Quiz Week 2 - Probability - Solutions

FIT5197 teaching team

Question 1

What is the probability of getting a numeric total of 4 or 6 from rolling two fair/unbiased dice?

Answer 1

Step 1. Define the random variable for events and have sample space: Let x denote rolling first dice and y denote rolling the second.

$$S_x = S_y = \{1, 2, 3, 4, 5, 6\}$$
$$S(x, y) = \{\forall x \in S_x, y \in S_y \quad (x, y)\}$$

Step 2. So it's easy to see total number of (x,y) pair is $6 * 6 = 36$. This is the denominator.

Step 3. Calculate for events $x + y = 4$ and $x + y = 6$.

- For event $x + y = 4$, there are (1,3), (3,1) , (2,2), three possible (x,y) pairs, so

$$P(x + y = 4) = \frac{3}{6 \times 6} = \frac{1}{12}$$

- Similarly, for event $x + y = 6$, there are (1,5), (2,4), (3,3), (4,2), (5,1) possible pairs, 5 in total.

$$P(x + y = 6) = \frac{5}{6 \times 6} = \frac{5}{36}$$

Question 2

Draw 5 cards from a deck of 52 cards (no jokers) without replacement, what is the probability of getting no red cards? Tip - google 'playing cards'

Answer 2

Fact 1: Draw 5 cards without replacement from 52 cards is a same event as "select (choose) 5 cards from 52 cards" denote this event as x

Fact 2: Draw 5 cards without replacement from deck getting no red cards means:

- Draw these 5 cards from {\color{red} all black} cards without replacement. Denote this event as y.
- Then do the counting for event x and y (from definition of n choose k)

$$freq(x) = \binom{52}{5} = \frac{52!}{5!47!}$$
$$freq(y) = \binom{26}{5} = \frac{26!}{5!21!}$$

So probability $P = \frac{freq(y)}{freq(x)} = \frac{26\times25\times24\times23\times22}{52\times51\times50\times49\times48}$

Attention: This is a different way to solve compared to the solution showed in lecture. You can think it by yourself.

Question 3

What is the probability of getting at least one ace when selecting 5 cards from a deck of 52 cards without replacement?

Answer 3

Event = At least one ace when selecting 5 cards

P(at least one A) = P(exactly one A) + P(exactly two As) + P(exactly three As) + P(exactly four As)

P(exactly one A) = $\frac{4}{52} \times \frac{48}{51} \times \frac{47}{50} \times \frac{46}{49} \times \frac{45}{48} \times \binom{5}{1} = 0.2994736$, where $\binom{5}{1} = \frac{5!}{4!1!} = 5$ means different ways of ordering one Ace in five places.

P(exactly two As) = $\frac{4}{52} \times \frac{3}{51} \times \frac{48}{50} \times \frac{47}{49} \times \frac{46}{48} \times \binom{5}{2} = 0.03992982$, where $\binom{5}{2} = \frac{5!}{3!2!} = 10$.

P(exactly three As) = $\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{48}{49} \times \frac{47}{48} \times \binom{5}{3} = 0.001736079$, where $\binom{5}{3} = \frac{5!}{2!3!} = 10$.

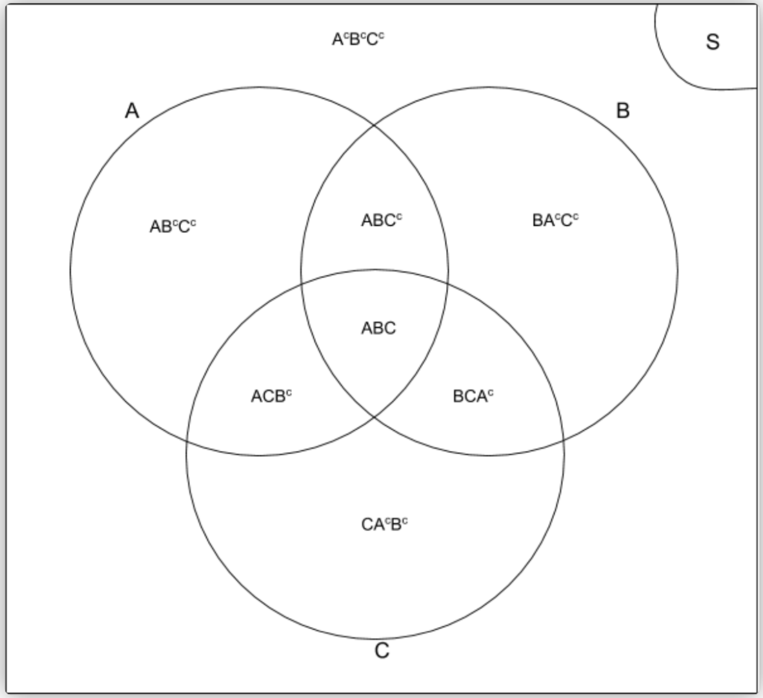
P(exactly four As) = $\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{48}{48} \times \binom{5}{4} = 0.00001846893$, where $\binom{5}{4} = \frac{5!}{1!4!} = 5$.

