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1a) First of all, likelihood function is specified.

$$L(a|X) = \prod_{i=1}^{n} a(1-x_i)^{a-1}$$

Then converted to log (ln) likelihood function. Because the log-likelihood function simplifies by turning products into sums.

$$L(a|X) = \log \prod_{i=1}^{n} a(1-x_i)^{a-1}$$

$$= \sum_{i=1}^{n} \left(\log(a) + (a-1)\log(1-x_i)\right)$$

For MLE we differentiate the log likelihood function with respect to a and set it equal to 0.

$$\frac{d}{da}\left[n\log a + (a-1)\sum_{i=1}^{n}\log(1-x_i)\right]$$

$$= \sum_{i=1}^{n}\log(1-x_i) + \frac{n}{a}$$

$$a = \frac{-n}{\sum_{i=1}^{n}\log(1-x_i)}$$

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1c) The posterior $= P(a|X) \propto \underbrace{p(X|a)}_{\text{likelihood function}} \cdot \underbrace{p(a)}_{\text{prior}}$

$$p(a) = \lambda \cdot a^{\lambda-1} \cdot e^{-\lambda a}$$

$$\log L(a) = \sum_{i=1}^{n} \log\left[a(1-x_i)^{a-1}\right]$$

$$\log p(a) = \log\left[\lambda \cdot a^{\lambda-1} \cdot e^{-\lambda a}\right]$$

$$\log p(a|X) = \log L(a) + \log p(a)$$

$$= n\log a + (a-1)\sum_{i=1}^{n} \log(1-x_i) + \log\lambda + (\lambda-1)\log a - \lambda a$$

Derivative of log posterior

$$= \frac{n}{a} + \sum_{i=1}^{n} \log(1-x_i) + \frac{\lambda-1}{a} - \lambda$$

$$\frac{n}{a} + \sum_{i=1}^{n} \log(1-x_i) + \frac{\lambda-1}{a} - \lambda = 0$$

$$a = \frac{n+\lambda-1}{\lambda - \sum_{i=1}^{n} \log(1-x_i)}$$