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FIT5047: Fundamentals of AI

Solving problems by searching
Chapters 3-5, 7

Problem solving: Learning objectives

- **Problem formulation**
- **Control strategies**
  - **Tentative**
    - > **Uninformed**:
      - – Backtracking [Chapter 7]
      - – Tree- and Graph search [Chapter 3]
    - > **Informed**: Greedy best-first search, A, A* [Chapter 3]
  - **Irrevocable**
    - > **Informed**: Hill climbing, Local beam search, Simulated annealing, Genetic algorithms [Chapter 4]
- **Adversarial search algorithms [Chapter 5]**
  - Optimal decisions
  - Minimax, α-β pruning

Assumptions about the environment

- **Observable**
- **Known**
- **Single/multi agent**
- **Deterministic**
- **Sequential**
- **Static/dynamic**
- **Discrete**

Problem-solving agents

Function Simple-Problem-Solving-Agent(*percept)*
returns *seq*
persistent: *state* – description of current world state
seq – action sequence
goal – a goal
problem – a problem formulation

initially null

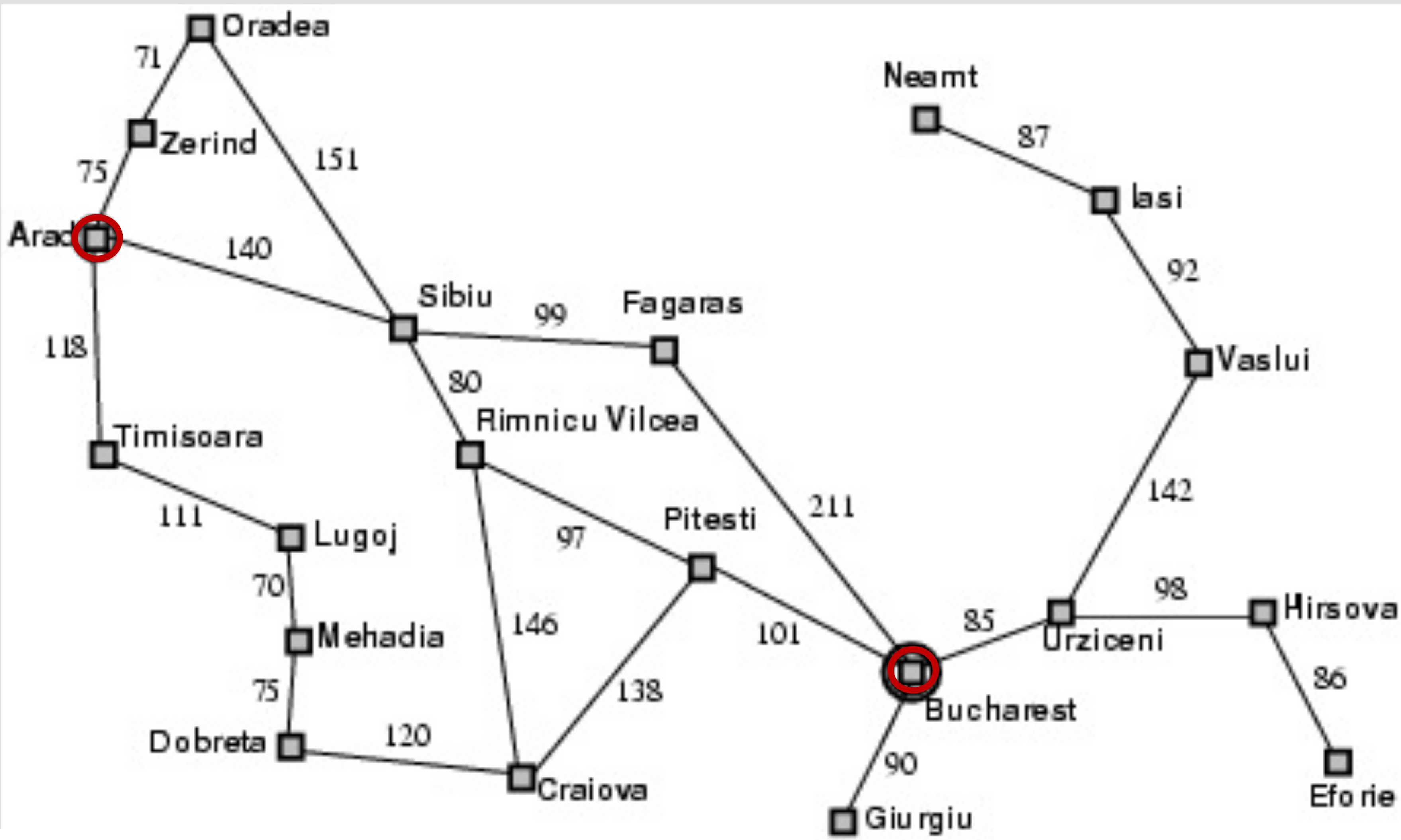
state ← UpdateState(*state,percept*)
goal ← FormulateGoal*(state)*
problem ← FormulateProblem(*state,goal*)
seq ← Search*(problem)*
return *seq*

Example: Romania

- ***On holiday in Romania; currently in Arad.***

- **Formulate goal:**
  - be in Bucharest
- **Formulate problem:**
  - states: various cities
  - actions: drive between cities
- **Find solution:**
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

Example: Romania



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Problem Formulation

Problem formulation (I)

- **Problem formulation comprises decisions about:**
  - which properties of the world matter
  - which actions are possible
  - how to represent world states and actions

Abstracting away from unnecessary detail is a key
➔ It can drastically reduce the size of the state/search space

Problem formulation (II)

- **Basic constituents**
  - States, Goals, Actions, Constraints
- **State space –** the set of all states reachable from the initial state by any sequence of actions
- **Path in the state space –** any sequence of actions leading from one state to another
- **Representing a problem**
  - Initial state
  - Operators (Actions) and transition model
  - Constraints
  - Goal test
  - Path cost function
- **A solution is a sequence of actions leading from the initial state to a goal state**

Problem formulation: Example

1. **initial state, e.g., "in Arad" – *In(Arad)***
2. **actions**
   - e.g., {*Go(Sibiu)*,*Go(Timisoara)*, … }

   transition model
   - e.g., *Result(In(Arad),Go(Zerind))* → *In(Zerind)*
3. **constraints – nil**
4. **goal test can be**
   - explicit, e.g., *In(Bucharest)*
   - implicit, e.g., *Checkmate(x)*
5. **path cost (additive)**
   - e.g., sum of distances, number of actions executed
   - *c(s,a,s')* is the step cost of taking action *a* at state *s* to reach state *s'*, assumed to be ≥ 0.

Problem formulation – 8 Puzzle (I)

Start

5	4	
6	1	8
7	3	2

End

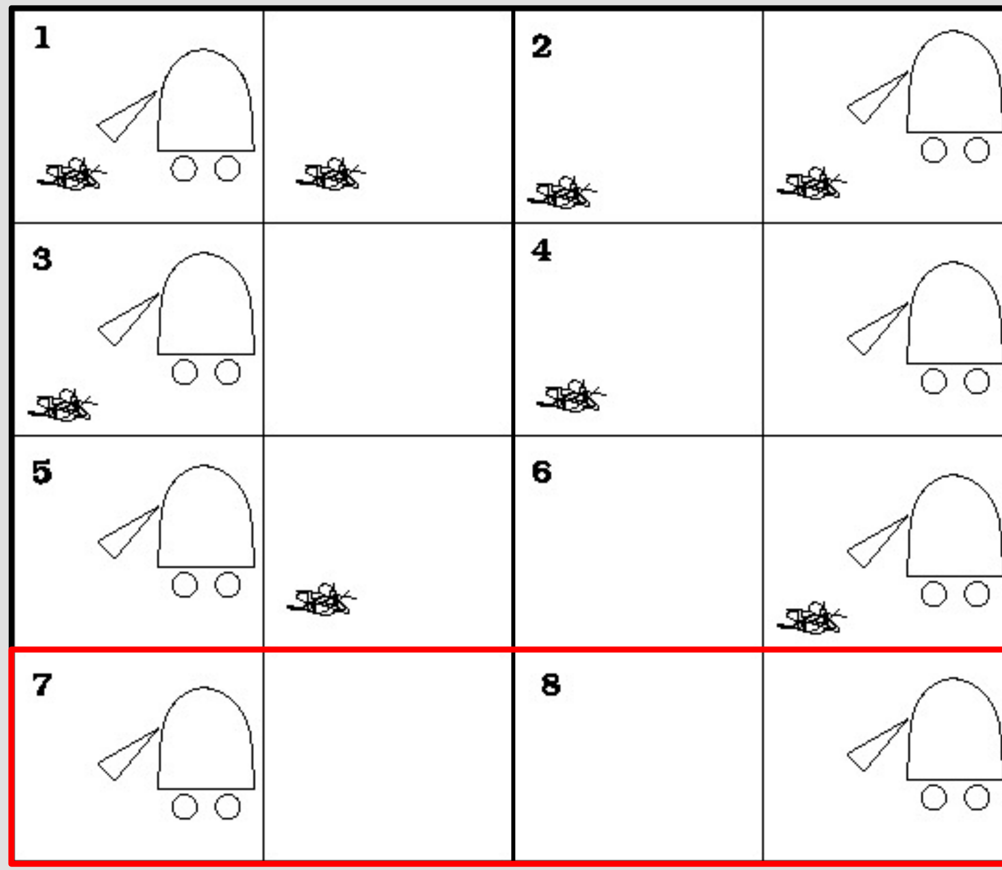
1	2	3
8		4
7	6	5

Problem formulation – 8 Puzzle (II)

- **States**
  - Location of each of the 8 tiles in one of the 9 squares
- **Operators**
  - Possible moves of blank tile
- **Constraints**
  - A tile cannot move out of bounds
- **Goal test**
  - Have we reached the goal configuration?
- **Path cost**
  - If we want to minimize the number of steps to reach the goal state, then all steps have the same cost

Problem formulation – Vacuum world

- **States**
  - 8 states shown
- **Operators**
  - Left, right, suck
- **Constraints**
  - Can't leave the world
- **Goal test**
  - States 7 and 8
- **Path cost**
  - E.g. each action costs 1



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Problem formulation: Missionaries & Cannibals (I)

- **Start state: 3 missionaries & 3 cannibals on one side of a river**
- **Goal state: 3 missionaries & 3 cannibals on the other side of the river**
- **Constraints:**
  – There is a boat that carries at most 2 people
  – The boat cannot travel empty
  – Cannibals should never outnumber missionaries

Problem formulation: Missionaries & Cannibals (II)

- **States**
  - 2-digit code (m,c) represents the number of m and c on start bank; 1 digit code represents boat position
  - Initial state (3,3) + boat position
- **Operators**
  - 1m1c, 2m, 2c, 1m, 1c
- **Constraints**
  - $[(c \leq m) \land (3\text{-}c \leq 3\text{-}m)] \lor m=3 \lor m=0$
- **Goal test**
  - (0,0)
- **Path cost**
  - Cost function: Minimize number of crossings

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Control Strategies

Classification of control strategies

- **Tentativeness**
  - Tentative, i.e., with reconsideration
  - Irrevocable, i.e., no reconsideration
- **Informedness**
  - Informed, i.e., use guidance on where to look for solutions
  - Uninformed, i.e., decide based ***only*** on problem definition

	Irrevocable	Tentative
Uninformed	**--**	**Backtrack, Tree- and Graph-Search (BFS, DFS, DLS, IDS, UCS)**
Informed	**Hill climbing, Local beam search, Simulated annealing, Genetic algorithms**	**Greedy best-first search, A, A***

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Tentative and Uninformed Search Algorithms:
Backtrack, Tree-/Graph-search

Tentative control strategies

- **Backtracking: keep track of one path only**
  - If we fail, go back to the last decision point and **erase the failed path**
  - Backtracking occurs when
    - > we reach a DEADEND state OR
    - > there are no more applicable rules OR
    - > we generate a previously encountered state description OR
    - > an arbitrary number of rules has been applied without reaching the goal
- **Graphsearch: keep track of several paths simultaneously**
  - Done using a structure called a ***search tree/graph***

Basic Backtracking algorithm

Procedure Backtrack (State)

1. **If Goal(State) Then return SUCCEED**
2. **If Deadend(State) Then return FAIL**
3. **Operators ← ApplicableOps(State)**
4. **Loop**
    1. **If** null(Operators) **Then** return FAIL
    2. Op ← Pop(Operators)
    3. State’ ← Op(State)
    4. Path ← Backtrack(State’)
    5. **If** Path=FAIL **Then** go Loop
    6. Return {Op, Path}

End

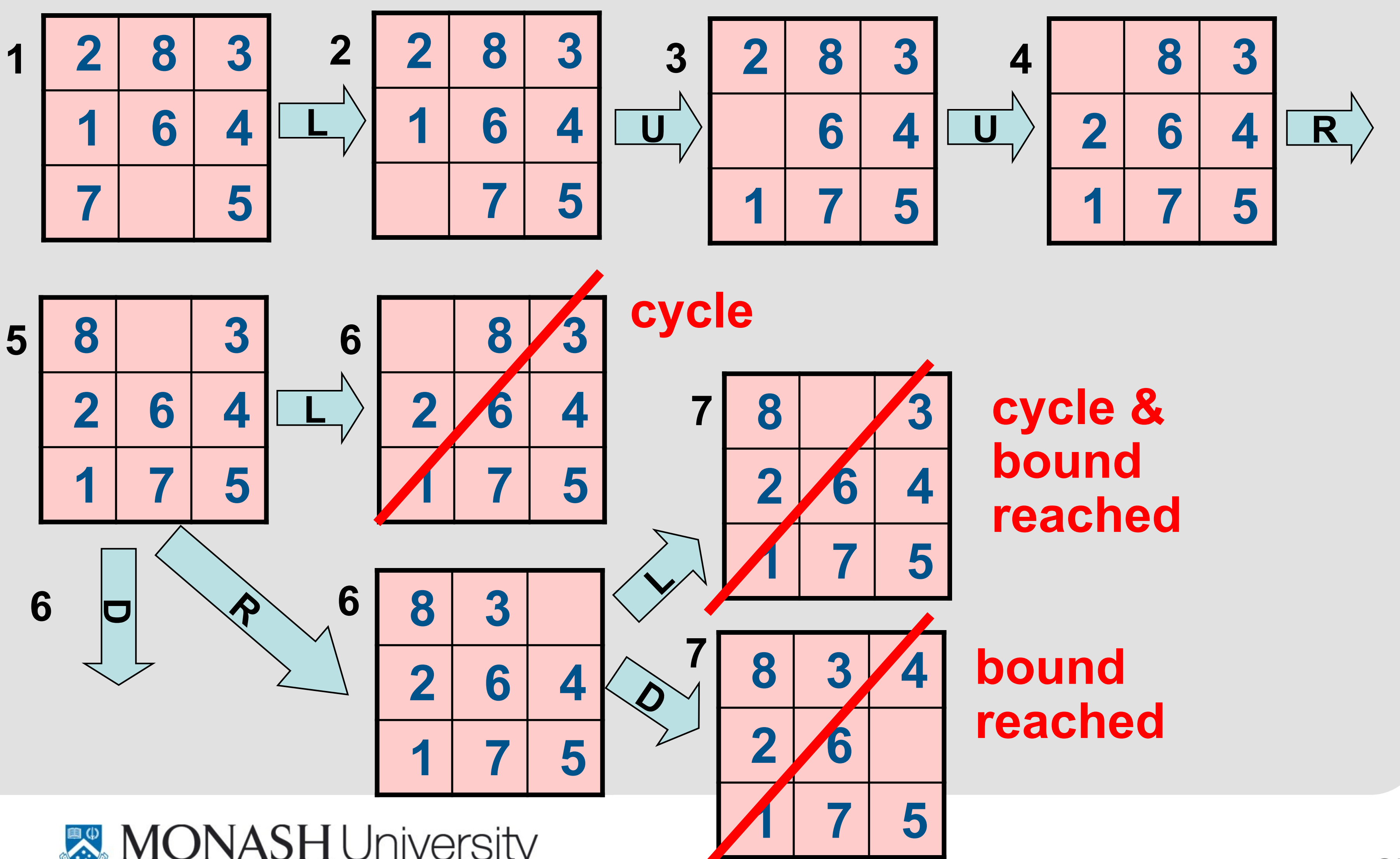
Backtracking algorithm

Procedure Backtrack1(StateList)

1. **State ← First(StateList)**
2. **If State ϵ RestOf(StateList) Then return FAIL**
3. **If Goal(State) Then return SUCCEED**
4. **If Deadend(State) Then return FAIL**
5. **If Length(StateList) > Bound Then return FAIL**
6. **Operators ← ApplicableOps(State)**
7. **Loop**
   1. **If** null(Ops) **Then** return FAIL
   2. Op ← Pop(Ops)
   3. State’ ← Op(State)
   4. StateList’ ← {State’, StateList}
   5. Path ← Backtrack1(StateList’)
   6. **If** Path=FAIL **Then** go Loop
   7. Return {Op, Path}

   End

Backtracking example – Bound = 6



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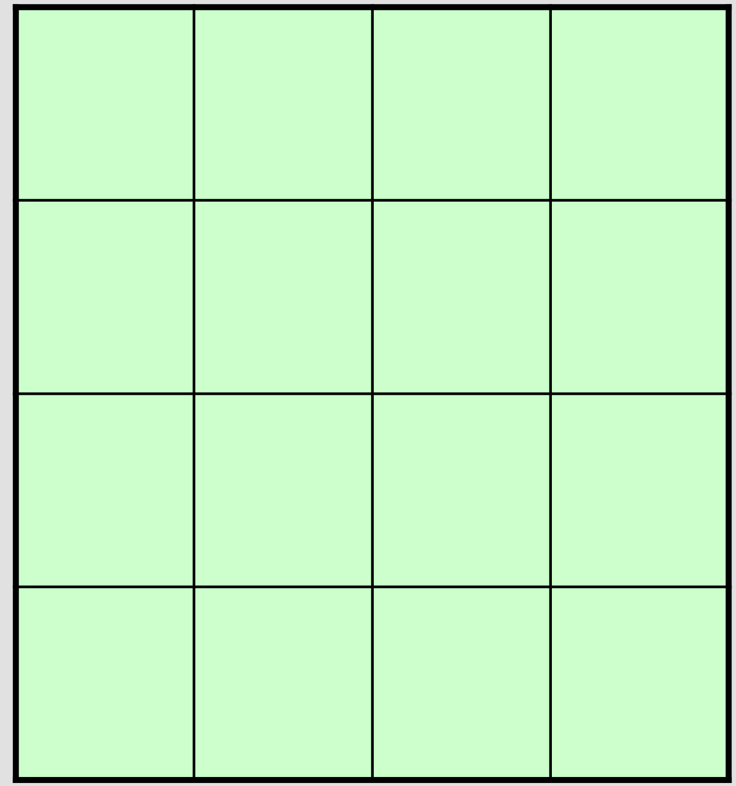
Backtracking example – 4 queens problem

