An Experimental Determination of the Ideal Gas Constant

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Abstract

The gas constant R is an essential proportionality constant used in relations such as the Boltzmann's constant relation, Van der Waals equation, and the Ideal Gas Law. In this study, the gas constant R has been experimentally determined using a simple method employing the use of a syringe as well as hanging weights of varying mass. The range of applied force for the syringe to maintain a position of static equilibrium was measured, holding the initial volume constant while varying the hanging masses. A theoretical analysis of the syringe-mass system resulted in a linear relation between the average change in volume and the inverse pressure, which was used to perform a linear fit of the volume-pressure data. The value of R, extracted from the resulting slope of the linear fit, was determined to be $R = 8.27 \pm 0.08\, J\, mol^{-1} K^{-1}$, a value in agreement and within the uncertainty of the accepted value of the ideal gas constant. This result underscores the validity of a simple and accessible method for quantifying a universal constant as well as the future educational contexts it can have aiding students in the comprehension of fundamental thermodynamic principles.

Introduction

In the study of thermodynamics, there are several universal constants of nature that are frequently employed. Seen not only in physics but also in chemistry, the Boltzmann constant, annotated as k_B, is the proportionality factor relating the average relative thermal energy of a particle in a gas with the thermodynamic temperature of the gas. This exact value is seen in several relations such as the Kelvin definition, Boltzmann entropy formula, and the Ideal Gas Law; the latter of which will be investigated during this study. The Boltzmann constant k_B, using Avogadro's number N_A, can produce a product of the ideal gas constant $R = k_B N_A$. This constant is most famously seen for its place in the Ideal Gas Law $PV = nRT$, where the constant acts as a scaling factor for the number of moles of gas and the temperature of the gas in the system. As of 2019, SI units were redefined in terms of the universal constants of nature, which have all been precisely defined to be a certain value. Which due to both the Boltzmann constant and Avogadro's Number being exact values, results in the ideal gas constant also being fixed at a value of precisely $8.31446261815324\, J\, K^{-1} mol^{-1}$. Due to this recent redefinition, any resulting measurements to high precision are resultingly trivial. The goal of this study is to do exactly that, experimentally measure the ideal gas constant. However, finding an effective method of quantifying the ideal constant can still have important educational implications, where these experiments can aid in a student's comprehension of thermodynamic principles and the Ideal Gas Law. Hence, in this work a simple, effective, and accessible method will be demonstrated, providing a theoretical analysis of the system used from which a relationship between the average volume and inverse pressure is obtained. The experimental procedure is then discussed involving hanging weights from a suspended syringe from which the required data sets can be collected. The resulting slope from the linear fit of this data is then used to extract a value of the gas constant R.

Methods

This procedure is centered around the goal of using an apparatus to measure pressure and volume to experimentally assess the gas constant R in the Ideal Gas Law, $PV = nRT$.
The analysis of this system will begin with the materials provided for this experiment.

(a) Apparatus Components

(b) Constructed Apparatus

Figure 1: **(a):** Components of the apparatus, consisting of supports ((1), (2), (3)), a 5Kg WH-BO5 Electronic Scale (4), hanging weights ranging from 100g-1000g (5), a caliper ranging from 1-160mm (6), a plastic syringe and stopper ranging from 0-10ml (7), and a 1m ruler (8). **(b):** Constructed apparatus showing the setup used for measurements and analysis. This setup was used in order to better hang the weights off of the syringe leading to more accurate data regarding the change in volume.

Required Measurements

The measurements needed for this experiment consist of recording values of the following: the mass of each hanging mass in order to find the force of gravity; the cross-sectional area of the syringe in order to find the pressure in the syringe from the hanging mass; the change in initial volume by hanging the weight on the syringe clip; the number of moles of air present in the syringe; and the ambient temperature of the room.

Theoretical Analysis

It is important to understand the forces involved for this procedure and a theoretical analysis of the given system will be important to properly analyze how to proceed with this experiment. By using free body diagrams, it becomes much easier to see the relationship between pressures.

