

# Precision Control of an Autonomous Surface Vessel



Alejandro Alonso García, Anders Egelund Kjeldal, Himal Kooverjee,  
Niels Skov Vestergaard, Noelia Villamarzo Arruñada

# Agenda



- ▶ **Introduction**
  - Use Case
- ▶ **System Description**
- ▶ **Model**
  - Reference Frames
  - Model Equations
  - Model Verification
- ▶ Control Approach
- ▶ Sensor Fusion
- ▶ Inner Controller
- ▶ Outer Controller
- ▶ Results
- ▶ Conclusion

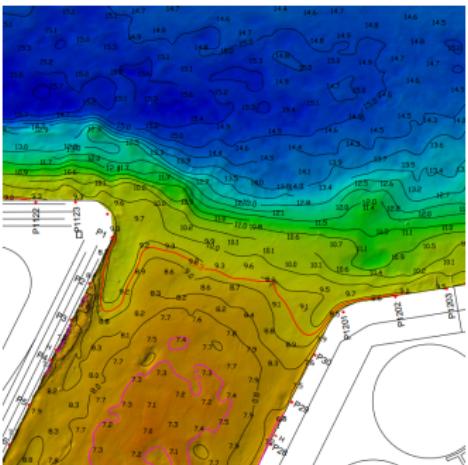
# Introduction



- ▶ Applications of an Autonomous Surface Vessel (ASV)
- ▶ Bathymetric Measurements
- ▶ Control of an ASV

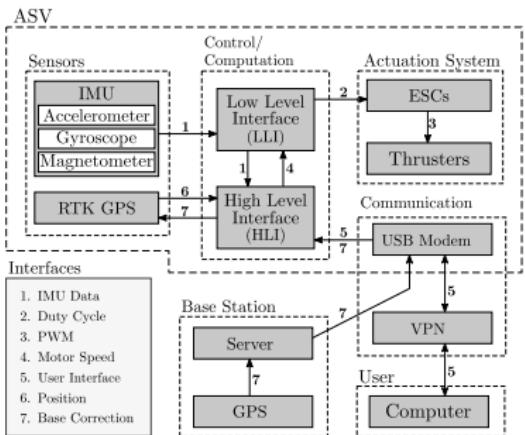
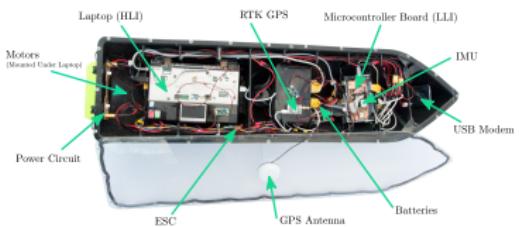
# Introduction

## Use Case



- ▶ Depth map used by Port of Aalborg
  - ▶ Problem: No recent knowledge of depths of the port
  - ▶ Solution: Automate smaller unmanned vessel

# System Description

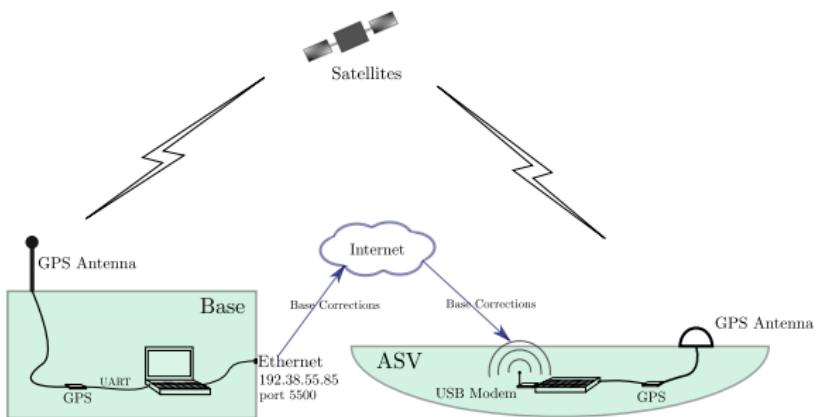


# System Description

## RTK GPS

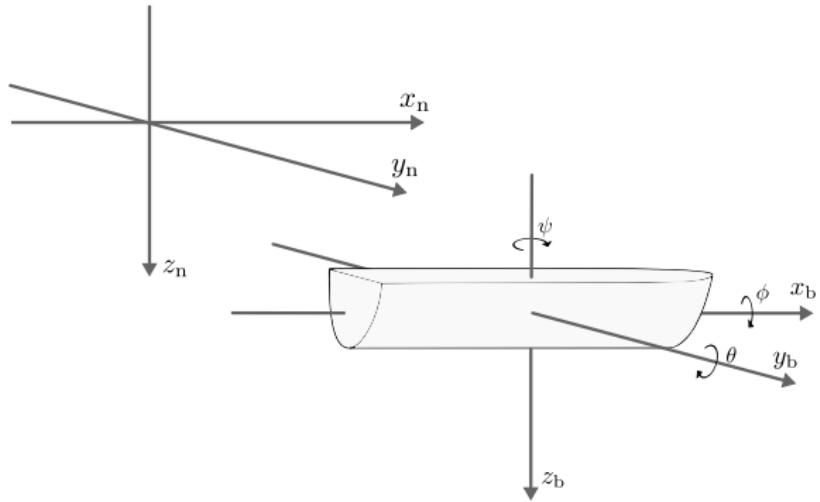


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# Model

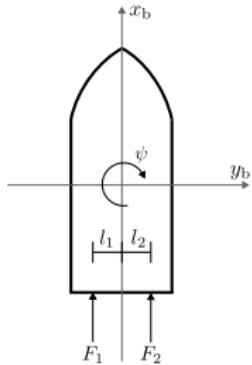
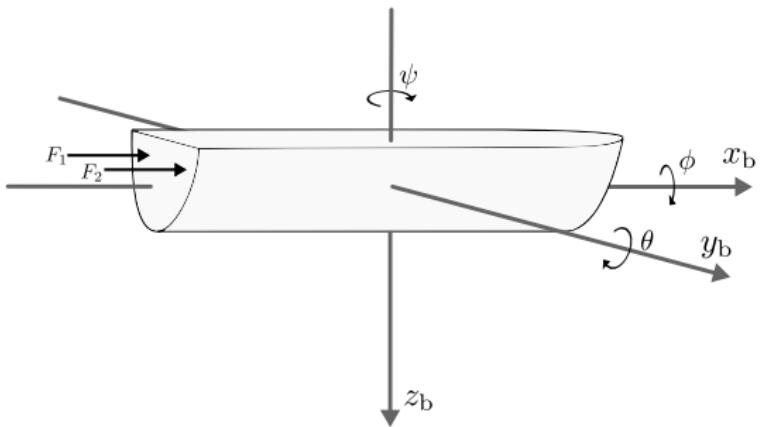
## Reference Frames



- ▶ Inertial Frame
- ▶ Body Frame

# Model

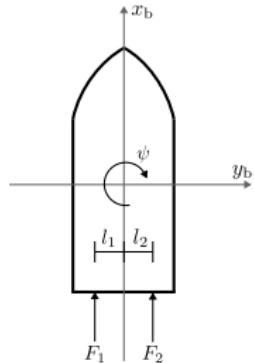
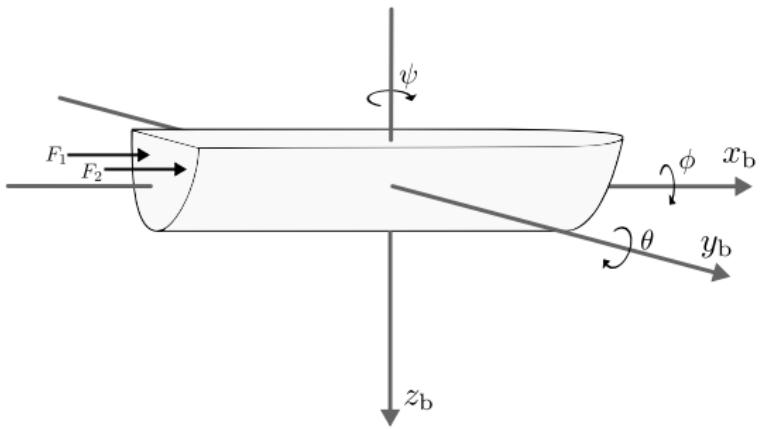
## Model Dynamics



