# Precision Control of an Autonomous Surface Vessel

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# Agenda



- **Introduction**
  - **Use Case**
- **System Description**
- **Model**
  - **Reference Frames**
  - **Model Equations**
  - **Model Verification**
- Control Approach
- Sensor Fusion
- Inner Controller
- Outer Controller
- Results
- Conclusion

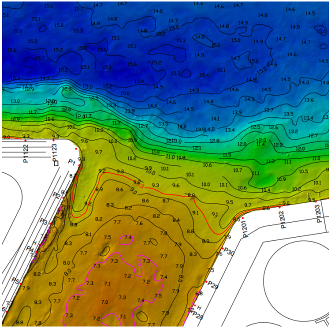
# Introduction



- Environmental monitoring
- Marine biological research
- Bathymetric measurements
- Control theory for an ASV

# Introduction

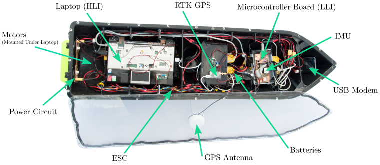
## Use Case

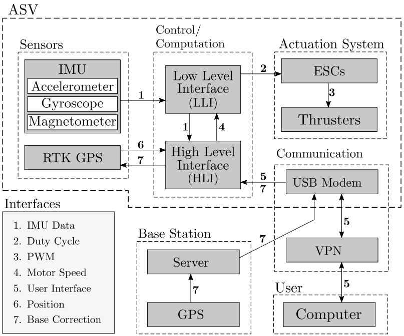


- Depth map used by Port of Aalborg
- Problem: No recent knowledge of depths of the port
- Solution: Automate smaller unmanned vessel

# System Description

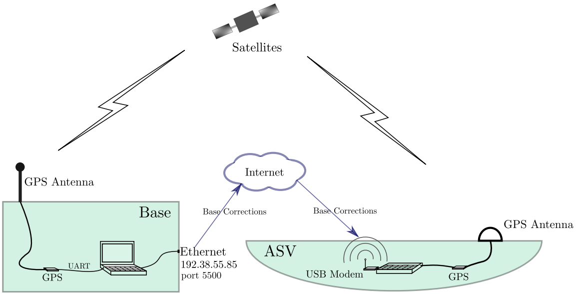


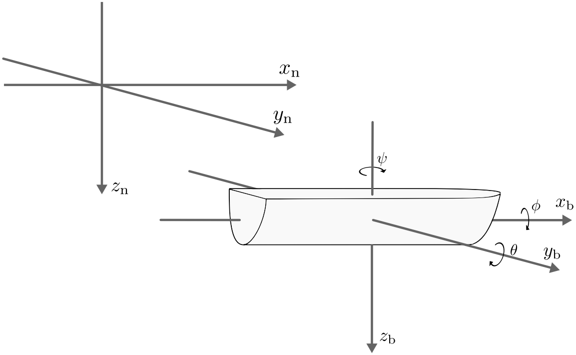


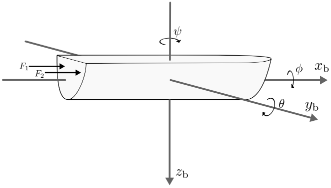
# System Description
## RTK GPS

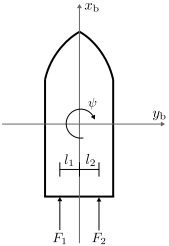







- Inertial Frame (NED)
- Body Frame

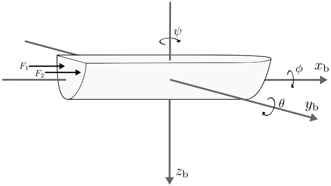


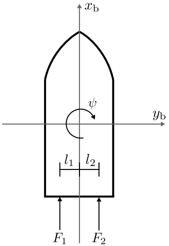




- Rigid Body Dynamics

$$\sum F = m\ddot{x}$$
$$\sum \tau = I\ddot{\theta}$$

- Hydrostatics
  - Buoyancy Force
- Hydrodynamics
  - Viscous Damping

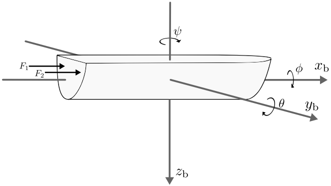


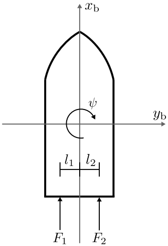




$$
\begin{aligned}
m\ddot{x}_\mathrm{b} &= F_1 + F_2 - d_{\dot{x}_\mathrm{b}}\dot{x}_\mathrm{b} + F_{x_\mathrm{b}} \\
m\ddot{y}_\mathrm{b} &= -d_{\dot{y}_\mathrm{b}}\dot{y}_\mathrm{b} + F_{y_\mathrm{b}} \\
m\ddot{z}_\mathrm{b} &= -d_{\dot{z}_\mathrm{b}}\dot{z}_\mathrm{b} + F_{z_\mathrm{b}} \\
I_\mathrm{x}\ddot{\phi} &= -d_{\dot{\phi}}\dot{\phi} + T_\phi \\
I_\mathrm{y}\ddot{\theta} &= -d_{\dot{\theta}}\dot{\theta} + T_\theta \\
I_\mathrm{z}\ddot{\psi} &= F_1 l_1 - F_2 l_2 - d_{\dot{\psi}}\dot{\psi}
\end{aligned}
$$

