Time Series Analysis Project

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Forecasting volatility of stock market using adaptive Fuzzy-GARCH model.

What is GARCH?

Autoregressive conditional heteroskedasticity (ARCH) is the condition that one or more data points in a series for which the variance of the current error term or innovation is a function of the actual sizes of the previous time periods' error terms: often the variance is related to the squares of the previous innovations.

If an autoregressive moving average model (ARMA model) is assumed for the error variance, the model is a **generalized autoregressive conditional heteroscedasticity**(**GARCH)** model.

The GARCH model uses prior conditional variances to estimate the degree of transmission of volatility; it is characterized
by a fat tail and excess kurtosis.

GARCH Model

the GARCH (*p*, *q*) model (where *p* is the order of the GARCH terms

$$l_t = \mu_t + a_t = \mu_t + \sigma_t \epsilon_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{t-i} + a_t + \sum_{j=1}^{q} b_j a_{t-j}$$

$$\sigma_t^2 = \gamma_0 + \sum_{i=0}^{r} \gamma_i a_{t-i}^2 + \sum_{j=1}^{s} \delta_j \sigma_{t-j}^2$$

where
$a_t = \sigma_t \epsilon_t$
and ϵ_t is iid(0,1)

So the model can be rewritten as

$$l_t = \mu_t + a_t = \mu_t + \sigma_t \epsilon_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i r_{t-i} + \sum_{j=1}^{q} b_j a_{t-j} + \sigma_t \epsilon_t$$

Importance

Why Fuzzy GARCH?

Stock market performance is time-varying and nonlinear, and exhibits properties of clustering.

Certain large changes tend to follow other large changes, and in general small changes tend to follow other small changes.

Methods of fuzzy modeling are promising techniques for describing complex dynamic systems.

Thus the method of functional fuzzy systems to analyze the clustering in the case of a GARCH model.

Fuzzy GARCH

$Rule^{(l)}$: IF $x_1(t)$ is F_{l1} and $\cdots$ and $x_n(t)$ is F_{ln}, THEN

$$h(t) = \alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j)$$

$$y(t) = \sqrt{h(t)}\varepsilon(t), \text{ for } l = 1, 2, \ldots, L$$

where $y(t)$ is output of system, F_{lj} for $j = 1, \ldots, n$ is the fuzzy set, L is the number of IF-THEN rules, and $x_1(t), x_2(t), \ldots, x_n(t)$ are the premise variables.

$$y(t) = \sqrt{h(t)}\varepsilon(t)$$
$$= \left\{ \sum_{l=1}^{L} g_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j) \right] \right\}^{0.5} \varepsilon(t)$$

where

$$u_l(x(t)) = \prod_{j=1}^{n} F_{lj}(x_j(t))$$

$$g_l(x(t)) = \frac{u_l(x(t))}{\sum_{l=1}^{L} u_l(x(t))}$$

$$x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$$

and where $F_{lj}(x_j(t))$ is the grade of membership of $x_j(t)$ in F_{lj}.

$$u_l(x(t)) = \prod_{j=1}^{n} F_{lj}(x_j(t)) = \prod_{j=1}^{n} \exp\left(-\frac{1}{2} \left(\frac{x_j(t) - c_{lj}}{\sigma_{lj}} \right)^2 \right)$$

where c_{lj}, and σ_{lj} are, respectively, the center and the spread

What is GA?

GA is a method for optimizing machine learning algorithms inspired by the processes of natural selection and genetic evolution

GA applies operators to a population of binary strings that encode the parameter space.

A parallel global search technique emulates natural genetic operators such as reproduction, crossover, and mutation.

At each generation, the algorithm explores different areas of the parameter space and then directs the search to the region where there is a high probability of finding improved performance.

