

**APPROXIMATING SOLUTIONS OF *PARTIAL DIFFERENTIAL EQUATIONS*
WITH PSUEDOSPECTRAL METHODS AND *SUPPORT VECTOR MACHINE***

A RESEARCH PAPER

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ABSTRAK

JUDUL ABSTRAK

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Abstrak merupakan penjelasan singkat dan padat tentang pekerjaan dan hasil penelitian TA, yang dituliskan secara teknis. Abstrak memiliki karakter tegas dan komprehensif, dan hanya dapat dituliskan setelah pekerjaan penelitian telah mencapai tahap tertentu, dan karenanya ada hasil penelitian yang dapat dilaporkan. Abstrak ditulis menjelang akhir penyelesaian penulisan buku TA.

Secara umum, abstrak memuat beberapa komponen penting, yaitu: konteks atau cakupan pekerjaan penelitian, tujuan penelitian, metodologi yang digunakan selama penelitian, hasil-hasil penting yang dapat ditambahkan dengan implikasinya, dan simpulan dari penelitian. Dengan demikian, suatu abstrak tidak dapat dituliskan apabila penelitian belum mencapai hasil tertentu, apalagi kalau penelitiannya pun belum dilakukan.

Panjang abstrak sebaiknya dicukupkan dalam satu halaman, termasuk kata kunci. Tiga kata kunci dipandang cukup, yang masing-masingnya memuat paduan kata utama, yang dapat merepresentasikan isi Abstrak. Halaman Abstrak tidak memuat informasi judul dan penulis, sehingga tidak secara langsung dapat digunakan sebagai lembaran Abstrak Sidang TA yang disediakan untuk hadirin, yang memerlukan tambahan (sekurangnya) dua informasi tersebut.

Kata kunci: Konsep Abstrak, Komponen Abstrak, Kata Kunci.

ABSTRACT

TITLE

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In general, Abstract is a translation of Abstrak. However, appropriate paraphrase may need some words or sentences whose meanings are close enough to those written in Abstrak.

Key words: Abstract Concepts, Abstract Components, Key Words.

PENGESAHAN

APPROXIMATING SOLUTIONS OF *PARTIAL DIFFERENTIAL EQUATIONS* WITH PSUEDOSPECTRAL METHODS AND *SUPPORT VECTOR MACHINE*

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1. Nama Penguji 1
2. Nama Penguji 2

PEDOMAN PENGGUNAAN BUKU TUGAS AKHIR

Buku Tugas Akhir Sarjana ini tidak dipublikasikan, namun terdaftar dan tersedia di Perpustakaan Institut Teknologi Bandung. Buku ini dapat diakses umum, dengan ketentuan bahwa penulis memiliki hak cipta dengan mengikuti aturan HaKI yang berlaku di Institut Teknologi Bandung. Referensi kepustakaan diperkenankan dicatat, tetapi pengutipan atau peringkasan hanya dapat dilakukan seizin penulis, dan harus disertai dengan kebiasaan ilmiah untuk menyebutkan sumbernya.

Memperbanyak atau menerbitkan sebagian atau seluruh buku Tugas Akhir harus atas izin Program Studi Sarjana Fisika, Fakultas Matematika dan Ilmu Pengetahuan Alam, Institut Teknologi Bandung.

*Tugas Akhir ini dipersembahkan untuk
Tuhan, Bangsa, dan Almamater*

KATA PENGANTAR

Kata pengantar berperan sebagai gerbang masuk bagi pembaca dan mendapat sajian ringkas tentang hal-hal terkait paparan pada buku Tugas Akhir (TA). Sajian ini sejatinya merupakan pengenalan umum bagi pembaca tentang isi tulisan. Hal ini berbeda dengan abstrak yang mendeskripsikan pekerjaan dan hasil penelitian secara lebih teknis.

Kata pengantar merupakan wadah penulis untuk mengenalkan dan mempromosikan pekerjaan dan hasil penelitian dengan bahasa yang sederhana, sehingga pembaca tertarik untuk menelusuri lebih jauh dengan mencermati seluruh paparan pada buku TA. Ini salah satu tujuan kata pengantar. Contoh paragraf yang mengantar pembaca pada isi Buku TA: *Template L^AT_EX* diberikan berikut ini.

Menuliskan pekerjaan dan hasil penelitian TA dalam suatu laporan buku TA memerlukan panduan standar. Panduan ini dibuat dalam beberapa dokumen, yang salah satunya adalah Buku TA: *Template L^AT_EX*. Suatu template adalah cetakan yang siap dituang oleh curahan buah pikiran yang keluar dari pengalaman dalam melakukan pekerjaan penelitian dan hasil-hasilnya. Mencermati cetakan yang memberikan sejumlah contoh dapat memperlancar penulisan laporan tersebut menjadi suatu produk, yaitu buku TA.

Tujuan lain dari Kata Pengantar adalah memberi tempat untuk menyampaikan rasa syukur dan terima kasih kepada banyak pihak, misalnya keluarga, staf akademik, staf tenaga kependidikan, teman, individu atau komunitas pemberi dukungan dan inspirasi, dan institusi pendukung pendanaan seperti pemberi beasiswa atau dana penelitian, atau pendukung akses fasilitas.

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Tidak ada sub bab/bagian pada Kata Pengantar, namun daftar rincian diperkenankan. Pada bagian identitas akhir, seperti berikut ini, dituliskan nama mahasiswa dan NIM, bukan *penulis*, dan tidak perlu ditandatangani. Berikut adalah contoh penulisan rincian yang berisi ucapan terima kasih:

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- Rekan-rekan Spectranova,

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DAFTAR NOTASI

Notasi	Arti
$F_{\mu\nu}$	Tensor Elektromagnetik
$R^\mu_{\alpha\nu\beta}$	Tensor Riemann
$\Gamma^\rho_{\mu\nu}$	Simbol Christoffel
$g_{\mu\nu}$	Tensor Metrik
A_μ	Medan Gauge
$R_{\mu\nu}$	Tensor Ricci
\mathcal{L}	Densitas Lagrangian
\hbar	Konstanta Planck Tereduksi
\mathbb{R}	Himpunan Bilangan Real

DAFTAR SINGKATAN

Notasi	Arti
FWHM	<i>Full width half maximum</i>
rms	<i>root mean square</i>
RFS	<i>Rotary forcespinning</i>
PVP	Polivinil pirolidon
SI	Satuan Internasional

DAFTAR GAMBAR

Gambar 1.1	Different presentations of ultrasonic scan data. The subject is a box containing several anomalies indicated by the yellow oval and red triangle. The first presentation is the A-Scan where the amplitude measured over time in one location. The second is B-Scan where the amplitudes over time is measured along a line. The results are then stacked with the vertical axis acting as time and the horizontal axis as position. The higher amplitudes correspond to lighter pixels. The third presentation is C-Scan which is a view of the amplitudes of reflected waves except the initial impulse and back wall reflection.	7
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CHAPTER I

INTRODUCTION

1.1 Background

Partial differential equations (PDE) are a common tool widely used in the modern scientific understanding and many engineering processes. This is because many systems can be described by the way they change and often this can be more intuitive. As an example, think of someone heating up a large frying pan on a gas stove. To describe how the pan heats up when the center is right above the burner, one can say that the center would heat up first followed by its surroundings. Eventually the pan would not heat up any further. Also notice that the edges of the pan would always be cooler than the center. However, once a more complex setup is introduced such as an uneven heat source or more complex materials with different heat rates of transferring heat it becomes much harder to describe how the pan heats up. A more useful way to describe the system is using the heat equation. For a temperature function $u(\mathbf{x}, t)$ of spatial coordinates vector \mathbf{x} and time t , the generalized heat equation is defined in eq. (1.1).

$$\frac{\partial u}{\partial t} = \nabla \cdot (\alpha \nabla u) \quad (1.1)$$

The divergence operator $\nabla \cdot$ denotes sum of all first spatial derivatives. In three dimensions this is $\nabla \cdot = \frac{\partial}{\partial x_1} + \frac{\partial}{\partial x_2} + \frac{\partial}{\partial x_3}$. And the gradient operator ∇ is the vector of all first spatial derivatives which in three dimensions is $\nabla f = \left[\frac{\partial f}{\partial x_1} \quad \frac{\partial f}{\partial x_2} \quad \frac{\partial f}{\partial x_3} \right]^T$. This equation says that the rate at which the temperature changes in time $\frac{\partial u}{\partial t}$ is proportional to how different the differences in temperature of a spot in the pan with its surroundings $\nabla^2 u$ multiplied with thermal diffusivity parameter α . One example of an insight this gives is that given a homogeneous material (α is constant) and the stove outputs heat at a constant rate the temperature will no longer change for a particular point once the temperature difference is constant. In other words once the function $u(\mathbf{x}, t)$ at some time $t > 0$ approaches a linear function in space, the temperature at all spots will no longer change in time. Mathematically this is because the time derivative is zero if the

second spatial derivatives are also zero. Intuitively this is because once the temperature distribution approaches linear, the current spot on the pan is outputting as much heat it is getting. This insight is an example of why PDEs are useful.