P(at least one ace) = 0.341158.

Question 4

There are 3 different magazines A, B and C offered for subscription for a community. 45% of the people in that community subscribe to A, 35% subscribe to B, 30% subscribe to C. 10% subscribe to A & B, 8% subscribe to A & C, 5% subscribe to B & C, 3% subscribe to all the three. Find the probability of: only subscribe to A; only subscribe to one magazine;

Answer 4



Community is divided into different groups according to their subscription to magazines i.e A-B-C. Each group is represented by a set in the Venn diagram. There can be people subscribed to more than one magazine, so those sets have intersections with each other to represent those people.

"A" as a set represents all the elements within set "A" and "A^c" represents all the elements out of the set "A". Intersections of sets will be shown with the notation as follow; AC^c = Intersection of set "A" and all the elements that are not in "C".

This notation is specifically selected as this is the notation in the M.Ross's book.

Outside of union of A, B, C; In other words, $A^cB^cC^c$ is the part that represents the people that are not subscribed to any magazine.

S is the universal set which covers everybody in the community.

In the next steps we will define all the given information using those represented areas and solve those equations for the unknown areas in the diagram. Once we find the values of the areas that are asked in the question, we will finalise our solution.

$$\begin{aligned} S(S) = S(ABC) + S(A^cBC) + S(AB^cC) + S(ABC^c) + S(AB^cC^c) \\ +S(A^cBC^c) + S(A^cB^cC) + S(A^cB^cC^c) = 100 \end{aligned} \tag{1}$$

$$S(A) = S(ABC) + S(AB^cC) + S(ABC^c) + S(AB^cC^c) = 0.45 \tag{2}$$

$$S(B) = S(ABC) + S(A^cBC) + S(ABC^c) + S(A^cBC^c) = 0.35 \tag{3}$$

$$S(C) = S(ABC) + S(A^cBC) + S(AB^cC) + S(A^cB^cC) = 0.30 \tag{4}$$

$$S(ABC) + S(ABC^c) = 0.10 \tag{5}$$

$$S(ABC) + S(AB^cC) = 0.08 \tag{6}$$

$$S(ABC) + S(A^cBC) = 0.05 \tag{7}$$

$$S(ABC) = 0.03 \tag{8}$$

Only subscribes to $A = S(AB^cC^c) = ?$

Only subscribes to one magazine = $S(AB^cC^c) + S(A^cBC^c) + S(A^cB^cC) = ?$

Using (8) and (5), (6), (7)

$$S(ABC^c) = 0.10 - 0.03 = 0.07 \tag{9}$$

$$S(AB^cC) = 0.08 - 0.03 = 0.05 \tag{10}$$

$$S(A^cBC) = 0.05 - 0.03 = 0.02 \tag{11}$$

Using (8), (9), (10), (11) in (2), (3), (4)

$$S(AB^cC^c) = 0.45 - (0.03 + 0.07 + 0.05) = 0.30 \tag{12}$$

$$S(A^cBC^c) = 0.35 - (0.03 + 0.07 + 0.02) = 0.23 \tag{13}$$

$$S(A^cB^cC) = 0.30 - (0.03 + 0.05 + 0.02) = 0.20 \tag{14}$$

$$\Rightarrow \quad S(AB^cC^c) = 0.30, \quad S(AB^cC^c) + S(A^cBC^c) + S(A^cB^cC) = 0.73$$

Question 5

1% of the population has X disease. A screening test accurately detects the disease for 90% if people with it. The test also indicates the disease for 15% of the people without it (the false positives). Suppose a person screened for the disease tests positive. What is the probability they have it?

Tips:

1. Read the description carefully.
2. Identify the events from question.

This is the way model any real problems. The word "model" means describe things/events in mathematics language.

Answer 5

There are two events, using random variables.

Y denotes people has X disease or not.

Z denotes screening test results for people: positive or negative.