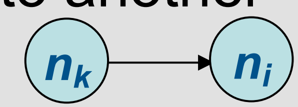
- **Start state:**
  - empty chess board
- **Goal state:**
  - 4 queens placed on chess board
- **Constraints:**
  - queens do not attack each other
- **Operators:**
  - place queen on tile (x,y)
- **Path cost: NA**



Graphsearch – Definitions

- **Graphsearch is a means of finding a path in a graph from a node representing the initial state to a node that satisfies the goal condition**

- **Definitions**
  - **Graph**: set of nodes
  - **Arcs**: connect between certain pairs of nodes
  - **Directed graph**: formed by arcs directed from a node to another
  - n_i is a ***child of*** n_k if
  - n_i is ***accessible from*** n_k if there is a path from n_k to n_i
  - **Expanding a node**: finding all its children
  - **Search Problem**: find a path between node *s* and a member of the ***goal set*** that represents states satisfying the goal condition



Search Tree

- **Tree – each node has at most one parent**
- **Root of search tree is the initial node**
- **Leaves are nodes without successors (“frontier”)**
- **At each step, choose one leaf node to *expand***

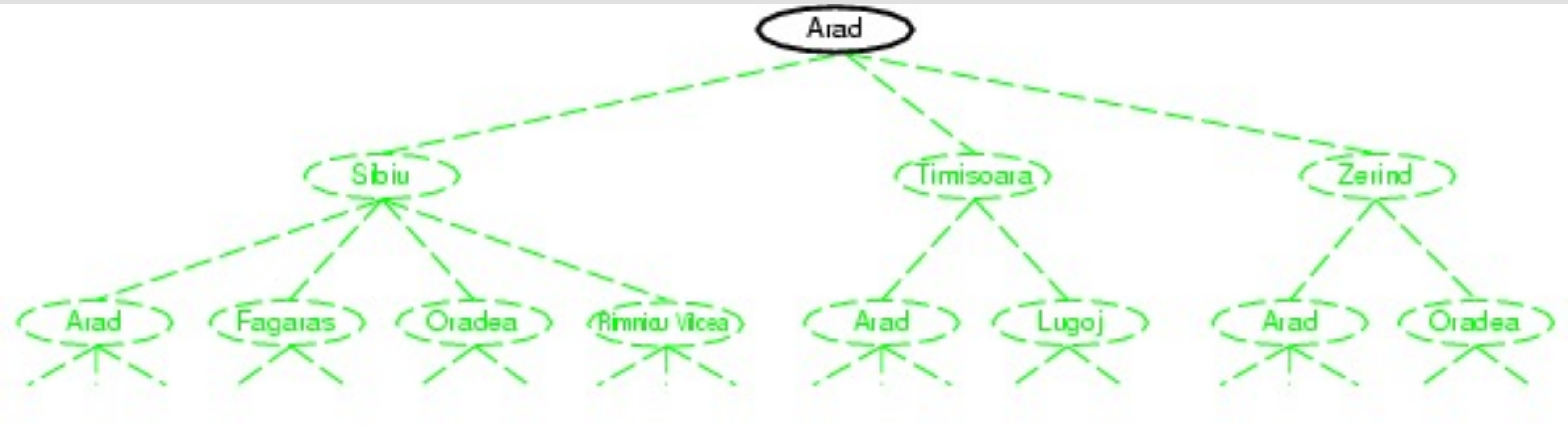
Basic Tree search algorithm

function TREE-SEARCH(*problem*) **returns** a solution or failure

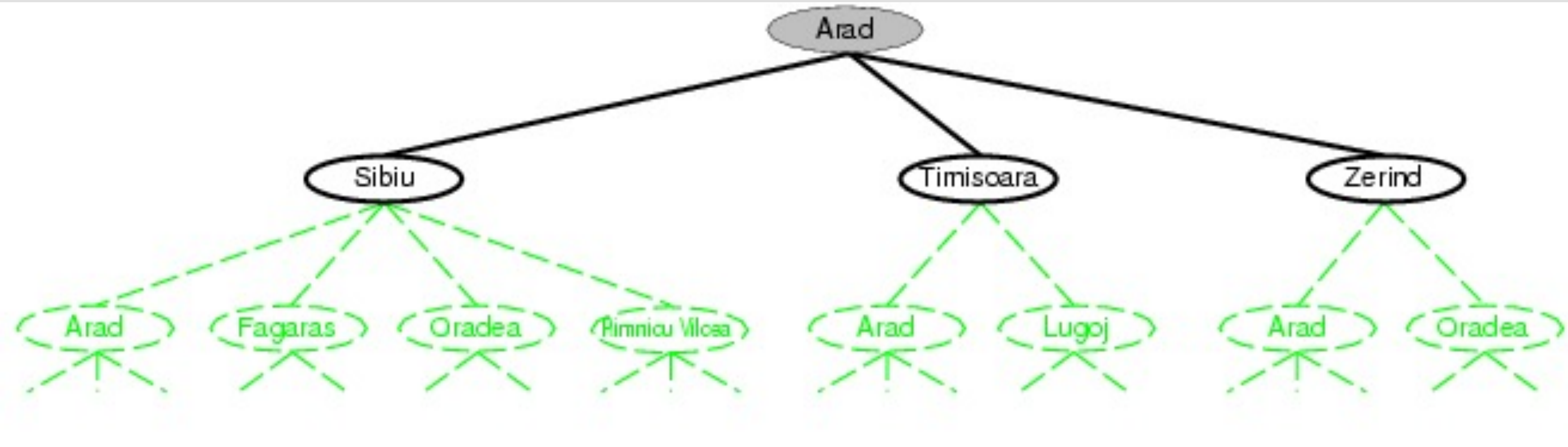
- Initialize the frontier using the initial state of *problem*
- **Loop**
  1. **if** the frontier is empty **then return** failure
  2. ***choose*** a leaf node and remove it from the frontier
  3. **if** the node contains a goal state **then return** the corresponding solution
  4. ***expand*** the chosen node, ***adding*** the resulting nodes to the frontier

end

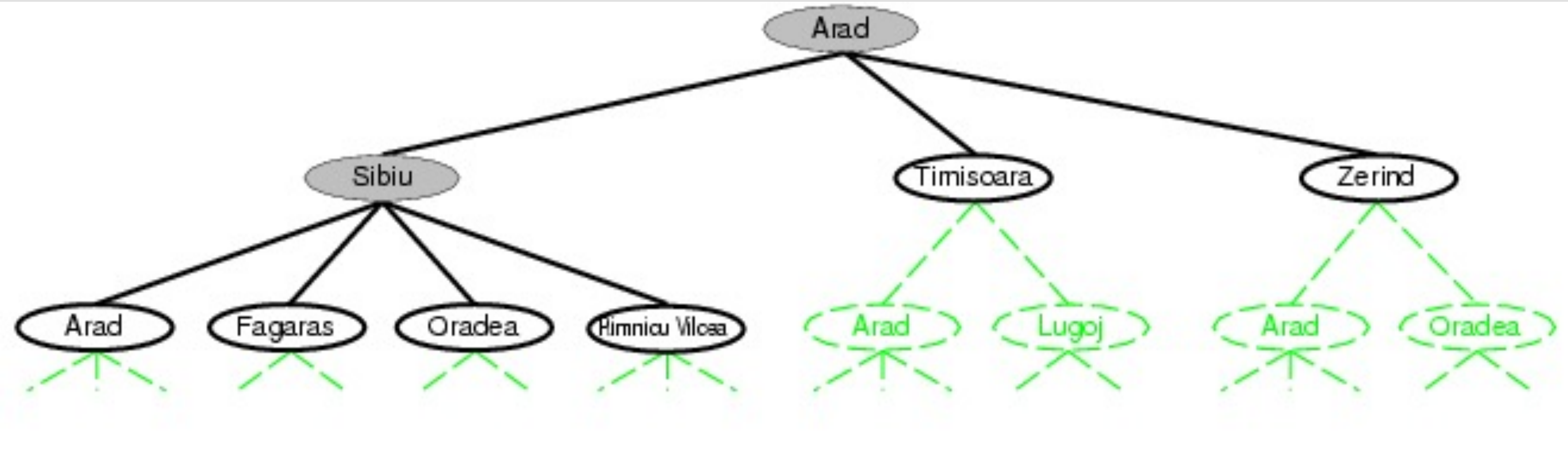
Example: Tree search (I)



Example: Tree search (II)

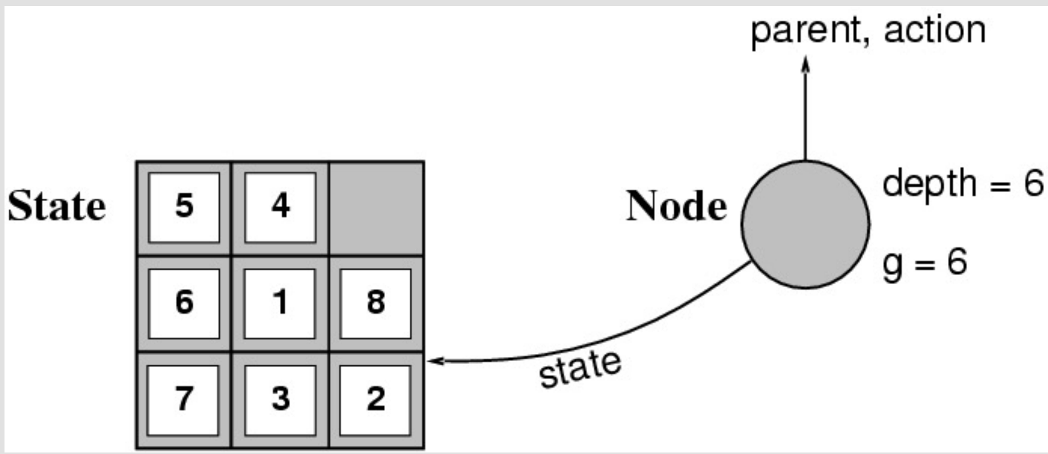


Example: Tree search (III)



Implementation: States vs. Nodes

- **state – a (representation of a) physical configuration**
- **node – a data structure that is part of a search tree**
  - includes *state, parent node, action, children, path cost g(x), depth*



- **The *Expand* function**
  - creates new nodes, fills in the various fields
  - uses ***SuccessorFn(Operators)*** to create the corresponding states

Searching graphs: Multiple paths to a node

- **Often search better is represented via graphs:**
  - There may be multiple paths to the same node
  - Improvement (due to search graph) depends on how costly it is to determine a node has already been visited

Graphsearch algorithm

function GRAPH-SEARCH(*problem*) **returns** a solution or failure

- Initialize the frontier using the initial state of *problem*
- Initialize the *explored set* (***closed***) to empty
- **Loop**
  1. **if** the frontier is empty **then return** failure
  2. ***choose*** a leaf node and remove it from the frontier
  3. **if** the node contains a goal state **then return** the corresponding solution
  4. add the node to the *explored set*
  5. ***expand*** the chosen node, ***merging*** the resulting nodes with the frontier or the *explored set*

end

Basic search algorithm: Key issues

- **Return a path or a node?**
- **Unboundedness:**
  - Tree search: because of loops
  - Graph/tree search: because the state space is infinite
- **Tree search: Repeated states**
  - Failure to detect repeated states can increase the complexity of a problem
- **How are nodes ordered? ➔ Search strategy**
  - Is the graph weighted or unweighted?
  - What is known about the "quality" of intermediate states?
  - Is the aim to find a ***minimal cost path*** or ***any path asap***?

Dealing with repeated states

- **3 ways to deal with repeated states (ordered by cost and effectiveness):**

  – Do not return to the state you just came from
  → don't generate successors with same state as a node's parent
  – Do not create paths with cycles in them
  → don't generate successors with same state as any ancestor
  – Do not generate any state that was ever generated before
  → Use hashset to check if state has been visited

Implementation of the Graphsearch algorithm

1. **Create a search graph *G* consisting only of the start node *s***
2. **OPEN ← *s***
3. **CLOSED ← Ø**
4. **Loop**
    1. **If** OPEN = Ø **Then** exit with failure
    2. *n* ← first node in OPEN
       Remove *n* from OPEN, put it in CLOSED
    3. **If** *n* = goal-node **Then** exit successfully with the solution obtained by tracing a path along the pointers from *n* to *s* in *G*
    4. **Expand node *n***, generating a set *M* of its children *that are not ancestors of n*. Put these members of *M* as children of *n* in *G*.
    5. Establish a pointer to *n* from those members of *M* *that were not already in G*. Add these members of *M* to OPEN. For each member of *M* already in *G*, decide whether or not to redirect its pointer to *n*.
    6. Reorder OPEN (according to an arbitrary scheme or merit)

End

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Tree and Graph Search Strategies

Search strategies

- **A search strategy is given by the order of node expansion**
- **Strategies are evaluated along several dimensions:**
  - **completeness**: does it always find a solution if one exists?
  - **time complexity**: maximum number of nodes generated
  - **space complexity**: maximum number of nodes in memory
  - **optimality**: does it always find a least-cost solution?
- **Time and space complexities are measured in terms of**
  - *b:* maximum branching factor of the search tree
  - *d:* depth of the least-cost solution
  - *m*: maximum depth of any path in the search space (may be ∞)

O Notation

- ***n*** **measures the size of the input**
- ***f*(*n*) is a function characterizing the worst-case complexity of an algorithm**
- ***O*(*f*(*n*)) is the set of all functions bounded from above by some positive multiple *k* of *f*(*n*)**

Example:
Let *n* be the number of items to be sorted, then

- **Bubble sort has worst case $k_1 n^2$; i.e., $O(n^2)$**
- **Heap sort has worst case $k_2 n \log n$; i.e., $O(n \log n)$**

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Tentative and Uninformed Search Strategies

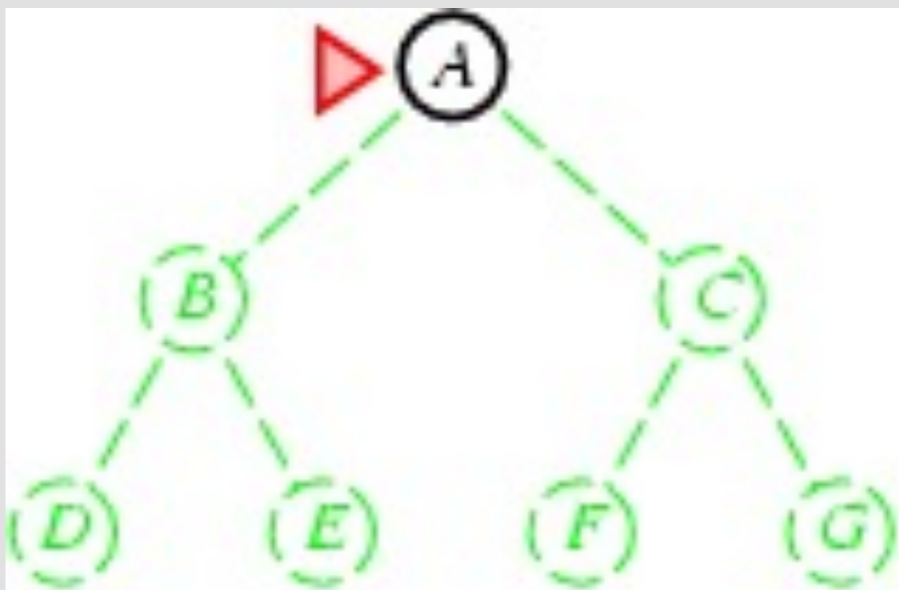
Uninformed search strategies

Uninformed search strategies use only the information available in the problem definition

- **Breadth-first search (BFS)**
- **Uniform-cost search (UCS)**
- **Depth-first search (DFS)**
- **Depth-limited search (DLS)**
- **Iterative deepening search (IDS)**

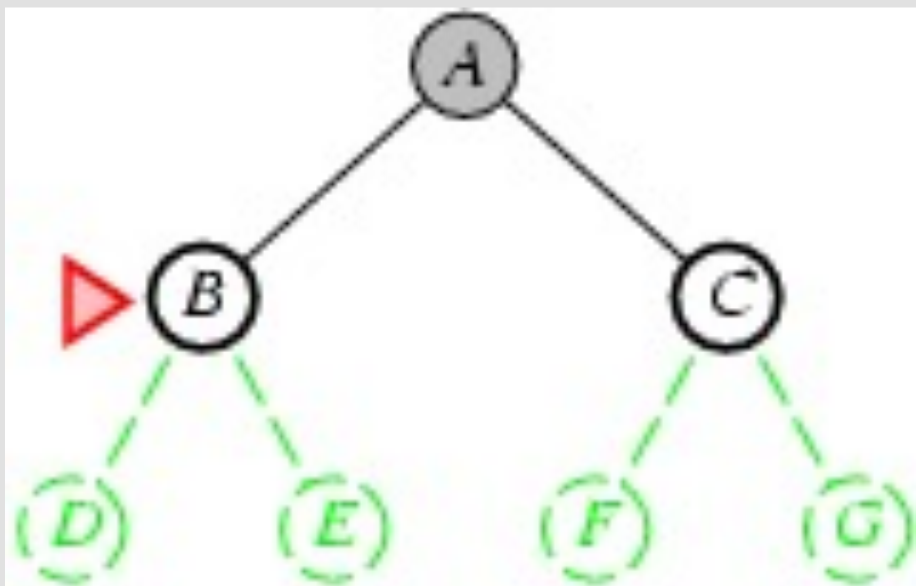
Breadth-first search (I)