# Model

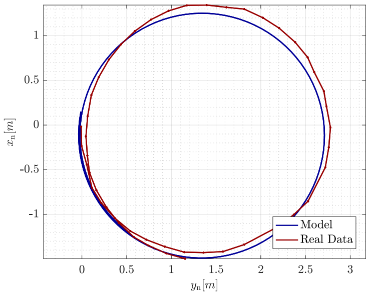
## Linearized Model Equations

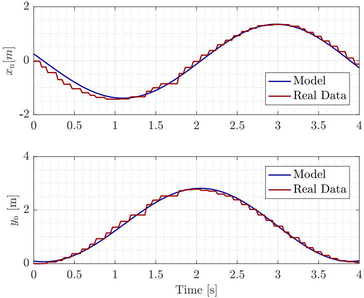



$$
\begin{aligned}
m\ddot{x}_b &= F_1 + F_2 - d_{\dot{x}_b}\dot{x}_b \\
m\ddot{y}_b &= -d_{\dot{y}_b}\dot{y}_b \\
m\ddot{z}_b &= -d_{\dot{z}_b}\dot{z}_b - \rho g A_{wp}\tilde{z}_n \\
I_x\ddot{\phi} &= -d_{\dot{\phi}}\dot{\phi} - \rho g V\overline{GM_T}\cdot\phi \\
I_y\ddot{\theta} &= -d_{\dot{\theta}}\dot{\theta} - \rho g V\overline{GM_L}\cdot\theta \\
I_z\ddot{\psi} &= F_1 l_1 - F_2 l_2 - d_{\dot{\psi}}\dot{\psi}
\end{aligned}
$$

- Verified model

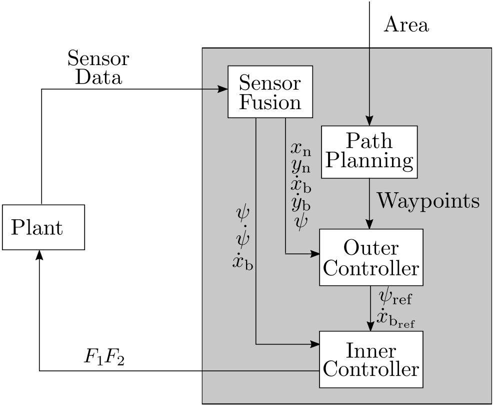


# Agenda

# Control Approach





# Sensor Fusion

## Structure

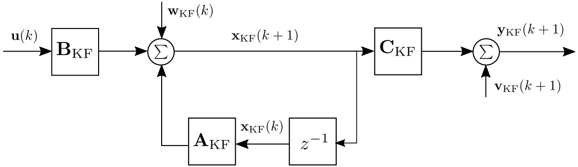


- Fuses GPS and IMU data
- Achieved using a Kalman filter
- Sensor fusion contains
  - Attitude
  - Position

# Sensor Fusion
## Signal Model



$$\mathbf{x}_{\mathrm{KF}}(k+1) = \mathbf{A}_{\mathrm{KF}}\mathbf{x}_{\mathrm{KF}}(k) + \mathbf{B}_{\mathrm{KF}}\mathbf{u}(k) + \mathbf{w}_{\mathrm{KF}}(k)$$
$$\mathbf{y}_{\mathrm{KF}}(k+1) = \mathbf{C}_{\mathrm{KF}}\mathbf{x}_{\mathrm{KF}}(k+1) + \mathbf{v}_{\mathrm{KF}}(k+1)$$




- w(k) and v(k) are assumed white Gaussian
- Matrices $\mathbf{Q}_{\mathrm{KF}}$ and $\mathbf{R}_{\mathrm{KF}}$ are the respective covariance matrices

$$\mathbf{u} = \begin{bmatrix} F_1 & F_2 \end{bmatrix}^{\mathrm{T}}$$

- Attitude

$$\mathbf{x}_{\mathrm{att}} = \begin{bmatrix} \phi & \theta & \psi & \dot{\phi} & \dot{\theta} & \dot{\psi} & \ddot{\phi} & \ddot{\theta} & \ddot{\psi} \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{y}_{\mathrm{att}} = \begin{bmatrix} \phi_{\mathrm{acc}} & \theta_{\mathrm{acc}} & \psi_{\mathrm{mag}} & \dot{\phi}_{\mathrm{gyro}} & \dot{\theta}_{\mathrm{gyro}} & \dot{\psi}_{\mathrm{gyro}} \end{bmatrix}^{\mathrm{T}}$$

- Position

$$\mathbf{x}_{\mathrm{pos}} = \begin{bmatrix} x_{\mathrm{n}} & y_{\mathrm{n}} & \dot{x}_{\mathrm{b}} & \dot{y}_{\mathrm{b}} & \ddot{x}_{\mathrm{b}} & \ddot{y}_{\mathrm{b}} \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{y}_{\mathrm{pos}} = \begin{bmatrix} x_{\mathrm{n,GPS}} & y_{\mathrm{n,GPS}} & \ddot{x}_{\mathrm{b,acc}} & \ddot{y}_{\mathrm{b,acc}} \end{bmatrix}^{\mathrm{T}}$$

- Step 0: Initialization

$$\hat{\mathbf{x}}_{\mathrm{KF}}(0|0) = \mathbf{0}$$
$$\mathbf{P}_{\mathrm{KF}}(0|0) = \mathbf{Q}_{\mathrm{KF}}$$