- ▶ Rigid Body Dynamics
- ▶ Hydrostatics
- ▶ Hydrodynamics

# Model

## Rigid Body Dynamics

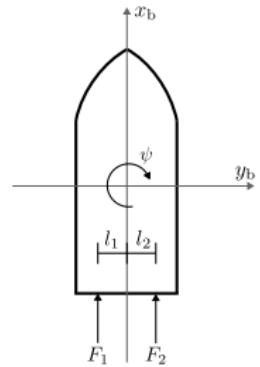
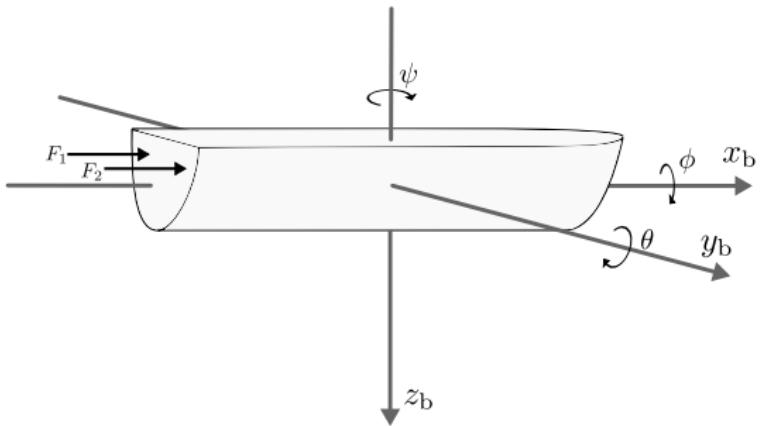


$$\sum F = m\ddot{x}$$

$$\sum \tau = I\ddot{\theta}$$

# Model

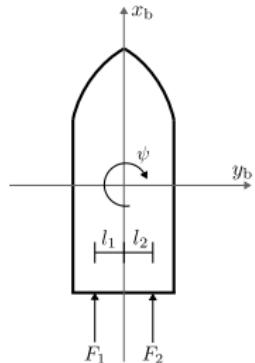
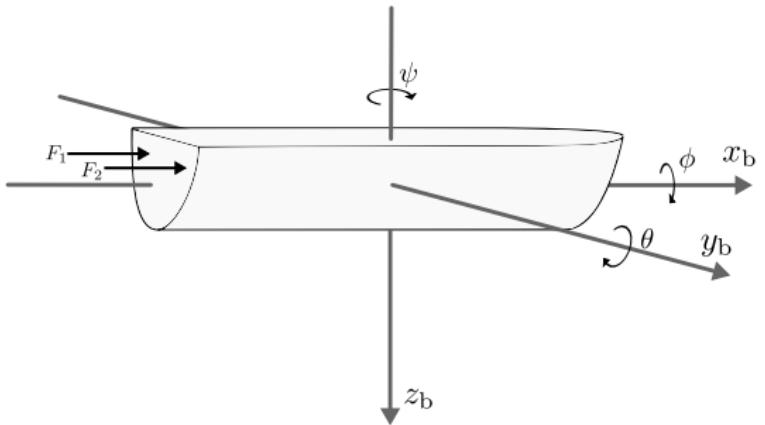
## Hydrostatics



- ▶ Buoyancy Force

# Model

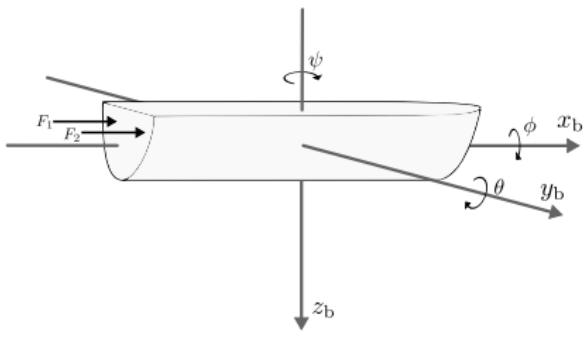
## Hydrodynamics



- ▶ Added mass
- ▶ Viscous Damping

# Model

## Model Equations



$$m\ddot{x}_b = F_1 + F_2 - d_{\dot{x}_b} \dot{x}_b + F_{x_b}$$

$$m\ddot{y}_b = -d_{\dot{y}_b} \dot{y}_b + F_{y_b}$$

$$m\ddot{z}_b = -d_{\dot{z}_b} \dot{z}_b + F_{z_b}$$

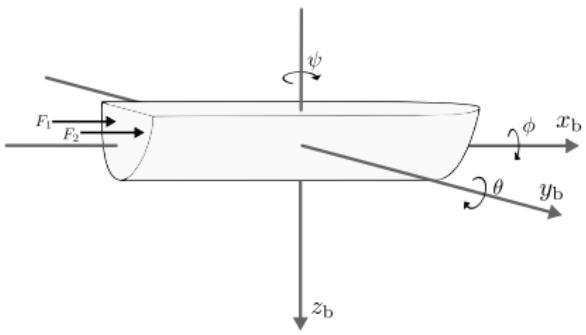
$$I_x \ddot{\phi} = -d_{\dot{\phi}} \dot{\phi} + T_\phi$$

$$I_y \ddot{\theta} = -d_{\dot{\theta}} \dot{\theta} + T_\theta$$

$$I_z \ddot{\psi} = F_1 l_1 - F_2 l_2 - d_{\dot{\psi}} \dot{\psi}$$

# Model

## Linearized Model Equations



$$m\ddot{x}_b = F_1 + F_2 - d_{\dot{x}_b} \dot{x}_b$$

$$m\ddot{y}_b = -d_{\dot{y}_b} \dot{y}_b$$

$$m\ddot{z}_b = -d_{\dot{z}_b} \dot{z}_b - \rho g A_w p \tilde{z}_n$$

$$I_x \ddot{\phi} = -d_{\dot{\phi}} \dot{\phi} - \rho g V \overline{GM_T} \cdot \phi$$

$$I_y \ddot{\theta} = -d_{\dot{\theta}} \dot{\theta} - \rho g V \overline{GM_L} \cdot \theta$$

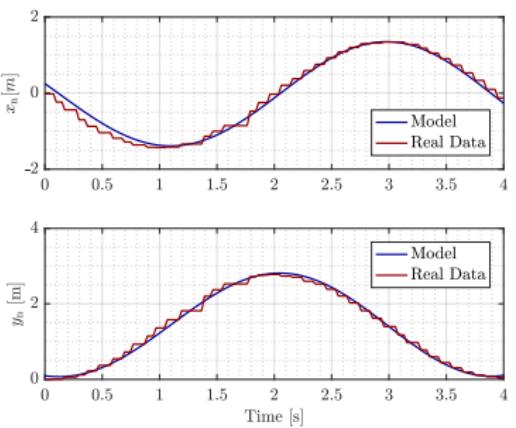
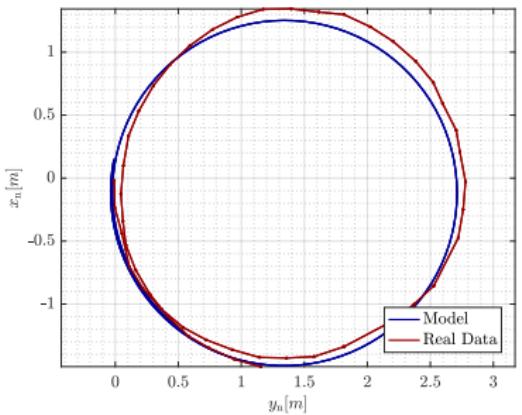
$$I_z \ddot{\psi} = F_1 I_1 - F_2 I_2 - d_{\dot{\psi}} \dot{\psi}$$

# Model

## Model Verification



### ► Verified Nonlinear Model

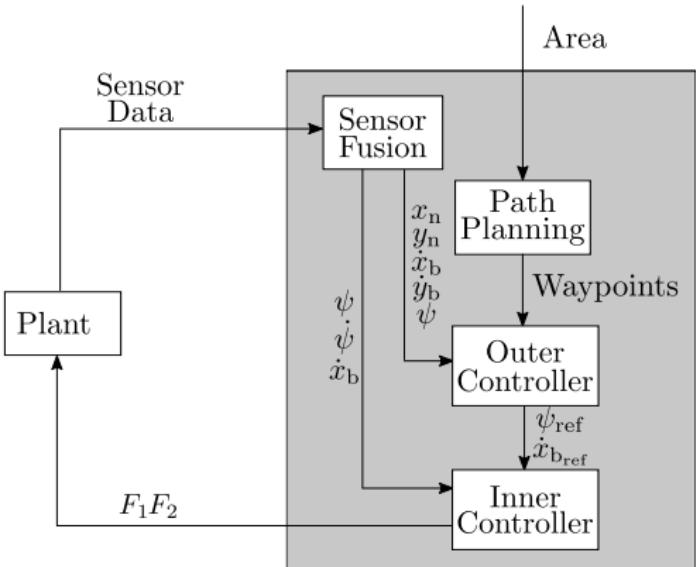


# Agenda



- ▶ Introduction
- ▶ System Description
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- ▶ **Control Approach**
- ▶ **Sensor Fusion**
  - Attitude Kalman Filter
  - Position Kalman Filter
- ▶ Inner Controller
- ▶ Outer Controller
- ▶ Results
- ▶ Conclusion

