Swiss-Cheese operad and Drinfeld center

Najib Idrissi

June 3rd, 2016 @ ETH Zürich

Laboratoire Paul Painlevé
Université de Lille 1 Sciences et Technologies

Outline

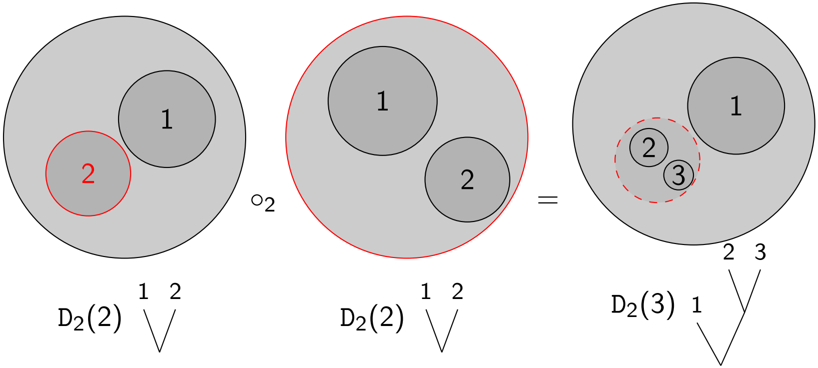
1. Background: Little disks and braids
2. The Swiss-Cheese operad
3. Rational model: Chords diagrams and Drinfeld associators

Outline

1. Background: Little disks and braids
2. The Swiss-Cheese operad
3. Rational model: Chords diagrams and Drinfeld associators

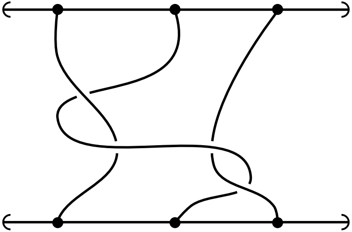
Little disks operad

The topological **operad** D_n [Boardman–Vogt, May] of little n-disks governs homotopy associative and commutative algebras:



Braid groups

Recall: pure braid group P_r

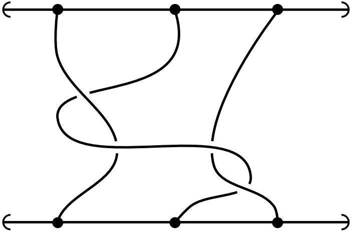


Proposition

$\mathtt{D}_2(r) \simeq \mathrm{Conf}_r(\mathbb{R}^2) \simeq K(P_r, 1)$

Braid groups

Recall: pure braid group P_r

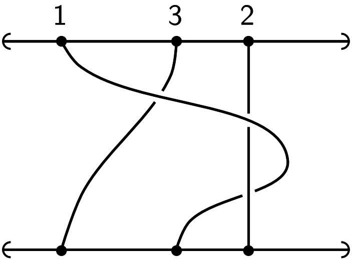


Proposition

$\mathtt{D}_2(r) \simeq \mathrm{Conf}_r(\mathbb{R}^2) \simeq K(P_r, 1) \implies \mathtt{D}_2 \simeq \mathrm{B}(\pi\mathtt{D}_2)$

Braid group**oids**

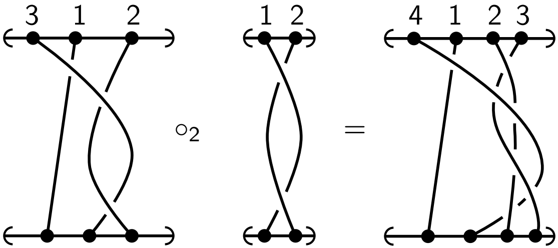
"Extension" of P_r: colored braid groupoid $\texttt{CoB}(r)$



$$\mathrm{ob}\,\texttt{CoB}(r) = \Sigma_r, \quad \mathrm{End}_{\texttt{CoB}(r)}(\sigma) \cong P_r$$

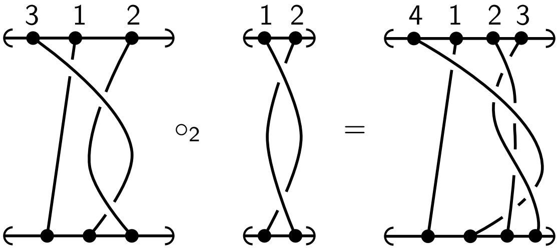
Cabling

"Cabling": insertion of a braid inside a strand



Cabling

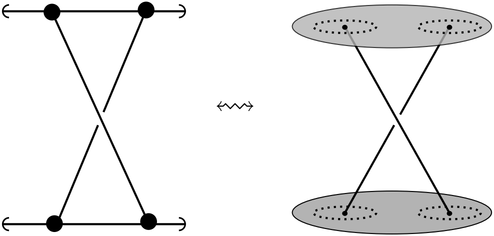
"Cabling": insertion of a braid inside a strand