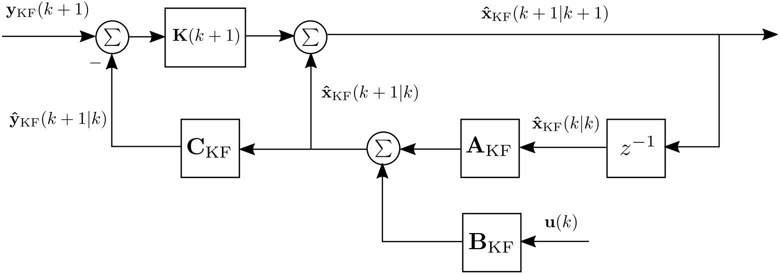
- Step 1: Prediction
- Step 2: Update

- Step 0: Initialization
- Step 1: Prediction

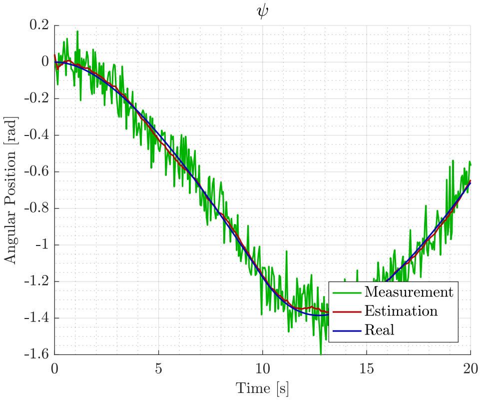
$$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k) = \mathbf{A}_{\mathrm{KF}}\hat{\mathbf{x}}_{\mathrm{KF}}(k|k) + \mathbf{B}_{\mathrm{KF}}\mathbf{u}(k)$$
$$\mathbf{P}(k+1|k) = \mathbf{A}_{\mathrm{KF}}\mathbf{P}(k|k)\mathbf{A}_{\mathrm{KF}}^{\mathrm{T}} + \mathbf{Q}_{\mathrm{KF}}$$

- Step 2: Update

$$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k+1) = \hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k) + \mathbf{K}(k+1)\left[\mathbf{y}_{\mathrm{KF}}(k+1) - \mathbf{C}_{\mathrm{KF}}\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k)\right]$$
$$\mathbf{P}(k+1|k+1) = \left[\mathbf{I} - \mathbf{K}(k+1)\mathbf{C}_{\mathrm{KF}}^{\mathrm{T}}\right]\mathbf{P}(k+1|k)$$



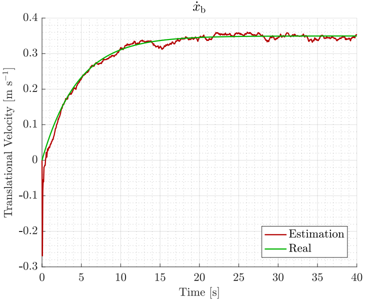
$\mathbf{y}_{\mathrm{KF}}(k+1)$
$-$
$\mathbf{K}(k+1)$
$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k+1)$
$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k)$
$\hat{\mathbf{y}}_{\mathrm{KF}}(k+1|k)$
$\mathbf{C}_{\mathrm{KF}}$
$\mathbf{A}_{\mathrm{KF}}$
$\hat{\mathbf{x}}_{\mathrm{KF}}(k|k)$
$z^{-1}$
$\mathbf{B}_{\mathrm{KF}}$
$\mathbf{u}(k)$

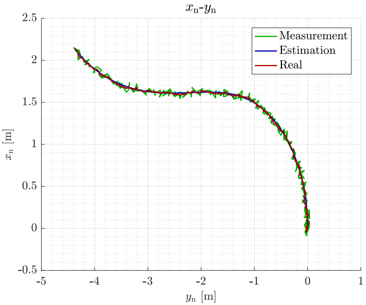


ψ
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
-1.6
Angular Position [rad]
0
5
10
15
20
Time [s]
Measurement
Estimation
Real

