1 Hierarchical models: data-analysis problems

1.1 Math tests

1.

2. Rewrite the distributions in terms of precision

$$(y_{ij}|\theta_i, \omega) \sim N(\theta_i, (\omega)^{-1})$$

$$(\theta_i|\omega, \lambda) \sim N(\mu, (\omega\lambda)^{-1})$$

Choose prior for parameters

$$\omega \sim \Gamma(\frac{d}{2}, \frac{\eta}{2})$$

$$\lambda \sim \Gamma(\frac{h}{2}, \frac{h}{2})$$

The joint distribution of everything is, suppose n data are grouped into m groups:

$$\text{constant} \times \omega^{n+m}\lambda^m e^{-\frac{\omega\sum_{ij}(y_{ij}-\theta_i)^2}{2} - \frac{\omega\lambda\sum_i(\theta_i-\mu)^2}{2} - \frac{\omega\eta}{2} - \frac{h\lambda}{2}} \omega^{\frac{d}{2}-1}\lambda^{\frac{h}{2}-1}$$

From which we have, suppose each group have g_i elements

$$(\omega|y, \lambda, \theta, \mu) \sim \Gamma(\frac{d}{2}+n+m, \frac{\eta}{2} + \frac{\sum_{ij}(y_{ij}-\theta_i)^2}{2} + \frac{\lambda\sum_i(\theta_i-\mu)^2}{2})$$

$$(\lambda|y, \omega, \theta, \mu) \sim \Gamma(\frac{h}{2}+m, \frac{h}{2} + \frac{\omega\sum_i(\theta_i-\mu)^2}{2})$$

$$(\theta_i|y, \omega, \lambda, \mu) \sim N(\frac{\lambda\mu + \sum_j y_{ij}}{\lambda + g_i}, \frac{1}{\omega(\lambda + g_i)})$$

$$(\mu|y, \omega, \lambda, \theta) \sim N(\frac{\sum_i \theta_i}{m}, \frac{1}{m\omega\lambda})$$

We will update the parameter according to these distribution. For the code, see mathtest.r.

3. See mathtest.r