$\Longrightarrow$ $\{\mathtt{CoB}(r)\}_{r\geq 1}$ is a symmetric operad in groupoids:

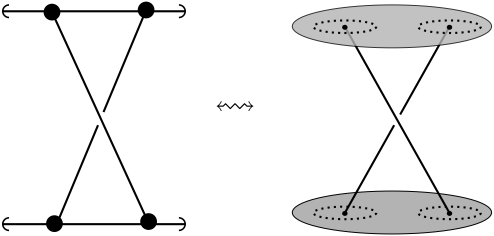
$$\circ_i : \mathtt{CoB}(k) \times \mathtt{CoB}(l) \to \mathtt{CoB}(k+l-1),\ 1 \leq i \leq k$$

Little disks and braids



$\mathtt{CoB}(r) \cong$ subgroupoid of $\pi\mathrm{D}_2(r)$

Little disks and braids

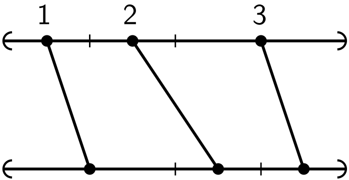


$$\mathtt{CoB}(r) \cong \text{subgroupoid of } \pi\mathtt{D}_2(r)$$

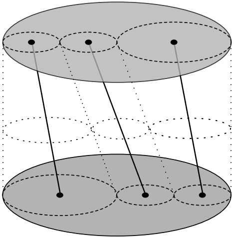
Problem: inclusion not compatible with operad structure

Little disks and braids (2)

Solution: parenthesized braids PaB

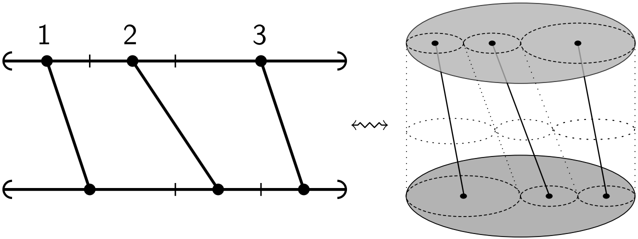


$\leftrightsquigarrow$



Little disks and braids (2)

Solution: parenthesized braids $\mathtt{PaB}$



Theorem (Fresse)

Operads $\pi \mathrm{D}_2$ and $\mathtt{CoB}$ are weakly equivalent.

$\pi \mathrm{D}_2 \xleftarrow{\sim} \mathtt{PaB} \xrightarrow{\sim} \mathtt{CoB}$ is a zigzag of weak equivalences of *operads*.

Algebras over categorical operads

$\mathtt{P} \in \mathsf{CatOp} \implies$ a $\mathtt{P}$-algebra is given by:

- A category C;
- For every object $x \in \operatorname{ob} \mathtt{P}(r)$, a *functor* $\bar{x} : \mathsf{C}^{\times r} \to \mathsf{C}$;
- For every morphism $f \in \operatorname{Hom}_{\mathtt{P}(r)}(x, y)$, a *natural transformation*

$$\begin{tikzcd} \mathsf{C}^{\times r} \arrow[r, bend left, "\bar{x}"] \arrow[r, bend right, "\bar{y}"'] \arrow[r, phantom, "\Downarrow \bar{f}"] & \mathsf{C} \end{tikzcd}$$

- $+$ compatibility with the action of symmetric groups and operadic composition.

Algebras over $\mathtt{CoB}$

For $\mathtt{P} = \mathtt{CoB}$, algebras are given by:

- A category C;
- $\sigma \in \operatorname{ob} \mathtt{CoB}(r) = \Sigma_r \rightsquigarrow \otimes_\sigma : \mathsf{C}^{\times r} \to \mathsf{C}$ s.t. $\otimes_{\mathrm{id}_1} = \mathrm{id}_{\mathsf{C}}$;

Algebras over $\mathtt{CoB}$

For $\mathtt{P} = \mathtt{CoB}$, algebras are given by:

- A category C;
- $\sigma \in \mathrm{ob}\,\mathtt{CoB}(r) = \Sigma_r \rightsquigarrow \otimes_\sigma : \mathsf{C}^{\times r} \to \mathsf{C}$ s.t. $\otimes_{\mathrm{id}_1} = \mathrm{id}_{\mathsf{C}}$;
- $\otimes_\sigma(X_1, \dots, X_n) = \otimes_{\mathrm{id}_r}(X_{\sigma(1)}, \dots, X_{\sigma(n)})$;

