Expert Systems with Applications 36 (2009) 11710–11717

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

A fuzzy GARCH model applied to stock market scenario using a genetic algorithm

Jui-Chung Hung *

Department of Information Technology, Ling Tung University, No. 1, Ling-Tung Road, Taichung 408, Taiwan, ROC

ARTICLE INFO

Keywords:
GARCH model
Fuzzy systems
Genetic algorithm

ABSTRACT

In this paper, we derive a new application of fuzzy systems designed for a generalized autoregression conditional heteroscedasticity (GARCH) model. In general, stock market performance is time-varying and nonlinear, and exhibits properties of clustering. The latter means simply that certain large changes tend to follow other large changes, and in general small changes tend to follow other small changes. This paper shows results from using the method of functional fuzzy systems to analyze the clustering in the case of a GARCH model.

The optimal parameters of the fuzzy membership functions and GARCH model are extracted using a genetic algorithm (GA). The GA method aims to achieve a global optimal solution with a fast convergence rate for this fuzzy GARCH model estimation problem. From the simulation results, we have determined that the performance is significantly improved if the leverage effect of clustering is considered in the GARCH model. The simulations use stock market data from the Taiwan weighted index (Taiwan) and the NASDAQ composite index (NASDAQ) to illustrate the performance of the proposed method.

1. Introduction

In analyzing time-dependent data, it is often the case that the conditional variances are not consistent with the assumption of homogeneity that is commonly associated with traditional econometrics models, especially those which treat financial data (Arciniegas & Rueda, 2008; Chan, 2002; Fama, 1965; Tsay, 2002). Mandelbrot (1963) discovered that conditional variance plays a role is in the phenomenon of volatility clustering. Volatility clustering means that large changes tend to follow other large changes, and small changes tend to follow small changes. Because of this phenomenon, Mandelbrot thought that the variance might change over time, that is, it would not be constant or homogeneous. Therefore, Engle (1982) proposed the autoregression conditional heteroscedasticity (ARCH) construct. Engle believed that conditional variances led to assumption of homogeneity. However, this approach proved impractical. He subsequently adopted a model in which conditional variances in time-dependent data were subject to influences from previous unexpected factors. Furthermore, he assumed that the conditional variances were functions of error terms, allowing them to change over time. He proposed that the ARCH Model could solve the biases and therefore address traditional econometrics models.

Building on Engle's ARCH(q) model, Bollerslev (1986) made use of the Autoregressive Moving Average (ARMA) model to introduce the GARCH model. The GARCH model uses prior conditional variances to estimate the degree of transmission of volatility; it is characterized by a fat tail and excess kurtosis. Its ability to explain the transmission of volatility is a key advantage of this approach. For these reasons, the GARCH model is frequently used to explore the returns and transmissions of volatility in time-dependent financial data sets. However, financial assets are easily impacted by both positive and negative information, and the impacted are existed the asymmetric. The GARCH model does not recognize transmissions of volatility that derive from the input of positive or negative information. Therefore, this model is not appropriate when the market is asymmetric.

To address this issue, other researchers introduced various asymmetric GARCH models. The GJR GARCH model was proposed by Glosten, Jagannathan, and Runkle (1993), while the exponential GARCH (EGARCH) was put forward by Nelson (1991). These models suggest that the negative relation between volatility and stock prices can be understood by the fact that an increase in unexpected volatility will increase the expected future volatility (assuming persistence). Some of these effects can be captured by modifications of linear models, but others demand nonlinear approaches. Unfortunately, because of their complexity, nonlinear models are in very limited use today.

During the past two decades, fuzzy systems including Sugeno systems (Takagi & Sugeno, 1985) and Mamdani systems are capable of approximating a wide range of functions. The Sugeno or functional fuzzy systems have recently found extensive application in a wide variety of industrial systems and consumer products and have attracted the attention of many control researchers due to their unique model-independent approach (Lee & Chen, 2008; Liu

* Tel.: +886 4 23892088; fax: +886 4 36002535.
E-mail address: juichung@seed.net.tw

0957-4174/$ - see front matter © 2009 Elsevier Ltd. All rights reserved.
doi:10.1016/j.eswa.2009.04.018

& Wang, 2007; Lee & Shin, 2003; Savran, 2007; Treesatayapun & Uatrongjit, 2005). This paper proposes a new class of GARCH models that are based on functional fuzzy systems. Methods of fuzzy modeling are promising techniques for describing complex dynamic systems. Combining the ease of implementation and convenience of linear models with an ability to capture complex system correlations, we propose that fuzzy models could also be a judicious choice for analyzing dynamic processes that feature time-dependent variances. In this paper, we combine GARCH models and functional fuzzy systems and we apply these new models to real-world financial markets using GA. The process of optimizing functional fuzzy systems and GARCH model parameters is highly complex and nonlinear. A GA-based parameter estimation algorithm is proposed to derive the optimal solution for the fuzzy GARCH model.

GA is a method for optimizing machine learning algorithms inspired by the processes of natural selection and genetic evolution (Goldberg, 1989; Crefensteet, 1986; Holland, 1962). GA applies operators to a population of binary strings that encode the parameter space. A parallel global search technique emulates natural genetic operators such as reproduction, crossover, and mutation. At each generation, the algorithm explores different areas of the parameter space and then directs the search to the region where there is a high probability of finding improved performance. Because GA simultaneously evaluates many points in a parameter space, it is more likely to ultimately converge on the global solution. In particular, there is no requirement that the search space is differentiable or continuous, and the algorithm can iterate several times on each data point. Accordingly, it is a very suitable approach for time-varying nonlinear functions (Zhou & Khotanzad, 2007).

