Precision Control of an Autonomous Surface Vessel

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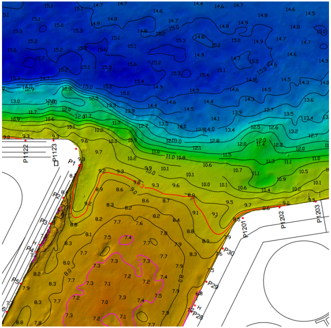
Agenda

Introduction

- Environmental monitoring
- Marine biological research
- Bathymetric measurements
- Control theory for an ASV

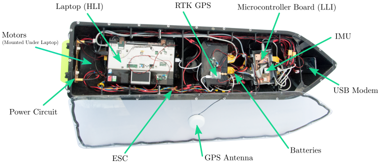
Introduction

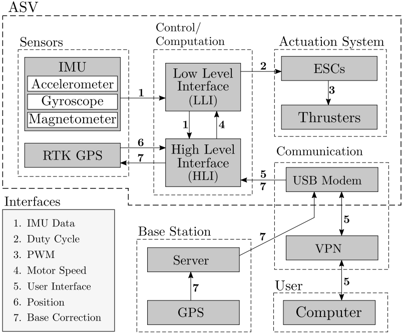
Use Case



- Depth map used by Port of Aalborg
- Problem: No recent knowledge of depths of the port
- Solution: Automate smaller unmanned vessel

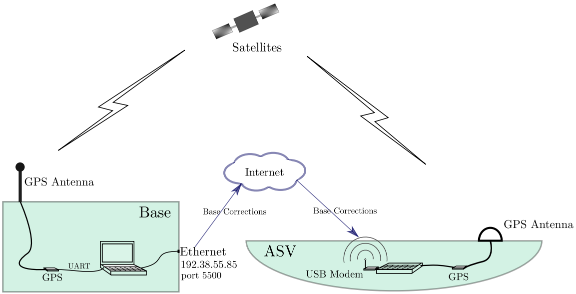
System Description

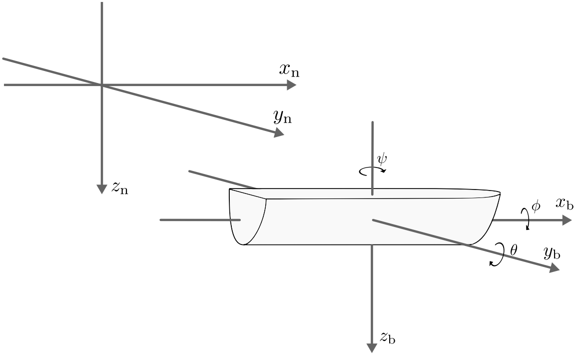




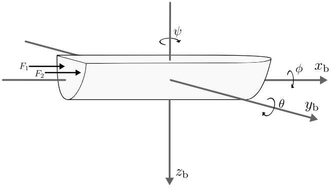
System Description

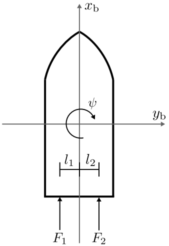
RTK GPS





- Inertial Frame (NED)
- Body Frame

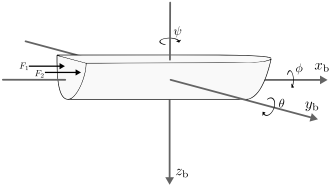


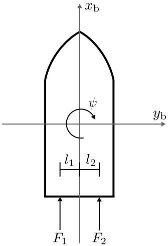


- Rigid Body Dynamics

$$\sum F = m\ddot{x}$$
$$\sum \tau = I\ddot{\theta}$$

- Hydrostatics
  - Buoyancy Force
- Hydrodynamics
  - Viscous Damping

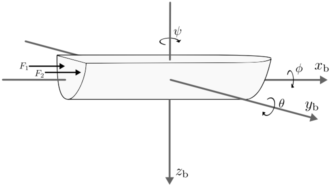


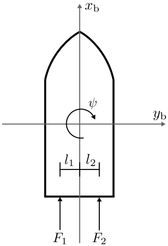


$$
\begin{aligned}
m\ddot{x}_\mathrm{b} &= F_1 + F_2 - d_{\dot{x}_\mathrm{b}}\dot{x}_\mathrm{b} + F_{x_\mathrm{b}} \\
m\ddot{y}_\mathrm{b} &= -d_{\dot{y}_\mathrm{b}}\dot{y}_\mathrm{b} + F_{y_\mathrm{b}} \\
m\ddot{z}_\mathrm{b} &= -d_{\dot{z}_\mathrm{b}}\dot{z}_\mathrm{b} + F_{z_\mathrm{b}} \\
I_\mathrm{x}\ddot{\phi} &= -d_{\dot{\phi}}\dot{\phi} + T_\phi \\
I_\mathrm{y}\ddot{\theta} &= -d_{\dot{\theta}}\dot{\theta} + T_\theta \\
I_\mathrm{z}\ddot{\psi} &= F_1 l_1 - F_2 l_2 - d_{\dot{\psi}}\dot{\psi}
\end{aligned}
$$

