SCOTS: Deflectomety for Deployable Space Reflectors

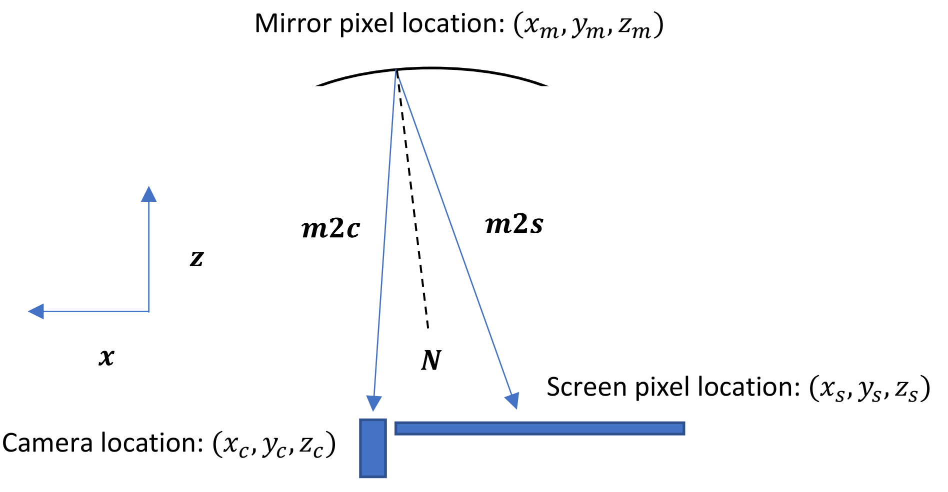
Principle

The SCOTS[1] test features:

1. An imaging camera and lens
2. Test specimen (spherical mirror)
3. Computer Monitor

The working principle is the displaying of a known phase screen on the monitor. The image of the phase screen is reflected in the mirror and acquired by the camera. The distortion of the phase screen corresponds to slope measurements in the reflector, allowing the reconstruction of the mirror shape from the acquired camera image.

Specifically, the goal is the exact identification of 2 geometric vectors for **each** *mirror pixel:* The *Mirror to Camera* ($\boldsymbol{m2c}$) and *Mirror to Screen* ($\boldsymbol{m2c}$) vectors. Consider the geometric layout below, illustrating a single mirror pixel with 2 corresponding geometric vectors:



Assume the two geometric vectors are found *for each mirror pixel*, then, the surface vector normal to the mirror are simply given as the average of the two vectors, normalized:

$$\boldsymbol{N} = \frac{|\boldsymbol{m2c}| + |\boldsymbol{m2s}|}{\boldsymbol{2}}$$

The (x, y) slope of the of mirror (w_x, w_y) may then be found by normalizing $\boldsymbol{N}$ and extracting the first and second coordinate:

$$w_x, w_y = |N|_1, |N|_2$$

And the surface may then be reconstructed from the slope data. The challenge is the accurate identification of the 9 geometric parameters in the above figure, for each mirror pixel.

Camera and Mirror Image Pixel Locations

The camera location parameters: (x_c, y_c, z_c) are given by geometric measurement of the system (specifically, the location of the camera *aperture*). The three values are constants.

The mirror pixel locations are also straightforward. First, the center of the mirror is found by geometric measurement of the system:

$$m_c = (x_{mc}, y_{mc}, z_{mc})$$

Then, the image scale of the camera is found by measuring the diameter of the mirror geometrically (D_m) and in pixels(D_{px}).

$$imageScale = \frac{D_{px}}{D_m}[px/mm]$$

The mirror pixel locations (x_m, y_m) are then found by:

$$x_m = x_{mc} + imageScale \cdot p_i$$

$$y_m = y_{mc} + imageScale \cdot p_j$$

Where (p_i, p_j) are the distances in pixels of the mirror pixel from the center of the mirror. That is, x_m and y_m are (n, m) matrices where (n, m) depends on the amount of image pixels in the mirror. $(n, m) \approx (200{,}200)$

z_m is found by geometric measurement and assumed constant (the sagitta of the mirror is ignored, may be changed in later revisions)