To use the genetic algorithm in the problem of fuzzy GARCH model parameter estimation, the relevant variables are first coded into a binary string called a chromosome. In each generation, three basic genetic operators (reproduction, crossover, and mutation) are performed to generate a new population with a constant population size. The chromosomes that remain after the population is reduced by the principle of survival of the fittest produce a better population candidate solution. The convergence of the proposed GA estimation scheme can be guaranteed via the theorem of the schema discussed in Holland (1975) and in Toroslu (2007). The estimation parameter that is obtained by the proposed estimation scheme ultimately converges to the optimal or near-optimal solution.

The rest of this paper is organized as follows. The next section describes the problem. Thereafter, section three presents the details of the fuzzy GARCH model. The fourth section discusses the proposed GA-based optimization of the fuzzy GARCH system. Experimental results that illustrate the effectiveness of the proposed method are provided in the fifth section. Conclusions are in the final section of the paper.

2. Problem description

Consider a ARCH(q) model that is defined as (Engle, 1982)

$$\begin{aligned} y(t) &= a(t) \\ a(t) &= \sqrt{h(t)}\varepsilon(t) \\ h(t) &= \alpha_0 + \sum_{i=1}^{q} \alpha_i a^2(t-i) \end{aligned} \tag{1}$$

where $y(t)$ is a random variable representing certain stock market data, $\varepsilon(t)$ is a zero mean and unit variance white noise random process, $h(t)$ is the conditional variance of $\varepsilon(t), t$ is the time index, and α_0, α_i are nonnegative; $\alpha_0 > 0, \alpha_i \geqslant 0$.

Bollerslev (1986) modified the conditional variance term in the ARCH(q) model, by assuming that the conditional variances are influenced not only by the squared error terms, but also by previous conditional variances. He incorporated previous conditional variances into the process for estimating transmission of volatility. The result was his proposed GARCH(p,q) model. The model is defined as

$$\begin{aligned} y(t) &= a(t) \\ a(t) &= \sqrt{h(t)}\varepsilon(t) \\ h(t) &= \alpha_0 + \sum_{i=1}^{q} \alpha_i a^2(t-i) + \sum_{j=1}^{p} \beta_j h(t-j) \end{aligned} \tag{2}$$

where $\alpha_0, \alpha_i,$ and β_j are unknown parameters that must be estimated. Without loss of generality, we assume

$$\begin{aligned} &\alpha_0 > 0, \alpha_i \geqslant 0; \quad i = 1,2,\ldots,q;\ q > 0 \\ &\beta_j \geqslant 0; \quad j = 1,2,\ldots,p;\ p > 0 \\ &\sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \end{aligned} \tag{3}$$

In generally, the GARCH model can produce relatively exact results for different financial data sets. Accordingly, the conventional GARCH model has long been considered the most popular instrument for analyzing time-dependent data that features time-varying conditional variance. However, the model does not perform well when the market features clustering behavior. Ignoring this fact can lead to poor prognostic characteristics. Consequently, we adopt functional fuzzy systems to propose a new fuzzy GARCH model.

3. Fuzzy GARCH model

Fuzzy logic systems are universal approximations that can uniformly estimate nonlinear continuous functions with arbitrary accuracy. The functional fuzzy model is a piecewise interpolation of several models that operates by means of membership functions. The fuzzy model is described by IF-THEN rules and will be employed here ensure the GARCH model can appropriately deal with the cluster problem. The l-th rule of the functional fuzzy system for GARCH is described by
$Rule^{(l)}$: IF $x_1(t)$ is F_{l1} and $\cdots$ and $x_n(t)$ is F_{ln}, THEN

$$\begin{aligned} h(t) &= \alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j) \\ y(t) &= \sqrt{h(t)}\varepsilon(t),\ \text{for } l = 1,2,\ldots,L \end{aligned} \tag{4}$$

where $y(t)$ is output of system, $F_{lj}$for$j = 1,\ldots,n$ is the fuzzy set, L is the number of IF-THEN rules, and $x_1(t), x_2(t), \ldots, x_n(t)$ are the premise variables. It is challenging to provide universal recommendations for choosing the set of explanatory variables for a fuzzy rule system (4). In this paper we used what is arguable a "natural" definition for them, namely the previous values of the time series

$$x_i(t) = y(t-i),\ \text{for } i = 1,2,\ldots,n \tag{5}$$

As seen in (5), the fuzzy model successfully captures the leveraging effects, which is resulted from the sign of the previous time series values.

The fuzzy system is inferred can be written as follows

$$\begin{aligned} h(t) &= \frac{\sum_{l=1}^{L} u_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j)\right]}{\sum_{l=1}^{L} u_l(x(t))} \\ &= \sum_{l=1}^{L} g_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j)\right] \end{aligned}$$

$$y(t) = \sqrt{h(t)}\varepsilon(t) = \left\{ \sum_{l=1}^{L} g_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j) \right] \right\}^{0.5} \varepsilon(t) \tag{6}$$

where

$$u_l(x(t)) = \prod_{j=1}^{n} F_{lj}(x_j(t))$$
$$g_l(x(t)) = \frac{u_l(x(t))}{\sum_{l=1}^{L} u_l(x(t))} \tag{7}$$
$$x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]$$

and where $F_{lj}(x_j(t))$ is the grade of membership of $x_j(t)$ in F_{lj}. In this paper we use a Gaussian membership function,

$$u_l(x(t)) = \prod_{j=1}^{n} F_{lj}(x_j(t)) = \prod_{j=1}^{n} \exp\left(-\frac{1}{2} \left(\frac{x_j(t) - c_{lj}}{\sigma_{lj}} \right)^2 \right) \tag{8}$$

where c_{lj}, and σ_{lj} are, respectively, the center and the spread of the l-th rule membership function corresponding to the j-th premise variable.