Many other fields in physics such as waves, quantum dynamics, fluid dynamics, elastics, and many more also define systems using PDEs. PDEs are also used outside the physical sciences. In Finance, the Black-Scholes-Merton or Black-Scholes equation models the dynamics of the financial market. In ecology, PDEs are used to model population growth which is useful for modelling location dependent carrying capacity. This is used to model species distribution which can help develop better conservation policies. In the social sciences, PDEs such as the cross-diffusion model is used to explore the evolution of the urban environment (Jin et al., 2023). Crowd dynamics is another field where researchers have found it useful to model using PDEs (Hughes, 2000; Mukherjee et al., 2015). These are used to model dynamics such as shock-waves, pathing, and crowd flow as a whole. There are a variety of applications, ranging from bird flocks, pedestrian crowds, to robotic swarms (Gong et al., 2023).

The scenario in which PDEs are used to simulate a system typically means computing an unknown quantity like the temperature field of an object over time from a set of known parameters such as initial conditions and material parameters such as thermal diffusivity in the case of the heat equation. This scenario is termed the forward problem. The inverse problem on the other hand solves for an unknown parameter using other known parameters and observations of the partial solution such as temperature field at the object's surface. Traditional methods used to solve these problems utilized knowledge of the exact equations to numerically solve PDEs. As an example, the Finite Difference Method (FDM) approximates the solution by substituting the partial derivatives with finite differences such as the first derivative approximation in eq. (1.2).

$$\frac{\partial f(x)}{\partial x} \approx \frac{f(x+h) - f(x)}{h} \quad (1.2)$$

Traditional numerical solvers have enjoyed many decades of development due to their long history. There are many general approaches to solve PDEs and many more very specific approaches. Other than FDM, some widely used approaches include Finite Element Methods (FEM), Finite Volume Methods (FVM), Collocation Methods,

and Spectral Methods. FEM and FDM are both mesh based approaches, meaning they rely on discretization of the computational domain <ADD DIAGRAM FOR THIS>. There are several challenges associated with this. First, irregular domains such as bone, spider silk, gravel, or other bio-inspired or aggregate materials pose a challenge due to the complexity of the domain geometry (Buoni & Petzold, 2007; Gaul et al., 1991; Jia et al., 2024). Also, irregular domains which create large deformations or mesh entanglement cause these methods to become ineffective (J.-S. Chen et al., 2017). While there are strategies to mitigate this, by definition they are an additional layer of difficulty to the process. Second, multiscale applications where the micro and macro scales are both important require fine meshes such that the small scale structures are adequately simulated. This creates meshes with very large number of points that are very resource intensive (Buoni & Petzold, 2007). Therefore, in problems where these issues important or resources are limited, mesh-free methods are preferred. Third, traditional methods require prior knowledge of analytic forms of PDEs. This is because the solvers use the equations to formulate the solution. A dealbreaker like this makes traditional numerical approaches unsuited to data dominant problems. Fourth, a single evaluation of traditional solvers may not be very costly, however multiple evaluations can add up. This is very apparent in inverse problems where the solver is evaluated multiple times to compute gradients in order to obtain parameter functions. These challenges altogether have motivated research into alternative methods that do not completely rely on prior knowledge, mesh free, and fast enough when computing solutions from parameters.

With the increasing prevalence of machine learning methods and their use in more and more fields, research into their use for scientific computing has taken off in recent years. While statistical modeling has already been widely used in areas such as physical constants, stellar population studies, risk assessments of events such as earthquakes and coronal mass ejections (Anselmo & Pardini, 2005; Berliner, 2003; Bernardi et al., 2022; Reinhardt et al., 2016; Spanos, 2006; Uzan, 2003), the dominant approach for forward modeling or inverse modeling has remained physics based numerical models. Machine learning has provided an alternative approach to model the solutions of PDEs. In their work, Aarts and Van Der Veer (2001) utilized neural networks to approximate each term of PDEs describing damped and undamped free vibrations and substituting them into the PDEs and associated initial conditions. The network parameters were then optimized to reduce the PDE residual and boundary condition loss using evolutionary

algorithms. This approach was taken in order to make machine learning models more transparent which at the time was being pursued because of the high cost of optimizing uncertainties in water management numerical simulators. However, since this method approximates the mapping between coordinates and the values of a function and their derivatives, retraining would be necessary for changes to the function itself. This could become very costly as retraining costs accumulate. A more recent approach that also utilizes soft constraints from PDE residuals is termed physics informed neural network (PINN) (Raissi et al., 2019). The authors propose a framework that leverages advancements in computing, namely automatic differentiation (AD) techniques made readily available by modern machine learning libraries. In general, PINNs use AD to compute each term of the PDE from the output of the model and this is then substituted into the PDE in order to compute the residuals and loss from boundary conditions. The network residual and boundary loss are then weighted and summed with the data loss. For a neural network $\hat{u}(\mathbf{x}, t)$ approximating the real solution $u(\mathbf{x}, t)$, the residual loss for the heat equation in eq. (1.1) is eq. (1.3). One issue with using AD to compute the residual is that the network input needs to be the independent variable (i.e. coordinates, time, etc.). This once again means that if one wants to compute a different solution, the network needs to be retrained.

$$\mathcal{L}_{PDE} = \left(\frac{\partial \hat{u}(\mathbf{x}, t)}{\partial t} - \nabla \cdot (\alpha \nabla \hat{u}(\mathbf{x}, t)) \right)^2 \quad (1.3)$$

Other works utilize convolutional neural networks (CNN) to compute the solution from input functions such as forcing terms or initial conditions. This approach generally means discretizing the functions on a grid and using these as training data. One study by Wang et al. (2020) predicts turbulent flow using spatial and temporal decomposition and a specialized U-Net, an architecture based on CNNs, to predict the velocity field from the decomposition of the previous velocity field. Part of the loss function is a regularization term for zero divergence in the velocity field to enforce incompressible fluid flow. This term was calculated using finite differences since auto differentiation is not applicable in this situation. Finite differences was also utilized in another CNN based fluid flow upscaling model by Gao et al. (2021b) to compute the residual terms of the steady incompressible Navier-Stokes equation. This model also inferred unknown physical parameters such as boundary conditions. However, this approach would mean the model would need to scale as a quadratic in 2D, cubic in 3D, and much steeper in

higher dimensions. Outside fluid dynamics, the combination of specialized CNNs and finite differences or another numerical differentiation method have been used for many other PDEs including Poisson’s equation for temperature fields (Gao et al., 2021a; Zhao et al., 2023), velocity models from seismic data (Muller et al., 2023), and seismic response of structures (Ni et al., 2022; Zhang et al., 2020). While the use of CNNs mean that discretization is implied, solutions of different initial conditions or parameter functions can be computed by inference and no retraining is required.

The mapping between discretized functions done by CNNs are related to an alternative approach that starts by viewing PDEs as operators, which are generalized mappings between spaces. One group of familiar operators are functions which maps between spaces of scalar values or vector values. PDEs on the other hand are operators that map between function spaces. A simple example is the derivative. The derivative takes in a function and returns the derivative of said function. In other words it is an operator that maps between the space of all functions to the space of derivatives of those functions. Another way to view operators starts by viewing functions as infinite dimensional vectors. Where elements in the vector are the function’s value evaluated at every point in space. The operator can be seen as a vector function mapping between these infinite dimensional vector spaces. Operators are important because a field of research has sprung up around this mathematical concept. Operator learning is the use of machine learning to learn operators using data driven approaches. As an analogy, function regression traditionally has been used to approximate the mapping between input values such as coordinates and output values of functions evaluated at said coordinates. In the case of operator learning, the mapping between function spaces are approximated. The aforementioned approaches using CNNs does this directly using the values of functions at discrete points. There are other approaches like DeepONet that does not require the uniform grid like CNNs (Lu et al., 2021). This architecture instead uses both input functions and coordinates as inputs. The output is the output function evaluated at the coordinates provided. This architecture is based on an extension for deep learning of the universal operator approximation theory for neural networks first proposed almost three decades ago at the time of writing by T. Chen and Chen (1995). The proposed architecture is composed of two sub-networks, where one termed the trunk $\hat{\mathbf{T}}(\mathbf{x}, t)$ learns the latent mapping for coordinates and the other network termed the branch $\hat{\mathbf{B}}(\mathbf{f})$ learns the latent mapping for the input function f . The two latent mappings are combined through a dot product to obtain the approximated output function value

