

1 Hierarchical models: data-analysis problems

1.1 Math tests

- 1.
2. Rewrite the distributions in terms of precision

$$(y_{ij}|\theta_i, \omega) \sim N(\theta_i, (\omega)^{-1})$$

$$(\theta_i|\omega, \lambda) \sim N(\mu, (\omega\lambda)^{-1})$$

Choose prior for parameters

$$\omega \sim \Gamma\left(\frac{d}{2}, \frac{\eta}{2}\right)$$

$$\lambda \sim \Gamma\left(\frac{h}{2}, \frac{h}{2}\right)$$

The joint distribution of everything is, suppose n data are grouped into m groups:

$$\text{constant} \times \omega^{n+m} \lambda^m e^{-\frac{\omega \sum_{ij} (y_{ij} - \theta_i)^2}{2} - \frac{\omega \lambda \sum_i (\theta_i - \mu)^2}{2} - \frac{\omega \eta}{2} - \frac{h \lambda}{2}} \omega^{\frac{d}{2}-1} \lambda^{\frac{h}{2}-1}$$

From which we have, suppose each group have g_i elements

$$(\omega|y, \lambda, \theta, \mu) \sim \Gamma\left(\frac{d+n+m}{2}, \frac{\eta}{2} + \frac{\sum_{ij} (y_{ij} - \theta_i)^2}{2} + \frac{\lambda \sum_i (\theta_i - \mu)^2}{2}\right)$$

$$(\lambda|y, \omega, \theta, \mu) \sim \Gamma\left(\frac{h+m}{2}, \frac{h}{2} + \frac{\omega \sum_i (\theta_i - \mu)^2}{2}\right)$$

$$(\theta_i|y, \omega, \lambda, \mu) \sim N\left(\frac{\lambda \mu + \sum_j y_{ij}}{\lambda + g_i}, \frac{1}{\omega(\lambda + g_i)}\right)$$

$$(\mu|y, \omega, \lambda, \theta) \sim N\left(\frac{\sum_i \theta_i}{m}, \frac{1}{m\omega\lambda}\right)$$

We will update the parameter according to these distribution. For the code, see mathtest.r.

3. See "mathtest\mathtest.r"

1.2 Price elasticity of demand