We assume

$$u_l(x(t)) \geqslant 0 \text{ and } \sum_{l=1}^{L} u_l(x(t)) > 0, \text{ for } l = 1, 2, \ldots, L$$

Therefore, we get

$$g_l(x(t)) \geqslant 0, \text{ for } l = 1, 2, \ldots, L \text{ and } \sum_{l=1}^{L} g_l(x(t)) = 1 \tag{9}$$

From Eqs. (6)–(9), we can express the output of the system as

$$y(t) = \sqrt{h(t)}\varepsilon(t) = \left\{ \frac{\sum_{l=1}^{L} u_l(x(t)) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j) \right]}{\sum_{l=1}^{L} u_l(x(t))} \right\}^{0.5} \varepsilon(t)$$
$$= \left\{ \frac{\sum_{l=1}^{L} \prod_{j=1}^{n} \exp\left(-\frac{1}{2} \left(\frac{x_j(t)-c_{lj}}{\sigma_{lj}} \right)^2 \right) \left[\alpha_0^l + \sum_{i=1}^{q} \alpha_i^l y^2(t-i) + \sum_{j=1}^{p} \beta_j^l h(t-j) \right]}{\sum_{l=1}^{L} \prod_{j=1}^{n} \exp\left(-\frac{1}{2} \left(\frac{x_j(t)-c_{lj}}{\sigma_{lj}} \right)^2 \right)} \right\}^{0.5} \varepsilon(t) \tag{10}$$

The fuzzy GARCH model in (10) is a general nonlinear time-varying equation of a type that has been used to model the behaviors of complex nonlinear dynamic system (Hung, 2007). A reasonable success criterion is to minimize the mean square error, the objective function of which is defined as

$$E = \sum_{i=1}^{N} [y(i) - \hat{y}(i)]^2 \tag{11}$$

where $y(i)$ for $i = 1, 2, \ldots, N$ represents data from the stock markets data, N is the number of data points, and $\hat{y}(i)$ is output of the fuzzy GARCH model in (10). Clearly, E is a highly nonlinear function of the center and spread of c_{lj}, and σ_{lj} for $l = 1, \ldots, L$ and $j = 1, \ldots, n$, the coefficients α_i^l for $l = 1, \ldots, L$ and $i = 0, \ldots, q$, and β_j^l for $l = 1, \ldots, L$ and $j = 1, \ldots, p$. This function may have several local minima. It is in general very difficult to find the global minimum of E in (11) using conventional methods. Therefore, in this paper, we will use GA to specify the center and spread of c_{lj}, σ_{lj} for $l = 1, \ldots, L$ and $j = 1, \ldots, n$, the coefficients α_i^l for $l = 1, \ldots, L$ and $i = 0, \ldots, q$, and β_j^l for $l = 1, \ldots, L$ and $j = 1, \ldots, p$ to solve the fuzzy GARCH problem in (11). In other words, we use a GA to search the parameter set $\boldsymbol{\theta} = (c_{11}, \ldots, c_{Ln}, \sigma_{11}, \ldots, \sigma_{Ln}, \alpha_0^1, \ldots, \alpha_q^L, \beta_1^1, \ldots, \beta_p^L)$ to minimize $E(\theta)$ in (11) from a global point of view.

4. GA-based fuzzy GARCH model

The GA is composed of three operations: (1) reproduction, (2) crossover, and (3) mutation. These operations are implemented by performing the basic tasks of copying strings, exchanging portions of strings, and changing the state of bits from 1's to 0's or vice versa. These operations ensure that the "fittest" members of the population survive and their information is preserved and combined to generate still better offspring. The result is an

Table 1
Search spaces and resolution of GA.

Function	Parameter numbers	Pattern lower bound L_i	Pattern upper bound K_i	Resolution R_i
α_i^l	$L \cdot (q+1)$	0	1	10^{-7}
β_j^1	$L \cdot p$	0	1	10^{-5}
c_{1j}	$L \cdot n$	−7%	7%	0.01%
σ_{1j}	$L \cdot n$	0	7%	0.01%