Then according to the description, we have:

$$P(Y = True) = 0.01 \tag{1}$$
$$P(Z = Positive|Y = True) = 0.9 \tag{2}$$
$$P(Z = Positive|Y = False) = 0.15 \tag{3}$$

and what we are asked to solve:

$$P(Y = True|Z = Positive) =? \tag{4}$$

Every time you are required to calculated Conditional probability , you should recall Bayes rule:

$$P(A|B) = \frac{P(A, B)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B)}$$

So we have:

$$(4) = \frac{P(Y = True, Z = Positive)}{P(Z = Positive)} = \frac{P(Z = Positive|Y = True) \cdot P(Y = True)}{P(Z = Positive)} = \frac{(2) \cdot (1)}{P(Z = Positive)}$$

in which we only need to solve $P(Z = Positive)$. Every time you want to solve probability of One variable with information of other variables, you should try marginal probability:

$$P(A) = \sum_{b\in\Omega} P(A, B = b) \quad \Omega \text{ is sample space of } B$$

Therefore:

$$P(Z = Positive) = \sum_{y\in\Omega} P(Z = Positive, Y = y) = \sum_{y\in\Omega} P(Z = Positive|Y = y) \cdot P(Y = y) \tag{5}$$

remind that $y \in \{True, False\}$

$$(5) = P(Z = Positive|Y = True) \cdot P(Y = True) + P(Z = Positive|Y = False) \cdot P(Y = False) = (2) \cdot (1) + (3) \cdot (1 - (1))$$

Then:

$$(4) = P(Y = True|Z = Positive) = \frac{0.9 \times 0.01}{0.9 \times 0.01 + 0.15 \times 0.99} \approx 0.057$$

Question 6

A jar contains ten marbles (marked from 1 to 10). Four marbles are drawn without replacement, the number noted. Find the probability of:

Smallest number is 5?

Biggest number is 5?

Answer 6

$\color{red}{\text{This question can be solved as a count problem.}}$ Firstly, we determine the denominator, which is the count of all possible drawns: $\color{red}{\text{chosen 4 from 10}}$

$$\binom{10}{4} = \frac{10!}{4!6!} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$$

1. Smallest number is 5. So number 5 is picked. This only contains 1 possible event. For other 3 marbles, they need to be picked up from numbers {6, 7, 8, 9, 10}, without replacement. Therefore, choose 3 from 5 numbers is
$$\binom{5}{3} = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$$

$$\Rightarrow \quad P = \frac{\binom{5}{3} \cdot 1}{\binom{10}{4}} = \frac{10}{210} = \frac{1}{21}$$

2. Similarly, biggest number is 5. So number 5 is picked. This only contains 1 possible event. For other 3 marbles, they need to be picked from numbers {1, 2, 3, 4}, without replacement. Therefore, choose 3 from 4 numbers:

$$\binom{4}{3} = \binom{4}{1} = 4$$

$$\Rightarrow \quad P = \frac{\binom{4}{3} \cdot 1}{\binom{10}{4}} = \frac{4}{210} = \frac{2}{105}$$

Question 7

A box contains n pairs of shoes (2n shoes in total). If 2r (with the assumption that $2r \leq n$) shoes are selected at random, find the probability for the following scenarios:

A0 = 'No matching pair'

A1 = 'only one matching pair'

A3 = 'exactly two matching pairs'

Answer 7

Probability of observing no matching pair can be calculated by dividing the number of possible ways for selecting non-matching pairs divided by all possible ways of selection. Hence:

$$P(no\ matching\ pair) = \frac{Number\ of\ possible\ ways\ for\ selecting\ 2r\ non\ matching\ pairs}{Total\ number\ of\ ways\ of\ selecting\ 2r\ shoes\ from\ 2n\ shoes}$$

First, we need to define our event space which will be the number of ways of all outcomes. We are selecting $2r$ shoes from $2n$ shoes so $\binom{2n}{2r}$ is the number of all possible ways. This will be same for all parts of the question.

No matching pair

Now we need to find the number of possible ways to select 2r shoes that none of them are matching with each other:

Because we don't want any matching shoes, **we will consider each pair as one shoe** and select $2r$ from only n shoes. Once you consider each pair as one shoe, you don't leave any chance for selecting the pair of the shoes. Hence $\binom{n}{2r}$.

By considering each pair as one shoe, we excluded the ways of selecting right or left shoe of each pair. To include that we need to multiply the number of ways by 2 for each selection as this selection can be right or left shoe. This makes 2^{2r} ways as we are selecting 2r shoes.

Hence the final probability of selecting no matching shoes will be:

$$\frac{(\binom{n}{2r} \times 2^{2r})}{\binom{2n}{2r}}$$

Only one matching pair

$$P(Only\ one\ matching\ pair) = \frac{Number\ of\ possible\ ways\ for\ selecting\ 2r\ with\ 1\ matching\ pairs}{Total\ number\ of\ ways\ of\ selecting\ 2r\ shoes\ from\ 2n\ shoes}$$

If there will be one matching pair, this means that we will select 2r-2 non-matching shoes and 2 matching shoes.

Again, we will consider each pair as one shoe for selecting $2r - 2$ non-pair shoes. Hence, we will select from n-1 shoes as one of the pairs will be selected in the next step, so it is excluded. $\binom{n-1}{2r-2}$.

By considering each pair as one shoe, we excluded the ways of selecting right or left shoe of each pair. To include that we need to multiply the number of ways by 2 for each selection as this selection can be right or left shoe. This makes 2^{2r-2} ways as we are selecting 2r-2 shoes.