$u(\mathbf{x}, t)$. This is formulated in eq. (1.4).

$$u(\mathbf{x}, t) \approx \hat{G}(\mathbf{f})(\mathbf{x}, t) = \hat{\mathbf{B}}(\mathbf{f}) \cdot \hat{\mathbf{T}}(\mathbf{x}, t) \quad (1.4)$$

Non-destructive testing (NDT) is critical process in the field of engineering and quality assurance that allows the evaluation of a subject without causing damage (Bochud et al., 2015; Howell, 2020; Maio & Fromme, 2022). This makes NDT crucial in many cases like construction and manufacturing where testing needs to be done on products without damaging it. The use of NDT allows for early detection of flaws and anomalies ensuring the reliability, longevity, and safety of structures and machinery. A widely used form of NDT, ultrasonic testing, propagates mechanical waves into the material. The ultrasonic waves, typically in the kHz to MHz range, travel through the material and interact with inhomogeneities in the material i.e. waves reflected by a cavity. This makes it possible to determine the location, size, and nature of deformities by analyzing the waves measured at predetermined locations. The measurements themselves result in a time series of wave amplitudes represented as displacement. The data can be presented as shown in Figure 1.1. The amplitude time series by itself is referred as an A-Scan. B-Scans take measurements at multiple locations to show a cross-section of the subject. C-Scans adds another dimension of measurements and presents a view of things that reflect the waves such as defects. Analysis of the ultrasonic data can then be carried out. When the material can be assumed homogeneous, ray based or time of flight based methods can be used. In pulse echo, the wave travels through the material and gets reflected by interfaces, whether the other side of the material, fractures within, or generally modulus/density discontinuities present in the material. This means the measurements would record a large amplitude initially from the impulse sent into the material. And then the reflected waves from the closest interface would arrive. This way, just by knowing the exact wave speed of the material, one could easily translate reflected wave arrival time into the positions of interfaces.

Ultrasonic NDT has advantages that make it one of the more widely used NDT methods (Felice & Fan, 2018; Howell, 2020; Niven, 2020). One advantage is that ultrasonic testing equipment is often portable making it suitable for cases where it is difficult or not possible to test in a controlled environment such as a laboratory. Inspections for infrastructure such as pipes or rail tracks are examples where it is costly and very disruptive to move the test subject to a testing location (**<empty citation>**).

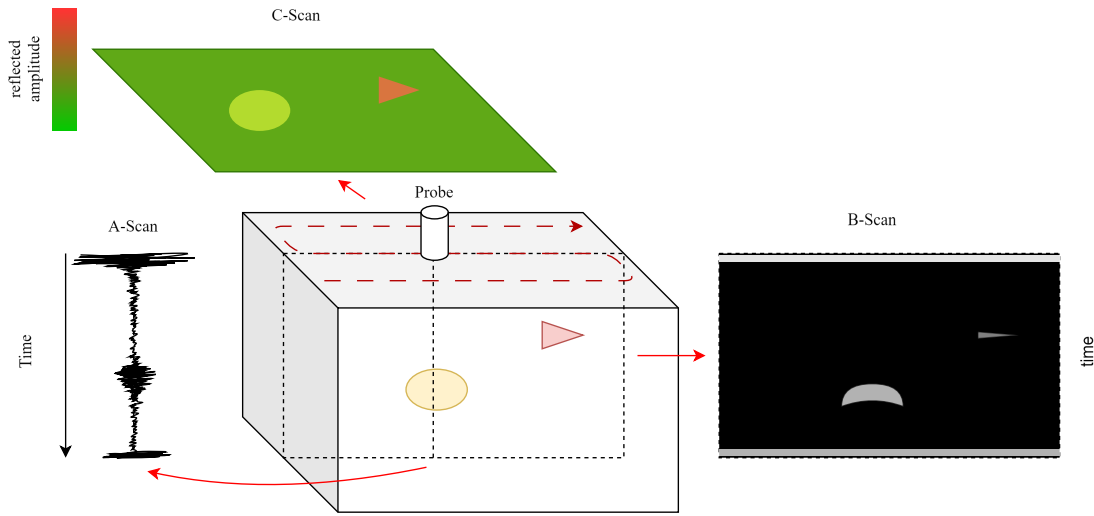


Figure 1.1: Different presentations of ultrasonic scan data. The subject is a box containing several anomalies indicated by the yellow oval and red triangle. The first presentation is the A-Scan where the amplitude measured over time in one location. The second is B-Scan where the amplitudes over time is measured along a line. The results are then stacked with the vertical axis acting as time and the horizontal axis as position. The higher amplitudes correspond to lighter pixels. The third presentation is C-Scan which is a view of the amplitudes of reflected waves except the initial impulse and back wall reflection.

Another advantage of ultrasonic testing is the low risk it poses in terms of safety. Unlike the ionizing radiation used in radiography, ultrasonic waves used in testing are harmless to humans. This means testing can be done with relatively less stringent safety precautions (<empty citation>). Additionally, the technology is relatively mature with a variety of equipment and supplier options. Furthermore, affordable options for inexpensive ultrasonic equipment is also available (<empty citation>). These advantages and the popularity of ultrasonic testing stemming from the aforementioned advantages highlights the important role ultrasonic testing plays in enabling affordable and reliable engineering.

The emergence of composites, sought after for their multi-functional properties, introduces challenges for inspection and analysis (Dwivedi et al., 2018; Ince et al., 2023; Niven, 2020; Yang et al., 2023). Different composites provide a range of benefits, from the strength to weight ratio of carbon fiber reinforced polymer composites to electromagnetic shielding materials. However, the anisotropic and complex characteristic of composites that give rise to the beneficial properties complicates wave propagation because of the location and directional dependence of material properties.

For example wave speed can be faster in certain directions and locations. Reflections will happen not just because of anomalies, but also expected features in the material itself. Extraneous signals from inhomogeneities anticipated in the material makes it harder to extract useful information on unknown defects. In order to enable reliable use of ultrasonic NDT on composite materials and other complex situations, a method to quantify and characterize the complicated behavior is required (Maio & Fromme, 2022).

This is where model based methods come into play. Advances in computation over recent decades has enabled the use of computational tools to revolutionize many fields. Numerical modeling, in particular, has an almost symbiotic relationship with the rise of computing; with the likes of ENIAC, MANIAC, and a plethora of other early electronic digital computers (Anderson, 1986). The ENIAC for example, promised to compute ballistic trajectories 10 times faster than the methods that were in used at the time. This progress allows the simulation and analysis of systems previously beyond reach. The effects range from prediction of weather systems and physics constrained problems like bleeding edge aerospace design to uncovering physical laws of nature.

Despite advances in computing power, numerical modeling of physical systems still remains an open problem with many challenges. Traditional methods such as Finite Element Method (FEM), Finite Difference Method (FDM), and Spectral Methods are limited to known operators and equations (Du et al., 2024; Fanaskov & Oseledets, 2023; Karniadakis et al., 2021; Lu et al., 2021). The modern data landscape further complicates traditional numerical modeling with its characteristic abundance of low quality, high quantity data, as some methods such as FDM are inherently sensitive to noise (De Aragão & Sava, 2021; Huntul et al., 2021). Noisy data also presents a formidable challenge in general because gleaning any meaningful information from noisy and incomplete data requires further filtering and interpretation. This issue is particularly troubling for inverse problems. This is because some inverse problems are unstable in addition to their ill-posedness which necessitates advanced techniques (Caubet, 2013; Mandache, 2001). The next layer of complexity is posed by the challenge of irregular domains. An example of this is fluid flow through materials such as gravel, sponges, even changing with time (Li & Ito, 2006). Due to the nature of irregular domains, mesh based methods struggle to adapt. Addressing these challenges is crucial for understanding a variety of problems in both scientific research

and engineering.