# Sensor Fusion
## Position Kalman Filter

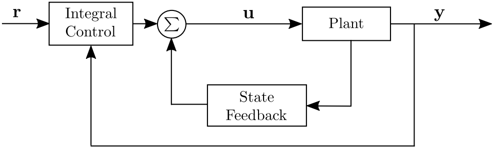




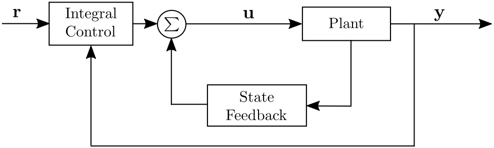
# Agenda

# Inner Controller

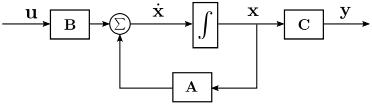


# Inner Controller



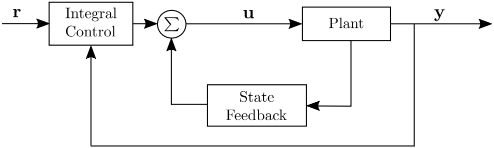



- Plant

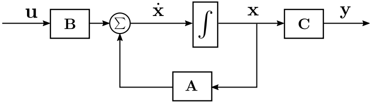


# Inner Controller






- Plant

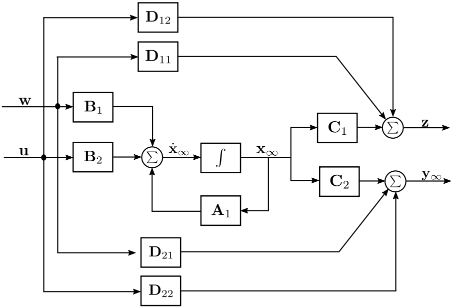



- Approaches
  - $\mathcal{H}_\infty$ Controller
  - Linear Quadratic Regulator

- Suboptimal $\mathcal{H}_\infty$ controller

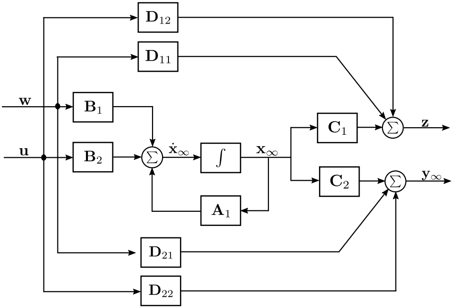
Find an internally stabilizing controller that provides a closed loop $\mathcal{H}_\infty$ norm less than some bound $\gamma$

- System structure



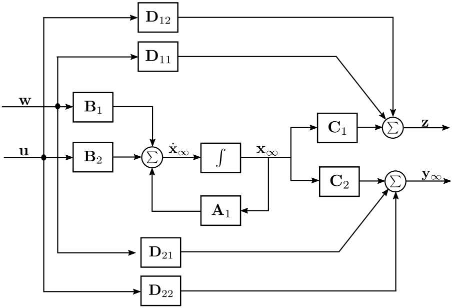

$$
\begin{aligned}
\dot{\mathbf{x}}_\infty(t) &= \mathbf{A}_1\mathbf{x}_\infty(t) + \mathbf{B}_1\mathbf{w}(t) + \mathbf{B}_2\mathbf{u}(t)\\
\mathbf{z}(t) &= \mathbf{C}_1\mathbf{x}_\infty(t) + \mathbf{D}_{11}\mathbf{w}(t) + \mathbf{D}_{12}\mathbf{u}(t)\\
\mathbf{y}_\infty(t) &= \mathbf{C}_2\mathbf{x}_\infty(t) + \mathbf{D}_{21}\mathbf{w}(t) + \mathbf{D}_{22}\mathbf{u}(t)
\end{aligned}
$$

- System structure



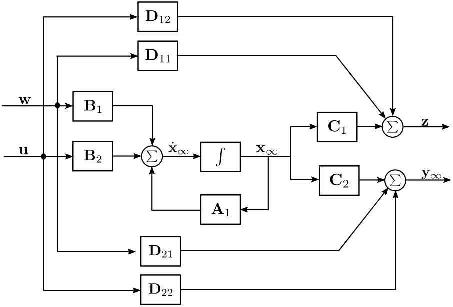

$$\mathbf{u}(t) = \begin{bmatrix} F_1 & F_2 \end{bmatrix}^{\mathrm{T}}$$

- System structure



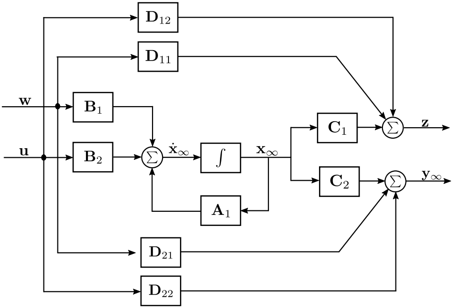

$$\mathbf{w}(t) = \begin{bmatrix} \psi_{\mathrm{ref}} & \dot{x}_{\mathrm{b,ref}} & F_{\mathrm{wc}} & \tau_{\mathrm{wc}} & F_{\mathrm{wave}} & \tau_{\mathrm{wave}} & n_\psi & n_{\dot{x}_{\mathrm{b}}} \end{bmatrix}^{\mathrm{T}}$$

- System structure



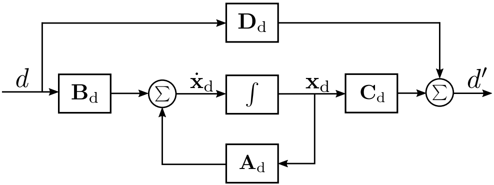

$$\mathbf{y}_\infty(t) = \begin{bmatrix} \psi & \dot{x}_\mathrm{b} & \mathbf{x}_\mathrm{I}^\mathrm{T} \end{bmatrix}^\mathrm{T}$$

- System structure



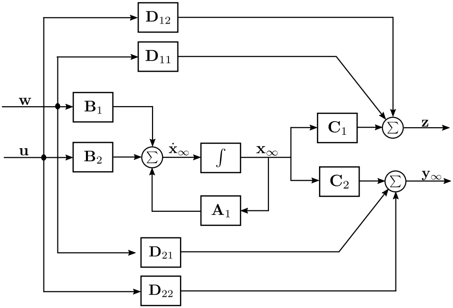

$$\mathbf{x}_\infty(t) = \begin{bmatrix} \psi & \dot{\psi} & \dot{x}_\mathrm{b} & x_{I_\psi} & x_{I_{\dot{x}_\mathrm{b}}} & x_{F_\mathrm{wc}} & x_{\tau_\mathrm{wc}} & x_{F_\mathrm{wave}} & x_{\tau_\mathrm{wave}} & x_{n_\psi} & x_{n_{\dot{x}_\mathrm{b}}} \end{bmatrix}^\mathrm{T}$$