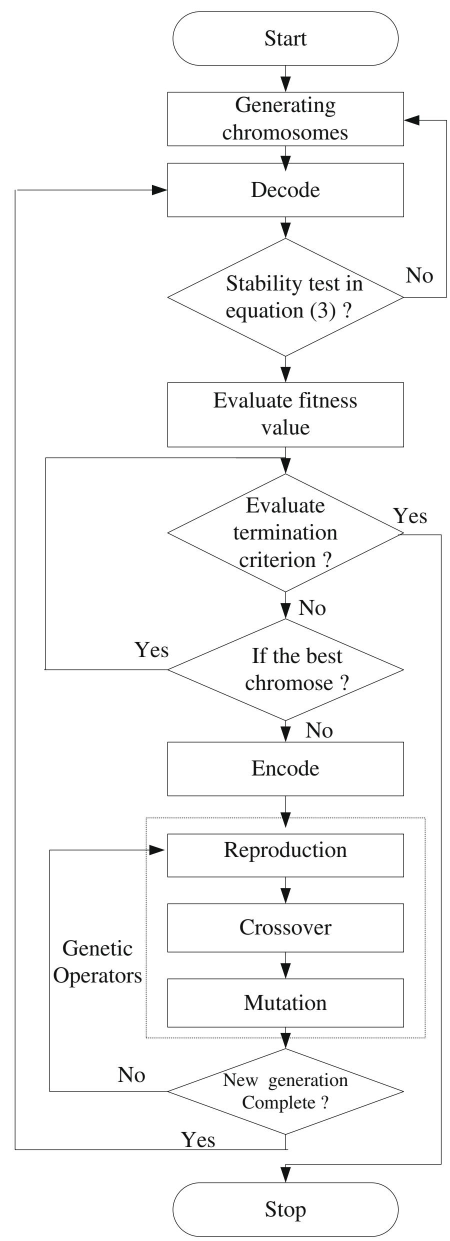

Fig. 1. Flowchart of the GA-based parameter estimation algorithm.

improvement in the next generation's performance. We describe the genetic algorithm in the following subsections (Goldberg, 1989).

4.1. Coding

We use GA to solve the problem of identifying the global minimum of the cost function $E(\boldsymbol{\theta})$. GA operates with a population of strings or chromosomes, not with the parameters themselves. Hence, to solve our problem, the parameter vector θ must first be coded into a string. For convenience and simplicity, the binary coding method is chosen. Consistent with this method, every element θ_i for $i = 1, 2, \ldots, L \times (2n + p + q + 1)$ of the parameter vector θ is coded as a string of length b_i, which consists of 0's and 1's. The choice of the bit number (b_i) for each parameter depends on the desired resolution R_i (see Table 1). For simplicity, with binary coding, the desired resolution can be calculated as

$$R_i = \frac{K_i - L_i}{2^{b_i} - 1}. \tag{12}$$

where K_i and L_i are the upper and lower bounds of the parameter θ_i. In our situation, $\boldsymbol{\theta}$ is coded as the following binary string,

$$\boldsymbol{\theta} = \overbrace{\underbrace{1\cdots11}_{c_{11}}}^{b_1}\cdots\overbrace{\underbrace{1\cdots00}_{\beta_p^L}}^{b_{L(2n+p+q+1)}} \tag{13}$$

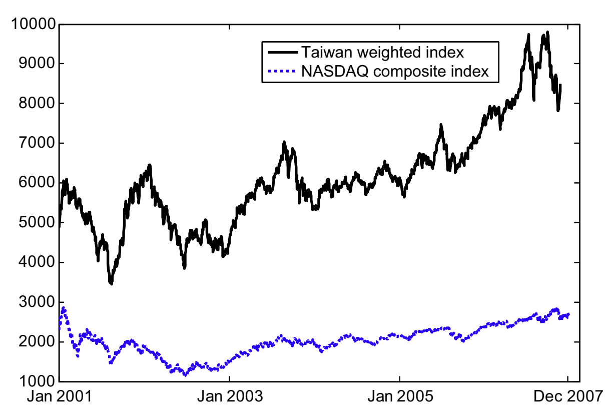

Fig. 2. Daily closing stock-price index for the Taiwan weighted index and the NASDAQ composite index.

4.2. Fitness function

The degree of fitness depends on the performance of the possible solution represented by that particular string. In our design problem, finding the minimum of $E(\theta)$ is equivalent to obtaining a maximum fitness value in the genetic searching process. Intuitively, we have a fitness function that is proportional to the cost function

$$Fit(\boldsymbol{\theta}) \propto \frac{1}{E(\boldsymbol{\theta})} \tag{14}$$

where $Fit(\boldsymbol{\theta})$ represents the fitness function. A number of methods (also known as fitness techniques) are appropriate in the context of this mapping. In this paper the relation between $Fit(\theta)$ and $E(\boldsymbol{\theta})$ be expressed as a linear equation as

$$Fit(\boldsymbol{\theta}) = Fit_w + \frac{Fit_b - Fit_w}{E_{\min} - E_{\max}}(E(\boldsymbol{\theta}) - E_{\max}) \tag{15}$$

where $E_{\min}$ and $E_{\max}$ denote the minimum and maximum values of the cost function $E(\boldsymbol{\theta})$, respectively. Fit_b and Fit_w are the prescribed best and worst fitness values, respectively. The three aforementioned operations are employed in GA to search for the global optimal solution (i.e., survival of the fittest) in (11) without becoming trapped at local minima (Crefensteet, 1986; Kouchakpour, Zaknich, & Braűnl, 2007).

4.3. Reproduction

Reproduction is a process by which individual strings are copied and placed in a mating pool for further genetic operations consistent with their fitness value. The probability of the *i*-th string with fitness value $Fit(\boldsymbol{\theta}_i)$ being selected for mating and reproduction in the next generation is:

$$PR_i = \frac{Fit_i}{\sum_{i=1}^{P} Fit_i} \tag{16}$$

where P is the population size specified by the designer (Kouchakpour et al., 2007). Once the strings are reproduced or copied for possible use in the next generation in a mating pool, they wait for the action of the other two operators, crossover and mutation.