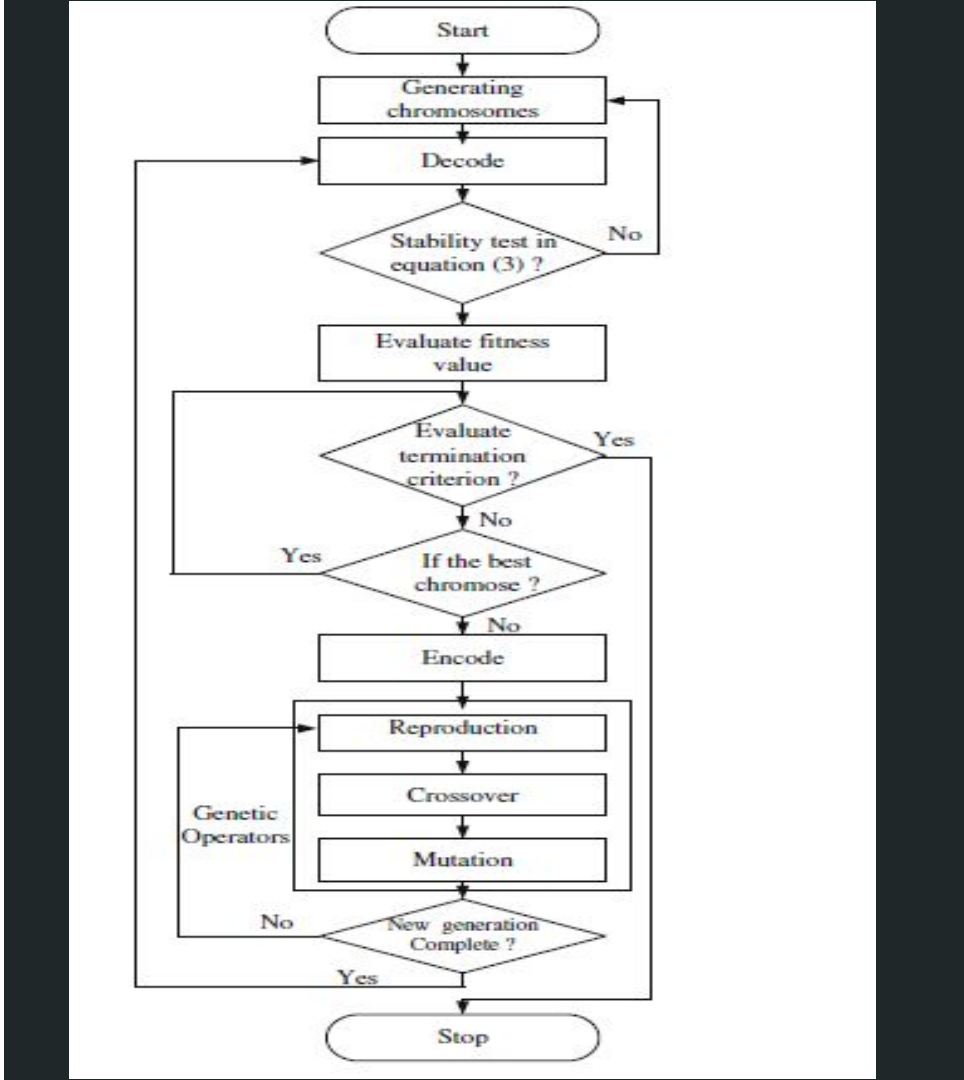
Why GA?

The process of optimizing functional fuzzy systems and GARCH model parameters is highly complex and nonlinear.

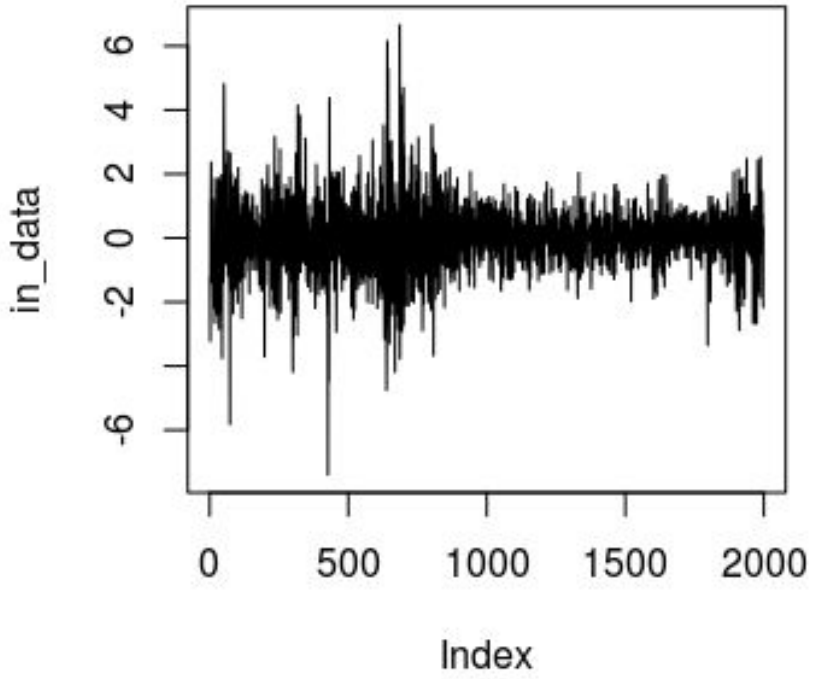
A GA-based parameter estimation algorithm is proposed to derive the optimal solution for the fuzzy GARCH model.