(a) Equillibrium Free Body Diagrams

Figure 2: **(a):** Free body diagrams illustrating the two equilibrium points that will be considered during the measurement of the change in initial volume. In this case $F_A = F_{atm}$, $F_s = F_{gas}$, F_g is the force of gravity, and F_f is the force of static friction. This free body diagram was directly taken from Jupyter Lab Notebook.

Analyzing the free body diagrams in figure 2 results in the following understanding of the pressure exerted by the gas as $P \propto F$.

$$P_{gas} = P_{atm} - P_g$$

Using this value of P_{gas} in the Ideal Gas Law, $PV = nRT$, results in the following equation:

$$V = \frac{1}{P_{gas}} nRT$$

This is a good starting point; however, static friction is still not considered in this version of the equation. This can be done by finding V_{avg} and replacing it into the Ideal Gas Law.

V_{avg} can be found calculated using the following equation:

$$\frac{1}{V_{avg}} = \frac{(\frac{1}{V_1} + \frac{1}{V_2})}{2}$$

Which can then be inverted resulting in the following:

$$V_{avg} = (\frac{1}{V_{avg}})^{-1}$$

Note: throughout all measurements and calculations, it is important to record any uncertainty associated. The two methods used will be through summation and through multiplication:

$$u[c] = \sqrt{(u[a])^2 + (u[b])^2} \text{ , } u[c] = \sqrt{(\frac{u[a]}{a})^2 + (\frac{u[b]}{b})^2}$$

This step of first finding the inverse average volume is important. Although $\frac{1}{P_{gas}}$ is being plotted with respect to V_{avg} it is important to note that the force initially recorded was from the hanging weight resulting in the force exerted being proportional to the inverse volume. If attempted in any other way an incorrect average volume would be calculated.

The resulting final equation will be the following:

$$V_{avg} = \frac{1}{P_{gas}} nR$$

Procedure

There are two general approaches to measure the force exerted by the hanging mass on the syringe. The first is by measuring the distance from the fulcrum using the provided ruler from both the syringe and the pin pressing on the scale. Due to torque, this will give a conversion factor from the force on the scale to the applied force on the syringe. The second is by measuring the mass on the scale and multiplying this mass by gravity, $9.807 m/s^2$, to get the gravitational force. For this experiment, the latter method will be used.

Once the apparatus is constructed as shown in figure 1 (b), the syringe is set to an initial volume of 2mL and then sealed with the stopper. From there, measurements can be recorded of the mass of each hanging weight from 200-600g (in Kg), the area of the syringe using calipers (in m^2), the temperature in the room (in K), the number of moles of air present in the initial volume (in mol), and the pressure on the syringe from the hanging mass (in Pa). It is important to record associated uncertainties for each measurement as rectangular or gaussian to properly display the accuracy of the data being recorded.

A hanging mass is then hooked onto the lower end of the syringe allowing the syringe volume to increase until equilibrium is reached. The change in initial volume can be recorded and labeled as V_1. This process is continued with each hanging weight in the given range, with the change in volume recorded as V_1 for each separate mass. Next, the hanging weight will be pulled past its point of equilibrium and released resulting in the mass moving upwards towards another, different equilibrium point. This change in volume is then recorded as V_2 and repeated for every mass used for the values of V_1. Relating pressure and volume with the Ideal Gas Law, $P \propto \frac{1}{V}$. With this relationship the next step taken is to find $\frac{1}{V_{avg}}$ which can be done with the results of V_1 and V_2 that have been recorded. At this point, it is now possible to find V_{avg} - this is talked about more in Theoretical Analysis*.

After the measurements are taken, the dependent and independent variables need to be determined. This can easily be done as we are able to change our pressure with the hanging weights and therefore the volume. This results in volume being dependent and pressure being independent, with the equation being plotted as $V_{avg} = \frac{1}{P_{gas}} nRT$.

