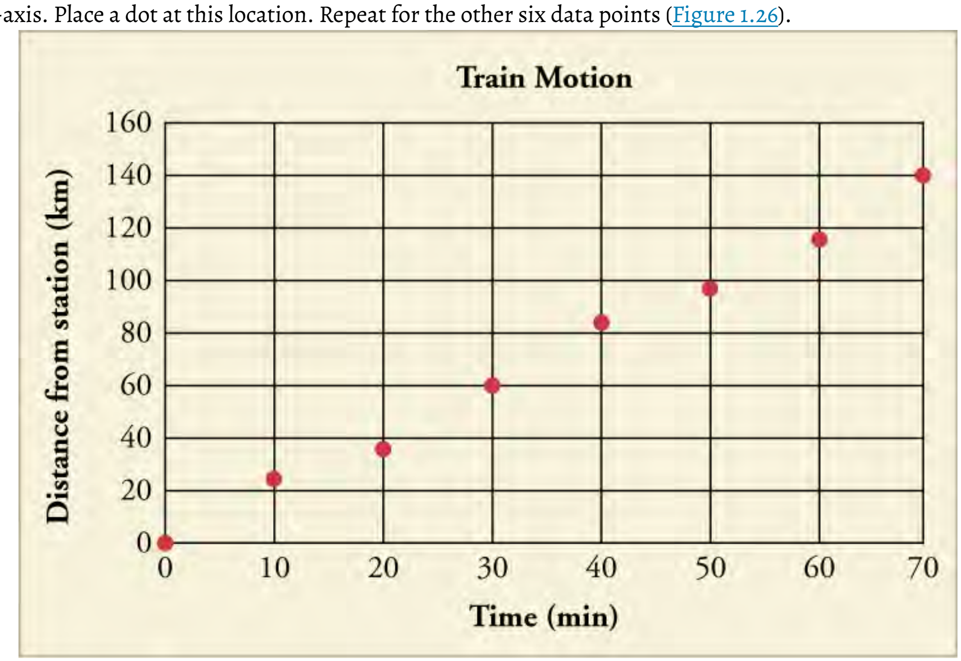


Figure 1.26 The graph of the train's distance from the station versus time from the exercise above.

5. Add a title to the top of the graph to state what the graph is describing, such as the y-axis parameter vs. the x-axis parameter. In the graph shown here, the title is *train motion*. It could also be titled distance of the train from the station vs. time.
6. Finally, with data points now on the graph, you should draw a trend line (Figure 1.27). The trend line represents the dependence you think the graph represents, so that the person who looks at your graph can see how close it is to the real data. In the present case, since the data points look like they ought to fall on a straight line, you would draw a straight line as the trend line. Draw it to come closest to all the points. Real data may have some inaccuracies, and the plotted points may not all fall on the trend line. In some cases, none of the data points fall exactly on the trend line.

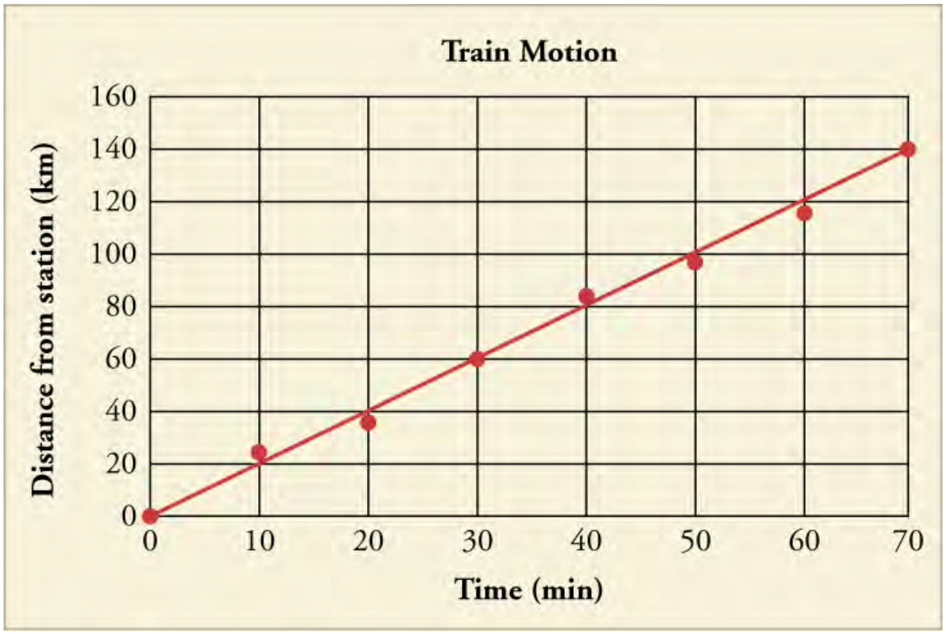


Figure 1.27 The completed graph with the trend line included.

Analyzing a Graph Using Its Equation

One way to get a quick snapshot of a dataset is to look at the equation of its trend line. If the graph produces a straight line, the equation of the trend line takes the form

$$y = mx + b.$$

The b in the equation is the y-intercept while the m in the equation is the **slope**. The y-intercept tells you at what y value the line intersects the y-axis. In the case of the graph above, the y-intercept occurs at 0, at the very beginning of the graph. The y-intercept, therefore, lets you know immediately where on the y-axis the plot line begins.

The m in the equation is the slope. This value describes how much the line on the graph moves up or down on the y-axis along the line's length. The slope is found using the following equation

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}.$$

In order to solve this equation, you need to pick two points on the line (preferably far apart on the line so the slope you calculate describes the line accurately). The quantities Y_2 and Y_1 represent the y-values from the two points on the line (not data points) that you picked, while X_2 and X_1 represent the two x-values of the those points.

What can the slope value tell you about the graph? The slope of a perfectly horizontal line will equal zero, while the slope of a perfectly vertical line will be undefined because you cannot divide by zero. A positive slope indicates that the line moves up the y-axis as the x-value increases while a negative slope means that the line moves down the y-axis. The more negative or positive the slope is, the steeper the line moves up or down, respectively. The slope of our graph in Figure 1.26 is calculated below based on the two endpoints of the line

$$\begin{aligned} m &= \frac{Y_2 - Y_1}{X_2 - X_1} \\ m &= \frac{(80\text{ km}) - (20\text{ km})}{(40\text{ min}) - (10\text{ min})} \\ m &= \frac{60\text{ km}}{30\text{ min}} \\ m &= 2.0\text{ km/min}. \end{aligned}$$

Equation of line: $y = (2.0\ \text{km/min})\,x + 0$

Because the x axis is time in minutes, we would actually be more likely to use the time t as the independent (x-axis) variable and write the equation as

$$y = (2.0 \ \ \text{km/min})\,t + 0. \tag{1.4}$$

The formula $y = mx + b$ only applies to **linear relationships**, or ones that produce a straight line. Another common type of line in physics is the **quadratic relationship**, which occurs when one of the variables is squared. One quadratic relationship in physics is the relation between the speed of an object its centripetal acceleration, which is used to determine the force needed to keep an object moving in a circle. Another common relationship in physics is the **inverse relationship**, in which one variable decreases whenever the other variable increases. An example in physics is Coulomb's law. As the distance between two charged objects increases, the electrical force between the two charged objects decreases. **Inverse proportionality**, such the relation between x and y in the equation

$$y = k/x,$$

for some number k, is one particular kind of inverse relationship. A third commonly-seen relationship is the **exponential relationship**, in which a change in the independent variable produces a proportional change in the dependent variable. As the value of the dependent variable gets larger, its rate of growth also increases. For example, bacteria often reproduce at an exponential rate when grown under ideal conditions. As each generation passes, there are more and more bacteria to reproduce. As a result, the growth rate of the bacterial population increases every generation (Figure 1.28).

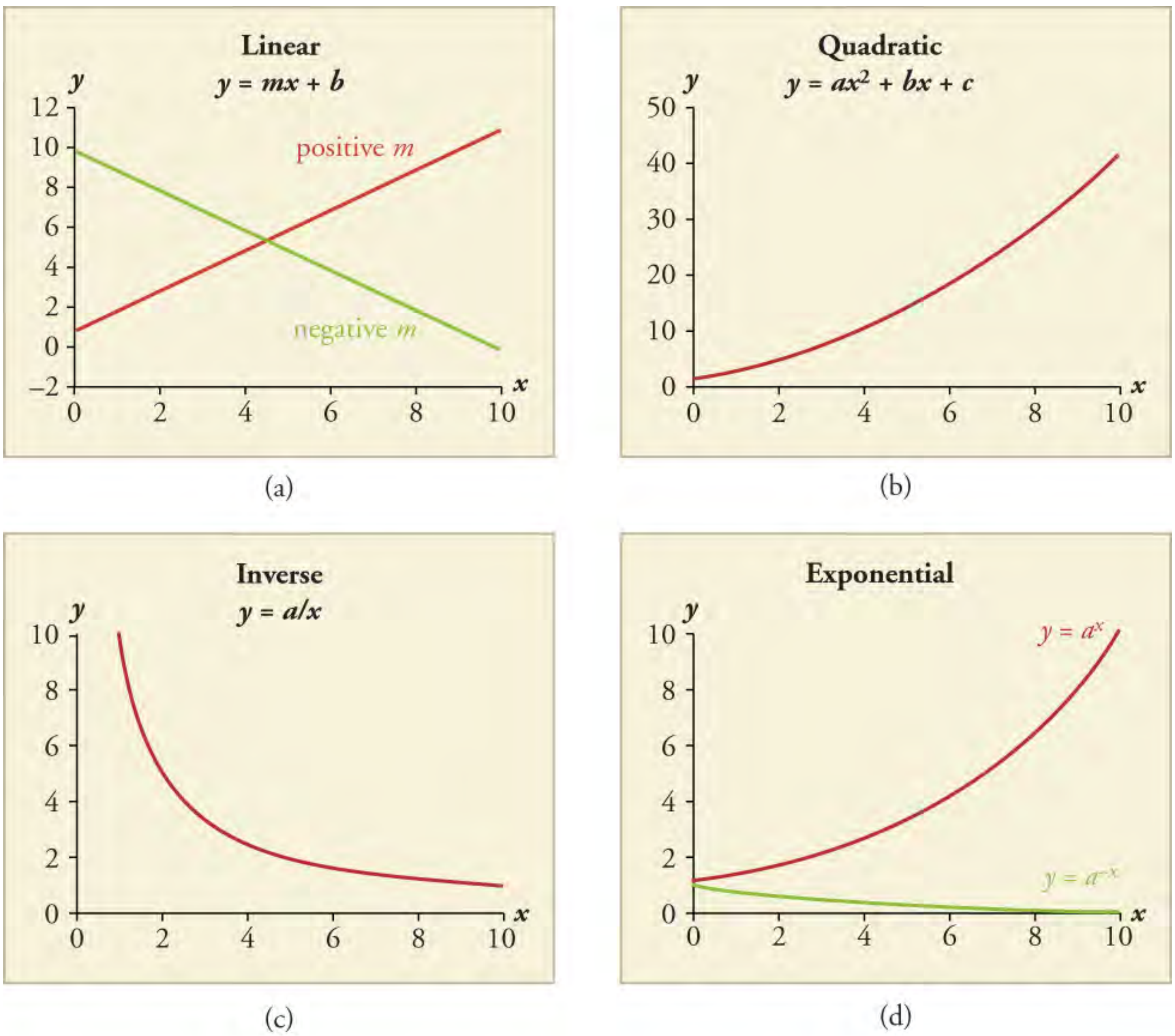


Figure 1.28 Examples of (a) linear, (b) quadratic, (c) inverse, and (d) exponential relationship graphs.

Using Logarithmic Scales in Graphing

Sometimes a variable can have a very large range of values. This presents a problem when you're trying to figure out the best scale to use for your graph's axes. One option is to use a **logarithmic (log) scale**. In a logarithmic scale, the value each mark labels is the previous mark's value multiplied by some constant. For a log base 10 scale, each mark labels a value that is 10 times the value of the mark before it. Therefore, a base 10 logarithmic scale would be numbered: 0, 10, 100, 1,000, etc. You can see how the logarithmic scale covers a much larger range of values than the corresponding linear scale, in which the marks would label the values 0, 10, 20, 30, and so on.

If you use a logarithmic scale on one axis of the graph and a linear scale on the other axis, you are using a **semi-log plot**. The Richter scale, which measures the strength of earthquakes, uses a semi-log plot. The degree of ground movement is plotted on a logarithmic scale against the assigned intensity level of the earthquake, which ranges linearly from 1-10 (Figure 1.29 **(a)**).

If a graph has both axes in a logarithmic scale, then it is referred to as a **log-log plot**. The relationship between the wavelength and frequency of electromagnetic radiation such as light is usually shown as a log-log plot (Figure 1.29 **(b)**). Log-log plots are also commonly used to describe exponential functions, such as radioactive decay.

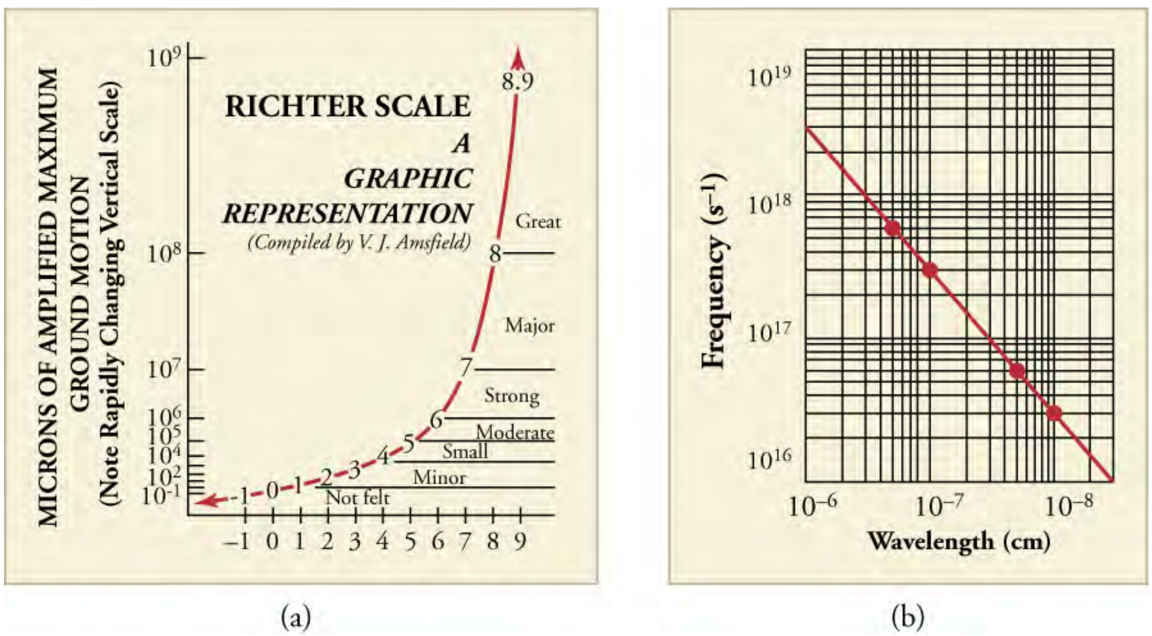


Figure 1.29 (a) The Richter scale uses a log base 10 scale on its *y-axis* (microns of amplified maximum ground motion). (b) The relationship between the frequency and wavelength of electromagnetic radiation can be plotted as a straight line if a log-log plot is used.

Virtual Physics

Graphing Lines

In this simulation you will examine how changing the slope and *y*-intercept of an equation changes the appearance of a plotted line. Select slope-intercept form and drag the blue circles along the line to change the line's characteristics. Then, play the line game and see if you can determine the slope or *y*-intercept of a given line.

Click to view content (https://phet.colorado.edu/sims/html/graphing-lines/latest/graphing-lines_en.html)

GRASP CHECK

How would the following changes affect a line that is neither horizontal nor vertical and has a positive slope?

1. increase the slope but keeping the *y*-intercept constant
2. increase the *y*-intercept but keeping the slope constant
   a. Increasing the slope will cause the line to rotate clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move vertically up on the graph without changing the line's slope.
   b. Increasing the slope will cause the line to rotate counter-clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move vertically up on the graph without changing the line's slope.
   c. Increasing the slope will cause the line to rotate clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

d. Increasing the slope will cause the line to rotate counter-clockwise around the y-intercept. Increasing the y-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

Check Your Understanding

12. Identify some advantages of metric units.

a. Conversion between units is easier in metric units.
b. Comparison of physical quantities is easy in metric units.
c. Metric units are more modern than English units.
d. Metric units are based on powers of 2.

13. The length of an American football field is 100 yd, excluding the end zones. How long is the field in meters? Round to the nearest 0.1 m.

a. 10.2 m
b. 91.4 m
c. 109.4 m
d. 328.1 m

14. The speed limit on some interstate highways is roughly 100 km/h. How many miles per hour is this if 1.0 mile is about 1.609 km?

a. 0.1 mi/h
b. 27.8 mi/h
c. 62 mi/h
d. 160 mi/h

15. Briefly describe the target patterns for accuracy and precision and explain the differences between the two.

a. Precision states how much repeated measurements generate the same or closely similar results, while accuracy states how close a measurement is to the true value of the measurement.
b. Precision states how close a measurement is to the true value of the measurement, while accuracy states how much repeated measurements generate the same or closely similar result.
c. Precision and accuracy are the same thing. They state how much repeated measurements generate the same or closely similar results.
d. Precision and accuracy are the same thing. They state how close a measurement is to the true value of the measurement.

KEY TERMS

accuracy how close a measurement is to the correct value for that measurement
ampere the SI unit for electrical current
atom smallest and most basic units of matter
classical physics physics, as it developed from the Renaissance to the end of the nineteenth century
constant a quantity that does not change
conversion factor a ratio expressing how many of one unit are equal to another unit
dependent variable the vertical, or y-axis, variable, which changes with (or is dependent on) the value of the independent variable
derived units units that are derived by combining the fundamental physical units
English units (also known as the customary or imperial system) system of measurement used in the United States; includes units of measurement such as feet, gallons, degrees Fahrenheit, and pounds
experiment process involved with testing a hypothesis
exponential relationship relation between variables in which a constant change in the independent variable is accompanied by change in the dependent variable that is proportional to the value it already had
fundamental physical units the seven fundamental physical units in the SI system of units are length, mass, time, electric current, temperature, amount of a substance, and luminous intensity
hypothesis testable statement that describes how something in the natural world works
independent variable the horizontal, or x-axis, variable, which is not influence by the second variable on the graph, the dependent variable
inverse proportionality a relation between two variables expressible by an equation of the form $y = k/x$ where k stays constant when x and y change; the special form of inverse relationship that satisfies this equation
inverse relationship any relation between variables where one variable decreases as the other variable increases
kilogram the SI unit for mass, abbreviated (kg)
linear relationships relation between variables that produce a straight line when graphed
log-log plot a plot that uses a logarithmic scale in both axes
logarithmic scale a graphing scale in which each tick on an axis is the previous tick multiplied by some value
meter the SI unit for length, abbreviated (m)
method of adding percents calculating the percent uncertainty of a quantity in multiplication or division by adding the percent uncertainties in the quantities being added or divided
model system that is analogous to the real system of interest in essential ways but more easily analyzed
modern physics physics as developed from the twentieth century to the present, involving the theories of relativity and quantum mechanics
observation step where a scientist observes a pattern or trend within the natural world
order of magnitude the size of a quantity in terms of its power of 10 when expressed in scientific notation
physics science aimed at describing the fundamental aspects of our universe—energy, matter, space, motion, and time
precision how well repeated measurements generate the same or closely similar results
principle description of nature that is true in many, but not all situations
quadratic relationship relation between variables that can be expressed in the form $y = ax^2 + bx + c$, which produces a curved line when graphed
quantum mechanics major theory of modern physics which describes the properties and nature of atoms and their subatomic particles
science the study or knowledge of how the physical world operates, based on objective evidence determined through observation and experimentation
scientific law pattern in nature that is true in all circumstances studied thus far
scientific methods techniques and processes used in the constructing and testing of scientific hypotheses, laws, and theories, and in deciding issues on the basis of experiment and observation
scientific notation way of writing numbers that are too large or small to be conveniently written in simple decimal form; the measurement is multiplied by a power of 10, which indicates the number of placeholder zeros in the measurement
second the SI unit for time, abbreviated (s)
semi-log plot A plot that uses a logarithmic scale on one axis of the graph and a linear scale on the other axis.
SI units International System of Units (SI); the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams; also known as the metric system
significant figures when writing a number, the digits, or number of digits, that express the precision of a measuring tool used to measure the number
slope the ratio of the change of a graph on the y axis to the change along the x-axis, the value of m in the equation of a line, $y = mx + b$
theory explanation of patterns in nature that is supported by much scientific evidence and verified multiple times by various groups of researchers
theory of relativity theory constructed by Albert Einstein which describes how space, time and energy are different

for different observers in relative motion

uncertainty a quantitative measure of how much measured values deviate from a standard or expected value

universal applies throughout the known universe

***y*-intercept** the point where a plot line intersects the y-axis

SECTION SUMMARY

1.1 Physics: Definitions and Applications

- Physics is the most fundamental of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Modern physics involves the theory of relativity, which describes how time, space and gravity are not constant in our universe can be different for different observers, and quantum mechanics, which describes the behavior of subatomic particles.
- Physics is the basis for all other sciences, such as chemistry, biology and geology, because physics describes the fundamental way in which the universe functions.

1.2 The Scientific Methods

- Science seeks to discover and describe the underlying order and simplicity in nature.
- The processes of science include observation, hypothesis, experiment, and conclusion.
- Theories are scientific explanations that are supported by a large body experimental results.
- Scientific laws are concise descriptions of the universe that are universally true.

1.3 The Language of Physics: Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- The four fundamental units we will use in this textbook are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.
- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

KEY EQUATIONS

1.3 The Language of Physics: Physical Quantities and Units

slope intercept form	$y = mx + b$
quadratic formula	$y = ax^2 + bx + c$
positive exponential formula	$y = a^x$
negative exponential formula	$y = a^{-x}$

CHAPTER REVIEW

Concept Items

1.1 Physics: Definitions and Applications

1. Which statement best compares and contrasts the aims and topics of natural philosophy had versus physics?
   a. Natural philosophy included all aspects of nature including physics.
   b. Natural philosophy included all aspects of nature excluding physics.
   c. Natural philosophy and physics are different.
   d. Natural philosophy and physics are essentially the

CHAPTER 2
Motion in One Dimension

Figure 2.1 Shanghai Maglev. At this rate, a train traveling from Boston to Washington, DC, a distance of 439 miles, could make the trip in under an hour and a half. Presently, the fastest train on this route takes over six hours to cover this distance. (Alex Needham, Public Domain)

Chapter Outline

INTRODUCTION Unless you have flown in an airplane, you have probably never traveled faster than 150 mph. Can you imagine traveling in a train like the one shown in Figure 2.1 that goes over 300 mph? Despite the high speed, the people riding in this train may not notice that they are moving at all unless they look out the window! This is because motion, even motion at 300 mph, is relative to the observer.

In this chapter, you will learn why it is important to identify a reference frame in order to clearly describe motion. For now, the motion you describe will be one-dimensional. Within this context, you will learn the difference between distance and displacement as well as the difference between speed and velocity. Then you will look at some graphing and problem-solving techniques.

2.1 Relative Motion, Distance, and Displacement

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe motion in different reference frames
- Define distance and displacement, and distinguish between the two
- Solve problems involving distance and displacement

Section Key Terms

displacement	distance	kinematics	magnitude
position	reference frame	scalar	vector

Defining Motion

Our study of physics opens with **kinematics**—the study of motion without considering its causes. Objects are in motion everywhere you look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects, atoms are always moving.

How do you know something is moving? The location of an object at any particular time is its **position**. More precisely, you need to specify its position relative to a convenient **reference frame**. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to Earth. To describe the position of a person in an airplane, for example, we use the airplane, not Earth, as the reference frame. (See Figure 2.2.) Thus, you can only know how fast and in what direction an object's position is changing against a background of something else that is either not moving or moving with a known speed and direction. The reference frame is the coordinate system from which the positions of objects are described.

Figure 2.2 Are clouds a useful reference frame for airplane passengers? Why or why not? (Paul Brennan, Public Domain)

Your classroom can be used as a reference frame. In the classroom, the walls are not moving. Your motion as you walk to the door, can be measured against the stationary background of the classroom walls. You can also tell if other things in the classroom are moving, such as your classmates entering the classroom or a book falling off a desk. You can also tell in what direction something is moving in the classroom. You might say, “The teacher is moving toward the door.” Your reference frame allows you to determine not only that something is moving but also the direction of motion.

You could also serve as a reference frame for others' movement. If you remained seated as your classmates left the room, you would measure their movement away from your stationary location. If you and your classmates left the room together, then your perspective of their motion would be change. You, as the reference frame, would be moving in the same direction as your other moving classmates. As you will learn in the **Snap Lab**, your description of motion can be quite different when viewed from different reference frames.

Snap Lab

Looking at Motion from Two Reference Frames

In this activity you will look at motion from two reference frames. Which reference frame is correct?

- Choose an open location with lots of space to spread out so there is less chance of tripping or falling due to a collision and/or loose basketballs.
- 1 basketball

Procedure

1. Work with a partner. Stand a couple of meters away from your partner. Have your partner turn to the side so that you are looking at your partner's profile. Have your partner begin bouncing the basketball while standing in place. Describe the motion of the ball.
2. Next, have your partner again bounce the ball, but this time your partner should walk forward with the bouncing ball. You will remain stationary. Describe the ball's motion.
3. Again have your partner walk forward with the bouncing ball. This time, you should move alongside your partner while continuing to view your partner's profile. Describe the ball's motion.
4. Switch places with your partner, and repeat Steps 1–3.

GRASP CHECK

How do the different reference frames affect how you describe the motion of the ball?

a. The motion of the ball is independent of the reference frame and is same for different reference frames.
b. The motion of the ball is independent of the reference frame and is different for different reference frames.
c. The motion of the ball is dependent on the reference frame and is same for different reference frames.
d. The motion of the ball is dependent on the reference frames and is different for different reference frames.

LINKS TO PHYSICS

History: Galileo's Ship

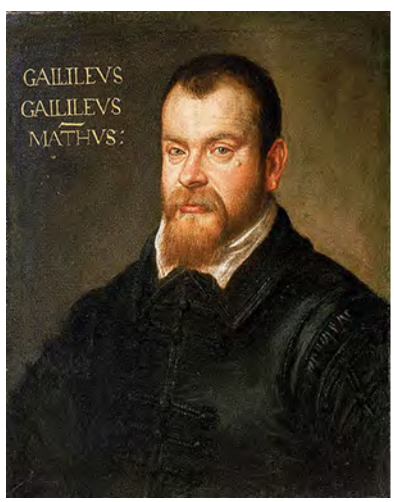


Figure 2.3 Galileo Galilei (1564–1642) studied motion and developed the concept of a reference frame. (Domenico Tintoretto)

The idea that a description of motion depends on the reference frame of the observer has been known for hundreds of years. The 17th-century astronomer Galileo Galilei (Figure 2.3) was one of the first scientists to explore this idea. Galileo suggested the following thought experiment: Imagine a windowless ship moving at a constant speed and direction along a perfectly calm sea. Is there a way that a person inside the ship can determine whether the ship is moving? You can extend this thought experiment by also imagining a person standing on the shore. How can a person on the shore determine whether the ship is moving?

Galileo came to an amazing conclusion. Only by looking at each other can a person in the ship or a person on shore describe the motion of one relative to the other. In addition, their descriptions of motion would be identical. A person inside the ship would describe the person on the land as moving past the ship. The person on shore would describe the ship and the person inside it as moving past. Galileo realized that observers moving at a constant speed and direction relative to each other describe motion in the same way. Galileo had discovered that a description of motion is only meaningful if you specify a reference frame.

GRASP CHECK

Imagine standing on a platform watching a train pass by. According to Galileo's conclusions, how would your description of motion and the description of motion by a person riding on the train compare?

a. I would see the train as moving past me, and a person on the train would see me as stationary.
b. I would see the train as moving past me, and a person on the train would see me as moving past the train.
c. I would see the train as stationary, and a person on the train would see me as moving past the train.
d. I would see the train as stationary, and a person on the train would also see me as stationary.

Distance vs. Displacement

As we study the motion of objects, we must first be able to describe the object's position. Before your parent drives you to school, the car is sitting in your driveway. Your driveway is the starting position for the car. When you reach your high school, the car has changed position. Its new position is your school.

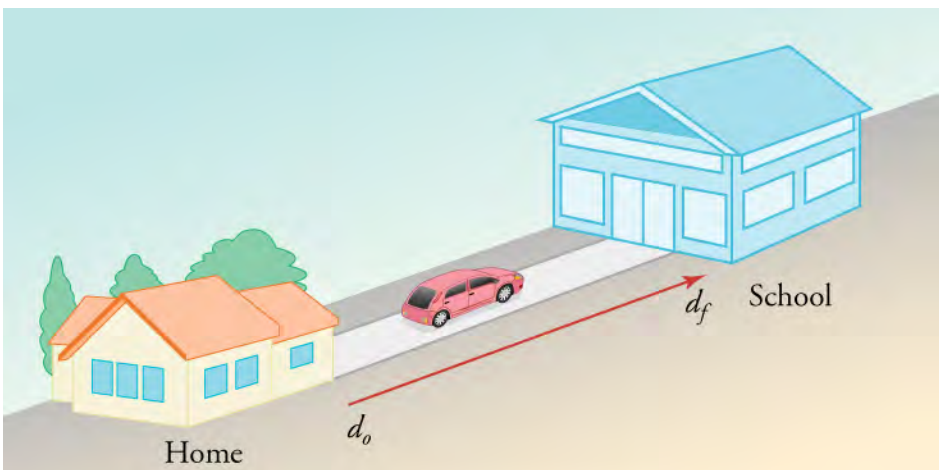


Figure 2.4 Your total change in position is measured from your house to your school.

Physicists use variables to represent terms. We will use **d** to represent car's position. We will use a subscript to differentiate between the initial position, $\mathbf{d}_0$, and the final position, $\mathbf{d}_f$. In addition, vectors, which we will discuss later, will be in bold or will have an arrow above the variable. Scalars will be italicized.

TIPS FOR SUCCESS

In some books, **x** or **s** is used instead of **d** to describe position. In $\mathbf{d}_0$, said *d naught*, the subscript 0 stands for *initial*. When we begin to talk about two-dimensional motion, sometimes other subscripts will be used to describe horizontal position, $\mathbf{d}_x$, or vertical position, $\mathbf{d}_y$. So, you might see references to $\mathbf{d}_{0x}$ and $\mathbf{d}_{fy}$.

Now imagine driving from your house to a friend's house located several kilometers away. How far would you drive? The **distance** an object moves is the length of the path between its initial position and its final position. The distance you drive to your friend's house depends on your path. As shown in Figure 2.5, distance is different from the length of a straight line between two points. The distance you drive to your friend's house is probably longer than the straight line between the two houses.

Figure 2.5 A short line separates the starting and ending points of this motion, but the distance along the path of motion is considerably longer.

We often want to be more precise when we talk about position. The description of an object's motion often includes more than just the distance it moves. For instance, if it is a five kilometer drive to school, the distance traveled is 5 kilometers. After dropping you off at school and driving back home, your parent will have traveled a total distance of 10 kilometers. The car and your parent will end up in the same starting position in space. The net change in position of an object is its **displacement**, or $\Delta\mathbf{d}$. The Greek letter delta, Δ, means *change in*.

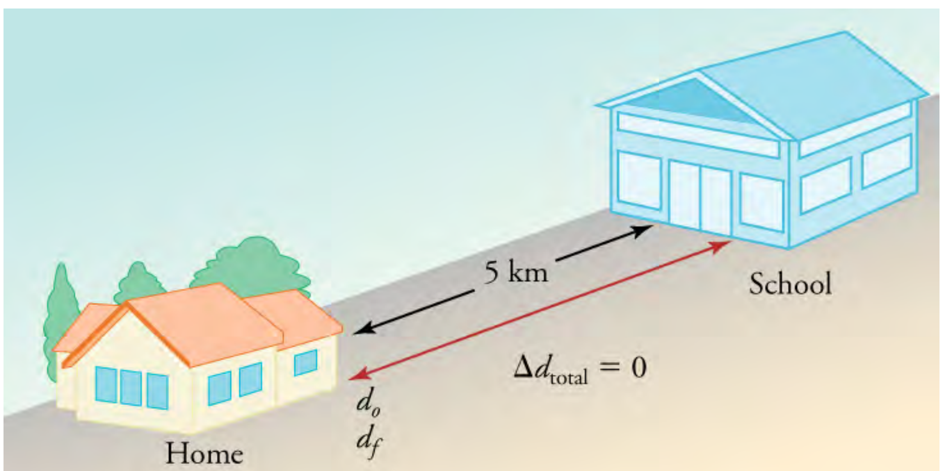


Figure 2.6 The total distance that your car travels is 10 km, but the total displacement is 0.

Snap Lab

Distance vs. Displacement

In this activity you will compare distance and displacement. Which term is more useful when making measurements?

- 1 recorded song available on a portable device
- 1 tape measure
- 3 pieces of masking tape
- A room (like a gym) with a wall that is large and clear enough for all pairs of students to walk back and forth without running into each other.

Procedure

1. One student from each pair should stand with their back to the longest wall in the classroom. Students should stand at least 0.5 meters away from each other. Mark this starting point with a piece of masking tape.
2. The second student from each pair should stand facing their partner, about two to three meters away. Mark this point

with a second piece of masking tape.
3. Student pairs line up at the starting point along the wall.
4. The teacher turns on the music. Each pair walks back and forth from the wall to the second marked point until the music stops playing. Keep count of the number of times you walk across the floor.
5. When the music stops, mark your ending position with the third piece of masking tape.
6. Measure from your starting, initial position to your ending, final position.
7. Measure the length of your path from the starting position to the second marked position. Multiply this measurement by the total number of times you walked across the floor. Then add this number to your measurement from step 6.
8. Compare the two measurements from steps 6 and 7.

GRASP CHECK

1. Which measurement is your total distance traveled?
2. Which measurement is your displacement?
3. When might you want to use one over the other?

a. Measurement of the total length of your path from the starting position to the final position gives the distance traveled, and the measurement from your initial position to your final position is the displacement. Use distance to describe the total path between starting and ending points,and use displacement to describe the shortest path between starting and ending points.
b. Measurement of the total length of your path from the starting position to the final position is distance traveled, and the measurement from your initial position to your final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.
c. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the total path between starting and ending points, and use displacement to describe the shortest path between starting and ending points.
d. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.

If you are describing only your drive to school, then the distance traveled and the displacement are the same—5 kilometers. When you are describing the entire round trip, distance and displacement are different. When you describe distance, you only include the **magnitude**, the size or amount, of the distance traveled. However, when you describe the displacement, you take into account both the magnitude of the change in position and the direction of movement.

In our previous example, the car travels a total of 10 kilometers, but it drives five of those kilometers forward toward school and five of those kilometers back in the opposite direction. If we ascribe the forward direction a positive (+) and the opposite direction a negative (–), then the two quantities will cancel each other out when added together.

A quantity, such as distance, that has magnitude (i.e., how big or how much) but does not take into account direction is called a **scalar**. A quantity, such as displacement, that has both magnitude and direction is called a **vector**.

WATCH PHYSICS

Vectors & Scalars

This video (http://openstax.org/l/28vectorscalar) introduces and differentiates between vectors and scalars. It also introduces quantities that we will be working with during the study of kinematics.

Click to view content (https://www.khanacademy.org/embed_video?v=ihNZlp7iUHE)

GRASP CHECK

How does this video (https://www.khanacademy.org/science/ap-physics-1/ap-one-dimensional-motion/ap-physics-foundations/v/introduction-to-vectors-and-scalars) help you understand the difference between distance and displacement? Describe the differences between vectors and scalars using physical quantities as examples.

a. It explains that distance is a vector and direction is important, whereas displacement is a scalar and it has no direction attached to it.
b. It explains that distance is a scalar and direction is important, whereas displacement is a vector and it has no direction attached to it.
c. It explains that distance is a scalar and it has no direction attached to it, whereas displacement is a vector and direction is important.
d. It explains that both distance and displacement are scalar and no directions are attached to them.

Displacement Problems

Hopefully you now understand the conceptual difference between distance and displacement. Understanding concepts is half the battle in physics. The other half is math. A stumbling block to new physics students is trying to wade through the math of physics while also trying to understand the associated concepts. This struggle may lead to misconceptions and answers that make no sense. Once the concept is mastered, the math is far less confusing.

So let's review and see if we can make sense of displacement in terms of numbers and equations. You can calculate an object's displacement by subtracting its original position, $\mathbf{d_o}$, from its final position $\mathbf{d_f}$. In math terms that means

$$\Delta\mathbf{d} = \mathbf{d}_\mathrm{f} - \mathbf{d}_0 .$$

If the final position is the same as the initial position, then $\Delta\mathbf{d} = 0$.

To assign numbers and/or direction to these quantities, we need to define an axis with a positive and a negative direction. We also need to define an origin, or *O*. In Figure 2.6, the axis is in a straight line with home at zero and school in the positive direction. If we left home and drove the opposite way from school, motion would have been in the negative direction. We would have assigned it a negative value. In the round-trip drive, $\mathbf{d}_\mathrm{f}$ and $\mathbf{d}_\mathrm{o}$ were both at zero kilometers. In the one way trip to school, $\mathbf{d}_\mathrm{f}$ was at 5 kilometers and $\mathbf{d}_\mathrm{o}$ was at zero km. So, $\Delta\mathbf{d}$ was 5 kilometers.

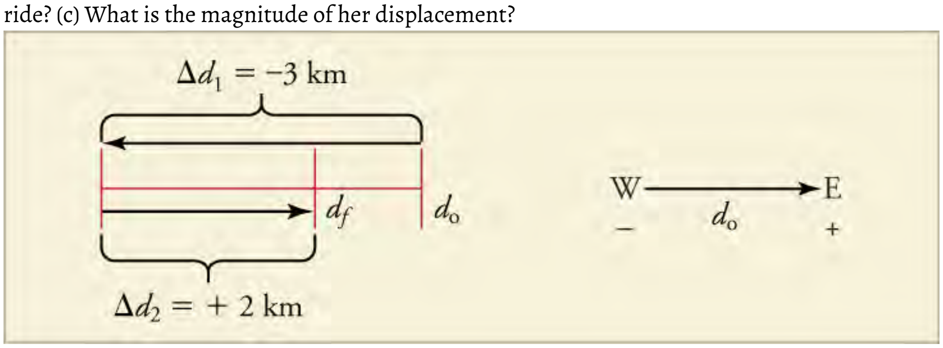
TIPS FOR SUCCESS

You may place your origin wherever you would like. You have to make sure that you calculate all distances consistently from your zero and you define one direction as positive and the other as negative. Therefore, it makes sense to choose the easiest axis, direction, and zero. In the example above, we took home to be zero because it allowed us to avoid having to interpret a solution with a negative sign.

