WEEK-4 - OPTIMIZATION:

→ Components of an optimization problem:

1. Objective function
2. Decision variables
3. Constraints.

→ Types of optimization problems:

1. Linear programming problem
2. Non-linear programming problem
   - Convex vs Non convex
3. Integer programming problem (linear and non linear)
4. Mixed integer linear programming problem
5. Mixed integer non-linear programming problem.

Non-Linear Optimization:

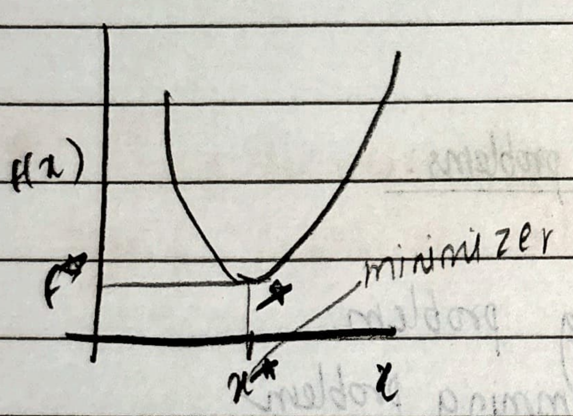
(1) Unconstrained case:

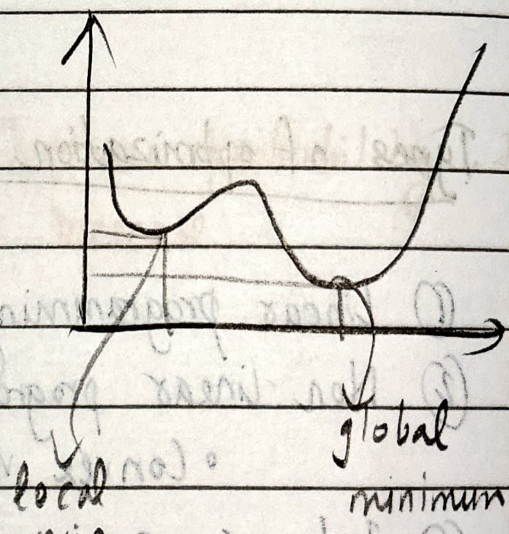
(i) Univariate:

decision variable

$$\min_{x} \left(f(x) \right) \quad x \in \mathbb{R}$$

→ objective function.





* first order necessary condition: $f'(x^*) = 0$
* Second order sufficiency condition: $f''(x^*) > 0$

Multivariate :

$$z = f(x_1, x_2 \ldots x_n)$$

eg: $z = \sqrt{x_1^2 + x_2^2}$

$$\min_x f(\bar{x})$$
$$\bar{x} \in R^n$$

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix}$$

↳ Gradient

Hessian

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & & \frac{\partial^2 f}{\partial x_1 x_n} \\ \vdots & & & \\ \frac{\partial^2 f}{\partial x_n \partial x_1} & \cdots & & \frac{\partial^2 f}{\partial x_n^2} \end{bmatrix}$$

* gradient of a function at a point is orthogonal to the contours
* Gradient points in the direction of greatest increase of the function.
* -ve gradient points in the dirn of greatest decrease of the function.
* Hessian is a symmetric matrix.

* Necessary condition for $\bar{x}^*$ to be the minimizer:

$$\nabla f(\bar{x}^*) = 0$$

* Sufficient condition:

$$(\bar{x} - \bar{x}^*)^T \nabla^2 f(\bar{x}^*) (\bar{x} - \bar{x}^*)$$

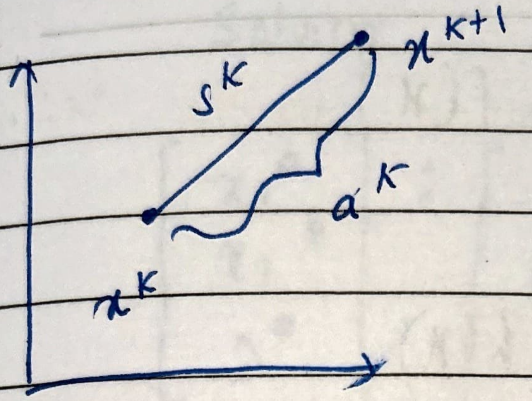
$$\nabla^2 f(\bar{x}^*) > 0$$

find Hessian
↓
find eigen values

if all +ve the hessian > 0

→ Unconstrained multivariate optimization – Descent direction and movement

→ Iterative



$$\boxed{x^{k+1} = x^k + \alpha^k s^k}$$

starting point

step length.

search dir^n

→ Steepest descent and optimum step size:

Minimize $f(x_1, x_2, \ldots x_n) = f(x)$

• Steepest descent:

- at iteration k starting posiint is x^k
- search dir^n s^k = Negative gradient of $f(x) = -\nabla f(x^k)$
- New point is $x^{k+1} = x^k + \alpha^k s^k$ where α^k is the value of α for which

$$f(x^{k+1}) = f(\alpha) = \text{is a minimum}$$