Finally, a linear regression and least squares fit are performed on the data set of the average volume versus the inverse pressure of the gas using the SciPy curvefit program. By using this equation, the resulting line yields a slope of nRT. Finally, by dividing this slope by the temperature and the number of moles previously calculated, the experimental value of the gas constant R can be extracted.

Discussion of Results

Initial Linear Fit

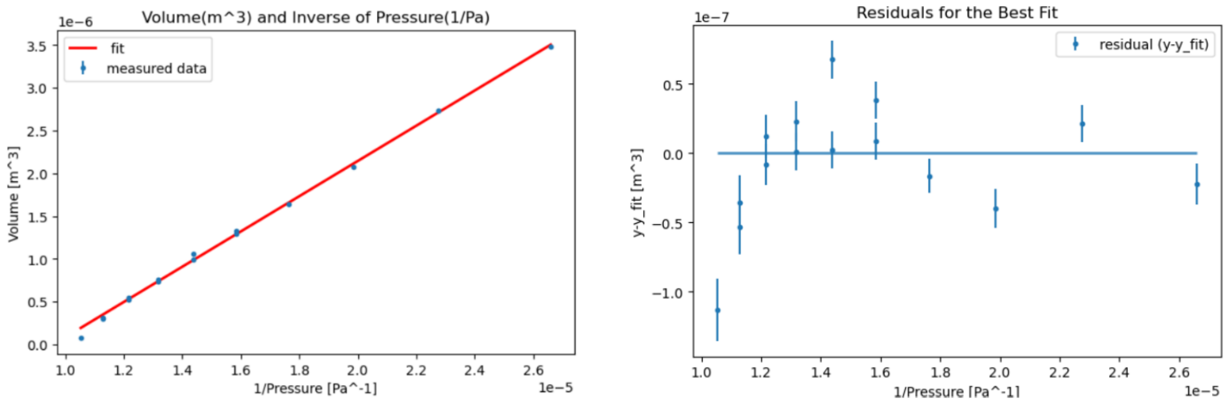

Figure 3: **(a):** Linear fit of the dataset of average volume and inverse pressure for the full range of available hanging weights. A least-squares fitting was done using SciPy curve fit program. For the full range of data, one can see there is a deviation from the linear model of the Ideal Gas Law in our data points. This deviation can be seen in the higher and lowest data points, the first of which likely being due to a poor seal from the syringe stopper while the latter is likely due to the mass being too light to be able to lower the syringe by a significant amount. **(b):** Residual plot of the full range of data for the average volume and inverse pressure plot. The deviations from the best fit line are made more apparent when looking at the residual plot, especially at the more extreme values.

From a measurement using calipers of the syringe, the cross-sectional area of the syringe piston was measured to be $1.539 \times 10^{-4} \pm 1.09 \times 10^{-5}\ m^2$. The initial volume of the syringe was set at $2.0 \pm 0.1 mL$, with the moles of air in the syringe calculated to be $8.92 \times 10^{-5} \pm 7.1 \times 10^{-6} mol$. Finally, using the thermometers found inside the laboratory the ambient temperature was determined to be $295.25 \pm 0.05\ K$. All of the resulting measurements and uncertainties will be employed throughout the paper in the extraction of the value of R.

From the linear fit, a slope (nRT) value of $2.061 \times 10^{-1} \pm 8.873 \times 10^{-4}\ J$ was extracted. From this slope an R value of $7.83 \pm 0.056\ J\ mol^{-1} K^{-1}$ was calculated, which although positive and consistent with the accepted R constant, this less than accurate value could be improved. Due to the ideal nature of the constant trying to be calculated, it was then decided to adjust the data used in our visualizations and calculations in order to better reflect the linear nature of the Ideal Gas Law and simultaneously remove any systematic errors from the experiment. Furthermore, many of the given uncertainties were either over or underestimated which can be seen in the strange pattern of the residual plot. Therefore, during the adjustment of the linear data further data points were taken in the adjusted range with the goal of improving the value of the best fit slope as well as the associated uncertainties.