Also, we need to include the number of ways to select one matching pairs. There are n pairs and selecting one is C(n,1)

Therefore, final probability becomes:

$$\frac{\binom{n-1}{2r-2} \times 2^{2r-2} \times \binom{n}{1}}{\binom{2n}{2r}}$$

Exactly two matching pairs

$$P(Only\ two\ matching\ pair) = \frac{Number\ of\ possible\ ways\ for\ selecting\ 2r\ with\ 2\ matching\ pairs}{Total\ number\ of\ ways\ of\ selecting\ 2r\ shoes\ from\ 2n\ shoes}$$

Very similar to previous example. Instead of excluding 2 shoes from the initial selection, we will exclude 4 shoes to make sure they are selected as pairs. Hence probability becomes:

$$\frac{\binom{n-2}{2r-4} \times 2^{2r-4} \times \binom{n}{2}}{\binom{2n}{2r}}$$

Simplified example

To make this easier to understand let's imagine we have red, green, blue and black pairs and we are selecting 4 shoes in total. So, all the shoes are (Right-red, Left-red, Right-blue, Left-blue, Right-green, Left-green, Right-black, Left-black)

So total number of possible selections are:

$$\binom{8}{4} = 70$$

Number of ways for selecting non-pair 4 shoes: To make sure they are not matching we will select 4 shoes each are different colours. There are 4 colours. Hence: $\binom{4}{4} = 1$; Basically we need to select 1 from each colour.

Then we should calculate the different ways of selecting from each colour; For example, for the black shoe, we can select right or left. This works for all colours the same way. Hence: 2^4 ways for selecting right to left.

Hence the final probability becomes:

$$\frac{\binom{4}{4} \times 2^4}{\binom{8}{4}} = \frac{16}{70}$$

Question 8

A certain virus infects one in every 400 people. A test used to detect the virus in a person is positive 85% of the time if the person has the virus and 5% of the time if the person does not have the virus. (This 5% result is called a false positive). Let A be the event "the person has the virus" and B be the event "the person tests positive".

a) Find the probability that a person has the virus given that they have tested positive, i.e. find $P(A|B)$. Round your answer to the nearest hundredth of a percent.

b) Find the probability that a person does not have the virus given that they test negative, i.e. find $P(A'|B')$. Round your answer to the nearest hundredth of a percent.

Hints:

Bayes theorem can be written in different ways by applying the different probability rules/laws.

$$P(X|Y) = \frac{P(X,Y)}{P(Y)} \quad [\textit{written as conditional probability}]$$

$$P(X|Y) = \frac{P(X) \cdot P(Y|X)}{P(Y)} \quad [\textit{product rule}: \ P(X,Y) = P(X) \cdot P(Y|X)]$$

$$P(X|Y) = \frac{P(X) \cdot P(Y|X)}{\sum_{x \in \Omega} P(Y|X=x) \cdot P(X=x)} \quad [\textit{sum rule}: \ P(Y) = \sum_{x \in \Omega} P(Y|X=x) \cdot P(X=x)]$$

Answer 8

First we define our events:

$$\begin{array}{cc} \text{test positive}(B) & \text{test negative}(\bar{B}) \\ \text{have virus}(A) & \text{don't have virus}(\bar{A}) \end{array}$$

The virus infects 1 in 400 people, so $P(A) = \frac{1}{400} = 0.0025$ and $P(\bar{A}) = 1 - P(A) = \frac{399}{400} = 0.9975$.

Also, a person tests positive 85% at the time if they have the virus,

so $P(test\ positive|have\ virus) = P(B|A) = 0.85$ and $P(test\ negative|have\ virus) = P(\bar{B}|A) = 1 - P(B|A) = 0.15$.

And, a person tests positive 5% of the time if they don't have the virus,

so $P(test\ positive|don't\ have\ virus) = P(B|\bar{A}) = 0.05$, and $P(test\ negative|don't\ have\ virus) = P(\bar{B}|\bar{A}) = 1 - P(B|\bar{A}) = 0.95$.

(a) Find $P(have\ virus|test\ positive) = P(A|B)$

We can note that Bayes theorem is just another way to write conditional probability. Since the conditional probability is $P(A|B) = \frac{P(A,B)}{P(B)}$ which equals $\frac{P(A) \cdot P(B|A)}{P(B)}$.

Using

$$P(A|B) = \frac{P(A,B)}{P(B)} = \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|\bar{A}) \cdot P(\bar{A})} \tag{1}$$

by applying product rule to numerator $P(A,B) = P(B|A) \cdot P(A)$

applying sum rule to denominator $\sum_{\alpha \in A} P(B|X = \alpha) \cdot P(X = \alpha)$, where in this case $A = (X = True|have\ virus)$ and $\bar{A} = (X = False|don't\ have\ virus)$.