Screen Pixel Locations

The identification of screen location corresponding to mirror pixel location is more involved. First, a pixel on the screen is identified which corresponds to the center mirror pixel location. This will be known as the zero phase point. From this point, 20 fringes are drawn on the screen according to:

$$A_x = 255 \cdot \sin(\frac{x}{canvasSize} \cdot 2\pi \cdot N + \phi)$$

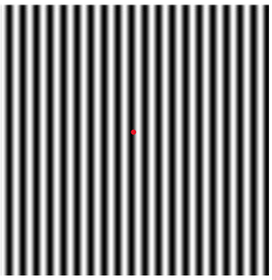
255 is the maximum 8-bit screen intensity, N is the amount of fringes and canvasSize is the size in pixels of the display window. x is the monitor pixel location which is zero in the zero phase point:

$$x = [-\frac{canvasSize}{2}, \frac{canvasSize}{2}]$$

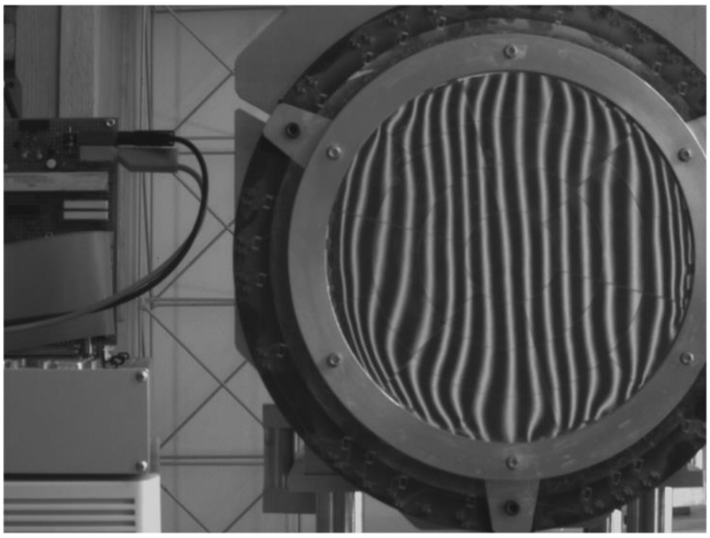
and ϕ is a phase addition. A total of 49 pictures are taken with increasing phase (the fringes “move” in small steps toward the right)

$$\phi = 0, \frac{\pi}{16}, \frac{2\pi}{16}, \frac{3\pi}{16} .. \frac{48\pi}{16}$$

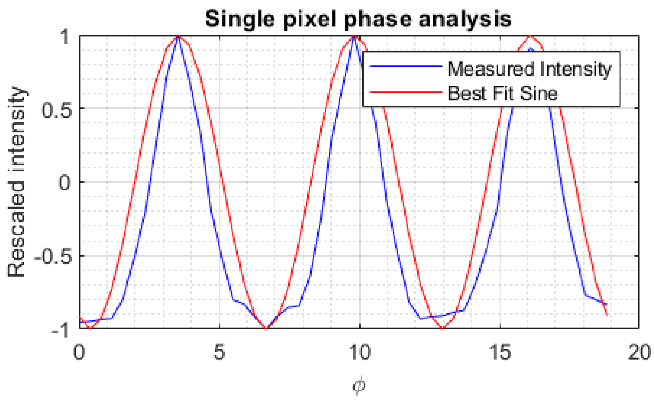
The Figure below shows the displayed fringes with the zero phase point marked in red:



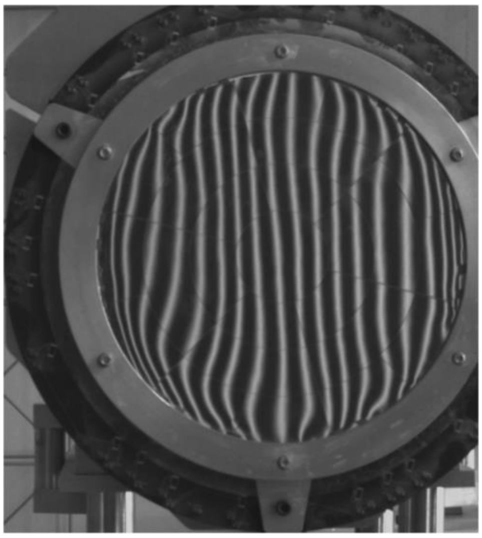
The next step is imaging the reflection in the mirror. The following shows the reflection of the fringes on the MATS 20cm demonstrator:

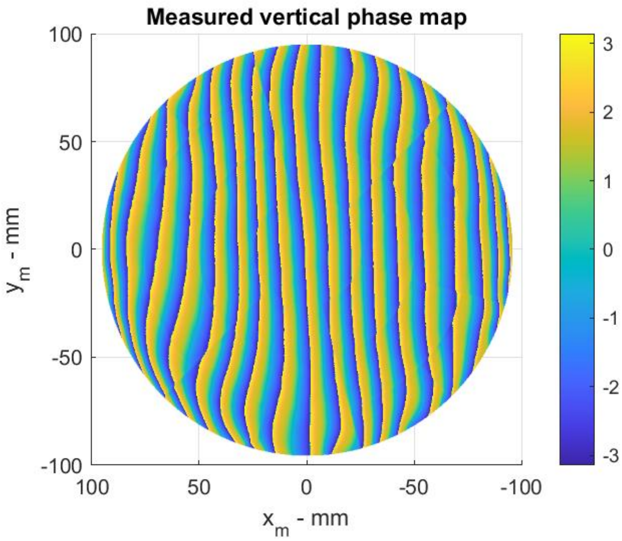


With the 49 images acquired, every individual pixel may be analyzed to find the corresponding phase in the original zero-phase image. Consider an example pixel located on the mirror, the intensity values are plotted for all 49 images, and the phase is found by a least squares approximation:



The nonlinearity of the screen intensity is clearly seen, but since the peaks and valleys are well-matching, the phase should be recovered well. When the phase is found for all mirror pixels, the resulting phase map can be plotted next to the reflected phase image:

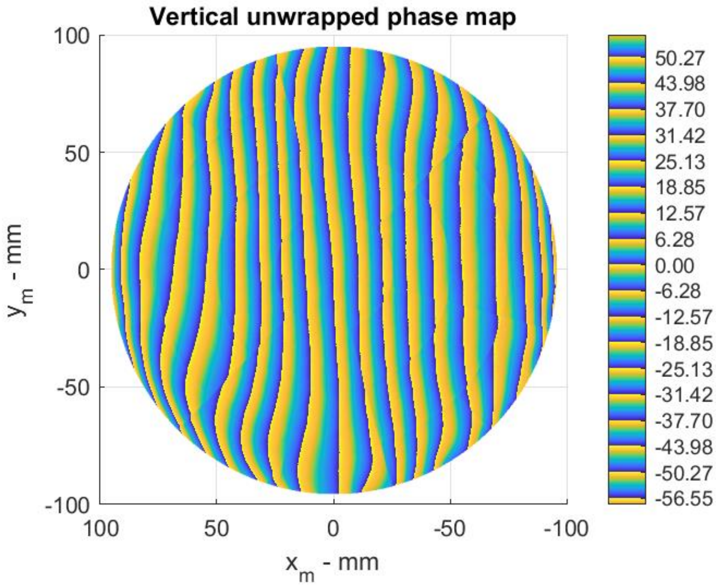




The next step is to unwrap the phase. The method explained at:

Unwrapping phase based on Ghiglia and Romero (1994) based on weighted and unweighted least-square method https://doi.org/10.1364/JOSAA.11.000107 implemented:
https://www.mathworks.com/matlabcentral/fileexchange/60345-2d-weighted-phase-unwrapping

is used. The resulting unwrapped phase map is:



Note that most details remain, although the phase is now monotonously increasing from left to right.

The process is repeated equivalently for fringes in the horizontal direction. The two resulting unwrapped phase maps give the relation between mirror phase and screen phase, and therefore mirror pixel and screen pixel. Thus, for each mirror pixel, the equivalent position on the monitor is known. From this equivalent screen position, and recalling the expression for the fringe map:

$$A_x = 255 \cdot \sin(\frac{x}{canvasSize} \cdot 2\pi \cdot N + \phi)$$

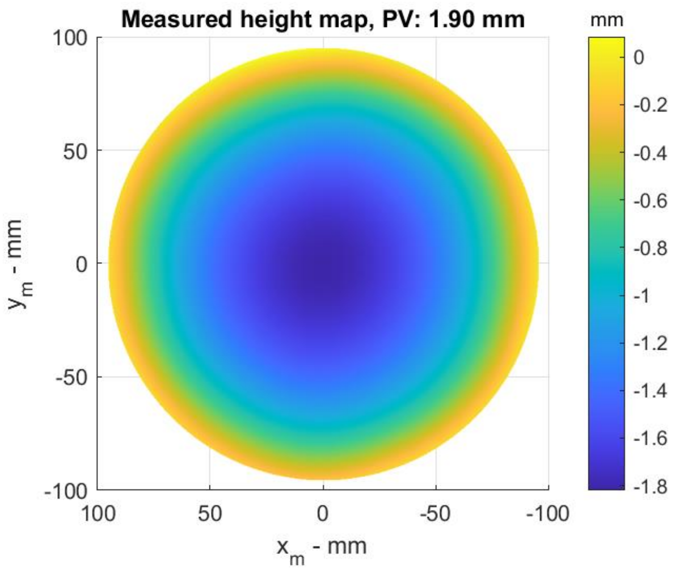
the geometric coordinates (x_s, x_y) can finally be calculated:

$$x_s = x_0 + phaseMap \cdot \frac{canvasSize}{N \cdot 2\pi \cdot screenPxScale}$$

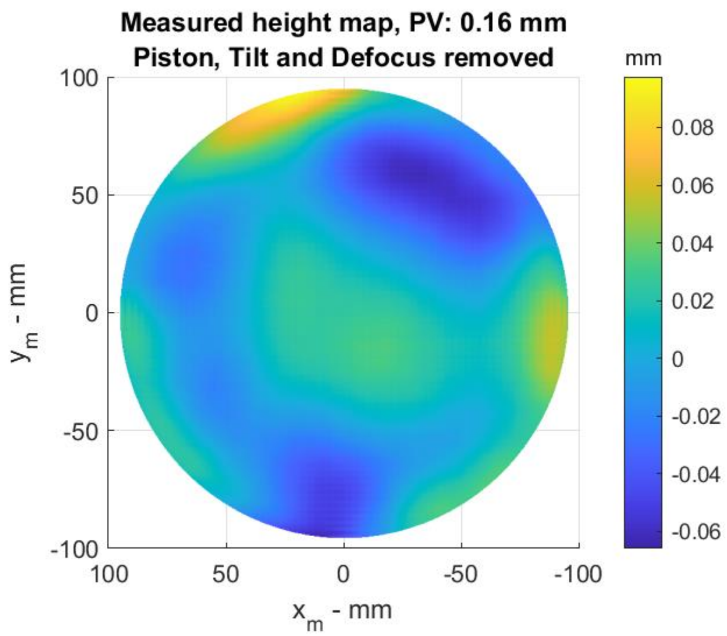
Where x_0 is the geometrically measured zero phase location and $screenPxScale$ is the conversion factor between pixels and distance $(\frac{mm}{px})$ on the monitor.

The y-direction calculation is equivalent.

With these calculations completed, all 9 geometric parameters are found for each individual mirror pixel, meaning the slope map of the reflector can be calculated as explained in the beginning. From the slope map, the mirror shape may be integrated:

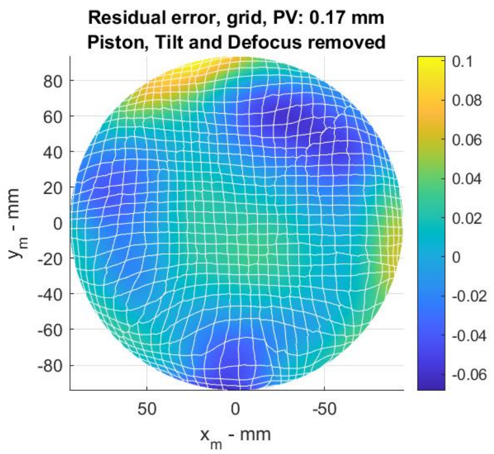


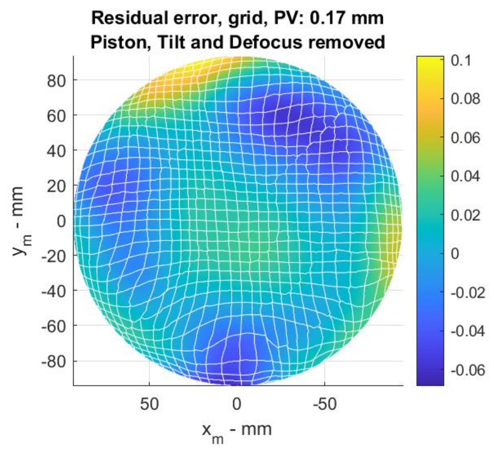
And finally subtracting the best fit Piston, Tip, Tilt and Defocus Zernike modes to better illustrate the shape (should be changed to best spherical fit in the future, since this is the best *parabolic* fit):