- **Expand shallowest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: FIFO – put successors at end of queue



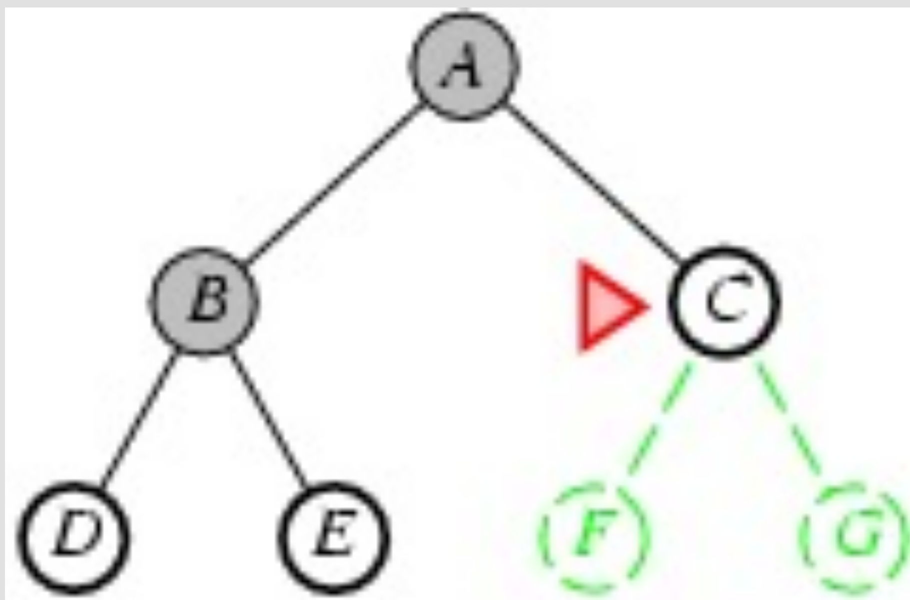
Breadth-first search (II)

- **Expand shallowest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: FIFO – put successors at end of queue



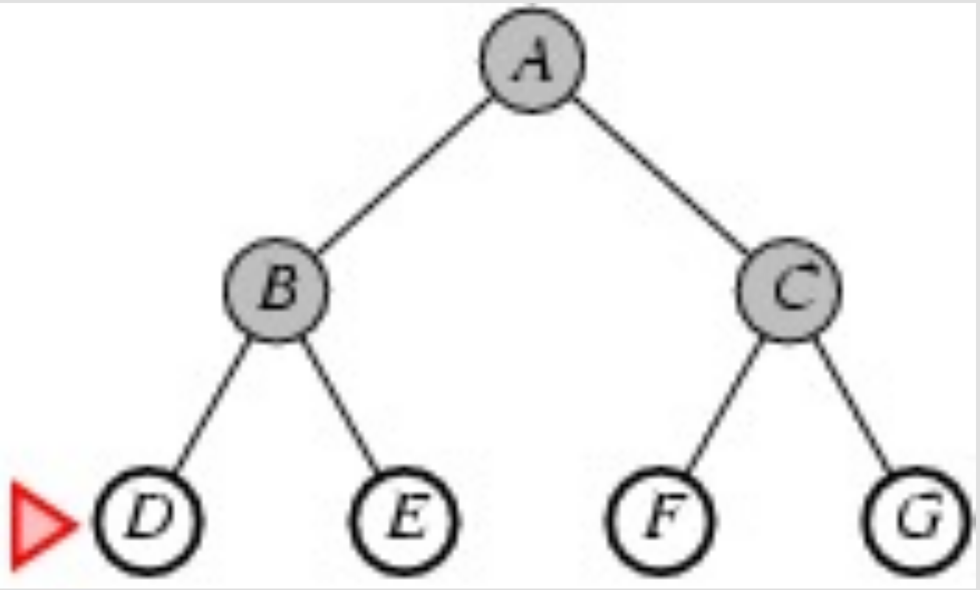
Breadth-first search (III)

- **Expand shallowest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: FIFO – put successors at **end** of queue

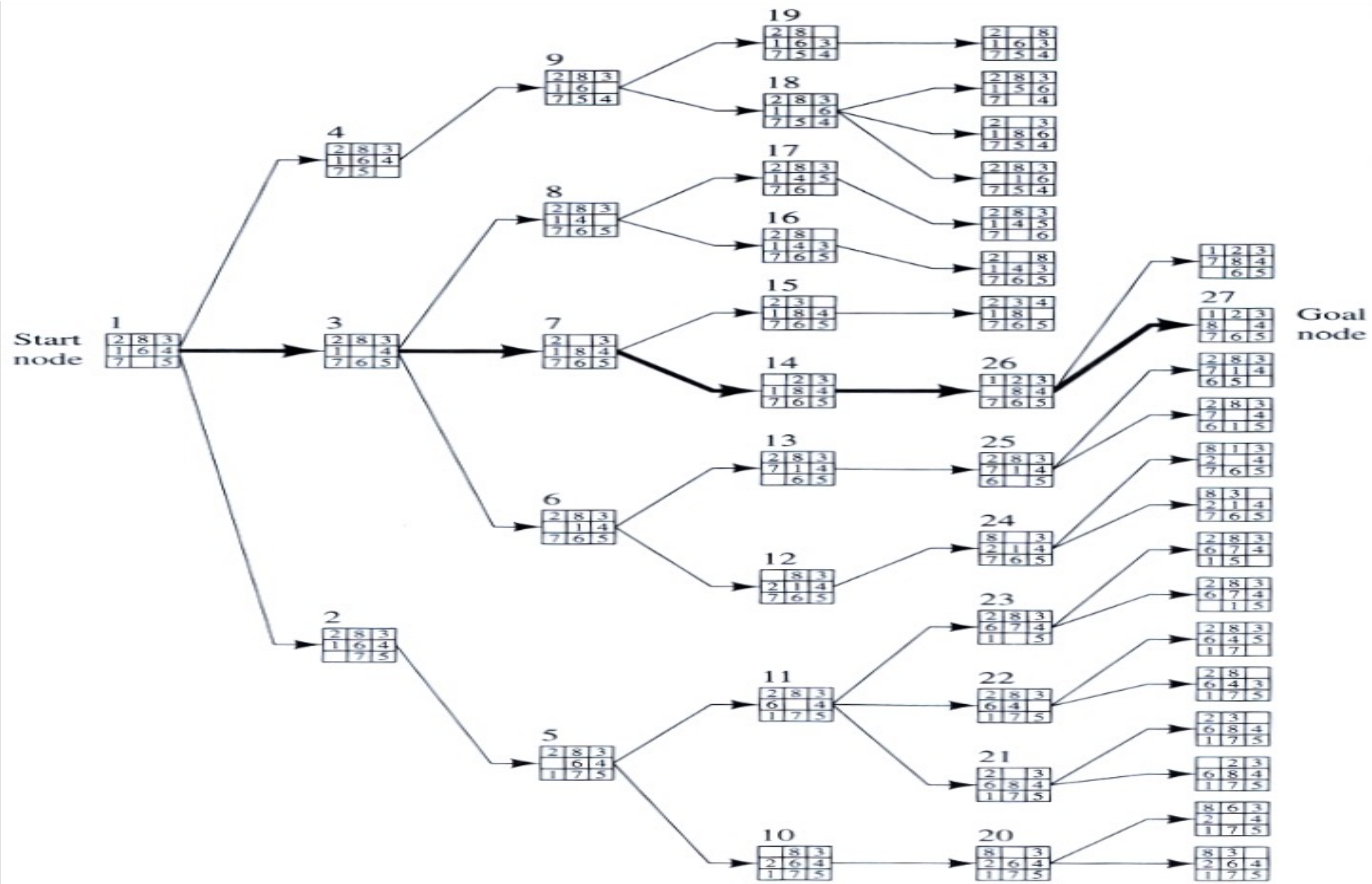


Breadth-first search (IV)

- **Expand shallowest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: FIFO – put successors at end of queue



BFS – Example



Properties of BFS

- **Complete?** Yes (if b is finite)
- **Time?** $b + b^2 + b^3 + \ldots + b^d = b\frac{b^d - 1}{b - 1} \to O(b^d)$
- **Space?** $O(b^d)$ (keeps every node in memory)
- **Optimal?** Yes (if all actions have the same cost)

Space is the bigger problem

The only tree/graphsearch algorithm that can stop when the goal node is reached

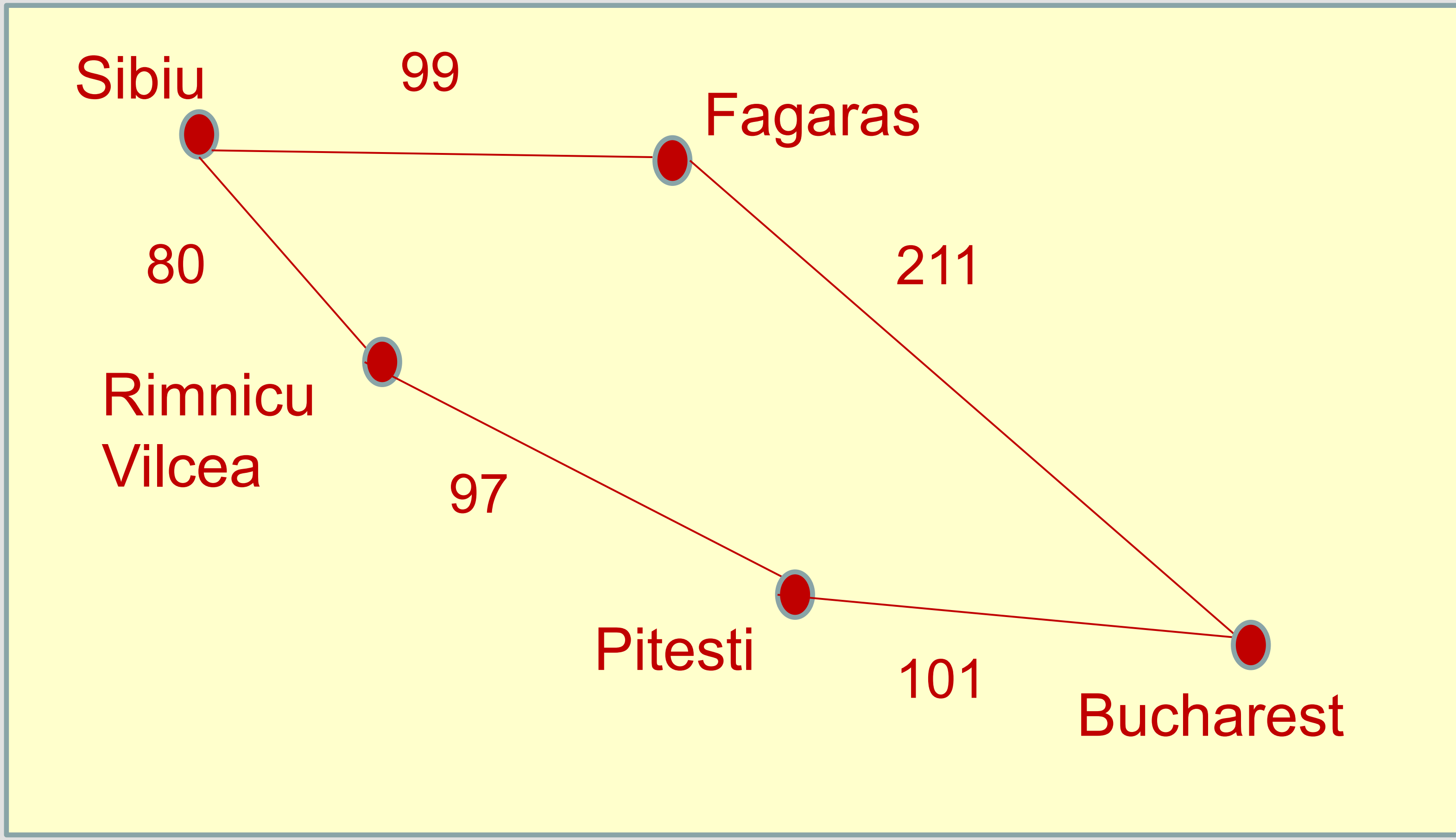
Uniform-cost search algorithm

function UNIFORM-COST-SEARCH(*problem*) **returns** a solution or failure

- Initialize the frontier using the initial state of *problem*
- **Loop**
  1. **if** the frontier is empty **then return** failure
  2. ***choose*** the lowest-cost node (*n*) in the frontier and remove it from the frontier -- i.e., the first node in OPEN
  3. **if** the node (called *n* in slide 38) contains a goal state **then return** the corresponding solution
  4. ***expand*** the chosen node (*n*) to obtain new nodes
     a. **if** new nodes are not in the frontier **then add** them to the frontier
     b. **else if** new nodes are in the frontier *with higher path cost* **then replace** old nodes in the frontier with the new nodes

end

UCS: Example


Sibiu
99
Fagaras
80
211
Rimnicu
Vilcea
97
Pitesti
101
Bucharest

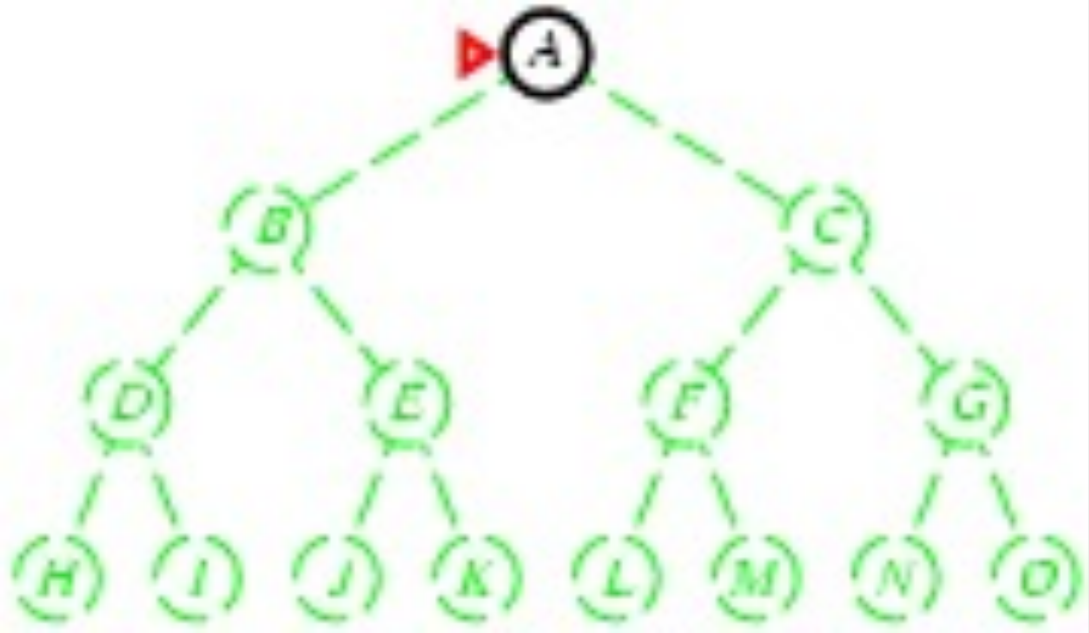
Properties of UCS

- **Almost equivalent to BFS if step costs are all equal**
- **Complete? Yes, if step cost ≥ ε**
- **Time?** $O(b^{1+floor(C^*/\varepsilon)})$
  - where C^* is the cost of the optimal solution
- **Space?** $O(b^{1+floor(C^*/\varepsilon)})$
- **Optimal? Yes: nodes are expanded in increasing order of *g(n)* = cost of path to node *n***

1. **When all step costs are the same, UCS does more work than BFS. Why?**
2. **When UCS selects a node for expansion, the optimal path to that node has been found.**

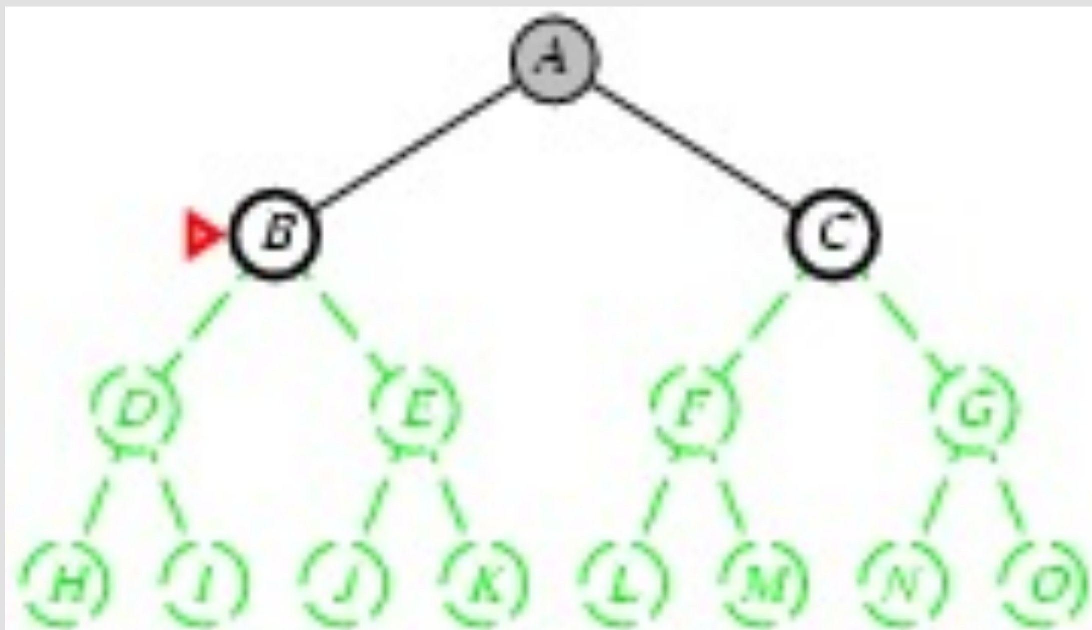
Depth-first search (I)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