Algebras over $\mathtt{CoB}$

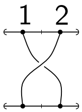
For $\mathtt{P} = \mathtt{CoB}$, algebras are given by:

- A category C;
- $\sigma \in \operatorname{ob}\mathtt{CoB}(r) = \Sigma_r \rightsquigarrow \otimes_\sigma : \mathsf{C}^{\times r} \to \mathsf{C}$ s.t. $\otimes_{\mathrm{id}_1} = \mathrm{id}_{\mathsf{C}}$;
- $\otimes_\sigma(X_1, \dots, X_n) = \otimes_{\mathrm{id}_r}(X_{\sigma(1)}, \dots, X_{\sigma(n)})$;
- $\otimes_{\mathrm{id}_2}(\otimes_{\mathrm{id}_2}(X, Y), Z) = \otimes_{\mathrm{id}_3}(X, Y, Z) = \otimes_{\mathrm{id}_2}(X, \otimes_{\mathrm{id}_2}(Y, Z))$...

Algebras over $\mathtt{CoB}$

For $\mathtt{P} = \mathtt{CoB}$, algebras are given by:

- A category C;
- $\sigma \in \operatorname{ob} \mathtt{CoB}(r) = \Sigma_r \rightsquigarrow \otimes_\sigma : \mathsf{C}^{\times r} \to \mathsf{C}$ s.t. $\otimes_{\mathrm{id}_1} = \mathrm{id}_{\mathsf{C}}$;
- $\otimes_\sigma(X_1, \dots, X_n) = \otimes_{\mathrm{id}_r}(X_{\sigma(1)}, \dots, X_{\sigma(n)})$;
- $\otimes_{\mathrm{id}_2}(\otimes_{\mathrm{id}_2}(X, Y), Z) = \otimes_{\mathrm{id}_3}(X, Y, Z) = \otimes_{\mathrm{id}_2}(X, \otimes_{\mathrm{id}_2}(Y, Z))$...
- $\beta \in \mathrm{Hom}_{\mathtt{CoB}(r)}(\sigma, \sigma')$ colored braid $\rightsquigarrow$ natural transformation $\beta_* : \otimes_\sigma \to \otimes_{\sigma'}$. For example:

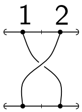


$$\rightsquigarrow \tau_{X,Y} : X \otimes Y \to Y \otimes X$$

Algebras over $\mathtt{CoB}$

For $\mathtt{P} = \mathtt{CoB}$, algebras are given by:

- A category C;
- $\sigma \in \operatorname{ob} \mathtt{CoB}(r) = \Sigma_r \rightsquigarrow \otimes_\sigma : \mathsf{C}^{\times r} \to \mathsf{C}$ s.t. $\otimes_{\mathrm{id}_1} = \mathrm{id}_{\mathsf{C}}$;
- $\otimes_\sigma(X_1, \dots, X_n) = \otimes_{\mathrm{id}_r}(X_{\sigma(1)}, \dots, X_{\sigma(n)})$;
- $\otimes_{\mathrm{id}_2}(\otimes_{\mathrm{id}_2}(X, Y), Z) = \otimes_{\mathrm{id}_3}(X, Y, Z) = \otimes_{\mathrm{id}_2}(X, \otimes_{\mathrm{id}_2}(Y, Z))\dots$
- $\beta \in \mathrm{Hom}_{\mathtt{CoB}(r)}(\sigma, \sigma')$ colored braid $\rightsquigarrow$ natural transformation $\beta_* : \otimes_\sigma \to \otimes_{\sigma'}$. For example:



$\rightsquigarrow \tau_{X,Y} : X \otimes Y \to Y \otimes X$

Theorem (MacLane, Joyal–Street)

An algebra over $\mathtt{CoB}$ is a braided monoidal category (strict, no unit).

Remarks

Extension of the theorem for parenthesized braids:

Theorem

An algebra over $\mathtt{PaB}$ is a braided monoidal category (no unit).

Unital versions $\mathtt{CoB}_{+}$ and $\mathtt{PaB}_{+}$:

Theorem

An algebra over $\mathtt{CoB}_{+}$ (resp. $\mathtt{PaB}_{+}$) is a strict (resp. non-strict) braided monoidal category with a strict (in both cases) unit.