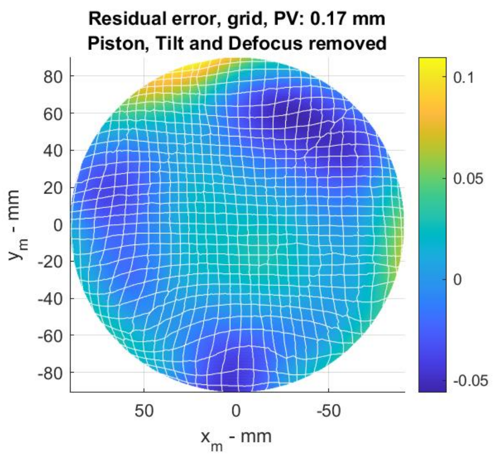
To qualify the test, 2 equal tests are performed at two different distances, to quantify repeatability & accuracy respectively.

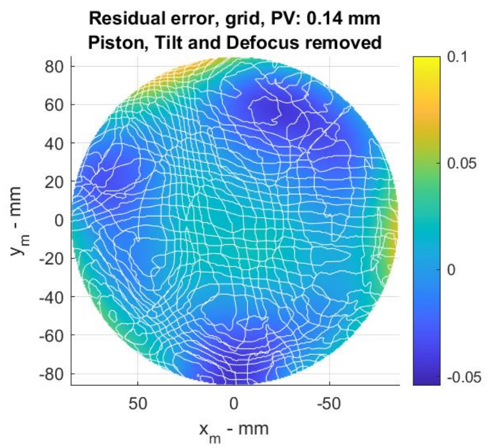
First, two tests are performed with screen and camera 1292mm from the mirror:



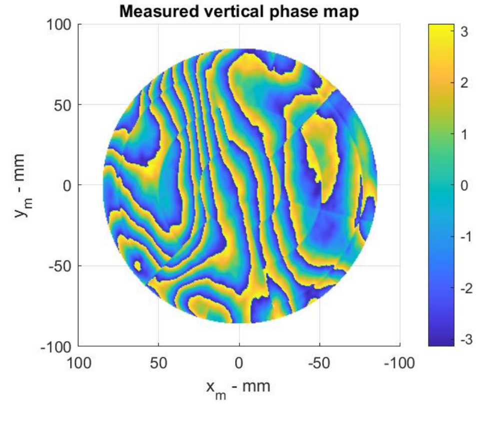


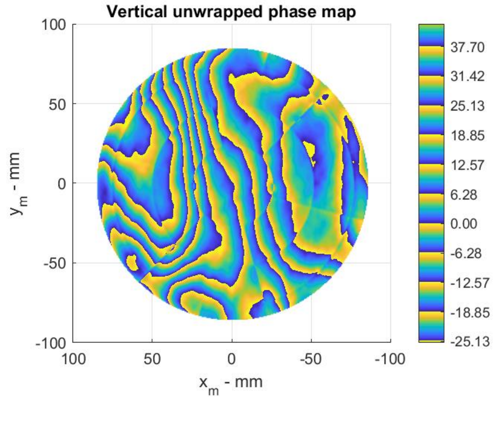
Secondly, a test is performed with both camera and screen 1192mm from the mirror(left), and one with the distance increased to 2150mm(right).





The longer the distance between mirror and screen, the more the errors are amplified in the reflected phase image. The 2150mm distance is slightly too far, as it can be seen the unwrapping algorithm does not capture everything perfectly:





Further, the geometric tolerances over the large distance is much looser, due to the lack of measurement equipment. In spite of this, the shape is still qualitatively very similar, with a small numeric difference.

The results seem to indicate that the test is quite robust to geometric variation in the test setup.