Model

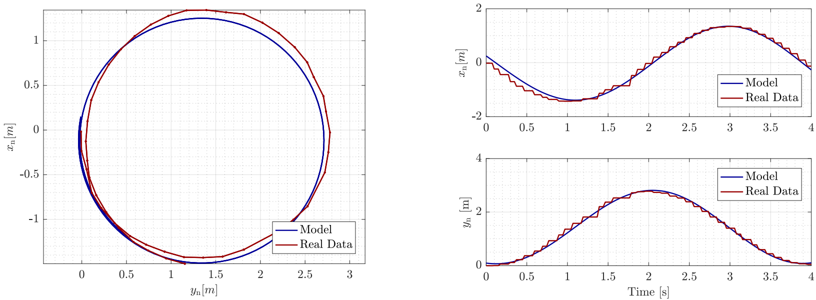
Linearized Model Equations





$$m\ddot{x}_\mathrm{b} = F_1 + F_2 - d_{\dot{x}_\mathrm{b}}\dot{x}_\mathrm{b}$$
$$m\ddot{y}_\mathrm{b} = -d_{\dot{y}_\mathrm{b}}\dot{y}_\mathrm{b}$$
$$m\ddot{z}_\mathrm{b} = -d_{\dot{z}_\mathrm{b}}\dot{z}_\mathrm{b} - \rho g A_\mathrm{wp}\tilde{z}_\mathrm{n}$$
$$I_\mathrm{x}\ddot{\phi} = -d_{\dot{\phi}}\dot{\phi} - \rho g V\overline{GM_T}\cdot\phi$$
$$I_\mathrm{y}\ddot{\theta} = -d_{\dot{\theta}}\dot{\theta} - \rho g V\overline{GM_L}\cdot\theta$$
$$I_\mathrm{z}\ddot{\psi} = F_1 l_1 - F_2 l_2 - d_{\dot{\psi}}\dot{\psi}$$

- Verified model



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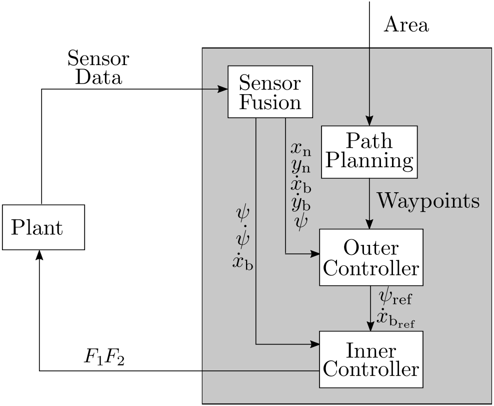
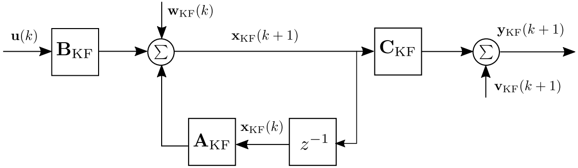
Control Approach



- Fuses GPS and IMU data
- Achieved using a Kalman filter
- Sensor fusion contains
  - Attitude
  - Position

$$\mathbf{x}_{\mathrm{KF}}(k+1) = \mathbf{A}_{\mathrm{KF}}\mathbf{x}_{\mathrm{KF}}(k) + \mathbf{B}_{\mathrm{KF}}\mathbf{u}(k) + \mathbf{w}_{\mathrm{KF}}(k)$$
$$\mathbf{y}_{\mathrm{KF}}(k+1) = \mathbf{C}_{\mathrm{KF}}\mathbf{x}_{\mathrm{KF}}(k+1) + \mathbf{v}_{\mathrm{KF}}(k+1)$$