WORKED EXAMPLE

Calculating Distance and Displacement

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?



Strategy

To solve this problem, we need to find the difference between the final position and the initial position while taking care to note the direction on the axis. The final position is the sum of the two displacements, $\Delta \mathbf{d}_1$ and $\Delta \mathbf{d}_2$.

Solution

a. Displacement: The rider's displacement is $\Delta \mathbf{d} = \mathbf{d}_\mathrm{f} - \mathbf{d}_0 = -1 \ \ \mathrm{km}$.
b. Distance: The distance traveled is 3 km + 2 km = 5 km.
c. The magnitude of the displacement is 1 km.

Discussion

The displacement is negative because we chose east to be positive and west to be negative. We could also have described the displacement as 1 km west. When calculating displacement, the direction mattered, but when calculating distance, the direction did not matter. The problem would work the same way if the problem were in the north–south or *y*-direction.

TIPS FOR SUCCESS

Physicists like to use standard units so it is easier to compare notes. The standard units for calculations are called *SI* units (International System of Units). SI units are based on the metric system. The SI unit for displacement is the meter (m), but sometimes you will see a problem with kilometers, miles, feet, or other units of length. If one unit in a problem is an SI unit and another is not, you will need to convert all of your quantities to the same system before you can carry out the calculation.

Practice Problems

1. On an axis in which moving from right to left is positive, what is the displacement and distance of a student who walks 32 m to the right and then 17 m to the left?
   a. Displacement is -15 m and distance is -49 *m*.
   b. Displacement is -15 m and distance is 49 m.
   c. Displacement is 15 m and distance is -49 m.
   d. Displacement is 15 m and distance is 49 m.

2. Tiana jogs 1.5 km along a straight path and then turns and jogs 2.4 km in the opposite direction. She then turns back and jogs 0.7 km in the original direction. Let Tiana's original direction be the positive direction. What are the displacement and distance she jogged?
   a. Displacement is 4.6 km,and distance is -0.2 km.
   b. Displacement is -0.2 km, and distance is 4.6 km.
   c. Displacement is 4.6 km, and distance is +0.2 km.
   d. Displacement is +0.2 km, and distance is 4.6 km.

WORK IN PHYSICS

Mars Probe Explosion

Figure 2.7 The Mars Climate Orbiter disaster illustrates the importance of using the correct calculations in physics. (NASA)

Physicists make calculations all the time, but they do not always get the right answers. In 1998, NASA, the National Aeronautics and Space Administration, launched the Mars Climate Orbiter, shown in Figure 2.7, a $125-million-dollar satellite designed to monitor the Martian atmosphere. It was supposed to orbit the planet and take readings from a safe distance. The American scientists made calculations in English units (feet, inches, pounds, etc.) and forgot to convert their answers to the standard metric SI units. This was a very costly mistake. Instead of orbiting the planet as planned, the Mars Climate Orbiter ended up flying into the Martian atmosphere. The probe disintegrated. It was one of the biggest embarrassments in NASA's history.

GRASP CHECK

In 1999 the Mars Climate Orbiter crashed because calculation were performed in English units instead of SI units. At one point the orbiter was just 187,000 feet above the surface, which was too close to stay in orbit. What was the height of the orbiter at this time in kilometers? (Assume 1 meter equals 3.281 feet.)

a. 16 km
b. 18 km
c. 57 km
d. 614 km

Check Your Understanding

3. What does it mean when motion is described as relative?
   a. It means that motion of any object is described relative to the motion of Earth.
   b. It means that motion of any object is described relative to the motion of any other object.
   c. It means that motion is independent of the frame of reference.
   d. It means that motion depends on the frame of reference selected.

4. If you and a friend are standing side-by-side watching a soccer game, would you both view the motion from the same reference frame?
   a. Yes, we would both view the motion from the same reference point because both of us are at rest in Earth's frame of reference.
   b. Yes, we would both view the motion from the same reference point because both of us are observing the motion from two points on the same straight line.
   c. No, we would both view the motion from different reference points because motion is viewed from two different points; the reference frames are similar but not the same.
   d. No, we would both view the motion from different reference points because response times may be different; so, the motion observed by both of us would be different.

5. What is the difference between distance and displacement?
   a. Distance has both magnitude and direction, while displacement has magnitude but no direction.
   b. Distance has magnitude but no direction, while displacement has both magnitude and direction.
   c. Distance has magnitude but no direction, while displacement has only direction.
   d. There is no difference. Both distance and displacement have magnitude and direction.

6. Which situation correctly identifies a race car's distance traveled and the magnitude of displacement during a one-lap car race?
   a. The perimeter of the race track is the distance, and the shortest distance between the start line and the finish line is the magnitude of displacement.
   b. The perimeter of the race track is the magnitude of displacement, and the shortest distance between the start and finish line is the distance.
   c. The perimeter of the race track is both the distance and magnitude of displacement.
   d. The shortest distance between the start line and the finish line is both the distance and magnitude of displacement.

7. Why is it important to specify a reference frame when describing motion?
   a. Because Earth is continuously in motion; an object at rest on Earth will be in motion when viewed from outer space.
   b. Because the position of a moving object can be defined only when there is a fixed reference frame.

c. Because motion is a relative term; it appears differently when viewed from different reference frames.
d. Because motion is always described in Earth's frame of reference; if another frame is used, it has to be specified with each situation.

2.2 Speed and Velocity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the average speed of an object
- Relate displacement and average velocity

Section Key Terms

average speed	average velocity	instantaneous speed
instantaneous velocity	speed	velocity

Speed

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we will look at time, speed, and velocity to expand our understanding of motion.

A description of how fast or slow an object moves is its speed. **Speed** is the rate at which an object changes its location. Like distance, speed is a scalar because it has a magnitude but not a direction. Because speed is a rate, it depends on the time interval of motion. You can calculate the elapsed time or the change in time, Δt, of motion as the difference between the ending time and the beginning time

$$\Delta t = t_f - t_0.$$

The SI unit of time is the second (s), and the SI unit of speed is meters per second (m/s), but sometimes kilometers per hour (km/h), miles per hour (mph) or other units of speed are used.

When you describe an object's speed, you often describe the average over a time period. **Average speed**, v_{avg}, is the distance traveled divided by the time during which the motion occurs.

$$v_{avg} = \frac{\text{distance}}{\text{time}}$$

You can, of course, rearrange the equation to solve for either distance or time

$$\text{time} = \frac{\text{distance}}{v_{avg}}.$$

$$\text{distance} = v_{avg} \times \text{time}$$

Suppose, for example, a car travels 150 kilometers in 3.2 hours. Its average speed for the trip is

$$\begin{aligned} v_{avg} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{150 \text{ km}}{3.2 \text{ h}} \\ &= 47 \text{ km/h}. \end{aligned}$$

A car's speed would likely increase and decrease many times over a 3.2 hour trip. Its speed at a specific instant in time, however, is its **instantaneous speed**. A car's speedometer describes its instantaneous speed.

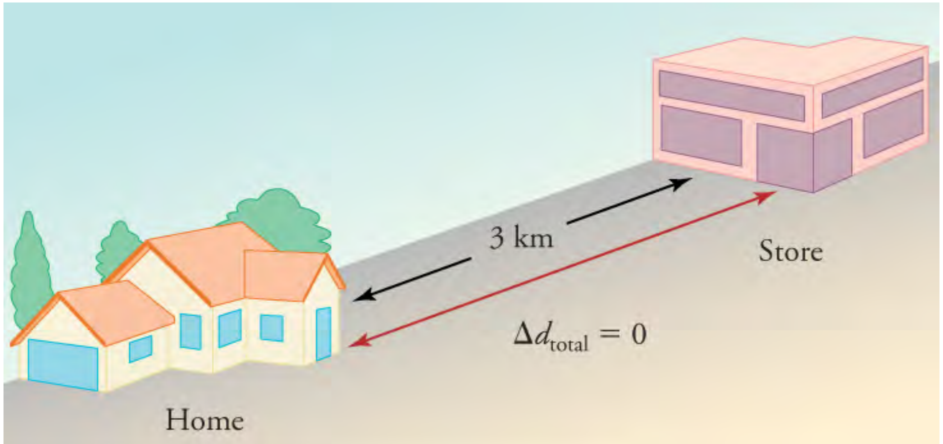


Figure 2.8 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, because there was no net change in position.

WORKED EXAMPLE

Calculating Average Speed

A marble rolls 5.2 m in 1.8 s. What was the marble's average speed?

Strategy

We know the distance the marble travels, 5.2 m, and the time interval, 1.8 s. We can use these values in the average speed equation.

Solution

$$v_{\text{avg}} = \frac{\text{distance}}{\text{time}} = \frac{5.2 \text{ m}}{1.8 \text{ s}} = 2.9 \text{ m/s}$$

Discussion

Average speed is a scalar, so we do not include direction in the answer. We can check the reasonableness of the answer by estimating: 5 meters divided by 2 seconds is 2.5 m/s. Since 2.5 m/s is close to 2.9 m/s, the answer is reasonable. This is about the speed of a brisk walk, so it also makes sense.

Practice Problems

8. A pitcher throws a baseball from the pitcher's mound to home plate in 0.46 s. The distance is 18.4 m. What was the average speed of the baseball?
   a. 40 m/s
   b. - 40 m/s
   c. 0.03 m/s
   d. 8.5 m/s

9. Cassie walked to her friend's house with an average speed of 1.40 m/s. The distance between the houses is 205 m. How long did the trip take her?
   a. 146 s
   b. 0.01 s
   c. 2.50 min
   d. 287 s

Velocity

The vector version of speed is velocity. **Velocity** describes the speed and direction of an object. As with speed, it is useful to describe either the average velocity over a time period or the velocity at a specific moment. **Average velocity** is displacement divided by the time over which the displacement occurs.

$$v_{avg} = \frac{\text{distance}}{\text{time}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_0}{t_f - t_0}$$

Velocity, like speed, has SI units of meters per second (m/s), but because it is a vector, you must also include a direction. Furthermore, the variable **v** for velocity is bold because it is a vector, which is in contrast to the variable *v* for speed which is italicized because it is a scalar quantity.

TIPS FOR SUCCESS

It is important to keep in mind that the average speed is not the same thing as the average velocity without its direction. Like we saw with displacement and distance in the last section, changes in direction over a time interval have a bigger effect on speed and velocity.

Suppose a passenger moved toward the back of a plane with an average velocity of –4 m/s. We cannot tell from the average velocity whether the passenger stopped momentarily or backed up before he got to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals such as those shown in Figure 2.9. If you consider infinitesimally small intervals, you can define **instantaneous velocity**, which is the velocity at a specific instant in time. Instantaneous velocity and average velocity are the same if the velocity is constant.

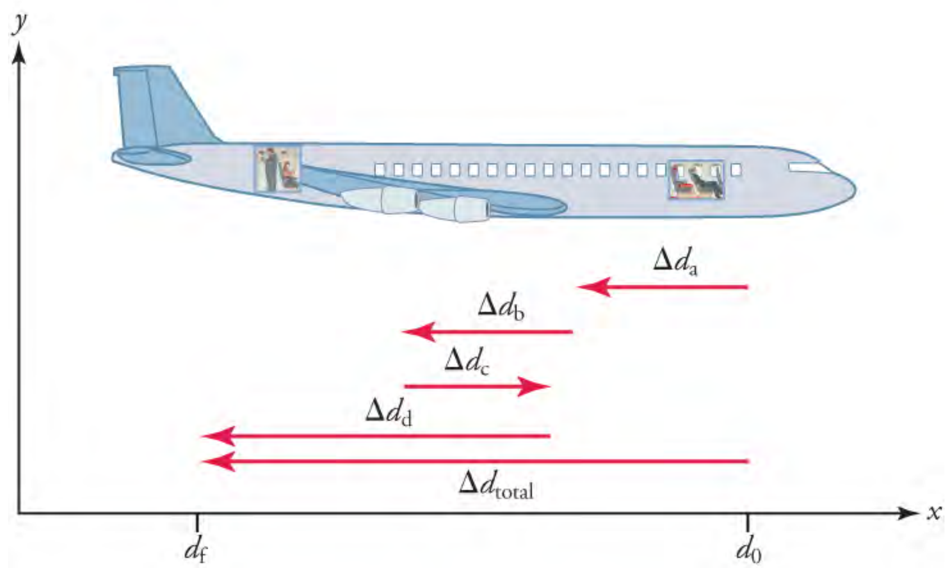


Figure 2.9 The diagram shows a more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

Earlier, you have read that distance traveled can be different than the magnitude of displacement. In the same way, speed can be different than the magnitude of velocity. For example, you drive to a store and return home in half an hour. If your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero because your displacement for the round trip is zero.

WATCH PHYSICS

Calculating Average Velocity or Speed

This video (http://openstax.org/l/28avgvelocity) reviews vectors and scalars and describes how to calculate average velocity and average speed when you know displacement and change in time. The video also reviews how to convert km/h to m/s.

Click to view content (https://www.khanacademy.org/embed_video?v=MAS6mBRZZXA)

GRASP CHECK

Which of the following fully describes a vector and a scalar quantity and correctly provides an example of each?

a. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.
b. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.
c. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.
d. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.

WORKED EXAMPLE

Calculating Average Velocity

A student has a displacement of 304 m north in 180 s. What was the student's average velocity?

Strategy

We know that the displacement is 304 m north and the time is 180 s. We can use the formula for average velocity to solve the problem.

Solution

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{304 \text{ m}}{180 \text{ s}} = 1.7 \text{ m/s north} \tag{2.1}$$

Discussion

Since average velocity is a vector quantity, you must include direction as well as magnitude in the answer. Notice, however, that the direction can be omitted until the end to avoid cluttering the problem. Pay attention to the significant figures in the problem. The distance 304 m has three significant figures, but the time interval 180 s has only two, so the quotient should have only two significant figures.

TIPS FOR SUCCESS

Note the way scalars and vectors are represented. In this book d represents distance and displacement. Similarly, v represents speed, and v represents velocity. A variable that is not bold indicates a scalar quantity, and a bold variable indicates a vector quantity. Vectors are sometimes represented by small arrows above the variable.

WORKED EXAMPLE

Solving for Displacement when Average Velocity and Time are Known

Layla jogs with an average velocity of 2.4 m/s east. What is her displacement after 46 seconds?

Strategy

We know that Layla's average velocity is 2.4 m/s east, and the time interval is 46 seconds. We can rearrange the average velocity formula to solve for the displacement.

Solution

$$\begin{aligned} \mathbf{v}_{\text{avg}} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ \Delta \mathbf{d} &= \mathbf{v}_{avg} \Delta t \\ &= (2.4 \text{ m/s})(46 \text{ s}) \\ &= 1.1 \times 10^2 \text{ m east} \end{aligned} \tag{2.2}$$

Discussion

The answer is about 110 m east, which is a reasonable displacement for slightly less than a minute of jogging. A calculator shows the answer as 110.4 m. We chose to write the answer using scientific notation because we wanted to make it clear that we only used two significant figures.

TIPS FOR SUCCESS

Dimensional analysis is a good way to determine whether you solved a problem correctly. Write the calculation using only units to be sure they match on opposite sides of the equal mark. In the worked example, you have m = (m/s)(s). Since seconds is in the denominator for the average velocity and in the numerator for the time, the unit cancels out leaving only m and, of course, m = m.

WORKED EXAMPLE

Solving for Time when Displacement and Average Velocity are Known

Phillip walks along a straight path from his house to his school. How long will it take him to get to school if he walks 428 m west with an average velocity of 1.7 m/s west?

Strategy

We know that Phillip's displacement is 428 m west, and his average velocity is 1.7 m/s west. We can calculate the time required for the trip by rearranging the average velocity equation.

Solution

$$\begin{aligned} \mathbf{v}_{\text{avg}} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ \Delta t &= \frac{\Delta \mathbf{d}}{\mathbf{v}_{\text{avg}}} \\ &= \frac{428\ \text{m}}{1.7\ \text{m/s}} \\ &= 2.5 \times 10^2\ \text{s} \end{aligned} \tag{2.3}$$

Discussion

Here again we had to use scientific notation because the answer could only have two significant figures. Since time is a scalar, the answer includes only a magnitude and not a direction.

Practice Problems

10. A trucker drives along a straight highway for 0.25 h with a displacement of 16 km south. What is the trucker's average velocity?
    a. 4 km/h north
    b. 4 km/h south
    c. 64 km/h north
    d. 64 km/h south

11. A bird flies with an average velocity of 7.5 m/s east from one branch to another in 2.4 s. It then pauses before flying with an average velocity of 6.8 m/s east for 3.5 s to another branch. What is the bird's total displacement from its starting point?
    a. 42 m west
    b. 6 m west
    c. 6 m east
    d. 42 m east

Virtual Physics

The Walking Man

In this simulation you will put your cursor on the man and move him first in one direction and then in the opposite direction. Keep the *Introduction* tab active. You can use the *Charts* tab after you learn about graphing motion later in this chapter. Carefully watch the sign of the numbers in the position and velocity boxes. Ignore the acceleration box for now. See if you can make the man's position positive while the velocity is negative. Then see if you can do the opposite.

[Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)](https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

Which situation correctly describes when the moving man's position was negative but his velocity was positive?

a. Man moving toward 0 from left of 0
b. Man moving toward 0 from right of 0
c. Man moving away from 0 from left of 0
d. Man moving away from 0 from right of 0

Check Your Understanding

12. Two runners travel along the same straight path. They start at the same time, and they end at the same time, but at the halfway mark, they have different instantaneous velocities. Is it possible for them to have the same average velocity for the trip?

a. Yes, because average velocity depends on the net or total displacement.
b. Yes, because average velocity depends on the total distance traveled.
c. No, because the velocities of both runners must remain the exactly same throughout the journey.
d. No, because the instantaneous velocities of the runners must remain same midway but can be different elsewhere.

13. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity, and under what circumstances are these two quantities the same?

a. Average speed. Both are the same when the car is traveling at a constant speed and changing direction.
b. Average speed. Both are the same when the speed is constant and the car does not change its direction.
c. Magnitude of average velocity. Both are same when the car is traveling at a constant speed.
d. Magnitude of average velocity. Both are same when the car does not change its direction.

14. Is it possible for average velocity to be negative?

a. Yes, in cases when the net displacement is negative.
b. Yes, if the body keeps changing its direction during motion.
c. No, average velocity describes only magnitude and not the direction of motion.
d. No, average velocity describes only the magnitude in the positive direction of motion.

2.3 Position vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope in position vs. time graphs
- Solve problems using position vs. time graphs

Section Key Terms

dependent variable	independent variable	tangent

Graphing Position as a Function of Time

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information, they also reveal relationships between physical quantities. In this section, we will investigate kinematics by analyzing graphs of position over time.

Graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against each other, the horizontal axis is usually considered the **independent variable**, and the vertical axis is the **dependent variable**. In algebra, you would have referred to the horizontal axis as the x-axis and the vertical axis as the y-axis. As in Figure 2.10, a straight-line graph has the general form $y = mx + b$.

Here m is the slope, defined as the rise divided by the run (as seen in the figure) of the straight line. The letter b is the y-intercept which is the point at which the line crosses the vertical, y-axis. In terms of a physical situation in the real world, these quantities will take on a specific significance, as we will see below. (Figure 2.10.)

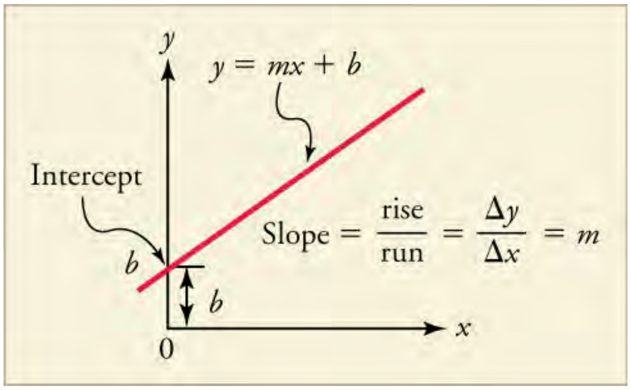


Figure 2.10 The diagram shows a straight-line graph. The equation for the straight line is y equals $mx + b$.

In physics, time is usually the independent variable. Other quantities, such as displacement, are said to depend upon it. A graph of position versus time, therefore, would have position on the vertical axis (dependent variable) and time on the horizontal axis (independent variable). In this case, to what would the slope and y-intercept refer? Let's look back at our original example when studying distance and displacement.

The drive to school was 5 km from home. Let's assume it took 10 minutes to make the drive and that your parent was driving at a constant velocity the whole time. The position versus time graph for this section of the trip would look like that shown in Figure 2.11.

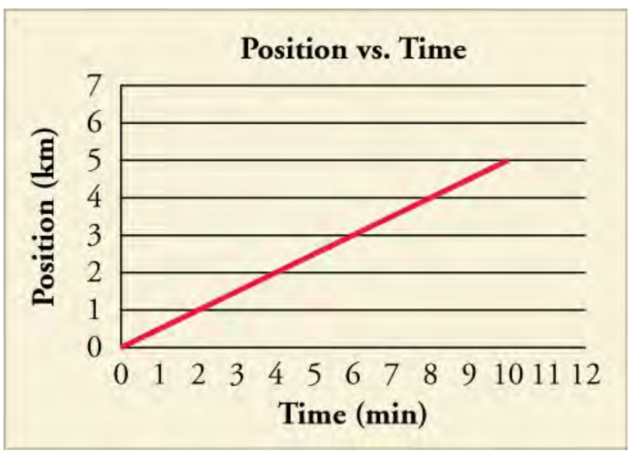


Figure 2.11 A graph of position versus time for the drive to school is shown. What would the graph look like if we added the return trip?

As we said before, $\mathbf{d}_0 = 0$ because we call home our O and start calculating from there. In Figure 2.11, the line starts at $\mathbf{d} = 0$, as well. This is the b in our equation for a straight line. Our initial position in a position versus time graph is always the place where the graph crosses the x-axis at $t = 0$. What is the slope? The *rise* is the change in position, (i.e., displacement) and the *run* is the change in time. This relationship can also be written

$$\frac{\Delta \mathbf{d}}{\Delta t}. \tag{2.4}$$

This relationship was how we defined average velocity. Therefore, the slope in a $\mathbf{d}$ versus t graph, is the average velocity.

TIPS FOR SUCCESS

Sometimes, as is the case where we graph both the trip to school and the return trip, the behavior of the graph looks different during different time intervals. If the graph looks like a series of straight lines, then you can calculate the average velocity for each time interval by looking at the slope. If you then want to calculate the average velocity for the entire trip, you can do a weighted average.

Let's look at another example. Figure 2.12 shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.

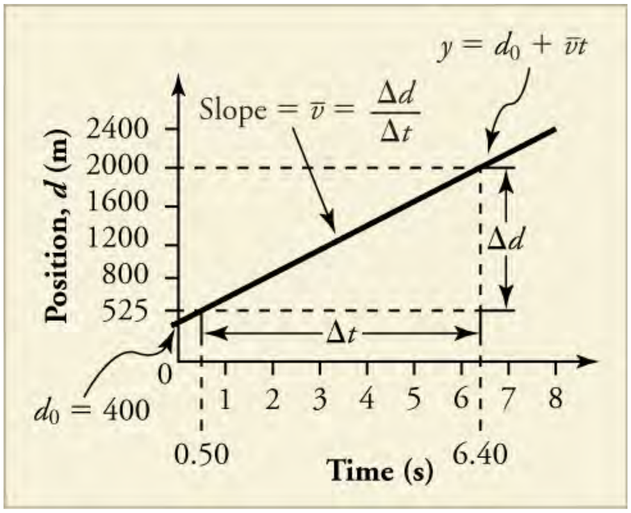


Figure 2.12 The diagram shows a graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph in Figure 2.12 is average velocity, $\mathbf{v}_{avg}$ and the intercept is displacement at time zero—that is, $\mathbf{d}_0$. Substituting these symbols into $y = mx + b$ gives

$$\mathbf{d} = \mathbf{v}t + \mathbf{d}_0 \tag{2.5}$$

or

$$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t. \tag{2.6}$$

Thus a graph of position versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation. From the figure we can see that the car has a position of 400 m at $t = 0$ s, 650 m at $t = 1.0$ s, and so on. And we can learn about the object's velocity, as well.

Snap Lab

Graphing Motion

In this activity, you will release a ball down a ramp and graph the ball's displacement vs. time.

- Choose an open location with lots of space to spread out so there is less chance for tripping or falling due to rolling balls.

- 1 ball
- 1 board
- 2 or 3 books
- 1 stopwatch
- 1 tape measure
- 6 pieces of masking tape
- 1 piece of graph paper
- 1 pencil

Procedure

1. Build a ramp by placing one end of the board on top of the stack of books. Adjust location, as necessary, until there is no obstacle along the straight line path from the bottom of the ramp until at least the next 3 m.
2. Mark distances of 0.5 m, 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp. Write the distances on the tape.

3. Have one person take the role of the experimenter. This person will release the ball from the top of the ramp. If the ball does not reach the 3.0 m mark, then increase the incline of the ramp by adding another book. Repeat this Step as necessary.
4. Have the experimenter release the ball. Have a second person, the timer, begin timing the trial once the ball reaches the bottom of the ramp and stop the timing once the ball reaches 0.5 m. Have a third person, the recorder, record the time in a data table.
5. Repeat Step 4, stopping the times at the distances of 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp.
6. Use your measurements of time and the displacement to make a position vs. time graph of the ball's motion.
7. Repeat Steps 4 through 6, with different people taking on the roles of experimenter, timer, and recorder. Do you get the same measurement values regardless of who releases the ball, measures the time, or records the result? Discuss possible causes of discrepancies, if any.

GRASP CHECK

True or False: The average speed of the ball will be less than the average velocity of the ball.

a. True
b. False

Solving Problems Using Position vs. Time Graphs

So how do we use graphs to solve for things we want to know like velocity?

WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car

Find the average velocity of the car whose position is graphed in Figure 1.13.

Strategy

The slope of a graph of d vs. t is average velocity, since slope equals rise over run.

$$\text{slope} = \frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v} \tag{2.7}$$

Since the slope is constant here, any two points on the graph can be used to find the slope.

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)
2. Substitute the **d** and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\begin{aligned} \mathbf{v} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ &= \frac{2000\ \text{m} - 525\ \text{m}}{6.4\ \text{s} - 0.50\ \text{s}}, \\ &= 250\ \text{m/s} \end{aligned} \tag{2.8}$$

Discussion

This is an impressively high land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 27 m/s or 96 km/h, but considerably shy of the record of 343 m/s or 1,234 km/h, set in 1997.

But what if the graph of the position is more complicated than a straight line? What if the object speeds up or turns around and goes backward? Can we figure out anything about its velocity from a graph of that kind of motion? Let's take another look at the jet-powered car. The graph in Figure 2.13 shows its motion as it is getting up to speed after starting at rest. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

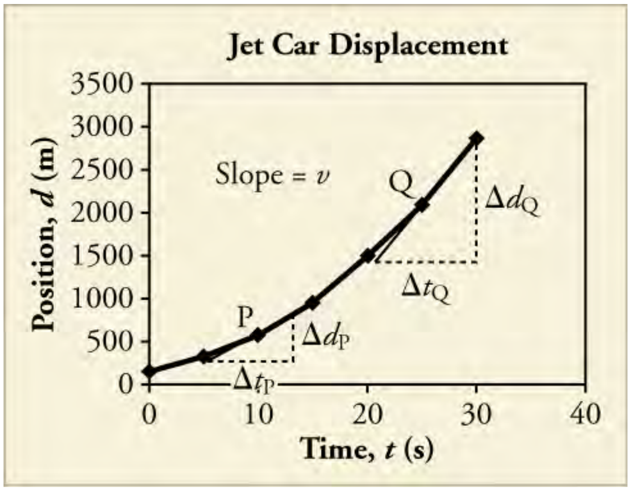


Figure 2.13 The diagram shows a graph of the position of a jet-powered car during the time span when it is speeding up. The slope of a distance versus time graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Figure 2.14 A U.S. Air Force jet car speeds down a track. (Matt Trostle, Flickr)

The graph of position versus time in Figure 2.13 is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line **tangent** to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.13. The average velocity is the net displacement divided by the time traveled.

WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car, Take Two

Calculate the instantaneous velocity of the jet car at a time of 25 s by finding the slope of the tangent line at point Q in Figure 2.13.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point.

Solution

1. Find the tangent line to the curve at $t = 25\ \text{s}$.
2. Determine the endpoints of the tangent. These correspond to a position of 1,300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, **v**.

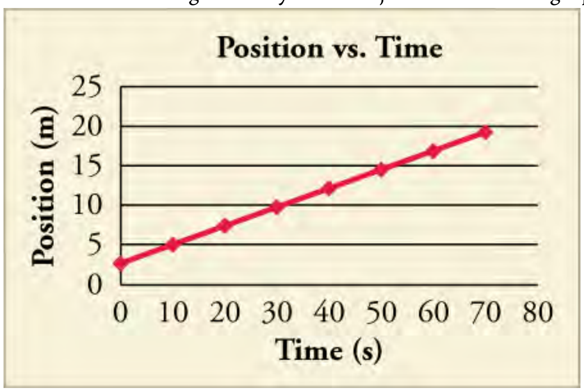
$$\begin{aligned}\text{slope} &= v_Q = \frac{\Delta d_Q}{\Delta t_Q} \\ &= \frac{(3120-1300)\ \text{m}}{(32-19)\ \text{s}} \\ &= \frac{1820\ \ \text{m}}{13\ \ \text{s}} \\ &= 140\ \ \text{m/s}\end{aligned} \tag{2.9}$$

Discussion

The entire graph of **v** versus *t* can be obtained in this fashion.

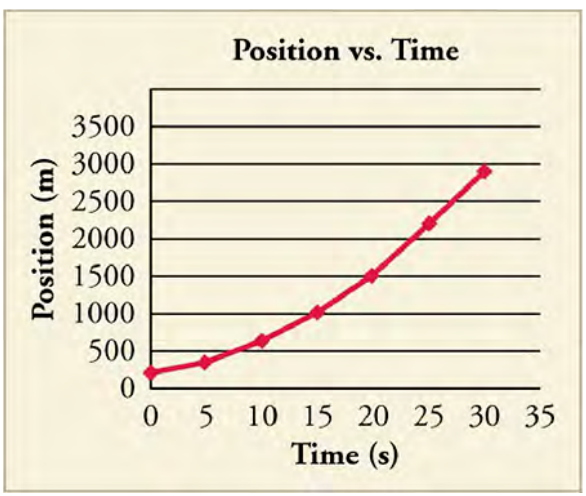
Practice Problems

15. Calculate the average velocity of the object shown in the graph below over the whole time interval.



a. 0.25 m/s
b. 0.31 m/s
c. 3.2 m/s
d. 4.00 m/s

16. True or False: By taking the slope of the curve in the graph you can verify that the velocity of the jet car is $115\ \text{m/s}$ at $t = 20\ \text{s}$.



a. True
b. False

Check Your Understanding

17. Which of the following information about motion can be determined by looking at a position vs. time graph that is a straight line?

a. frame of reference
b. average acceleration
c. velocity
d. direction of force applied

18. True or False: The position vs time graph of an object that is speeding up is a straight line.
a. True
b. False

2.4 Velocity vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope and area in velocity vs. time graphs
- Solve problems using velocity vs. time graphs

Section Key Terms

acceleration

Graphing Velocity as a Function of Time

Earlier, we examined graphs of position versus time. Now, we are going to build on that information as we look at graphs of velocity vs. time. Velocity is the rate of change of displacement. **Acceleration** is the rate of change of velocity; we will discuss acceleration more in another chapter. These concepts are all very interrelated.

Virtual Physics

Maze Game

In this simulation you will use a vector diagram to manipulate a ball into a certain location without hitting a wall. You can manipulate the ball directly with position or by changing its velocity. Explore how these factors change the motion. If you would like, you can put it on the *a* setting, as well. This is acceleration, which measures the rate of change of velocity. We will explore acceleration in more detail later, but it might be interesting to take a look at it here.

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GRASP CHECK

Click to view content (https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/#sim-maze-game)

a. The ball can be easily manipulated with displacement because the arena is a position space.
b. The ball can be easily manipulated with velocity because the arena is a position space.
c. The ball can be easily manipulated with displacement because the arena is a velocity space.
d. The ball can be easily manipulated with velocity because the arena is a velocity space.

What can we learn about motion by looking at velocity vs. time graphs? Let's return to our drive to school, and look at a graph of position versus time as shown in Figure 2.15.

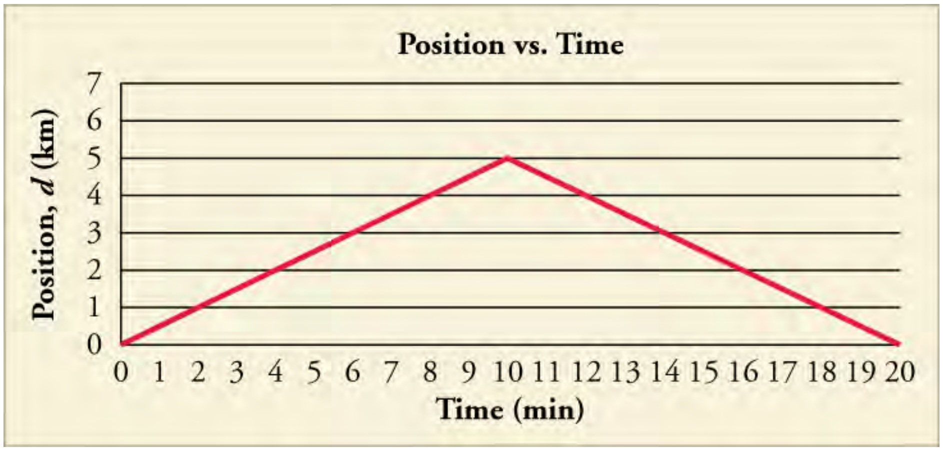

Figure 2.15 A graph of position versus time for the drive to and from school is shown.

We assumed for our original calculation that your parent drove with a constant velocity to and from school. We now know that the car could not have gone from rest to a constant velocity without speeding up. So the actual graph would be curved on either end, but let's make the same approximation as we did then, anyway.

TIPS FOR SUCCESS

It is common in physics, especially at the early learning stages, for certain things to be *neglected*, as we see here. This is because it makes the concept clearer or the calculation easier. Practicing physicists use these kinds of short-cuts, as well. It works out because usually the thing being *neglected* is small enough that it does not significantly affect the answer. In the earlier example, the amount of time it takes the car to speed up and reach its cruising velocity is very small compared to the total time traveled.

Looking at this graph, and given what we learned, we can see that there are two distinct periods to the car's motion—the way to school and the way back. The average velocity for the drive to school is 0.5 km/minute. We can see that the average velocity for the drive back is –0.5 km/minute. If we plot the data showing velocity versus time, we get another graph (Figure 2.16):

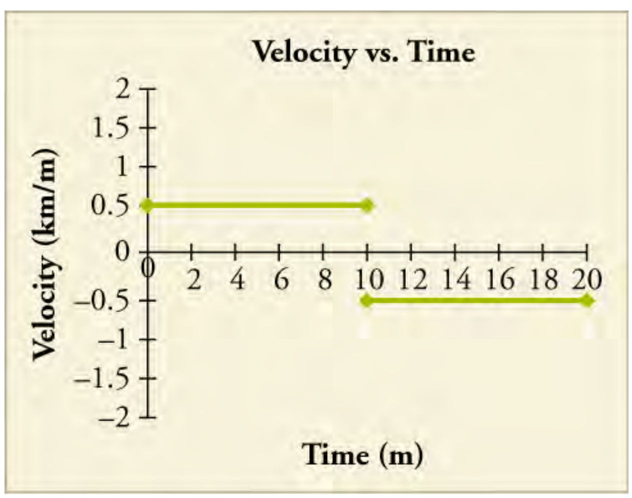

Figure 2.16 Graph of velocity versus time for the drive to and from school.

We can learn a few things. First, we can derive a **v** versus *t* graph from a **d** versus *t* graph. Second, if we have a straight-line position–time graph that is positively or negatively sloped, it will yield a horizontal velocity graph. There are a few other interesting things to note. Just as we could use a position vs. time graph to determine velocity, we can use a velocity vs. time graph to determine position. We know that $\mathbf{v} = \mathbf{d}/t$. If we use a little algebra to re-arrange the equation, we see that $\mathbf{d} = \mathbf{v} \times t$. In Figure 2.16, we have velocity on the *y*-axis and time along the *x*-axis. Let's take just the first half of the motion. We get 0.5 km/minute × 10 minutes. The units for *minutes* cancel each other, and we get 5 km, which is the displacement for the trip to school. If we calculate the same for the return trip, we get –5 km. If we add them together, we see that the net displacement for the whole trip is 0 km, which it should be because we started and ended at the same place.

TIPS FOR SUCCESS

You can treat units just like you treat numbers, so a km/km=1 (or, we say, it cancels out). This is good because it can tell us whether or not we have calculated everything with the correct units. For instance, if we end up with m × s for velocity instead of m/s, we know that something has gone wrong, and we need to check our math. This process is called dimensional analysis, and it is one of the best ways to check if your math makes sense in physics.

The area under a velocity curve represents the displacement. The velocity curve also tells us whether the car is speeding up. In our earlier example, we stated that the velocity was constant. So, the car is not speeding up. Graphically, you can see that the slope of these two lines is 0. This slope tells us that the car is not speeding up, or accelerating. We will do more with this information in a later chapter. For now, just remember that the area under the graph and the slope are the two important parts of the graph. Just like we could define a linear equation for the motion in a position vs. time graph, we can also define one for a velocity vs. time graph. As we said, the slope equals the acceleration, **a**. And in this graph, the *y*-intercept is $\mathbf{v}_0$. Thus, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$.

