MASTER'S THEOREM

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If $f(n) \in \theta(n^d)$ or $f(n) = c^* n^d$ where $d \geq 0$ in recurr-ence $T(n) = aT(n/b) + f(n)$ then

$$T(n) \in \begin{cases} \theta(n^d) & , \text{ if } (a < b^d) \\ \theta(n^d \log n) & , \text{ if } (a = b^d) \\ \cancel{\theta(n^{\log_b a})} & \\ \theta(n^{\log_b a}) & , \text{ if } (a > b^d) \end{cases}$$

(1) $T(n) = 8T(n/2) + 1000n^2$

$a = 8$, $b = 2$, $f(n) = 1000n^2 = c^* n^d$

$\therefore d = 2$

$b^d = 2^2 = 4$

Hence $a > b^d$

$\therefore T(n) \in \theta(n^{\log_b a})$

$\log_b a = \log_2 8$

$= 3$

$\therefore T(n) \in \theta(n^3)$

(2) $T(n) = 2T(n/2) + n^2$

$a = 2$, $b = 2$, $d = 2$

$b^d = 2^2 = 4$

Hence $a < b^d$

$\therefore T(n) \in \theta(n^d)$

$\Rightarrow T(n) \in \theta(n^2)$

(3) $T(n) = 2T(n/2) + 10n$

$a = 2$, $b = 2$, $d = 1$

$b^d = 2^1 = 2$

Hence $a = b^d$

$\therefore T(n) = \theta(n^d \log n)$

$\Rightarrow T(n) = \theta(n \log n)$