### 3.1.4 Linear combinations of random variables

Textbook correction

Sums of random variables arise naturally in many problems. In the health insurance example, the amount spent by the employee during her next five years of employment can be represented as $X_1 + X_2 + X_3 + X_4 + X_5$, where $X_1$ is the cost of the first year, $X_2$ the second year, etc. If the employee's domestic partner has health insurance with another employer, the total annual cost to the couple would be the sum of the costs for the employee ($X$) and for her partner ($Y$), or $X + Y$. In each of these examples, it is intuitively clear that the average cost would be the sum of the average of each term.

Sums of random variables represent a special case of linear combinations of variables.

---

**LINEAR COMBINATIONS OF RANDOM VARIABLES AND THEIR EXPECTED VALUES**

If $X$ and $Y$ are random variables, then a linear combination of the random variables is given by

$$aX + bY,$$

where $a$ and $b$ are constants. The mean of a linear combination of random variables is

$$E(aX + bY) = aE(X) + bE(Y) = a\mu_X + b\mu_Y.$$

---

The formula easily generalizes to a sum of any number of random variables. For example, the average health care cost for 5 years, given that the cost for services remains the same, is

$$E(X_1 + X_2 + X_3 + X_4 + X_5) = \cancel{E(5X_1)} = 5E(X_1) = (5)(1010) = \$5{,}050.$$

The formula implies that for a random variable $Z$, $E(a + Z) = a + E(Z)$. This could have been used when calculating the average health costs for the employee by defining $a$ as the fixed cost of the premium ($a = \$948$) and $Z$ as the cost of the physician visits. Thus, the total annual cost for a year could be calculated as: $E(a + Z) = a + E(Z) = \$948 + E(Z) = \$948 + .30(1 \times \$20) + .40(3 \times \$20) + .20(4 \times \$20) + 0.10(8 \times \$20) = \$1{,}010.00$.