Substitute the numbers into (1):

$$P(A|B) = \frac{0.85 \times 0.0025}{0.85 \times 0.0025 + 0.05 \times 0.9975} = 0.0409 = 4.09\%$$

Note this is low, primarily because only 1 in 400 people have the virus.

(b) Find $P(don't\ have\ virus|test\ negative) = P(\bar{A}|\bar{B})$

By Bayes theorem we have

$$P(\bar{A}|\bar{B}) = \frac{P(\bar{B}|\bar{A}) \cdot P(\bar{A})}{P(\bar{B})} = \frac{P(\bar{B}|\bar{A}) \cdot P(\bar{A})}{P(\bar{B}|A) \cdot P(A) + P(\bar{B}|\bar{A}) \cdot P(\bar{A})} \tag{2}$$

Substitute values from (2)

$$P(\bar{A}|\bar{B}) = \frac{0.95 \times 0.9975}{0.15 \times 0.0025 + 0.95 \times 0.9975} = 0.9996 = 99.96\%$$

Note this is high, primarily because most people don't have the virus and the probability of testing negative when not having the virus is also high.

More Fun

Let's redo (b) using the tree method.

We have the following from above:

$$P(\bar{B}|A) = 0.15$$
$$P(\bar{B}|\bar{A}) = 0.95$$
$$P(A) = 0.0025$$
$$P(\bar{A}) = 0.9975$$

Let's consider the possible outcomes:

Case 1. test negative and have virus $\bar{B} \cap A$

Case 2. test positive and have virus $B \cap A$

Case 3. test negative and don't have virus $\bar{B} \cap \bar{A}$

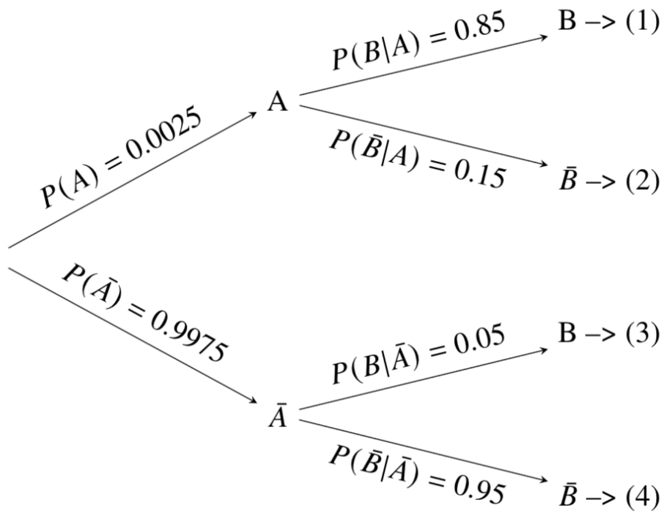
Case 4. test positive and don't have virus $B \cap \bar{A}$

Now we have values of $P(A), P(\bar{A})$, no values of $P(B), P(\bar{B})$. So we draw a tree where the root node branches to A and $\bar{A}$ (primary branch nodes), and the second branches end in B and $\bar{B}$.

The primary branches represent $P(A) = 0.0025$ and $P(\bar{A}) = 0.9975$,

and the secondary branches represent the conditional probabilities $P(B|A) = 1 - P(\bar{B}|A) = 0.85$, $P(\bar{B}|A) = 0.15$, $P(B|\bar{A}) = 1 - P(\bar{B}|\bar{A}) = 0.05$, $P(\bar{B}|\bar{A}) = 0.95$.

So the tree with assigned values:



Branch (1) <-> Case 2: $P(A, B) = P(B|A) \cdot P(A) = 0.002125$

Branch (2) <-> Case 1: $P(A, \bar{B}) = P(\bar{B}|A) \cdot P(A) = 0.000375$

Branch (3) <-> Case 4: $P(\bar{A}, B) = P(B|\bar{A}) \cdot P(\bar{A}) = 0.049875$

Branch (4) <-> Case 3: $P(\bar{A}, \bar{B}) = P(\bar{B}|\bar{A}) \cdot P(\bar{A}) = 0.947625$

Now we can calculate $P(\bar{A}|\bar{B})$ using $\frac{P(\bar{A},\bar{B})}{P(A,\bar{B})+P(\bar{A},\bar{B})} = \frac{0.947625}{0.000375+0.947625} = 0.9996 = 99.96\%$.

Using the same approach you can also solve $P(A|B)$.

