Tutorial - 4

Sol 1- $T(n) = 3T(n/2) + n^2$

$a = 3, b = 2 \quad f(n) = n^2$

a and b are constant and f(n) is +ve func

∴ Using master's theorem-

$c = \log_b a$

$= \log_2 3 = 1.58$

$n^c = n^{1.58}$

ca. $n^2 > n^{1.58}$

case 3:

$\boxed{T(n) = \Theta(n^2)}$

Sol 2- $T(n) = 4T(n/2) + n^2$

$a = 4 \quad b = 2 \quad f(n) = n^2.$

a and b are constant and f(n) is positive func.

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

Page No ______ Date

$n^c = n^2$

$n^c = f(n)$

case 2: $\boxed{T(n) = \Theta(n^2 \log n)}$

Sol 3 - $T(n) = T(n/2) + 2^n$

$a = 1, \; b = 2, \; f(n) = 2^n$

using master's theorem

$c = \log_b a = \log_2 1$

$c = 0$

$n^c = n^0 = 1$

$f(n) > c$

$\boxed{T(n) = \Theta(2^n)}$

Sol 4 - $T(n) = 2^n T(n/2) + n^2$

$a = 2^n, \; b = 2, \; f(n) = n^n$

a is not constant.

Master's theorem is not applicable.

Sol 5 - $T(n) = 16\, T(n/4) + n$

$a = 16, \; b = 4, \; f(n) = n.$

$c = \log_b a = \log_4 16 = \log_4 4^2 = 2.$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1:

$\boxed{T(n) = \Theta(n^2)}$

Sol 6 - $T(n) = 2T(n/2) + n\log n$

$a = 2 \quad b = 2 \quad f(n) = n\log n$

$c = \log_b a = \log_2 2 = 1$

Page No. ____ Date ____

$n^c = n^1 = n.$

$f(n) > n^c$

case 3:

$\boxed{T(n) = \theta(n \log n}$

Sol 7- $T(n) = 2T(n/2) + n/\log n.$

$a = 2,\ b = 2,\ f(n) = n/\log n.$

$c = \log_b a = \log_2 2 = 1.$

$n^c = n^1 = n.$ $\quad \because$ non-polynomial difference b/w $f(n)$ & n^c

$\therefore$ Master theorem is not applicable.

Sol 8- $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.5}$

$f(n) > n^c$

$\therefore$ case 3

$\Rightarrow \boxed{T_n = \Theta(n^{0.51})}$

Sol 9- $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5 \cdot b = 2 \quad f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable.

Sol 10- $T(n) = 16\,T(n/4) + n!$

$a = 16,\ b = 4,\ f(n) = n!$

$c = \log_b a = \log_4 16 = 2\log_4 4 = 2.$

$n^c = n^2$

$\because f(n) > n^c \quad \Rightarrow \boxed{T(n) = \Theta(n!)}$

Sol 12 – $\sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}$ $\quad b = 2$ $\quad f(n) = \log n$.

$\because a$ is not constant

$\therefore$ Master's theorem is not applicable.

Sol 13 – $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$ $\quad b = 3$ $\quad f(n) = \sqrt{n}$

$c = \log_b a = \log_3 3 = 1$.

$\because n^c = n^1 = n$.

$\therefore$ case 1

$\Rightarrow \boxed{T(n) = \Theta(n)}$

Sol 14 – $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$.

$c = \log_b a = \log_2 3 = 1.58$

$n^c = n^{1.58}$

$f(n) < n^c$

case 1:

$\Rightarrow \boxed{T(n) = \Theta(n^{1.58})}$

Sol 15 – $T(n) = 4T(n/2) + c \cdot n$

$a = 4$, $b = 2$, $f(n) = c \cdot n$.

$c = \log_b a = \log_2 4 = 2\log_2 2 = 2$.

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1

$\Rightarrow \boxed{T(n) = \Theta(n^2)}$

Sol 16 – $T(n) = 3T(n/4) + n\log n$

$a = 3$ $\quad b = 4$ $\quad f(n) = n\log n$

$c = \log_b a = \log_4 3 = 0.79$.

$n^c = n^{0.79}$

$\because f(n) > n^c$

$\therefore$ case 3

$T(n) = \Theta(n \log n)$

Page No.
Date

Sol 18 – $T(n) = 6T(n/3) + n^2 \log n$.

$a = 6$. $b = 3$ $f(n) = n^2 \log n$.

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$\because f(n) > n^c$ $\therefore$ case 3 $\Rightarrow$ $\boxed{T(n) = \Theta(n^2 \log n)}$

Sol 19 – $T(n) = 4T(n/2) + n/\log n$

$a = 4$ $b = 2$ $f(n) = n/\log n$

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2$

$n^c = n^2$

$\because f(n) < n^2$

$\therefore$ case 1 : $\Rightarrow$ $\boxed{T(n) = \Theta(n^2)}$

Sol 20 – $T(n) = 64T(n/8) - n^2 \log n$

$\because f(n)$ is –ve func.

$\therefore$ Master's theorem is not applicable.

Sol 22 – $T(n) = T(n/2) + n(2 - \cos n)$

$\because f(n)$ is not regular func.

$\therefore$ Master's theorem is not applicable.