But what if the velocity is not constant? Let's look back at our jet-car example. At the beginning of the motion, as the car is speeding up, we saw that its position is a curve, as shown in Figure 2.17.

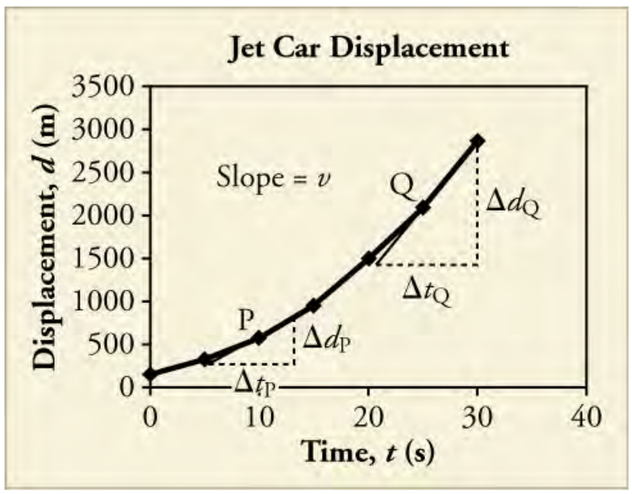

Figure 2.17 A graph is shown of the position of a jet-powered car during the time span when it is speeding up. The slope of a d vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

You do not have to do this, but you could, theoretically, take the instantaneous velocity at each point on this graph. If you did, you would get Figure 2.18, which is just a straight line with a positive slope.

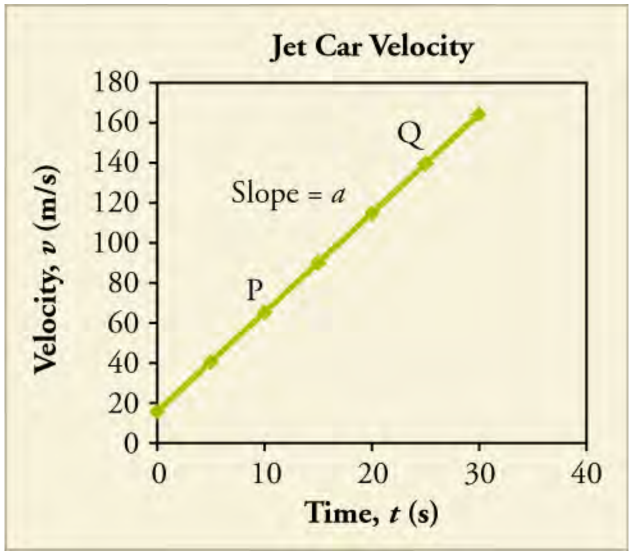

Figure 2.18 The graph shows the velocity of a jet-powered car during the time span when it is speeding up.

Again, if we take the slope of the velocity vs. time graph, we get the acceleration, the rate of change of the velocity. And, if we take the area under the slope, we get back to the displacement.

Solving Problems using Velocity–Time Graphs

Most velocity vs. time graphs will be straight lines. When this is the case, our calculations are fairly simple.

WORKED EXAMPLE

Using Velocity Graph to Calculate Some Stuff: Jet Car

Use this figure to (a) find the displacement of the jet car over the time shown (b) calculate the rate of change (acceleration) of the velocity. (c) give the instantaneous velocity at 5 s, and (d) calculate the average velocity over the interval shown.

Strategy

a. The displacement is given by finding the area under the line in the velocity vs. time graph.
b. The acceleration is given by finding the slope of the velocity graph.
c. The instantaneous velocity can just be read off of the graph.
d. To find the average velocity, recall that $\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_{\text{f}} - \mathbf{d}_0}{t_{\text{f}} - t_0}$

Solution

a.
 1. Analyze the shape of the area to be calculated. In this case, the area is made up of a rectangle between 0 and 20 m/s stretching to 30 s. The area of a rectangle is length × width. Therefore, the area of this piece is 600 m.
 2. Above that is a triangle whose base is 30 s and height is 140 m/s. The area of a triangle is 0.5 × length × width. The area of this piece, therefore, is 2,100 m.
 3. Add them together to get a net displacement of 2,700 m.

b.
 1. Take two points on the velocity line. Say, $t = 5$ s and $t = 25$ s. At $t = 5$ s, the value of $\mathbf{v} = 40$ m/s. At $t = 25$ s, $\mathbf{v} = 140$ m/s.
 2. Find the slope.
$$\begin{aligned} \mathbf{a} &= \frac{\Delta \mathbf{v}}{\Delta t} \\ &= \frac{100 \text{ m/s}}{20 \text{ s}} \\ &= 5 \text{ m/s}^2 \end{aligned}$$

c. The instantaneous velocity at $t = 5$ s, as we found in part (b) is just 40 m/s.

d.
 1. Find the net displacement, which we found in part (a) was 2,700 m.
 2. Find the total time which for this case is 30 s.
 3. Divide 2,700 m/30 s = 90 m/s.

Discussion

The average velocity we calculated here makes sense if we look at the graph. 100m/s falls about halfway across the graph and since it is a straight line, we would expect about half the velocity to be above and half below.

TIPS FOR SUCCESS

You can have negative position, velocity, and acceleration on a graph that describes the way the object is moving. You should never see a graph with negative time on an axis. Why?

Most of the velocity vs. time graphs we will look at will be simple to interpret. Occasionally, we will look at curved graphs of velocity vs. time. More often, these curved graphs occur when something is speeding up, often from rest. Let's look back at a more realistic velocity vs. time graph of the jet car's motion that takes this *speeding up* stage into account.

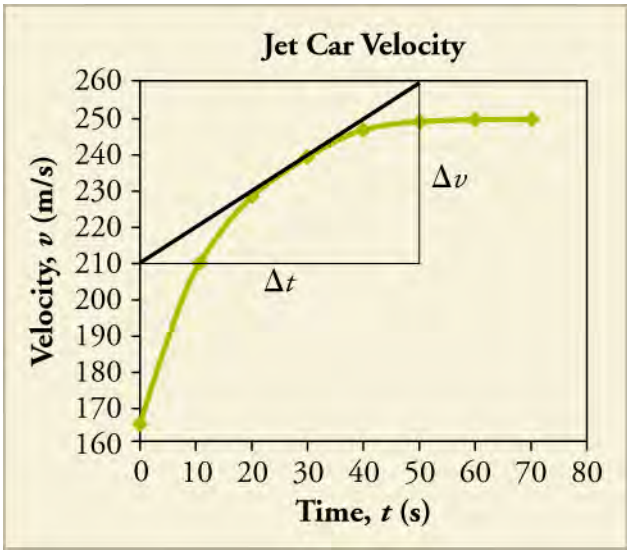

Figure 2.19 The graph shows a more accurate graph of the velocity of a jet-powered car during the time span when it is speeding up.

WORKED EXAMPLE

Using Curvy Velocity Graph to Calculate Some Stuff: jet car, Take Two

Use Figure 2.19 to (a) find the approximate displacement of the jet car over the time shown, (b) calculate the instantaneous acceleration at $t = 30$ s, (c) find the instantaneous velocity at 30 s, and (d) calculate the approximate average velocity over the interval shown.

Strategy

a. Because this graph is an undefined curve, we have to estimate shapes over smaller intervals in order to find the areas.
b. Like when we were working with a curved displacement graph, we will need to take a tangent line at the instant we are interested and use that to calculate the instantaneous acceleration.
c. The instantaneous velocity can still be read off of the graph.
d. We will find the average velocity the same way we did in the previous example.

Solution

a.
   1. This problem is more complicated than the last example. To get a good estimate, we should probably break the curve into four sections. $0 \rightarrow 10$ s, $10 \rightarrow 20$ s, $20 \rightarrow 40$ s, and $40 \rightarrow 70$ s.
   2. Calculate the bottom rectangle (common to all pieces). $165 \text{ m/s} \times 70 \text{ s} = 11{,}550$ m.
   3. Estimate a triangle at the top, and calculate the area for each section. Section 1 = 225 m; section 2 = 100 m + 450 m = 550 m; section 3 = 150 m + 1,300 m = 1,450 m; section 4 = 2,550 m.
   4. Add them together to get a net displacement of 16,325 m.

b. Using the tangent line given, we find that the slope is 1 m/s^2.

c. The instantaneous velocity at $t = 30$ s, is 240 m/s.
d. 1. Find the net displacement, which we found in part (a), was 16,325 m.
   2. Find the total time, which for this case is 70 s.
   3. Divide $\frac{16{,}325 \text{ m}}{70 \text{ s}} \sim 233 \text{ m/s}$

Discussion

This is a much more complicated process than the first problem. If we were to use these estimates to come up with the average velocity over just the first 30 s we would get about 191 m/s. By approximating that curve with a line, we get an average velocity of 202.5 m/s. Depending on our purposes and how precise an answer we need, sometimes calling a curve a straight line is a worthwhile approximation.

Practice Problems

19.

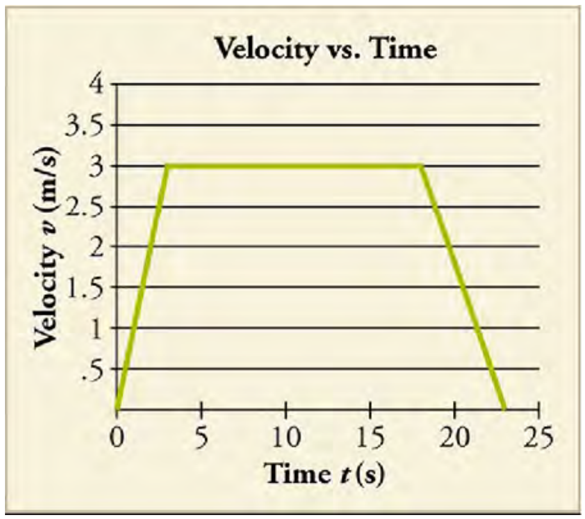

Figure 2.20

Consider the velocity vs. time graph shown below of a person in an elevator. Suppose the elevator is initially at rest. It then speeds up for 3 seconds, maintains that velocity for 15 seconds, then slows down for 5 seconds until it stops. Find the instantaneous velocity at $t = 10$ s and $t = 23$ s.

a. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 0 m/s.
b. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 3 m/s.
c. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 0 m/s.
d. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 1.5 m/s.

20.

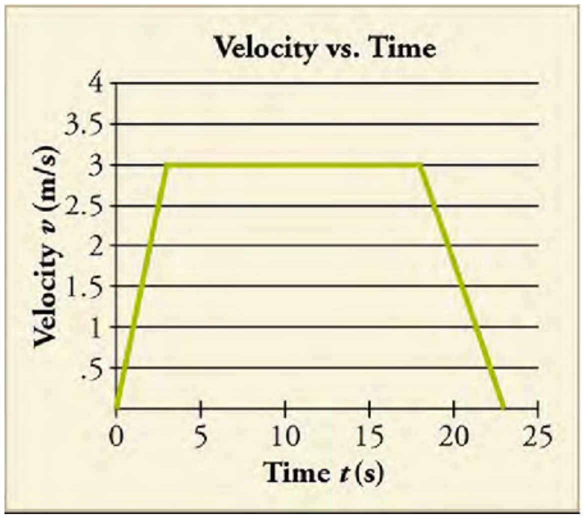

Figure 2.21

Calculate the net displacement and the average velocity of the elevator over the time interval shown.

a. Net displacement is 45 m and average velocity is 2.10 m/s.
b. Net displacement is 45 m and average velocity is 2.28 m/s.
c. Net displacement is 57 m and average velocity is 2.66 m/s.
d. Net displacement is 57 m and average velocity is 2.48 m/s.

Snap Lab

Graphing Motion, Take Two

In this activity, you will graph a moving ball's velocity vs. time.

- your graph from the earlier Graphing Motion Snap Lab!
- 1 piece of graph paper
- 1 pencil

Procedure

1. Take your graph from the earlier Graphing Motion Snap Lab! and use it to create a graph of velocity vs. time.
2. Use your graph to calculate the displacement.

GRASP CHECK

Describe the graph and explain what it means in terms of velocity and acceleration.

a. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was not accelerating.
b. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was accelerating.
c. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was not accelerating.
d. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was accelerating.

Check Your Understanding

21. What information could you obtain by looking at a velocity vs. time graph?

a. acceleration
b. direction of motion
c. reference frame of the motion

d. shortest path

22. How would you use a position vs. time graph to construct a velocity vs. time graph and vice versa?

a. Slope of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
b. Slope of position vs. time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.
c. Area of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
d. Area of position/time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.

KEY TERMS

acceleration the rate at which velocity changes

average speed distance traveled divided by time during which motion occurs

average velocity displacement divided by time over which displacement occurs

dependent variable the variable that changes as the independent variable changes

displacement the change in position of an object against a fixed axis

distance the length of the path actually traveled between an initial and a final position

independent variable the variable, usually along the horizontal axis of a graph, that does not change based on human or experimental action; in physics this is usually time

instantaneous speed speed at a specific instant in time

instantaneous velocity velocity at a specific instant in time

kinematics the study of motion without considering its causes

magnitude size or amount

position the location of an object at any particular time

reference frame a coordinate system from which the positions of objects are described

scalar a quantity that has magnitude but no direction

speed rate at which an object changes its location

tangent a line that touches another at exactly one point

vector a quantity that has both magnitude and direction

velocity the speed and direction of an object

SECTION SUMMARY

2.1 Relative Motion, Distance, and Displacement

- A description of motion depends on the reference frame from which it is described.
- The distance an object moves is the length of the path along which it moves.
- Displacement is the difference in the initial and final positions of an object.

2.2 Speed and Velocity

- Average speed is a scalar quantity that describes distance traveled divided by the time during which the motion occurs.
- Velocity is a vector quantity that describes the speed and direction of an object.
- Average velocity is displacement over the time period during which the displacement occurs. If the velocity is constant, then average velocity and instantaneous velocity are the same.

2.3 Position vs. Time Graphs

- Graphs can be used to analyze motion.
- The slope of a position vs. time graph is the velocity.
- For a straight line graph of position, the slope is the average velocity.
- To obtain the instantaneous velocity at a given moment for a curved graph, find the tangent line at that point and take its slope.

2.4 Velocity vs. Time Graphs

- The slope of a velocity vs. time graph is the acceleration.
- The area under a velocity vs. time curve is the displacement.
- Average velocity can be found in a velocity vs. time graph by taking the weighted average of all the velocities.

KEY EQUATIONS

2.1 Relative Motion, Distance, and Displacement

Displacement $\Delta \mathbf{d} = \mathbf{d_f} - \mathbf{d_0}$

2.2 Speed and Velocity

Average speed $v_{\text{avg}} = \frac{\textbf{distance}}{\textbf{time}}$

Average velocity $\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_{\text{f}} - \mathbf{d}_0}{t_{\text{f}} - t_0}$

2.3 Position vs. Time Graphs

Displacement $\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t$.

2.4 Velocity vs. Time Graphs

Velocity $\mathbf{v} = \mathbf{v}_0 + at$

Acceleration $\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$

CHAPTER 3
Acceleration

Figure 3.1 A plane slows down as it comes in for landing in St. Maarten. Its acceleration is in the opposite direction of its velocity. (Steve Conry, Flickr)

Chapter Outline

INTRODUCTION You may have heard the term *accelerator*, referring to the gas pedal in a car. When the gas pedal is pushed down, the flow of gasoline to the engine increases, which increases the car's velocity. Pushing on the gas pedal results in acceleration because the velocity of the car increases, and acceleration is defined as a change in velocity. You need two quantities to define velocity: a speed and a direction. Changing either of these quantities, or both together, changes the velocity. You may be surprised to learn that pushing on the brake pedal or turning the steering wheel also causes acceleration. The first reduces the *speed* and so changes the velocity, and the second changes the *direction* and also changes the velocity.

In fact, any change in velocity—whether positive, negative, directional, or any combination of these—is called an acceleration in physics. The plane in the picture is said to be accelerating because its velocity is decreasing as it prepares to land. To begin our study of acceleration, we need to have a clear understanding of what acceleration means.

3.1 Acceleration

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain acceleration and determine the direction and magnitude of acceleration in one dimension
- Analyze motion in one dimension using kinematic equations and graphic representations

Section Key Terms

average acceleration instantaneous acceleration negative acceleration

Defining Acceleration

Throughout this chapter we will use the following terms: *time*, *displacement*, *velocity*, and *acceleration*. Recall that each of these terms has a designated variable and SI unit of measurement as follows:

- Time: t, measured in seconds (s)
- Displacement: Δd, measured in meters (m)
- Velocity: v, measured in meters per second (m/s)
- Acceleration: a, measured in meters per second per second (m/s^2, also called meters per second squared)
- Also note the following:
  - Δ means *change in*
  - The subscript 0 refers to an initial value; sometimes subscript i is instead used to refer to initial value.
  - The subscript f refers to final value
  - A bar over a symbol, such as $\bar{a}$, means *average*

Acceleration is the change in velocity divided by a period of time during which the change occurs. The SI units of velocity are m/s and the SI units for time are s, so the SI units for acceleration are m/s^2. **Average acceleration** is given by

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

Average acceleration is distinguished from **instantaneous acceleration**, which is acceleration at a specific instant in time. The magnitude of acceleration is often not constant over time. For example, runners in a race accelerate at a greater rate in the first second of a race than during the following seconds. You do not need to know all the instantaneous accelerations at all times to calculate average acceleration. All you need to know is the change in velocity (i.e., the final velocity minus the initial velocity) and the change in time (i.e., the final time minus the initial time), as shown in the formula. Note that the average acceleration can be positive, negative, or zero. A **negative acceleration** is simply an acceleration in the negative direction.

Keep in mind that although acceleration points in the same direction as the *change* in velocity, it is not always in the direction of the velocity itself. When an object slows down, its acceleration is opposite to the direction of its velocity. In everyday language, this is called deceleration; but in physics, it is acceleration—whose direction happens to be opposite that of the velocity. For now, let us assume that motion to the right along the x-axis is *positive* and motion to the left is *negative*.

Figure 3.2 shows a car with positive acceleration in (a) and negative acceleration in (b). The arrows represent vectors showing both direction and magnitude of velocity and acceleration.

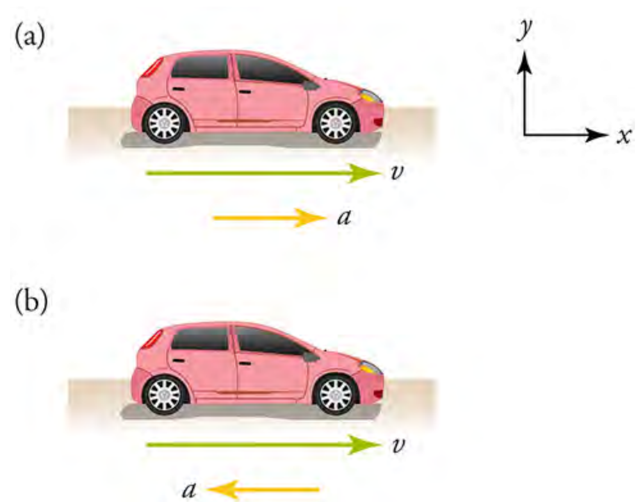

Figure 3.2 The car is speeding up in (a) and slowing down in (b).

Velocity and acceleration are both vector quantities. Recall that vectors have both magnitude and direction. An object traveling at a constant velocity—therefore having no acceleration—does accelerate if it changes direction. So, turning the steering wheel of a moving car makes the car accelerate because the velocity changes direction.

Virtual Physics

The Moving Man

With this animation in , you can produce both variations of acceleration and velocity shown in Figure 3.2, plus a few more variations. Vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Try changing acceleration from positive to negative while the man is moving. We will come back to this animation and look at the *Charts* view when we study graphical representation of motion.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

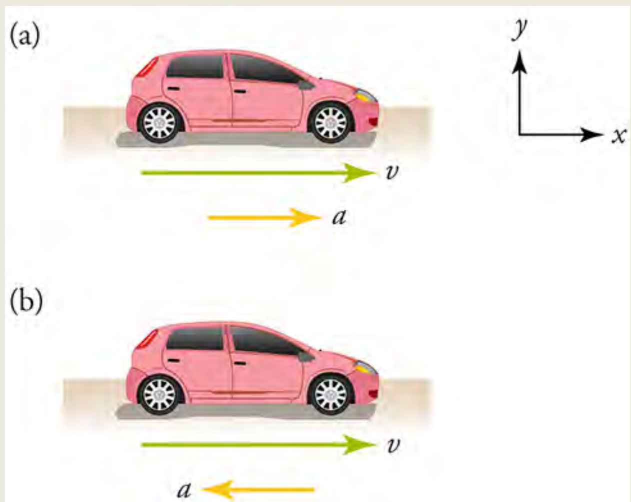

Figure 3.3

Which part, (a) or (b), is represented when the velocity vector is on the positive side of the scale and the acceleration vector is set on the negative side of the scale? What does the car's motion look like for the given scenario?

a. Part (a). The car is slowing down because the acceleration and the velocity vectors are acting in the opposite direction.
b. Part (a). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.
c. Part (b). The car is slowing down because the acceleration and velocity vectors are acting in the opposite directions.
d. Part (b). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.

Calculating Average Acceleration

Look back at the equation for average acceleration. You can see that the calculation of average acceleration involves three values: change in time, (Δt); change in velocity, (Δv); and acceleration (a).

Change in time is often stated as a time interval, and change in velocity can often be calculated by subtracting the initial velocity from the final velocity. Average acceleration is then simply change in velocity divided by change in time. Before you begin calculating, be sure that all distances and times have been converted to meters and seconds. Look at these examples of acceleration of a subway train.

WORKED EXAMPLE

An Accelerating Subway Train

A subway train accelerates from rest to 30.0 km/h in 20.0 s. What is the average acceleration during that time interval?

Strategy

Start by making a simple sketch.

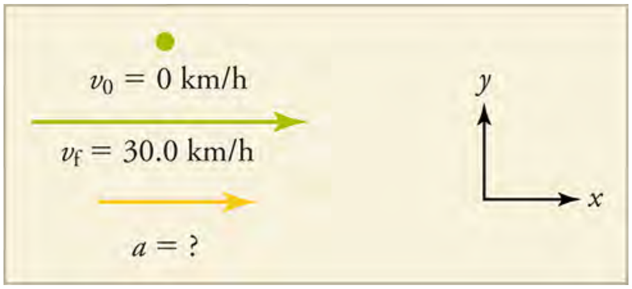

Figure 3.4

This problem involves four steps:

1. Convert to units of meters and seconds.
2. Determine the change in velocity.
3. Determine the change in time.
4. Use these values to calculate the average acceleration.

Solution

1. Identify the knowns. Be sure to read the problem for given information, which may not *look* like numbers. When the problem states that the train starts from rest, you can write down that the initial velocity is 0 m/s. Therefore, $v_0 = 0$; $v_f = 30.0$ km/h; and $\Delta t = 20.0$ s.
2. Convert the units.

$$\frac{30.0\text{ km}}{\text{h}} \times \frac{10^3\text{ m}}{1\text{ km}} \times \frac{1\text{ h}}{3600\text{ s}} = 8.333\ \frac{\text{m}}{\text{s}} \tag{3.1}$$

3. Calculate change in velocity, $\Delta v = v_f - v_0 = 8.333\text{ m/s} - 0 = +\,8.333\text{ m/s}$, where the plus sign means the change in velocity is to the right.
4. We know Δt, so all we have to do is insert the known values into the formula for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8.333\text{ m/s}}{20.00\text{ s}} = +0.417\frac{\text{m}}{\text{s}^2} \tag{3.2}$$

Discussion

The plus sign in the answer means that acceleration is to the right. This is a reasonable conclusion because the train starts from rest and ends up with a velocity directed to the right (i.e., positive). So, acceleration is in the same direction as the *change* in velocity, as it should be.

WORKED EXAMPLE

An Accelerating Subway Train

Now, suppose that at the end of its trip, the train slows to a stop in 8.00 s from a speed of 30.0 km/h. What is its average acceleration during this time?

Strategy

Again, make a simple sketch.

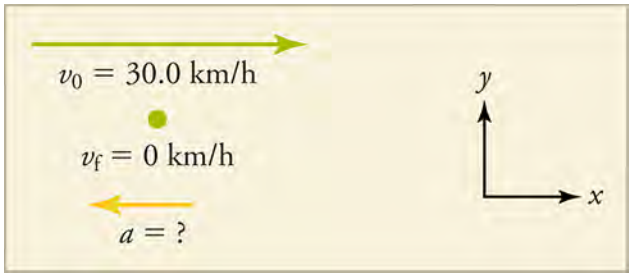

Figure 3.5

In this case, the train is decelerating and its acceleration is negative because it is pointing to the left. As in the previous example, we must find the change in velocity and change in time, then solve for acceleration.

Solution

1. Identify the knowns: $v_0 = 30.0$ km/h; $v_f = 0$; and $\Delta t = 8.00$ s.
2. Convert the units. From the first problem, we know that 30.0 km/h = 8.333 m/s.
3. Calculate change in velocity, $\Delta v = v_f - v_0 = 0 - 8.333\text{ m/s} = -8.333\text{ m/s}$, where the minus sign means that the change in velocity points to the left.
4. We know $\Delta t = 8.00$ s, so all we have to do is insert the known values into the equation for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-8.333\text{ m/s}}{8.00\text{ s}} = -1.04\frac{\text{m}}{\text{s}^2} \tag{3.3}$$

Discussion

The minus sign indicates that acceleration is to the left. This is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would reduce the velocity. Again, acceleration is in the same direction as the *change* in velocity, which is negative in this case. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

TIPS FOR SUCCESS

- It is easier to get plus and minus signs correct if you always assume that motion is away from zero and toward positive values on the *x*-axis. This way *v* always starts off being positive and points to the right. If speed is increasing, then acceleration is positive and also points to the right. If speed is decreasing, then acceleration is negative and points to the left.
- It is a good idea to carry two extra significant figures from step-to-step when making calculations. Do not round off with each step. When you arrive at the final answer, apply the rules of significant figures for the operations you carried out and round to the correct number of digits. Sometimes this will make your answer slightly more accurate.

Practice Problems

1. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

a. -0.23 m/s^2
b. -4.29 m/s^2
c. 0.23 m/s^2
d. 4.29 m/s^2

2. A women backs her car out of her garage with an acceleration of 1.40 m/s^2. How long does it take her to reach a speed of 2.00 m/s?

a. 0.70 s
b. 1.43 s
c. 2.80 s
d. 3.40 s

WATCH PHYSICS

Acceleration

This video shows the basic calculation of acceleration and some useful unit conversions.

Click to view content (https://www.khanacademy.org/embed_video?v=FOkQszg1-j8)

GRASP CHECK

Why is acceleration a vector quantity?

a. It is a vector quantity because it has magnitude as well as direction.
b. It is a vector quantity because it has magnitude but no direction.
c. It is a vector quantity because it is calculated from distance and time.
d. It is a vector quantity because it is calculated from speed and time.

GRASP CHECK

What will be the change in velocity each second if acceleration is 10 m/s/s?

a. An acceleration of $10\ \text{m/s/s}$ means that every second, the velocity increases by $10\ \text{m/s}$.
b. An acceleration of $10\ \text{m/s/s}$ means that every second, the velocity decreases by $10\ \text{m/s}$.
c. An acceleration of $10\ \text{m/s/s}$ means that every $10\ \text{seconds}$, the velocity increases by $10\ \text{m/s}$.
d. An acceleration of $10\ \text{m/s/s}$ means that every $10\ \text{seconds}$, the velocity decreases by $10\ \text{m/s}$.

Snap Lab

Measure the Acceleration of a Bicycle on a Slope

In this lab you will take measurements to determine if the acceleration of a moving bicycle is constant. If the acceleration is constant, then the following relationships hold: $\overline{v} = \frac{\Delta d}{\Delta t} = \frac{v_0+v_\mathrm{f}}{2}$ If $v_0 = 0$, then $v_\mathrm{f} = 2\overline{v}$ and $\overline{a} = \frac{v_\mathrm{f}}{\Delta t}$

You will work in pairs to measure and record data for a bicycle coasting down an incline on a smooth, gentle slope. The data will consist of distances traveled and elapsed times.

- Find an open area to minimize the risk of injury during this lab.

- stopwatch
- measuring tape
- bicycle

1. Find a gentle, paved slope, such as an incline on a bike path. The more gentle the slope, the more accurate your data will likely be.
2. Mark uniform distances along the slope, such as 5 m, 10 m, etc.
3. Determine the following roles: the bike rider, the timer, and the recorder. The recorder should create a data table to collect the distance and time data.
4. Have the rider at the starting point at rest on the bike. When the timer calls *Start*, the timer starts the stopwatch and the rider begins coasting down the slope on the bike without pedaling.
5. Have the timer call out the elapsed times as the bike passes each marked point. The recorder should record the times in the data table. It may be necessary to repeat the process to practice roles and make necessary adjustments.
6. Once acceptable data has been recorded, switch roles. Repeat Steps 3–5 to collect a second set of data.
7. Switch roles again to collect a third set of data.
8. Calculate average acceleration for each set of distance-time data. If your result for $\overline{a}$ is not the same for different pairs of Δv and Δt, then acceleration is not constant.
9. Interpret your results.

GRASP CHECK

If you graph the average velocity (y-axis) vs. the elapsed time (x-axis), what would the graph look like if acceleration is uniform?

a. a horizontal line on the graph
b. a diagonal line on the graph
c. an upward-facing parabola on the graph
d. a downward-facing parabola on the graph

Check Your Understanding

3. What are three ways an object can accelerate?
   a. By speeding up, maintaining constant velocity, or changing direction
   b. By speeding up, slowing down, or changing direction
   c. By maintaining constant velocity, slowing down, or changing direction
   d. By speeding up, slowing down, or maintaining constant velocity

4. What is the difference between average acceleration and instantaneous acceleration?
   a. Average acceleration is the change in displacement divided by the elapsed time; instantaneous acceleration is the acceleration at a given point in time.
   b. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in displacement divided by the elapsed time.
   c. Average acceleration is the change in velocity divided by the elapsed time; instantaneous acceleration is acceleration at a given point in time.
   d. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in velocity divided by the elapsed time.

5. What is the rate of change of velocity called?
   a. Time
   b. Displacement
   c. Velocity
   d. Acceleration

3.2 Representing Acceleration with Equations and Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the kinematic equations related to acceleration and illustrate them with graphs
- Apply the kinematic equations and related graphs to problems involving acceleration

Section Key Terms

acceleration due to gravity	kinematic equations	uniform acceleration

How the Kinematic Equations are Related to Acceleration

We are studying concepts related to motion: time, displacement, velocity, and especially acceleration. We are only concerned with motion in one dimension. The **kinematic equations** apply to conditions of constant acceleration and show how these concepts are related. **Constant acceleration** is acceleration that does not change over time. The first kinematic equation relates displacement d, average velocity $\overline{v}$, and time t.

$$d = d_0 + \overline{v}t \tag{3.4}$$

The initial displacement d_0 is often 0, in which case the equation can be written as $\overline{v} = \frac{d}{t}$

This equation says that average velocity is displacement per unit time. We will express velocity in meters per second. If we graph displacement versus time, as in Figure 3.6, the slope will be the velocity. Whenever a rate, such as velocity, is represented graphically, time is usually taken to be the independent variable and is plotted along the *x* axis.

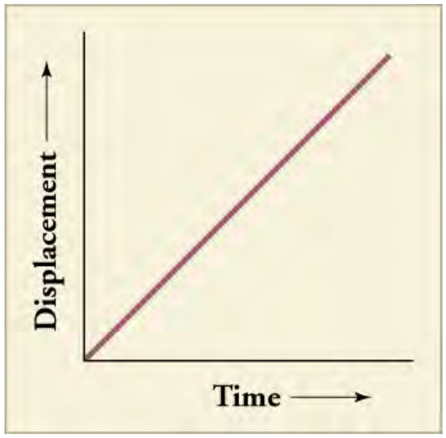

Figure 3.6 The slope of displacement versus time is velocity.

The second kinematic equation, another expression for average velocity $\overline{v}$, is simply the initial velocity plus the final velocity divided by two.

$$\overline{v} = \frac{v_0 + v_\mathrm{f}}{2} \tag{3.5}$$

Now we come to our main focus of this chapter; namely, the kinematic equations that describe motion with constant acceleration. In the third kinematic equation, acceleration is the rate at which velocity increases, so velocity at any point equals initial velocity plus acceleration multiplied by time

$$v = v_0 + at \text{ Also, if we start from rest } (v_0 = 0 \text{ }), \text{ we can write } a = \frac{v}{t} \tag{3.6}$$

Note that this third kinematic equation does not have displacement in it. Therefore, if you do not know the displacement and are not trying to solve for a displacement, this equation might be a good one to use.

The third kinematic equation is also represented by the graph in Figure 3.7.

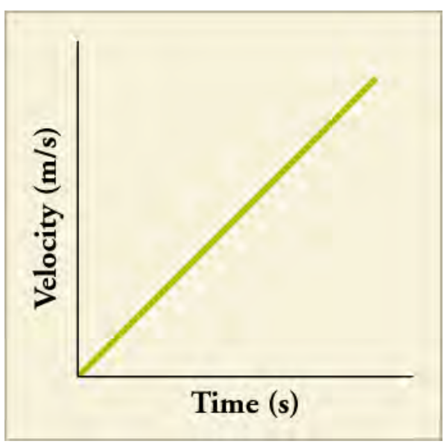

Figure 3.7 The slope of velocity versus time is acceleration.

The fourth kinematic equation shows how displacement is related to acceleration

$$d = d_0 + v_0 t + \frac{1}{2}at^2. \tag{3.7}$$

When starting at the origin, $d_0 = 0$ and, when starting from rest, $v_0 = 0$, in which case the equation can be written as

$$a = \frac{2d}{t^2}.$$

This equation tells us that, for constant acceleration, the slope of a plot of $2d$ versus t^2 is acceleration, as shown in Figure 3.8.

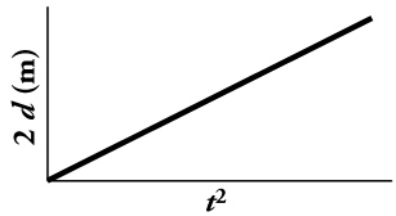

Figure 3.8 When acceleration is constant, the slope of $2d$ versus t^2 gives the acceleration.

The fifth kinematic equation relates velocity, acceleration, and displacement

$$v^2 = v_0^2 + 2a\,(d - d_0)\,. \qquad (3.8)$$

This equation is useful for when we do not know, or do not need to know, the time.

When starting from rest, the fifth equation simplifies to

$$a = \frac{v^2}{2d}.$$

According to this equation, a graph of velocity squared versus twice the displacement will have a slope equal to acceleration.

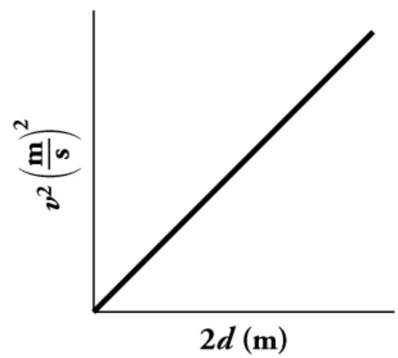

Figure 3.9

Note that, in reality, knowns and unknowns will vary. Sometimes you will want to rearrange a kinematic equation so that the knowns are the values on the axes and the unknown is the slope. Sometimes the intercept will not be at the origin (0,0). This will happen when v_0 or d_0 is not zero. This will be the case when the object of interest is already in motion, or the motion begins at some point other than at the origin of the coordinate system.

Virtual Physics

The Moving Man (Part 2)

Look at the Moving Man simulation again and this time use the *Charts* view. Again, vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Observe how the graphs of position, velocity, and acceleration vary with time. Note which are linear plots and which are not.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

On a velocity versus time plot, what does the slope represent?

a. Acceleration

b. Displacement
c. Distance covered
d. Instantaneous velocity

GRASP CHECK

On a position versus time plot, what does the slope represent?

a. Acceleration
b. Displacement
c. Distance covered
d. Instantaneous velocity

The kinematic equations are applicable when you have constant acceleration.

1. $d = d_0 + \bar{v}t$, or $\bar{v} = \frac{d}{t}$ when $d_0 = 0$
2. $\bar{v} = \frac{v_0 + v_f}{2}$
3. $v = v_0 + at$, or $a = \frac{v}{t}$ when $v_0 = 0$
4. $d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$
5. $v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$

Applying Kinematic Equations to Situations of Constant Acceleration

Problem-solving skills are essential to success in a science and life in general. The ability to apply broad physical principles, which are often represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Essential analytical skills will be developed by solving problems in this text and will be useful for understanding physics and science in general throughout your life.

Problem-Solving Steps

While no single step-by-step method works for every problem, the following general procedures facilitate problem solving and make the answers more meaningful. A certain amount of creativity and insight are required as well.

1. *Examine the situation to determine which physical principles are involved. It is vital to draw a simple sketch at the outset.* Decide which direction is positive and note that on your sketch.
2. *Identify the knowns: Make a list of what information is given or can be inferred from the problem statement.* Remember, not all given information will be in the form of a number with units in the problem. If something starts *from rest*, we know the initial velocity is zero. If something *stops*, we know the final velocity is zero.
3. *Identify the unknowns: Decide exactly what needs to be determined in the problem.*
4. *Find an equation or set of equations that can help you solve the problem.* Your list of knowns and unknowns can help here. For example, if time is not needed or not given, then the fifth kinematic equation, which does not include time, could be useful.
5. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made.
6. *Check the answer to see if it is reasonable: Does it make sense?* This final step is extremely important because the goal of physics is to accurately describe nature. To see if the answer is reasonable, check its magnitude, its sign, and its units. Are the significant figures correct?

Summary of Problem Solving

- Determine the knowns and unknowns.
- Find an equation that expresses the unknown in terms of the knowns. More than one unknown means more than one equation is needed.
- Solve the equation or equations.

- Be sure units and significant figures are correct.
- Check whether the answer is reasonable.

FUN IN PHYSICS

Drag Racing

Figure 3.10 Smoke rises from the tires of a dragster at the beginning of a drag race. (Lt. Col. William Thurmond. Photo courtesy of U.S. Army.)

The object of the sport of drag racing is acceleration. Period! The races take place from a standing start on a straight one-quarter-mile (402 m) track. Usually two cars race side by side, and the winner is the driver who gets the car past the quarter-mile point first. At the finish line, the cars may be going more than 300 miles per hour (134 m/s). The driver then deploys a parachute to bring the car to a stop because it is unsafe to brake at such high speeds. The cars, called dragsters, are capable of accelerating at 26 m/s^2. By comparison, a typical sports car that is available to the general public can accelerate at about 5 m/s^2.