Still don’t get it? Watch the videos here: [Addition Law, Multiplication Law and Bayes Theorem (https://www.onlinemathlearning.com/bayes-theorem.html)](https://www.onlinemathlearning.com/bayes-theorem.html)

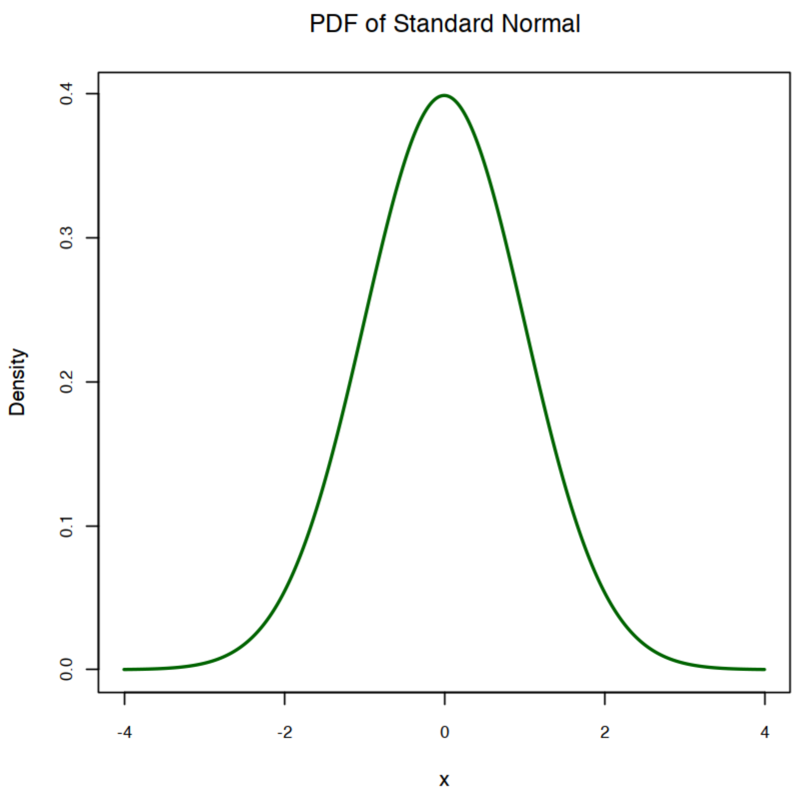
R Questions

Demonstrate PDF, CDF and Quantile plots for the standard normal distribution

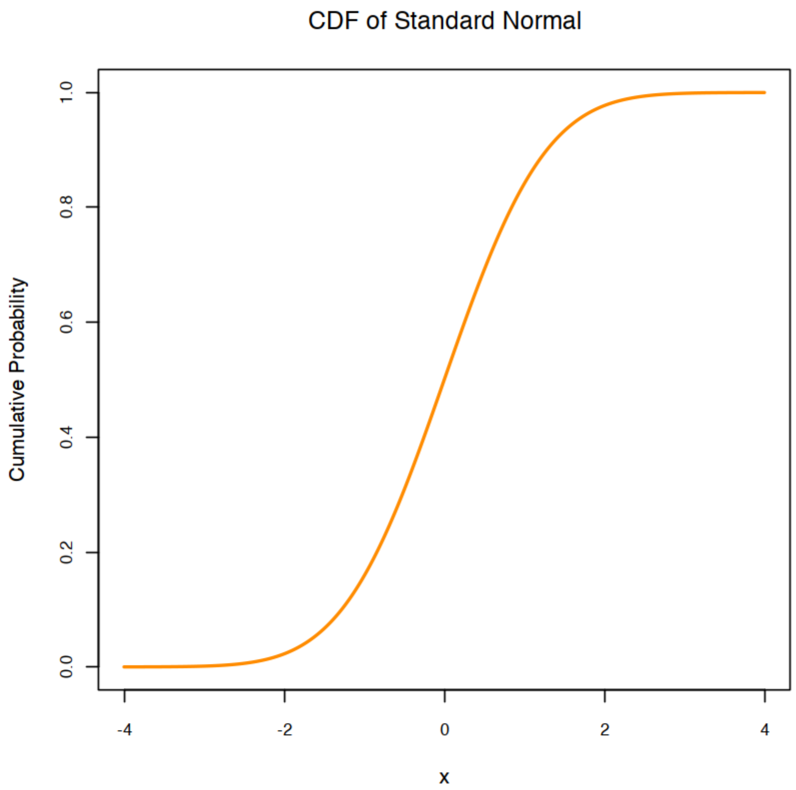
In [1]:
```
xseq<-seq(-4,4,.01)
densities<-dnorm(xseq, 0,1)
cumulative<-pnorm(xseq, 0, 1)

qseq<-seq(0,1,0.01)
quantile=qnorm(qseq,0,1)
```