# Control Approach



# Sensor Fusion

## Structure



- ▶ Fuses GPS and IMU data
- ▶ Achieved using a Kalman filter
- ▶ Sensor fusion contains
  - ▶ Attitude
  - ▶ Position

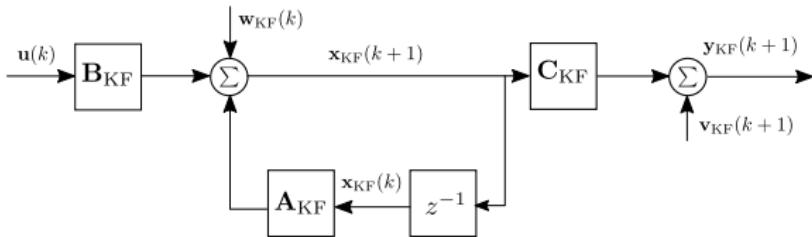
# Sensor Fusion

## Signal Model



$$\mathbf{x}_{\text{KF}}(k+1) = \mathbf{Ax}_{\text{KF}}(k) + \mathbf{B}_{\text{KF}}\mathbf{u}(k) + \mathbf{w}_{\text{KF}}(k)$$

$$\mathbf{y}_{\text{KF}}(k) = \mathbf{C}_{\text{KF}}\mathbf{x}_{\text{KF}}(k) + \mathbf{v}_{\text{KF}}(k)$$



- ▶  $w(k)$  and  $v(k)$  are assumed white Gaussian
- ▶ Matrices  $\mathbf{Q}_{\text{KF}}$  and  $\mathbf{R}_{\text{KF}}$  contain the respective covariances

# Sensor Fusion

## Signal Model - State and Measurement Vectors



### ► Attitude

$$\mathbf{x}_{\text{att}} = [\phi \quad \theta \quad \psi \quad \dot{\phi} \quad \dot{\theta} \quad \dot{\psi} \quad \ddot{\phi} \quad \ddot{\theta} \quad \ddot{\psi}]^T$$

$$\mathbf{y}_{\text{att}} = [\phi_{\text{acc}} \quad \theta_{\text{acc}} \quad \psi_{\text{mag}} \quad \dot{\phi}_{\text{gyro}} \quad \dot{\theta}_{\text{gyro}} \quad \dot{\psi}_{\text{gyro}}]^T$$

### ► Position

$$\mathbf{x}_{\text{pos}} = [x_n \quad y_n \quad \dot{x}_b \quad \dot{y}_b \quad \ddot{x}_b \quad \ddot{y}_b]^T$$

$$\mathbf{y}_{\text{pos}} = [x_{n,\text{GPS}} \quad y_{n,\text{GPS}} \quad \ddot{x}_{b,\text{acc}} \quad \ddot{y}_{b,\text{acc}}]^T$$

# Sensor Fusion

## Kalman Filter



- ▶ Step 0: Initialization

$$\hat{\mathbf{x}}_{\text{KF}}(0|0) = \mathbf{0}$$

$$\mathbf{P}_{\text{KF}}(0|0) = \mathbf{Q}_{\text{KF}}$$

- ▶ Step 1: Prediction
- ▶ Step 2: Update

# Sensor Fusion

## Kalman Filter



- ▶ Step 0: Initialization
- ▶ Step 1: Prediction

$$\hat{\mathbf{x}}_{\text{KF}}(k+1|k) = \mathbf{A}_{\text{KF}} \hat{\mathbf{x}}_{\text{KF}}(k|k) + \mathbf{B}_{\text{KF}} \mathbf{u}(k)$$

$$\mathbf{P}(k+1|k) = \mathbf{A}_{\text{KF}} \mathbf{P}(k|k) \mathbf{A}_{\text{KF}}^T + \mathbf{Q}_{\text{KF}}$$

- ▶ Step 2: Update

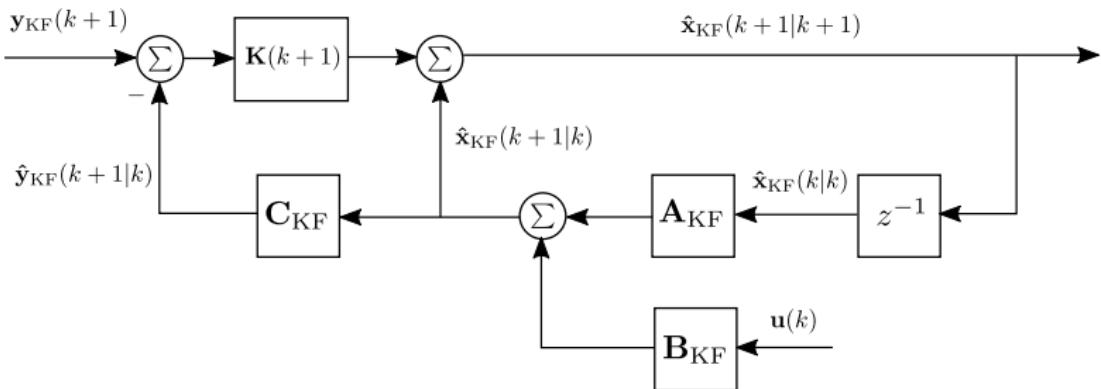
$$\hat{\mathbf{x}}_{\text{KF}}(k+1|k+1) = \hat{\mathbf{x}}_{\text{KF}}(k+1|k) + \mathbf{K}(k+1) [\mathbf{y}_{\text{KF}}(k+1) - \mathbf{C}_{\text{KF}} \hat{\mathbf{x}}_{\text{KF}}(k+1|k)]$$

$$\mathbf{P}(k+1|k+1) = \left[ \mathbf{I} - \mathbf{K}(k+1) \mathbf{C}_{\text{KF}}^T \right] \mathbf{P}(k+1|k)$$

$$\mathbf{K}(k+1) = \mathbf{P}(k+1|k) \mathbf{C}_{\text{KF}}^T \left[ \mathbf{C}_{\text{KF}} \mathbf{P}(k+1|k) \mathbf{C}_{\text{KF}}^T + \mathbf{R}_{\text{KF}} \right]^{-1}$$

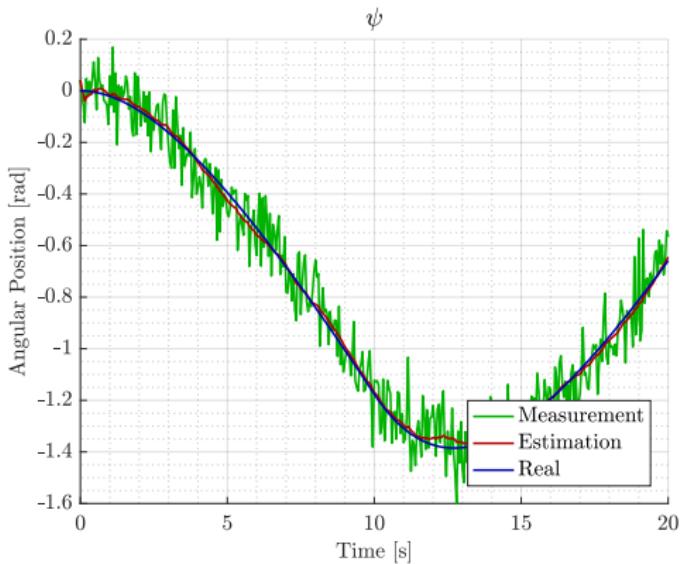
# Sensor Fusion

## Kalman Filter



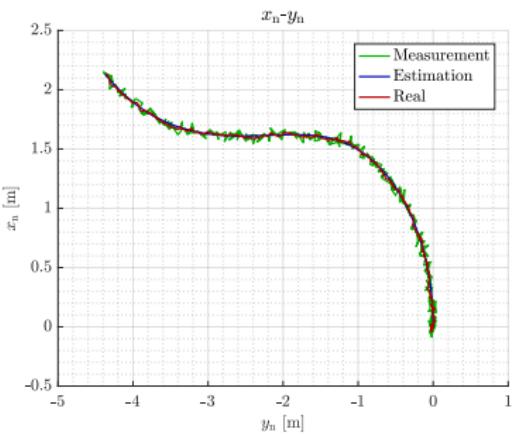
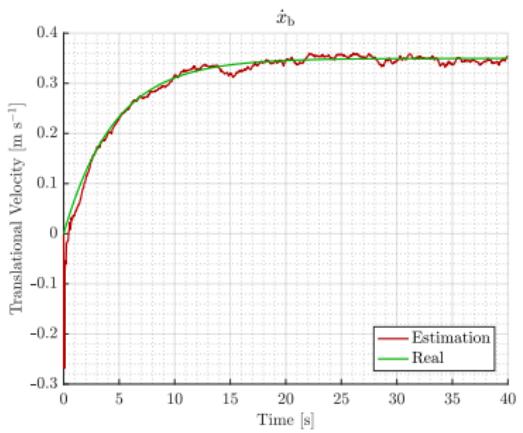
# Sensor Fusion