Several measurements are taken during each drag race:

- Reaction time is the time between the starting signal and when the front of the car crosses the starting line.
- Elapsed time is the time from when the vehicle crosses the starting line to when it crosses the finish line. The record is a little over 3 s.
- Speed is the average speed during the last 20 m before the finish line. The record is a little under 400 mph.

The video shows a race between two dragsters powered by jet engines. The actual race lasts about four seconds and is near the end of the video (https://openstax.org/l/28dragsters) .

GRASP CHECK

A dragster crosses the finish line with a velocity of 140 m/s. Assuming the vehicle maintained a constant acceleration from start to finish, what was its average velocity for the race?

a. 0 m/s
b. 35 m/s
c. 70 m/s
d. 140 m/s

WORKED EXAMPLE

Acceleration of a Dragster

The time it takes for a dragster to cross the finish line is unknown. The dragster accelerates from rest at 26 m/s^2 for a quarter mile (0.250 mi). What is the final velocity of the dragster?

Strategy

The equation $v^2 = v_0^2 + 2a(d - d_0)$ is ideally suited to this task because it gives the velocity from acceleration and displacement, without involving the time.

Solution

1. Convert miles to meters.

$$(0.250 \text{ mi}) \times \frac{1609 \text{ m}}{1 \text{ mi}} = 402 \text{ m} \tag{3.9}$$

2. Identify the known values. We know that $v_0 = 0$ since the dragster starts from rest, and we know that the distance traveled, $d - d_0$ is 402 m. Finally, the acceleration is constant at $a = 26.0 \text{ m/s}^2$.
3. Insert the knowns into the equation $v^2 = v_0^2 + 2a(d - d_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0 \frac{\text{m}}{\text{s}^2}\right)(402 \text{ m}) = 2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2} \tag{3.10}$$

Taking the square root gives us $v = \sqrt{2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2}} = 145 \frac{\text{m}}{\text{s}}$.

Discussion

145 m/s is about 522 km/hour or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values. We took the positive value because we know that the velocity must be in the same direction as the acceleration for the answer to make physical sense.

An examination of the equation $v^2 = v_0^2 + 2a(d - d_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on the magnitude of the acceleration and the distance over which it applies.
- For a given acceleration, a car that is going twice as fast does not stop in twice the distance—it goes much further before it stops. This is why, for example, we have reduced speed zones near schools.

Practice Problems

6. Dragsters can reach a top speed of 145 m/s in only 4.45 s. Calculate the average acceleration for such a dragster.
   a. -32.6 m/s^2
   b. 0 m/s^2
   c. 32.6 m/s^2
   d. 145 m/s^2

7. An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s^2. Assuming she can maintain that acceleration, what is her speed 2.40 s later?
   a. 4.50 m/s
   b. 10.8 m/s
   c. 19.6 m/s
   d. 44.1 m/s

Constant Acceleration

In many cases, acceleration is not uniform because the force acting on the accelerating object is not constant over time. A situation that gives constant acceleration is the acceleration of falling objects. When air resistance is not a factor, all objects near Earth's surface fall with an acceleration of about 9.80 m/s^2. Although this value decreases slightly with increasing altitude, it may be assumed to be essentially constant. The value of 9.80 m/s^2 is labeled g and is referred to as **acceleration due to gravity**. Gravity is the force that causes nonsupported objects to accelerate downward—or, more precisely, toward the center of Earth. The magnitude of this force is called the weight of the object and is given by mg where m is the mass of the object (in kg). In places other than on Earth, such as the Moon or on other planets, g is not 9.80 m/s^2, but takes on other values. When using g for the acceleration a in a kinematic equation, it is usually given a negative sign because the acceleration due to gravity is downward.

WORK IN PHYSICS

Effects of Rapid Acceleration

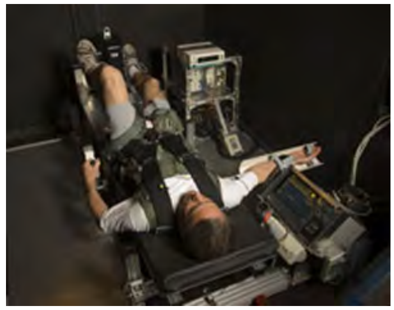

Figure 3.11 Astronauts train using G Force Simulators. (NASA)

When in a vehicle that accelerates rapidly, you experience a force on your entire body that accelerates your body. You feel this force in automobiles and slightly more on amusement park rides. For example, when you ride in a car that turns, the car applies a force on your body to make you accelerate in the direction in which the car is turning. If enough force is applied, you will accelerate at 9.80 m/s^2. This is the same as the acceleration due to gravity, so this force is called one G.

One G is the force required to accelerate an object at the acceleration due to gravity at Earth's surface. Thus, one G for a paper cup is much less than one G for an elephant, because the elephant is much more massive and requires a greater force to make it accelerate at 9.80 m/s^2. For a person, a G of about 4 is so strong that his or her face will distort as the bones accelerate forward through the loose flesh. Other symptoms at extremely high Gs include changes in vision, loss of consciousness, and even death. The space shuttle produces about 3 Gs during takeoff and reentry. Some roller coasters and dragsters produce forces of around 4 Gs for their occupants. A fighter jet can produce up to 12 Gs during a sharp turn.

Astronauts and fighter pilots must undergo G-force training in simulators. [The video (https://www.youtube.com/watch?v=n-8QHOUWECU)](https://www.youtube.com/watch?v=n-8QHOUWECU) shows the experience of several people undergoing this training.

People, such as astronauts, who work with G forces must also be trained to experience zero G—also called free fall or weightlessness—which can cause queasiness. NASA has an aircraft that allows it occupants to experience about 25 s of free fall. The aircraft is nicknamed the *Vomit Comet*.

GRASP CHECK

A common way to describe acceleration is to express it in multiples of g, Earth's gravitational acceleration. If a dragster accelerates at a rate of 39.2 m/s^2, how many g's does the driver experience?

a. 1.5 g
b. 4.0 g
c. 10.5 g
d. 24.5 g

WORKED EXAMPLE

Falling Objects

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity v_0 of 13 m/s.
(a) Calculate the position and velocity of the rock at 1.00, 2.00, and 3.00 seconds after it is thrown. Ignore the effect of air resistance.

Strategy

Sketch the initial velocity and acceleration vectors and the axes.

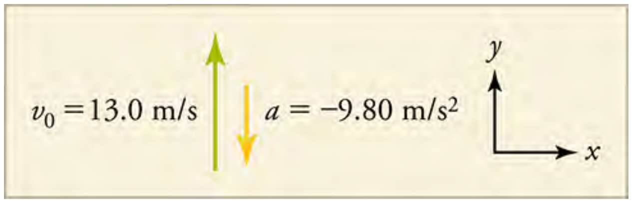

Figure 3.12 Initial conditions for rock thrown straight up.

List the knowns: time $t = 1.00$ s, 2.00 s, and 3.00 s; initial velocity $v_0 = 13$ m/s; acceleration $a = g = -9.80$ m/s^2; and position $y_0 = 0$ m

List the unknowns: y_1, y_2, and y_3; v_1, v_2, and v_3—where 1, 2, 3 refer to times 1.00 s, 2.00 s, and 3.00 s

Choose the equations.

$$d = d_0 + v_0 t + \frac{1}{2}at^2 \text{ becomes } y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{3.11}$$

$$v = v_0 + at \text{ becomes } v = v_0 + -gt \tag{3.12}$$

These equations describe the unknowns in terms of knowns only.

Solution

$$\begin{aligned}
y_1 &= 0 + (13.0 \text{ m/s})\,(1.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(1.00 \text{ s})^2}{2} = 8.10 \text{ m} \\
y_2 &= 0 + (13.0 \text{ m/s})\,(2.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(2.00 \text{ s})^2}{2} = 6.40 \text{ m} \\
y_3 &= 0 + (13.0 \text{ m/s})\,(3.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(3.00 \text{ s})^2}{2} = -5.10 \text{ m} \\
v_1 &= 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(1.00 \text{ s}) = 3.20 \text{ m/s} \\
v_2 &= 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(2.00 \text{ s}) = -6.60 \text{ m/s} \\
v_3 &= 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(3.00 \text{ s}) = -16.4 \text{ m/s}
\end{aligned}$$

Discussion

The first two positive values for y show that the rock is still above the edge of the cliff, and the third negative y value shows that it has passed the starting point and is below the cliff. Remember that we set *up* to be positive. Any position with a positive value is above the cliff, and any velocity with a positive value is an upward velocity. The first value for v is positive, so the rock is still on the way up. The second and third values for v are negative, so the rock is on its way down.

(b) Make graphs of position versus time, velocity versus time, and acceleration versus time. Use increments of 0.5 s in your graphs.

Strategy

Time is customarily plotted on the *x*-axis because it is the independent variable. Position versus time will not be linear, so calculate points for 0.50 s, 1.50 s, and 2.50 s. This will give a curve closer to the true, smooth shape.

Solution

The three graphs are shown in Figure 3.13.

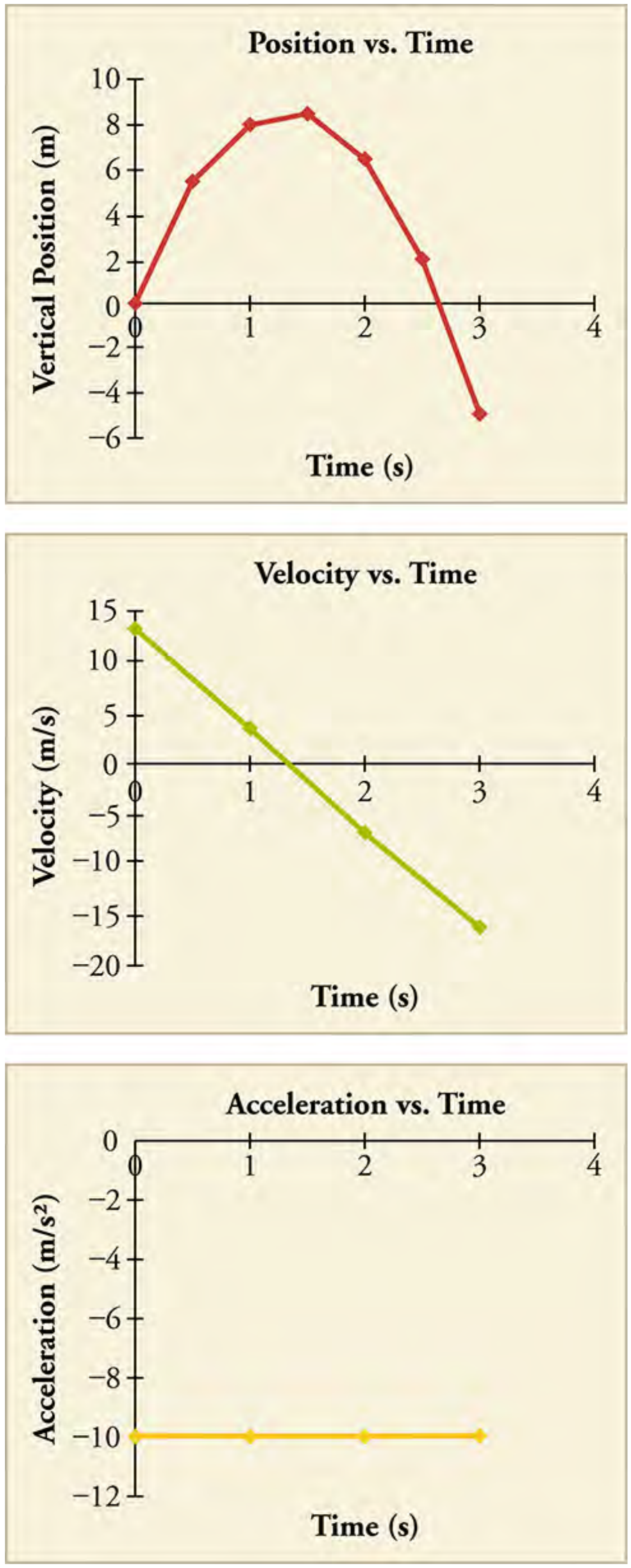

Figure 3.13

Discussion

- *y* vs. *t* does *not* represent the two-dimensional parabolic path of a trajectory. The path of the rock is straight up and straight down. The slope of a line tangent to the curve at any point on the curve equals the velocity at that point—i.e., the instantaneous velocity.

- Note that the v vs. t line crosses the vertical axis at the initial velocity and crosses the horizontal axis at the time when the rock changes direction and begins to fall back to Earth. This plot is linear because acceleration is constant.
- The a vs. t plot also shows that acceleration is constant; that is, it does not change with time.

Practice Problems

8. A cliff diver pushes off horizontally from a cliff and lands in the ocean 2.00 s later. How fast was he going when he entered the water?
   a. 0 m/s
   b. 19.0 m/s
   c. 19.6 m/s
   d. 20.0 m/s

9. A girl drops a pebble from a high cliff into a lake far below. She sees the splash of the pebble hitting the water 2.00 s later. How fast was the pebble going when it hit the water?
   a. 9.80 m/s
   b. 10.0 m/s
   c. 19.6 m/s
   d. 20.0 m/s

Check Your Understanding

10. Identify the four variables found in the kinematic equations.
   a. Displacement, Force, Mass, and Time
   b. Acceleration, Displacement, Time, and Velocity
   c. Final Velocity, Force, Initial Velocity, and Mass
   d. Acceleration, Final Velocity, Force, and Initial Velocity

11. Which of the following steps is always required to solve a kinematics problem?
   a. Find the force acting on the body.
   b. Find the acceleration of a body.
   c. Find the initial velocity of a body.
   d. Find a suitable kinematic equation and then solve for the unknown quantity.

12. Which of the following provides a correct answer for a problem that can be solved using the kinematic equations?
   a. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 8 m/s.
   b. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 16 m/s.
   c. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 2 m/s^2.
   d. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 0.5 m/s^2.

KEY TERMS

acceleration due to gravity acceleration of an object that is subject only to the force of gravity; near Earth's surface this acceleration is 9.80 m/s^2

average acceleration change in velocity divided by the time interval over which it changed

constant acceleration acceleration that does not change with respect to time

instantaneous acceleration rate of change of velocity at a specific instant in time

kinematic equations the five equations that describe motion in terms of time, displacement, velocity, and acceleration

negative acceleration acceleration in the negative direction

SECTION SUMMARY

3.1 Acceleration

- Acceleration is the rate of change of velocity and may be negative or positive.
- Average acceleration is expressed in m/s^2 and, in one dimension, can be calculated using $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$.

3.2 Representing Acceleration with Equations and Graphs

- The kinematic equations show how time, displacement, velocity, and acceleration are related for objects in motion.
- In general, kinematic problems can be solved by identifying the kinematic equation that expresses the unknown in terms of the knowns.
- Displacement, velocity, and acceleration may be displayed graphically versus time.

KEY EQUATIONS

3.1 Acceleration

Average acceleration	$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$

3.2 Representing Acceleration with Equations and Graphs

Average velocity	$d = d_0 + \overline{v}t$, or $\overline{v} = \frac{d}{t}$ when $d_0 = 0$
Average velocity	$\overline{v} = \frac{v_0 + v_f}{2}$
Velocity	$v = v_0 + at$, or when $v_0 = 0$
Displacement	$d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$
Acceleration	$v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{v^2}{2d}$ when $d_0 = 0$ and $v_0 = 0$

CHAPTER REVIEW

Concept Items

3.1 Acceleration

1. How can you use the definition of acceleration to explain the units in which acceleration is measured?
  a. Acceleration is the rate of change of velocity. Therefore, its unit is m/s^2.
  b. Acceleration is the rate of change of displacement. Therefore, its unit is m/s.
  c. Acceleration is the rate of change of velocity. Therefore, its unit is m^2/s.
  d. Acceleration is the rate of change of displacement. Therefore, its unit is m^2/s.

2. What are the SI units of acceleration?
  a. m^2/s
  b. cm^2/s
  c. m/s^2
  d. cm/s^2

3. Which of the following statements explains why a racecar going around a curve is accelerating, even if the speed is constant?
  a. The car is accelerating because the magnitude as well as the direction of velocity is changing.
  b. The car is accelerating because the magnitude of velocity is changing.
  c. The car is accelerating because the direction of velocity is changing.

CHAPTER 4
Forces and Newton's Laws of Motion

Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. (Credit: Jin Jang)

Chapter Outline

INTRODUCTION Isaac Newton (1642–1727) was a natural philosopher; a great thinker who combined science and philosophy to try to explain the workings of nature on Earth and in the universe. His laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance period of history to the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. Drawing upon earlier work by scientists Galileo Galilei and Johannes Kepler, Newton's laws of motion allowed motion on Earth and in space to be predicted mathematically. In this chapter you will learn about force as well as Newton's first, second, and third laws of motion.

4.1 Force

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Differentiate between force, net force, and dynamics
- Draw a free-body diagram

Section Key Terms

dynamics	external force	force
free-body diagram	net external force	net force

Defining Force and Dynamics

Force is the cause of motion, and motion draws our attention. Motion itself can be beautiful, such as a dolphin jumping out of the water, the flight of a bird, or the orbit of a satellite. The study of motion is called kinematics, but kinematics describes only the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws describe the way objects speed up, slow down, stay in motion, and interact with other objects. They are also universal laws: they apply everywhere on Earth as well as in space.

A force pushes or pulls an object. The object being moved by a force could be an inanimate object, a table, or an animate object, a person. The pushing or pulling may be done by a person, or even the gravitational pull of Earth. Forces have different magnitudes and directions; this means that some forces are stronger than others and can act in different directions. For example, a cannon exerts a strong force on the cannonball that is launched into the air. In contrast, a mosquito landing on your arm exerts only a small force on your arm.

When multiple forces act on an object, the forces combine. Adding together all of the forces acting on an object gives the total force, or **net force**. An **external force** is a force that acts on an object within the system *from outside* the system. This type of force is different than an internal force, which acts between two objects that are both within the system. **The net external force** combines these two definitions; it is the total combined external force. We discuss further details about net force, external force, and net external force in the coming sections.

In mathematical terms, two forces acting in opposite directions have opposite *signs* (positive or negative). By convention, the negative sign is assigned to any movement to the left or downward. If two forces pushing in opposite directions are added together, the larger force will be somewhat canceled out by the smaller force pushing in the opposite direction. It is important to be consistent with your chosen coordinate system within a problem; for example, if negative values are assigned to the downward direction for velocity, then distance, force, and acceleration should also be designated as being negative in the downward direction.

Free-Body Diagrams and Examples of Forces

For our first example of force, consider an object hanging from a rope. This example gives us the opportunity to introduce a useful tool known as a **free-body diagram**. A free-body diagram represents the object being acted upon—that is, the free body—as a single point. Only the forces acting *on* the body (that is, external forces) are shown and are represented by vectors (which are drawn as arrows). These forces are the only ones shown because only external forces acting on the body affect its motion. We can ignore any internal forces within the body because they cancel each other out, as explained in the section on Newton's third law of motion. Free-body diagrams are very useful for analyzing forces acting on an object.

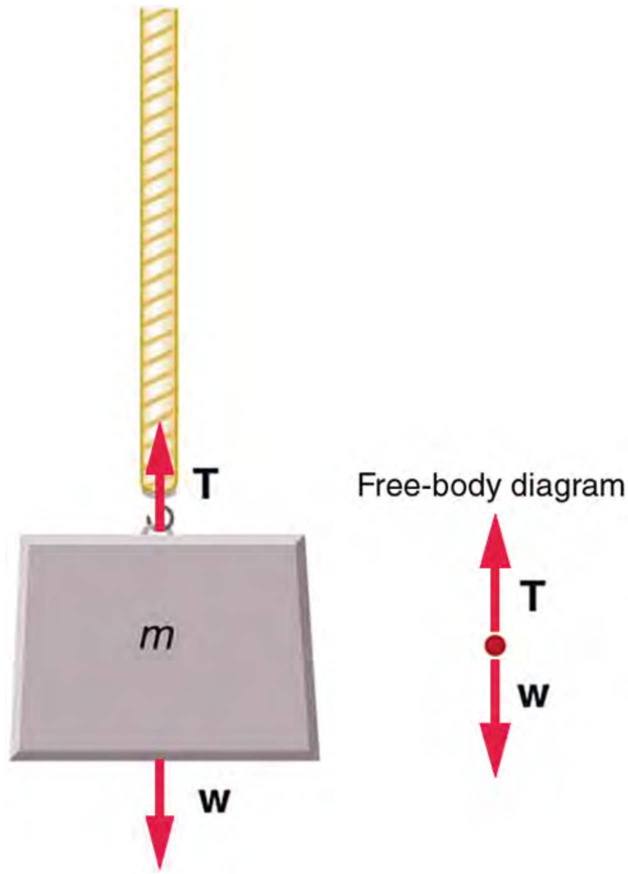

Figure 4.2 An object of mass, *m*, is held up by the force of tension.

Figure 4.2 shows the force of tension in the rope acting in the upward direction, opposite the force of gravity. The forces are indicated in the free-body diagram by an arrow pointing up, representing tension, and another arrow pointing down, representing gravity. In a free-body diagram, the lengths of the arrows show the relative magnitude (or strength) of the forces. Because forces are vectors, they add just like other vectors. Notice that the two arrows have equal lengths in Figure 4.2, which means that the forces of tension and weight are of equal magnitude. Because these forces of equal magnitude act in opposite directions, they are perfectly balanced, so they add together to give a net force of zero.

Not all forces are as noticeable as when you push or pull on an object. Some forces act without physical contact, such as the pull of a magnet (in the case of magnetic force) or the gravitational pull of Earth (in the case of gravitational force).

In the next three sections discussing Newton's laws of motion, we will learn about three specific types of forces: friction, the normal force, and the gravitational force. To analyze situations involving forces, we will create free-body diagrams to organize the framework of the mathematics for each individual situation.

TIPS FOR SUCCESS

Correctly drawing and labeling a free-body diagram is an important first step for solving a problem. It will help you visualize the problem and correctly apply the mathematics to solve the problem.

Check Your Understanding

1. What is kinematics?
   a. Kinematics is the study of motion.
   b. Kinematics is the study of the cause of motion.
   c. Kinematics is the study of dimensions.
   d. Kinematics is the study of atomic structures.

2. Do two bodies have to be in physical contact to exert a force upon one another?

a. No, the gravitational force is a field force and does not require physical contact to exert a force.
b. No, the gravitational force is a contact force and does not require physical contact to exert a force.
c. Yes, the gravitational force is a field force and requires physical contact to exert a force.
d. Yes, the gravitational force is a contact force and requires physical contact to exert a force.

3. What kind of physical quantity is force?
 a. Force is a scalar quantity.
 b. Force is a vector quantity.
 c. Force is both a vector quantity and a scalar quantity.
 d. Force is neither a vector nor a scalar quantity.

4. Which forces can be represented in a free-body diagram?
 a. Internal forces
 b. External forces
 c. Both internal and external forces
 d. A body that is not influenced by any force

4.2 Newton's First Law of Motion: Inertia

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's first law and friction, and
- Discuss the relationship between mass and inertia.

Section Key Terms

friction	inertia	law of inertia
mass	Newton's first law of motion	system

Newton's First Law and Friction

Newton's first law of motion states the following:

1. A body at rest tends to remain at rest.
2. A body in motion tends to remain in motion at a constant velocity unless acted on by a net external force. (Recall that *constant velocity* means that the body moves in a straight line and at a constant speed.)

At first glance, this law may seem to contradict your everyday experience. You have probably noticed that a moving object will usually slow down and stop unless some effort is made to keep it moving. The key to understanding why, for example, a sliding box slows down (seemingly on its own) is to first understand that a net external force acts on the box to make the box slow down. Without this net external force, the box would continue to slide at a constant velocity (as stated in Newton's first law of motion). What force acts on the box to slow it down? This force is called **friction**. Friction is an external force that acts opposite to the direction of motion (see Figure 4.3). Think of friction as a resistance to motion that slows things down.

Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it lifts the puck slightly, so the puck experiences very little friction as it moves over the surface. With friction almost eliminated, the puck glides along with very little change in speed. On a frictionless surface, the puck would experience no net external force (ignoring air resistance, which is also a form of friction). Additionally, if we know enough about friction, we can accurately predict how quickly objects will slow down.

Now let's think about another example. A man pushes a box across a floor at constant velocity by applying a force of +50 N. (The positive sign indicates that, by convention, the direction of motion is to the right.) What is the force of friction that opposes the motion? The force of friction must be –50 N. Why? According to Newton's first law of motion, any object moving at constant velocity has no net external force acting upon it, which means that the sum of the forces acting on the object must be zero. The mathematical way to say that no net external force acts on an object is $\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$. So if the man applies +50 N of force, then the force of friction must be –50 N for the two forces to add up to zero (that is, for the two forces to *cancel* each other). Whenever you encounter the phrase *at constant velocity*, Newton's first law tells you that the net external force is zero.

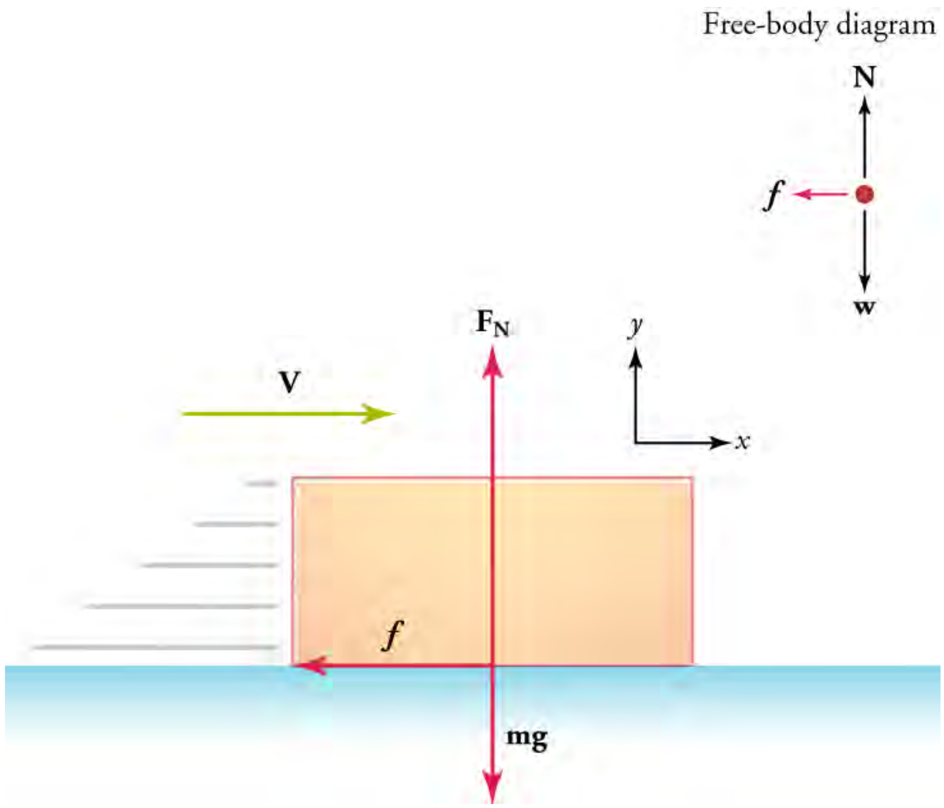

Figure 4.3 For a box sliding across a floor, friction acts in the direction opposite to the velocity.

The force of friction depends on two factors: the coefficient of friction and the normal force. For any two surfaces that are in contact with one another, the coefficient of friction is a constant that depends on the nature of the surfaces. The normal force is the force exerted by a surface that pushes on an object in response to gravity pulling the object down. In equation form, the force of friction is

$$\mathbf{f} = \mu\mathbf{N}, \tag{4.1}$$

where μ is the coefficient of friction and **N** is the normal force. (The coefficient of friction is discussed in more detail in another chapter, and the normal force is discussed in more detail in the section *Newton's Third Law of Motion*.)

Recall from the section on Force that a net external force acts from outside on the object of interest. A more precise definition is that it acts on the **system** of interest. A system is one or more objects that you choose to study. It is important to define the system at the beginning of a problem to figure out which forces are external and need to be considered, and which are internal and can be ignored.

For example, in Figure 4.4 (a), two children push a third child in a wagon at a constant velocity. The system of interest is the wagon plus the small child, as shown in part (b) of the figure. The two children behind the wagon exert external forces on this system ($\mathbf{F}_1$, $\mathbf{F}_2$). Friction *f* acting at the axles of the wheels and at the surface where the wheels touch the ground two other external forces acting on the system. Two more external forces act on the system: the weight **W** of the system pulling down and the normal force **N** of the ground pushing up. Notice that the wagon is not accelerating vertically, so Newton's first law tells us that the normal force balances the weight. Because the wagon is moving forward at a constant velocity, the force of friction must have the same strength as the sum of the forces applied by the two children.

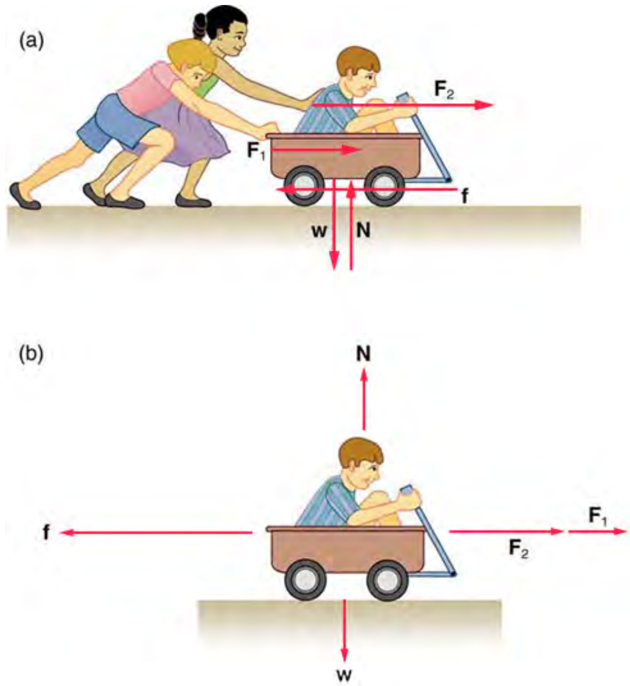

Figure 4.4 (a) The wagon and rider form a *system* that is acted on by external forces. (b) The two children pushing the wagon and child provide two external forces. Friction acting at the wheel axles and on the surface of the tires where they touch the ground provide an external force that act against the direction of motion. The weight **W** and the normal force **N** from the ground are two more external forces acting on the system. All external forces are represented in the figure by arrows. All of the external forces acting on the system add together, but because the wagon moves at a constant velocity, all of the forces must add up to zero.

Mass and Inertia

Inertia is the tendency for an object at rest to remain at rest, or for a moving object to remain in motion in a straight line with constant speed. This key property of objects was first described by Galileo. Later, Newton incorporated the concept of inertia into his first law, which is often referred to as the **law of inertia**.

As we know from experience, some objects have more inertia than others. For example, changing the motion of a large truck is more difficult than changing the motion of a toy truck. In fact, the inertia of an object is proportional to the mass of the object. **Mass** is a measure of the amount of matter (or *stuff*) in an object. The quantity or amount of matter in an object is determined by the number and types of atoms the object contains. Unlike weight (which changes if the gravitational force changes), mass does not depend on gravity. The mass of an object is the same on Earth, in orbit, or on the surface of the moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so mass is usually not determined this way. Instead, the mass of an object is determined by comparing it with the standard kilogram. Mass is therefore expressed in kilograms.

TIPS FOR SUCCESS

In everyday language, people often use the terms *weight* and *mass* interchangeably—but this is not correct. Weight is actually a force. (We cover this topic in more detail in the section *Newton's Second Law of Motion*.)

WATCH PHYSICS

Newton's First Law of Motion

This video contrasts the way we thought about motion and force in the time before Galileo's concept of inertia and Newton's first law of motion with the way we understand force and motion now.

[Click to view content (https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)](https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)

GRASP CHECK

Before we understood that objects have a tendency to maintain their velocity in a straight line unless acted upon by a net force, people thought that objects had a tendency to stop on their own. This happened because a specific force was not yet understood. What was that force?

a. Gravitational force
b. Electrostatic force
c. Nuclear force
d. Frictional force

Virtual Physics

Forces and Motion—Basics

In this simulation, you will first explore net force by placing blue people on the left side of a tug-of-war rope and red people on the right side of the rope (by clicking people and dragging them with your mouse). Experiment with changing the number and size of people on each side to see how it affects the outcome of the match and the net force. Hit the "Go!" button to start the match, and the “reset all” button to start over.

Next, click on the Friction tab. Try selecting different objects for the person to push. Slide the *applied force* button to the right to apply force to the right, and to the left to apply force to the left. The force will continue to be applied as long as you hold down the button. See the arrow representing friction change in magnitude and direction, depending on how much force you apply. Try increasing or decreasing the friction force to see how this change affects the motion.

Click to view content (https://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html)

GRASP CHECK

Click on the tab for the *Acceleration Lab and check the Sum of Forces* option. Push the box to the right and then release. Notice which direction the sum of forces arrow points after the person stops pushing the box and lets it continue moving to the right on its own. At this point, in which direction is the net force, the sum of forces, pointing? Why?

a. The net force acts to the right because the applied external force acted to the right.
b. The net force acts to the left because the applied external force acted to the left.
c. The net force acts to the right because the frictional force acts to the right.
d. The net force acts to the left because the frictional force acts to the left.

Check Your Understanding

5. What does Newton’s first law state?
   a. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant acceleration unless acted on by a net external force.
   b. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant velocity unless acted on by a net external force.
   c. The rate of change of momentum of a body is directly proportional to the external force applied to the body.
   d. The rate of change of momentum of a body is inversely proportional to the external force applied to the body.

6. According to Newton’s first law, a body in motion tends to remain in motion at a constant velocity. However, when you slide an object across a surface, the object eventually slows down and stops. Why?
   a. The object experiences a frictional force exerted by the surface, which opposes its motion.
   b. The object experiences the gravitational force exerted by Earth, which opposes its motion
   c. The object experiences an internal force exerted by the body itself, which opposes its motion.
   d. The object experiences a pseudo-force from the body in motion, which opposes its motion.

7. What is inertia?
   a. Inertia is an object's tendency to maintain its mass.
   b. Inertia is an object's tendency to remain at rest.
   c. Inertia is an object's tendency to remain in motion
   d. Inertia is an object's tendency to remain at rest or, if moving, to remain in motion.

8. What is mass? What does it depend on?
   a. Mass is the weight of an object, and it depends on the gravitational force acting on the object.
   b. Mass is the weight of an object, and it depends on the number and types of atoms in the object.
   c. Mass is the quantity of matter contained in an object, and it depends on the gravitational force acting on the object.
   d. Mass is the quantity of matter contained in an object, and it depends on the number and types of atoms in the object.

4.3 Newton's Second Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's second law, both verbally and mathematically
- Use Newton's second law to solve problems

Section Key Terms

freefall	Newton's second law of motion	weight

Describing Newton's Second Law of Motion

Newton's first law considered bodies at rest or bodies in motion at a constant velocity. The other state of motion to consider is when an object is moving with a changing velocity, which means a change in the speed and/or the direction of motion. This type of motion is addressed by **Newton's second law of motion**, which states how force causes changes in motion. Newton's second law of motion is used to calculate what happens in situations involving forces and motion, and it shows the mathematical relationship between force, mass, and *acceleration*. Mathematically, the second law is most often written as

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \text{ or } \Sigma\mathbf{F} = m\mathbf{a}, \tag{4.2}$$

where $\mathbf{F}_{\text{net}}$ (or $\sum\mathbf{F}$) is the net external force, m is the mass of the system, and $\mathbf{a}$ is the acceleration. Note that $\mathbf{F}_{\text{net}}$ and $\sum\mathbf{F}$ are the same because the net external force is the sum of all the external forces acting on the system.

First, what do we mean by *a change in motion*? A change in motion is simply a change in velocity: the speed of an object can become slower or faster, the direction in which the object is moving can change, or both of these variables may change. A change in velocity means, by definition, that an acceleration has occurred. Newton's first law says that only a nonzero net external force can cause a change in motion, so a net external force must cause an acceleration. Note that acceleration can refer to slowing down or to speeding up. Acceleration can also refer to a change in the direction of motion with no change in speed, because acceleration is the change in velocity divided by the time it takes for that change to occur, *and* velocity is defined by speed *and* direction.

From the equation $\text{F}_{\text{net}} = m\text{a}$, we see that force is directly proportional to both mass and acceleration, which makes sense. To accelerate two objects from rest to the same velocity, you would expect more force to be required to accelerate the more massive object. Likewise, for two objects of the same mass, applying a greater force to one would accelerate it to a greater velocity.

Now, let's rearrange Newton's second law to solve for acceleration. We get

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m} \text{ or } \mathbf{a} = \frac{\Sigma\mathbf{F}}{m}. \tag{4.3}$$

In this form, we can see that acceleration is directly proportional to force, which we write as

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{4.4}$$

where the symbol $\propto$ means *proportional to*.

This proportionality mathematically states what we just said in words: acceleration is directly proportional to the net external force. When two variables are directly proportional to each other, then if one variable doubles, the other variable must double. Likewise, if one variable is reduced by half, the other variable must also be reduced by half. In general, when one variable is multiplied by a number, the other variable is also multiplied by the same number. It seems reasonable that the acceleration of a system should be directly proportional to and in the same direction as the net external force acting on the system. An object experiences greater acceleration when acted on by a greater force.

It is also clear from the equation $\mathbf{a} = \mathbf{F}_{\text{net}}/m$ that acceleration is inversely proportional to mass, which we write as

$$\mathbf{a} \propto \frac{1}{m}. \tag{4.5}$$

Inversely proportional means that if one variable is multiplied by a number, the other variable must be *divided* by the same number. Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. This relationship is illustrated in Figure 4.5, which shows that a given net external force applied to a basketball produces a much greater acceleration than when applied to a car.