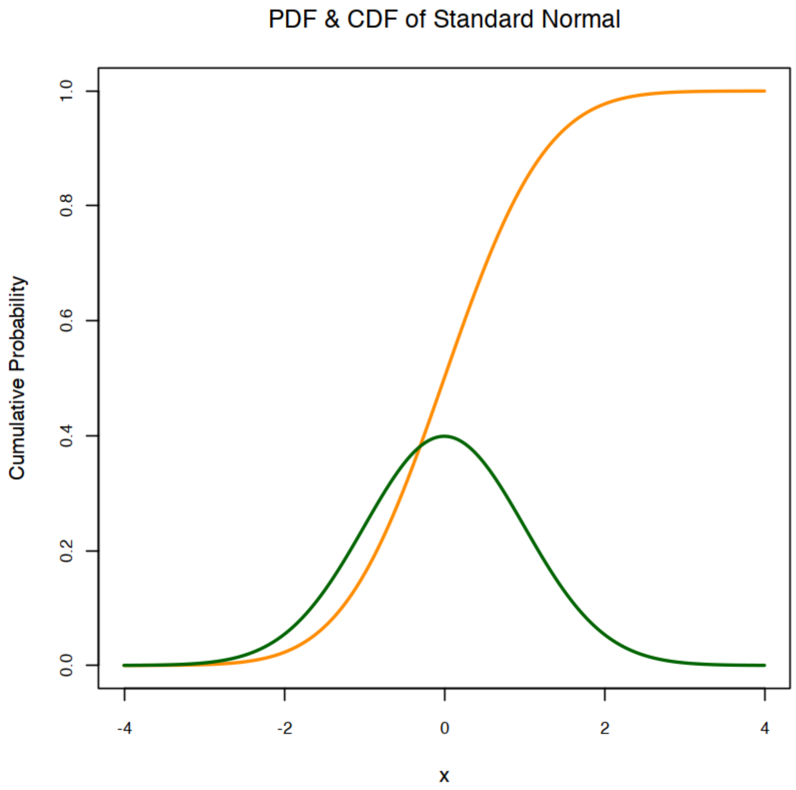
In [2]:
```
plot(xseq, densities, col="darkgreen", xlab="x", ylab="Density", type="l",lwd=2, cex=2, main="PDF of Standard Normal", cex
```



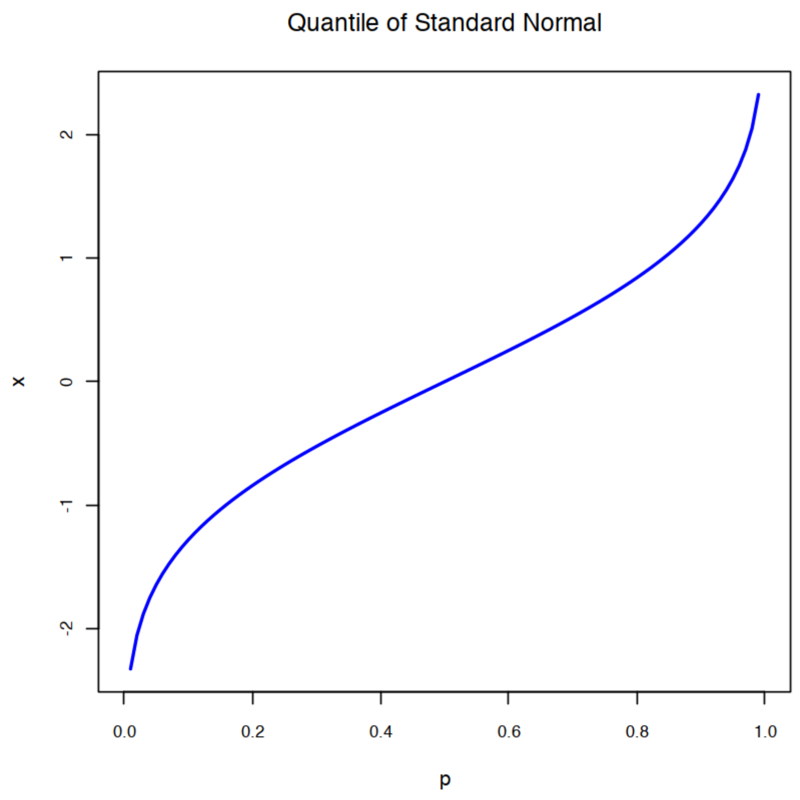
In [3]:
```
plot(xseq, cumulative, col="darkorange", xlab="x", ylab="Cumulative Probability",type="l",lwd=2, cex=2, main="CDF of Stand
```



In [4]:
```
plot(xseq, cumulative, col="darkorange", xlab="x", ylab="Cumulative Probability",type="l",lwd=2, cex=2, main="PDF & CDF of
lines(xseq, densities,col="darkgreen",lwd=2, cex=2)
```



In [5]:
```
plot(qseq, quantile, col="blue", xlab="p", ylab="x",type="l",lwd=2, cex=2, main="Quantile of Standard Normal", cex.axis=.8
```



Create a function in R to calculate and draw CDF from the following PDF:

$$f(x) = \begin{cases} 0.2, & x \in [0, 3) \\ 0.05, & x \in [3, 5) \\ 0.15, & x \in [5, 6) \\ 0.05, & x \in [7, 10) \\ 0, & x \in [10, \infty) \end{cases}$$