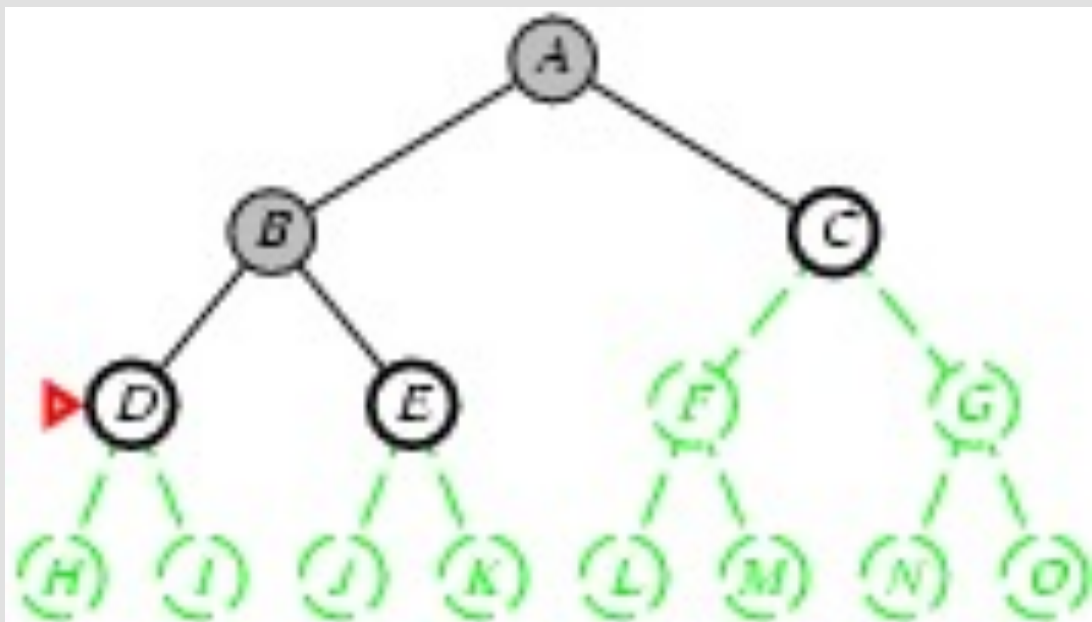
Depth-first search (II)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



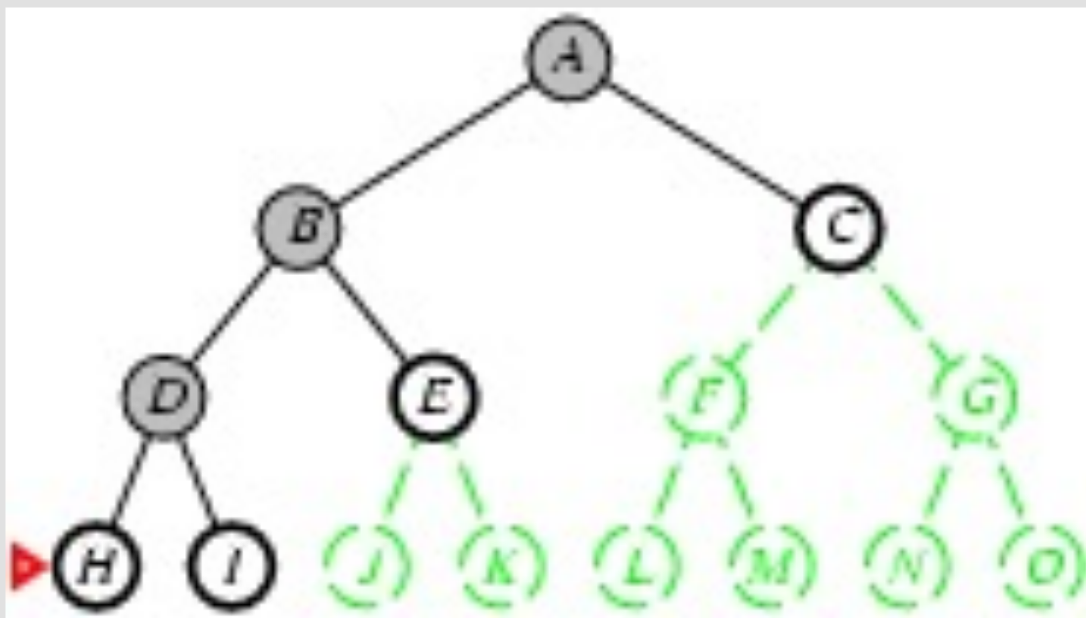
Depth-first search (III)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



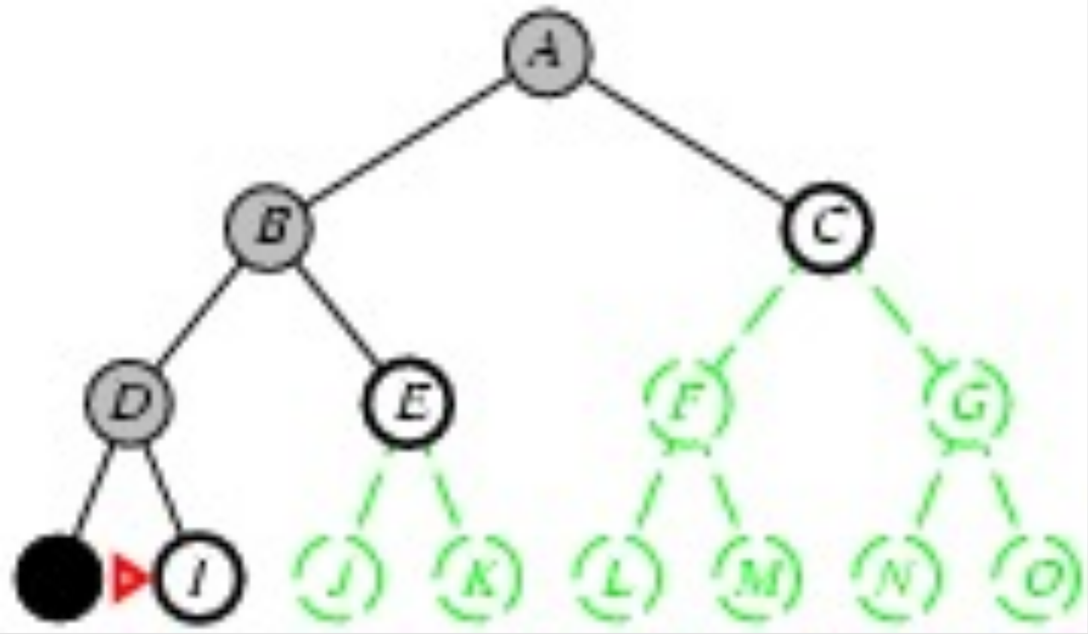
Depth-first search (IV)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



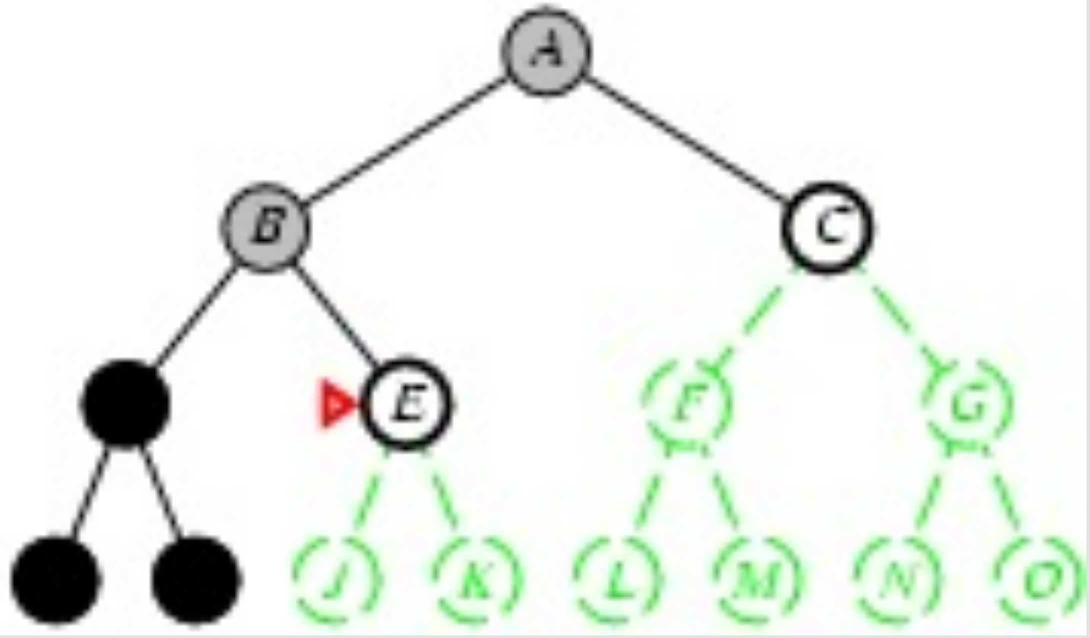
Depth-first search (V)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



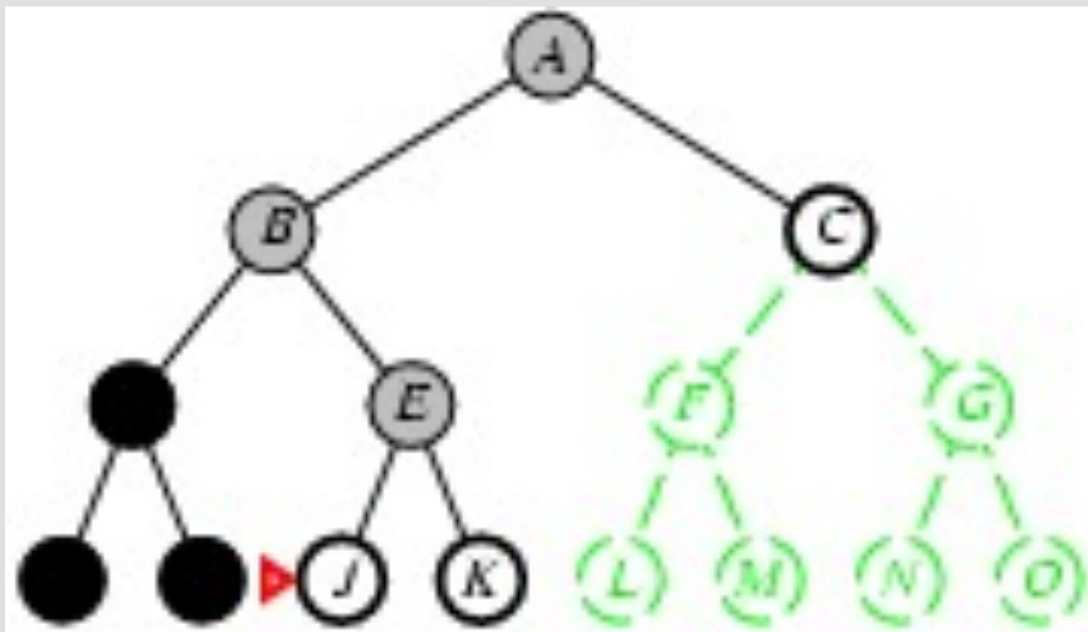
Depth-first search (VI)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



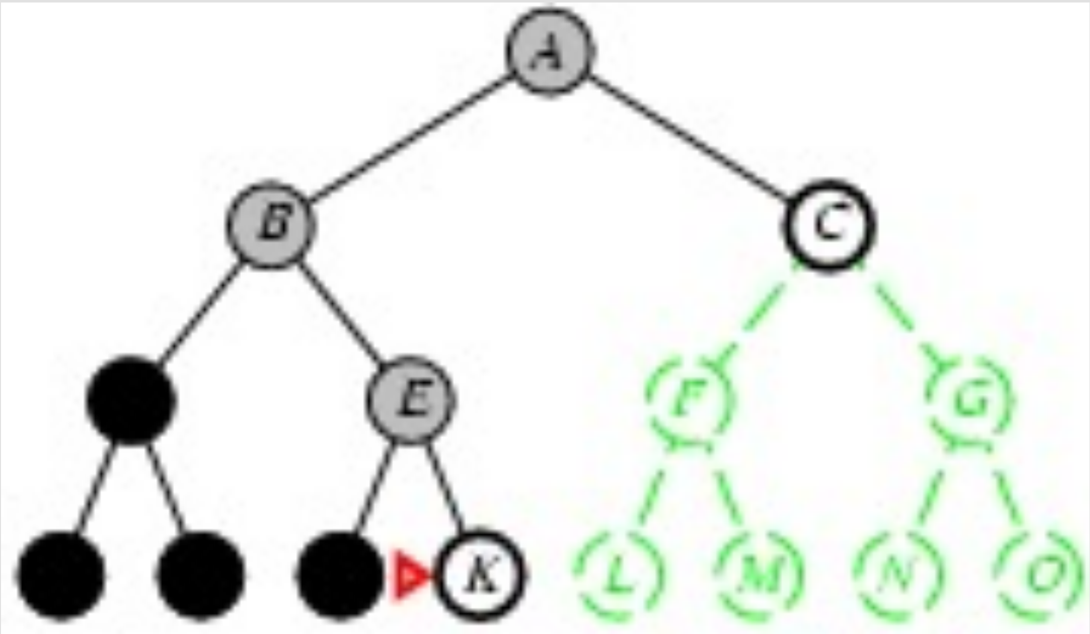
Depth-first search (VII)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



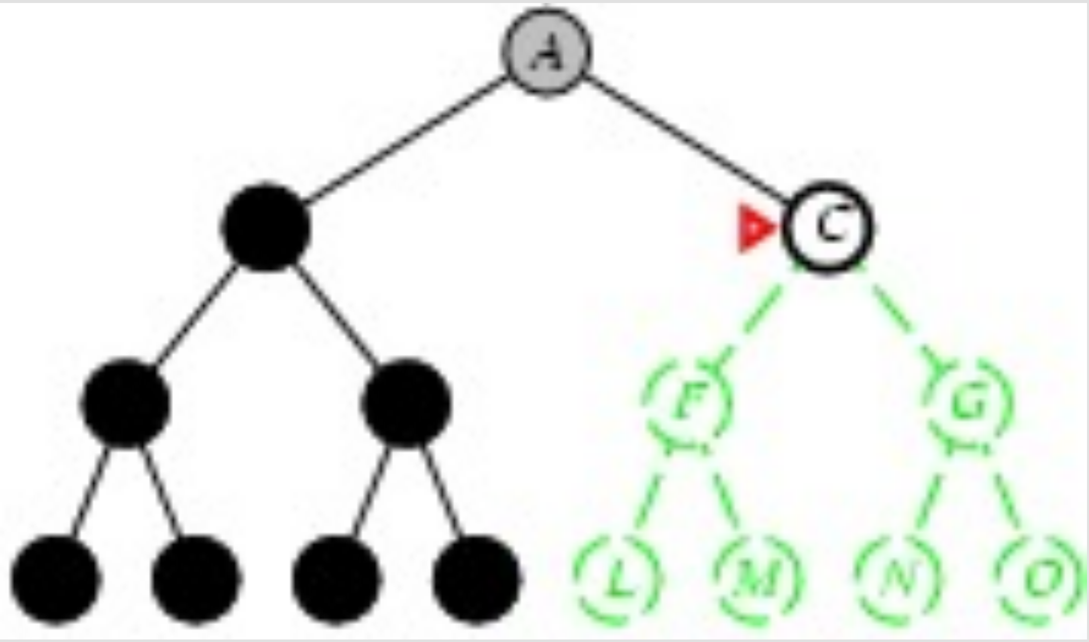
Depth-first search (VIII)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



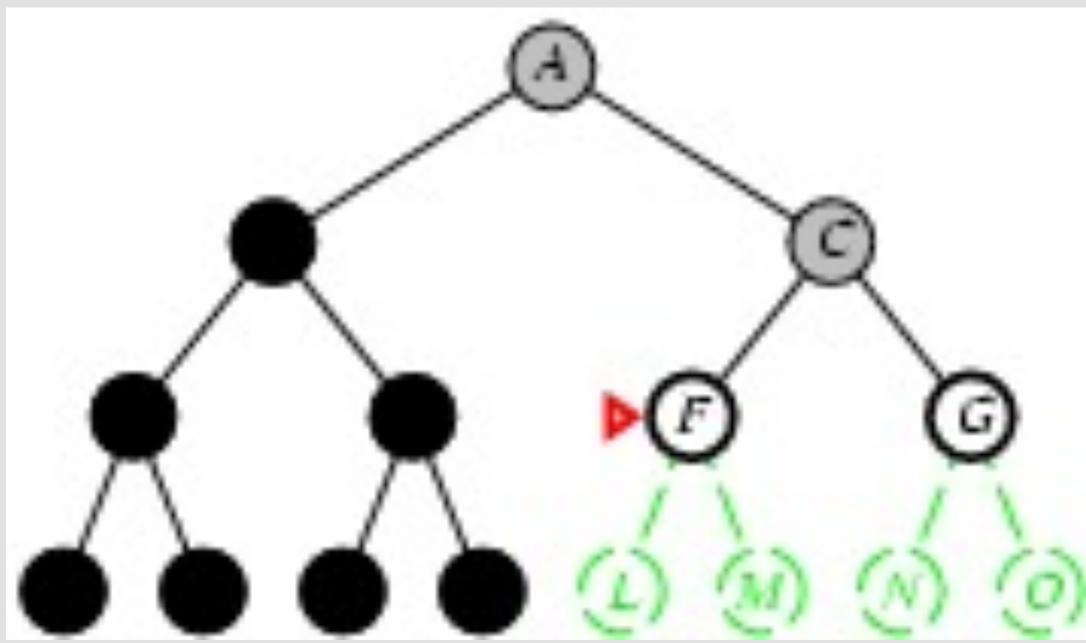
Depth-first search (IX)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