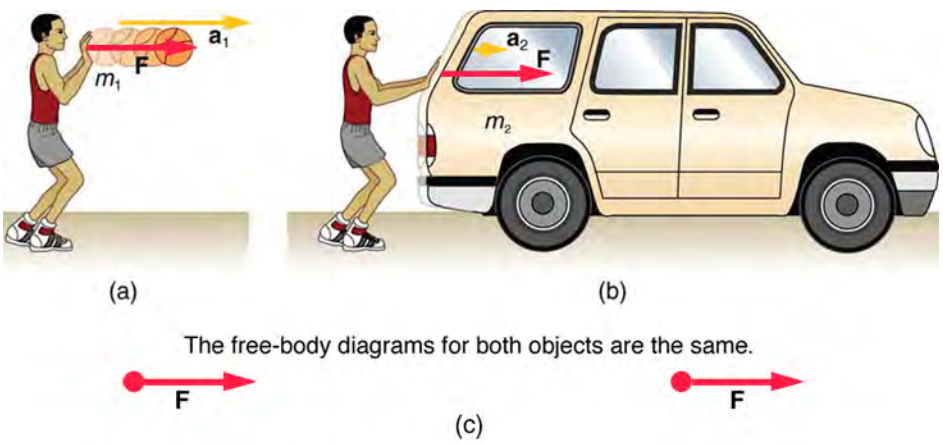

Figure 4.5 The same force exerted on systems of different masses produces different accelerations. (a) A boy pushes a basketball to make a pass. The effect of gravity on the ball is ignored. (b) The same boy pushing with identical force on a stalled car produces a far smaller acceleration (friction is negligible). Note that the free-body diagrams for the ball and for the car are identical, which allows us to compare the two situations.

Applying Newton's Second Law

Before putting Newton's second law into action, it is important to consider units. The equation $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units of mass, length, and time (recall that acceleration has units of length divided by time squared). The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1 m/s^2. That is, because $\mathbf{F}_{\text{net}} = m\mathbf{a}$, we have

$$1\ \text{N} = 1\ \text{kg} \times 1\text{m/s}^2 = 1\frac{\text{kg} \cdot \text{m}}{\text{s}^2}. \tag{4.6}$$

One of the most important applications of Newton's second law is to calculate **weight** (also known as the gravitational force), which is usually represented mathematically as $\mathbf{W}$. When people talk about gravity, they don't always realize that it is an acceleration. When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that the net external force acting on an object is responsible for the acceleration of the object. If air resistance is negligible, the net external force on a falling object is only the gravitational force (i.e., the weight of the object).

Weight can be represented by a vector because it has a direction. Down is defined as the direction in which gravity pulls, so weight is normally considered a downward force. By using Newton's second law, we can figure out the equation for weight.

Consider an object with mass m falling toward Earth. It experiences only the force of gravity (i.e., the gravitational force or weight), which is represented by W. Newton's second law states that $\mathbf{F}_{\text{net}} = m\mathbf{a}$. Because the only force acting on the object is the gravitational force, we have $\mathbf{F}_{\text{net}} = \mathbf{W}$. We know that the acceleration of an object due to gravity is $\mathbf{g}$, so we have $\mathbf{a} = \mathbf{g}$. Substituting these two expressions into Newton's second law gives

$$\mathbf{W} = m\mathbf{g}. \tag{4.7}$$

This is the equation for weight—the gravitational force on a mass m. On Earth, $\mathbf{g} = 9.80\ \text{m/s}^2$, so the weight (disregarding for now the direction of the weight) of a 1.0-kg object on Earth is

$$\mathbf{W} = m\mathbf{g} = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = 9.8\ \ \text{N}. \tag{4.8}$$

Although most of the world uses newtons as the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Recall that although gravity acts downward, it can be assigned a positive or negative value, depending on what the positive direction is in your chosen coordinate system. Be sure to take this into consideration when solving problems with weight. When the downward direction is taken to be negative, as is often the case, acceleration due to gravity becomes $\mathbf{g} = -9.8\ \text{m/s}^2$.

When the net external force on an object is its weight, we say that it is in **freefall**. In this case, the only force acting on the object is the force of gravity. On the surface of Earth, when objects fall downward toward Earth, they are never truly in freefall because there is always some upward force due to air resistance that acts on the object (and there is also the buoyancy force of air, which is similar to the buoyancy force in water that keeps boats afloat).

Gravity varies slightly over the surface of Earth, so the weight of an object depends very slightly on its location on Earth. Weight varies dramatically away from Earth's surface. On the moon, for example, the acceleration due to gravity is only $1.67\ \text{m/s}^2$. Because weight depends on the force of gravity, a 1.0-kg mass weighs 9.8 N on Earth and only about 1.7 N on the moon.

It is important to remember that weight and mass are very different, although they are closely related. Mass is the quantity of matter (how much *stuff*) in an object and does not vary, but weight is the gravitational force on an object and is proportional to the force of gravity. It is easy to confuse the two, because our experience is confined to Earth, and the weight of an object is essentially the same no matter where you are on Earth. Adding to the confusion, the terms mass and weight are often used interchangeably in everyday language; for example, our medical records often show our weight in kilograms, but never in the correct unit of newtons.

Snap Lab

Mass and Weight

In this activity, you will use a scale to investigate mass and weight.

- 1 bathroom scale
- 1 table

1. What do bathroom scales measure?
2. When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled.
3. The springs provide a measure of your weight (provided you are not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is now divided by 9.80 to give a reading in kilograms, which is a of mass. The scale detects weight but is calibrated to display mass.
4. If you went to the moon and stood on your scale, would it detect the same *mass* as it did on Earth?

GRASP CHECK

While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why?

a. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
b. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction opposite to your weight.
c. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
d. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on

you that acts in the direction opposite to your weight.

TIPS FOR SUCCESS

Only *net external force* impacts the acceleration of an object. If more than one force acts on an object and you calculate the acceleration by using only one of these forces, you will not get the correct acceleration for that object.

WATCH PHYSICS

Newton's Second Law of Motion

This video reviews Newton's second law of motion and how net external force and acceleration relate to one another and to mass. It also covers units of force, mass, and acceleration, and reviews a worked-out example.

Click to view content (https://www.khanacademy.org/embed_video?v=ou9YMWlJgkE)

GRASP CHECK

True or False—If you want to reduce the acceleration of an object to half its original value, then you would need to reduce the net external force by half.

a. True
b. False

WORKED EXAMPLE

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N parallel to the ground. The mass of the mower is 240 kg. What is its acceleration?

Figure 4.6

Strategy

Because $\mathbf{F}_{\mathbf{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law: $\mathbf{F}_{\mathbf{net}} = m\mathbf{a}$.

Solution

Solving Newton's second law for the acceleration, we find that the magnitude of the acceleration, $\mathbf{a}$, is $\mathbf{a} = \frac{\mathbf{F}_{\mathrm{net}}}{m}$. Entering the given values for net external force and mass gives

$$\mathbf{a} = \frac{51\ \ \mathrm{N}}{240\ \ \mathrm{kg}} \tag{4.9}$$

Inserting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$\mathbf{a} = \frac{51\ \text{kg} \cdot \text{m/s}^2}{240\ \text{kg}} = 0.21\ \text{m/s}^2. \tag{4.10}$$

Discussion

The acceleration is in the same direction as the net external force, which is parallel to the ground and to the right. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion, because we are given that the net external force is in the direction in which the person pushes. Also, the vertical forces must cancel if there is no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is reasonable for a person pushing a mower; the mower's speed must increase by 0.21 m/s every second, which is possible. The time during which the mower accelerates would not be very long because the person's top speed would soon be reached. At this point, the person could push a little less hard, because he only has to overcome friction.

WORKED EXAMPLE

What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on humans at high accelerations. Rocket sleds consisted of a platform mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust, **T**, for the four-rocket propulsion system shown below. The sled's initial acceleration is $49\ \text{m/s}^2$, the mass of the system is 2,100 kg, and the force of friction opposing the motion is 650 N.

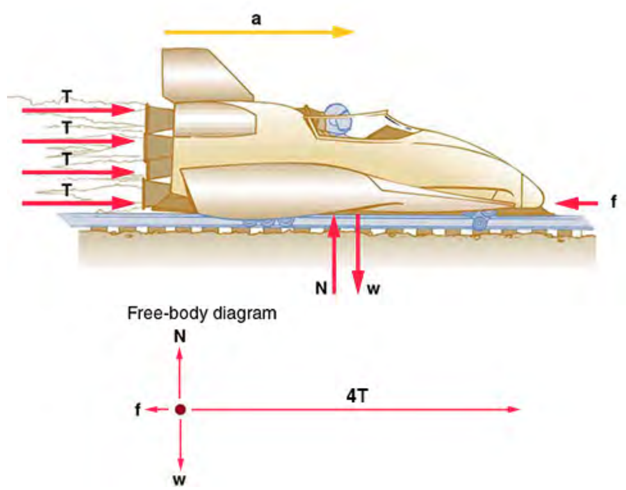

Figure 4.7

Strategy

The system of interest is the rocket sled. Although forces act vertically on the system, they must cancel because the system does not accelerate vertically. This leaves us with only horizontal forces to consider. We'll assign the direction to the right as the positive direction. See the free-body diagram in Figure 4.8.

Solution

We start with Newton's second law and look for ways to find the thrust **T** of the engines. Because all forces and acceleration are along a line, we need only consider the magnitudes of these quantities in the calculations. We begin with

$$\mathbf{F}_{\text{net}} = m\mathbf{a}, \tag{4.11}$$

where $\mathbf{F}_{\text{net}}$ is the net external force in the horizontal direction. We can see from Figure 4.8 that the engine thrusts are in the same direction (which we call the positive direction), whereas friction opposes the thrust. In equation form, the net external force is

$$\mathbf{F}_{\text{net}} = 4\mathbf{T} - \mathbf{f}. \tag{4.12}$$

Newton's second law tells us that $\mathbf{F}_{\text{net}} = m\mathbf{a}$, so we get

$$m\mathbf{a} = 4\mathbf{T} - \mathbf{f}. \tag{4.13}$$

After a little algebra, we solve for the total thrust 4**T**:

$$4\mathbf{T} = m\mathbf{a} + \mathbf{f}, \tag{4.14}$$

which means that the individual thrust is

$$\mathbf{T} = \frac{m\mathbf{a} + \mathbf{f}}{4}. \tag{4.15}$$

Inserting the known values yields

$$\mathbf{T} = \frac{(2100\text{ kg})(49\text{ m/s}^2) + 650\text{ N}}{4} = 2.6 \times 10^4\text{ N}. \tag{4.16}$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and to test the apparatus designed to protect fighter pilots in emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 **g**. (Recall that g, the acceleration due to gravity, is 9.80 m/s^2. An acceleration of 45 **g** is $45 \times 9.80\text{ m/s}^2$, which is approximately 440 m/s^2.) Living subjects are no longer used, and land speeds of 10,000 km/h have now been obtained with rocket sleds. In this example, as in the preceding example, the system of interest is clear. We will see in later examples that choosing the system of interest is crucial—and that the choice is not always obvious.

Practice Problems

9. If 1 N is equal to 0.225 lb, how many pounds is 5 N of force?
   a. 0.045 lb
   b. 1.125 lb
   c. 2.025 lb
   d. 5.000 lb

10. How much force needs to be applied to a 5-kg object for it to accelerate at 20 m/s²?
    a. 1 N
    b. 10 N
    c. 100 N
    d. 1,000 N

Check Your Understanding

11. What is the mathematical statement for Newton's second law of motion?
    a. $F = ma$
    b. $F = 2ma$
    c. $F = \frac{m}{a}$
    d. $F = ma^2$

12. Newton's second law describes the relationship between which quantities?
    a. Force, mass, and time
    b. Force, mass, and displacement
    c. Force, mass, and velocity
    d. Force, mass, and acceleration

13. What is acceleration?
    a. Acceleration is the rate at which displacement changes.
    b. Acceleration is the rate at which force changes.
    c. Acceleration is the rate at which velocity changes.

d. Acceleration is the rate at which mass changes.

4.4 Newton's Third Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's third law, both verbally and mathematically
- Use Newton's third law to solve problems

Section Key Terms

Newton's third law of motion	normal force	tension	thrust

Describing Newton's Third Law of Motion

If you have ever stubbed your toe, you have noticed that although your toe initiates the impact, the surface that you stub it on exerts a force back on your toe. Although the first thought that crosses your mind is probably "ouch, that hurt" rather than "this is a great example of Newton's third law," both statements are true.

This is exactly what happens whenever one object exerts a force on another—each object experiences a force that is the same strength as the force acting on the other object but that acts in the opposite direction. Everyday experiences, such as stubbing a toe or throwing a ball, are all perfect examples of Newton's third law in action.

Newton's third law of motion states that whenever a first object exerts a force on a second object, the first object experiences a force equal in magnitude but opposite in direction to the force that it exerts.

Newton's third law of motion tells us that forces always occur in pairs, and one object cannot exert a force on another without experiencing the same strength force in return. We sometimes refer to these force pairs as *action-reaction* pairs, where the force exerted is the action, and the force experienced in return is the reaction (although which is which depends on your point of view).

Newton's third law is useful for figuring out which forces are external to a system. Recall that identifying external forces is important when setting up a problem, because the external forces must be added together to find the net force.

We can see Newton's third law at work by looking at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 4.8. She pushes against the pool wall with her feet and accelerates in the direction opposite to her push. The wall has thus exerted on the swimmer a force of equal magnitude but in the direction opposite that of her push. You might think that two forces of equal magnitude but that act in opposite directions would cancel, *but they do not because they act on different systems.*

In this case, there are two different systems that we could choose to investigate: the swimmer or the wall. If we choose the swimmer to be the system of interest, as in the figure, then $\mathrm{F}_{\text{wall on feet}}$ is an external force on the swimmer and affects her motion. Because acceleration is in the same direction as the net external force, the swimmer moves in the direction of $\mathrm{F}_{\text{wall on feet}}$. Because the swimmer is our system (or object of interest) and not the wall, we do not need to consider the force $\mathrm{F}_{\text{feet on wall}}$ because it originates *from* the swimmer rather than *acting on* the swimmer. Therefore, $\mathrm{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathrm{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to the direction in which she wants to move.

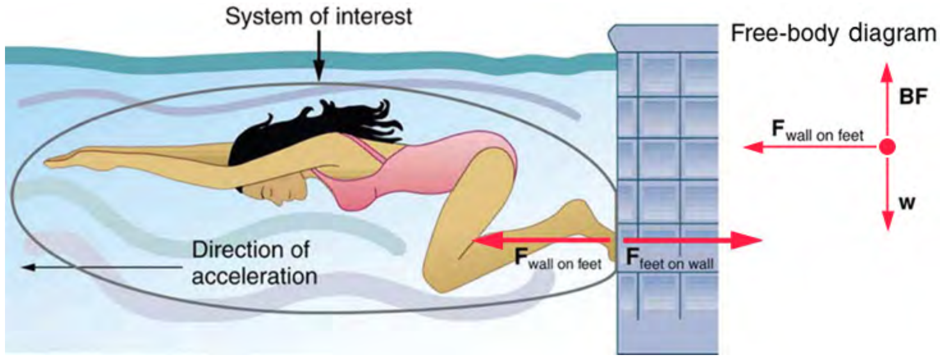

Figure 4.8 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means that the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition is the result of Newton's third law of motion, which dictates that the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but that acts in the direction opposite to the force that the swimmer exerts on the wall.

Other examples of Newton's third law are easy to find. As a teacher paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force in the forward direction on the teacher that causes him to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the car's wheels in reaction to the car's wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward.

Another example is the force of a baseball as it makes contact with the bat. Helicopters create lift by pushing air down, creating an upward reaction force. Birds fly by exerting force on air in the direction opposite that in which they wish to fly. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself forward in the water by ejecting water backward through a funnel in its body, which is similar to how a jet ski is propelled. In these examples, the octopus or jet ski push the water backward, and the water, in turn, pushes the octopus or jet ski forward.

Applying Newton's Third Law

Forces are classified and given names based on their source, how they are transmitted, or their effects. In previous sections, we discussed the forces called *push*, *weight*, and *friction*. In this section, applying Newton's third law of motion will allow us to explore three more forces: the **normal force**, **tension**, and **thrust**. However, because we haven't yet covered vectors in depth, we'll only consider one-dimensional situations in this chapter. Another chapter will consider forces acting in two dimensions.

The gravitational force (or weight) acts on objects at all times and everywhere on Earth. We know from Newton's second law that a net force produces an acceleration; so, why is everything not in a constant state of freefall toward the center of Earth? The answer is the normal force. The normal force is the force that a surface applies to an object to support the weight of that object; it acts perpendicular to the surface upon which the object rests. If an object on a flat surface is not accelerating, the net external force is zero, and the normal force has the same magnitude as the weight of the system but acts in the opposite direction. In equation form, we write that

$$\mathbf{N} = m\mathbf{g}. \tag{4.17}$$

Note that this equation is only true for a horizontal surface.

The word *tension* comes from the Latin word meaning *to stretch*. Tension is the force along the length of a flexible connector, such as a string, rope, chain, or cable. Regardless of the type of connector attached to the object of interest, one must remember that the connector can only pull (or *exert tension*) in the direction *parallel* to its length. Tension is a pull that acts parallel to the connector, and that acts in opposite directions at the two ends of the connector. This is possible because a flexible connector is simply a long series of action-reaction forces, except at the two ends where outside objects provide one member of the action-reaction forces.

Consider a person holding a mass on a rope, as shown in Figure 4.9.

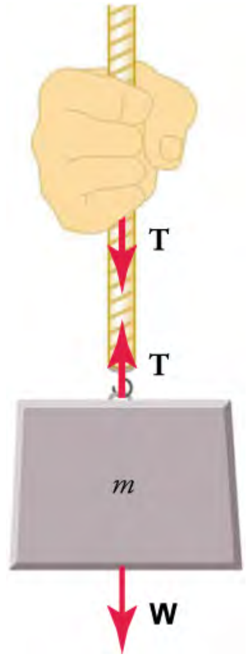

Figure 4.9 When a perfectly flexible connector (one requiring no force to bend it) such as a rope transmits a force **T**, this force must be parallel to the length of the rope, as shown. The pull that such a flexible connector exerts is a tension. Note that the rope pulls with equal magnitude force but in opposite directions to the hand and to the mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that transmits forces of equal magnitude between the two objects but that act in opposite directions.

Tension in the rope must equal the weight of the supported mass, as we can prove by using Newton's second law. If the 5.00 kg mass in the figure is stationary, then its acceleration is zero, so $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight **W** and the tension **T** supplied by the rope. Summing the external forces to find the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{T} - \mathbf{W} = 0, \tag{4.18}$$

where **T** and **W** are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. By substituting $m\mathbf{g}$ for $\mathbf{F}_{\text{net}}$ and rearranging the equation, the tension equals the weight of the supported mass, just as you would expect

$$\mathbf{T} = \mathbf{W} = m\mathbf{g}. \tag{4.19}$$

For a 5.00-kg mass (neglecting the mass of the rope), we see that

$$\mathbf{T} = m\mathbf{g} = (5.00\ \text{kg})(9.80\ \text{m/s}^2) = 49.0\ \text{N}. \tag{4.20}$$

Another example of Newton's third law in action is thrust. Rockets move forward by expelling gas backward at a high velocity. This means that the rocket exerts a large force backward on the gas in the rocket combustion chamber, and the gas, in turn, exerts a large force forward on the rocket in response. This reaction force is called *thrust*.

TIPS FOR SUCCESS

A common misconception is that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can expel exhaust gases more easily.

LINKS TO PHYSICS

Math: Problem-Solving Strategy for Newton's Laws of Motion

The basics of problem solving, presented earlier in this text, are followed here with specific strategies for applying Newton's laws of motion. These techniques also reinforce concepts that are useful in many other areas of physics.

First, identify the physical principles involved. If the problem involves forces, then Newton's laws of motion are involved, and it is important to draw a careful sketch of the situation. An example of a sketch is shown in Figure 4.10. Next, as in Figure 4.10, use vectors to represent all forces. Label the forces carefully, and make sure that their lengths are proportional to the magnitude of the forces and that the arrows point in the direction in which the forces act.

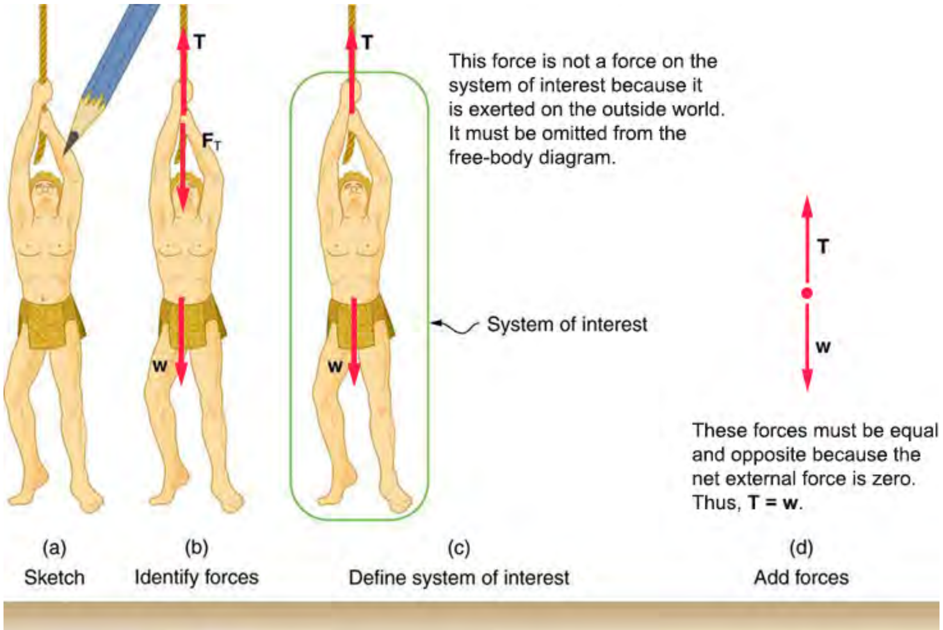

Figure 4.10 (a) A sketch of Tarzan hanging motionless from a vine. (b) Arrows are used to represent all forces. **T** is the tension exerted on Tarzan by the vine, $\mathbf{F}_\mathrm{T}$ is the force exerted on the vine by Tarzan, and **W** is Tarzan's weight (i.e., the force exerted on Tarzan by Earth's gravity). All other forces, such as a nudge of a breeze, are assumed to be negligible. (c) Suppose we are given Tarzan's mass and asked to find the tension in the vine. We define the system of interest as shown and draw a free-body diagram, as shown in (d). $\mathbf{F}_\mathrm{T}$ is no longer shown because it does not act on the system of interest; rather, $\mathbf{F}_\mathrm{T}$ acts on the outside world. (d) The free-body diagram shows only the external forces acting on Tarzan. For these to sum to zero, we must have $\mathbf{T} = \mathbf{W}$.

Next, make a list of knowns and unknowns and assign variable names to the quantities given in the problem. Figure out which variables need to be calculated; these are the unknowns. Now carefully define the system: which objects are of interest for the problem. This decision is important, because Newton's second law involves only external forces. Once the system is identified, it's possible to see which forces are external and which are internal (see Figure 4.10).

If the system acts on an object outside the system, then you know that the outside object exerts a force of equal magnitude but in the opposite direction on the system.

A diagram showing the system of interest and all the external forces acting on it is called a free-body diagram. Only external forces are shown on free-body diagrams, not acceleration or velocity. Figure 4.10 shows a free-body diagram for the system of interest.

After drawing a free-body diagram, apply Newton's second law to solve the problem. This is done in Figure 4.10 for the case of Tarzan hanging from a vine. When external forces are clearly identified in the free-body diagram, translate the forces into equation form and solve for the unknowns. Note that forces acting in opposite directions have opposite signs. By convention, forces acting downward or to the left are usually negative.

GRASP CHECK

If a problem has more than one system of interest, more than one free-body diagram is required to describe the external forces acting on the different systems.

a. True
b. False

WATCH PHYSICS

Newton's Third Law of Motion

This video explains Newton's third law of motion through examples involving push, normal force, and thrust (the force that propels a rocket or a jet).

Click to view content (https://www.openstax.org/l/astronaut)

GRASP CHECK

If the astronaut in the video wanted to move upward, in which direction should he throw the object? Why?

a. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., upward).
b. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., downward).
c. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., upward).
d. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., downward).

WORKED EXAMPLE

An Accelerating Subway Train

A physics teacher pushes a cart of demonstration equipment to a classroom, as in Figure 4.11. Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. To push the cart forward, the teacher's foot applies a force of 150 N in the opposite direction (backward) on the floor. Calculate the acceleration produced by the teacher. The force of friction, which opposes the motion, is 24.0 N.

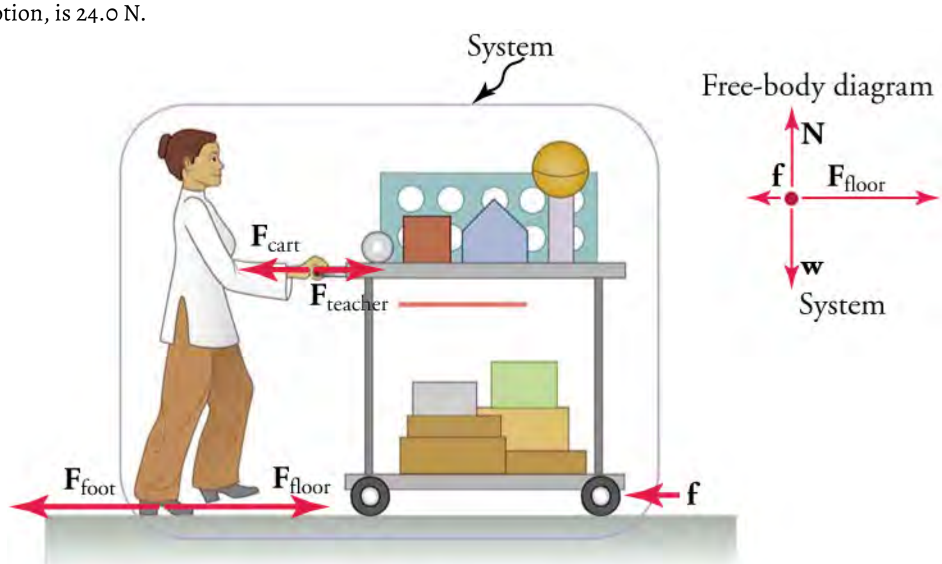

Figure 4.11

Strategy

Because they accelerate together, we define the system to be the teacher, the cart, and the equipment. The teacher pushes backward with a force $\mathbf{F}_{foot}$ of 150 N. According to Newton's third law, the floor exerts a forward force $\mathbf{F}_{floor}$ of 150 N on the system. Because all motion is horizontal, we can assume that no net force acts in the vertical direction, and the problem becomes one dimensional. As noted in the figure, the friction f opposes the motion and therefore acts opposite the direction of $\mathbf{F}_{floor}$.

We should not include the forces $\mathbf{F}_{\text{teacher}}$, $\mathbf{F}_{\text{cart}}$, or $\mathbf{F}_{\text{foot}}$ because these are exerted *by* the system, not *on* the system. We find the net external force by adding together the external forces acting on the system (see the free-body diagram in the figure) and then use Newton's second law to find the acceleration.

Solution

Newton's second law is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \tag{4.21}$$

The net external force on the system is the sum of the external forces: the force of the floor acting on the teacher, cart, and equipment (in the horizontal direction) and the force of friction. Because friction acts in the opposite direction, we assign it a negative value. Thus, for the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{F}_{\text{floor}} - \mathbf{f} = 150 \text{ N} - 24.0 \text{ N} = 126 \text{ N}. \tag{4.22}$$

The mass of the system is the sum of the mass of the teacher, cart, and equipment.

$$m = (65.0 + 12.0 + 7.0) \text{ kg} = 84 \text{ kg} \tag{4.23}$$

Insert these values of net F and m into Newton's second law to obtain the acceleration of the system.

$$\begin{aligned} \mathbf{a} &= \frac{\mathbf{F}_{\text{net}}}{m} \\ a &= \frac{126 \text{ N}}{84 \text{ kg}} = 1.5 \text{ m/s}^2 \end{aligned} \tag{4.24}$$

$$F_1 < F_2 \tag{4.25}$$

Discussion

None of the forces between components of the system, such as between the teacher's hands and the cart, contribute to the net external force because they are internal to the system. Another way to look at this is to note that the forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the teacher on the cart is of equal magnitude but in the opposite direction of the force exerted by the cart on the teacher. In this case, both forces act on the same system, so they cancel. Defining the system was crucial to solving this problem.

Practice Problems

14. What is the equation for the normal force for a body with mass m that is at rest on a horizontal surface?

a. N = m
b. N = mg
c. N = mv
d. N = g

15. An object with mass m is at rest on the floor. What is the magnitude and direction of the normal force acting on it?

a. N = mv in upward direction
b. N = mg in upward direction
c. N = mv in downward direction
d. N = mg in downward direction

Check Your Understanding

16. What is Newton's third law of motion?

a. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude and acts in the direction of the applied force.
b. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude and acts in the direction of the applied force.
c. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude but acts in the direction opposite the direction of the applied force.
d. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude but

acts in the direction opposite the direction of the applied force.

17. Considering Newton's third law, why don't two equal and opposite forces cancel out each other?

a. Because the two forces act in the same direction
b. Because the two forces have different magnitudes
c. Because the two forces act on different systems
d. Because the two forces act in perpendicular directions

KEY TERMS

dynamics the study of how forces affect the motion of objects and systems

external force a force acting on an object or system that originates outside of the object or system

force a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

free-body diagram a diagram showing all external forces acting on a body

freefall a situation in which the only force acting on an object is the force of gravity

friction an external force that acts in the direction opposite to the direction of motion

inertia the tendency of an object at rest to remain at rest, or for a moving object to remain in motion in a straight line and at a constant speed

law of inertia Newton's first law of motion: a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

mass the quantity of matter in a substance; measured in kilograms

net external force the sum of all external forces acting on an object or system

net force the sum of all forces acting on an object or system

Newton's first law of motion a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion the net external force, $\mathbf{F}_{\text{net}}$, on an object is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and also proportional to the object's mass, m; defined mathematically as $\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$.

Newton's third law of motion when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts

normal force the force that a surface applies to an object; acts perpendicular and away from the surface with which the object is in contact

system one or more objects of interest for which only the forces acting on them from the outside are considered, but not the forces acting between them or inside them

tension a pulling force that acts along a connecting medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force exerted on the object by the rope is called tension

thrust a force that pushes an object forward in response to the backward ejection of mass by the object; rockets and airplanes are pushed forward by a thrust reaction force in response to ejecting gases backward

weight the force of gravity, $\mathbf{W}$, acting on an object of mass m; defined mathematically as $\mathbf{W} = m\mathbf{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

SECTION SUMMARY

4.1 Force

- Dynamics is the study of how forces affect the motion of objects and systems.
- Force is a push or pull that can be defined in terms of various standards. It is a vector and so has both magnitude and direction.
- External forces are any forces outside of a body that act on the body. A free-body diagram is a drawing of all external forces acting on a body.

4.2 Newton's First Law of Motion: Inertia

- Newton's first law states that a body at rest remains at rest or, if moving, remains in motion in a straight line at a constant speed, unless acted on by a net external force. This law is also known as the law of inertia.
- Inertia is the tendency of an object at rest to remain at rest or, if moving, to remain in motion at constant velocity. Inertia is related to an object's mass.
- Friction is a force that opposes motion and causes an object or system to slow down.
- Mass is the quantity of matter in a substance.

4.3 Newton's Second Law of Motion

- Acceleration is a change in velocity, meaning a change in speed, direction, or both.
- An external force acts on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to the system's mass.
- In equation form, Newton's second law of motion is $\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$. This is sometimes written as $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$.
- The weight of an object of mass m is the force of gravity that acts on it. From Newton's second law, weight is

given by $\mathbf{W} = m\mathbf{g}$.
- If the only force acting on an object is its weight, then the object is in freefall.

4.4 Newton's Third Law of Motion

- Newton's third law of motion states that when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.
- When an object rests on a surface, the surface applies a force on the object that opposes the weight of the object. This force acts perpendicular to the surface and is called the normal force.
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension. When a rope supports the weight of an object at rest, the tension in the rope is equal to the weight of the object.
- Thrust is a force that pushes an object forward in response to the backward ejection of mass by the object. Rockets and airplanes are pushed forward by thrust.

KEY EQUATIONS

4.2 Newton's First Law of Motion: Inertia

Newton's first law of motion	$\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$

4.3 Newton's Second Law of Motion

Newton's second law of motion	$\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$
Newton's second law of motion to solve acceleration	$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$
Newton's second law of motion to solve weight	$\mathbf{W} = m\mathbf{g}$

4.4 Newton's Third Law of Motion

normal force for a nonaccelerating horizontal surface	$\mathbf{N} = m\mathbf{g}$
tension for an object at rest	$\mathbf{T} = m\mathbf{g}$

CHAPTER REVIEW

Concept Items

4.1 Force

1. What is dynamics?
   a. Dynamics is the study of internal forces.
   b. Dynamics is the study of forces and their effect on motion.
   c. Dynamics describes the motion of points, bodies, and systems without consideration of the cause of motion.
   d. Dynamics describes the effect of forces on each other.

2. Two forces acting on an object are perpendicular to one another. How would you draw these in a free-body diagram?
   a. The two force arrows will be drawn at a right angle to one another.
   b. The two force arrows will be pointing in opposite directions.
   c. The two force arrows will be at a 45° angle to one another.
   d. The two force arrows will be at a 180° angle to one another.

3. A free-body diagram shows the forces acting on an object. How is that object represented in the diagram?
   a. A single point
   b. A square box
   c. A unit circle
   d. The object as it is

4.2 Newton's First Law of Motion: Inertia

4. A ball rolls along the ground, moving from north to south. What direction is the frictional force that acts on the ball?
   a. North to south
   b. South to north
   c. West to east
   d. East to west

5. The tires you choose to drive over icy roads will create more friction with the road than your summer tires. Give another example where more friction is desirable.

c. One focus is the parent body and the other is located outside of the elliptical orbit, on the line on which is the semi-major axis of the ellipse.
d. One focus is on the line containing the semi-major axis of the ellipse, and the other is located anywhere on the elliptical orbit of the satellite.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Newton's law of universal gravitation and compare it to Einstein's theory of general relativity
- Perform calculations using Newton's law of universal gravitation

Section Key Terms

Einstein's theory of general relativity	gravitational constant	Newton's universal law of gravitation

Concepts Related to Newton's Law of Universal Gravitation

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 7.7. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner, Galileo Galilei, had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph. It had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose an explanation of the mechanism that caused them to follow these paths and not others.

Figure 7.7 The popular legend that Newton suddenly discovered the law of universal gravitation when an apple fell from a tree and hit him on the head has an element of truth in it. A more probable account is that he was walking through an orchard and wondered why all the apples fell in the same direction with the same acceleration. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Expressed in modern language, **Newton's universal law of gravitation** states that every object in the universe attracts every other object with a force that is directed along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This attraction is illustrated by Figure 7.8.

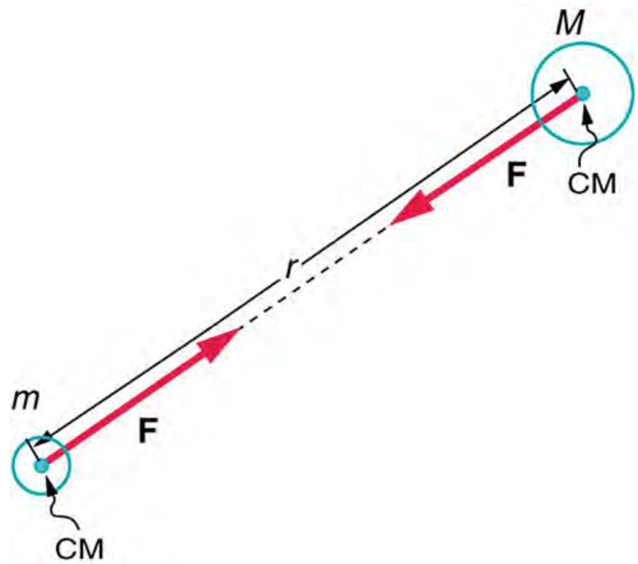

Figure 7.8 Gravitational attraction is along a line joining the centers of mass (CM) of the two bodies. The magnitude of the force on each body is the same, consistent with Newton's third law (action-reaction).

For two bodies having masses *m* and *M* with a distance *r* between their centers of mass, the equation for Newton's universal law of gravitation is

$$\mathbf{F} = G\frac{mM}{r^2}$$

where **F** is the magnitude of the gravitational force and *G* is a proportionality factor called the **gravitational constant**. *G* is a universal constant, meaning that it is thought to be the same everywhere in the universe. It has been measured experimentally to be $G = 6.673 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.

If a person has a mass of 60.0 kg, what would be the force of gravitational attraction on him at Earth's surface? *G* is given above, Earth's mass *M* is 5.97×10^{24} kg, and the radius *r* of Earth is 6.38×10^6 m. Putting these values into Newton's universal law of gravitation gives

$$\mathbf{F} = G\frac{mM}{r^2} = \left(6.673 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)\left(\frac{(60.0\ \mathrm{kg})\left(5.97 \times 10^{24}\ \mathrm{kg}\right)}{\left(6.38 \times 10^6\ \mathrm{m}\right)^2}\right) = 584\ \ \mathrm{N}$$

We can check this result with the relationship: $\mathbf{F} = m\mathbf{g} = (60\ \mathrm{kg})(9.8\mathrm{m/s^2}) = 588\ \mathrm{N}$

You may remember that **g**, the acceleration due to gravity, is another important constant related to gravity. By substituting **g** for **a** in the equation for Newton's second law of motion we get $\mathbf{F} = m\mathbf{g}$. Combining this with the equation for universal gravitation gives

$$m\mathbf{g} = G\frac{mM}{r^2}$$

Cancelling the mass *m* on both sides of the equation and filling in the values for the gravitational constant and mass and radius of the Earth, gives the value of *g*, which may look familiar.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)\left(\frac{5.98 \times 10^{24}\mathrm{kg}}{\left(6.38 \times 10^6\mathrm{m}\right)^2}\right) = 9.80\mathrm{m/s^2}$$

This is a good point to recall the difference between mass and weight. Mass is the amount of matter in an object; weight is the force of attraction between the mass within two objects. Weight can change because g is different on every moon and planet. An object's mass m does not change but its weight $m\mathbf{g}$ can.

Virtual Physics

Gravity and Orbits

Move the sun, Earth, moon and space station in this simulation to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies. Turn off gravity to see what would happen without it!