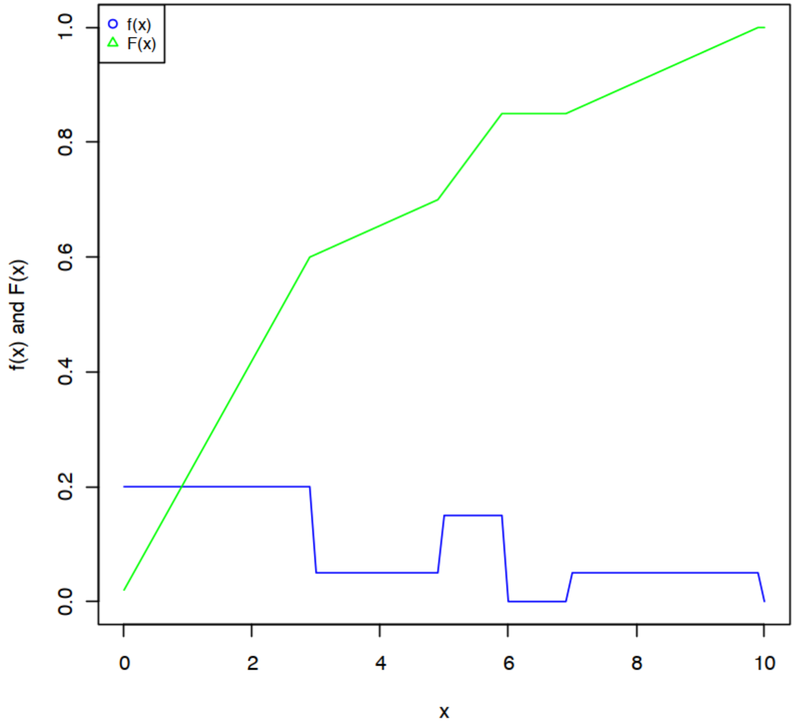
Draw the CDF and PDF in the same plot. Label the axes clearly and provide a legend.

In [6]:
```
fx <- function(x) {
  if(x >= 0 && x < 3) {
    res <-  0.2;
  } else if(x >=3 && x < 5) {
    res <- 0.05;
  } else if(x >= 5 && x < 6) {
    res <-  0.15;
  } else if(x >= 7 && x < 10) {
    res <-  0.05;
  } else {
    res <- 0;
  }
  return(res);
}
```

In [7]:
```
fx <- Vectorize(fx)
dx <- 0.1
x <- seq(0, 10, dx)
Fx <- function(x, dx = 0.1) {
  cumsum(fx(x)*dx)
}

# y1 <- fx(x)
# y2 <- Fx(x, dx)
# plot(x, y1, col="blue", ylim=c(0,1))
# lines(x, y2, col="green")

plot(fx, 0, 10, col="blue", ylim=c(0,1), ylab="f(x) and F(x)")
plot(Fx, 0, 10, col="green", add = T)
legend("topleft",c("f(x)","F(x)"),cex=.8,col=c("blue","green"),pch=c(1,2))
```



Create a function to calculate posterior probability using Bayes theorem.

In [8]:
```
bayes <- function(prob_a, prob_b_given_a, prob_b_given_not_a){
    prob_a_and_b <- prob_b_given_a * prob_a
    prob_not_a <- 1 - prob_a
    prob_b <- (prob_a * prob_b_given_a) + (prob_not_a * prob_b_given_not_a)
    prob_a_given_b <- prob_a_and_b/prob_b
    return(prob_a_given_b)
}
```

Bayes Theorem Question

A certain virus infects one in every 400 people. A test used to detect the virus in a person is positive 85% of the time if the person has the virus and 5% of the time if the person does not have the virus. (This 5% result is called a false positive). Let A be the event "the person has the virus" and B be the event "the person tests positive".

a) Find the probability that a person has the virus given that they have tested positive, i.e. find P(A|B). Round your answer to the nearest hundredth of a percent.

b) Find the probability that a person does not have the virus given that they test negative, i.e. find P(A'|B'). Round your answer to the nearest hundredth of a percent.

In [9]:
```
prob_a <- 1/400
prob_b_given_a <- 0.85
prob_b_given_not_a <- 0.05

prob_a_given_b <- bayes(prob_a, prob_b_given_a, prob_b_given_not_a)
prob_a_given_b
```

0.0408653846153846

In [10]:
```
prob_not_a <- 399/400
prob_not_b_given_not_a <- 0.95
prob_not_b_given_a <- 0.15

prob_not_a_given_not_b <- bayes(prob_not_a, prob_not_b_given_not_a, prob_not_b_given_a)
prob_not_a_given_not_b
```

0.999604430379747

In [ ]: