

WEEK-5 - Optimization.

→ Multivariate optimization with constraints.

$$\text{eg. min}_{x_1, x_2} 2x_1^2 + 4x_2^2 \quad f(x)$$

st

$$3x_1 + 2x_2 = 12 \quad h(x)$$

$$\boxed{-\nabla f(\bar{x}^*) = \lambda^* \nabla h(\bar{x}^*)}$$

in higher dimension, (more than one equality constraints)

$$\boxed{-\nabla f(\bar{x}^*) = \sum_{i=1}^m [\nabla h_i(\bar{x}^*)] \lambda_i^*}$$

gradient lies in the space spanned by the normal of the gradients.

$$\min -4x_1 = 3\lambda$$

$$-8x_2 = 2\lambda$$

$$3x_1 + 2x_2 - 12 = 0$$

Solving

$$\begin{bmatrix} x_1^* \\ x_2^* \\ \lambda^* \end{bmatrix} = \begin{bmatrix} 3.27 \\ 1.09 \\ -4.36 \end{bmatrix}$$

→ With inequality constraints

$$\min_{x_1, x_2} 2x_1^2 + 4x_2^2$$

st

$$3x_1 + 2x_2 \leq 12$$

general formulation,

$$\min_{\bar{x}} f(\bar{x}) \quad \text{st} \quad h_i(\bar{x}) = \bar{0}, \quad i=1, \dots, m$$

$$g_j(\bar{x}) \leq \bar{0}, \quad j=1, 2, \dots$$

→ Karush-Kuhn-Tucker.

- Necessary condition for \bar{x}^* to be the minimizer



KKT conditions has to be satisfied.

- Sufficient condition

$\nabla^2 L(\bar{x}^*)$ has to be positive definite.

$$\rightarrow \nabla f(\bar{x}^*) + \sum_{i=1}^l [\nabla h_i(\bar{x}^*)] \lambda_i$$

$$+ \sum_{i=1}^m [\nabla g_i(\bar{x}^*)] \mu_i = 0$$

$$h_j(\bar{x}^*) = 0$$

$$g_i(\bar{x}^*) \leq 0$$

$$\mu_i (g_i(\bar{x}^*)) = 0 \quad \mu_i \geq 0$$

eg:

$$L(x_1, x_2, \mu)$$

$$\min_{x_1, x_2} 2x_1^2 + 4x_2^2$$

$$x_1, x_2$$

st

$$3x_1 + 2x_2 \leq 12 \Rightarrow (a)$$

$$2x_1 + 5x_2 \geq 10 \Rightarrow (b)$$

$$x \leq 1 \Rightarrow (c)$$

Soln

Lagrangian

$$L(x_1, x_2, \mu_1, \mu_2, \mu_3) = 2x_1^2 + 4x_2^2 +$$

$$\mu_1 (3x_1 + 2x_2 - 12)$$

$$+ \mu_2 (10 - 2x_1 - 5x_2) + \mu_3 (x_1 - 1)$$

First order KKT conditions

$$4x_1 + 2\mu_1 - 2\mu_2 = 0$$

&

$$8x_2 + 2\mu_1 - 5\mu_2 = 0$$

$$M_1 (3x_1 + 2x_2 - 12) = 0$$

$$M_2 (10 - 2x_1 - 5x_2) = 0$$

$$M_3 (x_3 - 1) = 0$$

$$M_i \geq 0$$

INTRO to DATA SCIENCE:

⇒ Types of Problems:

- ① Classification problems
- ② Function approximation

→ Steps in Data Imputation

- ① Problem Definition
- ② Problem Characterization
- ③ Solution Conceptualization
- ④ Method Identification
- ⑤ Actualization
- ⑥ Assess assumptions