MECE 5397: Scientific Computing for Mechanical Engineers

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Solving Poisson’s Equation on a Rectangle

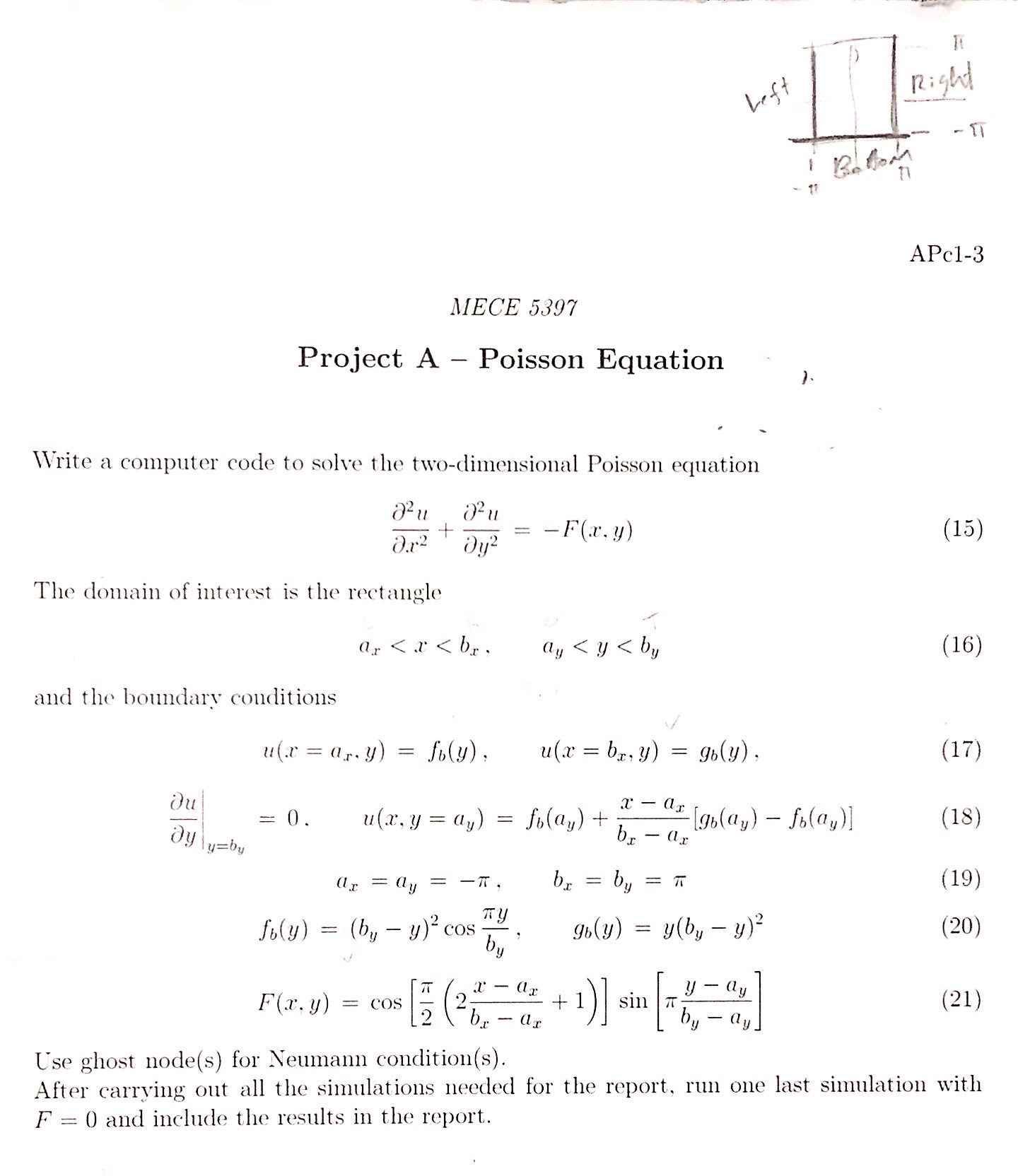
Project ID: **APc1-3**

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# Abstract

# Mathematical Statement of the Problem



# Discretization of the Equations

## Discretizing Poisson Equation

Rearranging for ui,j

For Gauss-Seidel Method

For Successive Over Relaxation Method

# Description of Numerical Methods Used

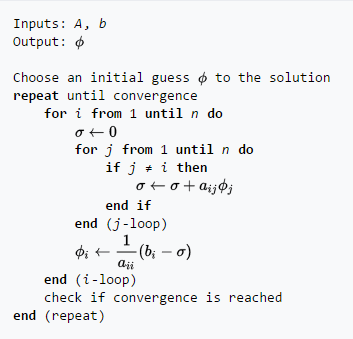
## Gauss-Seidel Method

The Gauss-Seidel Method is an iterative linear technique for solving square systems. It is defined by the form