## Attitude Kalman Filter



# Sensor Fusion

## Position Kalman Filter

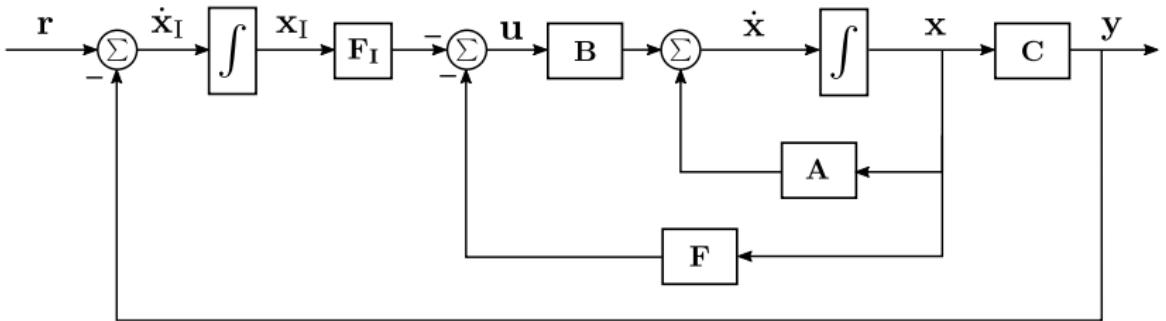


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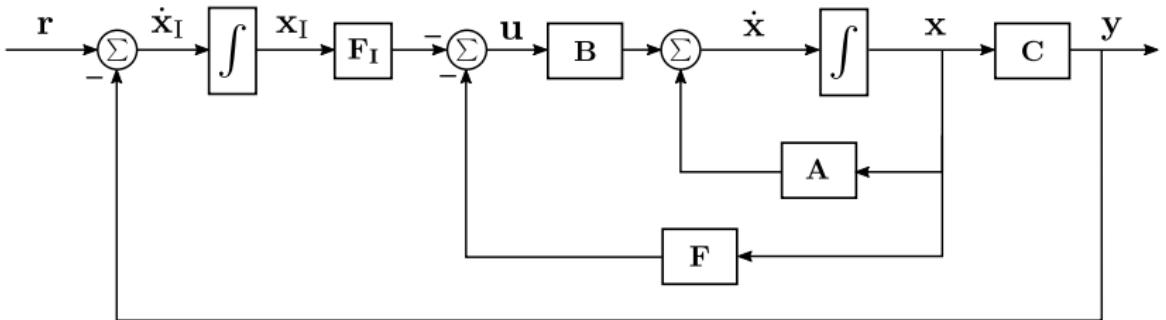


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# Inner Controller



# Inner Controller



- ▶  $\mathcal{H}_{\infty}$  Controller
- ▶ Linear Quadratic Regulator

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



- ▶ Suboptimal  $\mathcal{H}_\infty$  controller

Find an internally stabilizing controller that provides a closed loop  $\mathcal{H}_\infty$  norm less than some bound  $\gamma$

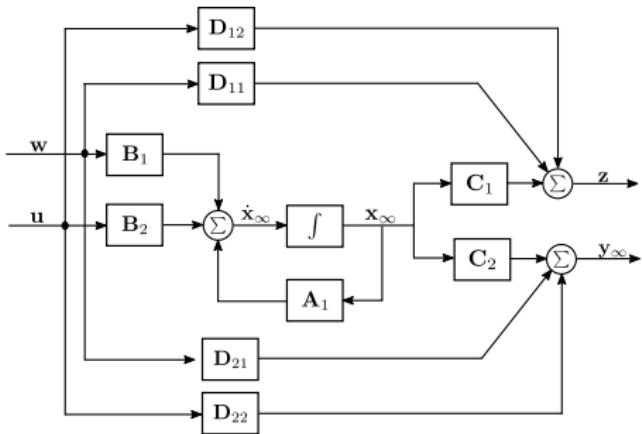
# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



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### ► System structure



$$\dot{\mathbf{x}}_\infty(t) = \mathbf{A}_1 \mathbf{x}_\infty(t) + \mathbf{B}_1 \mathbf{w}(t) + \mathbf{B}_2 \mathbf{u}(t)$$

$$\mathbf{z}(t) = \mathbf{C}_1 \mathbf{x}_\infty(t) + \mathbf{D}_{11} \mathbf{w}(t) + \mathbf{D}_{12} \mathbf{u}(t)$$

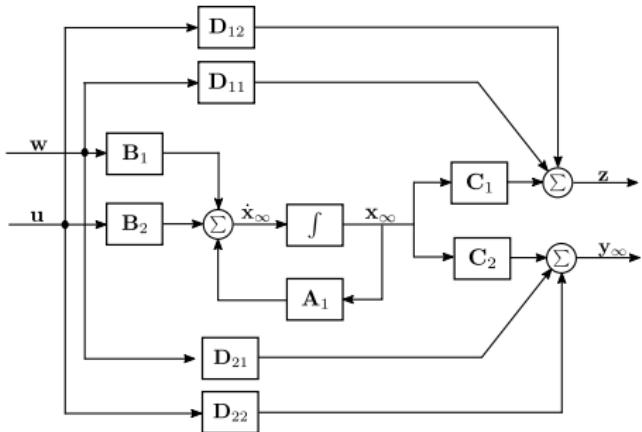
$$\mathbf{y}_\infty(t) = \mathbf{C}_2 \mathbf{x}_\infty(t) + \mathbf{D}_{21} \mathbf{w}(t) + \mathbf{D}_{22} \mathbf{u}(t)$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



### ► System structure



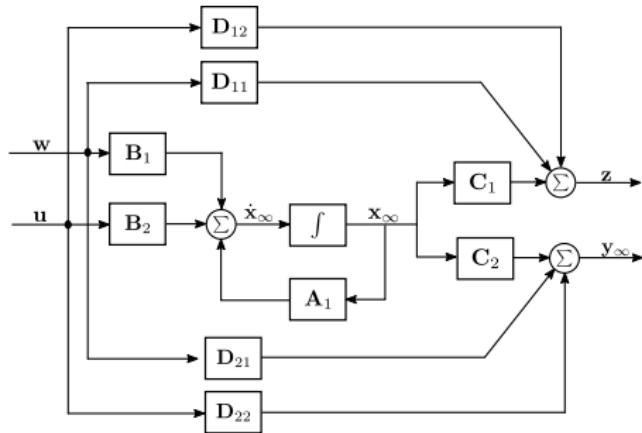
$$\mathbf{u}(t) = [F_1 \quad F_2]^T$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



## ► System structure



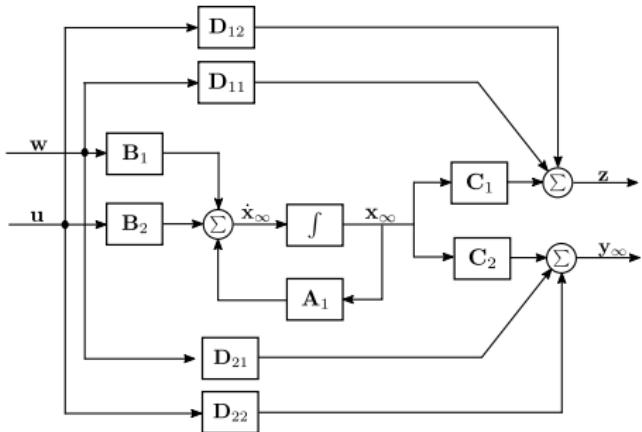
$$\mathbf{w}(t) = [\psi_{\text{ref}} \quad \dot{x}_{\text{b,ref}} \quad F_{\text{wc}} \quad \tau_{\text{wc}} \quad F_{\text{wave}} \quad \tau_{\text{wave}} \quad n_\psi \quad n_{\dot{x}_{\text{b}}}]^T$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



### ► System structure



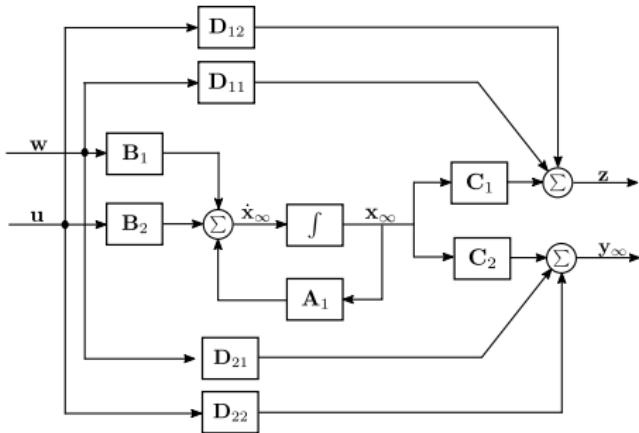
$$\mathbf{y}_{\infty}(t) = [\psi \quad \dot{x}_b \quad \mathbf{x}_I^T]^T$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



