We maintain an array "lastupdate" which records the last time (the number of iteration) each component of β is updated. Suppose at time t_c we visit a component of β, say β_i, we compare the current number of iteration with the value of lastupdate, which we will denote by t_l. If there is a difference, we will check the value of β_i. There are three possibilities.

1. If $\beta_i = 0$, we do not need to do anything.

2. Otherwise, assume for now that during all the updates we missed, β_i doesn't change sign. Then we can approximate all the updates we missed (do not include the update we want to do in current time t_c) by an integral:

$$
\begin{aligned}
& \pm \sum_{j=t_l}^{t_c} \frac{\lambda}{\sqrt{\lambda^2 (j - t_l + 1) + H_{t_l}}} \\
= \quad & \pm \lambda \frac{1}{\lambda^2} \int_{\lambda^2}^{(t_c - t_l + 1)\lambda^2} \frac{dx}{\sqrt{x + H}} \\
= \quad & \pm \frac{2}{\lambda} \left(\sqrt{(t_c - t_l + 1)\lambda^2 + H} - \sqrt{\lambda^2 + H} \right) \qquad (1)
\end{aligned}
$$

   Where $H_t = \sum_i^t g_i^2$.

   There is a special case where we only missed less than 2 update, in this case the integral is not a good approximation. We will update β_i in a normal way.

3. If during the updates we missed, β_i changes sign. That means that lasso "thinks" this component should not take a big value. Therefore we will take assert that $\beta_i = 0$.

However in practice we will not be able to differentiate second and third case. We will compute eq (1) and compare it with β_i.

To summarize the above in a pseudocode

Input: t_l,t_c,β_i,λ,H_{t_l} Return: β_i, H_{t_c}, $t_l = t_c - 1$
If $\beta_i = 0$ or $t_c - t_l <= 1$, return β_i, $H_{t_c} = H_{t_l}$ Else if $t_c - t_l - 1 <= 2$, use normal lasso update procedure to update β_i and H_{t_l} and return β_i, H_{t_l} Else Compute Δ =eq (1) If $\lvert\beta_i\rvert > \lvert\Delta\rvert$, return $\beta_i = sgn(\beta_i)(\lvert\beta_i\rvert - \lvert\Delta\rvert)$, $H_{t_l} = H_{t_c} + (t_c - t_l - 1)\lambda^2$. Else return $\beta_i = 0$, $H_{t_l} = H_{t_c}$