- w(k) and v(k) are assumed white Gaussian
- Matrices $\mathbf{Q}_{\mathrm{KF}}$ and $\mathbf{R}_{\mathrm{KF}}$ are the respective covariance matrices

$$\mathbf{u} = \begin{bmatrix} F_1 & F_2 \end{bmatrix}^{\mathrm{T}}$$

- Attitude

$$\mathbf{x}_{\mathrm{att}} = \begin{bmatrix} \phi & \theta & \psi & \dot{\phi} & \dot{\theta} & \dot{\psi} & \ddot{\phi} & \ddot{\theta} & \ddot{\psi} \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{y}_{\mathrm{att}} = \begin{bmatrix} \phi_{\mathrm{acc}} & \theta_{\mathrm{acc}} & \psi_{\mathrm{mag}} & \dot{\phi}_{\mathrm{gyro}} & \dot{\theta}_{\mathrm{gyro}} & \dot{\psi}_{\mathrm{gyro}} \end{bmatrix}^{\mathrm{T}}$$

- Position

$$\mathbf{x}_{\mathrm{pos}} = \begin{bmatrix} x_{\mathrm{n}} & y_{\mathrm{n}} & \dot{x}_{\mathrm{b}} & \dot{y}_{\mathrm{b}} & \ddot{x}_{\mathrm{b}} & \ddot{y}_{\mathrm{b}} \end{bmatrix}^{\mathrm{T}}$$
$$\mathbf{y}_{\mathrm{pos}} = \begin{bmatrix} x_{\mathrm{n,GPS}} & y_{\mathrm{n,GPS}} & \ddot{x}_{\mathrm{b,acc}} & \ddot{y}_{\mathrm{b,acc}} \end{bmatrix}^{\mathrm{T}}$$

- Step 0: Initialization

$$\hat{\mathbf{x}}_{\mathrm{KF}}(0|0) = \mathbf{0}$$
$$\mathbf{P}_{\mathrm{KF}}(0|0) = \mathbf{Q}_{\mathrm{KF}}$$

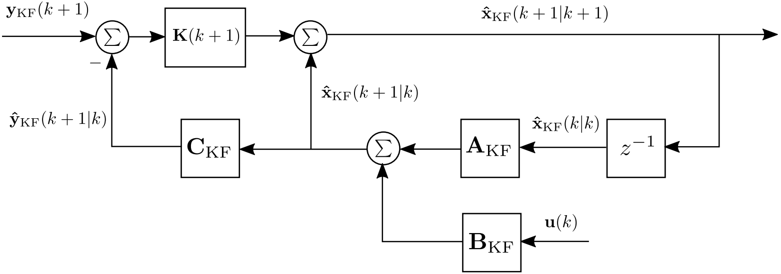
- Step 1: Prediction
- Step 2: Update

- Step 0: Initialization
- Step 1: Prediction

$$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k) = \mathbf{A}_{\mathrm{KF}}\hat{\mathbf{x}}_{\mathrm{KF}}(k|k) + \mathbf{B}_{\mathrm{KF}}\mathbf{u}(k)$$
$$\mathbf{P}(k+1|k) = \mathbf{A}_{\mathrm{KF}}\mathbf{P}(k|k)\mathbf{A}_{\mathrm{KF}}^{\mathrm{T}} + \mathbf{Q}_{\mathrm{KF}}$$

- Step 2: Update

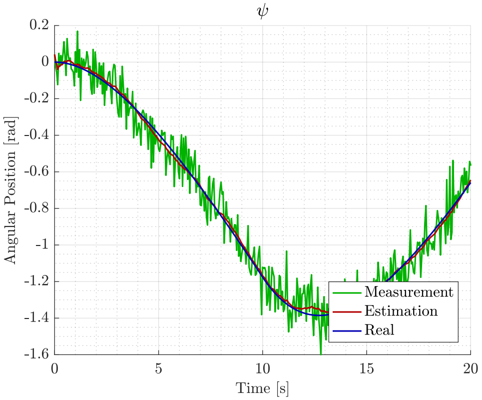
$$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k+1) = \hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k) + \mathbf{K}(k+1)\left[\mathbf{y}_{\mathrm{KF}}(k+1) - \mathbf{C}_{\mathrm{KF}}\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k)\right]$$
$$\mathbf{P}(k+1|k+1) = \left[\mathbf{I} - \mathbf{K}(k+1)\mathbf{C}_{\mathrm{KF}}^{\mathrm{T}}\right]\mathbf{P}(k+1|k)$$