- Disturbance model




$$\frac{d'}{d} = \frac{a}{s+a} \rightarrow \dot{d}' = -ad' + ad \rightarrow \begin{cases} \dot{x}_\mathrm{d} = -ax_\mathrm{d} + ad \\ d' = x_\mathrm{d} \end{cases}$$

- System structure




$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}_\infty^{\mathrm{T}} & \mathbf{u}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

- Controller design parameters ($\gamma$, $\mathbf{C}_1$, $\mathbf{D}_{12}$)

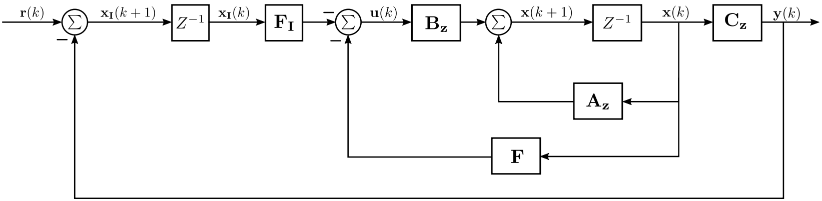
$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{W}_{\mathrm{x}} & \mathbf{0}_{3x2} & \mathbf{0}_{3x2} & \mathbf{0}_{3x2} & \mathbf{0}_{3x2} \\ \mathbf{0}_{2x3} & \mathbf{W}_{\mathrm{I}} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} \\ \mathbf{0}_{2x3} & \mathbf{0}_{2x2} & \mathbf{W}_{\mathrm{wc}} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} \\ \mathbf{0}_{2x3} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{W}_{\mathrm{wave}} & \mathbf{0}_{2x2} \\ \mathbf{0}_{2x3} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{W}_{\mathrm{noise}} \\ \mathbf{0}_{2x3} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} \\ \mathbf{0}_{2x3} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} \end{bmatrix} \qquad \mathbf{D}_{12} = \begin{bmatrix} \mathbf{0}_{2x3} \\ \mathbf{0}_{2x2} \\ \mathbf{0}_{2x2} \\ \mathbf{0}_{2x2} \\ \mathbf{0}_{2x2} \\ \mathbf{W}_{\mathrm{u}} \end{bmatrix}$$

- Feedback gain

$$\mathbf{X}_\infty = Ric \begin{bmatrix} \mathbf{A}_1 & \gamma^{-2}\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} - \mathbf{B}_2\mathbf{B}_2^{\mathrm{T}} \\ -\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 & -\mathbf{A}_1^{\mathrm{T}} \end{bmatrix}$$

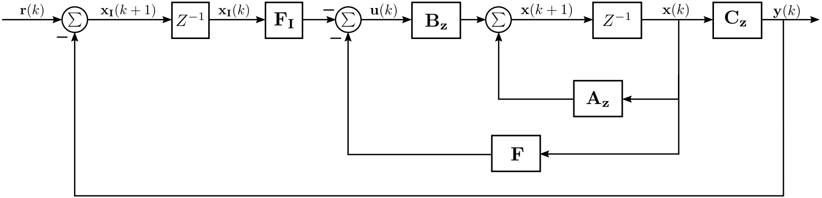
$$\mathbf{F}_\infty = -\mathbf{B}_2^{\mathrm{T}}\mathbf{X}_\infty$$

# Agenda


$\mathbf{r}(k)$
$-$
$\mathbf{x_I}(k+1)$
$Z^{-1}$
$\mathbf{x_I}(k)$
$\mathbf{F_I}$
$-$
$-$
$\mathbf{u}(k)$
$\mathbf{B_z}$
$\mathbf{x}(k+1)$
$Z^{-1}$
$\mathbf{x}(k)$
$\mathbf{C_z}$
$\mathbf{y}(k)$
$\mathbf{A_z}$
$\mathbf{F}$

# Inner Controller
## Linear Quadratic Controller Design






$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}_{\mathrm{I}}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\mathrm{z},3\mathrm{x}3} & \mathbf{0}_{3\mathrm{x}2} \\ -\mathbf{C}_{\mathrm{z},2\mathrm{x}3} & \mathbf{I}_{2\mathrm{x}2} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_{\mathrm{I}}(k) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{\mathrm{z},3\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}2} \end{bmatrix} \mathbf{u}(k) + \begin{bmatrix} \mathbf{0}_{3\mathrm{x}2} \\ \mathbf{I}_{2\mathrm{x}2} \end{bmatrix} \mathbf{r}(k)$$

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{C}_{\mathrm{z},2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_{\mathrm{I}}(k) \end{bmatrix}$$

- Discrete cost function

$$\mathcal{J}_{\mathrm{z}} = \sum_{k=0}^{\infty} \mathbf{x}^{\mathrm{T}}(k)\mathbf{Q}_{\mathrm{z}}\mathbf{x}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}_{\mathrm{z}}\mathbf{u}(k)$$

- Continuous cost function

$$\mathcal{J} = \int_0^\infty \mathbf{x}^\mathrm{T}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^\mathrm{T}(t)\mathbf{R}\mathbf{u}(t)\; dt$$