Outline

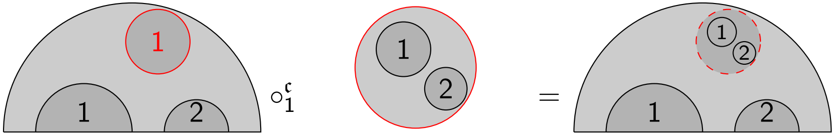
1. Background: Little disks and braids
2. The Swiss-Cheese operad
3. Rational model: Chords diagrams and Drinfeld associators

Definition of the Swiss-Cheese operad

The Swiss-Cheese operad $\mathtt{SC}$ [Voronov, 1999] governs a $\mathtt{D}_2$-algebra acting on a $\mathtt{D}_1$-algebra. It's a *colored* operad, with two colors $\mathfrak{c}$ ("closed" $\leftrightarrow \mathtt{D}_2$) and $\mathfrak{o}$ ("open" $\leftrightarrow \mathtt{D}_1$).

Definition of the Swiss-Cheese operad

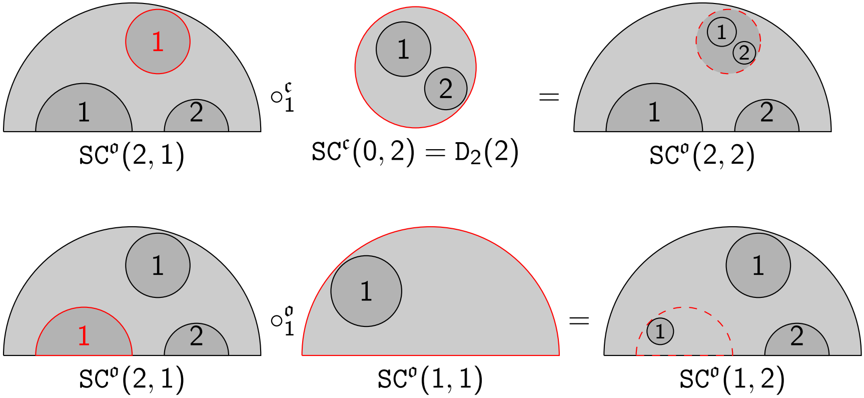
The Swiss-Cheese operad $\mathtt{SC}$ [Voronov, 1999] governs a $\mathtt{D}_2$-algebra acting on a $\mathtt{D}_1$-algebra. It's a colored operad, with two colors $\mathfrak{c}$ ("closed" $\leftrightarrow \mathtt{D}_2$) and $\mathfrak{o}$ ("open" $\leftrightarrow \mathtt{D}_1$).



$\mathtt{SC}^{\mathfrak{o}}(2,1)$ $\quad$ $\mathtt{SC}^{\mathfrak{c}}(0,2) = \mathtt{D}_2(2)$ $\quad$ $\mathtt{SC}^{\mathfrak{o}}(2,2)$

Definition of the Swiss-Cheese operad

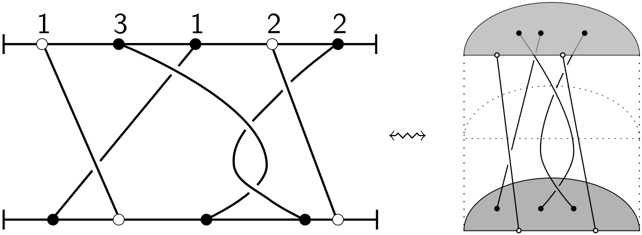
The Swiss-Cheese operad $\mathtt{SC}$ [Voronov, 1999] governs a $\mathtt{D}_2$-algebra acting on a $\mathtt{D}_1$-algebra. It's a colored operad, with two colors $\mathfrak{c}$ ("closed" $\leftrightarrow \mathtt{D}_2$) and $\mathfrak{o}$ ("open" $\leftrightarrow \mathtt{D}_1$).



The operad CoPB

Idea

Extend CoB to build a colored operad weakly equivalent to πSC.

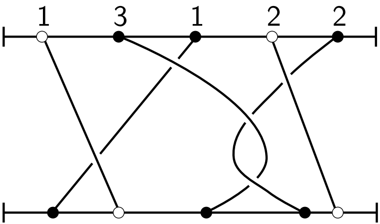


$\mathtt{CoPB}(2,3)$

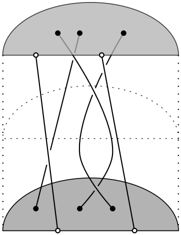
The operad $\mathtt{CoPB}$

Idea

Extend $\mathtt{CoB}$ to build a colored operad weakly equivalent to $\pi\mathsf{SC}$.



