FIT5216: Modelling Discrete Optimization Problems
Inclass Task 16: Magic Squares

1 Problem Statement

Given an $n \times n$ square the aim is to fill it with the numbers $1..n^2$ so that

- each number appears exactly once
- The sum of each row is the same
- The sum of each column is the same
- The sum of each major diagonal is the same

An example 3×3 magic square is

4	3	8
9	5	1
2	7	6

Note that each column and row and major diagonal adds to 15!

Data and decisions for the problem is defined as follows:

```
int: n;
set of int: ROW = 1..n;
set of int: COL = 1..n;
set of int: NUM = 1..n*n;
array[ROW,COL] of var NUM: x;
```

1.1 Stage A

Build a model `magic.mzn` for this problem. Use it to determine the number of solutions for size $n = 3, 4, 5$.

Add symmetry breaking to remove symmetric solutions. Count how many solutions now. Do the relative numbers make sense!

What about fining a solution to $n = 6, 7, ...$? Does symmetry breaking help?

1.2 Stage B

Modify your program to `magic_opt.mzn` to maximize the weighted sum of the top left corner where the value is 4 times the top left corner + 2 times the two square orthogonal to it plus the value of the diagonal neighbour.

For our example 3×3 solution the value is

4×4	2×3	0×8
2×9	1×5	0×1
0×2	0×7	0×6

for a total of 40.

Add *correct* symmetry breaking constraints for this problem.

1.3 Stage C

Build a program `magic_ll.mzn` to find the single lex greatest solution, i.e. when thinking of the 2d array as a 1d array. You may have to run the program multiple times, using the output of the previous run as input.

Does symmetry breaking help or hurt you solve this problem?

As a help, here are the correct answers

$n = 3$	[8,3,4,1,5,9,6,7,2]
$n = 4$	[16,15,2,1,5,3,14,12,4,10,7,13,9,6,11,8]
$n = 5$	[24,24,13,2,1,23,4,7,20,11,3,10,16,15,21,5,19,17,6,18,9,8,12,22,14]

2 Instructions

Edit the provided `mzn` model files to solve the problems described above. Your implementations can be tested locally by using the *Run* icon in the MiniZinc IDE or by using,

```
minizinc ./modelname.mzn ./datafile.dzn
```

at the command line.