### ► System structure



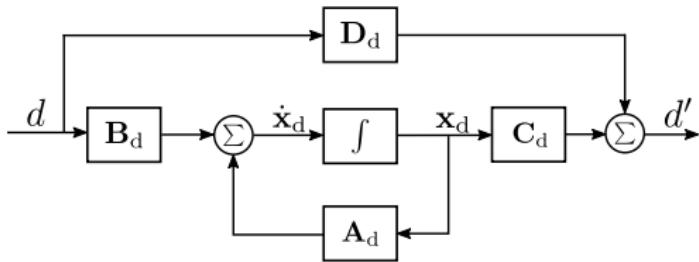
$$\mathbf{x}_\infty(t) = \begin{bmatrix} \psi & \dot{\psi} & \dot{x}_b & X_{I_\psi} & X_{I_{\dot{x}_b}} & X_{F_{wc}} & X_{\tau_{wc}} & X_{F_{wave}} & X_{\tau_{wave}} & X_{n_\psi} & X_{n_{\dot{x}_b}} \end{bmatrix}^T$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



- ▶ Disturbance model



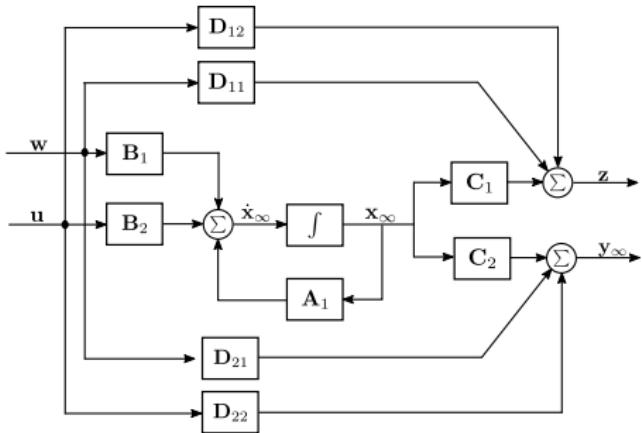
$$\frac{d'}{d} = \frac{a}{s+a} \rightarrow \dot{d}' = -ad' + ad \rightarrow \begin{cases} \dot{x}_d = -ax_d + ad \\ d' = x_d \end{cases}$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



### ► System structure



$$\mathbf{z}(t) = [\mathbf{x}_\infty^T \quad \mathbf{u}^T]^T$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



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- Controller design parameters ( $\gamma$ ,  $\mathbf{C}_1$ ,  $\mathbf{D}_{12}$ )

$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{W}_x & \mathbf{0}_{3 \times 2} & \mathbf{0}_{3 \times 2} & \mathbf{0}_{3 \times 2} & \mathbf{0}_{3 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{W}_I & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} & \mathbf{W}_{w_c} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{W}_{\text{wave}} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{W}_{\text{noise}} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 3} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 2} \end{bmatrix} \quad \mathbf{D}_{12} = \begin{bmatrix} \mathbf{0}_{2 \times 3} \\ \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} \\ \mathbf{0}_{2 \times 2} \\ \mathbf{W}_u \end{bmatrix}$$

# Inner Controller

## $\mathcal{H}_\infty$ Controller Design



- ▶ Feedback gain

$$\mathbf{X}_\infty = Ric \begin{bmatrix} \mathbf{A}_1 & \gamma^{-2} \mathbf{B}_1 \mathbf{B}_1^T - \mathbf{B}_2 \mathbf{B}_2^T \\ -\mathbf{C}_1^T \mathbf{C}_1 & -\mathbf{A}_1^T \end{bmatrix}$$

$$\mathbf{F}_\infty = -\mathbf{B}_2^T \mathbf{X}_\infty$$

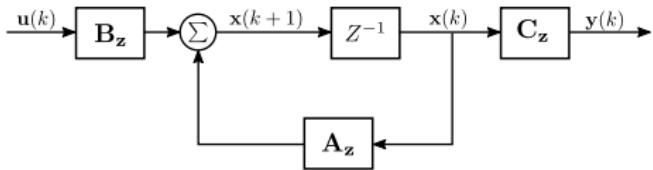
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# Inner Controller

## Linear Quadratic Controller Design

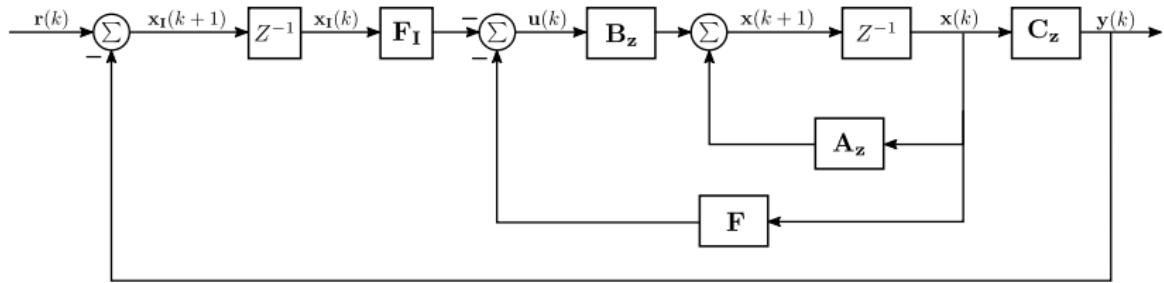


$$\mathbf{x}(k+1) = \mathbf{A}_z \mathbf{x}(k) + \mathbf{B}_z \mathbf{u}(k)$$

$$\mathbf{y}(k) = \mathbf{C}_z \mathbf{x}(k) + \mathbf{D}_z \mathbf{u}(k)$$

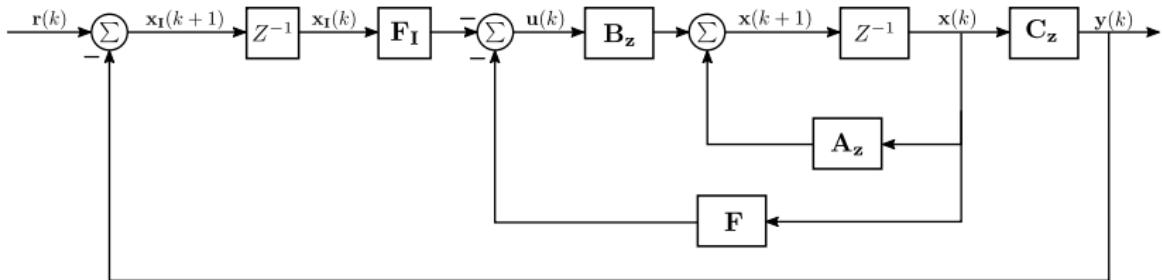
# Inner Controller

## Linear Quadratic Controller Design



# Inner Controller

## Linear Quadratic Controller Design



$$\mathbf{x}_e(k+1) = \mathbf{A}_e \mathbf{x}_e(k) + \mathbf{B}_e \mathbf{u}(k) + \mathbf{r}(k)$$

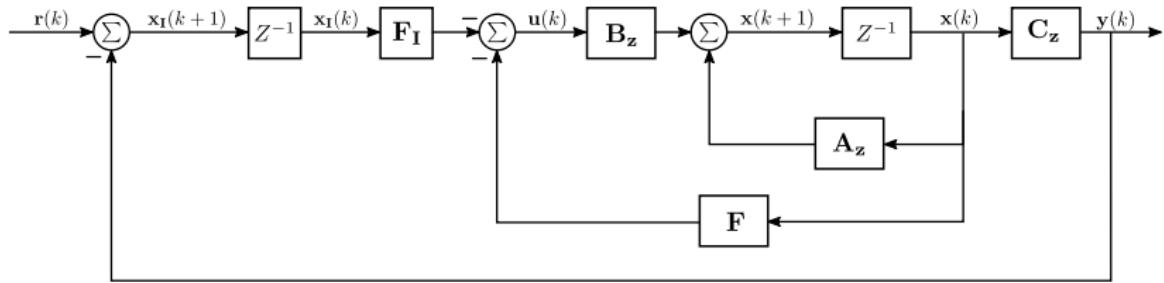
$$\mathbf{y}(k) = \mathbf{C}_e \mathbf{x}_e(k)$$

# Inner Controller

## Linear Quadratic Controller Design



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$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}_I(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{z,3 \times 3} & \mathbf{0}_{3 \times 2} \\ -\mathbf{C}_{z,2 \times 3} & \mathbf{I}_{2 \times 2} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_I(k) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{z,3 \times 2} \\ \mathbf{0}_{2 \times 2} \end{bmatrix} \mathbf{u}(k) + \begin{bmatrix} \mathbf{0}_{3 \times 2} \\ \mathbf{I}_{2 \times 2} \end{bmatrix} \mathbf{r}(k)$$

$$\mathbf{y}(k) = [\mathbf{C}_{z,2 \times 3} \quad \mathbf{0}_{2 \times 2}] \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_I(k) \end{bmatrix}$$

