MECE 5397: Scientific Computing for Mechanical Engineers

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Solving Poisson’s Equation on a Rectangle

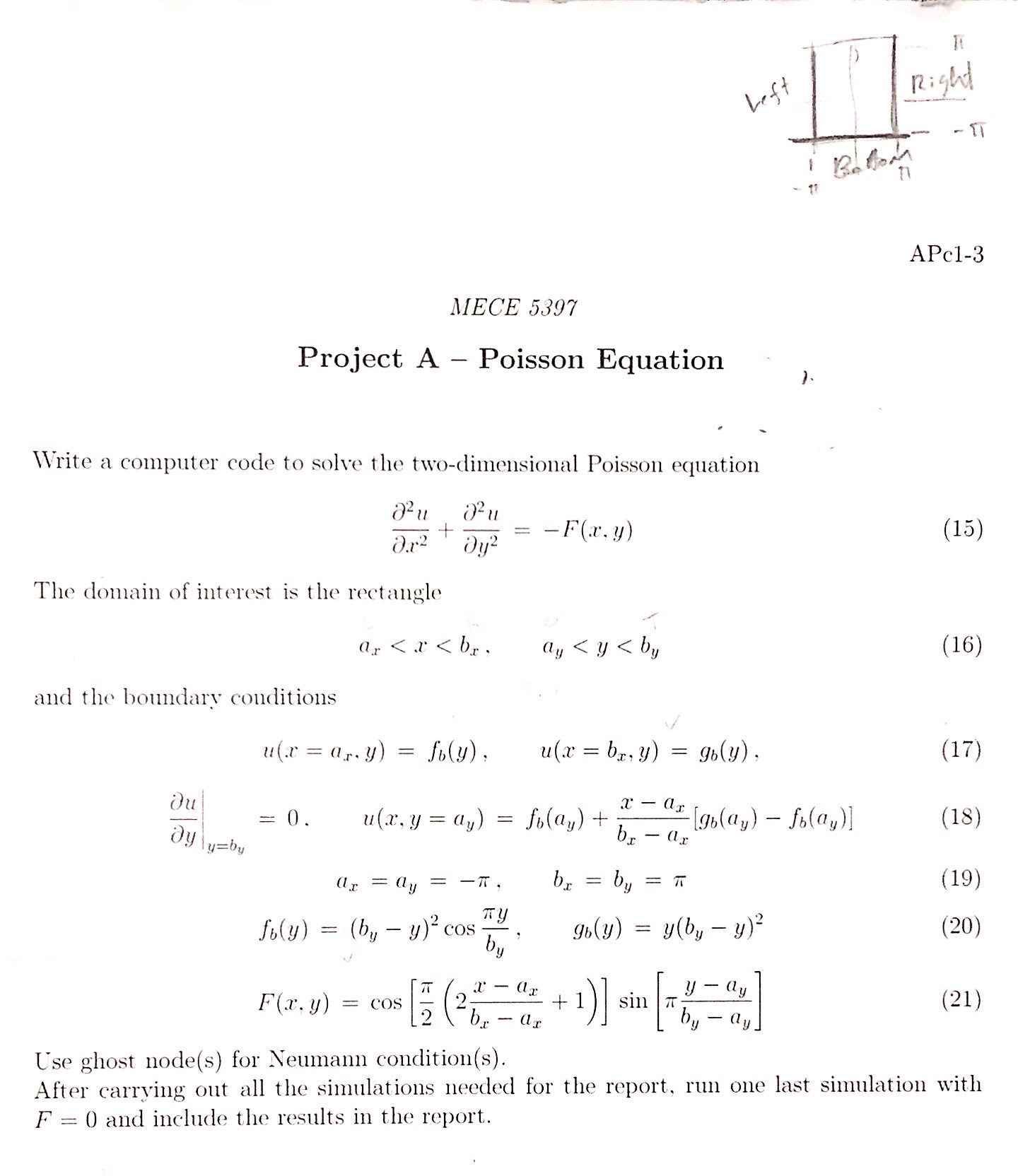
Project ID: **APc1-3**

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# Abstract

# Mathematical Statement of the Problem



# Discretization of the Equations

## Discretizing Poisson Equation

Rearranging for ui,j

For Gauss-Seidel Method

For Successive Over Relaxation Method

# Description of Numerical Methods Used

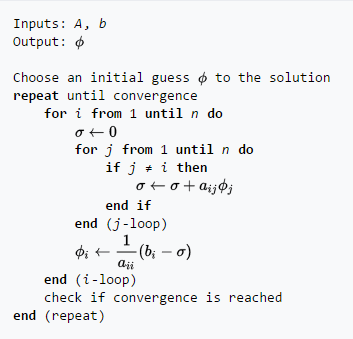
## Gauss-Seidel Method

The Gauss-Seidel Method is an iterative linear technique for solving square systems. It is defined by the form

