Here we give the ADMM algorithm for fitting the lasso problem:

$$\min_\beta \frac{1}{2n}(X\beta - y)^2 + \lambda|\beta|$$

We frame this question in ADMM way:

$$\min_{\beta,\alpha} \frac{1}{2n}(X\beta - y)^2 + \lambda|\alpha|$$

$$\text{under condition } \beta - \alpha = 0$$

We write the augmented Lagrangian

$$\frac{1}{2n}(X\beta - y)^2 + \lambda|\alpha| + y^T(\beta - \alpha) + \frac{\rho}{2}(\beta - \alpha)^2$$

The algorithm follows

$$\beta^{k+1} = \arg\min_\beta \frac{1}{2n}(X\beta - y)^2 + \frac{\rho}{2}(\beta - v^k)^2$$

$$\alpha^{k+1} = \arg\min_\alpha \lambda|\alpha| + \frac{\rho}{2}(\alpha - w^k)^2$$

$$u^{k+1} = u^k + \beta^{k+1} - \alpha^{k+1}$$

$$v^k = \alpha^k - u^k$$

$$w^k = \beta^{k+1} + u^k$$

Update for both β and α are solvable problems

$$\beta^{k+1} = \arg\min_\beta \frac{1}{2n}(X\beta - y)^2 + \frac{\rho}{2}(\beta + v^k)^2 = (\frac{1}{n}X^TX + \rho I)^{-1}(\frac{1}{n}X^Ty + \rho v^k)$$

$$\alpha^{k+1} = \arg\min_\alpha \lambda|\alpha| + \frac{\rho}{2}(\alpha - w^k)^2 = S_{\frac{\lambda}{\rho}}(w^k)$$