- Continuous cost function

$$\mathcal{J} = \int_0^\infty \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)\ dt$$

$$Q = diag\left(\frac{1}{{\psi_{\max}}^2}\ ,\ \frac{1}{{\dot{\psi}_{\max}}^2}\ ,\ \frac{1}{{\dot{x}_{b,\max}}^2}\ ,\ \frac{1}{{x_{\mathrm{I},\psi,\max}}^2}\ ,\ \frac{1}{{x_{\mathrm{I},\dot{x}_b,\max}}^2}\right)$$

# Inner Controller
## Linear Quadratic Controller Design



- Continuous cost function

$$\mathcal{J} = \int_0^{\infty} \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)\; dt$$

$$Q = diag\left(\frac{1}{{\psi_{\mathrm{max}}}^2}, \frac{1}{{\dot{\psi}_{\mathrm{max}}}^2}, \frac{1}{{\dot{x}_{b,\mathrm{max}}}^2}, \frac{1}{{x_{\mathrm{I},\psi,\mathrm{max}}}^2}, \frac{1}{{x_{\mathrm{I},\dot{x}_b,\mathrm{max}}}^2}\right)$$

$$R = diag\left(\frac{1}{{F_{1_{\mathrm{max}}}}^2}, \frac{1}{{F_{2_{\mathrm{max}}}}^2}\right)$$

- Discretize the cost function to get $\mathbf{Q}_z$ and $\mathbf{R}_z$
- Solve the discrete-time algebraic Riccati equation
- Get the feedback gains

$$\mathbf{u}(k) = -\begin{bmatrix} \mathbf{F} & \mathbf{F}_\mathrm{I} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_\mathrm{I}(k) \end{bmatrix}$$

Simulation of LQR and $\mathcal{H}_\infty$ design

- Disturbances from wind and current
  - $\pm$1.5 N along $\dot{x}_b$
  - $\pm$1.5 N·m around $z_b$
- The waves are sinusoidal, with frequency varying between 0-10 Hz

Simulation of LQR and $\mathcal{H}_\infty$ design

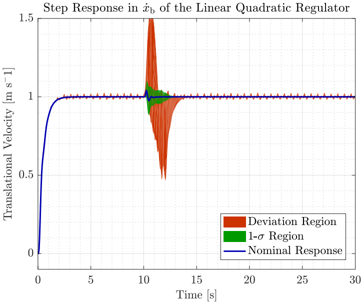
- Disturbances from wind and current
  - $\pm 1.5$ N along $\dot{x}_b$
  - $\pm 1.5$ N·m around $z_b$
- The waves are sinusoidal, with frequency varying between 0-10 Hz
- The parameters are varied $\pm 20\%$
  - Mass, $m$
  - Moment of inertia, $I_z$, around $z_b$
  - The damping coefficients $d_x$ and $d_\psi$
  - The lengths $l_1$ and $l_2$

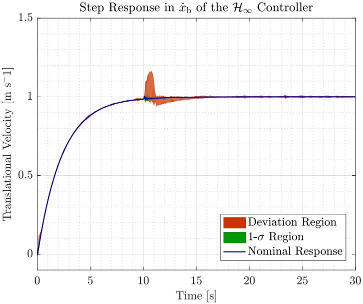
Simulation of LQR and $\mathcal{H}_\infty$ design

- Disturbances from wind and current
  - $\pm 1.5$ N along $\dot{x}_b$
  - $\pm 1.5$ N·m around $z_b$
- The waves are sinusoidal, with frequency varying between 0-10 Hz
- The parameters are varied $\pm 20\%$
  - Mass, $m$
  - Moment of inertia, $I_z$, around $z_b$
  - The damping coefficients $d_x$ and $d_\psi$
  - The lengths $l_1$ and $l_2$
- Monte Carlo simulations with 1000 realizations

# Inner Controller
## Comparison of the Controllers



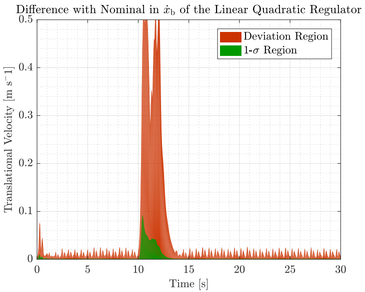


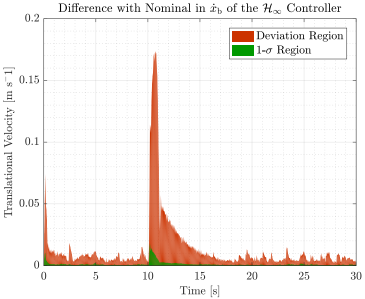



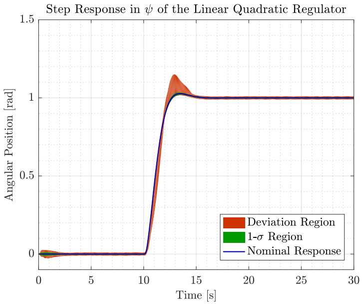

- LQR gives a faster response
- $\mathcal{H}_\infty$ controller is more robust to disturbances
- The disturbance at 10 s is due to the change in reference in $\psi$