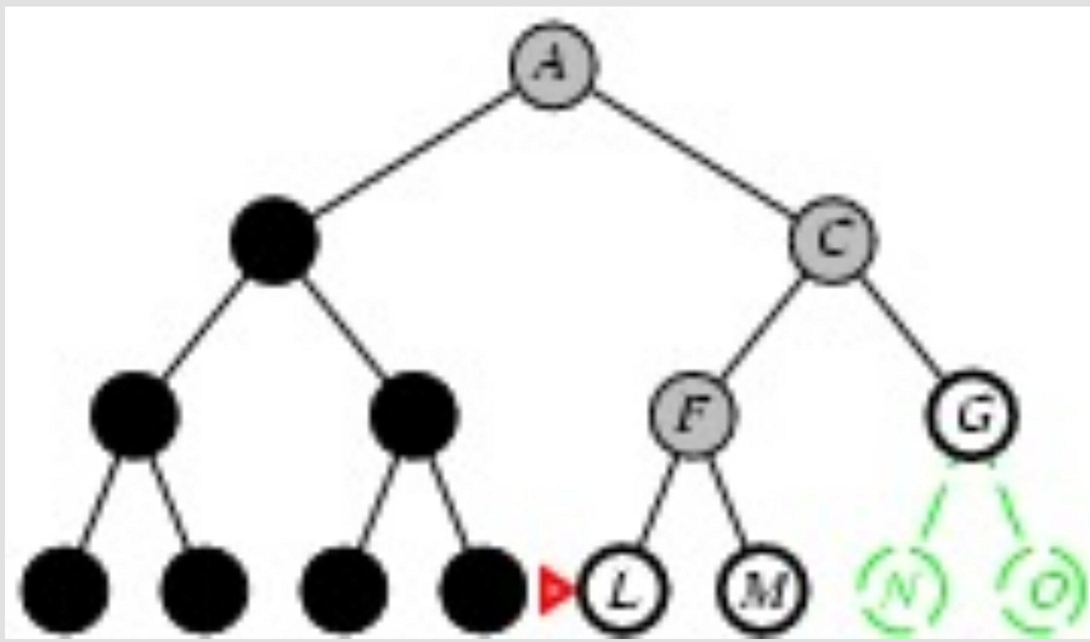
Depth-first search (X)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  – QUEUEING-FN: LIFO – insert successors in front of queue



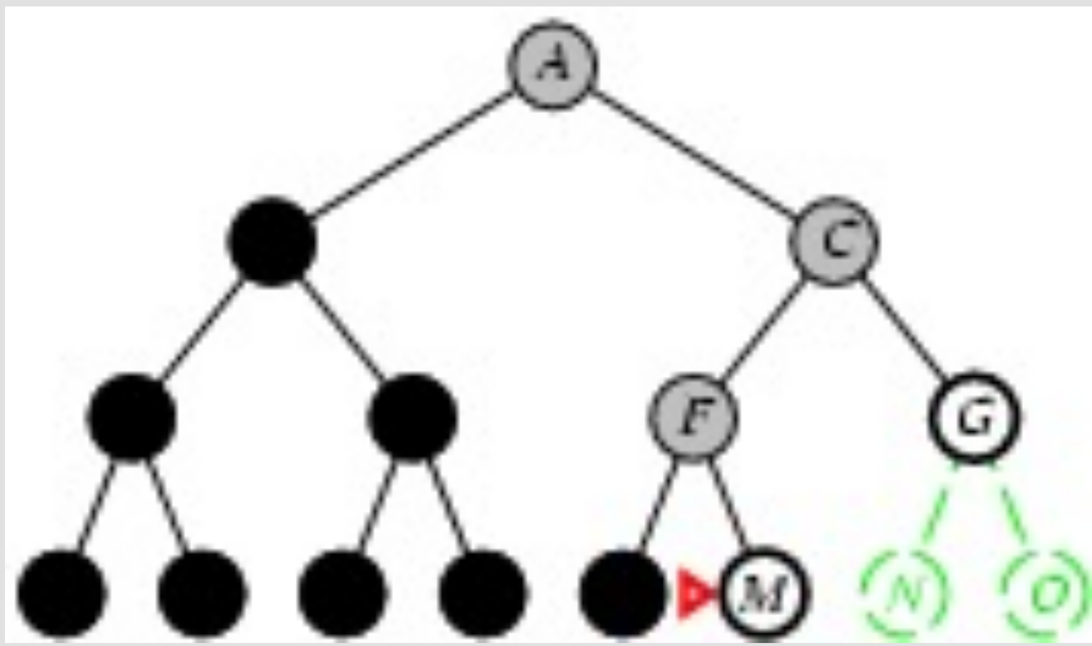
Depth-first search (XI)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



Depth-first search (XII)

- **Expand the deepest unexpanded node**
- **Implementation: managing the frontier**
  - QUEUEING-FN: LIFO – insert successors in front of queue



Properties of DFS

- **Complete?**
  - Infinite-state spaces: No
  - Finite-state spaces: Yes, if we check for ancestors
- **Time?** $O(b^m)$, **terrible if *m* is much larger than *d***
- **Space?** $O(bm)$, **i.e., linear space**
- **Optimal? No**

When all step costs are the same, will DFS find the optimal path?

Depth-limited search

- **DFS with depth limit *L***
  - nodes at depth *L* have no successors
  - returns *cut-off* if no solution is found
- **Complete? No if $d > L$**
- **Time?** $b + b^2 + b^3 + \ldots + b^L = b\frac{b^L - 1}{b-1} \rightarrow O(b^L)$
- **Space?** $O(bL)$
- **Optimal? No**

When all step costs are the same, will DLS find the optimal path?

Iterative deepening DFS

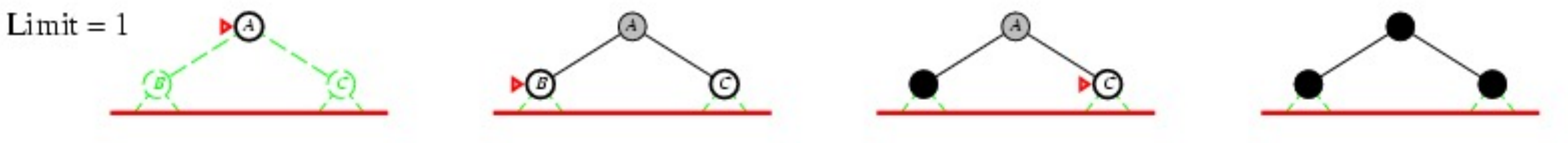
function ITERATIVE-DEEPENING-DF-SEARCH(*problem*) **returns** a solution or failure

- Initialize the frontier using the initial state of *problem*
- **For** *depth* ← 0 **to** ∞
  - *result* ← DEPTH-LIMITED-SEARCH(*problem*,*depth*)
  - **if** *result* ≠ *cut-off* **then return** *result*
- **end**

indicates failure

Iterative deepening search *depth*=0

Iterative deepening search *depth*=1



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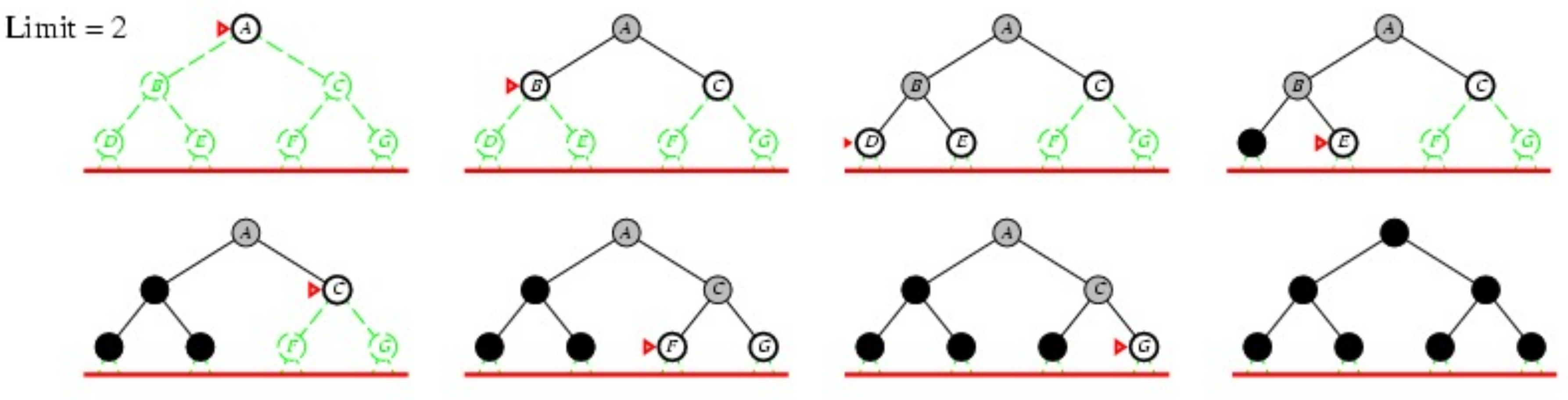
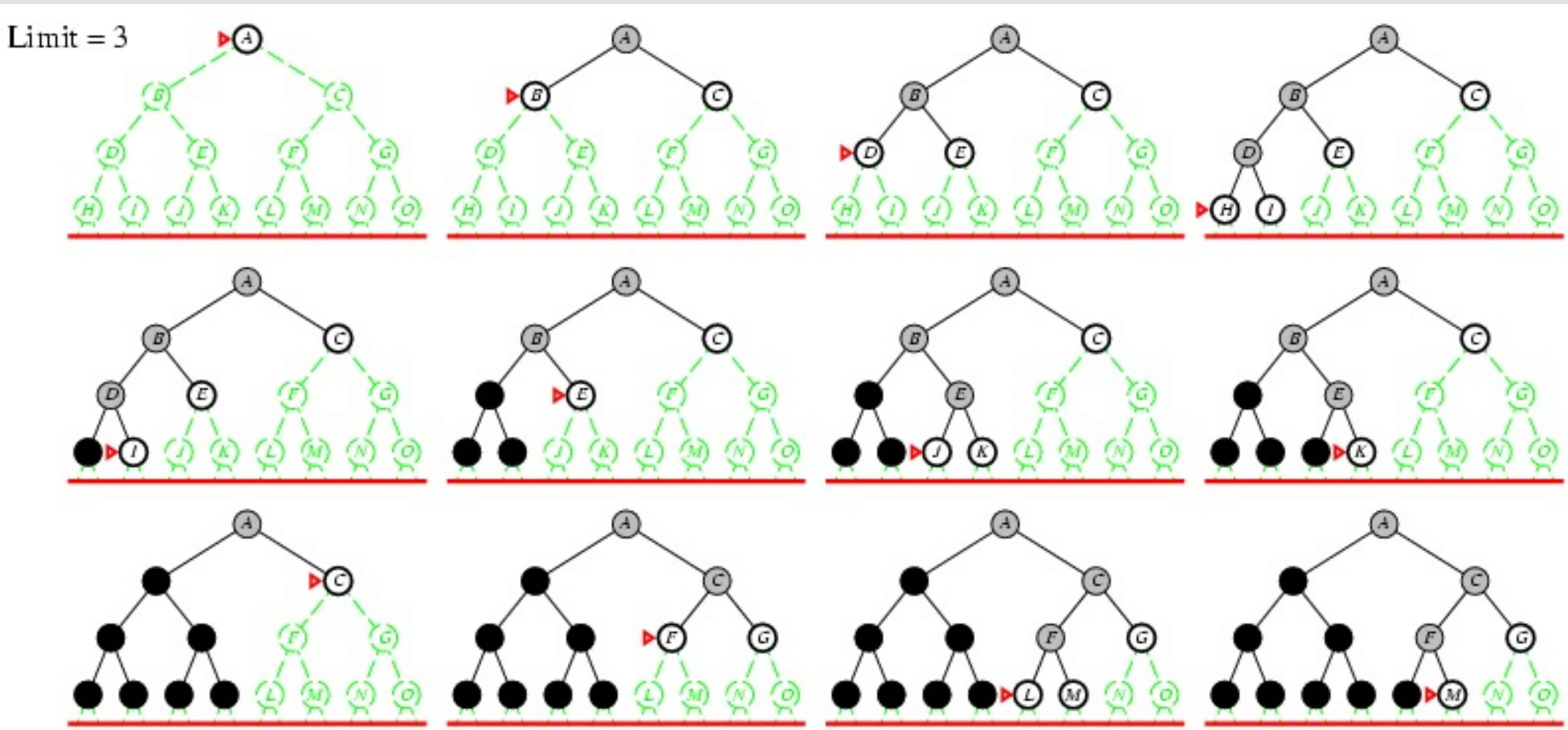
Iterative deepening search *depth*=2



Iterative deepening search *depth*=3



Iterative deepening search – Generated nodes

- **Number of nodes generated in a depth-limited search to depth d with branching factor b:**

$$N_{DLS} = b + b^2 + b^3 + \ldots + b^d = b\,\frac{b^d - 1}{b - 1} \rightarrow O(b^d)$$

- **Number of nodes generated in an iterative deepening search to depth d with branching factor b:**

$$N_{IDS} = db + (d-1)b^2 + \cdots + 3b^{d-2} + 2b^{d-1} + b^d \rightarrow O(b^d)$$

- **Example: For $b = 10$, $d = 6$,**
  - $N_{DLS} = 1 + 10 + 100 + 1{,}000 + 10{,}000 + 100{,}000 = 111{,}111$
  - $N_{IDS} = 6 + 50 + 400 + 3{,}000 + 20{,}000 + 100{,}000 = 123{,}456$
  - Overhead $= \dfrac{123{,}456 - 111{,}111}{111{,}111} = 11\%$

Properties of IDS

- **Complete?** **Yes**
- **Time?**
  $$db + (d-1)b^2 + \cdots + 3b^{d-2} + 2b^{d-1} + b^d \rightarrow O(b^d)$$
- **Space?** $O(bd)$
- **Optimal?** **Yes, if step costs are identical**

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Informed Search Strategies: Best-first Search (BFS)

Heuristic (informed) graphsearch procedures

- **Use *Heuristic Information* (domain dependent information) to help reduce the search**
  – Evaluation function – a real valued function used to compute the "promise" of a node

Heuristic graphsearch: Definitions (I)

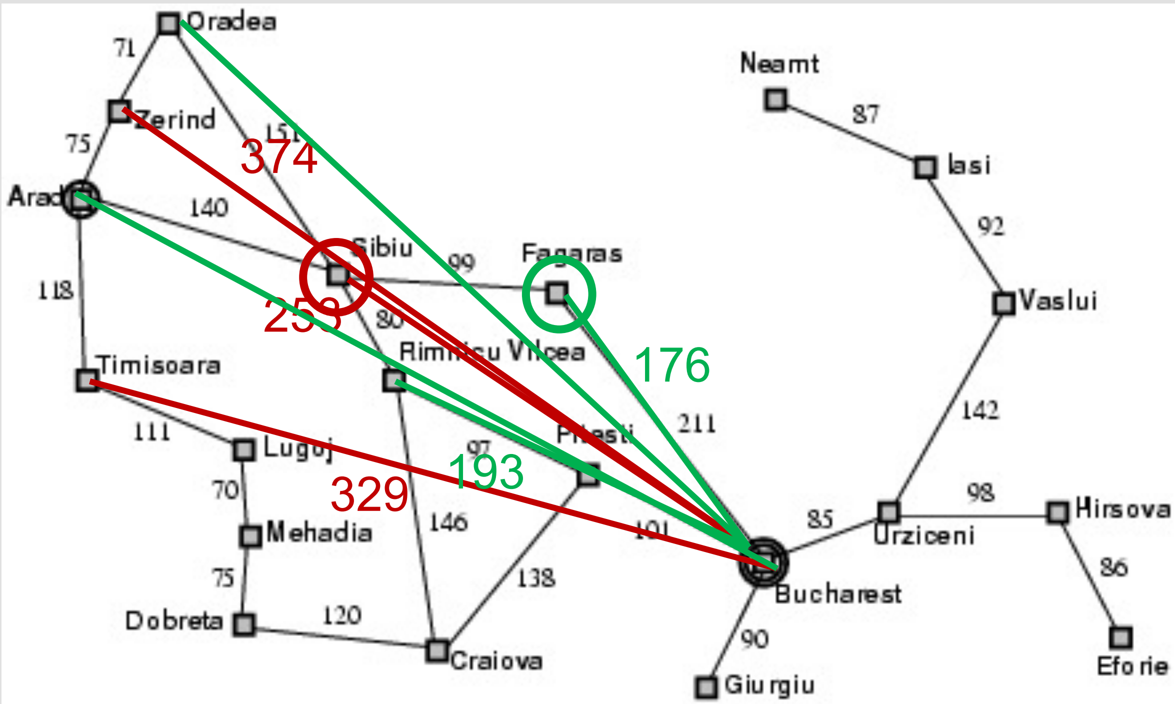
- $k(n_i, n_j)$ – minimal cost path between n_i and n_j
- $h^*(n) = min\{k(n,t_i)\}$
  minimum $k(n,t_i)$ over the set of goal nodes $\{t_i\}$
- $g^*(n) = k(s,n)$
  minimum cost from the start node s to n
- $f^*(n)=g^*(n)+h^*(n)$
  cost of an optimal path constrained to go through n
- $f^*(s)=h^*(s)$
  cost of an unconstrained optimal path from s to a goal

Heuristic graphsearch: Definitions (II)

- $f(n)$ – **<u>estimate</u> of the minimal cost path constrained to go through node** n
- $g(n)$ – **<u>estimate</u> of** $g^*(n)$ that satisfies $g(n) \geq 0$
  Usual choice: Cost of the path in the search tree/graph from s **to** n ➔ $g(n) \geq g^*(n)$
- $h(n)$ – **heuristic function**
  Estimate of $h^*(n)$ which satisfies that $h(n) \geq 0$

Greedy BFS

- **Expands the node that is closest to the goal among the *current options***
  - $f(n) = h(n)$
  - Example: $h_{SLD}(n)$ = Straight-Line Distance to the goal