$\rightsquigarrow$



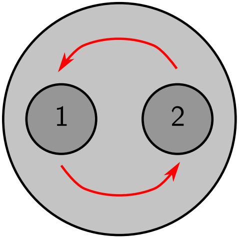
$\mathtt{CoPB}(2,3)$

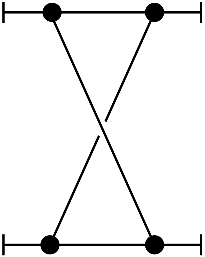
Theorem (I.)

$$\pi\mathsf{SC} \xleftarrow{\sim} \mathtt{PaPB} \xrightarrow{\sim} \mathtt{CoPB}.$$

Braidings and semi-braidings

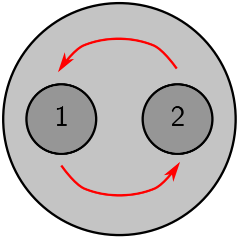
In $\mathtt{D}_2$ / $\mathtt{CoB}$: braiding = homotopy commutativity

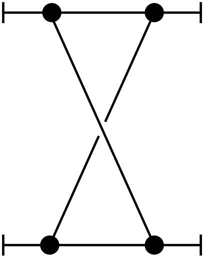




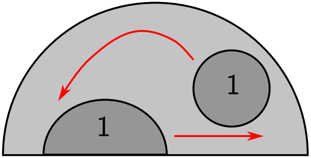
Braidings and semi-braidings

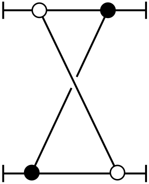
In $\mathtt{D}_2$ / $\mathtt{CoB}$: braiding = homotopy commutativity





In $\mathtt{SC}$ / $\mathtt{CoPB}$: half-braiding = “central” morphism





Drinfeld center

C: monoidal category $\rightsquigarrow$ ΣC bicategory with one object
$\rightsquigarrow$ Drinfeld center $\mathcal{Z}(\mathsf{C}) := \mathrm{End}(\mathrm{id}_{\Sigma \mathsf{C}})$

- objects: (X, Φ) with $X \in \mathsf{C}$ and $\Phi : (X \otimes -) \xrightarrow{\cong} (- \otimes X)$ ("half-braiding") ;
- $\{$morphisms $(X, \Phi) \to (Y, \Psi)\} = \{$morphisms $X \to Y$ compatible with Φ and $\Psi\}$.

Theorem (Drinfeld, Joyal–Street 1991, Majid 1991)

$\mathcal{Z}(\mathsf{C})$ is a braided monoidal category with:

$$(X, \Phi) \otimes (Y, \Psi) = (X \otimes Y, (\Psi \otimes 1) \circ (1 \otimes \Phi)),$$

$$\tau_{(X,\Phi),(Y,\Psi)} = \Phi_Y.$$

Voronov's theorem

Recall:

$$H_*(\mathtt{D}_1) = \mathtt{Ass}, \quad H_*(\mathtt{D}_2) = \mathtt{Ger}$$

Theorem (Voronov, Hoefel)

An algebra over $H_*(\mathtt{SC})$ is given by:

- An associative algebra A ;
- A Gerstenhaber algebra B ;
- A central morphism of commutative algebras $B \to Z(A)$.

(Voronov's original version: $B \otimes A \to A$ instead $B \to A$)

Algebras over $\mathtt{CoPB}$

Theorem (I.)

An algebra over $\mathtt{CoPB}$ is given by:

- A (strict non-unital) monoidal category N ;
- A (strict non-unital) braided monoidal category M ;
- A (strict) braided monoidal functor $F : \mathsf{M} \to \mathcal{Z}(\mathsf{N})$.

$\to$ categorical version of Voronov's theorem

Algebras over CoPB

Theorem (I.)

An algebra over CoPB is given by:

- A (strict non-unital) monoidal category N ;
- A (strict non-unital) braided monoidal category M ;
- A (strict) braided monoidal functor $F : \mathsf{M} \to \mathcal{Z}(\mathsf{N})$.

$\to$ categorical version of Voronov's theorem

Like CoB: non-strict and/or unitary versions of the theorem.

Algebras over CoPB

Theorem (I.)

An algebra over CoPB is given by:

- A (strict non-unital) monoidal category N ;
- A (strict non-unital) braided monoidal category M ;
- A (strict) braided monoidal functor $F : \mathsf{M} \to \mathcal{Z}(\mathsf{N})$.