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where the procedure is generally continued until the changes between x and the next iteration of x are below a set tolerance.

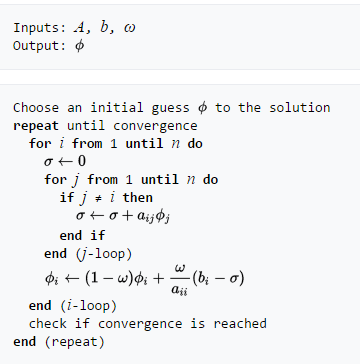
Convergence for Gauss-Seidel method are dependent on the matrix A. It converges if A is symmetric positive-definite or A is strictly diagonally dominant. Even if conditions are not satisfied, Gauss-Seidel converges sometimes.



## Successive Over Relaxation

The successive over-relaxation (SOR) is a variant of the Gauss-Seidel method for solving a linear system of equations, resulting in faster convergence. C:\Users\nkiwaich\Pictures\4.PNG where w is the relaxation factor, thusC:\Users\nkiwaich\Pictures\Capture1.PNG

Where w is the choice for relaxation, and depends of the coefficient matrix. Usually, w is greater than 0 and smaller than 2.



# Technical Specifications of Computer Used

* Processor – Intel Core i7-3770S CPU @ 3.10GHz
* RAM - 8 GB
* Hard Drive - 500 GB
* Graphics Card - any with DisplayPort/HDMI or DVI support - desktop only
* Monitor – Dell OptiPlex widescreen LCD with DisplayPort/HDMI or DVI support

# Results

## Graphs

For the Project given, we obtained a contour plot as shown

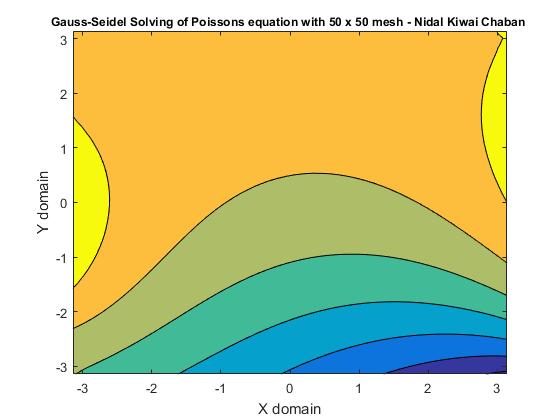


Figure 1-Contour Plot

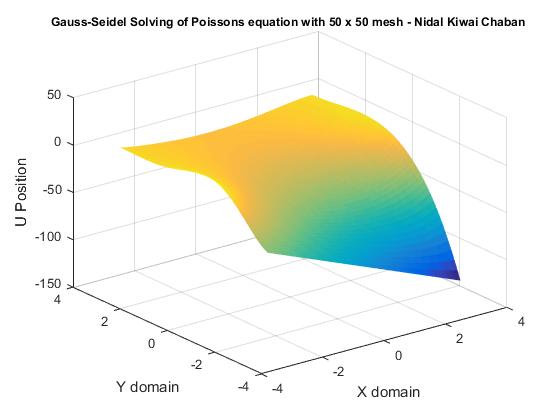


Figure 2-Surface Plot

## Parameters used in simulations

* Input Parameters
  + Number of nodes n for n x n mesh size
  + ax = ay = -π
  + bx = by = π
  + Boundary Conditions as stated
* Output Parameters
  + Number of iterations
  + Elapsed time
  + Mean of u (for grid independence)