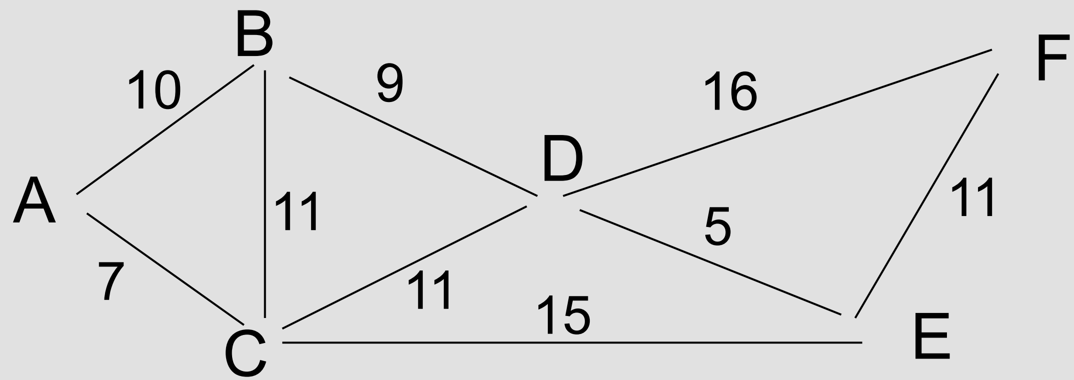
Properties of Greedy BFS

- **Complete?**
  - Infinite-state spaces: No
  - Finite-state spaces: Yes, if we check for ancestors
- **Time?** $O(b^m)$
- **Space?** $O(b^m)$
- **Optimal?** **No**

Algorithm A

- **Graphsearch using the evaluation function** $f(n) = g(n) + h(n)$
- $g(n) \geq g^*(n)$
- $h(n) \geq 0$
- **Algorithm A expands next the node in the frontier with the smallest value of** $f(n)$

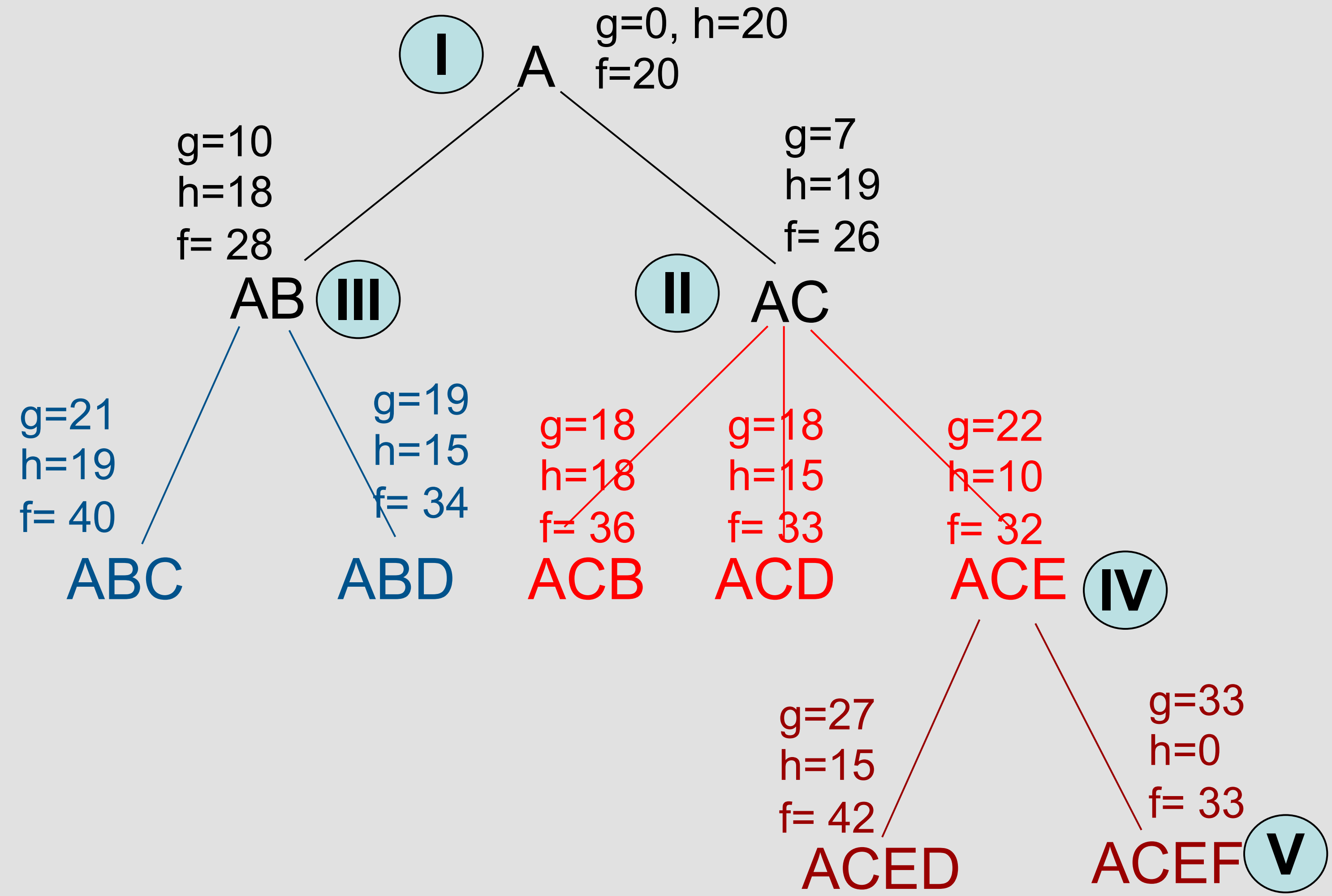
Algorithm A example: Shortest path (I)



ROAD DISTANCES						
	A	B	C	D	E	F
A		10	7			
B			11	9		
C				11	15	
D					5	16
E						11

AIR DISTANCES						
	A	B	C	D	E	F
A		4	3	8	12	20
B			6	5	9	18
C				7	10	19
D					5	15
E						10

Algorithm A example: Shortest path (II)



A* = A + Admissibility (+ Consistency)

- ***Admissibility* of h:**
  If $\forall n \;\; h(n) \leq h^*(n)$
  then A* finds an optimal solution (if one exists)
- ***Monotonicity (Consistency)* of h:**
  If $\forall n \;\; h(n) \leq c(n, m) + h(m)$
  where m is any child of n
  then A* has found an optimal path to any node in the frontier that it selects for expansion
- **Optimality of A***
  – General graphsearch (Nilsson and classnotes) is optimal and terminates (provided there is a solution) if $h(n)$ is admissible
  – Restricted graphsearch (Russell & Norvig) is optimal and terminates if $h(n)$ is consistent

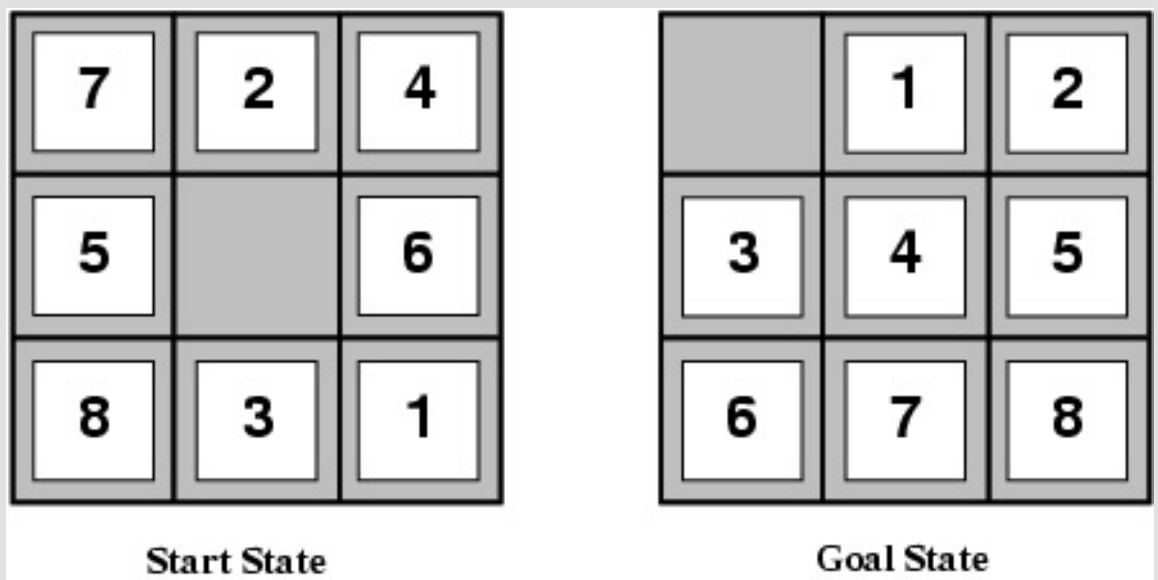
Properties of A and A*

	A	A*
Complete?	Yes	Yes
Time?	$O(b^d)$	$O(b^\Delta)$, where $\Delta \alpha \max\lvert h-h^*\rvert$
Space?	$O(b^d)$	$O(b^\Delta)$
Optimal?	No	Yes

Admissible heuristics: 8 Puzzle

- ***$h_1(n)$*** **= number of misplaced tiles**
- ***$h_2(n)$*** **= total *Manhattan distance* (# of squares from desired location of each tile)**

- **$h_1(S) = ?$**
- **$h_2(S) = ?$**



Relaxed problems

- **A problem with fewer restrictions on the actions is called a *relaxed problem***
- **The cost of an optimal solution to a relaxed problem is an admissible heuristic for the original problem**
- **Examples:**
  – If the rules of the 8-puzzle are relaxed so that a tile can move anywhere, then $h_1(n)$ gives the shortest solution
  – If the rules are relaxed so that a tile can move to any adjacent square, then $h_2(n)$ gives the shortest solution

Dominance

- **Given two <u>admissible heuristics</u> h_1 and h_2, if $h_2(n) \geq h_1(n)$ for all n then h_2 <u>dominates</u> h_1**
  - → h_2 is better for search
- **If we have several admissible heuristics $h_1, h_2, \ldots, h_n$, none of which dominates the rest of them, then we can take the maximum:**

$$h(i) = \max\{h_1(i), h_2(i), \ldots, h_n(i)\}$$

Measuring performance

Performance is often measured by the *effective branching factor (EBF) b^** of a search algorithm

- **If N nodes are generated, the branching factor that a uniform tree of depth d would have to have to contain N+1 nodes is:**

$$N + 1 = 1 + b^* + (b^*)^2 + \ldots + (b^*)^d = \frac{b^{d+1} - 1}{b - 1}$$

Approximating with $b = 2$

$$N + 1 = b^{d+1} - 1 \rightarrow \sqrt[d+1]{N + 2} = b$$

Example: $N=52, d=5 \rightarrow b^* \approx 1.9$

➔ **Experimental measurements of b^* on a small set of problems can provide an idea of a heuristic's usefulness**

– A good heuristic yields $b^* \approx 1$

Summary: Treesearch & Graphsearch

- **When an agent is not clear on which immediate action is best, it can consider possible sequences of actions: search**
- **Before solutions can be found, the agent must formulate a goal and a problem, which consist of:**
  – the initial state; a set of operators; a set of constraints; a goal test function; a path cost function
- **A single general search algorithm can be used to solve any search problem**
- **Different search strategies yield different search algorithms, which are judged on the basis of:**
  – completeness; optimality; time complexity; space complexity

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Irrevocable Search Algorithms

Local search algorithms

- **In many optimization problems, the goal state is the solution**
- **State space = set of complete configurations**
- **Find configuration satisfying constraints, e.g., n-queens problem**
- **In such cases, we can use local search algorithms**
  - keep a single current state, and try to improve it

Example: *n*-Queens problem

- **Put *n* queens on an *n* × *n* board with no two queens on the same row, column or diagonal**

Hill climbing algorithm

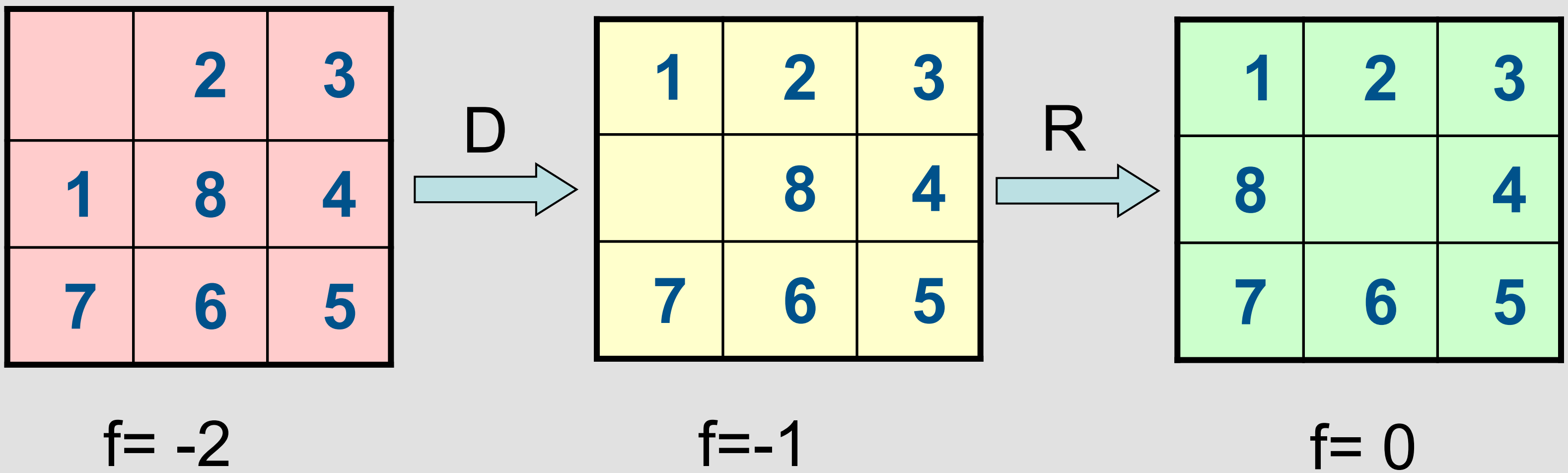
Procedure Hill Climbing(current-state)

1. **If current-state = goal-state Then return it**
2. **Else until a solution is found or no more operators can be applied do**
   a. Select an operator that has not been applied yet to current-state and apply it to generate new-state
   b. Evaluate new-state:
      - **i.** **If** new-state = goal-state **Then** return it and quit
      - **ii.** **Elseif** new-state is better than current-state **Then** current-state ← new-state

Steepest ascent hill-climbing: select the best operator

Hill Climbing – Example 8 Puzzle (I)

- **f = - { number of tiles out of place }**



Hill Climbing – Example 8 Puzzle (II)

- **f = - { number of tiles out of place }**

Current

1	2	5
	8	4
7	6	3

f= -2

Goal

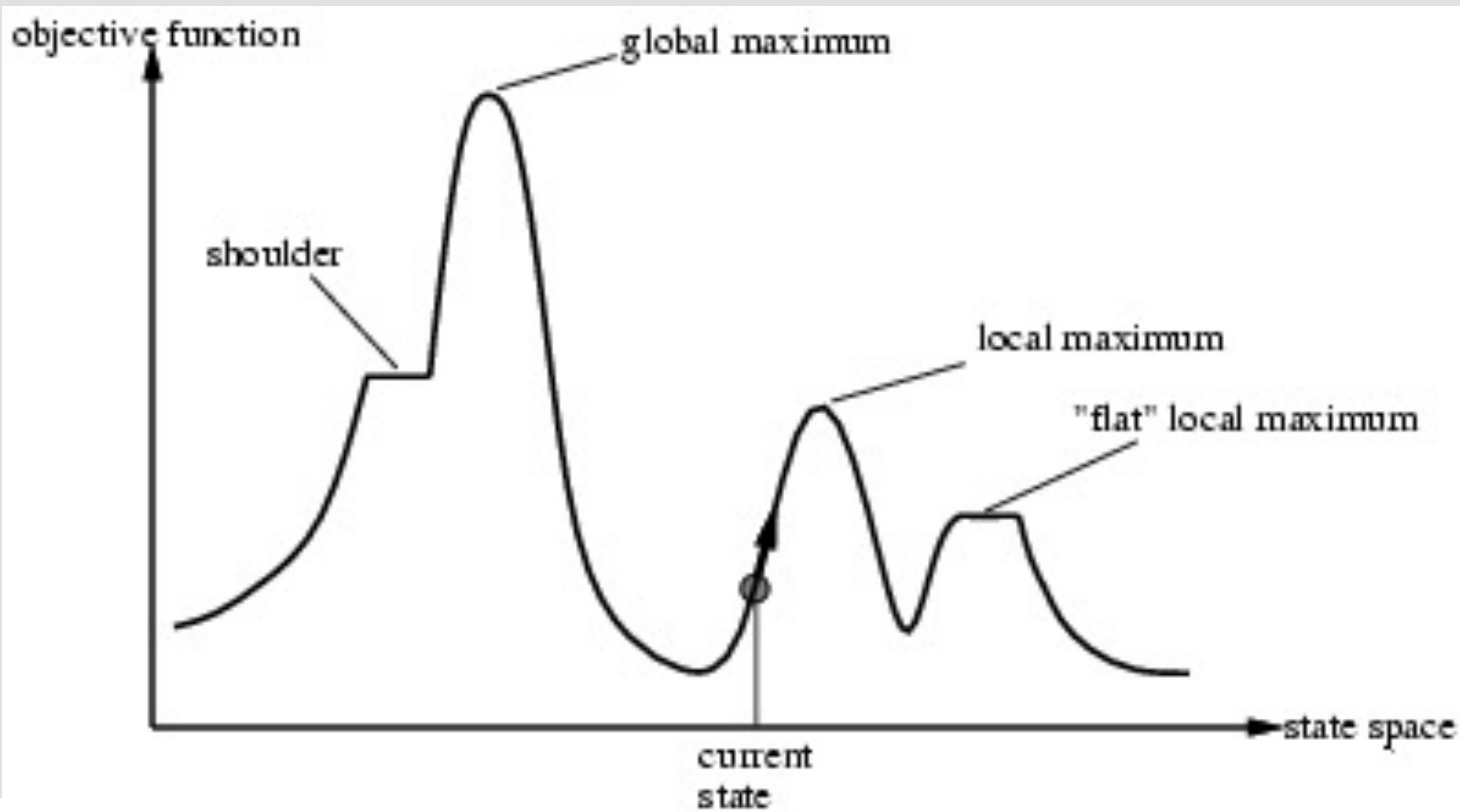
1	2	3
	8	4
7	6	5

f= 0

Stuck in local maximum

Hill-climbing search

- **Problem: depending on the initial state, the algorithm can get stuck in local maxima**