[Click to view content (https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)](https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)

GRASP CHECK

Why doesn't the Moon travel in a smooth circle around the Sun?

a. The Moon is not affected by the gravitational field of the Sun.
b. The Moon is not affected by the gravitational field of the Earth.
c. The Moon is affected by the gravitational fields of both the Earth and the Sun, which are always additive.
d. The moon is affected by the gravitational fields of both the Earth and the Sun, which are sometimes additive and sometimes opposite.

Snap Lab

Take-Home Experiment: Falling Objects

In this activity you will study the effects of mass and air resistance on the acceleration of falling objects. Make predictions (hypotheses) about the outcome of this experiment. Write them down to compare later with results.

- Four sheets of $8\ \text{-}1/2 \times 11$ -inch paper

Procedure

- Take four identical pieces of paper.
  - Crumple one up into a small ball.
  - Leave one uncrumpled.
  - Take the other two and crumple them up together, so that they make a ball of exactly twice the mass of the other crumpled ball.
  - Now compare which ball of paper lands first when dropped simultaneously from the same height.
    1. Compare crumpled one-paper ball with crumpled two-paper ball.
    2. Compare crumpled one-paper ball with uncrumpled paper.

GRASP CHECK

Why do some objects fall faster than others near the surface of the earth if all mass is attracted equally by the force of gravity?

a. Some objects fall faster because of air resistance, which acts in the direction of the motion of the object and exerts more force on objects with less surface area.
b. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with less surface area.
c. Some objects fall faster because of air resistance, which acts in the direction of motion of the object and exerts more force on objects with more surface area.
d. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with more surface area.

It is possible to derive Kepler's third law from Newton's law of universal gravitation. Applying Newton's second law of motion to angular motion gives an expression for centripetal force, which can be equated to the expression for force in the universal gravitation equation. This expression can be manipulated to produce the equation for Kepler's third law. We saw earlier that the expression r^3/T^2 is a constant for satellites orbiting the same massive object. The derivation of Kepler's third law from Newton's law of universal gravitation and Newton's second law of motion yields that constant:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

where M is the mass of the central body about which the satellites orbit (for example, the sun in our solar system). The usefulness of this equation will be seen later.

The universal gravitational constant G is determined experimentally. This definition was first done accurately in 1798 by English scientist Henry Cavendish (1731–1810), more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most) by using an apparatus like that in Figure 7.9. Remarkably, his value for G differs by less than 1% from the modern value.

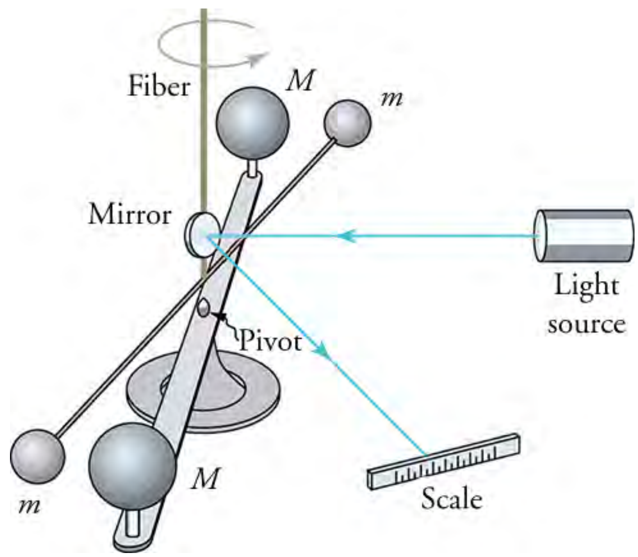

Figure 7.9 Cavendish used an apparatus like this to measure the gravitational attraction between two suspended spheres (*m*) and two spheres on a stand (*M*) by observing the amount of torsion (twisting) created in the fiber. The distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Einstein's Theory of General Relativity

Einstein's theory of general relativity explained some interesting properties of gravity not covered by Newton's theory. Einstein based his theory on the postulate that acceleration and gravity have the same effect and cannot be distinguished from each other. He concluded that light must fall in both a gravitational field and in an accelerating reference frame. Figure 7.10 shows this effect (greatly exaggerated) in an accelerating elevator. In Figure 7.10**(a)**, the elevator accelerates upward in zero gravity. In Figure 7.10**(b)**, the room is not accelerating but is subject to gravity. The effect on light is the same: it "falls" downward in both situations. The person in the elevator cannot tell whether the elevator is accelerating in zero gravity or is stationary and subject to gravity. Thus, gravity affects the path of light, even though we think of gravity as acting between masses, while photons are massless.

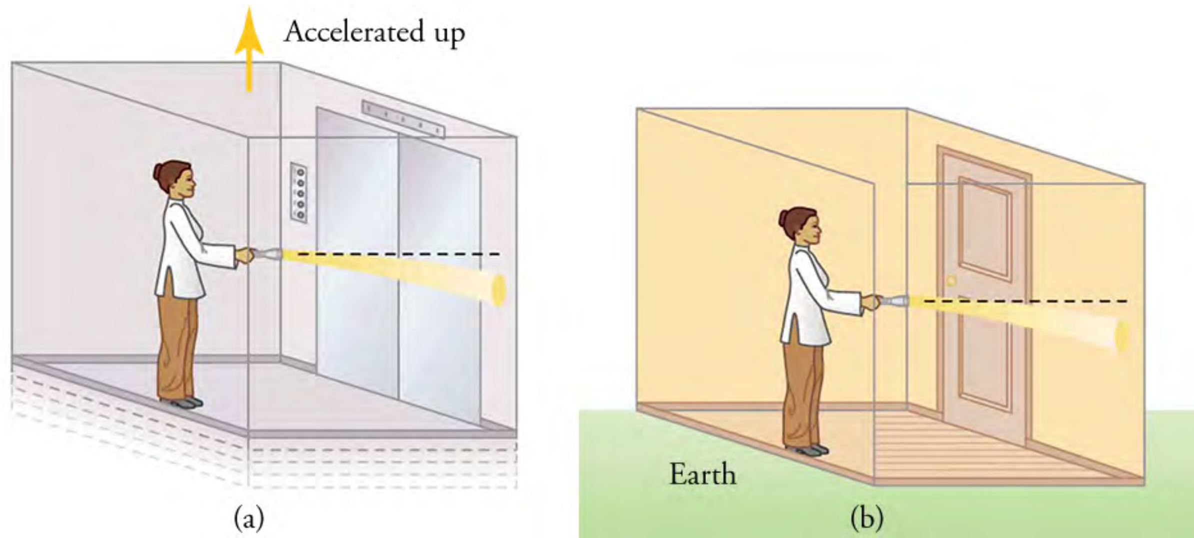

Figure 7.10 (a) A beam of light emerges from a flashlight in an upward-accelerating elevator. Since the elevator moves up during the time the light takes to reach the wall, the beam strikes lower than it would if the elevator were not accelerated. (b) Gravity must have the same effect on light, since it is not possible to tell whether the elevator is accelerating upward or is stationary and acted upon by gravity.

Einstein's theory of general relativity got its first verification in 1919 when starlight passing near the sun was observed during a solar eclipse. (See Figure 7.11.) During an eclipse, the sky is darkened and we can briefly see stars. Those on a line of sight nearest the sun should have a shift in their apparent positions. Not only was this shift observed, but it agreed with Einstein's predictions well within experimental uncertainties. This discovery created a scientific and public sensation. Einstein was now a folk hero as well as a very great scientist. The bending of light by matter is equivalent to a bending of space itself, with light following the curve. This is another radical change in our concept of space and time. It is also another connection that any particle with mass *or* energy (e.g., massless photons) is affected by gravity.

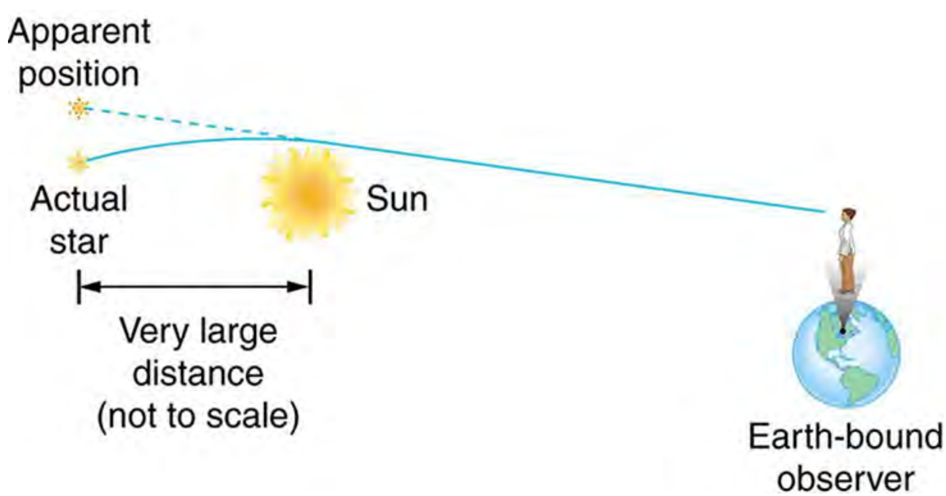

Figure 7.11 This schematic shows how light passing near a massive body like the sun is curved toward it. The light that reaches the Earth then seems to be coming from different locations than the known positions of the originating stars. Not only was this effect observed, but the amount of bending was precisely what Einstein predicted in his general theory of relativity.

To summarize the two views of gravity, Newton envisioned gravity as a tug of war along the line connecting any two objects in the universe. In contrast, Einstein envisioned gravity as a bending of space-time by mass.

BOUNDLESS PHYSICS

NASA gravity probe B

NASA's Gravity Probe B (GP-B) mission has confirmed two key predictions derived from Albert Einstein's general theory of relativity. The probe, shown in Figure 7.12 was launched in 2004. It carried four ultra-precise gyroscopes designed to measure two effects hypothesized by Einstein's theory:

- The geodetic effect, which is the warping of space and time by the gravitational field of a massive body (in this case, Earth)
- The frame-dragging effect, which is the amount by which a spinning object pulls space and time with it as it rotates

Figure 7.12 Artist concept of Gravity Probe B spacecraft in orbit around the Earth. (credit: NASA/MSFC)

Both effects were measured with unprecedented precision. This was done by pointing the gyroscopes at a single star while orbiting Earth in a polar orbit. As predicted by relativity theory, the gyroscopes experienced very small, but measureable, changes in the direction of their spin caused by the pull of Earth's gravity.

The principle investigator suggested imagining Earth spinning in honey. As Earth rotates it drags space and time with it as it would a surrounding sea of honey.

GRASP CHECK

According to the general theory of relativity, a gravitational field bends light. What does this have to do with time and space?

a. Gravity has no effect on the space-time continuum, and gravity only affects the motion of light.
b. The space-time continuum is distorted by gravity, and gravity has no effect on the motion of light.
c. Gravity has no effect on either the space-time continuum or on the motion of light.
d. The space-time continuum is distorted by gravity, and gravity affects the motion of light.

Calculations Based on Newton's Law of Universal Gravitation

TIPS FOR SUCCESS

When performing calculations using the equations in this chapter, use units of kilograms for mass, meters for distances, newtons for force, and seconds for time.

The mass of an object is constant, but its weight varies with the strength of the gravitational field. This means the value of **g** varies from place to place in the universe. The relationship between force, mass, and acceleration from the second law of motion can be written in terms of **g**.

$$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$$

In this case, the force is the weight of the object, which is caused by the gravitational attraction of the planet or moon on which the object is located. We can use this expression to compare weights of an object on different moons and planets.

WATCH PHYSICS

Mass and Weight Clarification

This video shows the mathematical basis of the relationship between mass and weight. The distinction between mass and weight are clearly explained. The mathematical relationship between mass and weight are shown mathematically in terms of the equation for Newton's law of universal gravitation and in terms of his second law of motion.

Click to view content (https://www.khanacademy.org/embed_video?v=IuBoeDihLUc)

GRASP CHECK

Would you have the same mass on the moon as you do on Earth? Would you have the same weight?

a. You would weigh more on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
b. You would weigh less on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.
c. You would weigh less on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
d. You would weigh more on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.

Two equations involving the gravitational constant, G, are often useful. The first is Newton's equation, $\mathbf{F} = G\frac{mM}{r^2}$. Several of the values in this equation are either constants or easily obtainable. **F** is often the weight of an object on the surface of a large object with mass M, which is usually known. The mass of the smaller object, m, is often known, and G is a universal constant with the same value anywhere in the universe. This equation can be used to solve problems involving an object on or orbiting Earth or other massive celestial object. Sometimes it is helpful to equate the right-hand side of the equation to $m\mathbf{g}$ and cancel the m on both sides.

The equation $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ is also useful for problems involving objects in orbit. Note that there is no need to know the mass of the object. Often, we know the radius r or the period T and want to find the other. If these are both known, we can use the equation to calculate the mass of a planet or star.

WATCH PHYSICS

Mass and Weight Clarification

This video demonstrates calculations involving Newton's universal law of gravitation.

Click to view content (https://www.khanacademy.org/embed_video?v=391txUI76gM)

GRASP CHECK

Identify the constants g and G.

a. g and G are both the acceleration due to gravity
b. g is acceleration due to gravity on Earth and G is the universal gravitational constant.
c. g is the gravitational constant and G is the acceleration due to gravity on Earth.
d. g and G are both the universal gravitational constant.

WORKED EXAMPLE

Change in g

The value of **g** on the planet Mars is 3.71 m/s^2. If you have a mass of 60.0 kg on Earth, what would be your mass on Mars? What would be your weight on Mars?

Strategy

Weight equals acceleration due to gravity times mass: $\mathbf{W} = m\mathbf{g}$. An object's mass is constant. Call acceleration due to gravity on Mars $\mathbf{g}_M$ and weight on Mars $\mathbf{W}_M$.

Solution

Mass on Mars would be the same, 60 kg.

$$\mathbf{W}_M = m\mathbf{g}_M = (60.0\text{ kg})\left(3.71\text{m/s}^2\right) = 223\text{ N} \tag{7.4}$$

Discussion

The value of **g** on any planet depends on the mass of the planet and the distance from its center. If the material below the surface varies from point to point, the value of **g** will also vary slightly.

WORKED EXAMPLE

Earth's g at the Moon

Find the acceleration due to Earth's gravity at the distance of the moon.

$$\begin{aligned} G &= 6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2 \\ \text{Earth-moon distance} &= 3.84 \times 10^8\text{ m} \\ \text{Earth's mass} &= 5.98 \times 10^{24}\text{ kg} \end{aligned} \tag{7.5}$$

Express the force of gravity in terms of g.

$$\text{F} = \text{W} = m\text{a} = mg \tag{7.6}$$

Combine with the equation for universal gravitation.

$$m\mathbf{g} = mG\frac{M}{r^2} \tag{7.7}$$

Solution

Cancel m and substitute.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(\frac{5.98 \times 10^{24}\text{ kg}}{\left(3.84 \times 10^8\text{ m}\right)^2}\right) = 2.70 \times 10^{-3}\text{m/s}^2 \tag{7.8}$$

Discussion

The value of **g** for the moon is 1.62 m/s². Comparing this value to the answer, we see that Earth's gravitational influence on an object on the moon's surface would be insignificant.

Practice Problems

6. What is the mass of a person who weighs 600 N?
   a. 6.00 kg
   b. 61.2 kg
   c. 600 kg
   d. 610 kg

7. Calculate Earth's mass given that the acceleration due to gravity at the North Pole is 9.830m/s^2 and the radius of the Earth is 6371 km from pole to center.
   a. 5.94×10^{17} kg
   b. 5.94×10^{24} kg
   c. 9.36×10^{17} kg
   d. 9.36×10^{24} kg

Check Your Understanding

8. Some of Newton's predecessors and contemporaries also studied gravity and proposed theories. What important advance did Newton make in the study of gravity that the other scientists had failed to do?
   a. He gave an exact mathematical form for the theory.

b. He added a correction term to a previously existing formula.
c. Newton found the value of the universal gravitational constant.
d. Newton showed that gravitational force is always attractive.

9. State the law of universal gravitation in words only.

a. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the square of the distance between them.
b. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
c. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the distance between them.
d. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the distance between them.

10. Newton's law of universal gravitation explains the paths of what?

a. A charged particle
b. A ball rolling on a plane surface
c. A planet moving around the sun
d. A stone tied to a string and whirled at constant speed in a horizontal circle

CHAPTER 9
Work, Energy, and Simple Machines

Figure 9.1 People on a roller coaster experience thrills caused by changes in types of energy. (Jonrev, Wikimedia Commons)

Chapter Outline

INTRODUCTION Roller coasters have provided thrills for daring riders around the world since the nineteenth century. Inventors of roller coasters used simple physics to build the earliest examples using railroad tracks on mountainsides and old mines. Modern roller coaster designers use the same basic laws of physics to create the latest amusement park favorites. Physics principles are used to engineer the machines that do the work to lift a roller coaster car up its first big incline before it is set loose to roll. Engineers also have to understand the changes in the car's energy that keep it speeding over hills, through twists, turns, and even loops.

What exactly is energy? How can changes in force, energy, and simple machines move objects like roller coaster cars? How can machines help us do work? In this chapter, you will discover the answer to this question and many more, as you learn about work, energy, and simple machines.

9.1 Work, Power, and the Work–Energy Theorem

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe and apply the work–energy theorem
- Describe and calculate work and power

Section Key Terms

energy	gravitational potential energy	joule	kinetic energy	mechanical energy
potential energy	power	watt	work	work–energy theorem

The Work–Energy Theorem

In physics, the term **work** has a very specific definition. Work is application of force, $\mathbf{f}$, to move an object over a distance, d, in the direction that the force is applied. Work, W, is described by the equation

$$W = \mathbf{f}d.$$

Some things that we typically consider to be work are not work in the scientific sense of the term. Let's consider a few examples. Think about why each of the following statements is true.

- Homework *is not* work.
- Lifting a rock upwards off the ground *is* work.
- Carrying a rock in a straight path across the lawn at a constant speed *is not* work.

The first two examples are fairly simple. Homework is not work because objects are not being moved over a distance. Lifting a rock up off the ground is work because the rock is moving in the direction that force is applied. The last example is less obvious. Recall from the laws of motion that force is *not* required to move an object at constant velocity. Therefore, while some force may be applied to keep the rock up off the ground, no net force is applied to keep the rock moving forward at constant velocity.

Work and **energy** are closely related. When you do work to move an object, you change the object's energy. You (or an object) also expend energy to do work. In fact, energy can be defined as the ability to do work. Energy can take a variety of different forms, and one form of energy can transform to another. In this chapter we will be concerned with **mechanical energy**, which comes in two forms: **kinetic energy** and **potential energy**.

- Kinetic energy is also called energy of motion. A moving object has kinetic energy.
- Potential energy, sometimes called stored energy, comes in several forms. **Gravitational potential energy** is the stored energy an object has as a result of its position above Earth's surface (or another object in space). A roller coaster car at the top of a hill has gravitational potential energy.

Let's examine how doing work on an object changes the object's energy. If we apply force to lift a rock off the ground, we increase the rock's potential energy, *PE*. If we drop the rock, the force of gravity increases the rock's kinetic energy as the rock moves downward until it hits the ground.

The force we exert to lift the rock is equal to its weight, w, which is equal to its mass, m, multiplied by acceleration due to gravity, $\mathbf{g}$.

$$\mathbf{f} = w = m\mathbf{g}$$

The work we do on the rock equals the force we exert multiplied by the distance, d, that we lift the rock. The work we do on the rock also equals the rock's gain in gravitational potential energy, PE_e.

$$W = PE_e = \mathbf{f}m\mathbf{g}$$

Kinetic energy depends on the mass of an object and its velocity, $\mathbf{v}$.

$$KE = \frac{1}{2}m\mathbf{v}^2$$

When we drop the rock the force of gravity causes the rock to fall, giving the rock kinetic energy. When work done on an object increases only its kinetic energy, then the net work equals the change in the value of the quantity $\frac{1}{2}m\mathbf{v}^2$. This is a statement of the **work–energy theorem**, which is expressed mathematically as

$$W = \Delta KE = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2.$$

The subscripts $_2$ and $_1$ indicate the final and initial velocity, respectively. This theorem was proposed and successfully tested by James Joule, shown in Figure 9.2.

Does the name Joule sound familiar? The **joule** (J) is the metric unit of measurement for both work and energy. The measurement of work and energy with the same unit reinforces the idea that work and energy are related and can be converted into one another. 1.0 J = 1.0 N•m, the units of force multiplied by distance. 1.0 N = 1.0 k•m/s^2, so 1.0 J = 1.0 k•m^2/s^2. Analyzing the units of the term (1/2)$m\mathbf{v}^2$ will produce the same units for joules.

Figure 9.2 The joule is named after physicist James Joule (1818–1889). (C. H. Jeens, Wikimedia Commons)

WATCH PHYSICS

Work and Energy

This video explains the work energy theorem and discusses how work done on an object increases the object's KE.

Click to view content (https://www.khanacademy.org/embed_video?v=2WS1sG9fhOk)

GRASP CHECK

True or false—The energy increase of an object acted on only by a gravitational force is equal to the product of the object's weight and the distance the object falls.

a. True
b. False

Calculations Involving Work and Power

In applications that involve work, we are often interested in how fast the work is done. For example, in roller coaster design, the amount of time it takes to lift a roller coaster car to the top of the first hill is an important consideration. Taking a half hour on the ascent will surely irritate riders and decrease ticket sales. Let's take a look at how to calculate the time it takes to do work.

Recall that a rate can be used to describe a quantity, such as work, over a period of time. **Power** is the rate at which work is done. In this case, rate means *per unit of time*. Power is calculated by dividing the work done by the time it took to do the work.

$$P = \frac{W}{t}$$

Let's consider an example that can help illustrate the differences among work, force, and power. Suppose the woman in Figure 9.3 lifting the TV with a pulley gets the TV to the fourth floor in two minutes, and the man carrying the TV up the stairs takes five minutes to arrive at the same place. They have done the same amount of work $(\mathbf{f}d)$ on the TV, because they have moved the same mass over the same vertical distance, which requires the same amount of upward force. However, the woman using the pulley has generated more power. This is because she did the work in a shorter amount of time, so the denominator of the power formula, t, is smaller. (For simplicity's sake, we will leave aside for now the fact that the man climbing the stairs has also done work on himself.)

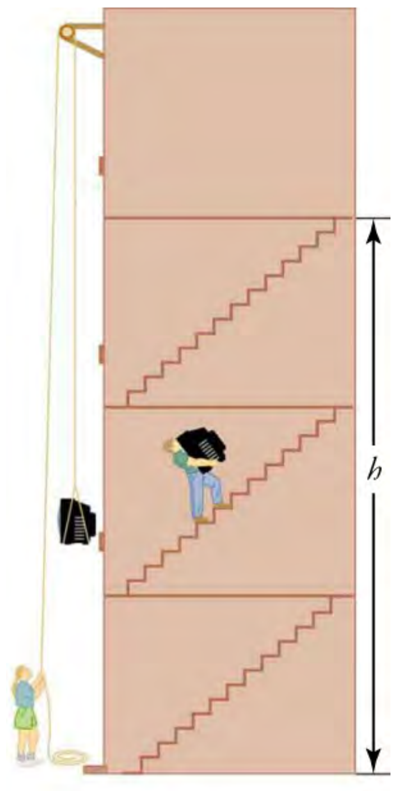

Figure 9.3 No matter how you move a TV to the fourth floor, the amount of work performed and the potential energy gain are the same.

Power can be expressed in units of **watts** (W). This unit can be used to measure power related to any form of energy or work. You have most likely heard the term used in relation to electrical devices, especially light bulbs. Multiplying power by time gives the amount of energy. Electricity is sold in kilowatt-hours because that equals the amount of electrical energy consumed.

The watt unit was named after James Watt (1736–1819) (see Figure 9.4). He was a Scottish engineer and inventor who discovered how to coax more power out of steam engines.

Figure 9.4 Is James Watt thinking about watts? (Carl Frederik von Breda, Wikimedia Commons)

LINKS TO PHYSICS

Watt's Steam Engine

James Watt did not invent the steam engine, but by the time he was finished tinkering with it, it was more useful. The first steam engines were not only inefficient, they only produced a back and forth, or reciprocal, motion. This was natural because pistons move in and out as the pressure in the chamber changes. This limitation was okay for simple tasks like pumping water or mashing potatoes, but did not work so well for moving a train. Watt was able build a steam engine that converted reciprocal motion to circular motion. With that one innovation, the industrial revolution was off and running. The world would never be the same. One of Watt's steam engines is shown in Figure 9.5. The video that follows the figure explains the importance of the steam engine in the industrial revolution.

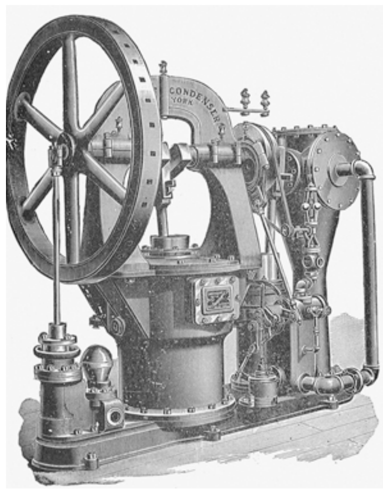

Figure 9.5 A late version of the Watt steam engine. (Nehemiah Hawkins, Wikimedia Commons)

WATCH PHYSICS

Watt's Role in the Industrial Revolution

This video demonstrates how the watts that resulted from Watt's inventions helped make the industrial revolution possible and allowed England to enter a new historical era.

Click to view content (https://www.youtube.com/embed/zhL5DCizj5c)

GRASP CHECK

Which form of mechanical energy does the steam engine generate?

a. Potential energy
b. Kinetic energy
c. Nuclear energy
d. Solar energy

Before proceeding, be sure you understand the distinctions among force, work, energy, and power. Force exerted on an object over a distance does work. Work can increase energy, and energy can do work. Power is the rate at which work is done.

WORKED EXAMPLE

Applying the Work–Energy Theorem

An ice skater with a mass of 50 kg is gliding across the ice at a speed of 8 m/s when her friend comes up from behind and gives her a push, causing her speed to increase to 12 m/s. How much work did the friend do on the skater?

Strategy

The work–energy theorem can be applied to the problem. Write the equation for the theorem and simplify it if possible.

$$W = \Delta \mathrm{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$$

$$\text{Simplify to } W = \frac{1}{2}m(\mathbf{v}_2^2 - \mathbf{v}_1^2)$$

Solution

Identify the variables. $m = 50$ kg,

$$\mathbf{v}_2 = 12\frac{\mathrm{m}}{\mathrm{s}}, \text{ and } \mathbf{v}_1 = 8\frac{\mathrm{m}}{\mathrm{s}} \tag{9.1}$$

Substitute.

$$W = \frac{1}{2}50(12^2 - 8^2) = 2,000 \ \mathrm{J} \tag{9.2}$$

Discussion

Work done on an object or system increases its energy. In this case, the increase is to the skater's kinetic energy. It follows that the increase in energy must be the difference in KE before and after the push.

TIPS FOR SUCCESS

This problem illustrates a general technique for approaching problems that require you to apply formulas: Identify the unknown and the known variables, express the unknown variables in terms of the known variables, and then enter all the known values.

Practice Problems

1. How much work is done when a weightlifter lifts a 200 N barbell from the floor to a height of 2 m?
   a. 0 J
   b. 100 J
   c. 200 J
   d. 400 J

2. Identify which of the following actions generates more power. Show your work.
   - carrying a 100 N TV to the second floor in 50 s or
   - carrying a 24 N watermelon to the second floor in 10 s?

   a. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as work done times the time interval.
   b. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as the ratio of work done to the time interval.
   c. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as work done times the time interval.
   d. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as the ratio of work done and the time interval.

Check Your Understanding

3. Identify two properties that are expressed in units of joules.
   a. work and force
   b. energy and weight
   c. work and energy
   d. weight and force

4. When a coconut falls from a tree, work W is done on it as it falls to the beach. This work is described by the equation

$$W = Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2. \tag{9.3}$$

Identify the quantities F, d, m, v_1, and v_2 in this event.

a. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
b. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
c. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.
d. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.

9.2 Mechanical Energy and Conservation of Energy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the law of conservation of energy in terms of kinetic and potential energy
- Perform calculations related to kinetic and potential energy. Apply the law of conservation of energy

Section Key Terms

law of conservation of energy

Mechanical Energy and Conservation of Energy

We saw earlier that mechanical energy can be either potential or kinetic. In this section we will see how energy is transformed from one of these forms to the other. We will also see that, in a closed system, the sum of these forms of energy remains constant.

Quite a bit of potential energy is gained by a roller coaster car and its passengers when they are raised to the top of the first hill. Remember that the *potential* part of the term means that energy has been stored and can be used at another time. You will see that this stored energy can either be used to do work or can be transformed into kinetic energy. For example, when an object that has gravitational potential energy falls, its energy is converted to kinetic energy. Remember that both work and energy are expressed in joules.

Refer back to . The amount of work required to raise the TV from point A to point B is equal to the amount of gravitational potential energy the TV gains from its height above the ground. This is generally true for any object raised above the ground. If all the work done on an object is used to raise the object above the ground, the amount work equals the object's gain in gravitational potential energy. However, note that because of the work done by friction, these energy–work transformations are never perfect. Friction causes the loss of some useful energy. In the discussions to follow, we will use the approximation that transformations are frictionless.

Now, let's look at the roller coaster in Figure 9.6. Work was done on the roller coaster to get it to the top of the first rise; at this point, the roller coaster has gravitational potential energy. It is moving slowly, so it also has a small amount of kinetic energy. As the car descends the first slope, its PE is converted to KE. At the low point much of the original PE has been transformed to KE, and speed is at a maximum. As the car moves up the next slope, some of the KE is transformed back into PE and the car slows down.

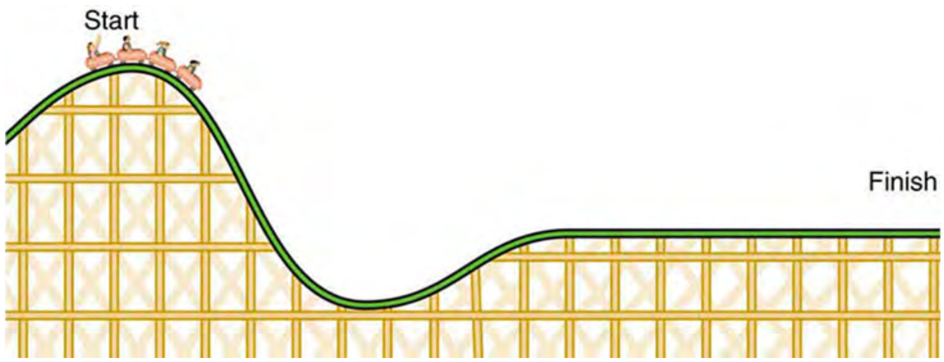

Figure 9.6 During this roller coaster ride, there are conversions between potential and kinetic energy.

Virtual Physics

Energy Skate Park Basics

This simulation shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in *KE* and *PE* by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation.

Click to view content (http://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

GRASP CHECK

This simulation (http://phet.colorado.edu/en/simulation/energy-skate-park-basics (http://phet.colorado.edu/en/simulation/energy-skate-park-basics)) shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in KE and PE by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation. The bar graphs show how KE and PE are transformed back and forth. Which statement best explains what happens to the mechanical energy of the system as speed is increasing?

a. The mechanical energy of the system increases, provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
b. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
c. The mechanical energy of the system increases provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.
d. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.

On an actual roller coaster, there are many ups and downs, and each of these is accompanied by transitions between kinetic and potential energy. Assume that no energy is lost to friction. At any point in the ride, the total mechanical energy is the same, and it is equal to the energy the car had at the top of the first rise. This is a result of the **law of conservation of energy**, which says that, in a closed system, total energy is conserved—that is, it is constant. Using subscripts 1 and 2 to represent initial and final energy, this law is expressed as

$$KE_1 + PE_1 = KE_2 + PE_2.$$

Either side equals the total mechanical energy. The phrase *in a closed system* means we are assuming no energy is lost to the surroundings due to friction and air resistance. If we are making calculations on dense falling objects, this is a good assumption. For the roller coaster, this assumption introduces some inaccuracy to the calculation.

Calculations involving Mechanical Energy and Conservation of Energy

TIPS FOR SUCCESS

When calculating work or energy, use units of meters for distance, newtons for force, kilograms for mass, and seconds for time. This will assure that the result is expressed in joules.

WATCH PHYSICS

Conservation of Energy

This video discusses conversion of *PE* to *KE* and conservation of energy. The scenario is very similar to the roller coaster and the skate park. It is also a good explanation of the energy changes studied in the snap lab.

Click to view content (https://www.khanacademy.org/embed_video?v=kw_4Loo1HR4)

GRASP CHECK

Did you expect the speed at the bottom of the slope to be the same as when the object fell straight down? Which statement best explains why this is not exactly the case in real-life situations?

a. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
b. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
c. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.
d. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.

WORKED EXAMPLE

Applying the Law of Conservation of Energy

A 10 kg rock falls from a 20 m cliff. What is the kinetic and potential energy when the rock has fallen 10 m?

Strategy

Choose the equation.

$$KE_1 + PE_1 = KE_2 + PE_2 \tag{9.4}$$

$$KE = \frac{1}{2}m\mathbf{v}^2; \quad PE = m\mathbf{g}h \tag{9.5}$$

$$\frac{1}{2}m\mathbf{v}_1^2 + m\mathbf{g}h_1 = \frac{1}{2}m\mathbf{v}_2^2 + m\mathbf{g}h_2 \tag{9.6}$$

List the knowns.

$m = 10\text{ kg}$, $\mathbf{v}_1 = 0$, $\mathbf{g} = 9.80$

$$\frac{\text{m}}{\text{s}^2}, \tag{9.7}$$

$h_1 = 20\text{ m}$, $h_2 = 10\text{ m}$

Identify the unknowns.

KE_2 and PE_2

Substitute the known values into the equation and solve for the unknown variables.

Solution

$$PE_2 = mgh_2 = 10\,(9.80)\,10 = 980 \text{ J} \tag{9.8}$$

$$KE_2 = PE_2 - (KE_1 + PE_1) = 980 - \{[0 - [10\,(9.80)\,20]]\} = 980 \text{ J} \tag{9.9}$$

Discussion

Alternatively, conservation of energy equation could be solved for $\mathbf{v}_2$ and KE_2 could be calculated. Note that m could also be eliminated.

TIPS FOR SUCCESS

Note that we can solve many problems involving conversion between KE and PE without knowing the mass of the object in question. This is because kinetic and potential energy are both proportional to the mass of the object. In a situation where $KE = PE$, we know that $m\mathbf{g}h = (1/2)m\mathbf{v}^2$.
Dividing both sides by m and rearranging, we have the relationship

$2\mathbf{g}h = \mathbf{v}^2$.

Practice Problems

5. A child slides down a playground slide. If the slide is 3 m high and the child weighs 300 N, how much potential energy does the child have at the top of the slide? (Round g to $10\ m/s^2$.)
   a. 0 J
   b. 100 J
   c. 300 J
   d. 900 J

6. A 0.2 kg apple on an apple tree has a potential energy of 10 J. It falls to the ground, converting all of its PE to kinetic energy. What is the velocity of the apple just before it hits the ground?
   a. 0 m/s
   b. 2 m/s
   c. 10 m/s
   d. 50 m/s

Snap Lab

Converting Potential Energy to Kinetic Energy

In this activity, you will calculate the potential energy of an object and predict the object's speed when all that potential energy has been converted to kinetic energy. You will then check your prediction.

You will be dropping objects from a height. Be sure to stay a safe distance from the edge. Don't lean over the railing too far. Make sure that you do not drop objects into an area where people or vehicles pass by. Make sure that dropping objects will not cause damage.

You will need the following:

Materials for each pair of students:
- Four marbles (or similar small, dense objects)
- Stopwatch

Materials for class:
- Metric measuring tape long enough to measure the chosen height
- A scale

Instructions

Procedure

1. Work with a partner. Find and record the mass of four small, dense objects per group.
2. Choose a location where the objects can be safely dropped from a height of at least 15 meters. A bridge over water with a safe pedestrian walkway will work well.
3. Measure the distance the object will fall.
4. Calculate the potential energy of the object before you drop it using $PE = m\mathbf{g}h = (9.80)mh$.
5. Predict the kinetic energy and velocity of the object when it lands using $PE = KE$ and so, $m\mathbf{g}h = \frac{m\mathbf{v}^2}{2}$; $\mathbf{v} = \sqrt{2(9.80)h} = 4.43\sqrt{h}$.
6. One partner drops the object while the other measures the time it takes to fall.
7. Take turns being the dropper and the timer until you have made four measurements.
8. Average your drop multiplied by and calculate the velocity of the object when it landed using $\mathbf{v} = \mathbf{a}t = \mathbf{g}t = (9.80)t$.
9. Compare your results to your prediction.

GRASP CHECK

Galileo's experiments proved that, contrary to popular belief, heavy objects do not fall faster than light objects. How do the equations you used support this fact?

a. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
b. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
c. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.
d. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.

Check Your Understanding

7. Describe the transformation between forms of mechanical energy that is happening to a falling skydiver before his parachute opens.
   a. Kinetic energy is being transformed into potential energy.
   b. Potential energy is being transformed into kinetic energy.
   c. Work is being transformed into kinetic energy.
   d. Kinetic energy is being transformed into work.

8. True or false—If a rock is thrown into the air, the increase in the height would increase the rock's kinetic energy, and then the increase in the velocity as it falls to the ground would increase its potential energy.
   a. True
   b. False

9. Identify equivalent terms for *stored energy* and *energy of motion*.
   a. Stored energy is potential energy, and energy of motion is kinetic energy.
   b. Energy of motion is potential energy, and stored energy is kinetic energy.
   c. Stored energy is the potential as well as the kinetic energy of the system.
   d. Energy of motion is the potential as well as the kinetic energy of the system.

9.3 Simple Machines

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe simple and complex machines
- Calculate mechanical advantage and efficiency of simple and complex machines

Section Key Terms

complex machine	efficiency output	ideal mechanical advantage	inclined plane	input work
lever	mechanical advantage	output work	pulley	screw
simple machine	wedge	wheel and axle		

Simple Machines

Simple machines make work easier, but they do not decrease the amount of work you have to do. Why can't simple machines change the amount of work that you do? Recall that in closed systems the total amount of energy is conserved. A machine cannot increase the amount of energy you put into it. So, why is a simple machine useful? Although it cannot change the amount of work you do, a simple machine can change the amount of force you must apply to an object, and the distance over which you apply the force. In most cases, a simple machine is used to reduce the amount of force you must exert to do work. The down side is that you must exert the force over a greater distance, because the product of force and distance, $\mathbf{f}d$, (which equals work) does not change.