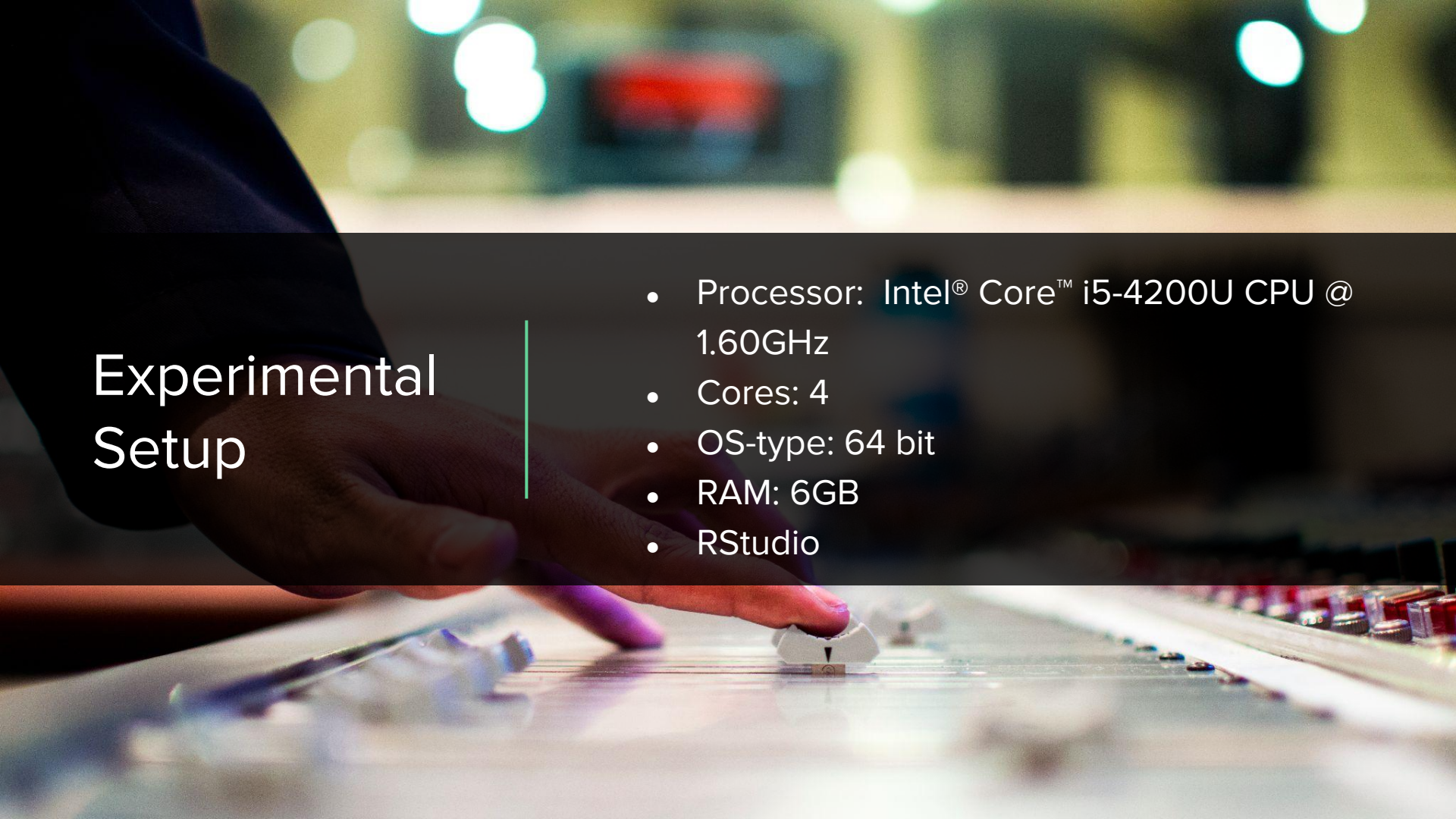
GA is a method for optimizing machine learning algorithms inspired by the processes of natural selection and genetic evolution