Local beam search

- **Keep track of *k* states rather than just one**
- **Start with *k* randomly generated states**
- **At each iteration, all the successors of all *k* states are generated**
  - If any one is a goal state, then return it and stop
  - Else select the *k* best successors from the complete list and repeat

Simulated annealing

- **Based on the physical process of annealing**
- **Idea: escape local maxima/minima by allowing some "bad" moves, but gradually decrease their frequency**
- ***Temperature* (*T*) – the temperature at which the annealing takes place**
- ***Annealing schedule* – the rate at which the temperature is lowered**

Simulated annealing algorithm

Procedure Simulated Annealing(current-state)

1. **If** current-state = goal-state **Then** return it and quit
2. BestSoFar←current-state
3. Initialize ***T*** according to the annealing schedule
4. Until no more operators can be applied do
   a. Select an operator that has not been applied yet to the current-state and apply it to generate a new-state
   b. Evaluate the new-state. Compute:
   ΔE = Value(current-state) - Value(new-state)
      i. **If** new-state = goal-state **Then** return it and quit
      ii. **Elseif** $\Delta E<0$ (new-state is better than current-state) **Then** current-state←new-state
      If new-state is better than BestSoFar **Then** BestSoFar←new-state
      iii. **Else** with probability $Pr=e^{-\Delta E/T}$ current-state←new-state
   c. Revise ***T*** according to the annealing schedule
   d. If ***T***=0 Then return BestSoFar

Maximization problem

Simulated annealing algorithm

Procedure Simulated Annealing(current-state)

1. **If** current-state = goal-state **Then** return it and quit
2. BestSoFar←current-state
3. Initialize ***T*** according to the annealing schedule
4. Until no more operators can be applied do
   a. Select an operator that has not been applied yet to current-state and apply it to generate new-state
   b. Evaluate new-state. Compute:
      ΔE = Value(current-state) - Value(new-state)
      i. **If** new-state = goal-state **Then** return it and quit
      ii. **Elseif** ΔE<0 (new-state is better than current-state) **Then** current-state←new-state
          If new-state is better than BestSoFar **Then** BestSoFar←new-state
      iii. **Else** with probability $Pr=e^{-\Delta E/T}$ current-state←new-state
   c. Revise ***T*** according to the annealing schedule
   d. If ***T***=0 Then return BestSoFar

Maximization problem

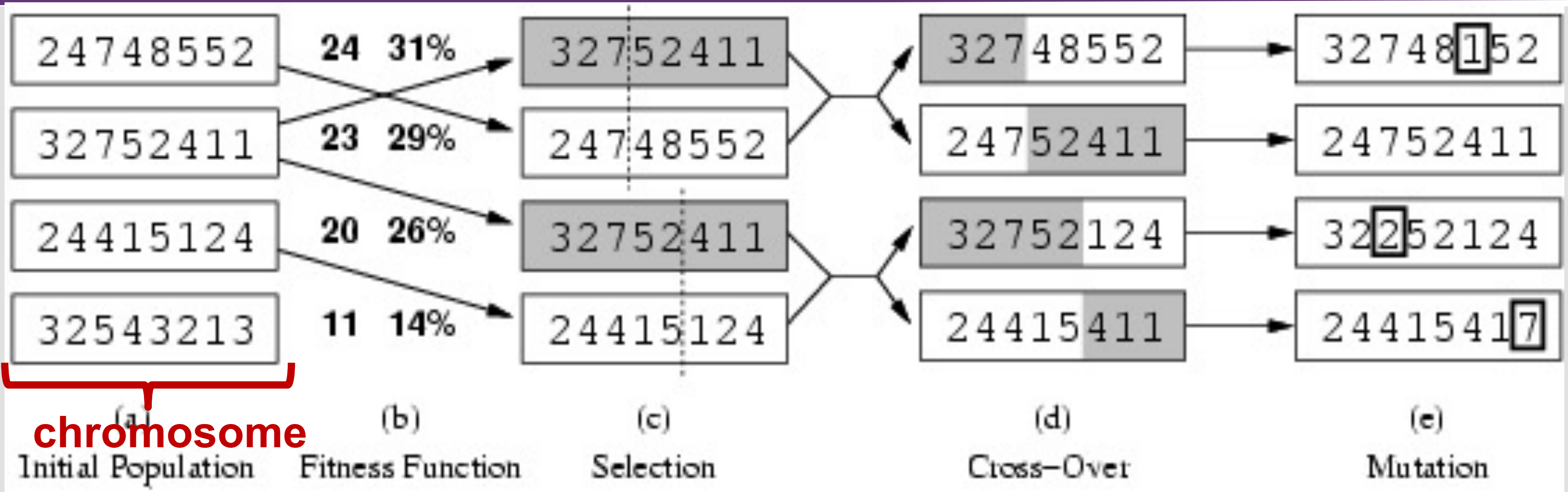
About simulated annealing search

- **One can prove: If *T* decreases slowly enough, then simulated annealing search will find a global optimum with probability approaching 1**
- **Widely used in VLSI layout & airline scheduling**

Genetic algorithms

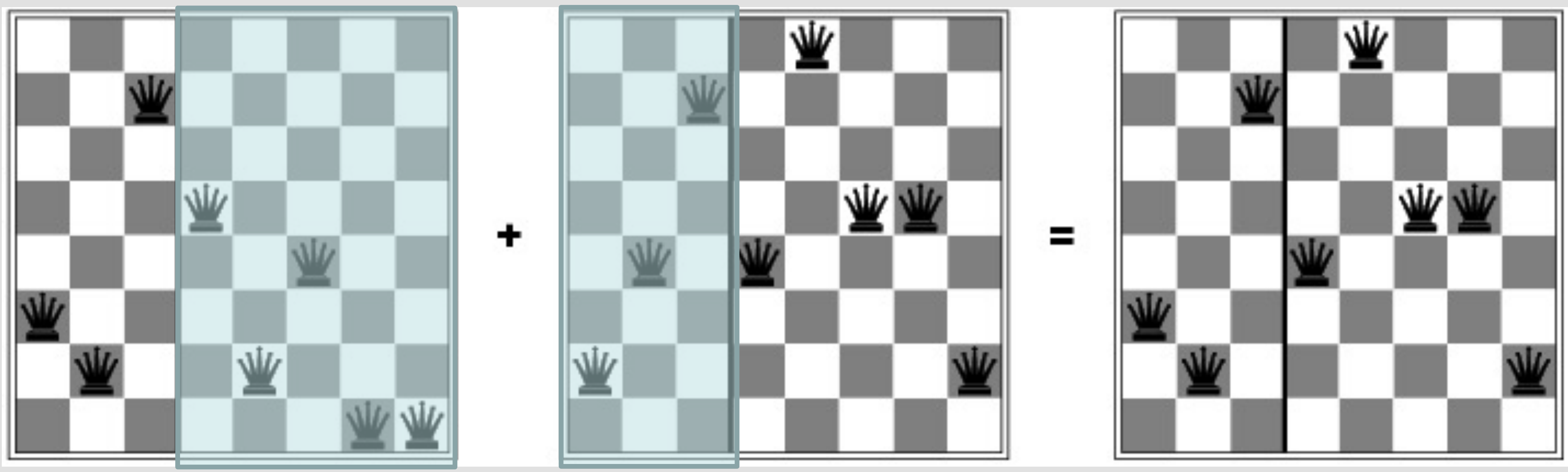
- **Start with a *population* of *k* randomly generated states**
- **Typically, a state (*chromosome*) is represented as a string over a finite alphabet of *genes* (often a string of 0s and 1s)**
- **A successor state is generated by combining two parent states**
- **Evaluation function (*fitness function*):**
  - Higher values for better states
- **Produce the next generation of states by *selection, crossover* and *mutation***

GAs: Example 8-Queens problem (I)

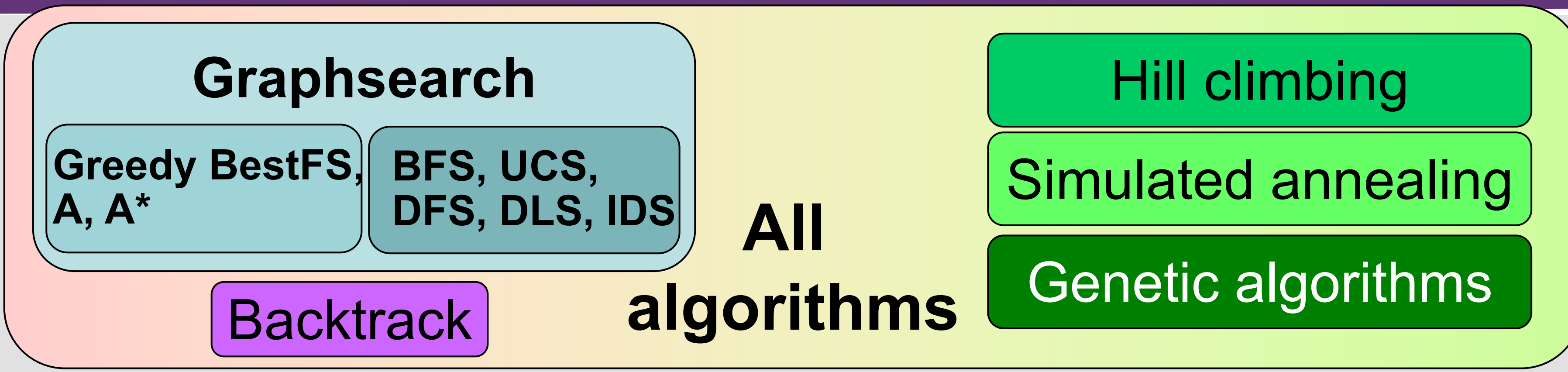


- **Representation:**
  - **Gene**: row # (between 1 and 8) of the queen that is in column i
  - **Chromosome**: 1 gene per column (8 genes per chromosome)
- **Fitness function: number of non-attacking pairs of queens (min = 0, max = 8 × 7/2 = 28)**
  - Probability of selection: $\frac{24}{24+23+20+11}$=31%, $\frac{23}{24+23+20+11}$=29%

GA Crossover: Example 8-Queens problem



Search algorithms – A perspective



- **Informedness in Graphsearch depends on g and h**
  - A $f(n) = g(n)+h(n)$ $(g(n) \geq g^*(n),\ h(n) \geq 0)$
  - A* $(g(n) \geq g^*(n),\ 0 \leq h(n) \leq h^*(n))$
- **Uninformed Graphsearch**
  - BFS є A* when $g(n)$=depth and $h(n)=0$
  - UCS є A* with $g(n)\geq 0$ and $h(n)=0$
  - DFS, DLS, IDS є Graphsearch, DFS, DLS, IDS $\notin$ A
- **Informed Graphsearch**
  - Greedy best-first search with $g(n)=0$ and $h(n)\geq 0$ $\notin$ A

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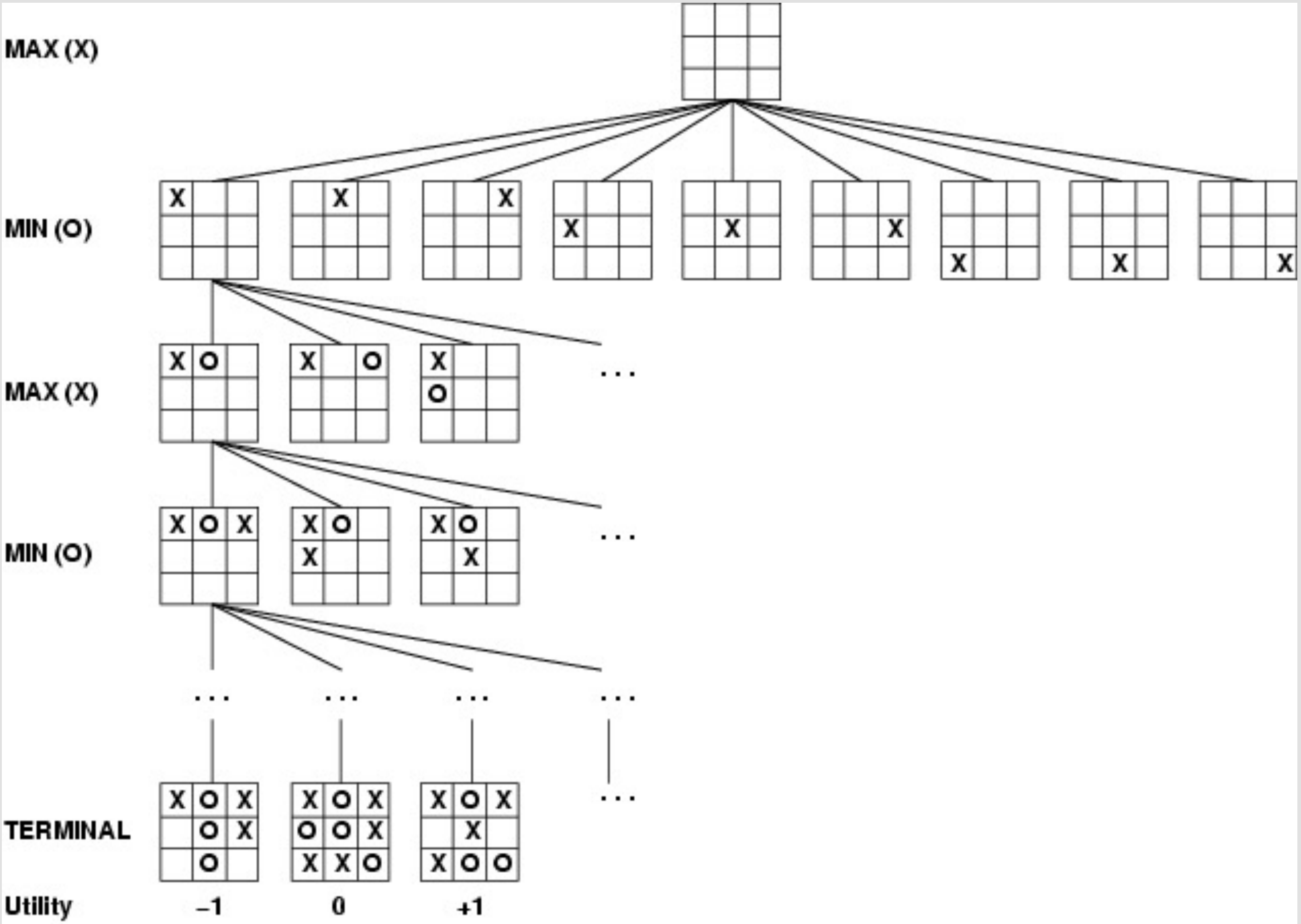
Adversarial
Search Algorithms

Searching game trees

- **Two person turn-based perfect information games**
- **Conventions:**
  - Players are MAX and MIN
    - > A good position for MAX has a value > 0 (winning is often ∞)
    - > A good position for MIN has a value < 0 (winning is often $-\infty$)
  - Goal: find a winning strategy for MAX
    - > For all nodes where it is MIN's move next, show that MAX can win from **every** position to which MIN might move
    - > For all nodes where it is MAX's move next, show that MAX can win from **some/one** position to which MAX might move

Game tree (2-player, Deterministic, Turns)

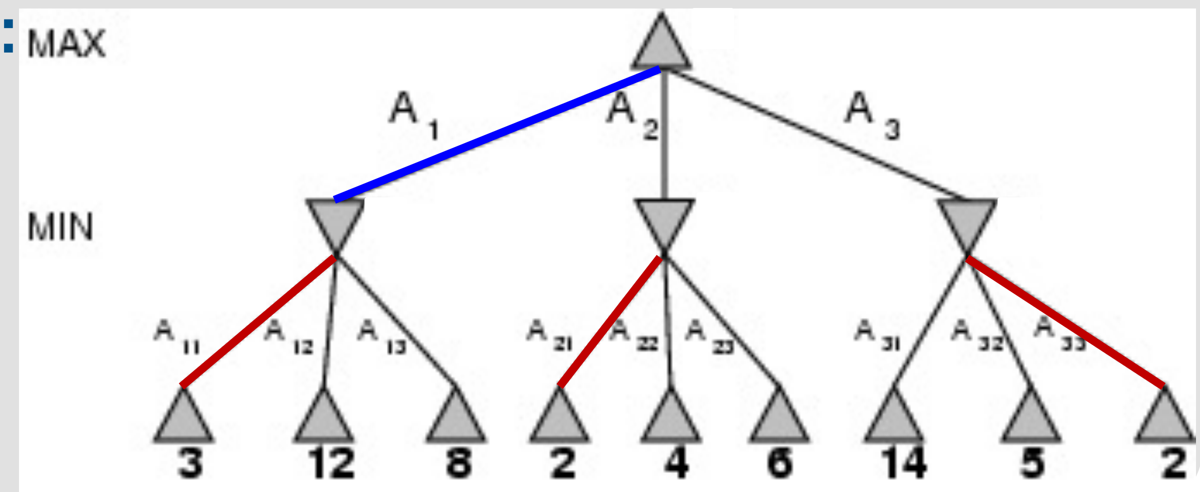


Games versus Search problems

- **Unpredictable opponent → must specify a move for every possible opponent reply**
- **Time limits: not all games can be searched to the end → find a good first move**

Minimax ideas

- **If MAX (blue choices) were to choose among tip nodes, she would take the node with the largest value**
- **If MIN (red choices) were to choose among tip nodes, he would take the node with the smallest value**
- **Choose move to the position with highest *minimax value*: best achievable payoff against best opponent's play**
- **Eg, 2-player game:**



