Define

$$S_{1\lambda}(y) = \arg\ \min_{\theta \geq 0} \frac{1}{2}\theta^2 - (y-\lambda)\theta$$

$$S_{2\lambda}(y) = \arg\ \min_{\theta \leq 0} \frac{1}{2}\theta^2 - (y+\lambda)\theta$$

Then $S_\lambda(y)$ is one of $S_{1\lambda}(y)$ and $S_{2\lambda}(y)$ which gives the smaller target function. Assuming $\lambda > 0$. When $y > \lambda$,

$$S_{1\lambda}(y) = y - \lambda$$

$$S_{2\lambda}(y) = 0$$

$$\frac{1}{2}S_{1\lambda}(y)^2 - (y-\lambda)S_{1\lambda}(y) - \frac{1}{2}S_{2\lambda}(y)^2 + (y+\lambda)S_{2\lambda}(y) = -\frac{1}{2}(y-\lambda)^2 < 0$$

So $S_\lambda(y) = y - \lambda$

If $-\lambda < y < \lambda$

$$S_{1\lambda}(y) = 0$$

$$S_{2\lambda}(y) = 0$$

So $S_\lambda(y) = 0$

If $y < \lambda$

$$S_{1\lambda}(y) = 0$$

$$S_{2\lambda}(y) = y + \lambda$$

$$\frac{1}{2}S_{1\lambda}(y)^2 - (y-\lambda)S_{1\lambda}(y) - \frac{1}{2}S_{2\lambda}(y)^2 + (y+\lambda)S_{2\lambda}(y) = \frac{1}{2}(y+\lambda)^2 > 0$$

So $S_\lambda(y) = y + \lambda$