4.4. Crossover

Crossover provides a mechanism for strings to mix and match desirable qualities through a random process. After reproduction,

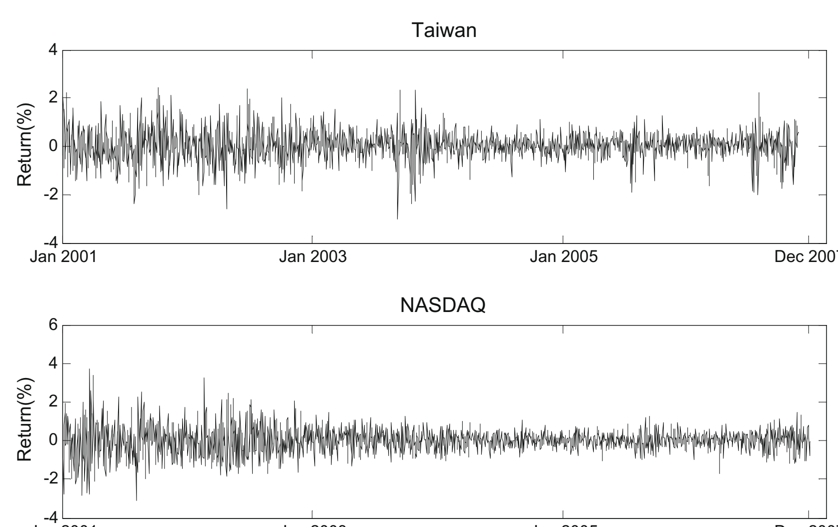

Fig. 3. Daily return.

simple crossover proceeds in three steps. First, two newly-reproduced strings are selected from the mating pool, created by reproduction in the previous generation. Second, a position that includes the two strings is selected at random. The third step involves exchanging all the characters by following the crossing sit. When combined with reproduction, crossover is an effective means of exchanging information and combining various elements high-quality solutions.

4.5. Mutation

Reproduction and crossover yield the bulk of the processing power of genetic algorithms. However, the third operation, mutation, enhances the ability of genetic algorithms to search for the optimal solution. Mutation is the occasional flip of each bit of a chromosome from 1 to 0, or vice versa. Mutation should be used sparingly, as it is a random search operator and, at high mutation

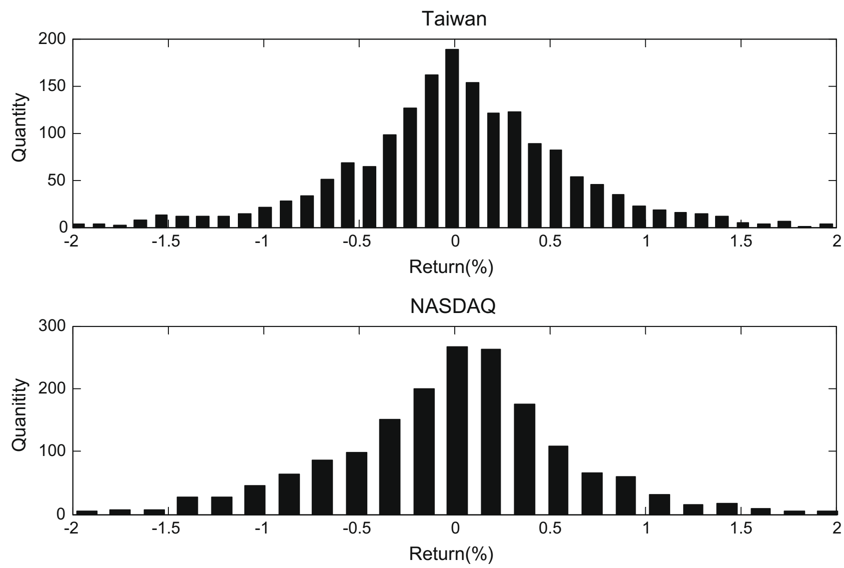

Fig. 4. The probability distribution for the Taiwan weighted index and the NASDAQ composite index.

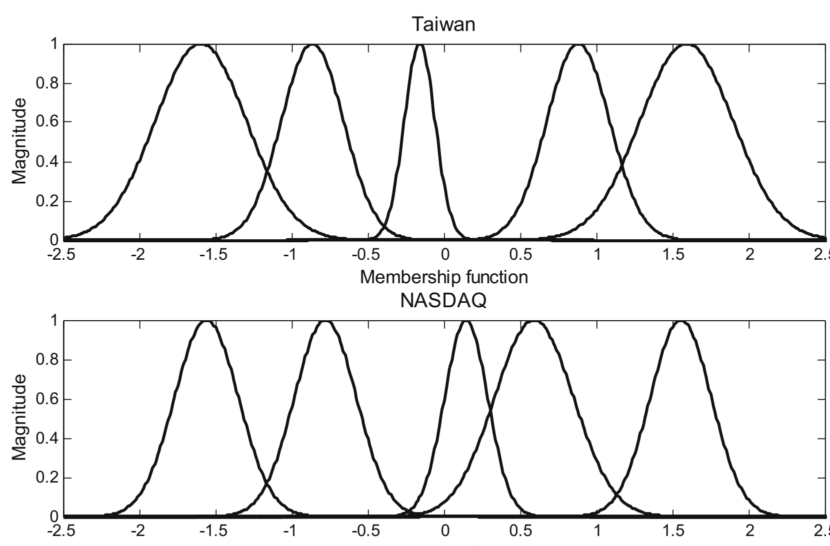

Fig. 5. The membership function for the Taiwan weighted index and the NASDAQ composite index.