# Inner Controller

## Linear Quadratic Controller Design



$$\mathcal{J}_z = \sum_{k=0}^{\infty} \mathbf{x}^T(k) \mathbf{Q}_z \mathbf{x}(k) + \mathbf{u}^T(k) \mathbf{R}_z \mathbf{u}(k)$$

# Inner Controller

## Linear Quadratic Controller Design



$$\mathcal{J} = \int_0^{\infty} \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) \ dt$$

# Inner Controller

## Linear Quadratic Controller Design



$$\mathcal{J} = \int_0^{\infty} \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) \, dt$$

$$Q = \left( \frac{1}{\psi_{\max}^2}, \frac{1}{\dot{\psi}_{\max}^2}, \frac{1}{\dot{x}_{b,\max}^2}, \frac{1}{x_{I,\psi,\max}^2}, \frac{1}{x_{I,\dot{\psi},\max}^2}, \frac{1}{x_{I,\dot{x}_b,\max}^2} \right)$$

# Inner Controller

## Linear Quadratic Controller Design



$$\mathcal{J} = \int_0^{\infty} \mathbf{x}^T(t) \mathbf{Q} \mathbf{x}(t) + \mathbf{u}^T(t) \mathbf{R} \mathbf{u}(t) \, dt$$

$$Q = \left( \frac{1}{\psi_{\max}^2}, \frac{1}{\dot{\psi}_{\max}^2}, \frac{1}{\dot{x}_{b,\max}^2}, \frac{1}{x_{I,\psi,\max}^2}, \frac{1}{x_{I,\dot{\psi},\max}^2}, \frac{1}{x_{I,\dot{x}_b,\max}^2} \right)$$

$$R = \left( \frac{1}{F_{1\max}^2}, \frac{1}{F_{2\max}^2} \right)$$

$$\mathbf{u} = - [\mathbf{F} \quad \mathbf{F}_I] \begin{bmatrix} \mathbf{x} \\ \mathbf{x}_I \end{bmatrix}$$

# Inner Controller

## Comparison of the Controllers



### Simulation of LQR and robust design

- ▶ Disturbances from wind and waves
  - ▶  $\pm 1.5$  N along  $\dot{x}_b$
  - ▶  $\pm 1.5$  N·m around  $z_b$
  - ▶ The frequency for both varies between 0-10 Hz

# Inner Controller

## Comparison of the Controllers



### Simulation of LQR and robust design

- ▶ Disturbances from wind and waves
  - ▶  $\pm 1.5$  N along  $\dot{x}_b$
  - ▶  $\pm 1.5$  N·m around  $z_b$
  - ▶ The frequency for both varies between 0-10 Hz
- ▶ The parameters are varied  $\pm 20\%$ 
  - ▶ Mass,  $m$
  - ▶ Moment of inertia,  $I_z$ , around  $z_b$
  - ▶ The damping coefficients  $d_x$  and  $d_y$
  - ▶ The lengths  $l_1$  and  $l_2$

# Inner Controller

## Comparison of the Controllers

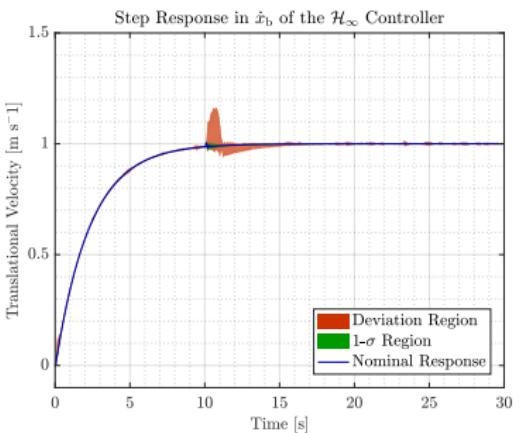
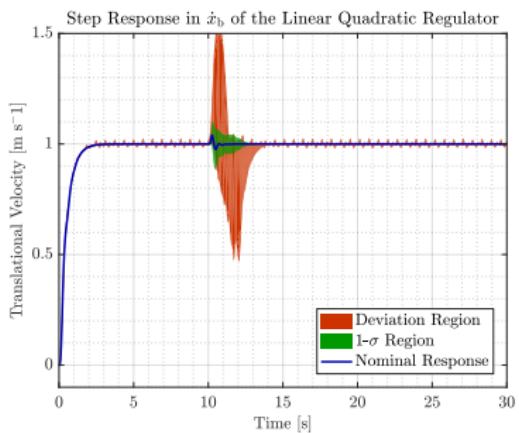


### Simulation of LQR and robust design

- ▶ Disturbances from wind and waves
  - ▶  $\pm 1.5$  N along  $\dot{x}_b$
  - ▶  $\pm 1.5$  N·m around  $z_b$
  - ▶ The frequency for both varies between 0-10 Hz
- ▶ The parameters are varied  $\pm 20\%$ 
  - ▶ Mass,  $m$
  - ▶ Moment of inertia,  $I_z$ , around  $z_b$
  - ▶ The damping coefficients  $d_x$  and  $d_y$
  - ▶ The lengths  $l_1$  and  $l_2$
- ▶ Monte Carlo simulations with 1000 realizations

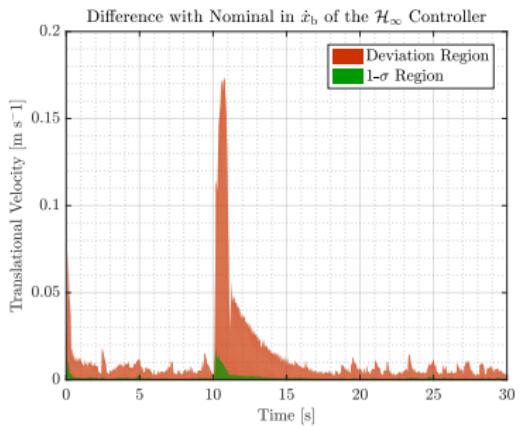
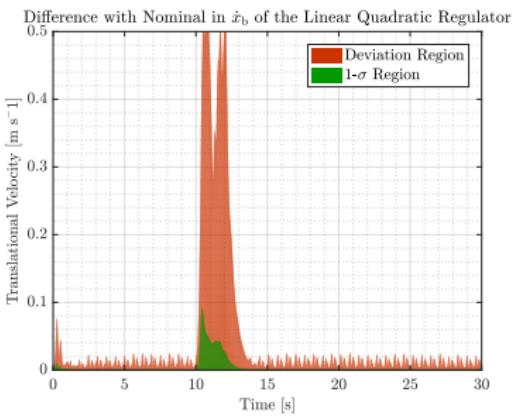
# Inner Controller

## Comparison of the Controllers



# Inner Controller

## Comparison of the Controllers

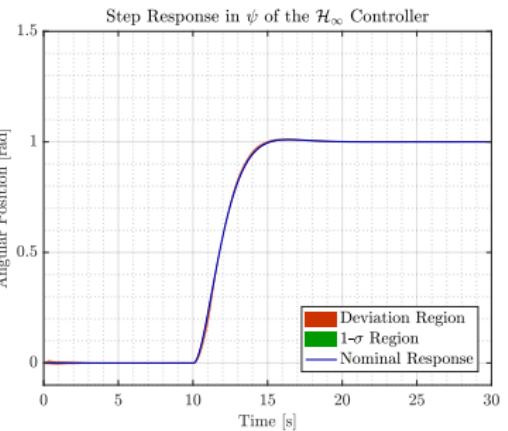
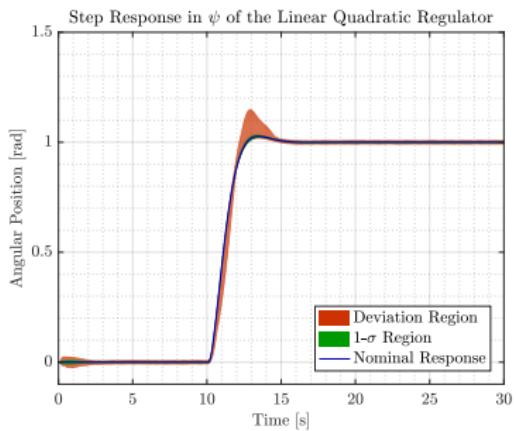