Let's examine how this works in practice. In Figure 9.7(a), the worker uses a type of **lever** to exert a small force over a large distance, while the pry bar pulls up on the nail with a large force over a small distance. Figure 9.7(b) shows the how a lever works mathematically. The effort force, applied at $\mathbf{F_e}$, lifts the load (the resistance force) which is pushing down at $\mathbf{F}_r$. The triangular pivot is called the fulcrum; the part of the lever between the fulcrum and $\mathbf{F}_e$ is the effort arm, L_e; and the part to the left is the resistance arm, L_r. The **mechanical advantage** is a number that tells us how many times a simple machine multiplies the effort force. The **ideal mechanical advantage**, *IMA*, is the mechanical advantage of a perfect machine with no loss of useful work caused by friction between moving parts. The equation for *IMA* is shown in Figure 9.7(b).

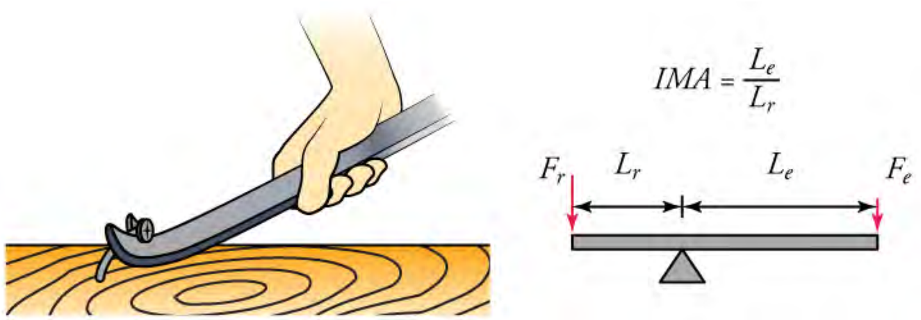

Figure 9.7 (a) A pry bar is a type of lever. (b) The ideal mechanical advantage equals the length of the effort arm divided by the length of the resistance arm of a lever.

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Getting back to conservation of energy, for any simple machine, the work put into the machine, W_i, equals the work the machine puts out, W_o. Combining this with the information in the paragraphs above, we can write

$$W_i = W_o$$
$$\mathbf{F}_e d_e = \mathbf{F}_r d_r$$
$$\text{If } \mathbf{F}_e < \mathbf{F}_r, \text{ then } d_e > d_r.$$

The equations show how a simple machine can output the same amount of work while reducing the amount of effort force by increasing the distance over which the effort force is applied.

WATCH PHYSICS

Introduction to Mechanical Advantage

This video shows how to calculate the *IMA* of a lever by three different methods: (1) from effort force and resistance force; (2) from the lengths of the lever arms, and; (3) from the distance over which the force is applied and the distance the load moves.

Click to view content (https://www.youtube.com/embed/pfzJ-z5Ij48)

GRASP CHECK

Two children of different weights are riding a seesaw. How do they position themselves with respect to the pivot point (the fulcrum) so that they are balanced?

a. The heavier child sits closer to the fulcrum.
b. The heavier child sits farther from the fulcrum.
c. Both children sit at equal distance from the fulcrum.
d. Since both have different weights, they will never be in balance.

Some levers exert a large force to a short effort arm. This results in a smaller force acting over a greater distance at the end of the resistance arm. Examples of this type of lever are baseball bats, hammers, and golf clubs. In another type of lever, the fulcrum is at the end of the lever and the load is in the middle, as in the design of a wheelbarrow.

The simple machine shown in Figure 9.8 is called a **wheel and axle.** It is actually a form of lever. The difference is that the effort arm can rotate in a complete circle around the fulcrum, which is the center of the axle. Force applied to the outside of the wheel causes a greater force to be applied to the rope that is wrapped around the axle. As shown in the figure, the ideal mechanical advantage is calculated by dividing the radius of the wheel by the radius of the axle. Any crank-operated device is an example of a wheel and axle.

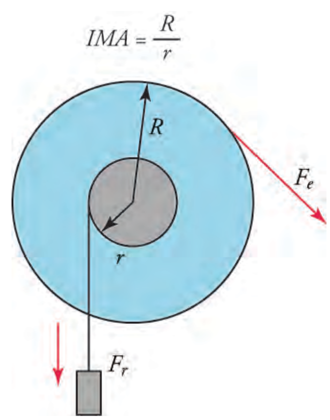

Figure 9.8 Force applied to a wheel exerts a force on its axle.

An **inclined plane** and a **wedge** are two forms of the same simple machine. A wedge is simply two inclined planes back to back. Figure 9.9 shows the simple formulas for calculating the *IMA*s of these machines. All sloping, paved surfaces for walking or driving are inclined planes. Knives and axe heads are examples of wedges.

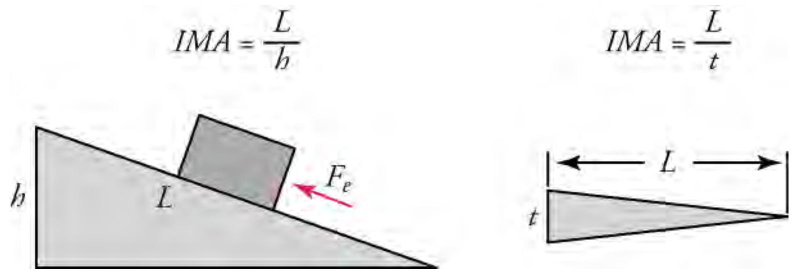

Figure 9.9 An inclined plane is shown on the left, and a wedge is shown on the right.

The **screw** shown in Figure 9.10 is actually a lever attached to a circular inclined plane. Wood screws (of course) are also examples of screws. The lever part of these screws is a screw driver. In the formula for *IMA*, the distance between screw threads is called *pitch* and has the symbol *P*.

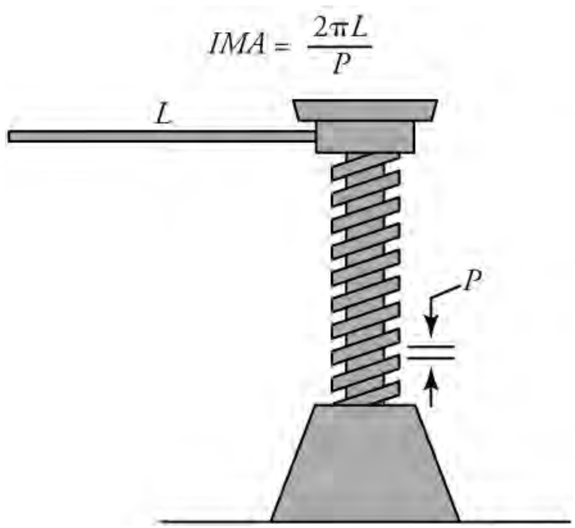

Figure 9.10 The screw shown here is used to lift very heavy objects, like the corner of a car or a house a short distance.

Figure 9.11 shows three different **pulley** systems. Of all simple machines, mechanical advantage is easiest to calculate for pulleys. Simply count the number of ropes supporting the load. That is the *IMA*. Once again we have to exert force over a longer distance to multiply force. To raise a load 1 meter with a pulley system you have to pull *N* meters of rope. Pulley systems are often used to raise flags and window blinds and are part of the mechanism of construction cranes.

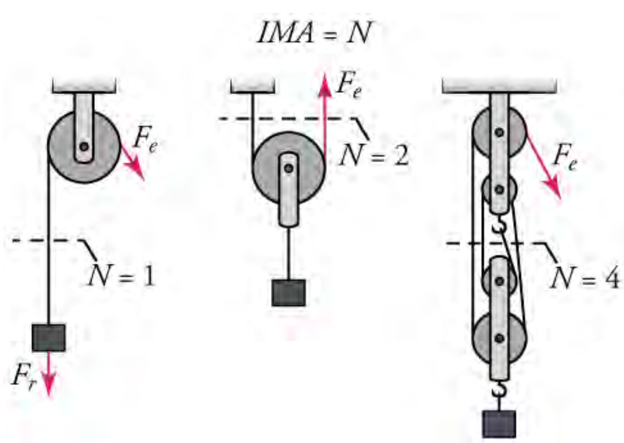

Figure 9.11 Three pulley systems are shown here.

WATCH PHYSICS

Mechanical Advantage of Inclined Planes and Pulleys

The first part of this video shows how to calculate the *IMA* of pulley systems. The last part shows how to calculate the *IMA* of an inclined plane.

Click to view content (https://www.khanacademy.org/embed_video?v=vSsK7Rfa3yA)

GRASP CHECK

How could you use a pulley system to lift a light load to great height?

a. Reduce the radius of the pulley.
b. Increase the number of pulleys.
c. Decrease the number of ropes supporting the load.
d. Increase the number of ropes supporting the load.

A **complex machine** is a combination of two or more simple machines. The wire cutters in Figure 9.12 combine two levers and two wedges. Bicycles include wheel and axles, levers, screws, and pulleys. Cars and other vehicles are combinations of many machines.

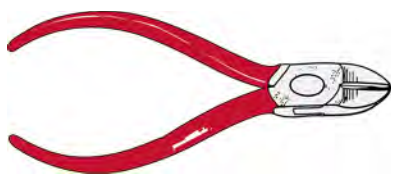

Figure 9.12 Wire cutters are a common complex machine.

Calculating Mechanical Advantage and Efficiency of Simple Machines

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Refer back to the discussions of each simple machine for the specific equations for the *IMA* for each type of machine.

No simple or complex machines have the actual mechanical advantages calculated by the *IMA* equations. In real life, some of the applied work always ends up as wasted heat due to friction between moving parts. Both the **input work** (W_i) and **output work** (W_o) are the result of a force, **F**, acting over a distance, d.

$$W_i = \mathbf{F}_i d_i \text{and} W_o = \mathbf{F}_o d_o$$

The **efficiency output** of a machine is simply the output work divided by the input work, and is usually multiplied by 100 so that it is expressed as a percent.

$$\%\ \text{efficiency} = \frac{W_o}{W_i} \times 100$$

Look back at the pictures of the simple machines and think about which would have the highest efficiency. Efficiency is related to friction, and friction depends on the smoothness of surfaces and on the area of the surfaces in contact. How would lubrication affect the efficiency of a simple machine?

WORKED EXAMPLE

Efficiency of a Lever

The input force of 11 N acting on the effort arm of a lever moves 0.4 m, which lifts a 40 N weight resting on the resistance arm a distance of 0.1 m. What is the efficiency of the machine?

Strategy

State the equation for efficiency of a simple machine, $\%\text{ efficiency} = \frac{W_o}{W_i} \times 100$, and calculate W_o and W_i. Both work values are the product *Fd*.

Solution

$W_i = \mathbf{F}_i d_i = (11)(0.4) = 4.4\text{ J}$ and $W_o = \mathbf{F}_o d_o = (40)(0.1) = 4.0\text{ J}$, then $\%\text{ efficiency} = \frac{W_o}{W_i} \times 100 = \frac{4.0}{4.4} \times 100 = 91\%$

Discussion

Efficiency in real machines will always be less than 100 percent because of work that is converted to unavailable heat by friction and air resistance. W_o and W_i can always be calculated as a force multiplied by a distance, although these quantities are not always as obvious as they are in the case of a lever.

Practice Problems

10. What is the IMA of an inclined plane that is 5 m long and 2 m high?

a. 0.4
b. 2.5
c. 0.4 m
d. 2.5 m

11. If a pulley system can lift a 200N load with an effort force of 52 N and has an efficiency of almost 100 percent, how many ropes are supporting the load?

a. 1 rope is required because the actual mechanical advantage is 0.26.
b. 1 rope is required because the actual mechanical advantage is 3.80.
c. 4 ropes are required because the actual mechanical advantage is 0.26.
d. 4 ropes are required because the actual mechanical advantage is 3.80.

Check Your Understanding

12. True or false—The efficiency of a simple machine is always less than 100 percent because some small fraction of the input work is always converted to heat energy due to friction.

a. True
b. False

13. The circular handle of a faucet is attached to a rod that opens and closes a valve when the handle is turned. If the rod has a diameter of 1 cm and the IMA of the machine is 6, what is the radius of the handle?

A. 0.08 cm
B. 0.17 cm
C. 3.0 cm
D. 6.0 cm

KEY TERMS

complex machine a machine that combines two or more simple machines
efficiency output work divided by input work
energy the ability to do work
gravitational potential energy energy acquired by doing work against gravity
ideal mechanical advantage the mechanical advantage of an idealized machine that loses no energy to friction
inclined plane a simple machine consisting of a slope
input work effort force multiplied by the distance over which it is applied
joule the metric unit for work and energy; equal to 1 newton meter (N•m)
kinetic energy energy of motion
law of conservation of energy states that energy is neither created nor destroyed
lever a simple machine consisting of a rigid arm that pivots on a fulcrum
mechanical advantage the number of times the input force is multiplied
mechanical energy kinetic or potential energy
output work output force multiplied by the distance over which it acts
potential energy stored energy
power the rate at which work is done
pulley a simple machine consisting of a rope that passes over one or more grooved wheels
screw a simple machine consisting of a spiral inclined plane
simple machine a machine that makes work easier by changing the amount or direction of force required to move an object
watt the metric unit of power; equivalent to joules per second
wedge a simple machine consisting of two back-to-back inclined planes
wheel and axle a simple machine consisting of a rod fixed to the center of a wheel
work force multiplied by distance
work–energy theorem states that the net work done on a system equals the change in kinetic energy

SECTION SUMMARY

9.1 Work, Power, and the Work–Energy Theorem

- Doing work on a system or object changes its energy.
- The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.
- Power is the rate at which work is done.

9.2 Mechanical Energy and Conservation of Energy

- Mechanical energy may be either kinetic (energy of motion) or potential (stored energy).
- Doing work on an object or system changes its energy.
- Total energy in a closed, isolated system is constant.

9.3 Simple Machines

- The six types of simple machines make work easier by changing the **f***d* term so that force is reduced at the expense of increased distance.
- The ratio of output force to input force is a machine's mechanical advantage.
- Combinations of two or more simple machines are called complex machines.
- The ratio of output work to input work is a machine's efficiency.

KEY EQUATIONS

9.1 Work, Power, and the Work–Energy Theorem

equation for work	$W = \mathbf{f}d$
force	$\mathbf{f} = w = m\mathbf{g}$
work equivalencies	$W = PE_e = \mathbf{f}m\mathbf{g}$
kinetic energy	$KE = \frac{1}{2}m\mathbf{v}^2$
work–energy theorem	$W = \Delta \text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$
power	$P = \frac{W}{t}$

CHAPTER 14
Sound

Figure 14.1 This tree fell some time ago. When it fell, particles in the air were disturbed by the energy of the tree hitting the ground. This disturbance of matter, which our ears have evolved to detect, is called sound. (B.A. Bowen Photography)

Chapter Outline

INTRODUCTION If a tree falls in a forest (see Figure 14.1) and no one is there to hear it, does it make a sound? The answer to this old philosophical question depends on how you define sound. If sound only exists when someone is around to perceive it, then the falling tree produced no sound. However, in physics, we know that colliding objects can disturb the air, water or other matter surrounding them. As a result of the collision, the surrounding particles of matter began vibrating in a wave-like fashion. This is a sound wave. Consequently, if a tree collided with another object in space, no one would hear it, because no sound would be produced. This is because, in space, there is no air, water or other matter to be disturbed and produce sound waves. In this chapter, we'll learn more about the wave properties of sound, and explore hearing, as well as some special uses for sound.

14.1 Speed of Sound, Frequency, and Wavelength

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Relate the characteristics of waves to properties of sound waves
- Describe the speed of sound and how it changes in various media
- Relate the speed of sound to frequency and wavelength of a sound wave

Section Key Terms

rarefaction sound

Properties of Sound Waves

Sound is a wave. More specifically, sound is defined to be a disturbance of matter that is transmitted from its source outward. A disturbance is anything that is moved from its state of equilibrium. Some sound waves can be characterized as periodic waves, which means that the atoms that make up the matter experience simple harmonic motion.

A vibrating string produces a sound wave as illustrated in Figure 14.2, Figure 14.3, and Figure 14.4. As the string oscillates back and forth, part of the string's energy goes into compressing and expanding the surrounding air. This creates slightly higher and lower pressures. The higher pressure… regions are compressions, and the low pressure regions are **rarefactions**. The pressure disturbance moves through the air as longitudinal waves with the same frequency as the string. Some of the energy is lost in the form of thermal energy transferred to the air. You may recall from the chapter on waves that areas of compression and rarefaction in longitudinal waves (such as sound) are analogous to crests and troughs in transverse waves.

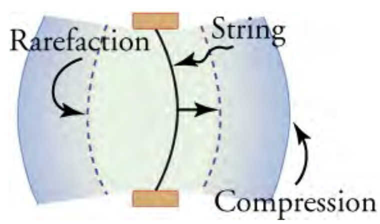

Figure 14.2 A vibrating string moving to the right compresses the air in front of it and expands the air behind it.

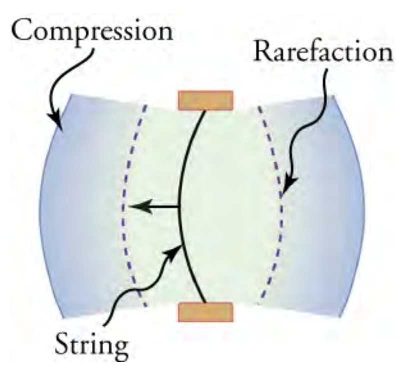

Figure 14.3 As the string moves to the left, it creates another compression and rarefaction as the particles on the right move away from the string.

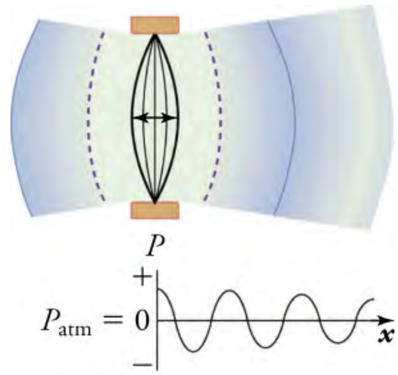

Figure 14.4 After many vibrations, there is a series of compressions and rarefactions that have been transmitted from the string as a sound wave. The graph shows gauge pressure (P_{gauge}) versus distance *x* from the source. Gauge pressure is the pressure relative to atmospheric pressure; it is positive for pressures above atmospheric pressure, and negative for pressures below it. For ordinary, everyday sounds, pressures vary only slightly from average atmospheric pressure.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But some of the energy is also absorbed by objects, such as the eardrum in Figure 14.5, and some of the energy is converted to thermal energy in the air. Figure 14.4 shows a graph of gauge pressure versus distance from the vibrating string. From this figure, you can see that the compression of a longitudinal wave is analogous to the peak of a transverse wave, and the rarefaction of a longitudinal wave is analogous to the trough of a transverse wave. Just as a transverse wave alternates between peaks and troughs, a longitudinal wave alternates between compression and rarefaction.

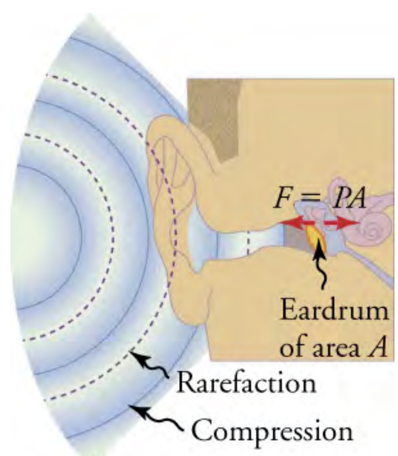

Figure 14.5 Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are then interpreted by the brain.

The Speed of Sound

The speed of sound varies greatly depending upon the medium it is traveling through. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. The greater the density of a medium, the slower the speed of sound. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases. Table 14.1 shows the speed of sound in various media. Since temperature affects density, the speed of sound varies with the temperature of the medium through which it's traveling to some extent, especially for gases.

Medium	v_w (m/s)
Gases at 0 °C	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at 20 °C	
Ethanol	1160
Mercury	1450
Water, fresh	1480
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Table 14.1 Speed of Sound in Various Media

The Relationship Between the Speed of Sound and the Frequency and Wavelength of a Sound Wave

Figure 14.6 When fireworks explode in the sky, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (Dominic Alves, Flickr)

Sound, like all waves, travels at certain speeds through different media and has the properties of frequency and wavelength. Sound travels much slower than light—you can observe this while watching a fireworks display (see Figure 14.6), since the flash of an explosion is seen before its sound is heard.

The relationship between the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v = f\lambda, \tag{14.1}$$

where v is the speed of sound (in units of m/s), f is its frequency (in units of hertz), and λ is its wavelength (in units of meters). Recall that wavelength is defined as the distance between adjacent identical parts of a wave. The wavelength of a sound, therefore, is the distance between adjacent identical parts of a sound wave. Just as the distance between adjacent crests in a transverse wave is one wavelength, the distance between adjacent compressions in a sound wave is also one wavelength, as shown in Figure 14.7. The frequency of a sound wave is the same as that of the source. For example, a tuning fork vibrating at a given frequency would produce sound waves that oscillate at that same frequency. The frequency of a sound is the number of waves that pass a point per unit time.

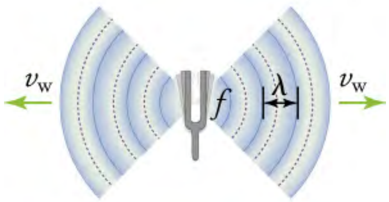

Figure 14.7 A sound wave emanates from a source vibrating at a frequency f, propagates at v, and has a wavelength λ .

One of the more important properties of sound is that its speed is nearly independent of frequency. If this were not the case, and high-frequency sounds traveled faster, for example, then the farther you were from a band in a football stadium, the more the sound from the low-pitch instruments would lag behind the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed.

Recall that $v = f\lambda$, and in a given medium under fixed temperature and humidity, v is constant. Therefore, the relationship between f and λ is inverse: The higher the frequency, the shorter the wavelength of a sound wave.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and maintains the frequency of the original source. If v changes and f remains the same, then the wavelength λ must change. Since $v = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Virtual Physics

Sound

Click to view content (https://www.openstax.org/l/28sound)

This simulation lets you see sound waves. Adjust the frequency or amplitude (volume) and you can see and hear how the wave changes. Move the listener around and hear what she hears. Switch to the Two Source Interference tab or the Interference by Reflection tab to experiment with interference and reflection.

TIPS FOR SUCCESS

Make sure to have audio enabled and set to Listener rather than Speaker, or else the sound will not vary as you move the listener around.

GRASP CHECK

In the first tab, Listen to a Single Source, move the listener as far away from the speaker as possible, and then change the frequency of the sound wave. You may have noticed that there is a delay between the time when you change the setting and the time when you hear the sound get lower or higher in pitch. Why is this?

a. Because, intensity of the sound wave changes with the frequency.
b. Because, the speed of the sound wave changes when the frequency is changed.
c. Because, loudness of the sound wave takes time to adjust after a change in frequency.
d. Because it takes time for sound to reach the listener, so the listener perceives the new frequency of sound wave after a delay.

Is there a difference in the amount of delay depending on whether you make the frequency higher or lower? Why?

a. Yes, the speed of propagation depends only on the frequency of the wave.
b. Yes, the speed of propagation depends upon the wavelength of the wave, and wavelength changes as the frequency changes.
c. No, the speed of propagation depends only on the wavelength of the wave.
d. No, the speed of propagation is constant in a given medium; only the wavelength changes as the frequency changes.

Snap Lab

Voice as a Sound Wave

In this lab you will observe the effects of blowing and speaking into a piece of paper in order to compare and contrast different sound waves.

- sheet of paper
- tape
- table

Instructions

Procedure

1. Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table, for example.
2. Gently blow air near the edge of the bottom of the sheet and note how the sheet moves.
3. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves.
4. Interpret the results.

GRASP CHECK

Which sound wave property increases when you are speaking more loudly than softly?

a. amplitude of the wave
b. frequency of the wave
c. speed of the wave

d. wavelength of the wave

WORKED EXAMPLE

What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in conditions where sound travels at 348.7 m/s.

STRATEGY

To find wavelength from frequency, we can use $v = f\lambda$.

Solution

(1) Identify the knowns. The values for v and f are given.

(2) Solve the relationship between speed, frequency and wavelength for λ .

$$\lambda = \frac{v}{f}. \tag{14.2}$$

(3) Enter the speed and the minimum frequency to give the maximum wavelength.

$$\lambda_{\text{max}} = \frac{348.7 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m} \approx 20 \text{ m (1 sig. figure)} \tag{14.3}$$

(4) Enter the speed and the maximum frequency to give the minimum wavelength.

$$\lambda_{\text{min}} = \frac{348.7 \text{ m/s}}{20,000 \text{ Hz}} = 0.017 \text{ m} \approx 2 \text{ cm (1 sig. figure)} \tag{14.4}$$

Discussion

Because the product of f multiplied by λ equals a constant velocity in unchanging conditions, the smaller f is, the larger λ must be, and vice versa. Note that you can also easily rearrange the same formula to find frequency or velocity.

Practice Problems

1. What is the speed of a sound wave with frequency 2000 Hz and wavelength 0.4 m?
   a. 5×10^3 m/s
   b. 3.2×10^2 m/s
   c. 2×10^{-4} m/s
   d. 8×10^2 m/s

2. Dogs can hear frequencies of up to 45 kHz. What is the wavelength of a sound wave with this frequency traveling in air at 0°C?
   a. 2.0×10^7 m
   b. 1.5×10^7 m
   c. 1.4×10^2 m
   d. 7.4×10^{-3} m

LINKS TO PHYSICS

Echolocation

Figure 14.8 A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

Echolocation is the use of reflected sound waves to locate and identify objects. It is used by animals such as bats, dolphins and whales, and is also imitated by humans in SONAR—Sound Navigation and Ranging—and echolocation technology.

Bats, dolphins and whales use echolocation to navigate and find food in their environment. They locate an object (or obstacle) by emitting a sound and then sensing the reflected sound waves. Since the speed of sound in air is constant, the time it takes for the sound to travel to the object and back gives the animal a sense of the distance between itself and the object. This is called *ranging*. Figure 14.8 shows a bat using echolocation to sense distances.

Echolocating animals identify an object by comparing the relative intensity of the sound waves returning to each ear to figure out the angle at which the sound waves were reflected. This gives information about the direction, size and shape of the object. Since there is a slight distance in position between the two ears of an animal, the sound may return to one of the ears with a bit of a delay, which also provides information about the position of the object. For example, if a bear is directly to the right of a bat, the echo will return to the bat's left ear later than to its right ear. If, however, the bear is directly ahead of the bat, the echo would return to both ears at the same time. For an animal without a sense of sight such as a bat, it is important to know *where* other animals are as well as *what* they are; their survival depends on it.

Principles of echolocation have been used to develop a variety of useful sensing technologies. SONAR, is used by submarines to detect objects underwater and measure water depth. Unlike animal echolocation, which relies on only one transmitter (a mouth) and two receivers (ears), manmade SONAR uses many transmitters and beams to get a more accurate reading of the environment. Radar technologies use the echo of radio waves to locate clouds and storm systems in weather forecasting, and to locate aircraft for air traffic control. Some new cars use echolocation technology to sense obstacles around the car, and warn the driver who may be about to hit something (or even to automatically parallel park). Echolocation technologies and training systems are being developed to help visually impaired people navigate their everyday environments.

GRASP CHECK

If a predator is directly to the left of a bat, how will the bat know?

a. The echo would return to the left ear first.
b. The echo would return to the right ear first.

Check Your Understanding

3. What is a rarefaction?
   a. Rarefaction is the high-pressure region created in a medium when a longitudinal wave passes through it.
   b. Rarefaction is the low-pressure region created in a medium when a longitudinal wave passes through it.
   c. Rarefaction is the highest point of amplitude of a sound wave.
   d. Rarefaction is the lowest point of amplitude of a sound wave.

4. What sort of motion do the particles of a medium experience when a sound wave passes through it?
   a. Simple harmonic motion

   b. Circular motion
   c. Random motion
   d. Translational motion

5. What does the speed of sound depend on?
   a. The wavelength of the wave
   b. The size of the medium
   c. The frequency of the wave
   d. The properties of the medium

6. What property of a gas would affect the speed of sound traveling through it?
   a. The volume of the gas
   b. The flammability of the gas
   c. The mass of the gas
   d. The compressibility of the gas

14.2 Sound Intensity and Sound Level

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Relate amplitude of a wave to loudness and energy of a sound wave
- Describe the decibel scale for measuring sound intensity
- Solve problems involving the intensity of a sound wave
- Describe how humans produce and hear sounds

Section Key Terms

amplitude	decibel	hearing	loudness
pitch	sound intensity	sound intensity level	

Amplitude, Loudness and Energy of a Sound Wave

Figure 14.9 Noise on crowded roadways like this one in Delhi makes it hard to hear others unless they shout. (Lingaraj G J, Flickr)

In a quiet forest, you can sometimes hear a single leaf fall to the ground. But in a traffic jam filled with honking cars, you may have to shout just so the person next to you can hear Figure 14.9.The loudness of a sound is related to how energetically its source is vibrating. In cartoons showing a screaming person, the cartoonist often shows an open mouth with a vibrating uvula (the hanging tissue at the back of the mouth) to represent a loud sound coming from the throat. Figure 14.10 shows such a cartoon depiction of a bird loudly expressing its opinion.

A useful quantity for describing the loudness of sounds is called **sound intensity**. In general, the intensity of a wave is the power per unit area carried by the wave. Power is the rate at which energy is transferred by the wave. In equation form, intensity I is

$$I = \frac{P}{A}, \tag{14.5}$$

where P is the power through an area A. The SI unit for I is W/m^2. The intensity of a sound depends upon its pressure amplitude.

The relationship between the intensity of a sound wave and its pressure amplitude (or pressure variation Δp) is

$$I = \frac{(\Delta p)^2}{2\rho v_w}, \tag{14.6}$$

where ρ is the density of the material in which the sound wave travels, in units of kg/m^3, and v is the speed of sound in the medium, in units of m/s. Pressure amplitude has units of pascals (Pa) or N/m^2. Note that Δp is half the difference between the maximum and minimum pressure in the sound wave.

We can see from the equation that the intensity of a sound is proportional to its amplitude squared. The pressure variation is proportional to the amplitude of the oscillation, and so I varies as $(\Delta p)^2$. This relationship is consistent with the fact that the sound wave is produced by some vibration; the greater its pressure amplitude, the more the air is compressed during the vibration. Because the power of a sound wave is the rate at which energy is transferred, the energy of a sound wave is also proportional to its amplitude squared.

TIPS FOR SUCCESS

Pressure is usually denoted by capital P, but we are using a lowercase p for pressure in this case to distinguish it from power P above.

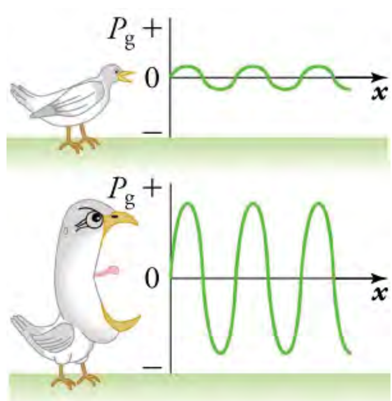

Figure 14.10 Graphs of the pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters.

The Decibel Scale

You may have noticed that when people talk about the loudness of a sound, they describe it in units of **decibels** rather than watts per meter squared. While sound intensity (in W/m^2) is the SI unit, the **sound intensity level** in decibels (dB) is more relevant for how humans perceive sounds. The way our ears perceive sound can be more accurately described by the logarithm of the intensity of a sound rather than the intensity of a sound directly. The sound intensity level β is defined to be

$$\beta\ (\text{dB}) = 10 \log_{10}\left(\frac{I}{I_0}\right), \tag{14.7}$$

where I is sound intensity in watts per meter squared, and $I_0 = 10^{-12}\ W/m^2$ is a reference intensity. I_0 is chosen as the reference point because it is the lowest intensity of sound a person with normal hearing can perceive. The decibel level of a sound having an intensity of $10^{-12}\ W/m^2$ is $\beta = 0$ dB, because $\log_{10} 1 = 0$. That is, the threshold of human hearing is 0 decibels.

Each factor of 10 in intensity corresponds to 10 dB. For example, a 90 dB sound compared with a 60 dB sound is 30 dB greater, or three factors of 10 (that is, 10^3 times) as intense. Another example is that if one sound is 10^7 as intense as another, it is 70 dB higher.

Since β is defined in terms of a ratio, it is unit-less. The unit called *deci*bel (dB) is used to indicate that this ratio is multiplied by 10. The sound intensity level is not the same as sound intensity—it tells you the *level* of the sound relative to a reference intensity rather than the actual intensity.

Snap Lab

Feeling Sound

In this lab, you will play music with a heavy beat to literally feel the vibrations and explore what happens when the volume changes.

- CD player or portable electronic device connected to speakers
- rock or rap music CD or mp3
- a lightweight table

Procedure

1. Place the speakers on a light table, and start playing the CD or mp3.
2. Place your hand gently on the table next to the speakers.
3. Increase the volume and note the level when the table just begins to vibrate as the music plays.
4. Increase the reading on the volume control until it doubles. What has happened to the vibrations?

GRASP CHECK

Do you think that when you double the volume of a sound wave you are doubling the sound intensity level (in dB) or the sound intensity (in $\mathrm{W/m^2}$)? Why?

a. The sound intensity in $\mathrm{W/m^2}$, because it is a closer measure of how humans perceive sound.
b. The sound intensity level in dB because it is a closer measure of how humans perceive sound.
c. The sound intensity in $\mathrm{W/m^2}$ because it is the only unit to express the intensity of sound.
d. The sound intensity level in dB because it is the only unit to express the intensity of sound.

Solving Sound Wave Intensity Problems

WORKED EXAMPLE

Calculating Sound Intensity Levels: Sound Waves

Calculate the sound intensity level in decibels for a sound wave traveling in air at 0 °C and having a pressure amplitude of 0.656 Pa.

STRATEGY

We are given Δp, so we can calculate I using the equation $I = \frac{(\Delta p)^2}{2\rho v}$. Using I, we can calculate β straight from its definition in $\beta\ (dB) = 10\ \log_{10}\left(\frac{I}{I_0}\right)\beta\ (\mathrm{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)$.

Solution

(1) Identify knowns:

Sound travels at 331 m/s in air at 0 °C.

Air has a density of 1.29 kg/m³ at atmospheric pressure and 0°C.

(2) Enter these values and the pressure amplitude into $I = \frac{(\Delta p)^2}{2\rho v_w}$.

$$I = \frac{(\Delta p)^2}{2\rho v_w} = \frac{(0.656\ \ \mathrm{Pa})^2}{2\left(1.29\ \mathrm{kg/m^3}\right)(331\ \ \mathrm{m/s})} = 5.04 \times 10^{-4}\ \ \mathrm{W/m^2}.$$

(3) Enter the value for I and the known value for I_0 into $\beta\ (\mathrm{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)$. Calculate to find the sound intensity level in decibels.

$$10\ \log_{10}\left(5.04 \times 10^8\right) = 10\,(8.70)\ \ \mathrm{dB} = 87.0\ \ \mathrm{dB}.$$

Discussion

This 87.0 dB sound has an intensity five times as great as an 80 dB sound. So a factor of five in intensity corresponds to a difference of 7 dB in sound intensity level. This value is true for any intensities differing by a factor of five.

WORKED EXAMPLE

Change Intensity Levels of a Sound: What Happens to the Decibel Level?

Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher.

STRATEGY

You are given that the ratio of two intensities is 2 to 1, and are then asked to find the difference in their sound levels in decibels. You can solve this problem using of the properties of logarithms.

Solution

(1) Identify knowns:

The ratio of the two intensities is 2 to 1, or: $\frac{I_2}{I_1} = 2.00$.

We want to show that the difference in sound levels is about 3 dB. That is, we want to show

$$\beta_2 - \beta_1 = 3 \text{ dB}. \tag{14.8}$$

Note that

$$\log_{10} b - \log_{10} a = \log_{10}\left(\frac{b}{a}\right). \tag{14.9}$$

(2) Use the definition of β to get

$$\beta_2 - \beta_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right) = 10 \log_{10} 2.00 = 10\,(0.301) \text{ dB}. \tag{14.10}$$

Therefore, $\beta_2 - \beta_1 = 3.01$ dB.

Discussion

This means that the two sound intensity levels differ by 3.01 dB, or about 3 dB, as advertised. Note that because only the ratio I_2/I_1 is given (and not the actual intensities), this result is true for any intensities that differ by a factor of two. For example, a 56.0 dB sound is twice as intense as a 53.0 dB sound, a 97.0 dB sound is half as intense as a 100 dB sound, and so on.

Practice Problems

7. Calculate the intensity of a wave if the power transferred is 10 W and the area through which the wave is transferred is 5 square meters.
   a. 200 W / m^2
   b. 50 W / m^2
   c. 0.5 W / m^2
   d. 2 W / m^2

8. Calculate the sound intensity for a sound wave traveling in air at 0°C and having a pressure amplitude of 0.90 Pa.
   a. $1.8 \times 10^{-3}\,\text{W/m}^2$
   b. $4.2 \times 10^{-3}\,\text{W/m}^2$
   c. $1.1 \times 10^{3}\,\text{W/m}^2$
   d. $9.5 \times 10^{-4}\,\text{W/m}^2$

Hearing and Voice

People create sounds by pushing air up through their lungs and through elastic folds in the throat called vocal cords. These folds open and close rhythmically, creating a pressure buildup. As air travels up and past the vocal cords, it causes them to vibrate. This vibration escapes the mouth along with puffs of air as sound. A voice changes in pitch when the muscles of the larynx relax or tighten, changing the tension on the vocal chords. A voice becomes louder when air flow from the lungs increases, making the amplitude of the sound pressure wave greater.

Hearing is the perception of sound. It can give us plenty of information—such as pitch, loudness, and direction. Humans can normally hear frequencies ranging from approximately 20 to 20,000 Hz. Other animals have hearing ranges different from that of humans. Dogs can hear sounds as high as 45,000 Hz, whereas bats and dolphins can hear up to 110,000 Hz sounds. You may have noticed that dogs respond to the sound of a dog whistle which produces sound out of the range of human hearing.