$\to$ categorical version of Voronov's theorem

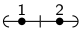
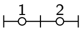
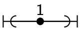
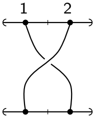
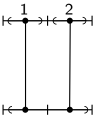
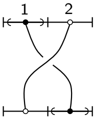
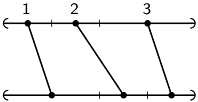
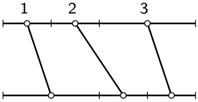
Like CoB: non-strict and/or unitary versions of the theorem.

Remark

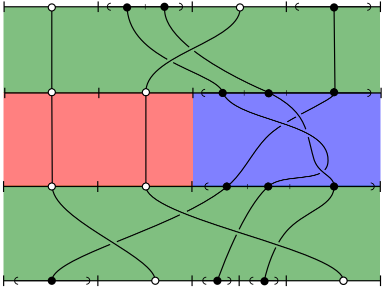
Mirrors results of Ayala–Francis–Tanaka and Ginot from the realm of ∞-categories and factorization algebras.

Generators

We present PaPB by generators and relations:

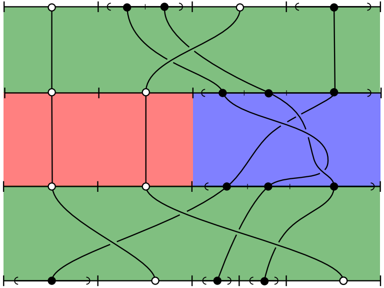
$\mu_{\mathfrak{c}} \in \operatorname{ob} \mathtt{PaB}(2)$	$\mu_{\mathfrak{o}} \in \operatorname{ob} \mathtt{PaPB}(2,0)$	$f \in \operatorname{ob} \mathtt{PaPB}(0,1)$	$\tau \in \mathtt{PaB}(2)$
			
$p \in \mathtt{PaPB}(0,2)$	$\psi \in \mathtt{PaPB}(1,1)$	$\alpha_{\mathfrak{c}} \in \mathtt{PaB}(3)$	$\alpha_{\mathfrak{o}} \in \mathtt{PaPB}(3,0)$
			

Idea of the proof



All morphisms can be split in four parts.

Idea of the proof



All morphisms can be split in four parts. The image of a morphism is well-defined thanks to:

- Coherence theorems of MacLane and Epstein;
- Adaptation of the proofs the theorem on PaP and the theorem on PaB;

Outline

1. Background: Little disks and braids
2. The Swiss-Cheese operad
3. Rational model: Chords diagrams and Drinfeld associators

Chord diagrams operad

Drinfeld–Kohno Lie algebra ("infinitesimal version" of pure braids):

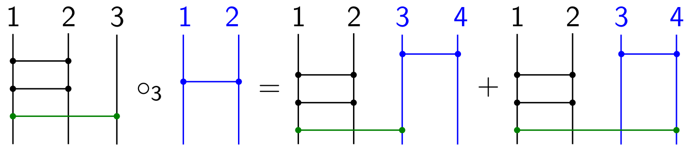
$$\mathfrak{p}(r) = \mathbb{L}(t_{ij})_{1 \le i \neq j \le r} / \langle t_{ij} - t_{ji}, [t_{ij}, t_{kl}], [t_{ik}, t_{ij} + t_{jk}] \rangle.$$

Chord diagrams operad

Drinfeld–Kohno Lie algebra ("infinitesimal version" of pure braids):

$$\mathfrak{p}(r) = \mathbb{L}(t_{ij})_{1 \leq i \neq j \leq r} / \langle t_{ij} - t_{ji}, [t_{ij}, t_{kl}], [t_{ik}, t_{ij} + t_{jk}] \rangle.$$

$\rightarrow$ operad:



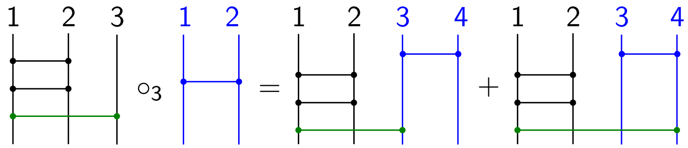
$$t_{13} t_{12} t_{12} \circ_3 t_{12} \in \mathbb{U}\mathfrak{p}(4)$$

Chord diagrams operad

Drinfeld–Kohno Lie algebra (“infinitesimal version” of pure braids):

$$\mathfrak{p}(r) = \mathbb{L}(t_{ij})_{1 \le i \ne j \le r} / \langle t_{ij} - t_{ji}, [t_{ij}, t_{kl}], [t_{ik}, t_{ij} + t_{jk}] \rangle.$$