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where the procedure is generally continued until the changes between x and the next iteration of x are below a set tolerance.

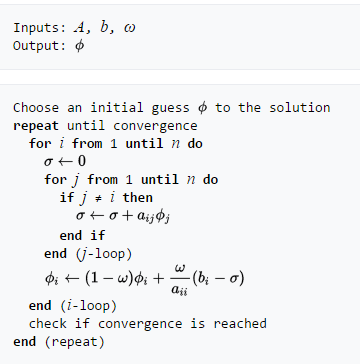
Convergence for Gauss-Seidel method are dependent on the matrix A. It converges if A is symmetric positive-definite or A is strictly diagonally dominant. Even if conditions are not satisfied, Gauss-Seidel converges sometimes.



## Successive Over Relaxation

The successive over-relaxation (SOR) is a variant of the Gauss-Seidel method for solving a linear system of equations, resulting in faster convergence. C:\Users\nkiwaich\Pictures\4.PNG where w is the relaxation factor, thusC:\Users\nkiwaich\Pictures\Capture1.PNG

Where w is the choice for relaxation, and depends of the coefficient matrix. Usually, w is greater than 0 and smaller than 2.



# Technical Specifications of Computer Used

* Processor – Intel Core i7-3770S CPU @ 3.10GHz
* RAM - 8 GB
* Hard Drive - 500 GB
* Graphics Card - any with DisplayPort/HDMI or DVI support - desktop only
* Monitor – Dell OptiPlex widescreen LCD with DisplayPort/HDMI or DVI support

# Results

## Graphs

For the Project given, we obtained a contour plot as shown

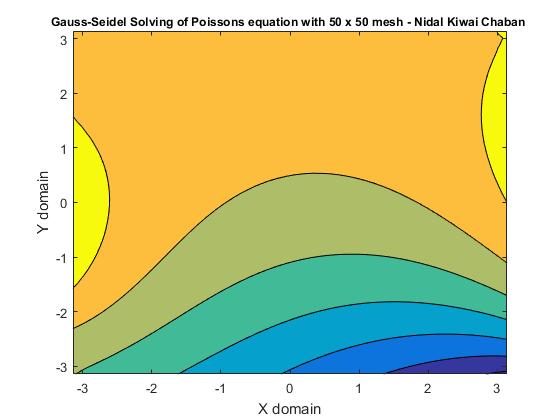


Figure 1-Contour Plot

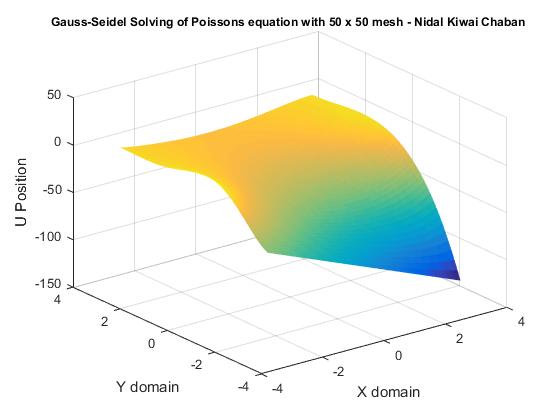


Figure 2-Surface Plot

## Parameters used in simulations

* Input Parameters
  + Number of nodes n for n x n mesh size
  + ax = ay = -π
  + bx = by = π
  + Boundary Conditions as stated
* Output Parameters
  + Number of iterations
  + Elapsed time
  + Mean of u (for grid independence)