# Inner Controller

## Comparison of the Controllers

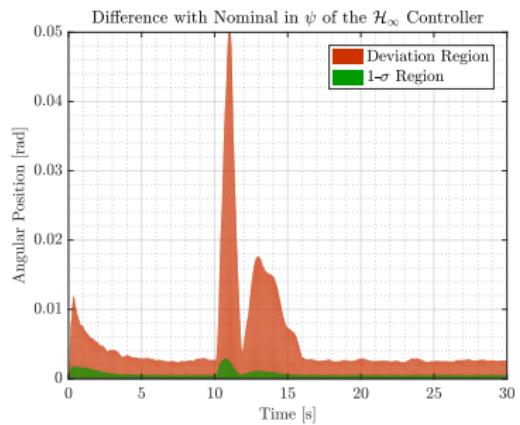
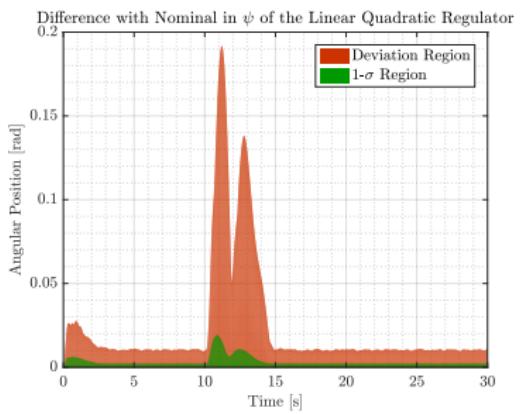


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# Inner Controller

## Comparison of the Controllers



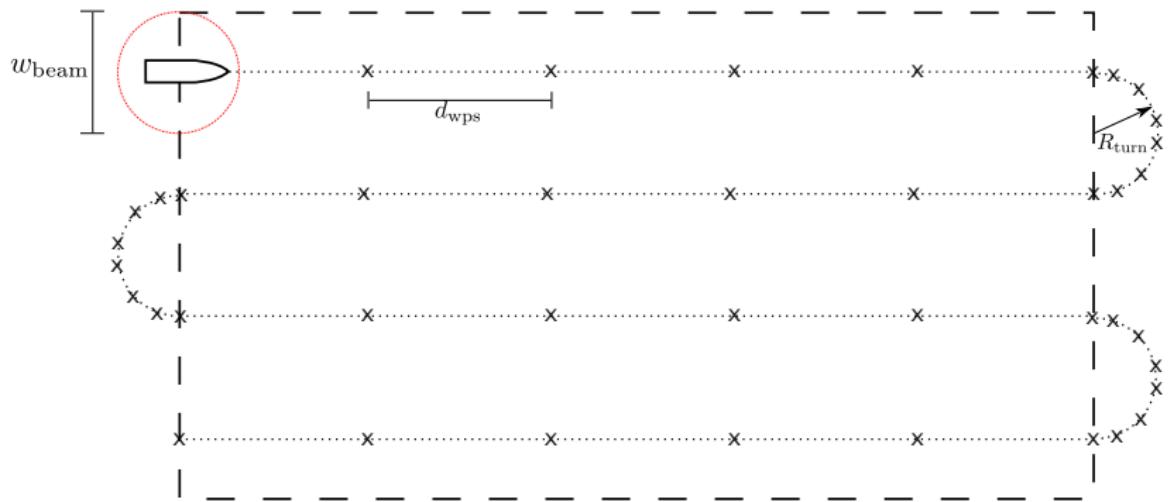
# Agenda



- ▶ Introduction
- ▶ System Description
- ▶ Model
- ▶ Control Approach
- ▶ Sensor Fusion
- ▶ Inner Controller
- ▶ **Outer Controller**
  - Path Generation Algorithm
  - Path Following Algorithm
- ▶ **Results**
  - Controller Results
  - Implementation Results
- ▶ Conclusion

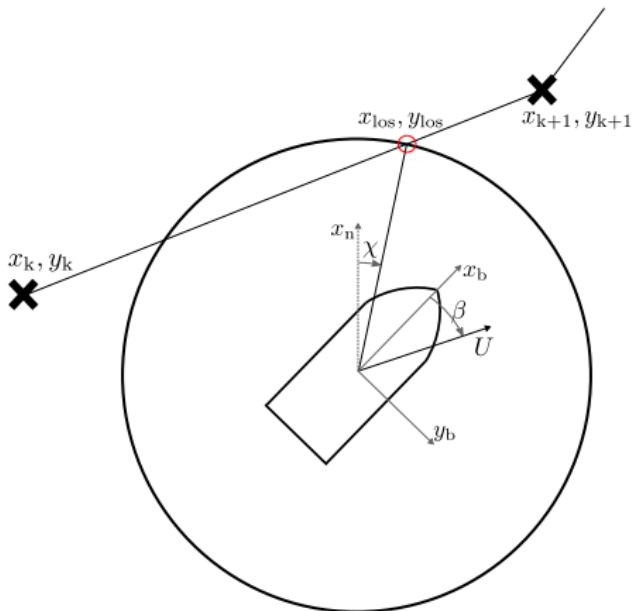
# Outer Controller

## Path Generation Algorithm



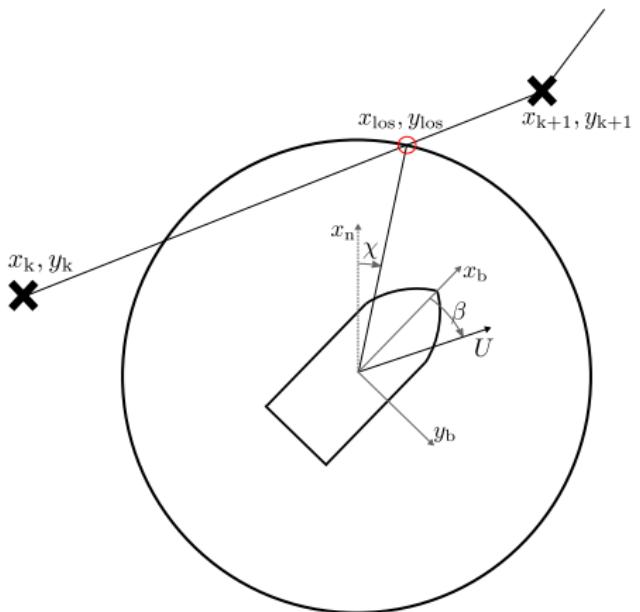
# Outer Controller

## Path Following Algorithm



# Outer Controller

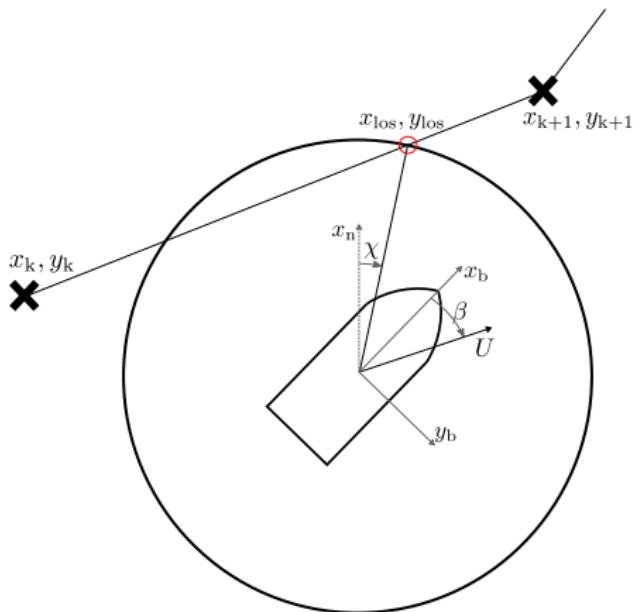
## Path Following Algorithm



$$\chi = \arctan \left( \frac{y_{\text{LOS}} - y_n}{x_{\text{LOS}} - x_n} \right)$$

# Outer Controller

## Path Following Algorithm

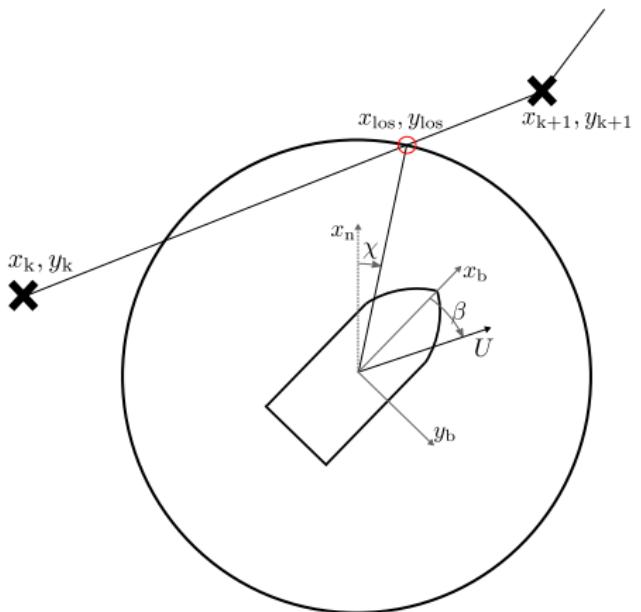


$$\chi = \arctan \left( \frac{y_{\text{LOS}} - y_n}{x_{\text{LOS}} - x_n} \right)$$

