FIT5047 Second Theory Assignment

Due date: 16th April 2022 (23:55).

Evaluation: 100 marks = 5%.

Submission: Moodle.

1 Propositional Logic - Syntax and Semantics

[5+5+5+5 = 20]

Decide whether the following propositional logic sentences are either valid, satisfiable (but not valid), or unsatisfiable. Justify your answer.

(a) $(A \leftrightarrow B) \wedge (\neg(A \wedge \neg C) \vee (B \leftrightarrow D))$ [5]

(b) $(A \wedge B) \leftrightarrow ((B \rightarrow C) \rightarrow (C \wedge D))$ [5]

Convert the following propositional logic sentences into Conjunctive Normal Form (CNF):

(c) $A \rightarrow ((A \rightarrow B) \vee (B \rightarrow C))$ [5]

(d) $(A \wedge B) \leftrightarrow ((B \rightarrow C) \rightarrow ((C \wedge A) \vee A))$ [5]

2 Propositional Logic - Resolution

[10+10 = 20]

Use resolution-refutation to decide whether or not the following goals can be proved, from the Knowledge Bases given below.

(a) Goal: $B \rightarrow \neg A$ [10]

$1 : C \rightarrow B$

$2 : \neg(B \vee (\neg C \wedge D))$

$3 : (C \rightarrow D) \wedge (D \rightarrow C)$

$4 : \neg A \rightarrow (D \leftrightarrow C)$

(b) Goal: $\neg A \rightarrow B$ [10]

$1 : (B \rightarrow \neg C) \wedge \neg(A \rightarrow B)$

$2 : (B \vee \neg C) \rightarrow (C \wedge A)$

$3 : (C \wedge (\neg C \vee D)) \rightarrow D$

$4 : (D \wedge B) \leftrightarrow (A \rightarrow D)$

3 Propositional Logic - Forward/Backward Chaining

[2+2+2+4+10 = 20]

This question is about Horn clauses as well as Forward and Backward chaining.

Goal: H

R1: $G \rightarrow A$

R2: $(A \wedge B \wedge C) \rightarrow D$

R3: $(E \wedge B) \rightarrow C$

R4: $D \rightarrow H$

R5: $(C \wedge H) \rightarrow F$

R6: $G \rightarrow B$

G

E

Apply forward and backward reasoning to the Horn clauses above to prove the goal.

For Forward chaining, after each rule application, show the values to be inserted in the columns named:

(a) AGENDA [2]

(b) COUNT [2]

(c) INFERRED [2]

in the table provided below, in the way seen in the lectures. Also,

(d) show the search graph resulting from the application of the rules. [4]

AGENDA	**COUNT**						**INFERRED**
	R1	**R2**	**R3**	**R4**	**R5**	**R6**	

For Backward chaining:

(e) provide a sequence of rules that can be used to prove the goal. [10]

4 First Order Logic - Substitution and Unification

[6+6+4+4 = 20]

In the following exercises, assume that $x, y, w, z, \ldots$ are variables, $A, B, C, D, \ldots$ are constants, $f, g, h, \ldots$ are functions, and $P_1, P_2, P_3 \ldots$ are predicate symbols.

(a) Calculate the composition of substitutions $s_1 s_2$ and $s_2 s_1$, given the two substitutions $s_1 = \{y|w, z|A\}$ and $s_2 = \{w|C, y|z\}$ [6]

(b) Apply the composed substitutions $s_1 s_2$ and $s_2 s_1$ of part (a) to $P_1(f(w), g(y), z)$, that is, calculate predicates $P_1(f(w), g(y), z)s_1 s_2$ and $P_1(f(w), g(y), z)s_2 s_1$. [6]

The following questions are about unification.

Do these expressions unify? If they do, give their mgu. And, if they do not unify, then indicate where and why unification fails.

(c) $P_1(x, f(x, y), g(x, w))$ and $P_1(A, f(w, B), g(w, x))$ [4]

(d) $P_1(x, f(x), A, A)$ and $P_1(y, f(A), y)$ [4]

5 First Order Logic - Resolution

[11+9 = 20]

Use resolution to check whether the goal "G" below can be proved. To do so, show:

(a) how you construct all necessary clauses to apply resolution, and [11]

(b) which most general unifiers are used (in case a proof can be found). [9]

$$\text{G} : \exists x, y, w \left(\neg\Big(\big(S(w) \wedge Q(y)\big) \leftrightarrow \big(P(x) \rightarrow S(w)\big)\Big)\right)$$

$$1 : \forall x, z \left(\big(\forall y\ (Q(y) \rightarrow \neg R(z))\big) \wedge \big(\forall y\ (P(x) \rightarrow Q(y))\big)\right)$$

$$2 : \forall z \left(\big(\forall y\ (Q(y) \vee \neg R(z))\big) \rightarrow \big(\forall x\ (R(z) \wedge P(x))\big)\right)$$

$$3 : \exists x, y\ \big(P(x) \vee Q(y)\big)$$