The problem posed by model-based ultrasonic testing combines several of these challenges. First, visualizing properties of materials such as density from measurements of ultrasonic waves that interacted with the material is an inverse problem. This is because the effect is used to infer the cause. The challenge with inverse problems is that in many cases, the result is very sensitive to differences in the parameters (Tarantola, 2005); small variation in density distribution can result in very different measurements. As such, additional noise or inaccuracies in measurement can massively impact the predicted parameter. This very issue has been a long standing challenge in the geophysics community. The full waveform inversion (FWI) procedure used to determine properties of the earth, requires certain techniques such as a minimization criteria that mitigates this sensitivity (Virieux & Operto, 2009). Second, complex or irregular domains are a challenge for traditional mesh based methods as the domain may vary in time (Bürchner et al., 2023; Li & Ito, 2006). Costly mesh generation hampers the performance of mesh based methods. In addition, some approaches like total focusing method (TFM), traveltime tomography (TT) and reverse time migration (RTM) are not able to provide more than a rough estimate of defects. Third, for some materials, there are little to no knowledge present in literature on how waves propagate through the material. Continued research has alleviated this issue by advanced our understanding of wave propagation and materials. Gordoa and Pickering (2021) presented an analytic solution to ultrasonic wave propagation in bubbly liquids. **<empty citation><empty citation>** . However, these solutions are not available for all situations and materials, as such is the case when a novel material is created. In some of these cases data collection may be easier to do compared to deriving from first principles.

Advances in computing technology and the integration of non-destructive testing have revolutionized the aerospace industry (Gupta et al., 2022; Jacob & Raddatz, 2022). This has had the effect of enabling rapid advancements in cutting-edge technologies such as vertical rocket landings, 3-D printed spacecraft, and in-space manufacturing to name a few (Corrado et al., 2023). These technologies addressed several key challenges with space exploration, namely cost, mission flexibility & resilience, and environmental considerations. NDT plays a crucial role in ensuring the safety and quality of the innovations. The novelty of innovations means they do not have the luxury of being

a proven solution.

Bagian ini mendeskripsikan gambaran umum, konteks, dan posisi penelitian TA dalam konstelasi perkembangan pengetahuan yang telah dicapai. Penjelasan yang dituliskan menjadi penting karena dengan landasan yang kuat, maka pekerjaan penelitian dapat terarah dilakukan. Hal ini lebih spesifik dan tegas disampaikan pada sub-sub bab berikutnya.

Beberapa pustaka utama yang berperan dominan dapat disampaikan di sini untuk memberi gambaran tentang letak penelitian TA dalam konstelasi keilmuan yang dicapai. Hasil-hasil dari pustaka terbaru dapat menopang Latar Belakang ini menjadi lebih kuat.

Sangat wajar apabila isi sub bab setelah Latar Belakang ini mengalami penyesuaian saat sejumlah hasil penelitian sudah diperoleh dan dianalisis. Pada dasarnya, hal ini dimungkinkan apabila ada penyesuaian kecil, karena fokus penelitian sejatinya sudah jelas sedari awal, namun hasil-hasil yang diperoleh dapat memperbaharui beberapa butir isi sub bab. Oleh karena itu, finalisasi isi Pendahuluan ini biasanya dilakukan menjelang akhir pembuatan laporan penelitian yang dituangkan dalam buku TA.

1.2 Rumusan dan Batasan Masalah

Bagian ini menjadi salah satu bagian penting dalam Pendahuluan. Setelah paparan Latar Belakang, maka masalah yang diangkat pada pekerjaan penelitian perlu dirumuskan dengan baik. Perumusan ini sebaiknya dibahasakan tidak dalam bentuk kalimat pertanyaan, melainkan kalimat aktif, dan dapat memuat lebih dari satu rumusan.

Sejalan dengan ini, setiap masalah yang diangkat selalu memiliki batas. Ada batasan, asumsi, atau kriteria yang menjadi pembatas atas masalah yang diangkat dalam penelitian TA, sehingga arah penelitian dapat fokus. Batasan ini perlu dituliskan secara tegas, dan dapat saja memuat lebih dari satu.

1.3 Tujuan

Bagian ini secara tegas menuliskan tujuan pekerjaan penelitian TA, yang dapat memuat lebih dari satu. Pemilihan kata kerja pada Tujuan ini sangat penting karena

menggambarkan arah fokus dari jalinan upaya yang dilakukan.

1.4 Metodologi

Di sini disampaikan metodologi yang diterapkan pada pekerjaan penelitian TA. Beberapa di antaranya adalah pengamatan dan akuisisi data, eksperimen numerik, studi pustaka, teoretik atau analitik, dan semi analitik dengan komplemen numerik.

1.5 Sistematika Penulisan

Bagian ini adalah penutup Bab I yang menyampaikan secara ringkas isi setiap bab. Karena pembaca sudah sampai akhir Bab I, yang berarti sudah mengetahui isinya, maka tidak perlu ditulis lagi rincian Bab I. Sebaiknya langsung dituliskan secara ringkas isi rincian bab-bab selanjutnya, misalnya, *Setelah Pendahuluan pada Bab I ini, Bab II akan mengulas tentang ...*

Apabila diperlukan, dapat dituliskan konvensi khusus yang digunakan pada penulisan naskah buku TA ini, misalnya tanda titik menggantikan tanda desimal karena alasan kemudahan dan kejelasan dalam formulasi matematika.

CHAPTER II

KAJIAN PUSTAKA

Bab ini mengulas secara rinci konsep-konsep dasar yang berkaitan dengan pekerjaan penelitian TA dan deskripsi studi pustaka yang dilakukan. Judul bab tidak harus seperti yang dituliskan, melainkan dapat lebih fleksibel yang mencerminkan isi paparan pada bab ini. Demikian halnya dengan judul sub bab.

2.1 Least Squares Support Vector Machine

An arguably fundamental model widely used whether in pedagogical settings or otherwise is the support vector machine. It dates back to works by Vapnik & Lerner in 1963 (Recognition of Patterns with help of Generalized Portraits) and V. N. Vapnik & A. Ya. Chervonenkis in 1964 (A note on one class of perceptrons/On a perceptron class). As the development on SVM continued, what originally was a model for classification of separable data generalized to regression tasks as well Vapnik (2000 The Nature of Statistical Learning Theory).

To derive the least squares support vector regression (J.A. Suykens LSSVM Book) model we start with the linear expression:

$$y = W^T \mathbf{x} + b \quad (2.1)$$

$$\min_{W,e} J(W,e) = \frac{1}{2} W^T W + C \frac{1}{2} \sum_{k=1}^n e_k \quad (2.2)$$

Such that

$$y_k = W^T \mathbf{x}_k + b + e_k \quad k = 1, \dots, n \quad (2.3)$$

$$L(W, b, e; \alpha) = \frac{1}{2} W^\top W + C \frac{1}{2} \sum_{k=1}^n e_k + \sum_{k=1}^n \alpha_k (W^\top \mathbf{x}_k + b + e_k - y_k) \quad (2.4)$$

Derive the KKT system

$$\begin{aligned} \frac{\partial L}{\partial W} = 0 &\rightarrow W = \sum_{k=1}^n \alpha_k x_k \\ \frac{\partial L}{\partial b} = 0 &\rightarrow \sum_{k=1}^n \alpha_k = 0 \\ \frac{\partial L}{\partial e_k} = 0 &\rightarrow \alpha_k = C e_k \quad k = 1, \dots, n \\ \frac{\partial L}{\partial \alpha_k} = 0 &\rightarrow W^\top \mathbf{x}_k + b + e_k - y_k = 0 \quad k = 1, \dots, n \end{aligned} \quad (2.5)$$

After eliminating W and e , with $\mathbf{1}_n = \langle 1, \dots, 1 \rangle$, $\mathbf{y} = [y_1, \dots, y_n]$, and $\alpha = [\alpha_1, \dots, \alpha_n]$ the solution is as follows in block matrix notation

$$\begin{bmatrix} 0 & \mathbf{1}_n^\top \\ \mathbf{1}_n & \Omega + \frac{I}{C} \end{bmatrix} \begin{bmatrix} b \\ \alpha \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{y} \end{bmatrix} \quad (2.6)$$

The solution in eq. (2.6)

Algorithm 1 LSSVR Training

Input: $\mathbf{X}, \mathbf{y}, \gamma$ **Output:** α, b

```
1:  $\Omega \leftarrow []$   $\triangleright$  Construct matrix of inner products in high dimensional space
2: for  $k = 0 \rightarrow n$  do
3:   for  $l = 0 \rightarrow n$  do
4:      $\Omega_{k,l} \leftarrow K(\mathbf{X}_k, \mathbf{X}_l)$ 
5:   end for
6: end for
7:  $\mathbf{H} \leftarrow \Omega + \frac{\mathbf{I}}{\gamma}$ 
8:  $\mathbf{A} \leftarrow []$   $\triangleright$  Construct left hand side matrix
9:  $\mathbf{A}_{0,0} \leftarrow 0$ 
10: for  $k = 0 \rightarrow n$  do
11:    $\mathbf{A}_{k+1,0} \leftarrow 1$ 
12:    $\mathbf{A}_{0,k+1} \leftarrow 1$ 
13: end for
14: for  $k = 0 \rightarrow n$  do
15:   for  $l = 0 \rightarrow n$  do
16:      $\mathbf{A}_{k+1,l+1} \leftarrow \mathbf{H}_{k,l}$ 
17:   end for
18: end for
19:  $\mathbf{B} \leftarrow []$   $\triangleright$  Construct left hand side of the equation
20:  $\mathbf{B}_0 \leftarrow 0$ 
21: for  $k = 0 \rightarrow n$  do
22:    $\mathbf{B}_{k+1} \leftarrow \mathbf{y}_k$ 
23: end for
24:  $\mathbf{A}^\dagger \leftarrow pseudoInverse(\mathbf{A})$   $\triangleright$  Compute solution using pseudo inverse
25:  $\mathbf{S} \leftarrow \mathbf{A}^\dagger \mathbf{B}$ 
26:  $b \leftarrow \mathbf{S}_0$ 
27: for  $k = 0 \rightarrow n$  do
28:    $\alpha_k \leftarrow \mathbf{S}_{k+1}$ 
29: end for
```

The basic psuedocode from the LSSVM Equation for function regression is defined in LSSVR Training for a training set of length n , features \mathbf{X} , and labels \mathbf{y} . Training the LSSVM means computing the values of langrange multipliers α and bias b . K is the kernel function used to compute the inner products in high dimensional space, here we asume the RBF kernel. \mathbf{A} is a matrix of size $n + 1$ by $n + 1$. \mathbf{H} is a matrix of size n by n . \mathbf{I} is the identity. \mathbf{B} is a vector of size $n + 1$. \mathbf{S} is a vector of size $n + 1$.

After training the model can be used for prediction of unseen points. The pseudocode for prediction is shown in LSSVR Prediction for prediction features \mathbf{U} . The trained model uses the learned multipliers of training points α , the training points themselves \mathbf{X} , and the bias b .

Algorithm 2 LSSVR Prediction

Input: $\mathbf{U}, \alpha, \mathbf{X}, b$

Output: \mathbf{v}

```

1:  $\Omega \leftarrow []$  ▷ Construct matrix of inner products in high dimensional space
2: for  $k = 0 \rightarrow m$  do
3:   for  $l = 0 \rightarrow n$  do
4:      $\Omega_{k,l} \leftarrow K(\mathbf{U}_k, \mathbf{X}_l)$ 
5:   end for
6: end for
7:  $\mathbf{v} \leftarrow \Omega\alpha + \mathbf{1}_m b$ 

```

In this exmple we will be using the function $2x^2 + 4$. The values of this function can be seen in Table 2.1.

No	x	y
1	0.0	4.0
2	0.33...	4.22...
3	0.66...	4.88...
4	1.0	6.0

Table 2.1: Example data of function $2x^2 + 4$

$$\text{For } \Omega_{i,j} = K(x_i, x_j) = \exp\left(-\frac{\|x_i - x_j\|^2}{2\sigma^2}\right)$$

For example, with $\sigma = 1$, $x_i = 0.0$, & $x_j = 0.33 \dots$

$$\begin{aligned}
K(0.0, 0.33) &= \exp\left(-\frac{\|0.0 - 0.33\|^2}{2(1)^2}\right) \\
&= \exp\left(-\frac{0.33^2}{2}\right) \\
&= 0.9460
\end{aligned} \tag{2.7}$$

$$\Omega \leftarrow \begin{bmatrix} 1.0000 & 0.9460 & 0.8007 & 0.6065 \\ 0.9460 & 1.0000 & 0.9460 & 0.8007 \\ 0.8007 & 0.9460 & 1.0000 & 0.9460 \\ 0.6065 & 0.8007 & 0.9460 & 1.0000 \end{bmatrix} \quad (2.8)$$

$$\mathbf{I}_{\frac{1}{\gamma}} \rightarrow \mathbf{I}_{\frac{1}{5}} \rightarrow \begin{bmatrix} 0.2000 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.2000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.2000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.2000 \end{bmatrix} \quad (2.9)$$

$$\Omega + \mathbf{I}_{\frac{1}{5}} \rightarrow H \rightarrow \begin{bmatrix} 1.2000 & 0.9460 & 0.8007 & 0.6065 \\ 0.9460 & 1.2000 & 0.9460 & 0.8007 \\ 0.8007 & 0.9460 & 1.2000 & 0.9460 \\ 0.6065 & 0.8007 & 0.9460 & 1.2000 \end{bmatrix} \quad (2.10)$$

$$A \rightarrow \begin{bmatrix} 0.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 1.0000 & 1.2000 & 0.9460 & 0.8007 & 0.6065 \\ 1.0000 & 0.9460 & 1.2000 & 0.9460 & 0.8007 \\ 1.0000 & 0.8007 & 0.9460 & 1.2000 & 0.9460 \\ 1.0000 & 0.6065 & 0.8007 & 0.9460 & 1.2000 \end{bmatrix} \quad (2.11)$$

$$B \rightarrow \begin{bmatrix} 0.0000 \\ 4.0000 \\ 4.2222 \\ 4.8889 \\ 6.0000 \end{bmatrix} \quad (2.12)$$

$$A^\dagger \rightarrow \begin{bmatrix} -0.8994 & 0.4348 & 0.0652 & 0.0652 & 0.4348 \\ 0.4348 & 2.0686 & -1.6490 & -0.5292 & 0.1096 \\ 0.0652 & -1.6490 & 3.3774 & -1.1992 & -0.5292 \\ 0.0652 & -0.5292 & -1.1992 & 3.3774 & -1.6490 \\ 0.4348 & 0.1096 & -0.5292 & -1.6490 & 2.0686 \end{bmatrix} \quad (2.13)$$

$$A^\dagger B \rightarrow S \rightarrow \begin{bmatrix} 4.9421 \\ -0.6177 \\ -1.3737 \\ -0.5625 \\ 2.5538 \end{bmatrix} \quad (2.14)$$

$$b \rightarrow 4.9421 \quad (2.15)$$

$$\alpha \rightarrow \begin{bmatrix} -0.6177 \\ -1.3737 \\ -0.5625 \\ 2.5538 \end{bmatrix} \quad (2.16)$$

Prediction

$$U \rightarrow \begin{bmatrix} 0.3 \\ 0.2 \\ 0.5 \end{bmatrix} \quad (2.17)$$

$$\Omega \rightarrow \begin{bmatrix} 0.9560 & 0.9994 & 0.9350 & 0.7827 \\ 0.9802 & 0.9912 & 0.8968 & 0.7261 \\ 0.8825 & 0.9862 & 0.9862 & 0.8825 \end{bmatrix} \quad (2.18)$$

$$\Omega\alpha \rightarrow \begin{bmatrix} -0.4904 \\ -0.6170 \\ -0.2008 \end{bmatrix} \quad (2.19)$$

$$\Omega\alpha + b\mathbf{1}_m \rightarrow v \rightarrow \begin{bmatrix} 4.4516 \\ 4.3251 \\ 4.7413 \end{bmatrix} \quad (2.20)$$

Where $\mathbf{1}_m$ is a vector of 1s with the length of U .

2.2 Sub Bab B

Suatu penelitian tidak dapat lepas dari capaian pengetahuan dan pemahaman yang sudah dipublikasikan. Deskripsi tentang capaian ini menjadi penting karena selain menunjukkan tingkat pemahaman mahasiswa, juga mengetahui tempat pekerjaan penelitian TA dalam konstelasi capaian tersebut. Studi pustaka dan paparan hasilnya dapat memperkaya wawasan tentang topik yang diangkat pada penelitian TA.

2.3 Membuat Persamaan

Secara prinsip, suatu persamaan menyatu dalam kalimat. Letak persamaan dapat berada di awal, tengah, atau akhir kalimat. Dengan demikian, pada akhir persamaan harus diberikan tanda baca, misalnya koma, titik koma, atau titik, yang menekankan kehadiran persamaan dalam kalimat. Tidak semua persamaan harus diberi nomor. Persamaan yang dirujuk pada naskah TA saja yang harus diberi nomor. Kode awal penomoran ini adalah nomor urut bab, termasuk untuk persamaan pada Lampiran, dengan urutan alfabet kapital.

Setiap notasi harus unik atau tunggal, sehingga arti setiap notasi adalah unik atau tunggal juga. Arti satu notasi harus dituliskan segera ketika notasi tersebut muncul, dan tidak diulang lagi setelahnya.