$\mathbf{y}_{\mathrm{KF}}(k+1)$
$\mathbf{K}(k+1)$
$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k+1)$
$-$
$\hat{\mathbf{x}}_{\mathrm{KF}}(k+1|k)$
$\hat{\mathbf{y}}_{\mathrm{KF}}(k+1|k)$
$\mathbf{C}_{\mathrm{KF}}$
$\mathbf{A}_{\mathrm{KF}}$
$\hat{\mathbf{x}}_{\mathrm{KF}}(k|k)$
z^{-1}
$\mathbf{B}_{\mathrm{KF}}$
$\mathbf{u}(k)$
$\sum$

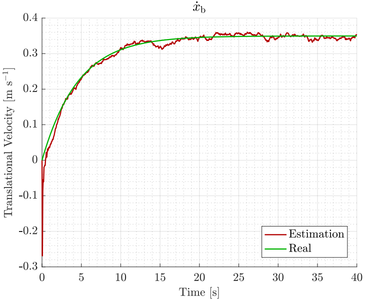
Sensor Fusion

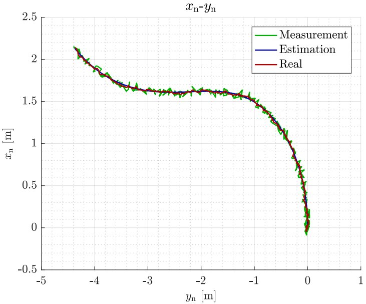
Attitude Kalman Filter



Sensor Fusion

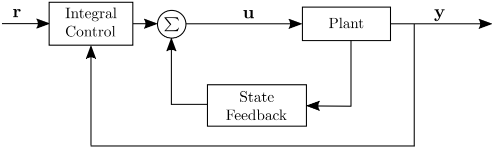
Position Kalman Filter



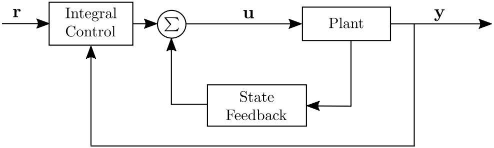


Agenda

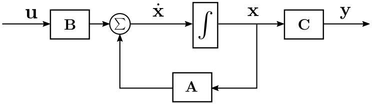
Inner Controller



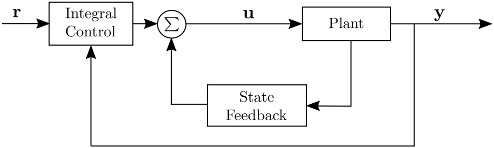
Inner Controller



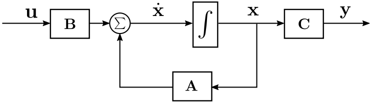
- Plant



Inner Controller



- Plant

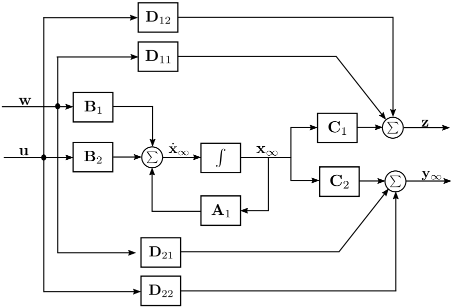


- Approaches
  - $\mathcal{H}_\infty$ Controller
  - Linear Quadratic Regulator

- Suboptimal $\mathcal{H}_\infty$ controller

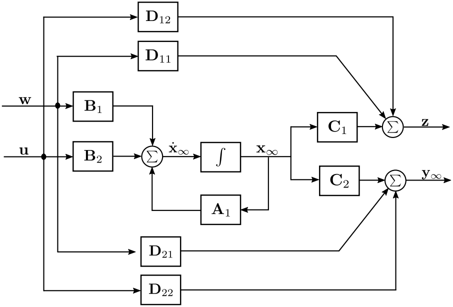
Find an internally stabilizing controller that provides a closed loop $\mathcal{H}_\infty$ norm less than some bound γ

- System structure



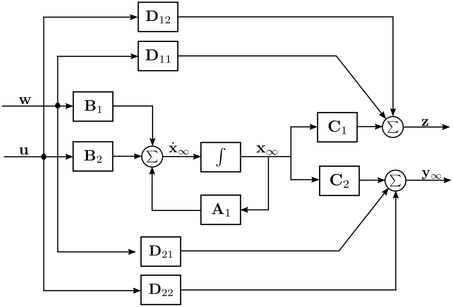
$$
\begin{aligned}
\dot{\mathbf{x}}_\infty(t) &= \mathbf{A}_1\mathbf{x}_\infty(t) + \mathbf{B}_1\mathbf{w}(t) + \mathbf{B}_2\mathbf{u}(t) \\
\mathbf{z}(t) &= \mathbf{C}_1\mathbf{x}_\infty(t) + \mathbf{D}_{11}\mathbf{w}(t) + \mathbf{D}_{12}\mathbf{u}(t) \\
\mathbf{y}_\infty(t) &= \mathbf{C}_2\mathbf{x}_\infty(t) + \mathbf{D}_{21}\mathbf{w}(t) + \mathbf{D}_{22}\mathbf{u}(t)
\end{aligned}
$$