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Minimax algorithm

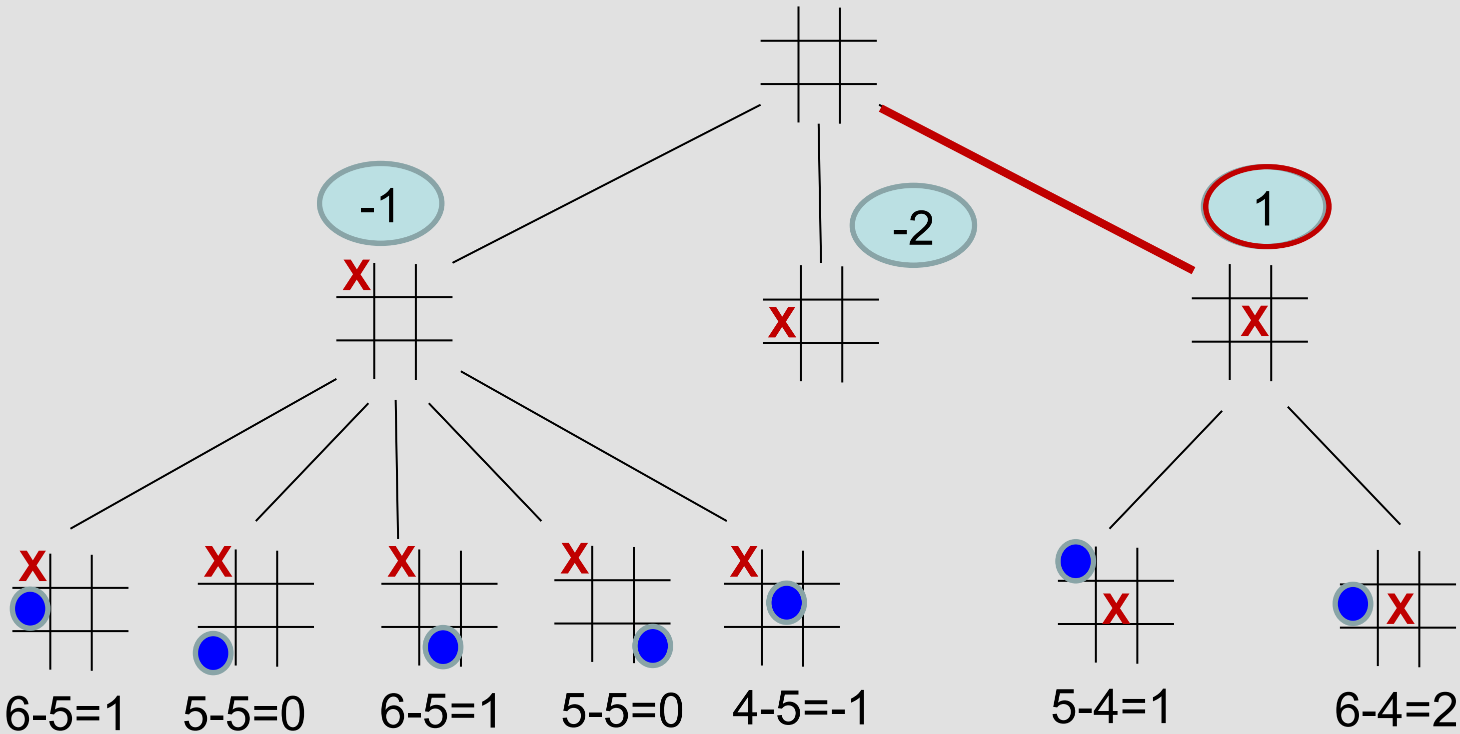
```
function MINIMAX-DECISION(state) returns an action
   return arg max_{a ∈ ACTIONS(s)} MIN-VALUE(RESULT(state, a))
```

```
function MAX-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s, a)))
   return v
```

```
function MIN-VALUE(state) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← ∞
   for each a in ACTIONS(state) do
      v ← MIN(v, MAX-VALUE(RESULT(s, a)))
   return v
```

Minimax example: Tic-Tac-Toe

- **Evaluation function:**
  { # of rows, columns, diagonals available to MAX –
  # of rows, columns, diagonals available to MIN }



Properties of Minimax

Minimax does *Complete Depth First Exploration*

– All paths are explored to depth *m*

- **Complete?** **Yes (if tree is finite)**
- **Optimal?** **Yes (against an optimal opponent)**
- **Time complexity?** $O(b^m)$
- **Space complexity?** $O(bm)$ **(depth-first exploration)**

- **E.g., for chess, $b \approx 35$, $m \approx 100$ for "reasonable" games → exact solution completely infeasible**

Resource limits

- **Suppose we have 100 secs per move, and we explore 10^4 nodes/sec → 10^6 nodes per move**

- **Standard approach:**
  - **Cutoff test –** depth limit (perhaps add quiescence search)
  - **Evaluation function –** estimates desirability of a position
    > E.g., for chess typically a linear weighted sum of features
    $\mathrm{Eval(s)} = \mathrm{w_1\, f_1(s) + w_2\, f_2(s) + \ldots + w_n\, f_n(s)}$,
    where $\mathrm{w_1} = 9$ and
    $\mathrm{f_1(s) = (\#\ of\ white\ queens) - (\#\ of\ black\ queens)}$
  - **Forward pruning**
    > beam search that looks only at n-best moves

Definitions: α and β values

- **α-value of a MAX node – current <u>largest</u> final backed-up value of its successors**
  - α-value is the <u>lower</u> bound of a MAX backed-up value
- **β-value of a MIN node – current <u>smallest</u> final backed-up value of its successors**
  - β-value is the <u>upper</u> bound of a MIN backed-up value

α-β Procedure

- **Rules for discontinuing the search:**
  - **α cut-off**: search can be discontinued below any **MIN** node having a β-value ≤ **α-value** of **any** of its MAX node ancestors
    - > **Final backed-up value** of this MIN node is set to its β-value
  - **β cut-off**: search can be discontinued below any **MAX** node having an α-value ≥ **β-value** of **any** of its MIN node ancestors
    - > **Final backed-up value** of this MAX node is set to its α-value

Termination condition

- **All the successors of the start node are given final backed-up values**
- **The best first move is that which creates the successor with the highest backed-up value**

The α-β algorithm (I)

```
function ALPHA-BETA-SEARCH(state) returns an action
   v ← MAX-VALUE(state, −∞, +∞)
   return the action in ACTIONS(state) with value v
```

```
function MAX-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← −∞
   for each a in ACTIONS(state) do
      v ← MAX(v, MIN-VALUE(RESULT(s,a), α, β))
      if v ≥ β then return v
      α ← MAX(α, v)
   return v
```

β cut-off

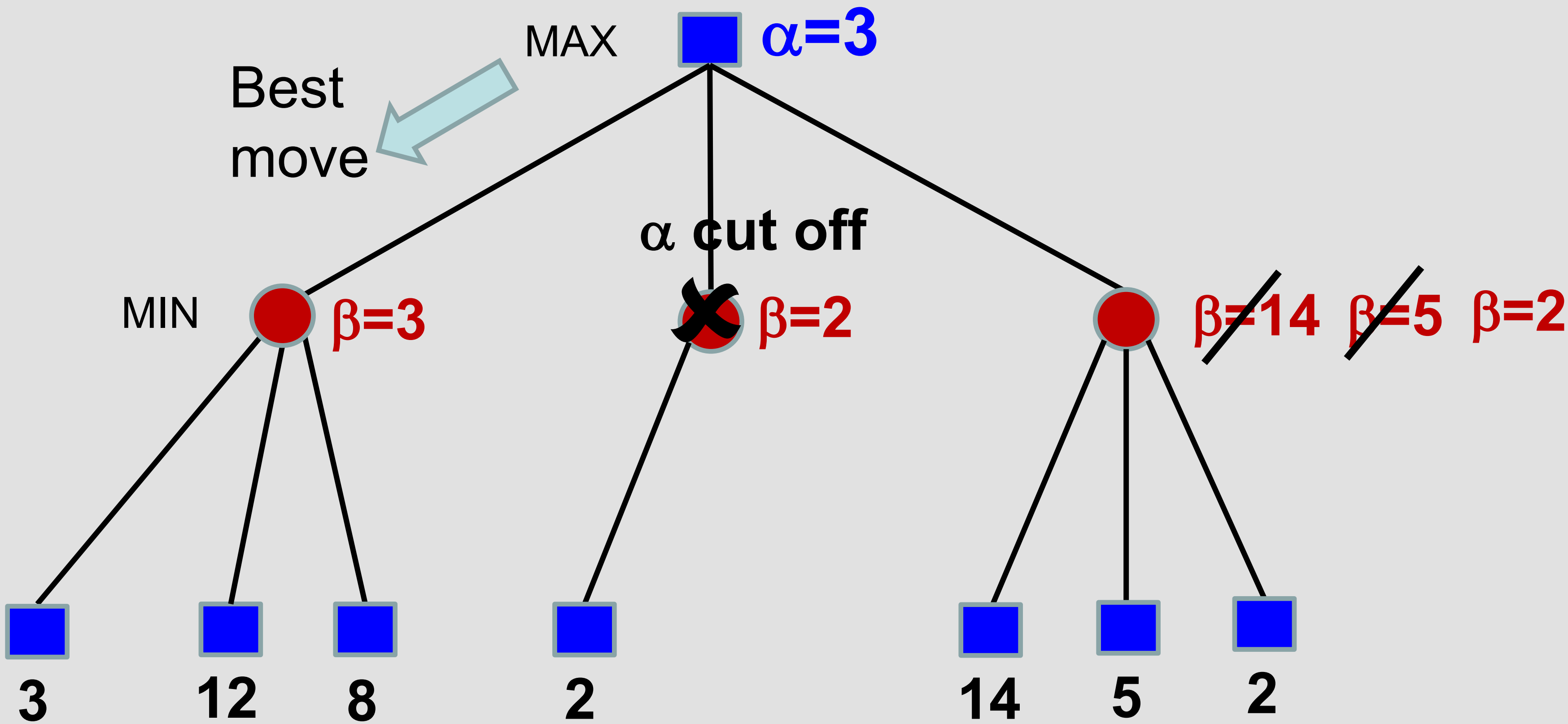
```
function MIN-VALUE(state, α, β) returns a utility value
   if TERMINAL-TEST(state) then return UTILITY(state)
   v ← +∞
   for each a in ACTIONS(state) do
      v ← MIN(v, MAX-VALUE(RESULT(s,a) , α, β))
      if v ≤ α then return v
      β ← MIN(β, v)
   return v
```

α cut-off

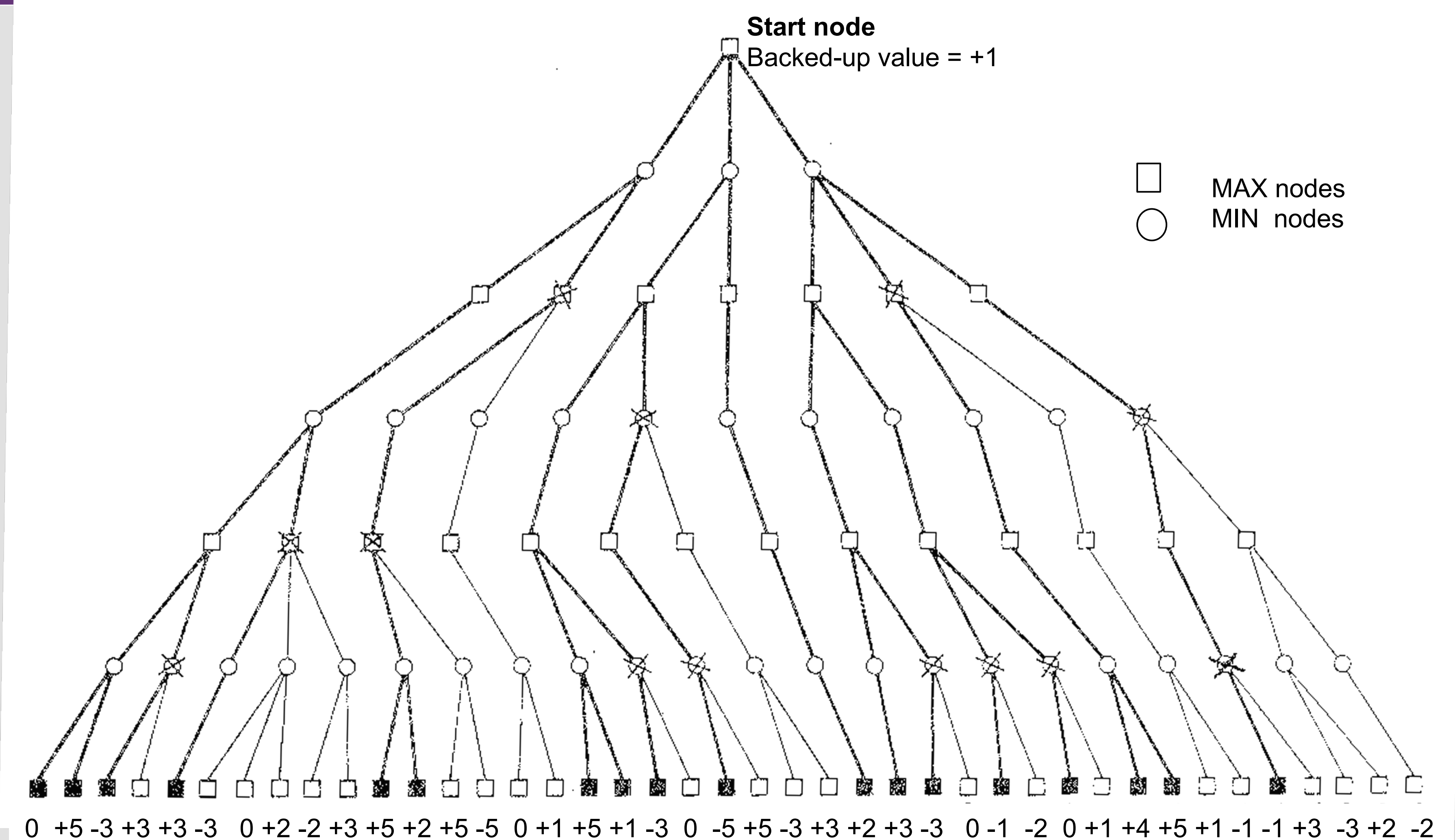
α = the value of the best choice (i.e., **highest-value**) we have found so far at any choice point along the path for MAX.

β = the value of the best choice (i.e., **lowest-value**) we have found so far at any choice point along the path for MIN.

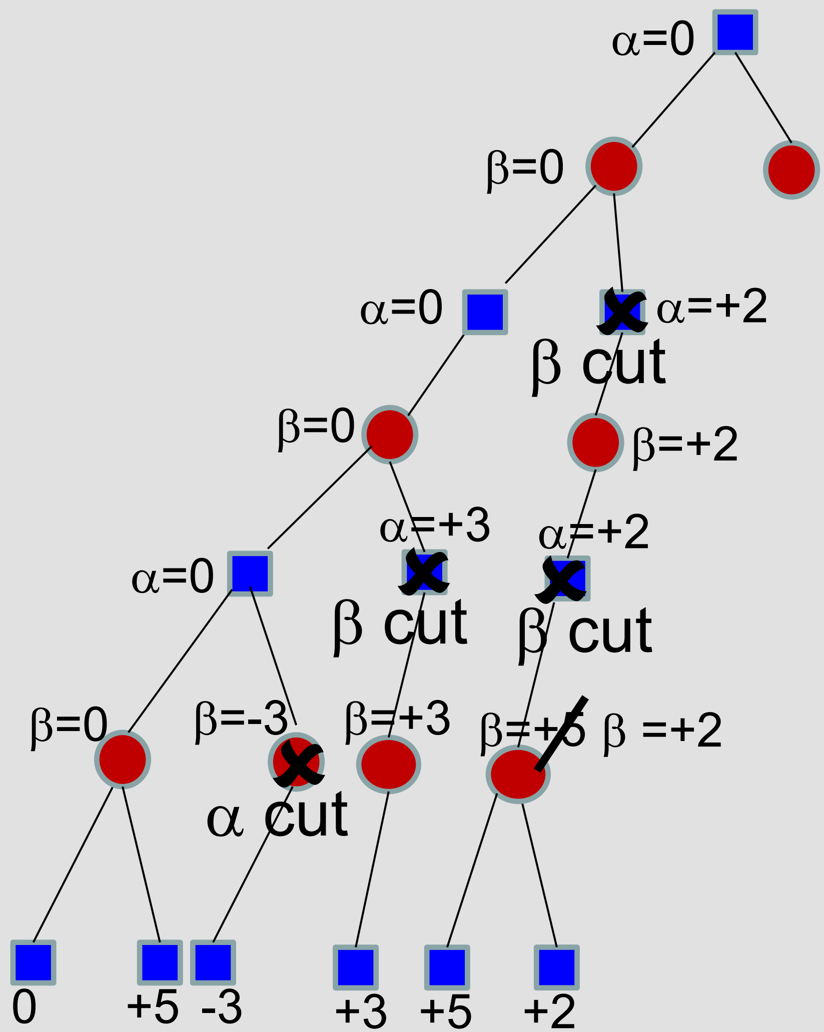
α-β Pruning – Example



α-β Pruning – Large example



α-β Pruning – Part of Large Example



Move ordering

- **The effectiveness of the αβ algorithm depends on the order in which states are examined**
- **With perfect ordering, time complexity = $O(b^{m/2})$**

  → depth of search can be doubled
- **Adding dynamic ordering schemes brings us close to the theoretical limit**

Deterministic games in practice

- **Checkers**: Chinook defeated the world champion in an abbreviated game in 1990. It uses αβ search combined with a pre-computed database defining perfect play for 39 trillion endgame positions.
- **Chess**: Deep Blue defeated human world champion Garry Kasparov in a six-game match in 1997. Deep Blue searches 30 billion positions per move (200 million per second), normally searching to depth 14, and extending the search up to depth 40 for promising options. Heuristics reduce the EBF to about 3.
- **Othello**: In 1997, a computer defeated the world champion 6-0. Humans are no match for computers.
- **Go**: $b > 361$, which is too large for αβ. In 2016, AlphaGo, which uses Deep Learning, beat the world champion 4-1.

Summary: Adversarial Search

Games illustrate important points about AI

- **Perfection is unattainable → must approximate**
- **Force us to think about what to think about, e.g., nodes to keep/discard**

Reading

- **Russell, S. and Norvig, P. (2010), *Artificial Intelligence – A Modern Approach* (3rd ed), Prentice Hall**
  - Chapter 7, Sections 7.1, 7.3 (only backtrack algorithm)
  - Chapter 3 (excluding 3.5.3, 3.5.4, 3.6.3, 3.6.4)
  - Chapter 4, Section 4.1
  - Chapter 5, Sections 5.1-5.4

Next lecture topic

- **Lecture Topic 4**
  - Knowledge representation