2.3.1 Contoh Persamaan Sederhana

Persamaan (2.21) mendeskripsikan dinamika fungsi gelombang $\psi(\vec{r}, t)$ di bawah pengaruh potensial $V(\vec{r})$ dan dituliskan sebagai berikut:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \vec{\nabla}^2 \psi + V(\vec{r}) \psi, \quad (2.21)$$

dengan m adalah massa partikel dan \hbar merupakan konstanta Planck tereduksi.

Di akhir persamaan (2.21) diberi koma karena berada ditengah kalimat. Untuk merujuk ke persamaan yang telah ditulis, gunakan perintah `\ref{}`.

Untuk menuliskan beberapa set persamaan yang masih terhubung, gunakan `\subequations{}`. Misal kita punya persamaan diferensial terkopel, kita bisa tuliskan

$$\frac{dy}{dt} = -x, \quad (2.22a)$$

$$\frac{dx}{dt} = -y. \quad (2.22b)$$

Kalau perlu matriks, kita bisa tulis seperti berikut.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2.23)$$

2.4 Referensi dan Citation

Sitasi dapat dimasukkan ke dalam Tugas Akhir seperti ini **Fujita1996**. Untuk sitasi dengan beberapa sumber, dapat dituliskan juga **hohen1964; Kim2006**. Atau untuk tiga sumber berarti **kongkanand2006; kresse1999; Leibb1993**.

CHAPTER III

METODOLOGI PENELITIAN

Secara umum, metode penelitian yang digunakan pada pekerjaan penelitian disampaikan pada bab ini. Judul bab tidak harus seperti yang dituliskan. Dalam kata lain bisa diubah sesuai kebutuhan.

3.1 Computational Model

The proposed model derives inspiration from DeepONet, Neural Spectral methods, and Spectral Neural Operators. The method uses LSSVR to map between the input coefficients/functions and the solution coefficients. This mapping is the surrogate of the operator in the spectral domain. To exploit the nature of the problem, the kernel chosen has particular characteristics that makes it useful for the spectral/function domain. These are kernels such as the maximum distance between two functions or something that lies in the spectral domain favoring low/high frequencies depending on the particular problem.

$$\min_{W_b, W_t, \xi} \quad \frac{1}{2} \|W_b\|_F^2 + \frac{1}{2} \|W_t\|_F^2 + \frac{c}{2} \sum_{i=1}^N \xi_i^2 \quad (3.1a)$$

$$\text{s.t.} \quad y_i = (W_b f_i)^\top (W_t v_i) + \xi_i, \quad i = 1, \dots, N \quad (3.1b)$$

Converting the optimization problem above to the lagrange formulation

$$\begin{aligned} L(W_b, W_t, \xi; \alpha) = & \frac{1}{2} \|W_b\|_F^2 + \frac{1}{2} \|W_t\|_F^2 + \frac{c}{2} \sum_{i=1}^N \xi_i^2 \\ & + \sum_{i=1}^N \alpha_i (y_i - \xi_i - (W_b f_i)^\top (W_t v_i)) \end{aligned} \quad (3.2)$$

Taking partial derivatives with respect to

$$\frac{\partial L(W_b, b_b, W_t, b_t, \xi; \alpha)}{\partial W_b} = 0$$

$$\frac{2}{2}W_b + 0 + 0 - \sum_{i=1}^l \alpha_i ((W_t v_i + b_t) f_i^\top) = 0$$

$$W_b - \sum_{i=1}^l \alpha_i ((W_t v_i + b_t) f_i^\top) = 0$$

$$W_b = \sum_{i=1}^l \alpha_i (W_t v_i f_i^\top + b_t f_i^\top) \quad (3.3)$$

$$W_b = \sum_{i=1}^l (\alpha_i W_t v_i f_i^\top) + \sum_{i=1}^l (\alpha_i b_t f_i^\top)$$

$$W_b = W_t \sum_{i=1}^l (\alpha_i v_i f_i^\top) + b_t \sum_{i=1}^l (\alpha_i f_i^\top)$$

$$W_b = W_t \sum_{i=1}^l (\alpha_i v_i f_i^\top) + b_t \sum_{i=1}^l (\alpha_i f_i^\top)$$

$$\begin{aligned}
& \frac{\partial L(W_b, b_b, W_t, b_t, \xi; \alpha)}{\partial W_t} = 0 \\
& 0 + \frac{2}{2}W_t + 0 - \sum_{i=1}^l \alpha_i((W_b f_i + b_b)v_i^\top) = 0 \\
& W_t - \sum_{i=1}^l \alpha_i((W_b f_i + b_b)v_i^\top) = 0 \\
& W_t = \sum_{i=1}^l \alpha_i(W_b f_i v_i^\top + b_b v_i^\top) \\
& W_t = \sum_{i=1}^l \alpha_i((\sum_{j=1}^l \alpha_j(W_t v_j f_j^\top + b_t f_j^\top))f_i v_i^\top + b_b v_i^\top) \\
& W_t = \sum_{i=1}^l (\sum_{j=1}^l (\alpha_i \alpha_j W_t v_j f_j^\top f_i v_i^\top + \alpha_i \alpha_j b_t f_j^\top f_i v_i^\top) + \alpha_i b_b v_i^\top) \\
& W_t = \sum_{i=1}^l (\sum_{j=1}^l (\alpha_i \alpha_j W_t v_j f_j^\top f_i v_i^\top) + \sum_{j=1}^l (\alpha_i \alpha_j b_t f_j^\top f_i v_i^\top) + \alpha_i b_b v_i^\top) \\
& W_t - \sum_{i=1}^l \sum_{j=1}^l \alpha_i \alpha_j W_t v_j f_j^\top f_i v_i^\top = \sum_{i=1}^l (\sum_{j=1}^l (\alpha_i \alpha_j b_t f_j^\top f_i v_i^\top) + \alpha_i b_b v_i^\top) \\
& W_t (I - \sum_{i=1}^l \sum_{j=1}^l \alpha_i \alpha_j v_j f_j^\top f_i v_i^\top) = \sum_{i=1}^l (\sum_{j=1}^l (\alpha_i \alpha_j b_t f_j^\top f_i v_i^\top) + \alpha_i b_b v_i^\top) \\
& W_t = \sum_{i=1}^l (\sum_{j=1}^l (\alpha_i \alpha_j b_t f_j^\top f_i v_i^\top) + \alpha_i b_b v_i^\top) (I - \sum_{i=1}^l \sum_{j=1}^l \alpha_i \alpha_j v_j f_j^\top f_i v_i^\top)^{-1} \\
& W_t = \sum_{i=1}^l (\alpha_i (\sum_{j=1}^l (\alpha_j b_t f_j^\top f_i) + b_b) v_i^\top) (I - \sum_{i=1}^l \sum_{j=1}^l \alpha_i \alpha_j v_j f_j^\top f_i v_i^\top)^{-1}
\end{aligned} \tag{3.4}$$

$$\begin{aligned}
& \frac{\partial L(W_b, b_b, W_t, b_t, \xi; \alpha)}{\partial b_b} = 0 \\
& 0 + 0 + 0 - \sum_{i=1}^l \alpha_i(W_t v_i + b_t) = 0 \\
& \sum_{i=1}^l \alpha_i(W_t v_i + b_t) = 0
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
\frac{\partial L(W_b, b_b, W_t, b_t, \xi; \alpha)}{\partial b_t} &= 0 \\
0 + 0 + 0 - \sum_{i=1}^l \alpha_i (W_b f_i + b_b) &= 0 \\
\sum_{i=1}^l \alpha_i (W_b f_i + b_b) &= 0
\end{aligned} \tag{3.6}$$

$$\begin{aligned}
\frac{\partial L(W_b, b_b, W_t, b_t, \xi; \alpha)}{\partial \xi} &= 0 \\
0 + 0 + \frac{2c}{2} \sum_{i=1}^l \xi_i - \sum_{i=1}^l \alpha_i &= 0 \\
\sum_{i=1}^l c \xi_i - \alpha_i &= 0 \\
c \xi_i &= \alpha_i
\end{aligned} \tag{3.7}$$

$$\begin{aligned}
\frac{\partial L(W_b, b_b, W_t, b_t, \xi; \alpha)}{\partial \alpha} &= 0 \\
0 + 0 + 0 + \sum_{i=1}^l y_i - \xi_i - (W_b f_i + b_b)^\top (W_t v_i + b_t) &= 0 \\
\sum_{i=1}^l y_i - \xi_i - (W_b f_i + b_b)^\top (W_t v_i + b_t) &= 0
\end{aligned} \tag{3.8}$$

3.2 Sub Bab α

Misal kita mau masukin tabel, kita bisa juga.

