Quiz Week 3 - Expectation - Questions

FIT5197 teaching team

Question 1

X is a discrete random variable over $\chi \in \{1, 2, 3\}$ with probability mass function $P(X = 1) = 0.5,\ P(X = 2) = 0.4,\ P(X = 3) = 0.1$

(a) What is $E[\ln X]$?

(b) What is $E\left[\ln\left(\frac{1}{P(X)}\right)\right]$?

Question 2

Consider the triangular distribution $p(x) = 1 - |x|$ defined on the interval $x \in [-1, 1]$ (Tip: draw $p(x)$)

(a) What is $E[X]$?

(b) What is $E[X^2]$?

Question 3

$P(X = 1) = 0.5, P(X = 2) = 0.4, P(X = 3) = 0.1$

What is $V[X]$?

Question 4

Show $V[X] = E[X^2] - E[X]^2$

Question 5

$P(X = x,\ Y = y)$ is defined as

	X = 1	X = 2	X = 3
Y = 1	0.05	0.15	0.1
Y = 2	0.25	0.15	0.3

What is $cov(X, Y)$?

Question 6

The wealth of an individual is random variable with probability density function:

$$f(x) = \frac{C}{x^{\alpha+1}},\ x\ \in [2, \infty),\ \alpha > 1$$

Furthermore you are given the following integral:

$$\int_2^{\infty} \frac{1}{x^n} = \begin{cases} \infty, & if\ n \leq 1 \\ \frac{2^{1-n}}{n-1}, & if\ n > 1 \end{cases}$$

(a) What is the value of C to make the distribution normalise to 1?

(b) What is the mean of x?

Question 7

Let $E[Z] = 1$ and $E[Z^2] = 6$, $E[Y] = -2$ and $E[Y^2] = 5$, and Z and Y are independent, then what is $V[3Z + 2Y]$?

Question 8

In a lottery a four-digit number is chosen at random from the range 0000-9999. A lottery ticket costs $2. You win $50 if your ticket matches the last two digits but not the last three, $500 if your ticket matches the last three digits but not last four, and $5,000 if your ticket matches all four digits. What is the expected payoff on a lottery ticket? How much money does the lotto make on average per ticket sold?

R hackers mini power punch challenge

Using numerical integration in R with the built-in 'integrate' function, calculate the expected value of the mean and variance for a normal distribution with mean 2 and standard deviation 2.