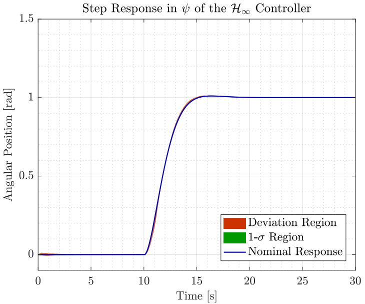
# Inner Controller

## Comparison of the Controllers



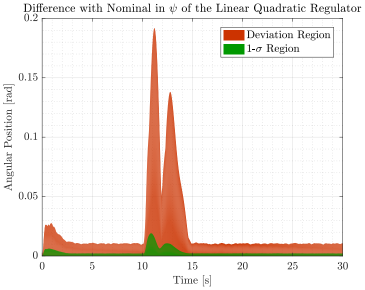


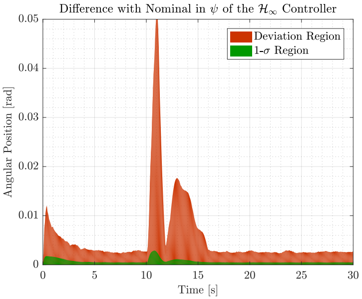




- LQR gives a faster response
- $\mathcal{H}_\infty$ controller is more robust to disturbances and it is faster than when tracking $\dot{x}_\mathrm{b}$

# Inner Controller
## Comparison of the Controllers





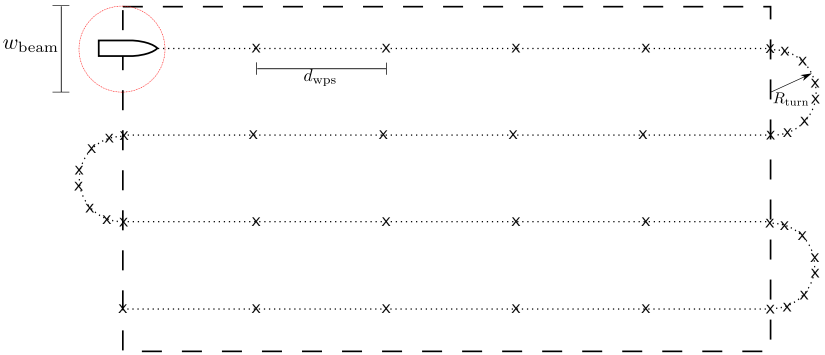
# Agenda



- Introduction
- System Description
- Model
- Control Approach
- Sensor Fusion
- Inner Controller
- **Outer Controller**
  - **Path Generation Algorithm**
  - **Path Following Algorithm**
- **Results**
  - **Controller Results**
  - **Implementation Results**
- **Conclusion**

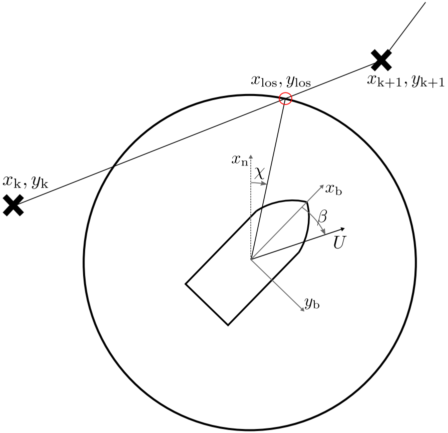
# Outer Controller
## Path Generation Algorithm





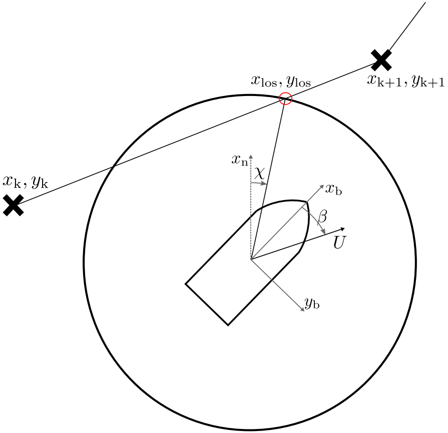
# Outer Controller

## Path Following Algorithm



# Outer Controller

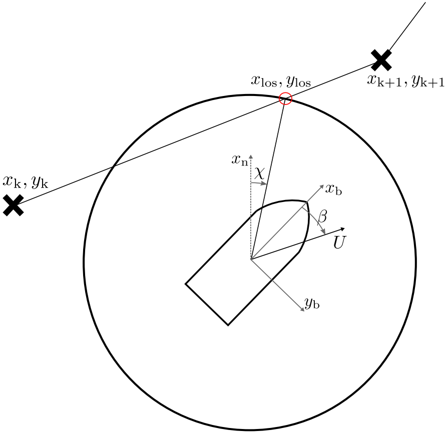
## Path Following Algorithm




$$\chi = \arctan\left(\frac{y_{\mathrm{LOS}} - y_{\mathrm{n}}}{x_{\mathrm{LOS}} - x_{\mathrm{n}}}\right)$$