Table 3.1: Tabel Sederhana Pertama

G	$\dim G$	$\dim F$
$SU(N)$	$N^2 - 1$	N
$SO(N)$	$\frac{1}{2}N(N - 1)$	N
$Sp(N)$	$N(2N + 1)$	$2N$
E_6	78	27
E_7	133	56
E_8	248	248
F_4	52	6
G_2	14	7

3.3 Sub Bab β

Atau bisa juga seperti berikut.

Table 3.2: Tabel Sederhana Kedua

G	$\dim G$	$\dim F$
$SU(N)$	$N^2 - 1$	N
$SO(N)$	$\frac{1}{2}N(N - 1)$	N
$Sp(N)$	$N(2N + 1)$	$2N$
E_6	78	27
E_7	133	56
E_8	248	248
F_4	52	6
G_2	14	7

CHAPTER IV

HASIL DAN ANALISIS

4.1 Sub Bab Hasil

Bab ini memaparkan pekerjaan penelitian dan, terutama, hasil-hasilnya, untuk dianalisis. Secara komprehensif bab ini merepresentasikan curahan pemikiran dan kemampuan mahasiswa dalam menjalani pekerjaan penelitian, yang hasil-hasilnya dapat dipertanggungjawabkan. Banyak pendukung yang diperlukan dalam penulisan bab ini, seperti skema penting pengolahan data, penurunan model matematika, asumsi khusus, tabulasi hasil dan analisis, dan gambar atau grafik yang membantu dalam paparan analisis. Judul bab dan sub bab disesuaikan dengan isi paparan.

4.2 Memasukkan Gambar

Setiap gambar harus dirujuk pada naskah TA, termasuk gambar pada Lampiran, menggunakan huruf pertama kapital (G) dan nomor gambar, tidak berdasarkan posisi relatifnya (misalnya di bawah ini atau sebelum ini). Format gambar yang umum adalah jpg, png, dan postscript (ps atau eps). Ukuran huruf pada nama sumbu dan label gambar/grafik harus cukup besar dan jelas, demikian halnya dengan angka pada sumbu. Gambar dan grafik dapat berwarna dengan pilihan warna yang tegas dan jelas.

4.2.1 Contoh Gambar Sederhana

Contoh menginput gambar pada buku TA pada Apabila ada dua gambar, kita juga

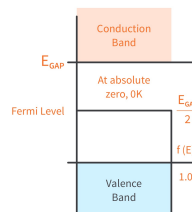


Figure 4.1: Tingkatan Fermi pada Bahan Semikonduktor

bisa menaruh keduanya berdampingan.



Figure 4.2: Dengan menempatkan gambar (a) dan (b), pembaca akan lebih mudah membandingkan keduanya.

CHAPTER V

SIMPULAN DAN SARAN

5.1 Simpulan

Bab ini merupakan pamungkas berupa rincian rangkuman yang merupakan simpulan dari analisis yang telah dilakukan. Simpulan ini menyajikan sejumlah hal penting yang disampaikan secara ringkas, padat, dan utuh, yang menjawab tujuan penelitian yang dituliskan pada Bab Pendahuluan. Sangat mungkin ada beberapa konsekuensi dan implikasi yang ditimbulkan dari simpulan yang dihasilkan, yang sepatutnya menjadi perhatian pada penelitian berikutnya. Judul bab dapat disesuaikan, namun umumnya ada *Simpulan* yang memang mendominasi isi bab ini.

5.2 Saran

Sejumlah ide yang muncul ketika melaksanakan penelitian TA dapat menjadi bahan atau topik untuk pekerjaan selanjutnya. Hal ini dapat berupa perbaikan atau ragam lain dari apa yang telah dilakukan sepanjang penelitian. Sub bab ini menjadi sumber informasi penting bagi, utamanya mahasiswa, yang akan melakukan penelitian lanjutan.

Bibliography

- Aarts, L. P., & Van Der Veer, P. (2001). Neural Network Method for Solving Partial Differential Equations. *Neural Processing Letters*, 14(3), 261–271. <https://doi.org/10.1023/A:1012784129883>
- Anderson, H. L. (1986). Scientific uses of the MANIAC. *Journal of Statistical Physics*, 43(5-6), 731–748. <https://doi.org/10.1007/BF02628301>
- Anselmo, L., & Pardini, C. (2005). Computational methods for reentry trajectories and risk assessment. *Advances in Space Research*, 35(7), 1343–1352. <https://doi.org/10.1016/j.asr.2005.04.089>
- Berliner, L. M. (2003). Physical-statistical modeling in geophysics. *Journal of Geophysical Research: Atmospheres*, 108(D24), 2002JD002865. <https://doi.org/10.1029/2002JD002865>
- Bernardi, M., Sánchez, H. D., Sheth, R. K., Brownstein, J. R., & Lane, R. R. (2022). Stellar population analysis of MaNGA early-type galaxies: IMF dependence and systematic effects. *Monthly Notices of the Royal Astronomical Society*, 518(3), 4713–4733. <https://doi.org/10.1093/mnras/stac3287>
- Bochud, N., Gomez, A., Rus, G., & Peinado, A. (2015). A sparse digital signal model for ultrasonic nondestructive evaluation of layered materials. *Ultrasonics*, 62, 160–173. <https://doi.org/10.1016/j.ultras.2015.05.013>
- Buoni, M., & Petzold, L. (2007). An efficient, scalable numerical algorithm for the simulation of electrochemical systems on irregular domains. *Journal of Computational Physics*, 225(2), 2320–2332. <https://doi.org/10.1016/j.jcp.2007.03.025>
- Bürchner, T., Kopp, P., Kollmannsberger, S., & Rank, E. (2023). Immersed boundary parametrizations for full waveform inversion. *Computer Methods in Applied Mechanics and Engineering*, 406, 115893. <https://doi.org/10.1016/j.cma.2023.115893>
- Caubet, F. (2013). Instability of an Inverse Problem for the Stationary Navier–Stokes Equations. *SIAM Journal on Control and Optimization*, 51(4), 2949–2975. <https://doi.org/10.1137/110836857>
- Chen, J.-S., Hillman, M., & Chi, S.-W. (2017). Meshfree Methods: Progress Made after 20 Years. *Journal of Engineering Mechanics*, 143(4), 04017001. [https://doi.org/10.1061/\(ASCE\)EM.1943-7889.0001176](https://doi.org/10.1061/(ASCE)EM.1943-7889.0001176)

- Chen, T., & Chen, H. (1995). Universal approximation to nonlinear operators by neural networks with arbitrary activation functions and its application to dynamical systems. *IEEE Transactions on Neural Networks*, 6(4), 911–917. <https://doi.org/10.1109/72.392253>
- Corrado, L., Cropper, M., & Rao, A. (2023). Space exploration and economic growth: New issues and horizons. *Proceedings of the National Academy of Sciences*, 120(43), e2221341120. <https://doi.org/10.1073/pnas.2221341120>
- De Aragão, O. A., & Sava, P. (2021). Explicit Control of Numerical Dispersion and Instability in Elastic Wavefield Modeling and Inversion. *82nd EAGE Annual Conference & Exhibition*, 1–5. <https://doi.org/10.3997/2214-4609.202112415>
- Du, Y., Chalapathi, N., & Krishnapriyan, A. S. (2024). Neural spectral methods: Self-supervised learning in the spectral domain. *The Twelfth International Conference on Learning Representations*. <https://openreview.net/forum?id=2DbVeua6a>
- Dwivedi, S. K., Vishwakarma, M., & Soni, P. (2018). Advances and Researches on Non Destructive Testing: A Review. *Materials Today: Proceedings*, 5(2), 3690–3698. <https://doi.org/10.1016/j.matpr.2017.11.620>
- Fanaskov, V. S., & Oseledets, I. V. (2023). Spectral Neural Operators. *Doklady Mathematics*, 108(S2), S226–S232. <https://doi.org/10.1134/S1064562423701107>
- Felice, M. V., & Fan, Z. (2018). Sizing of flaws using ultrasonic bulk wave testing: A review. *Ultrasonics*, 88, 26–42. <https://doi.org/10.1016/j.ultras.2018.03.003>
- Gao, H., Sun, L., & Wang, J.-X. (2021a). PhyGeoNet: Physics-informed geometry-adaptive convolutional neural networks for solving parameterized steady-state PDEs on irregular domain. *Journal of Computational Physics*, 428, 110079. <https://doi.org/10.1016/j.jcp.2020.110079>
- Gao, H., Sun, L., & Wang, J.-X. (2021b). Super-resolution and denoising of fluid flow using physics-informed convolutional neural networks without high-resolution labels. *Physics of Fluids*, 33(7), 073603. <https://doi.org/10.1063/5.0054312>
- Gaul, L., Klein, P., & Plenge, M. (1991). Simulation of wave propagation in irregular soil domains by BEM and associated small scale experiments. *Engineering Analysis with Boundary Elements*, 8(4), 200–205. [https://doi.org/10.1016/0955-7997\(91\)90014-K](https://doi.org/10.1016/0955-7997(91)90014-K)