$\to$ operad:



$$t_{13} t_{12} t_{12} \circ_3 t_{12} \in \mathbb{U}\mathfrak{p}(4)$$

Mal’cev completion:

$$\widehat{\mathtt{CD}} = \mathbb{G}\hat{\mathbb{U}}\hat{\mathfrak{p}}$$

$\to$ operad in the category of complete group(oid)s

Drinfeld associators

Drinfeld associators $(\mu \in \mathbb{Q}^\times)$:

$$\mathrm{Ass}^\mu(\mathbb{Q}) = \{\phi : \mathtt{PaB}_+ \to \widehat{\mathtt{CD}}_+ \mid \phi(\tau) = e^{\mu t_{12}/2}\}$$

If $\phi \in \mathrm{Ass}^\mu(\mathbb{Q})$, then:

$$\Phi(t_{12}, t_{23}) := \phi(\alpha) \in \mathbb{G}(\mathbb{Q}[[t_{12}, t_{23}]])$$

satisfies the usual equations (pentagon, hexagon)

Drinfeld associators

Drinfeld associators ($\mu \in \mathbb{Q}^\times$) :

$$\mathrm{Ass}^\mu(\mathbb{Q}) = \{\phi : \mathtt{PaB}_+ \to \widehat{\mathtt{CD}}_+ \mid \phi(\tau) = e^{\mu t_{12}/2}\}$$

If $\phi \in \mathrm{Ass}^\mu(\mathbb{Q})$, then:

$$\Phi(t_{12}, t_{23}) := \phi(\alpha) \in \mathbb{G}(\mathbb{Q}[[t_{12}, t_{23}]])$$

satisfies the usual equations (pentagon, hexagon)

Theorem (Drinfeld)

$$\mathrm{Ass}^\mu(\mathbb{Q}) \neq \varnothing$$

ϕ induces a *rational equivalence* $\pi(\mathtt{D}_2)_+ \simeq \mathtt{PaB}_+ \xrightarrow{\sim_{\mathbb{Q}}} \widehat{\mathtt{CD}}_+$

Formality

Theorem (Kontsevich, 1999; Tamarkin, 2003, $n = 2$)

The operad $\mathtt{D}_n$ is formal: $C_*(\mathtt{D}_n) \simeq H_*(\mathtt{D}_n)$.

Formality

Theorem (Kontsevich, 1999; Tamarkin, 2003, $n = 2$)

The operad $\mathtt{D}_n$ is formal: $C_*(\mathtt{D}_n) \simeq H_*(\mathtt{D}_n)$.

Rational homotopy theory: $H^*(\mathtt{P})$ vs Sullivan forms $\Omega^*(\mathtt{P})$

Theorem (Fresse–Willwacher 2015)

$\mathtt{D}_n \simeq_{\mathbb{Q}} \langle H^*(\mathtt{D}_n) \rangle^{\mathbb{L}} \implies \mathtt{D}_n$ is formal over $\mathbb{Q}$.

Formality

Theorem (Kontsevich, 1999; Tamarkin, 2003, $n = 2$)

The operad $\mathtt{D}_n$ is formal: $C_*(\mathtt{D}_n) \simeq H_*(\mathtt{D}_n)$.

Rational homotopy theory: $H^*(\mathtt{P})$ vs Sullivan forms $\Omega^*(\mathtt{P})$

Theorem (Fresse–Willwacher 2015)

$\mathtt{D}_n \simeq_{\mathbb{Q}} \langle H^*(\mathtt{D}_n)\rangle^{\mathbb{L}} \implies \mathtt{D}_n$ is formal over $\mathbb{Q}$.

In low dimensions:

- $\pi\mathtt{D}_1 \simeq_{\mathbb{Q}} \pi\langle H^*(\mathtt{D}_1)\rangle^{\mathbb{L}} \simeq \mathtt{PaP}$;
- Tamarkin: $\mathrm{Ass}(\mathbb{Q}) \neq \varnothing \implies \pi\mathtt{D}_2 \simeq_{\mathbb{Q}} \pi\langle H^*(\mathtt{D}_2)\rangle^{\mathbb{L}} \simeq \widehat{\mathtt{CD}}$.