Adjusted Linear Fit

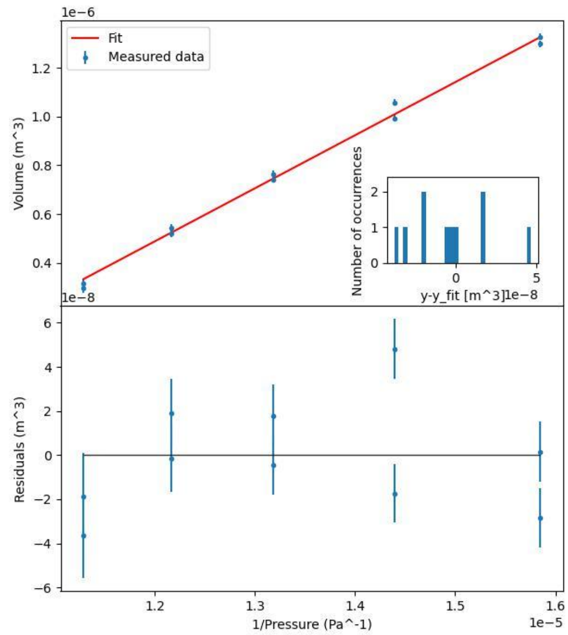

Figure 4: **(a):** Linear fit of the adjusted dataset (200-600g) of average volume and inverse pressure for the full range of available hanging weights. A least-squares fitting was done using SciPy curve fit program. For the adjusted range of data, one can see the previous deviation in linearity has been significantly reduced. A further set over the same values were recorded for the adjusted range to increase precision and reduce the uncertainty of our slope and the calculation for R. **(b):** Residual plot of the adjusted range of data for the average volume and inverse pressure plot. Around one third of the points agreed with the best fit with the remaining points scattered randomly. **(c):** Histogram of the residuals showing the frequency of how often an uncertainty value was associated with a data point.

From the linear fit, a slope (nRT) value of $2.179 \times 10^{-1} \pm 3.028 \times 10^{-3}\ J$ was extracted. From this slope an R value of $8.27 \pm 0.08\ J\ mol^{-1}K^{-1}$ was calculated, which is a far more precise value than in our full data set being close to 99.5% of the accepted R constant of $8.31446 \ldots\ J\ mol^{-1}K^{-1}$. Based on the results of this final data set it can be seen that there is a very apparent linear relationship between the average volume and inverse pressure with our method. This relationship demonstrates a positive agreement with the accepted ideal gas constant R. Improved uncertainties were reflected in our final data set as it can be seen in our residual plot that there is no evidence of overestimation.

Gas Constant Conclusion Paragraph

In summary, we conducted an experiment of measuring the gas constant R using an inexpensive method consisting of hanging weights and a syringe. The data of external force exerted on the syringe was converted to the pressure exerted by the gas inside the syringe, with data for volume being taken based on the change from the initial starting value. This data was fit using a linear least-squares model, where R was then extracted from the slope of this dataset resulting in the measured value of R to be $8.27 \pm 0.08 \frac{J}{K\,mol}$. Our obtained value agrees with the accepted value of R showcasing that our method used is sufficiently accurate for future use in different educational contexts or thermodynamic experiments. The limitations of our experimental method can be seen in our expanded data set as there is a noticeable deviation from the ideal linear model in this unadjusted range which, even after adjusted, results in the associated uncertainty being considerably larger than theoretical value. Future work may continue; however, shifting away from the calculation of the R constant may be an ideal direction due to the 2019 SI unit redefinition. Future experiments could instead peer into the non-ideal interactions and properties of air, or even calculating local external pressure. Looking ahead, possible future steps could include a more precise evaluation of systematic uncertainties which may lead to a more precise experimental value, or possibly reveal more non-ideal characteristics in our theoretical model.

Citations

Gas constant. (n.d.). https://www.hellenicaworld.com/Science/Physics/en/GasConstant.html

Helmenstine, A. (2023, September 3). *Boltzmann Constant Definition and units*. Science Notes and Projects. https://sciencenotes.org/boltzmann-constant-definition-and-units/