第 8 次大作业

索引

1.

依题意，$f(X)=-\ln\big(p_3p_4x_1x_2(1-p_1x_1-p_2x_2)\big)=-\ln\big(p_3p_4g(X)\big)$。$g(X)=x_1x_2(1-p_1x_1-p_2x_2)$达到最大值即$f(X)$达到最小值。令

$$\frac{\partial g}{\partial x_1}=x_1-2p_1x_1x_2-p_2{x_2}^2=0$$
$$\frac{\partial g}{\partial x_2}=x_2-p_1{x_1}^2-2p_2x_1x_2=0$$

得到

$$x_1=\frac{1}{3p_1}$$
$$x_2=\frac{1}{3p_2}$$

于是

$$g(X)_{\max}=\frac{1}{27p_1p_2}$$
$$f(X)_{\min}=-\ln\frac{p_3p_4}{27p_1p_2}=3\ln 3+\ln(p_1)+\ln(p_2)-\ln(p_3)-\ln(p_4)$$

2.

多维情形的牛顿迭代法:$X_{k+1} = X_k - \mathrm{H}_f{}^{-1}(X_k)\nabla f(X_k)$。其中海森矩阵的逆可以求出，为

$$\mathrm{H}_f{}^{-1} = \frac{1}{f''_{11}f''_{22} - f''_{12}f''_{21}}\begin{bmatrix} f''_{22} & -f''_{12} \\ -f''_{21} & f''_{11} \end{bmatrix}$$

将各导数算出带入迭代即可。

3.

根据约束方程，$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{a_{11}a_{22}-a_{21}a_{12}}\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}\begin{bmatrix} b -_1 - a_{13}x_3 \\ b_2 - a_{23}x_3 \end{bmatrix}$，代入$f(X)$令$\frac{\partial f}{\partial x_3} = 0$可得

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{h_1(a_{12}a_{23} - a_{22}a_{13})^2 + h_2(a_{13}a_{21} - a_{23}a_{11})^2 + h_3(a_{11}a_{22} - a_{21}a_{12})^2} \cdot$$

$$\begin{bmatrix} -h_2(a_{13}a_{21} - a_{23}a_{11})(a_{23}b_1 - a_{13}b_2) + h_3(a_{11}a_{22} - a_{21}a_{12})(a_{22}b_1 - a_{12}b_2) \\ -h_3(a_{11}a_{22} - a_{21}a_{12})(a_{21}b_1 - a_{11}b_2) + h_1(a_{12}a_{23} - a_{22}a_{13})(a_{23}b_1 - a_{13}b_2) \\ -h_1(a_{12}a_{23} - a_{22}a_{13})(a_{22}b_1 - a_{12}b_2) + h_2(a_{13}a_{21} - a_{23}a_{11})(a_{21}b_1 - a_{11}b_2) \end{bmatrix}$$

得到X以后带入算出$f(X)$即可。