Flow


Start
Generating chromosomes
Decode
Stability test in equation (3) ?
No
Evaluate fitness value
Evaluate termination criterion ?
Yes
No
Yes
If the best chromose ?
No
Encode
Reproduction
Crossover
Genetic Operators
Mutation
No
New generation Complete ?
Yes
Stop

Stock Market Da





Experimental Setup

- Processor: Intel® Core™ i5-4200U CPU @ 1.60GHz
- Cores: 4
- OS-type: 64 bit
- RAM: 6GB
- RStudio

Fuzzy GARCH Parameters

Parameter	**Range/value**
alpha0	0-1
alpha1	0-1
beta1	0-1
L	5

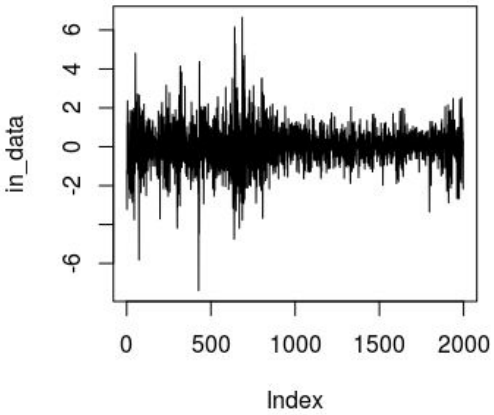
GA Parameters

Parameter	Value
Population	30
Max Iterations	100
P-Crossover	0.8
P-Mutation	0.1

Comparison

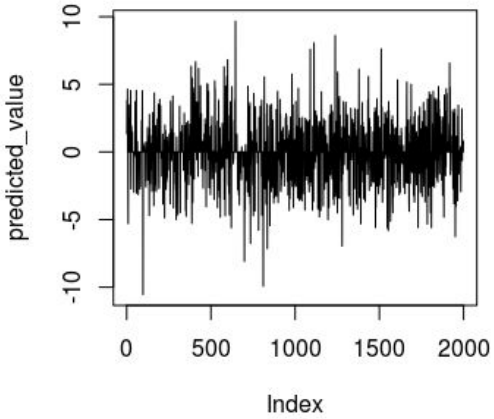
	FUZZY GARCH USING GA	**GARCH**
RSE-Root Squared Error	6302.962	11053

Actual Data



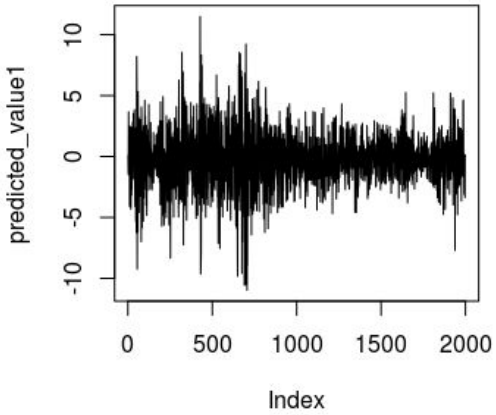
Mean: 0.00810018
Range:-7.396247 to 6.660540

Fuzzy GARCH using GA



Mean:0.01674563
Range: -10.334832 to 9.674232

GARCH



Mean: -0.07577152
Range: -9.595869 to 11.137105

Thank You