## Effect of Number of Points Used For Discretization

For Gauss-Seidel

|  |  |  |
| --- | --- | --- |
| Gauss-Seidel Method | | |
| Mesh size | Iterations for tol=1e-6 | Elapsed time (seconds) |
| 10x10 | 34 | .037481 |
| 20x20 | 769 | .5911 |
| 50x50 | 4992 | 5.66963 |
| 100x100 | 20976 | 34.6258 |
| 200x200 | Excessive time consumed | |
| 1000x1000 |
| 5000x5000 |

For Successive Over-Relaxation

|  |  |  |
| --- | --- | --- |
| Successive Over-Relaxation Method | | |
| Mesh size | Iterations for tol=1e-6 | Elapsed time (seconds) |
| 10x10 | 8 | .008049 |
| 20x20 | 269 | .158379 |
| 50x50 | 1811 | 1.503082 |
| 100x100 | 7547 | 12.2652 |
| 200x200 | 33060 | 164.638 |
| 1000x1000 | Excessive time consumed | |
| 5000x5000 |

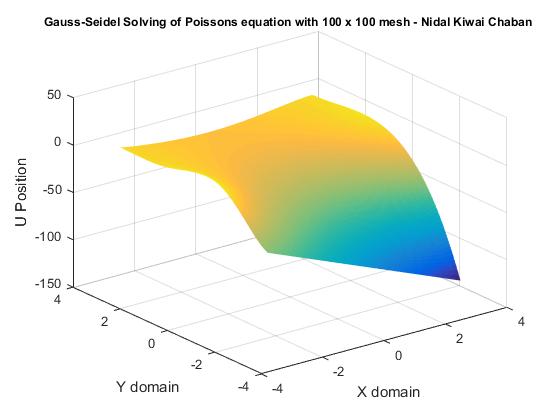


Figure 3-Surface plot of 100x100 mesh for Gauss-Seidel

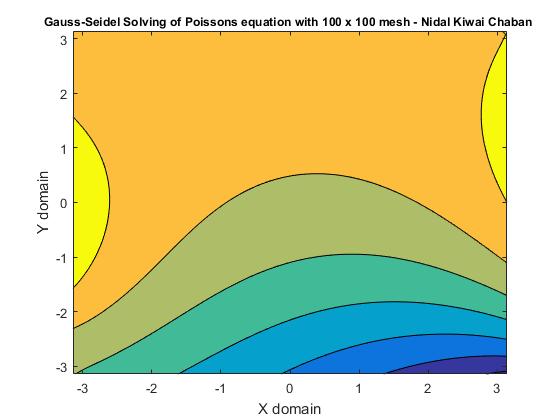


Figure 4-Contour plot of 100x100 mesh for Gauss-Seidel

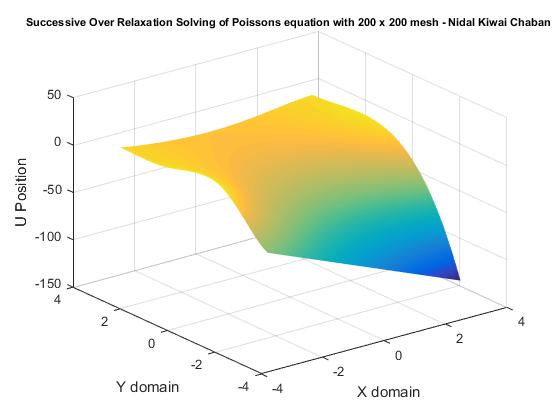


Figure 5-Surface Plot of 200 x 200 mesh of SOR

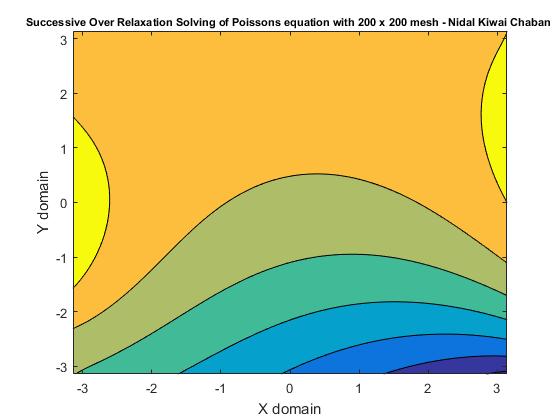


Figure 6-Contour Plot of 200x200 mesh for SOR

As it is shown, the finer the mesh is, the more accurate and the more iterations are required to come to convergence. This is due to the need for every node to come within tolerance to be considered a satisfactory simulation run.