- Gong, X., Herty, M., Piccoli, B., & Visconti, G. (2023). Crowd Dynamics: Modeling and Control of Multiagent Systems. *Annual Review of Control, Robotics, and Autonomous Systems*, 6(1), 261–282. <https://doi.org/10.1146/annurev-control-060822-123629>
- Gordoa, P., & Pickering, A. (2021). Ultrasonic Waves in Bubbly Liquids: An Analytic Approach. *Mathematics*, 9(11), 1309. <https://doi.org/10.3390/math9111309>
- Gupta, M., Khan, M. A., Butola, R., & Singari, R. M. (2022). Advances in applications of Non-Destructive Testing (NDT): A review. *Advances in Materials and Processing Technologies*, 8(2), 2286–2307. <https://doi.org/10.1080/2374068X.2021.1909332>
- Howell, P. A. (2020). *Nondestructive evaluation (nde) methods and capabilities handbook*. <https://ntrs.nasa.gov/citations/20200002023>
- Hughes, R. (2000). The flow of large crowds of pedestrians. *Mathematics and Computers in Simulation*, 53(4-6), 367–370. [https://doi.org/10.1016/S0378-4754\(00\)00228-7](https://doi.org/10.1016/S0378-4754(00)00228-7)
- Huntul, M. J., Abbas, M., & Baleanu, D. (2021). An inverse problem of reconstructing the time-dependent coefficient in a one-dimensional hyperbolic equation. *Advances in Difference Equations*, 2021(1), 452. <https://doi.org/10.1186/s13662-021-03608-1>
- Ince, J. C., Peerzada, M., Mathews, L. D., Pai, A. R., Al-qatatsheh, A., Abbasi, S., Yin, Y., Hameed, N., Duffy, A. R., Lau, A. K., & Salim, N. V. (2023). Overview of emerging hybrid and composite materials for space applications. *Advanced Composites and Hybrid Materials*, 6(4), 130. <https://doi.org/10.1007/s42114-023-00678-5>
- Jacob, G., & Raddatz, F. (2022, April 18). Data fusion for the efficient NDT of challenging aerospace structures: A review. In N. G. Meyendorf, C. Niezrecki, & S. Farhangdoust (Eds.), *NDE 4.0, Predictive Maintenance, and Communication and Energy Systems in a Globally Networked World* (p. 8). SPIE. <https://doi.org/10.1117/12.2612357>
- Jia, Y., Liu, K., & Zhang, X. S. (2024). Modulate stress distribution with bio-inspired irregular architected materials towards optimal tissue support. *Nature Communications*, 15(1), 4072. <https://doi.org/10.1038/s41467-024-47831-2>

- Jin, M., Wang, L., Ge, F., & Yan, J. (2023). Detecting the interaction between urban elements evolution with population dynamics model. *Scientific Reports*, 13(1), 12367. <https://doi.org/10.1038/s41598-023-38979-w>
- Karniadakis, G. E., Kevrekidis, I. G., Lu, L., Perdikaris, P., Wang, S., & Yang, L. (2021). Physics-informed machine learning. *Nature Reviews Physics*, 3(6), 422–440. <https://doi.org/10.1038/s42254-021-00314-5>
- Li, Z., & Ito, K. (2006). *The immersed interface method: Numerical solutions of PDEs involving interfaces and irregular domains*. SIAM, Society for Industrial and Applied Mathematics.
OCLC: ocm64594525.
- Lu, L., Jin, P., Pang, G., Zhang, Z., & Karniadakis, G. E. (2021). Learning nonlinear operators via DeepONet based on the universal approximation theorem of operators. *Nature Machine Intelligence*, 3(3), 218–229. <https://doi.org/10.1038/s42256-021-00302-5>
- Maio, L., & Fromme, P. (2022). On ultrasound propagation in composite laminates: Advances in numerical simulation. *Progress in Aerospace Sciences*, 129, 100791. <https://doi.org/10.1016/j.paerosci.2021.100791>
- Mandache, N. (2001). Exponential instability in an inverse problem for the Schrödinger equation. *Inverse Problems*, 17(5), 1435–1444. <https://doi.org/10.1088/0266-5611/17/5/313>
- Mukherjee, S., Goswami, D., & Chatterjee, S. (2015). A Lagrangian Approach to Modeling and Analysis of a Crowd Dynamics. *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 45(6), 865–876. <https://doi.org/10.1109/TSMC.2015.2389763>
- Muller, A. P. O., Costa, J. C., Bom, C. R., Klatt, M., Faria, E. L., de Albuquerque, M. P., & de Albuquerque, M. P. (2023). Deep pre-trained FWI: Where supervised learning meets the physics-informed neural networks. *Geophysical Journal International*, 235(1), 119–134. <https://doi.org/10.1093/gji/ggad215>
- Ni, P., Sun, L., Yang, J., & Li, Y. (2022). Multi-End Physics-Informed Deep Learning for Seismic Response Estimation. *Sensors*, 22(10), 3697. <https://doi.org/10.3390/s22103697>
- Niven, R. (2020). *Full waveform inversion of ultrasonic phased array data for the tomographic reconstruction of heterogeneous media for NDT*. <https://doi.org/10.48730/V9HJ-Y070>

- Raissi, M., Perdikaris, P., & Karniadakis, G. (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. *Journal of Computational Physics*, 378, 686–707. <https://doi.org/10.1016/j.jcp.2018.10.045>
- Reinhardt, J. C., Chen, X., Liu, W., Manchev, P., & Paté-Cornell, M. E. (2016). Asteroid Risk Assessment: A Probabilistic Approach. *Risk Analysis*, 36(2), 244–261. <https://doi.org/10.1111/risa.12453>
- Spanos, A. (2006). Where do statistical models come from? Revisiting the problem of specification. In *Institute of Mathematical Statistics Lecture Notes - Monograph Series* (pp. 98–119). Institute of Mathematical Statistics. <https://doi.org/10.1214/0749217060000000419>
- Tarantola, A. (2005). *Inverse problem theory: And methods for model parameter estimation*. SIAM, Society for Industrial and Applied Mathematics.
- Uzan, J.-P. (2003). The fundamental constants and their variation: Observational and theoretical status. *Reviews of Modern Physics*, 75(2), 403–455. <https://doi.org/10.1103/RevModPhys.75.403>
- Virieux, J., & Operto, S. (2009). An overview of full-waveform inversion in exploration geophysics. *GEOPHYSICS*, 74(6), WCC1–WCC26. <https://doi.org/10.1190/1.3238367>
- Wang, R., Kashinath, K., Mustafa, M., Albert, A., & Yu, R. (2020). Towards Physics-informed Deep Learning for Turbulent Flow Prediction. *Proceedings of the 26th ACM SIGKDD International Conference on Knowledge Discovery & Data Mining*, 1457–1466. <https://doi.org/10.1145/3394486.3403198>
- Yang, H., Yang, L., Yang, Z., Shan, Y., Gu, H., Ma, J., Zeng, X., Tian, T., Ma, S., & Wu, Z. (2023). Ultrasonic detection methods for mechanical characterization and damage diagnosis of advanced composite materials: A review. *Composite Structures*, 324, 117554. <https://doi.org/10.1016/j.compstruct.2023.117554>
- Zhang, R., Liu, Y., & Sun, H. (2020). Physics-guided convolutional neural network (PhyCNN) for data-driven seismic response modeling. *Engineering Structures*, 215, 110704. <https://doi.org/10.1016/j.engstruct.2020.110704>
- Zhao, X., Gong, Z., Zhang, Y., Yao, W., & Chen, X. (2023). Physics-informed convolutional neural networks for temperature field prediction of heat source layout without labeled data. *Engineering Applications of Artificial Intelligence*, 117, 105516. <https://doi.org/10.1016/j.engappai.2022.105516>

LAMPIRAN A

KODE PROGRAM

1.1 PROGRAM SATU

1.2 PROGRAM DUA

LAMPIRAN B

GAMBAR-GAMBAR