We want to solve the lasso problem

$$\min_x \frac{1}{2}|y-x|_2^2 + \frac{\lambda}{2}|Dx|_1$$

Written in ADMM form

$$\min_{x,z} \frac{1}{2}|y-x|_2^2 + \frac{\lambda}{2}|z|_1$$

$$\text{subject to } Dx = z$$

The augumented Lagrangian

$$L_{\text{aug}} = \frac{1}{2}|y-x|_2^2 + \frac{\lambda}{2}|z|_1 + \frac{\rho}{2}(Dx-z)^2 + \alpha^T(Dx-z)$$

ADMM solution update

$$v^k = z^k - u^k$$

$$x^{k+1} = \arg\min_x \frac{1}{2}|x-y|_2^2 + \frac{\rho}{2}|Dx - v^k|_2^2 = x^{k+1} = (I + \rho D^T D)^{-1}(y + \rho D^T v^k)$$

$$w^k = Dx^{k+1} + u^k$$

$$z^{k+1} = \arg\min_z \frac{\lambda}{2}|z| + \frac{\rho}{2}|z - w^k|_2^2 = y^{k+1} = S_{\frac{\lambda}{2\rho}}(w^k)$$

$$u^{k+1} = u^k + Dx^{k+1} - z^{k+1}$$