Table 2
The center and spread of the membership function as evaluated using a genetic-based algorithm for the Taiwan weighted index and the NASDAQ composite index.

		$l = 1$	$l = 2$	$l = 3$	$l = 4$	$l = 5$
Taiwan	Center $(c_{11}, \ldots c_{51})$	−1.5965	−0.8637	−0.1598	0.8812	1.5918
	Spread $(\rho_{11}, \ldots, \rho_{51})$	0.3006	0.2047	0.1012	0.2064	0.3059
NASDAQ	Center $(c_{11}, \ldots, c_{51})$	−1.559	−0.7787	0.1457	0.5912	1.5543
	Spread $(\rho_{11}, \ldots, \rho_{51})$	0.21	0.2041	0.1416	0.2592	0.2006

rates, the algorithm degrades to a random search (Crefensteet, 1986).

4.6. Design procedure

Based on the aforementioned analysis, the design procedure for the fuzzy GARCH model was divided into the following steps:

Step 0: input the received data $y(k)$.
Step 1: generate random population of P chromosomes.
Step 2: decode each string into the corresponding parameter vector.
Step 3: use the stability test in (3) to specify the parameter domain of the coefficients of the GARCH model. If this fails, renew the chromosome.
Step 4: calculate the fitness values according to (15).
Step 5: keep the best chromosomes intact for the next generation.
Step 6: use reproduction, crossover, and mutation to generate new chromosomes in the next generation.

Repeat the procedures from Step 2 to Step 6 until a suitable parameter set is finally arrived at obtained. The flowchart of the design procedure for the fuzzy GARCH model is presented in Fig. 1.

5. Simulation

Our data set comprised daily closing values for two stock indices over the period 1 January 2001 through 31 December 2007. We chose the Taiwan weighted index and the NASDAQ composite index (Fig. 2). We generated daily stock returns by taking the logarithmic difference of the daily stock index and multiplying it by 100, i.e., $y(t) = 100 \times (\log S(t) - \log S(t-1))$, where $S(t)$ is the daily closing price at time t. The logarithmic difference data of the Taiwan weighted index and of the NASDAQ composite index are given in Fig. 3.

Fig. 3 also shows these indexes in the context of volatility clustering as characterized by Fama (1965). According to research by

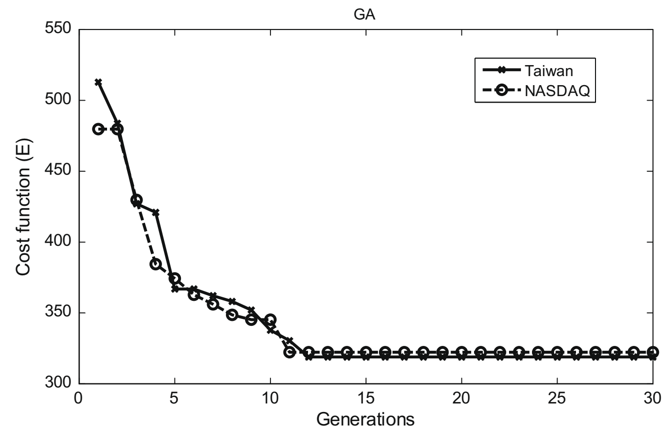

Fig. 6. The cost function $E(\boldsymbol{\theta})$ as evaluated using the genetic-based algorithm for the Taiwan weighted index and the NASDAQ composite index.

Bollerslev (1986), the GARCH (1,1) model describes the volatility of financial and economic data well. Therefore, this paper uses $p = 1, q = 1$ to estimate the relevant parameters. The other parameters related to the fuzzy systems and GA are defined as follows:

$$P = 300,\ T = 30,\ p_c = 0.9,\ p_m = 0.01$$
$$n = 1,\ L = 5 \tag{17}$$

where p_c is the crossover rate, p_m is the mutation rate, T is the number of generations, P is the population size, n is the number of premise variables, and L is the number of rules. The simulated results of the conventional method were obtained by averaging 20 independent Monte Carlo (MC) runs.

The probability distribution of the Taiwan weighted index and the NASDAQ composite index are shown in Fig. 4. It is clear that both the Taiwan weighted index and the NASDAQ composite index feature not only extended tails but also excess kurtosis.

Table 3
Parameter estimates for the Taiwan weighted index with the proposed method, GARCH, GJR GARCH, and EGARCH.

		Parameter	Value	Standard error	T statistic
Proposed method	$l = 1$	α_0^1	0.042094	0.007155	0.5.8826
		α_1^1	0.26662	0.046844	5.6917
		β_1^1	0.55601	0.053003	10.4902
	$l = 2$	α_0^2	0.004742	0.001361	.4833
		α_1^2	0.061884	0.011544	5.3608
		β_1^2	0.91137	0.016109	56.8738
	$l = 3$	α_0^3	0.020854	0.049433	0.4219
		α_1^3	0.02611	0.044906	0.044
		β_1^3	0.6575	0.07776	8.454
	$l = 4$	α_0^4	0.000638	0.000415	1.5352
		α_1^4	0.038224	0.008857	4.3154
		β_1^4	0.9574	0.009111	0.009111
	$l = 5$	α_0^5	0.0015927	0.000756	2.1053
		α_1^5	0.088168	0.011498	7.668
		β_1^5	0.90804	0.008891	102.122
GARCH		α_0	0.003794	0.012419	2.4725
		α_1	0.066795	0.007511	8.8927
		β_1	0.9244	0.008632	0.008632
GJR		α_0	0.005546	0.001284	4.3172
		α_1	0.045209	0.009819	4.604
		β_1	0.91317	0.0096698	94.4361
		Leverage	0.053587	0.014441	3.7108
EGARCH		α_0	−0.019872	0.006317	−3.1457
		α_1	0.15699	0.015913	9.8657
		β_1	0.97802	0.004609	212.1877
		Leverage	−0.052772	0.010813	−4.8805