## Effect of Number of Points Used For Discretization

For Gauss-Seidel

|  |  |  |
| --- | --- | --- |
| Gauss-Seidel Method | | |
| Mesh size | Iterations for tol=1e-6 | Elapsed time (seconds) |
| 10x10 | 34 | .037481 |
| 20x20 | 769 | .5911 |
| 50x50 | 4992 | 5.66963 |
| 100x100 | 20976 | 34.6258 |
| 200x200 | Excessive time consumed | |
| 1000x1000 |
| 5000x5000 |

For Successive Over-Relaxation

|  |  |  |
| --- | --- | --- |
| Successive Over-Relaxation Method | | |
| Mesh size | Iterations for tol=1e-6 | Elapsed time (seconds) |
| 10x10 | 8 | .008049 |
| 20x20 | 269 | .158379 |
| 50x50 | 1811 | 1.503082 |
| 100x100 | 7547 | 12.2652 |
| 200x200 | 33060 | 164.638 |
| 1000x1000 | Excessive time consumed | |
| 5000x5000 |

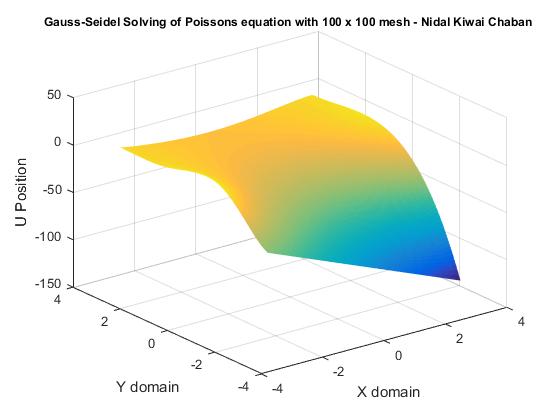


Figure 3-Surface plot of 100x100 mesh for Gauss-Seidel

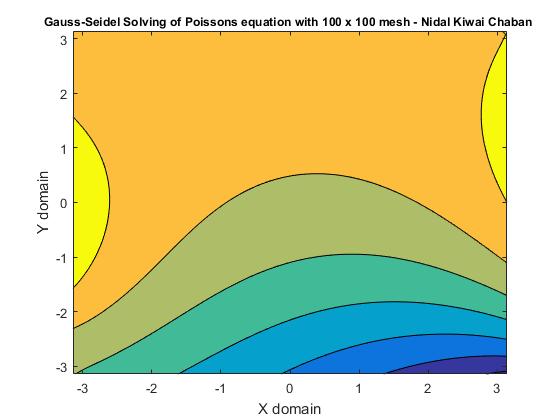


Figure 4-Contour plot of 100x100 mesh for Gauss-Seidel

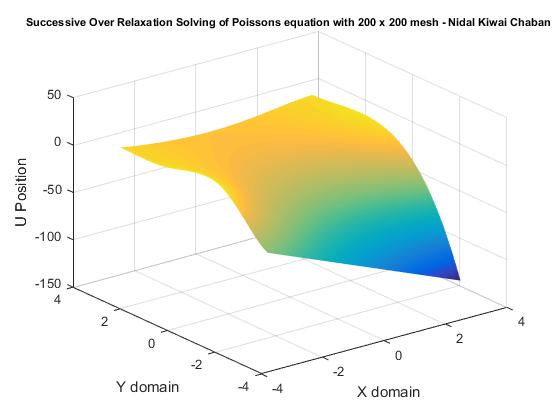


Figure 5-Surface Plot of 200 x 200 mesh of SOR

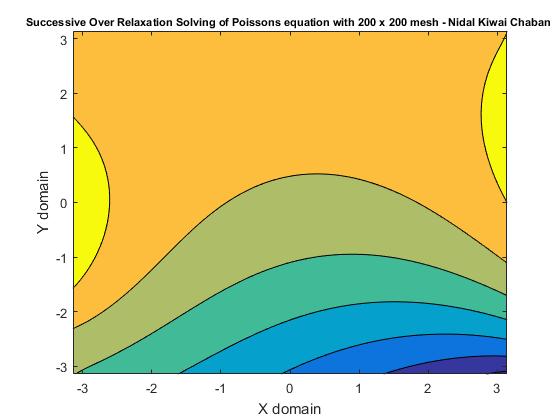


Figure 6-Contour Plot of 200x200 mesh for SOR

As it is shown, the finer the mesh is, the more accurate and the more iterations are required to