# Outer Controller

## Path Following Algorithm



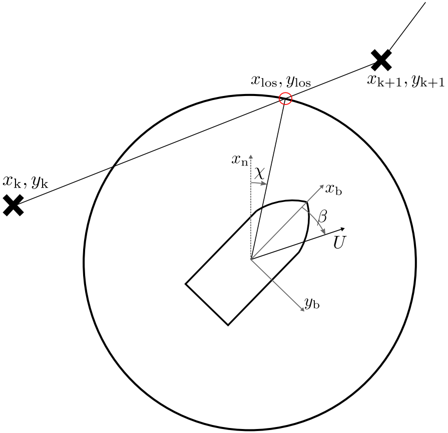

$$\chi = \text{arctan}\left(\frac{y_{\rm LOS} - y_{\rm n}}{x_{\rm LOS} - x_{\rm n}}\right)$$

$$\beta = \text{arctan}\left(\frac{\dot{y}_{\rm b}}{\dot{x}_{\rm b}}\right)$$

# Outer Controller

## Path Following Algorithm



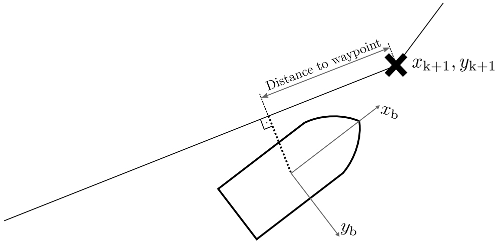



$$\chi = \arctan\left(\frac{y_{\mathrm{LOS}} - y_{\mathrm{n}}}{x_{\mathrm{LOS}} - x_{\mathrm{n}}}\right)$$

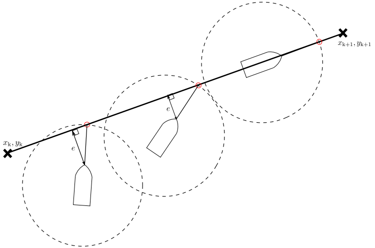
$$\beta = \arctan\left(\frac{\dot{y}_{\mathrm{b}}}{\dot{x}_{\mathrm{b}}}\right)$$

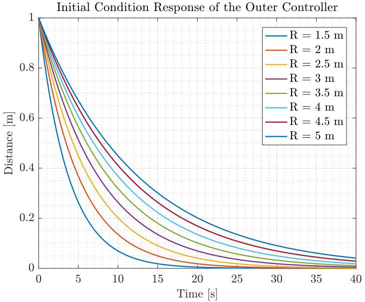
$$\psi_{\mathrm{ref}} = \chi - \beta$$

- Change active waypoints

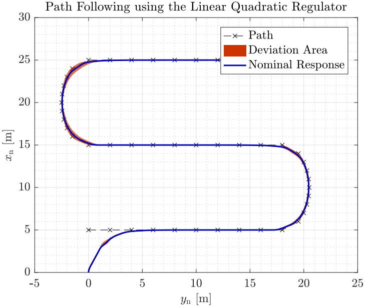


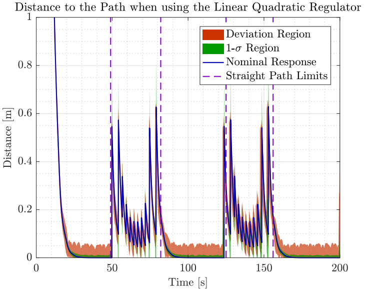
- Convergence to the path



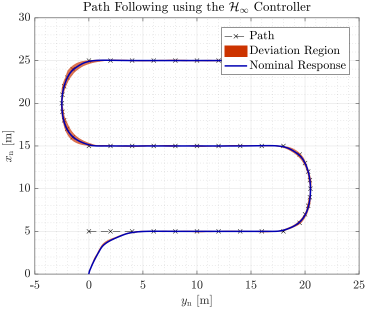



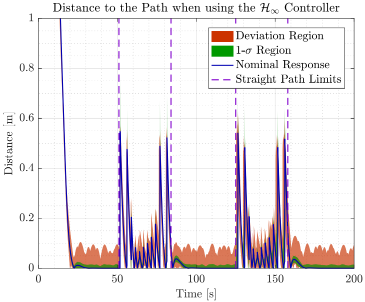
- LQR as inner controller



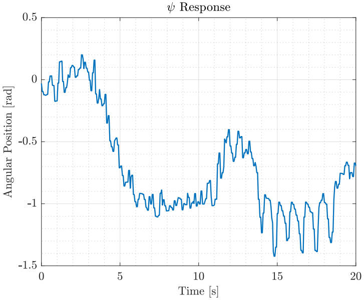



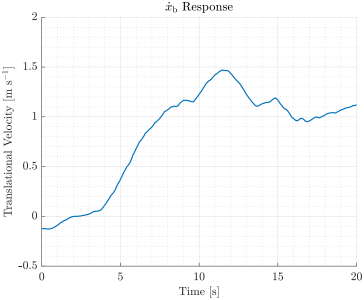
- Robust controller as inner controller



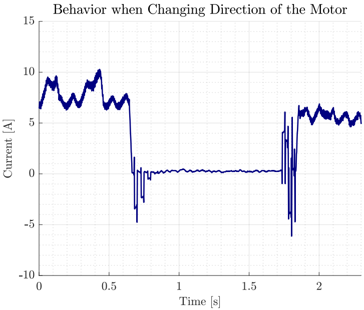



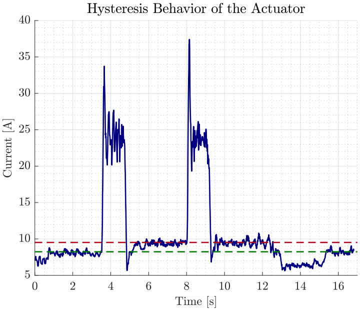
- Inner controller test



- Actuator tests



# Conclusion



- The estimator has been tuned and tested through simulation.
- The controller has also been analyzed though simulations that include disturbances, noise and varying parameters.
- The simulated results have not been fully replicated in the real vessel, but they show a promising behavior of the control system.

# Precision Control of an Autonomous Surface Vessel

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