Non-formality

$H_*(\mathtt{SC}) = \mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+$ is a "Voronov product"

Non-formality

$H_*(\mathtt{SC}) = \mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+$ is a "Voronov product"

$$\begin{aligned} & H^*(\mathtt{SC}) \cong \left(\mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+\right)^* \cong \mathtt{Ger}_+^* \otimes_0 \mathtt{Ass}_+^* \\ \implies & \langle H^*(\mathtt{SC})\rangle^{\mathbb{L}} \simeq \langle \mathtt{Ger}_+^* \rangle^{\mathbb{L}} \times_0 \langle \mathtt{Ass}_+^* \rangle^{\mathbb{L}} \\ \implies & \pi \langle H^*(\mathtt{SC})\rangle^{\mathbb{L}} \simeq_{\mathbb{Q}} \widehat{\mathtt{CD}}_+ \times_0 \mathtt{PaP}_+ \end{aligned}$$

Non-formality

$H_*(\mathtt{SC}) = \mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+$ is a "Voronov product"

$$\begin{aligned} & H^*(\mathtt{SC}) \cong \left(\mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+\right)^* \cong \mathtt{Ger}_+^* \otimes_0 \mathtt{Ass}_+^* \\ \implies & \langle H^*(\mathtt{SC})\rangle^{\mathbb{L}} \simeq \langle \mathtt{Ger}_+^* \rangle^{\mathbb{L}} \times_0 \langle \mathtt{Ass}_+^* \rangle^{\mathbb{L}} \\ \implies & \pi \langle H^*(\mathtt{SC})\rangle^{\mathbb{L}} \simeq_{\mathbb{Q}} \widehat{\mathtt{CD}}_+ \times_0 \mathrm{PaP}_+ \end{aligned}$$

Theorem (Livernet, 2015)

SC is not formal.

$$\implies \pi \mathrm{SC} \not\simeq_{\mathbb{Q}} \pi \langle H^*(\mathtt{SC})\rangle^{\mathbb{L}} \simeq_{\mathbb{Q}} \widehat{\mathtt{CD}}_+ \times_0 \mathrm{PaP}_+$$

Non-formality

$H_*(\mathtt{SC}) = \mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+$ is a "Voronov product"

$$
\begin{aligned}
& H^*(\mathtt{SC}) \cong (\mathtt{Ger}_+ \otimes_0 \mathtt{Ass}_+)^* \cong \mathtt{Ger}_+^* \otimes_0 \mathtt{Ass}_+^* \\
\implies & \langle H^*(\mathtt{SC}) \rangle^{\mathbb{L}} \simeq \langle \mathtt{Ger}_+^* \rangle^{\mathbb{L}} \times_0 \langle \mathtt{Ass}_+^* \rangle^{\mathbb{L}} \\
\implies & \pi \langle H^*(\mathtt{SC}) \rangle^{\mathbb{L}} \simeq_{\mathbb{Q}} \widehat{\mathtt{CD}}_+ \times_0 \mathtt{PaP}_+
\end{aligned}
$$

Theorem (Livernet, 2015)

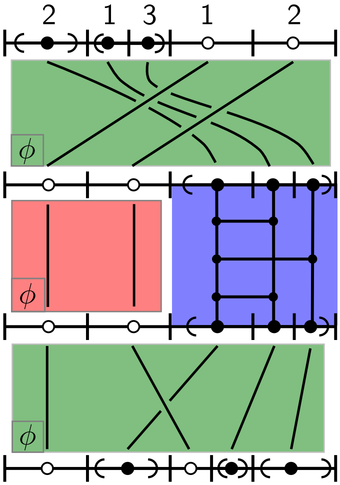
SC is not formal.

$$\implies \pi\mathtt{SC} \not\simeq_{\mathbb{Q}} \pi \langle H^*(\mathtt{SC}) \rangle^{\mathbb{L}} \simeq_{\mathbb{Q}} \widehat{\mathtt{CD}}_+ \times_0 \mathtt{PaP}_+$$

Remark

Not known if $\mathtt{SC}^{\mathrm{vor}} \simeq_{\mathbb{Q}}^{???} \langle H^*(\mathtt{SC}^{\mathrm{vor}}) \rangle^{\mathbb{L}} \simeq_{\mathbb{Q}} \langle \mathtt{Ger}^* \rangle^{\mathbb{L}} \times \langle \mathtt{Ass}^* \rangle^{\mathbb{L}}$

Rational model of $\pi\mathsf{SC}_+$



By reusing the proof of the previous theorem, we build a new operad $\mathtt{Pa}\widehat{\mathtt{PCD}}^{\phi}_{+}$ (for a given $\phi \in \mathsf{Ass}^{\mu}(\mathbb{Q})$).

Theorem (I.)

$$\pi\mathsf{SC}_+ \simeq_{\mathbb{Q}} \mathtt{Pa}\widehat{\mathtt{PCD}}^{\phi}_{+}.$$

Thanks!

Thank you for your attention!

arXiv:1507.06844

These slides to be available soon at
http://math.univ-lille1.fr/~idrissi