The fuzzy membership functions of the Taiwan weighted index and the NASDAQ composite index are shown in Fig. 5. The parameters of the Taiwan weighted index and NASDAQ composite index consistent with their respective membership functions are provided in Table 2. From Fig. 5 and Table 2, it is clear that the financial markets are asymmetric and that the influence of negative information is greater than that of positive information.

The cost function $E(\boldsymbol{\theta})$ obtained by means of the genetic algorithm applied to the Taiwan weighted index and the NASDAQ composite index are depicted in Fig. 6. It is important to note from this figure that the cost functions of the GA-based estimation method are exponential and rapidly converge at the beginning of each generation.

Tables 3 and 4 give the parameter estimates associated with the Taiwan weighted index and the NASDAQ composite index for the proposed model, the GARCH model, the GJR GARCH model, and the EGARCH model. Figs. 7 and 8 show the innovations (residues) of the Taiwan weighted index and the NASDAQ composite index for each of the aforementioned models. As Figs. 7 and 8 show, the residues in the proposed method are smaller than those of the others models. Our empirical results indicate that the stock markets data is asymmetric and varies nonlinearly with time. The proposed model performs better than the others, as shown by the residue data.

Table 4
Parameter estimates for the NASDAQ composite index as evaluated using the proposed method, GARCH, GJR GARCH, and EGARCH.

		Parameter	Value	Standard error	T statistic
Proposed method	$l = 1$	α_0^1	2×10^{-7}	0.0013947	0.0001
		α_1^1	0.21698	0.044286	4.8996
		β_1^1	0.78302	0.040304	19.4278
	$l = 2$	α_0^2	2×10^{-7}	0.0011277	0.0002
		α_1^2	0.11785	0.021345	0.0213
		β_1^2	0.88215	0.018762	0.018
	$l = 3$	α_0^3	0.0004704	0.0006371	0.7384
		α_1^3	0.013092	0.0084043	1.5577
		β_1^3	0.98257	0.012073	81.3866
	$l = 4$	α_0^4	2×10^{-7}	0.0004281	0.0005
		α_1^4	0.083576	0.01415	0.01415
		β_1^4	0.91642	0.012724	72.0251
	$l = 5$	α_0^5	0.0005834	0.0004140	1.4089
		α_1^5	0.033847	0.008174	4.1407
		β_1^5	0.95193	0.00716	132.9499
GARCH		α_0	0.0011245	0.0004094	2.746
		α_1	0.034397	0.0062414	5.5111
		β_1	0.96129	0.006653	144.4855
GJR		α_0	0.0012793	0.0004451	2.8740
		α_1	0.008348	0.0043056	1.9390
		β_1	0.95889	0.006252	144.1408
		Leverage	0.055483	0.01151	4.8206
EGARCH		α_0	−0.0068132	0.011946	−5.7033
		α_1	0.30983	0.047718	6.4929
		β_1	0.91305	0.013479	67.7383
		Leverage	−0.10358	0.022112	−4.6843

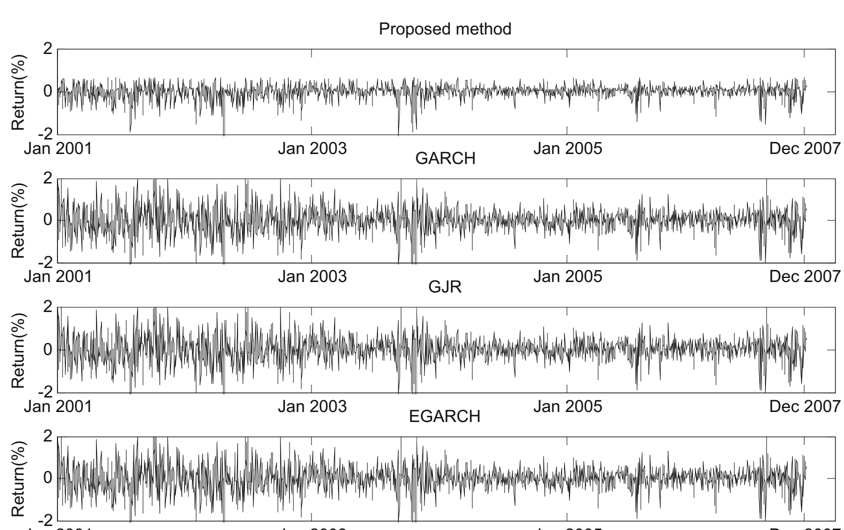

Fig. 7. The innovations (residuals) of the Taiwan weighted index as evaluated using the proposed method, GARCH, GJR GARCH, and EGARCH.

