Quiz Week 4 - distributions - Questions

FIT5197 teaching team

Question 1

Given the following sample of weights $w \in \{66, 54, 69, 77, 55, 97, 79, 52, 56, 60, 64, 102\}$ and assuming the random variable defining the weights is normally distributed, i.e. $W \sim N(\mu, \sigma^2)$

(a). estimate the mean μ using the sample mean.

(b). estimate the variance σ^2 using the sample variance.

(c). using the estimate of the mean and variance from (a) and (b) to determine $P(36 < w < 100)$

Question 2

You toss an unfair coin 11 times where the chance of tossing a head is 0.6. What is the combined probability of getting exactly 5 or exactly 6 heads?

Question 3

Two cafes are side by side and are open 7 days a week.

(a) The first cafe sells cappucinos at $5 each and has 30 customers a day. Half the customers buy a cappucino, the rest don't. What is the distribution of dollar sales made by the first cafe in a week?

(b) The second cafe sells cafe lattes at $6 each. Customers arrive at a rate of 60 a day and all the customers buy a cafe latte. What is the distribution of dollar sales made by the second cafe in a week?

(c) Now the Poisson distribution is approximately Gaussian when the rate is larger, and this applies moderately well when the rate is 15 or more. Give Gaussian approximations to the distributions of weekly dollar sales for each of the two cafes.

Question 4

The continuous version of the uniform distribution has the following probability density function:

$$p(x|a,b) = \begin{cases} 0, & x < a \\ \frac{1}{b-a}, & a \leq x \leq b \\ 0, & x > b \end{cases}$$

Show that

$$E[X] = \frac{a+b}{2}$$

and that

$$V[X] = \frac{(b-a)^2}{12}$$

Question 5

Suppose that a binary message—either 0 or 1—must be transmitted by electromagnetic transmission from location A to location B. However, the data sent over the transmission channel are subject to a channel noise disturbance and so to reduce the possibility of error, the value 2 is sent over the channel when the message is 1 and the value -2 is sent when the message is 0. If x is the value sent at location A then R, the value received at location B, is given by $R = x + N$, where N is the channel noise disturbance. When the message is received at location B, the receiver decodes it according to the following rule: \ if $R \geq .5$, then 1 is concluded \ if $R < .5$, then 0 is concluded \ (a) Assuming the channel noise N follows a standard normal distribution, determine the error probabilities of incorrectly determining a 0 if a 1 was sent and incorrectly determining a 1 if a 0 was sent, i.e. $P(\text{error}|1 \text{ was sent})$ and $P(\text{error}|0 \text{ was sent})$.

(b) Now consider that an additional Gaussian noise source, M, with mean $\mu = 0$ and standard deviation $\sigma = 2$ is influencing the channel such that $R = x + N + M$. What are the error probabilities, $P(\text{error}|1 \text{ was sent})$ and $P(\text{error}|0 \text{ was sent})$, under these conditions? \

R questions

1. Create an R program to visualise with clear labelling the following distributions using the given R functions:

1. Gaussian Distribution with mean equal to 5 and standard deviation equal to 2 => Compute and display the PDF using dnorm and the CDF using pnorm
2. Bernoulli Distribution with probability of success equal to 0.5 => Compute and display the approximate PMF using rbinom to simulate drawing m=100,000 samples from the Bernoulli distribution
3. Binomial Distribution with n=20 trials and probability of success equal to 0.3 => Compute and display the approximate PMF using rbinom to simulate drawing m=100,000 samples from the Binomial distribution and the exact PMF using dbinom.
4. Poisson Distribution with rate parameter equal to 5 => Compute and display the approximate PMF using rpois to simulate drawing m=100,000 samples from the Poisson distribution and the exact PMF using dpois.

2. Suppose that IQ scores have a bell-shaped distribution with a mean of 100 and a standard deviation of 15.

1. What percentage of people should have an IQ score between 85 and 115?
2. What percentage of people should have an IQ score between 70 and 130?
3. What percentage of people should have an IQ score of more than 130?
4. A person with an IQ score greater than 145 is considered a genius. Does the empirical rule support this statement? Explain