Sounds below 20 Hz are called infrasound, whereas those above 20,000 Hz are ultrasound. The perception of frequency is called **pitch**, and the perception of intensity is called **loudness**.

The way we hear involves some interesting physics. The sound wave that hits our ear is a pressure wave. The ear converts sound waves into electrical nerve impulses, similar to a microphone.

Figure 14.11 shows the anatomy of the ear with its division into three parts: the outer ear or ear canal; the middle ear, which runs from the eardrum to the cochlea; and the inner ear, which is the cochlea itself. The body part normally referred to as the ear is technically called the pinna.

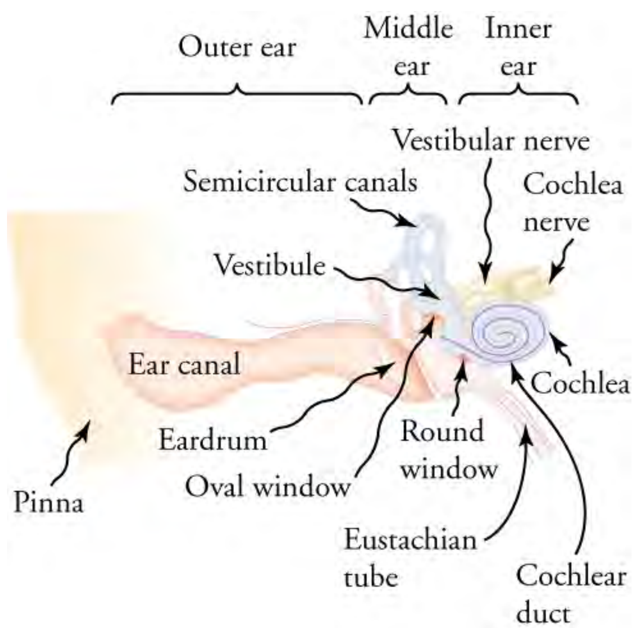

Figure 14.11 The illustration shows the anatomy of the human ear.

The outer ear, or ear canal, carries sound to the eardrum protected inside of the ear. The middle ear converts sound into mechanical vibrations and applies these vibrations to the cochlea. The lever system of the middle ear takes the force exerted on the eardrum by sound pressure variations, amplifies it and transmits it to the inner ear via the oval window. Two muscles in the middle ear protect the inner ear from very intense sounds. They react to intense sound in a few milliseconds and reduce the force transmitted to the cochlea. This protective reaction can also be triggered by your own voice, so that humming during a fireworks display, for example, can reduce noise damage.

Figure 14.12 shows the middle and inner ear in greater detail. As the middle ear bones vibrate, they vibrate the cochlea, which contains fluid. This creates pressure waves in the fluid that cause the tectorial membrane to vibrate. The motion of the tectorial membrane stimulates tiny cilia on specialized cells called hair cells. These hair cells, and their attached neurons, transform the motion of the tectorial membrane into electrical signals that are sent to the brain.

The tectorial membrane vibrates at different positions based on the frequency of the incoming sound. This allows us to detect the pitch of sound. Additional processing in the brain also allows us to determine which direction the sound is coming from (based on comparison of the sound's arrival time and intensity between our two ears).

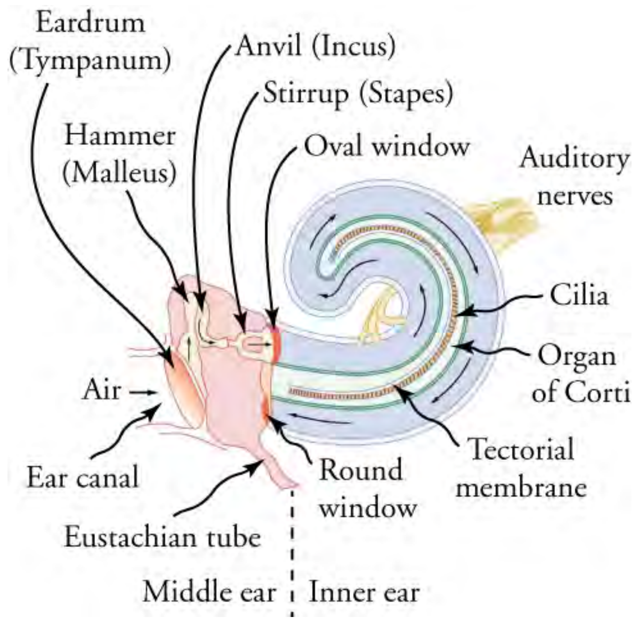

Figure 14.12 The inner ear, or cochlea, is a coiled tube about 3 mm in diameter and 3 cm in length when uncoiled. As the stapes vibrates against the oval window, it creates pressure waves that travel through fluid in the cochlea. These waves vibrate the tectorial membrane, which bends the cilia and stimulates nerves in the organ of Corti. These nerves then send information about the sound to the brain.

FUN IN PHYSICS

Musical Instruments

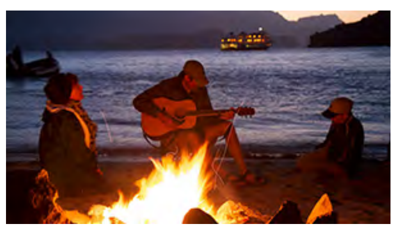

Figure 14.13 Playing music, also known as “rocking out”, involves creating vibrations using musical instruments. (John Norton)

Yet another way that people make sounds is through playing musical instruments (see the previous figure). Recall that the perception of frequency is called pitch. You may have noticed that the pitch range produced by an instrument tends to depend upon its size. Small instruments, such as a piccolo, typically make high-pitch sounds, while larger instruments, such as a tuba, typically make low-pitch sounds. High-pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds, just as a large instrument creates long-wavelength sounds.

Most of us have excellent relative pitch, which means that we can tell whether one sound has a different frequency from another. We can usually distinguish one sound from another if the frequencies of the two sounds differ by as little as 1 Hz. For example, 500.0 and 501.5 Hz are noticeably different.

Musical notes are particular sounds that can be produced by most instruments, and are the building blocks of a song. In Western music, musical notes have particular names, such as A-sharp, C, or E-flat. Some people can identify musical notes just by listening to them. This rare ability is called *perfect*, or *absolute, pitch*.

When a violin plays middle C, there is no mistaking it for a piano playing the same note. The reason is that each instrument produces a distinctive set of frequencies and intensities. We call our perception of these combinations of frequencies and intensities the *timbre* of the sound. It is more difficult to quantify timbre than loudness or pitch. Timbre is more subjective. Evocative adjectives such as dull, brilliant, warm, cold, pure, and rich are used to describe the timbre of a sound rather than quantities with units, which makes for a difficult topic to dissect with physics. So the consideration of timbre takes us into the realm of perceptual psychology, where higher-level processes in the brain are dominant. This is also true for other perceptions of sound, such as music and noise. But as a teenager, you are likely already aware that one person's music may be another person's noise.

GRASP CHECK

If you turn up the volume of your stereo, will the pitch change? Why or why not?

a. No, because pitch does not depend on intensity.
b. Yes, because pitch is directly related to intensity.

Check Your Understanding

9. What is sound intensity?

a. Intensity is the energy per unit area carried by a wave.
b. Intensity is the energy per unit volume carried by a wave.
c. Intensity is the power per unit area carried by a wave.
d. Intensity is the power per unit volume carried by a wave.

10. How is power defined with reference to a sound wave?

a. Power is the rate at which energy is transferred by a sound wave.
b. Power is the rate at which mass is transferred by a sound wave.
c. Power is the rate at which amplitude of a sound wave changes.
d. Power is the rate at which wavelength of a sound wave changes.

11. What word or phrase is used to describe the loudness of sound?

a. frequency or oscillation
b. intensity level or decibel
c. timbre
d. pitch

12. What is the mathematical expression for sound intensity level β?

a. $\beta\,(\mathrm{dB}) = 10\log_{10}\left(\frac{I_0}{I}\right)$
b. $\beta\,(\mathrm{dB}) = 20\log_{10}\left(\frac{I}{I_0}\right)$
c. $\beta(dB) = 20\log_{10}\left(\frac{I_0}{I}\right)$
d. $\beta\,(\mathrm{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right)$

13. What is the range frequencies that humans are capable of hearing?

a. $20\text{ Hz to}200,000\text{ Hz}$
b. 2 Hz to $50,000\text{ Hz}$
c. 2 Hz to $2,000\text{ Hz}$
d. 20 Hz to $20,000\text{ Hz}$

14. How do humans change the pitch of their voice?

a. Relaxing or tightening their glottis
b. Relaxing or tightening their uvula
c. Relaxing or tightening their tongue
d. Relaxing or tightening their larynx

References

Nave, R. Vocal sound production—HyperPhysics. Retrieved from http://hyperphysics.phy-astr.gsu.edu/hbase/music/voice.html

14.3 Doppler Effect and Sonic Booms

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the Doppler effect of sound waves
- Explain a sonic boom
- Calculate the frequency shift of sound from a moving object by the Doppler shift formula, and calculate the speed of an object by the Doppler shift formula

Section Key Terms

Doppler effect	sonic boom

The Doppler Effect of Sound Waves

The **Doppler effect** is a change in the observed pitch of a sound, due to relative motion between the source and the observer. An example of the Doppler effect due to the motion of a source occurs when you are standing still, and the sound of a siren coming from an ambulance shifts from high-pitch to low-pitch as it passes by. The closer the ambulance is to you, the more sudden the shift. The faster the ambulance moves, the greater the shift. We also hear this shift in frequency for passing race cars, airplanes, and trains. An example of the Doppler effect with a stationary source and moving observer is if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by.

What causes the Doppler effect? Let's compare three different scenarios: Sound waves emitted by a stationary source (Figure 14.14), sound waves emitted by a moving source (Figure 14.15), and sound waves emitted by a stationary source but heard by moving observers (Figure 14.16). In each case, the sound spreads out from the point where it was emitted.

If the source and observers are stationary, then observers on either side see the same wavelength and frequency as emitted by the source. But if the source is moving and continues to emit sound as it travels, then the air compressions (crests) become closer together in the direction in which it's traveling and farther apart in the direction it's traveling away from. Therefore, the wavelength is shorter in the direction the source is moving (on the right in Figure 14.15), and longer in the opposite direction (on the left in Figure 14.15).

Finally, if the observers move, as in Figure 14.16, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency (and therefore shorter wavelength), and the person moving away from the source receives them at a lower frequency (and therefore longer wavelength).

Figure 14.14 Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.

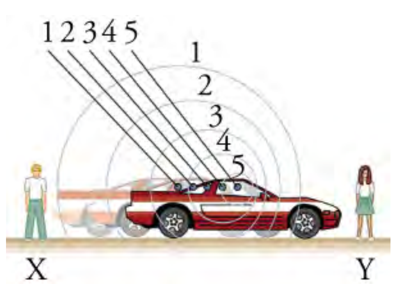

Figure 14.15 Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is

reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.

Figure 14.16 The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v = f\lambda$, where v is the fixed speed of sound. The sound moves in a medium and has the same speed v in that medium whether the source is moving or not. Therefore, f multiplied by λ is a constant. Because the observer on the right in Figure 14.15 receives a shorter wavelength, the frequency she perceives must be higher. Similarly, the observer on the left receives a longer wavelength and therefore perceives a lower frequency.

The same thing happens in Figure 14.16. A higher frequency is perceived by the observer moving toward the source, and a lower frequency is perceived by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the perceived frequency. Relative motion apart decreases the perceived frequency. The greater the relative speed is, the greater the effect.

WATCH PHYSICS

Introduction to the Doppler Effect

This video explains the Doppler effect visually.

Click to view content (https://www.openstax.org/l/28doppler)

GRASP CHECK

If you are standing on the sidewalk facing the street and an ambulance drives by with its siren blaring, at what point will the frequency that you observe most closely match the actual frequency of the siren?

a. when it is coming toward you
b. when it is going away from you
c. when it is in front of you

For a stationary observer and a moving source of sound, the frequency (f_{obs}) of sound perceived by the observer is

$$f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right), \tag{14.11}$$

where f_s is the frequency of sound from a source, v_s is the speed of the source along a line joining the source and observer, and v_w is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer.

TIPS FOR SUCCESS

Rather than just memorizing rules, which are easy to forget, it is better to think about the rules of an equation intuitively. Using a minus sign in $f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$ will decrease the denominator and increase the observed frequency, which is consistent with the expected outcome of the Doppler effect when the source is moving toward the observer. Using a plus sign will increase the denominator and decrease the observed frequency, consistent with what you would expect for the source moving away from the observer. This may be more helpful to keep in mind rather than memorizing the fact that "the minus sign is used for motion toward the observer and the plus sign for motion away from the observer."

Note that the greater the speed of the source, the greater the Doppler effect. Similarly, for a stationary source and moving observer, the frequency perceived by the observer f_{obs} is given by

$$f_{obs} = f_s \left(\frac{v_w \pm v_{obs}}{v_w} \right), \tag{14.12}$$

where v_{obs} is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus sign is for motion away from the source.

Sonic Booms

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_{s}. The greater the plane's speed, v_{s}, the greater the Doppler shift and the greater the value of f_{obs}. Now, as v_{s} approaches the speed of sound, v_{w}, f_{obs} approaches infinity, because the denominator in $f_{obs} = f_s \left(\frac{v_w}{v_w - v_s} \right)$ approaches zero.

This result means that at the speed of sound, in front of the source, each wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is theoretically infinite. If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the source when it was approaching are stacked up with those from it when receding, creating a **sonic boom**. A sonic boom is a constructive interference of sound created by an object moving faster than sound.

An aircraft creates two sonic booms, one from its nose and one from its tail (see Figure 14.17). During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not observe the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive enough to break windows. Because of this, supersonic flights are banned over populated areas of the United States.

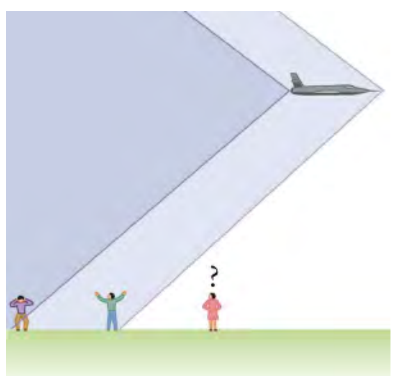

Figure 14.17 Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Solving Problems Using the Doppler Shift Formula

WATCH PHYSICS

Doppler Effect Formula for Observed Frequency

This video explains the Doppler effect formula for cases when the source is moving toward the observer.

Click to view content (https://www.openstax.org/l/28dopplerform)

GRASP CHECK

Let's say that you have a rare phobia where you are afraid of the Doppler effect. If you see an ambulance coming your way, what would be the best strategy to minimize the Doppler effect and soothe your Doppleraphobia?

a. Stop moving and become stationary till it passes by.
b. Run toward the ambulance.
c. Run alongside the ambulance.

WATCH PHYSICS

Doppler Effect Formula When Source is Moving Away

This video explains the Doppler effect formula for cases when the source is moving away from the observer.

Click to view content (https://www.openstax.org/l/28doppleraway)

GRASP CHECK

Sal uses two different formulas for the Doppler effect-one for when the source is moving toward the observer and another for when the source is moving away. However, in this textbook we use only one formula. Explain.

a. The combined formula that can be used is, Use (+) when the source is moving toward the observer and (–) when the source is moving away from the observer.
b. The combined formula that can be used is, $f_{obs} = f_s\left(\frac{v_w \pm v_s}{v_w}\right)$. Use (+) when the source is moving away from the observer and (–) when the source is moving toward the observer.
c. The combined formula that can be used is, $f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$. Use (+) when the source is moving toward the observer and (–) when the source is moving away from the observer.
d. The combined formula that can be used is, $f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$. Use (+) when the source is moving away from the observer and (–) when the source is moving toward the observer.

WORKED EXAMPLE

Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150 Hz horn is moving at 35 m/s in still air on a day when the speed of sound is 340 m/s. What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

Strategy

To find the observed frequency, $f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$ must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train.

Solution

(1) Enter known values into $f_{obs} = f_s\left(\frac{v_w}{v_w - v_s}\right)$ to calculate the frequency observed by a stationary person as the train approaches:

$$f_{obs} = f_s\left(\frac{v_w}{v_w - v_s}\right) = (150\ \text{Hz})\left(\frac{340\ \text{m/s}}{340\ \text{m/s} - 35\ \text{m/s}}\right) = 167\ \text{Hz} \approx 170\ \text{Hz (2 sig. figs.)}$$

(2) Use the same equation but with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{obs} = f_s\left(\frac{v_w}{v_w + v_s}\right) = (150\ \text{Hz})\left(\frac{340\ \text{m/s}}{340\ \text{m/s} + 35\ \text{m/s}}\right) = 136\ \text{Hz} \approx 140\ \text{Hz (2 sig. figs.)}$$

Discussion

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining the train and the observer. In both cases, the shift is significant and easily noticed. Note that the shift is approximately 20 Hz for motion toward and approximately 10 Hz for motion away. The shifts are not symmetric.

Practice Problems

15. What is the observed frequency when the source having frequency $3.0\ \text{kHz}$ is moving towards the observer at a speed of $1.0 \times 10^2\ \text{m/s}$ and the speed of sound is $331\ \text{m/s}$?
   a. $3.0\ \text{kHz}$
   b. $3.5\ \text{kHz}$
   c. $2.3\ \text{kHz}$
   d. $4.3\ \text{kHz}$

16. A train is moving away from you at a speed of $50.0\ \text{m/s}$. If you are standing still and hear the whistle at a frequency of $305\ \text{Hz}$, what is the actual frequency of the produced whistle? (Assume speed of sound to be $331\ \text{m/s}$.)
   a. $259\ \text{Hz}$
   b. $205\ \text{Hz}$
   c. $405\ \text{Hz}$
   d. $351\ \text{Hz}$

Check Your Understanding

17. What is the Doppler effect?
   a. The Doppler effect is a change in the observed speed of a sound due to the relative motion between the source and the observer.
   b. The Doppler effect is a change in the observed frequency of a sound due to the relative motion between the source and the observer.
   c. The Doppler effect is a change in the observed intensity of a sound due to the relative motion between the source and the observer.
   d. The Doppler effect is a change in the observed timbre of a sound, due to the relative motion between the source and the observer.

18. Give an example of the Doppler effect caused by motion of the source.
   a. The sound of a vehicle horn shifts from low-pitch to high-pitch as we move towards it.
   b. The sound of a vehicle horn shifts from low-pitch to high-pitch as we move away from it.
   c. The sound of a vehicle horn shifts from low-pitch to high-pitch as it passes by.
   d. The sound of a vehicle horn shifts from high-pitch to low-pitch as it passes by.

19. What is a sonic boom?
   a. It is a destructive interference of sound created by an object moving faster than sound.
   b. It is a constructive interference of sound created by an object moving faster than sound.
   c. It is a destructive interference of sound created by an object moving slower than sound.
   d. It is a constructive interference of sound created by an object moving slower than sound.

20. What is the relation between speed of source and value of observed frequency when the source is moving towards the observer?
   a. They are independent of each other.
   b. The greater the speed, the greater the value of observed frequency.
   c. The greater the speed, the smaller the value of observed frequency.
   d. The speed of the sound is directly proportional to the square of the frequency observed.

14.4 Sound Interference and Resonance

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe resonance and beats
- Define fundamental frequency and harmonic series
- Contrast an open-pipe and closed-pipe resonator
- Solve problems involving harmonic series and beat frequency

Section Key Terms

beat	beat frequency	damping	fundamental	harmonics
natural frequency	overtones	resonance	resonate	

Resonance and Beats

Sit in front of a piano sometime and sing a loud brief note at it while pushing down on the sustain pedal. It will sing the same note back at you—the strings that have the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. This is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency.

A driving force (such as your voice in the example) puts energy into a system at a certain frequency, which is not necessarily the same as the natural frequency of the system. Over time the energy dissipates, and the amplitude gradually reduces to zero- this is called **damping**. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**, and a system being driven at its natural frequency is said to **resonate**.

Most of us have played with toys where an object bobs up and down on an elastic band, something like the paddle ball suspended from a finger in Figure 14.18. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.

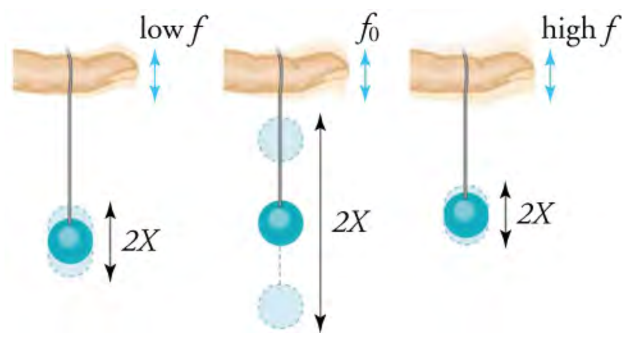

Figure 14.18 The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

Another example is that when you tune a radio, you adjust its resonant frequency so that it oscillates only at the desired station's broadcast (driving) frequency. Also, a child on a swing is driven (pushed) by a parent at the swing's natural frequency to reach the maximum amplitude (height). In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance.

Figure 14.19 Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises.

All sound resonances are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle to the recognizability of a great singer's voice, resonance and standing waves play a vital role in sound.

Interference happens to all types of waves, including sound waves. In fact, one way to support that something *is a wave* is to observe interference effects. Figure 14.19 shows a set of headphones that employs a clever use of sound interference to cancel noise. To get destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise.

In addition to resonance, superposition of waves can also create beats. **Beats** are produced by the superposition of two waves with slightly different frequencies but the same amplitude. The waves alternate in time between constructive interference and destructive interference, giving the resultant wave an amplitude that varies over time. (See the resultant wave in Figure 14.20).

This wave fluctuates in amplitude, or beats, with a frequency called the **beat frequency**. The equation for beat frequency is

$$f_B = |f_1 - f_2|, \tag{14.13}$$

where f_1 and f_2 are the frequencies of the two original waves. If the two frequencies of sound waves are similar, then what we hear is an average frequency that gets louder and softer at the beat frequency.

TIPS FOR SUCCESS

Don't confuse the beat frequency with the regular frequency of a wave resulting from superposition. While the beat frequency is given by the formula above, and describes the frequency of the beats, the actual frequency of the wave resulting from superposition is the average of the frequencies of the two original waves.

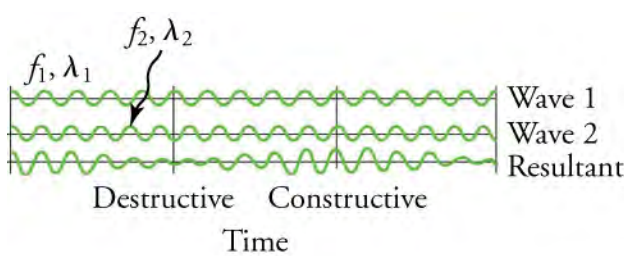

Figure 14.20 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

Virtual Physics

Wave Interference

Click to view content (https://www.openstax.org/l/28interference)

For this activity, switch to the Sound tab. Turn on the Sound option, and experiment with changing the frequency and amplitude, and adding in a second speaker and a barrier.

GRASP CHECK

According to the graph, what happens to the amplitude of pressure over time. What is this phenomenon called, and what causes it ?

a. The amplitude decreases over time. This phenomenon is called damping. It is caused by the dissipation of energy.
b. The amplitude increases over time. This phenomenon is called feedback. It is caused by the gathering of energy.
c. The amplitude oscillates over time. This phenomenon is called echoing. It is caused by fluctuations in energy.

Fundamental Frequency and Harmonics

Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in Figure 14.21, Figure 14.22, and Figure 14.23. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.

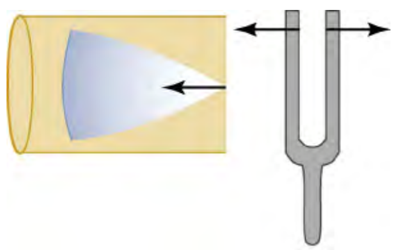

Figure 14.21 Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube.

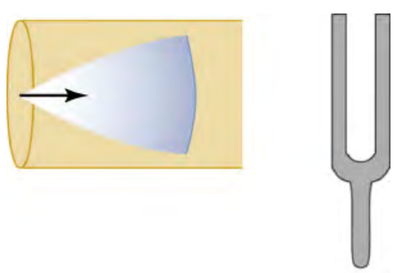

Figure 14.22 Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube.

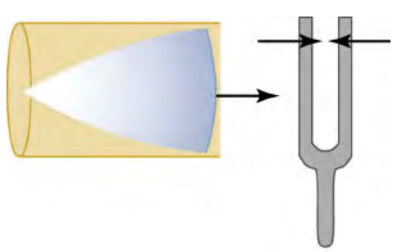

Figure 14.23 Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube *L* is just right, the disturbance gets back to the tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates.

The standing wave formed in the tube has its maximum air displacement (an antinode) at the open end, and no displacement (a node) at the closed end. Recall from the last chapter on waves that motion is unconstrained at the antinode, and halted at the node. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; therefore, $\lambda = 4L$. This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in Figure 14.24.

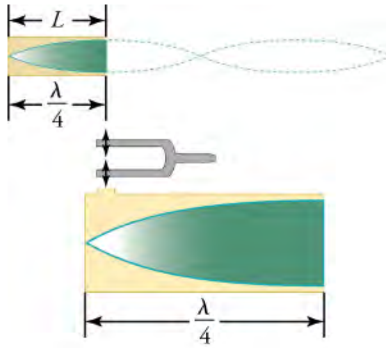

Figure 14.24 The same standing wave is created in the tube by a vibration introduced near its closed end.

Since maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube see Figure 14.25). Here the standing wave has three-fourths of its wavelength in the tube, or $L = (3/4)\lambda'$, so that $\lambda' = 4L/3$. There is a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube.

We use specific terms for the resonances in any system. The lowest resonant frequency is called the **fundamental**, while all higher resonant frequencies are called **overtones**. All resonant frequencies are multiples of the fundamental, and are called **harmonics**. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. Figure 14.26 shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

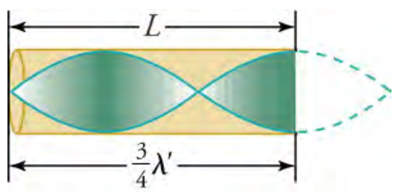

Figure 14.25 Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is shorter, with three-fourths λ' equaling the length of the tube, so that $\lambda' = 4L/3$. This higher-frequency vibration is the first overtone.

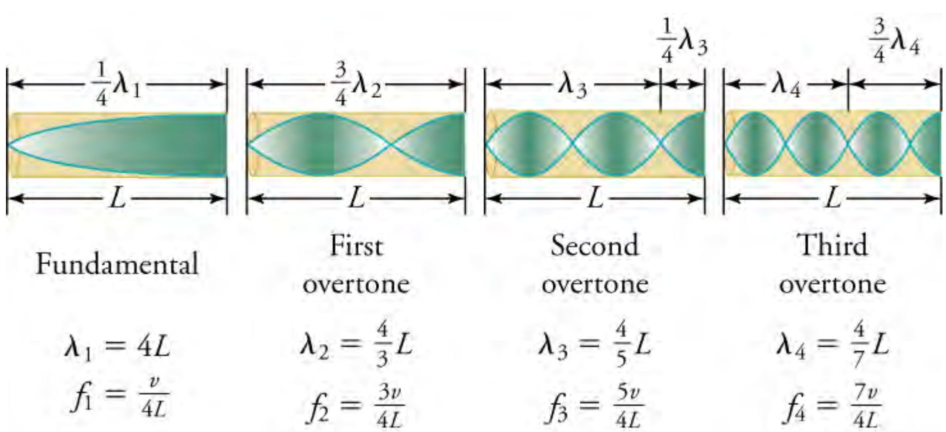

Figure 14.26 The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

The fundamental and overtones can be present at the same time in a variety of combinations. For example, the note middle C on a trumpet sounds very different from middle C on a clarinet, even though both instruments are basically modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different. This mix is what gives musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones.

Open-Pipe and Closed-Pipe Resonators

The resonant frequencies of a tube closed at one end (known as a closed-pipe resonator) are $f_n = n\frac{v}{4L},\ n = 1, 3, 5...,$

where f_1 is the fundamental, f_3 is the first overtone, and so on. Note that the resonant frequencies depend on the speed of sound v and on the length of the tube L.

Another type of tube is one that is *open* at both ends (known as an open-pipe resonator). Examples are some organ pipes, flutes, and oboes. The air columns in tubes open at both ends have maximum air displacements at both ends. (See Figure 14.27). Standing waves form as shown.

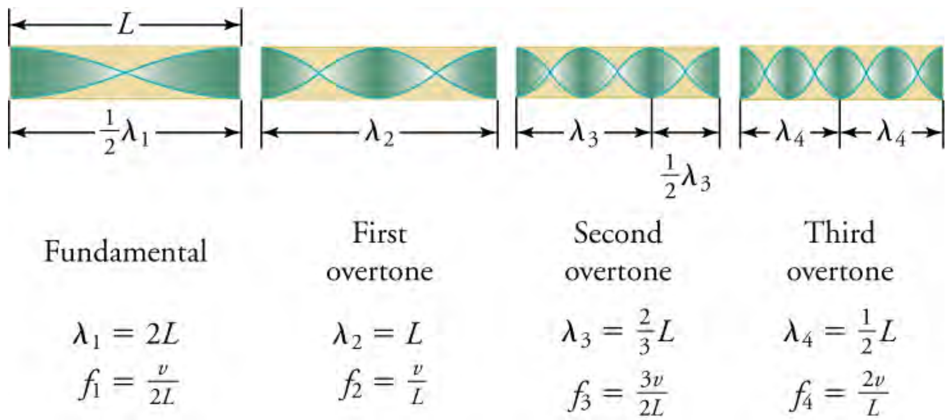

Figure 14.27 The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end.

The resonant frequencies of an open-pipe resonator are

$$f_n = n\frac{v}{2L},\ n = 1, 2, 3...,$$

where f_1 is the fundamental, f_2 is the first overtone, f_3 is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones.

Middle C, for example, would sound richer played on an open tube since it has more overtones. An open-pipe resonator has more overtones than a closed-pipe resonator because it has even multiples of the fundamental as well as odd, whereas a closed tube has only odd multiples.

In this section we have covered resonance and standing waves for wind instruments, but vibrating strings on stringed instruments also resonate and have fundamentals and overtones similar to those for wind instruments.

Solving Problems Involving Harmonic Series and Beat Frequency

WORKED EXAMPLE

Finding the Length of a Tube for a Closed-Pipe Resonator

If sound travels through the air at a speed of 344 m/s, what should be the length of a tube closed at one end to have a fundamental frequency of 128 Hz?

Strategy

The length L can be found by rearranging the equation $f_n = n\frac{v}{4L}$.

Solution

(1) Identify knowns.

- The fundamental frequency is 128 Hz.
- The speed of sound is 344 m/s.

(2) Use $f_n = n\frac{v_w}{4L}$ to find the fundamental frequency ($n = 1$).

$$f_1 = \frac{v}{4L} \tag{14.14}$$

(3) Solve this equation for length.

$$L = \frac{v}{4f_1} \tag{14.15}$$

(4) Enter the values of the speed of sound and frequency into the expression for L.

$$L = \frac{v}{4f_1} = \frac{344\ \text{ m/s}}{4\,(128\ \text{ Hz})} = 0.672\ \text{ m} \tag{14.16}$$

Discussion

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and therefore, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

WORKED EXAMPLE

Finding the Third Overtone in an Open-Pipe Resonator

If a tube that's open at both ends has a fundamental frequency of 120 Hz, what is the frequency of its third overtone?

Strategy

Since we already know the value of the fundamental frequency (n = 1), we can solve for the third overtone (n = 4) using the equation $f_n = n\frac{v}{2L}$.

Solution

Since fundamental frequency (n = 1) is

$$f_1 = \frac{v}{2L}, \tag{14.17}$$

and

$$f_4 = 4\frac{v}{2L}, f_4 = 4f_1 = 4(120\text{ Hz})\text{=}480\text{ Hz.} \tag{14.18}$$

Discussion

To solve this problem, it wasn't necessary to know the length of the tube or the speed of the air because of the relationship between the fundamental and the third overtone. This example was of an open-pipe resonator; note that for a closed-pipe resonator, the third overtone has a value of n = 7 (not n = 4).

WORKED EXAMPLE

Using Beat Frequency to Tune a Piano

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). If a piano tuner hears two beats per second, and the tuning fork has a frequency of 256 Hz, what are the possible frequencies of the piano?

Strategy

Since we already know that the beat frequency f_B is 2, and one of the frequencies (let's say f_2) is 256 Hz, we can use the equation $f_B = |f_1 - f_2|$ to solve for the frequency of the piano f_1.

Solution

Since $f_B = |f_1 - f_2|$,

we know that either $f_B = f_1 - f_2$ or $-f_B = f_1 - f_2$.

Solving for f_1,

$$f_1 = f_B + f_2 \text{ or } f_1 = -f_B + f_2. \tag{14.19}$$

Substituting in values,

$$f_1 = 2 + 256 \text{ Hz or } f_1 = -2 + 256 \text{ Hz} \tag{14.20}$$

So,

$$f_1 = 258 \text{ Hz or } 254 \text{ Hz.} \tag{14.21}$$

Discussion

The piano tuner might not initially be able to tell simply by listening whether the frequency of the piano is too high or too low and must tune it by trial and error, making an adjustment and then testing it again. If there are even more beats after the adjustment, then the tuner knows that he went in the wrong direction.

Practice Problems

21. Two sound waves have frequencies 250 Hz and 280 Hz. What is the beat frequency produced by their superposition?

a. 290 Hz
b. 265 Hz
c. 60 Hz
d. 30 Hz

22. What is the length of a pipe closed at one end with fundamental frequency 350 Hz? (Assume the speed of sound in air is 331 m/s.)

a. 26 cm
b. 26 m
c. 24 m
d. 24 cm

Check Your Understanding

23. What is damping?

a. Over time the energy increases and the amplitude gradually reduces to zero. This is called damping.
b. Over time the energy dissipates and the amplitude gradually increases. This is called damping.
c. Over time the energy increases and the amplitude gradually increases. This is called damping.
d. Over time the energy dissipates and the amplitude gradually reduces to zero. This is called damping.

24. What is resonance? When can you say that the system is resonating?

a. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.
b. The phenomenon of driving a system with a frequency higher than its natural frequency is called resonance, and a system being driven at its natural frequency does not resonate.
c. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency does not resonate.
d. The phenomenon of driving a system with a frequency higher than its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.

25. In the tuning fork and tube experiment, in case a standing wave is formed, at what point on the tube is the maximum disturbance from the tuning fork observed? Recall that the tube has one open end and one closed end.

a. At the midpoint of the tube
b. Both ends of the tube
c. At the closed end of the tube
d. At the open end of the tube

26. In the tuning fork and tube experiment, when will the air column produce the loudest sound?

a. If the tuning fork vibrates at a frequency twice that of the natural frequency of the air column.
b. If the tuning fork vibrates at a frequency lower than the natural frequency of the air column.
c. If the tuning fork vibrates at a frequency higher than the natural frequency of the air column.
d. If the tuning fork vibrates at a frequency equal to the natural frequency of the air column.

27. What is a closed-pipe resonator?

a. A pipe or cylindrical air column closed at both ends
b. A pipe with an antinode at the closed end
c. A pipe with a node at the open end
d. A pipe or cylindrical air column closed at one end

28. Give two examples of open-pipe resonators.

a. piano, violin
b. drum, tabla
c. rlectric guitar, acoustic guitar
d. flute, oboe

KEY TERMS

amplitude the amount that matter is disrupted during a sound wave, as measured by the difference in height between the crests and troughs of the sound wave.

beat a phenomenon produced by the superposition of two waves with slightly different frequencies but the same amplitude

beat frequency the frequency of the amplitude fluctuations of a wave

damping the reduction in amplitude over time as the energy of an oscillation dissipates

decibel a unit used to describe sound intensity levels

Doppler effect an alteration in the observed frequency of a sound due to relative motion between the source and the observer

fundamental the lowest-frequency resonance

harmonics the term used to refer to the fundamental and its overtones

hearing the perception of sound

loudness the perception of sound intensity

natural frequency the frequency at which a system would oscillate if there were no driving and no damping forces

overtones all resonant frequencies higher than the fundamental

pitch the perception of the frequency of a sound

rarefaction a low-pressure region in a sound wave

resonance the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate to drive a system at its natural frequency

sonic boom a constructive interference of sound created by an object moving faster than sound

sound a disturbance of matter that is transmitted from its source outward by longitudinal waves

sound intensity the power per unit area carried by a sound wave

sound intensity level the level of sound relative to a fixed standard related to human hearing

SECTION SUMMARY

14.1 Speed of Sound, Frequency, and Wavelength

- Sound is one type of wave.
- Sound is a disturbance of matter that is transmitted from its source outward in the form of longitudinal waves.
- The relationship of the speed of sound v, its frequency f, and its wavelength λ is given by $v = f\lambda$, which is the same relationship given for all waves.
- The speed of sound depends upon the medium through which the sound wave is travelling.
- In a given medium at a specific temperature (or density), the speed of sound v is the same for all frequencies and wavelengths.

14.2 Sound Intensity and Sound Level

- The intensity of a sound is proportional to its amplitude squared.
- The energy of a sound wave is also proportional to its amplitude squared.
- Sound intensity level in decibels (dB) is more relevant for how humans perceive sounds than sound intensity (in W/m^2), even though sound intensity is the SI unit.
- Sound intensity level is not the same as sound intensity—it tells you the *level* of the sound relative to a reference intensity rather than the actual intensity.
- Hearing is the perception of sound and involves that transformation of sound waves into vibrations of parts within the ear. These vibrations are then transformed into neural signals that are interpreted by the brain.
- People create sounds by pushing air up through their lungs and through elastic folds in the throat called vocal cords.

14.3 Doppler Effect and Sonic Booms

- The Doppler effect is a shift in the observed frequency of a sound due to motion of either the source or the observer.
- The observed frequency is greater than the actual source's frequency when the source and the observer are moving closer together, either by the source moving toward the observer or the observer moving toward the source.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.

14.4 Sound Interference and Resonance

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- Beats occur when waves of slightly different frequencies are superimposed.
- In air columns, the lowest-frequency resonance is called the fundamental, whereas all higher resonant frequencies are called overtones. Collectively, they are