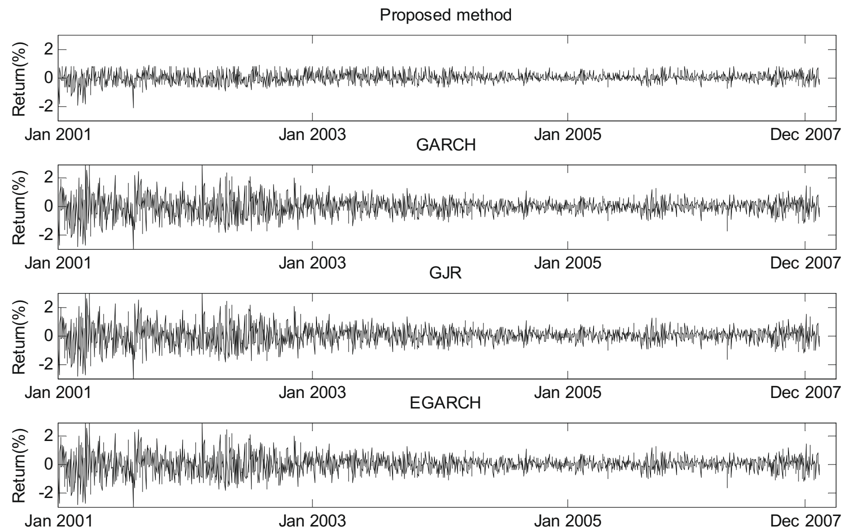

Fig. 8. The innovations (residuals) of the NASDAQ composite index as evaluated using the proposed method, GARCH, GJR GARCH, and EGARCH.

6. Conclusions

Empirical evidence demonstrates that the financial market data that we used is nonlinear and time-variance. For these reasons, this paper proposes a new method that we call the fuzzy GARCH model, a very nonlinear and highly complex approach. We used a GA-based design method to estimate parameters for the fuzzy GARCH model. Our simulation results indicate that the proposed method offers significant performance improvements. Furthermore, the GA algorithm was able to search for peaks parallel to the parameter space, and it was therefore not necessary to specify any initial conditions in order to achieve improved results.

Acknowledgement

The author is grateful to the anonymous referees, whose constructive and helpful comments led to significant improvements in the manuscript. This research was supported by the National Science Council under Grant Number NSC 97-2221-E-275-006.

References

Arciniegas, A. I., & Rueda, I. E. A. (2008). Forecasting short-term power prices in the Ontario Electricity Market (OEM) with a fuzzy logic based inference system. *Utilities Policy, 16*(3), 39–48.
Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometric, 31*(4), 307–327.
Chan, H. N. (2002). *Time series applications to finance*. USA: John Wiley & Sons.
Crefensteet, J. J. (1986). Optimization of control parameters for genetic algorithms. *IEEE Transactions on Systems, Man and Cybernetics, 1*(1), 122–128.
Engle, R. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of United Kingdom inflation. *Econometrica, 50*(7), 987–1007.
Fama, E. F. (1965). The behavior of stock market price. *Journal of Business, 38*, 34–105.
Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance, 48*(12), 1779–1801.
Goldberg, D. E. (1989). *Genetic algorithms in search optimization and machine learning*. MA: Addition Wesley.
Holland, J. H. (1962). Outline for a logical theory of adaptive systems. *Journal of the ACM, 9*(3), 297–314.
Holland, J. H. (1975). *Adaptive in natural and artificial systems*. Ann Arbor, MI: Univ. Mich. Press.
Hung, J. C. (2007). Intelligent threshold GARCH model applied to stock market of transmissions that volatility. In *International conference on convergence information technology* (pp. 1523–1528).
Kouchakpour, P., Zaknich, A., & Braűnl, T. (2007). Population variation in genetic programming. *Information Sciences, 17*(9), 3438–3452.
Lee, L. W., & Chen, S. M. (2008). Fuzzy risk analysis based on fuzzy numbers with different shapes and different deviations. *Expert Systems with Applications, 4*(5), 2763–2771.
Lee, W. C., & Shin, Y. C. (2003). Construction of fuzzy systems using least-squares method and genetic algorithm. *Fuzzy sets and systems, 3*(8), 297–323.
Liu, Y. J., & Wang, W. (2007). Adaptive fuzzy control for a class of uncertain nonaffine nonlinear systems. *Information Sciences, 18*(9), 3901–3917.
Mandelbrot, B. (1963). The variation of certain speculative prices. *Journal of Business of the University of Chicargo, 36*, 394–419.
Nelson, D. B. (1991). Conditional heteroskedasticity in asset returns: A new approach. *Econometrica, 59*(3), 347–370.
Savran, A. (2007). An adaptive recurrent fuzzy system for nonlinear identification. *Applied Soft Computing, 2*(3), 593–600.
Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man and Cybernetics*, 116–132.
Toroslu, I. H., & Arslanoglu, Y. (2007). Genetic algorithm for the personnel assignment problem with multiple objectives. *Information Sciences, 3*(2), 787–803.
Treesatayapun, C., & Uatrongjit, S. (2005). Adaptive controller with fuzzy rules emulated structure and its applications. *Engineering Applications of Artificial Intelligence, 5*(8), 603–615.
Tsay, R. S. (2002). *Analysis of financial time series*. USA: John Wiley & Sons.
Zhou, E., & Khotanzad, A. (2007). Fuzzy classifier design using genetic algorithms. *Pattern Recognition, 12*(12), 3401–3414.