$$\beta = \arctan \left( \frac{\dot{y}_b}{\dot{x}_b} \right)$$

# Outer Controller

## Path Following Algorithm



$$\chi = \arctan \left( \frac{y_{\text{LOS}} - y_n}{x_{\text{LOS}} - x_n} \right)$$

$$\beta = \arctan \left( \frac{\dot{y}_b}{\dot{x}_b} \right)$$

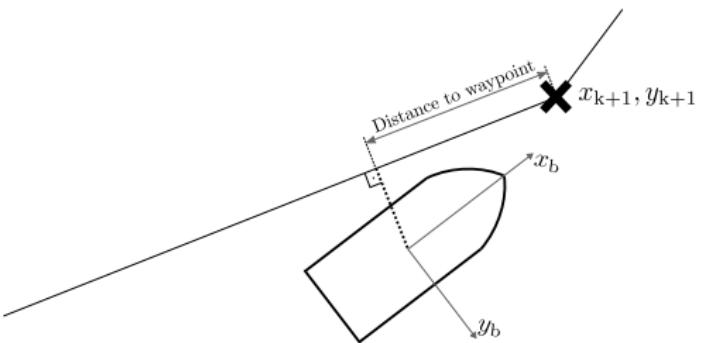
$$\psi_{\text{ref}} = \chi - \beta$$

# Outer Controller

## Path Following Algorithm



- ▶ Change active waypoints

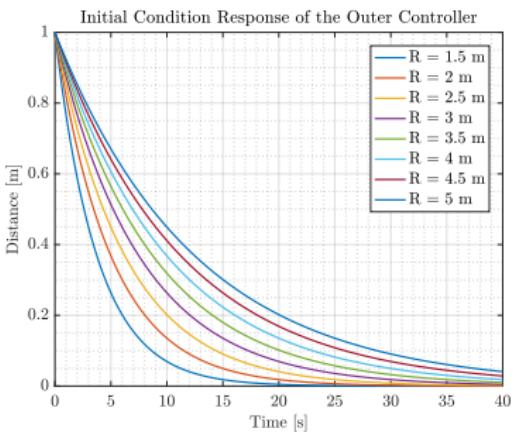
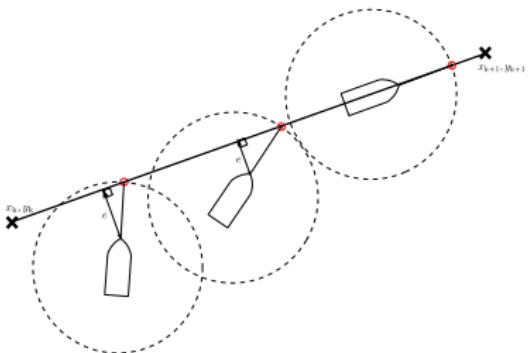


# Outer Controller

## Path Following Algorithm



- ▶ Convergence to the path

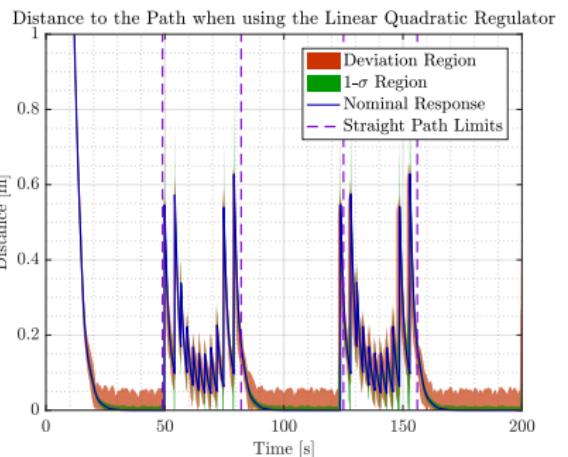
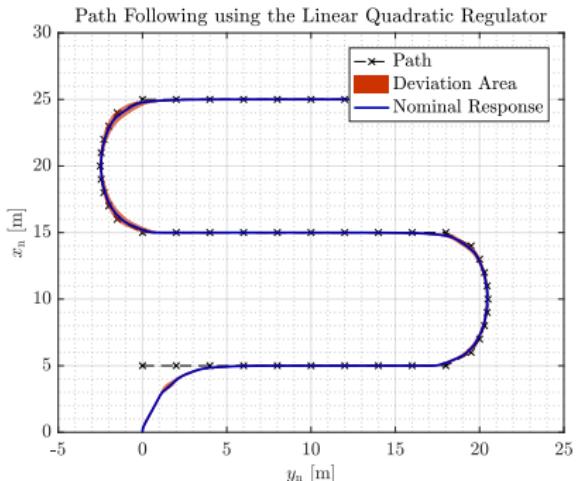


# Results

## Controller Results



### ► LQR as inner controller

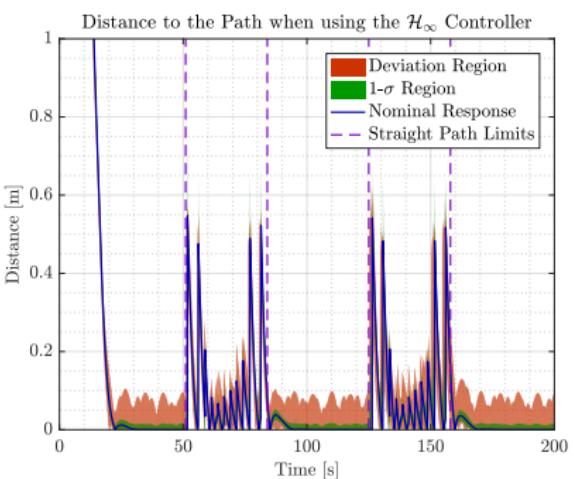
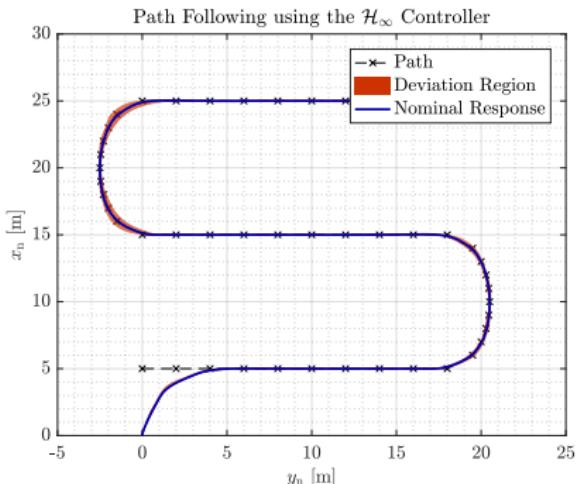


# Results

## Controller Results



- ▶ Robust controller as inner controller

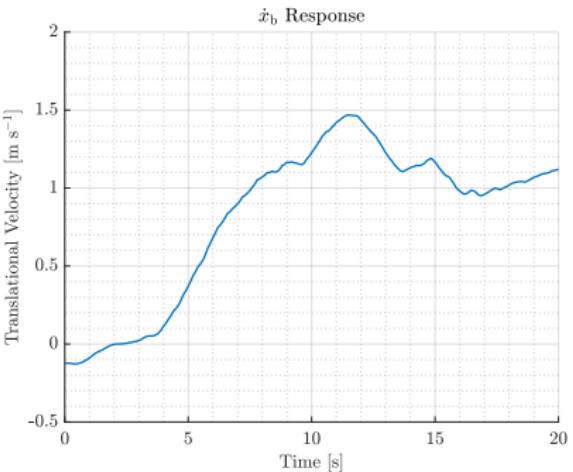
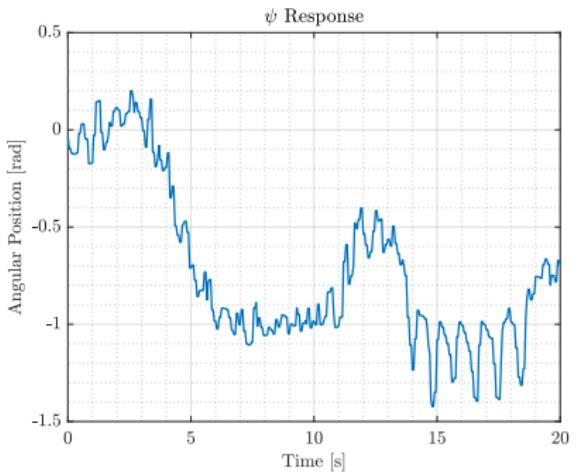


# Results

## Implementation Results



### ► Inner controller test



# Conclusion



- ▶ The estimator has been tuned and tested through simulation.
- ▶ The controller has also been analyzed though simulations that include disturbances, noise and varying parameters.
- ▶ The simulated results have not been fully replicated in the real vessel, but they show a promising behavior of the control system.

# Precision Control of an Autonomous Surface Vessel



AALBORG UNIVERSITY  
DENMARK