Tutorial – 2

Q1.
```
void func (int n)
{
  int j=1, i=0;
  While (i<n)
  { i = i+j;
    j++;
  }
}
```

i	j
0	1
1	2
3	3
6	4
10	5

$= 0, 1, 3, 6 \ldots$

$n = 0+1+2+3+\ldots k$

$n = \frac{k(k+1)}{2} \Rightarrow n \cong k^2$

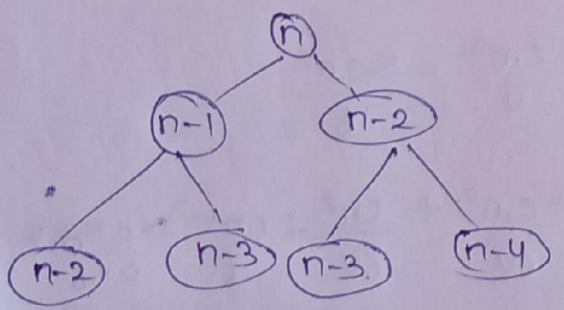
$k \cong \sqrt{n}$

Time complexity $= O(\sqrt{n})$

Sol2 – Recurrence relation

$T(n) = T(n-1) + T(n-2) + 1$

using recurrence tree method –



— 1

— 2

— 2^2

— 2^n

Time-complexity $= 1+2+\ldots 2^n$

$= \frac{1(2^{n+1}-1)}{2-1}$

$= 2^{n+1}-1$

T.C. $= O(2^n)$

Space complexity = Space complexity of fiboraci series using recursion is proportional to height of recurrence tree.

$= O(n)$

Sol 3 1) $n\log n$ — for (i to n)

```
{ for(j=1; j<=n; j*=2)
    O(1)
}
```

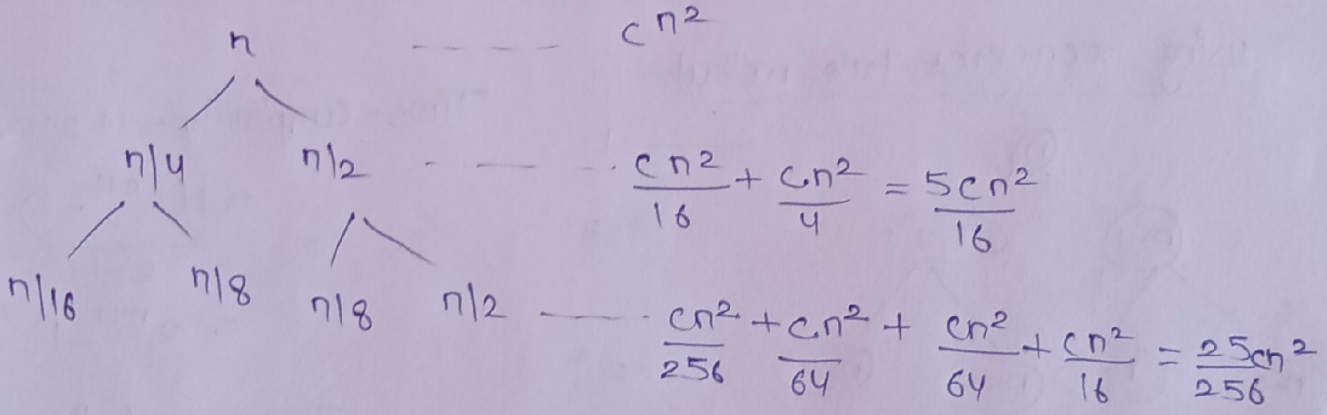
2) n^3 — for(i to n)

```
{ for(j to n)
  { for (k to n)
    { O(1)
    }
  }
}
```

3) $\log(\log n)$ —

```
int i=n
while (i>0)
{ i = √i;
}
```

Sol 4 — $T(n) = T(n/4) + T(n/2) + cn^2$



$$T(n) = c\left(n^2 + \frac{5n^2}{16} + \frac{25n^2}{256} + \dots\right)$$

$r = 5/16$, $Sn = 1/(1-r)$

$$T(n) = cn^2\left(1 + \frac{5}{16} + \frac{25}{256} + \dots\right)$$

$$= cn^2 \cdot (1/(1-5/16)) = O(n^2)$$

Sol 5 — int fun(int n){

```
for(int i=1; i<=n; i++){
    for(int j=1; j<n; j+=i)
        { O(1)
        }
}
}
```

i	j	times
1	1 to n	(n-1)
2	1 to n	(n-1)/2
3	1 to n	(n-1)/3
⋮	⋮	
n	1 ⟶ n	(n-1)/n
		nlogn

T.C = O(nlogn)

Sol 6 — for(int i=2; i<=n; i=pow(i,k)

```
{
  O(1)
}
```

$i = 2, 2^k, 2^{k^2}, 2^{k^3} \ldots 2^{k^x}$

$n = 2^{k^x}$

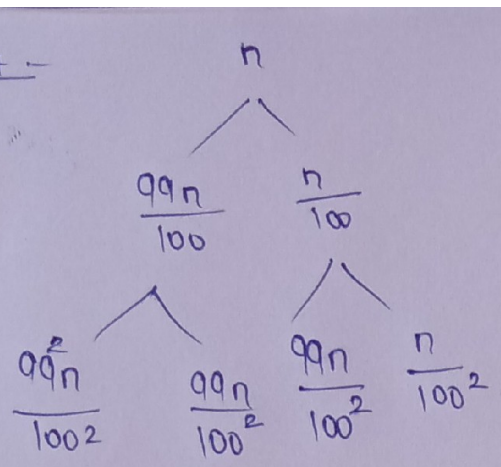
$\log n = k^x \log 2$

$\frac{\log(\log n)}{\log 2} = x \log k$

$x = \frac{\log \log n}{\log 2 * \log k}$

T.C = O(log(logn))

Sol 7 -



Taking longest branch $= \frac{99n}{100}$

$T.C = \log \frac{100n}{99}$

$T.C \simeq \log n$

$= O(\log n)$

$$n = \left(\frac{99}{100}\right)^k$$

$$k = \log\left(\frac{100n}{99}\right)$$

$$T(n) = n\left(\log \frac{100}{99}\right)^n / 100$$

$$= O(n \log_{99} n)$$

Sol 8 - i) $1 < \log\log n < \log n < \sqrt{n}\ (\text{root}(n)) < n < n\log n < n^2 < 2^n < 2^{2^n} < 4^n < n!$

ii) $1 < \log\log n < \sqrt{(\log n)} < \log n < \log 2n < 2\log n < n < 2n < 4n < n\log n < n^2 < \log(n!) < 2^{2^n} < n!$

iii) $96 < \log_8 n < \log_2 n < 5n < n\log(n) < n\log_8 n < 8n^2 < 7n^3 < \log n! < 8^{2^n} < n!$