- System structure



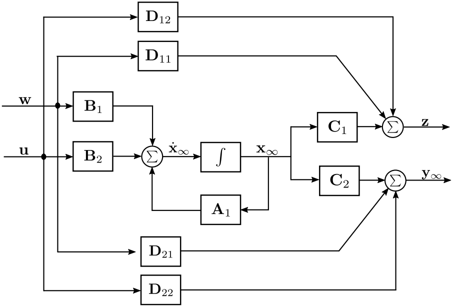
$$\mathbf{u}(t) = \begin{bmatrix} F_1 & F_2 \end{bmatrix}^{\mathrm{T}}$$

- System structure



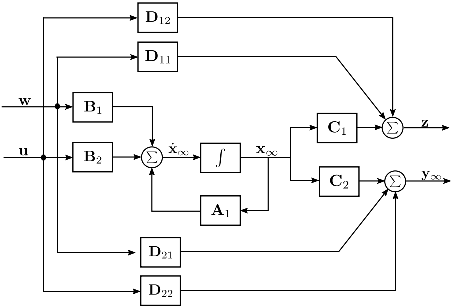
$$\mathbf{w}(t) = \begin{bmatrix} \psi_{\mathrm{ref}} & \dot{x}_{\mathrm{b,ref}} & F_{\mathrm{wc}} & \tau_{\mathrm{wc}} & F_{\mathrm{wave}} & \tau_{\mathrm{wave}} & n_\psi & n_{\dot{x}_{\mathrm{b}}} \end{bmatrix}^{\mathrm{T}}$$

- System structure



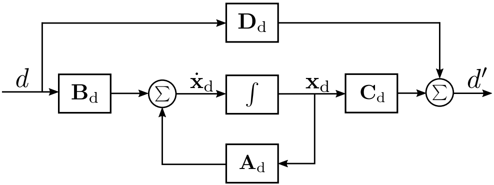
$$\mathbf{y}_\infty(t) = \begin{bmatrix} \psi & \dot{x}_\mathrm{b} & \mathbf{x}_\mathrm{I}^\mathrm{T} \end{bmatrix}^\mathrm{T}$$

- System structure



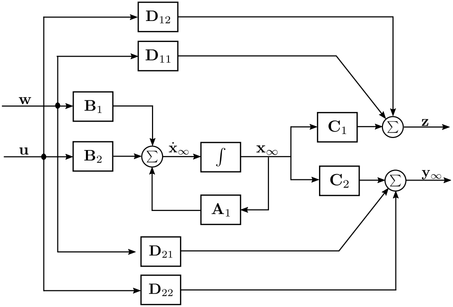
$$\mathbf{x}_\infty(t) = \begin{bmatrix} \psi & \dot{\psi} & \dot{x}_\mathrm{b} & x_{I_\psi} & x_{I_{\dot{x}_\mathrm{b}}} & x_{F_\mathrm{wc}} & x_{\tau_\mathrm{wc}} & x_{F_\mathrm{wave}} & x_{\tau_\mathrm{wave}} & x_{n_\psi} & x_{n_{\dot{x}_\mathrm{b}}} \end{bmatrix}^\mathrm{T}$$

- Disturbance model



$$\frac{d'}{d} = \frac{a}{s+a} \rightarrow \dot{d}' = -ad' + ad \rightarrow \begin{cases} \dot{x}_\mathrm{d} = -ax_\mathrm{d} + ad \\ d' = x_\mathrm{d} \end{cases}$$

- System structure



$$\mathbf{z}(t) = \begin{bmatrix} \mathbf{x}_\infty^{\mathrm{T}} & \mathbf{u}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

- Controller design parameters (γ, $\mathbf{C}_1$, $\mathbf{D}_{12}$)

$$\mathbf{C}_1 = \begin{bmatrix} \mathbf{W}_\mathrm{x} & \mathbf{0}_{3\mathrm{x}2} & \mathbf{0}_{3\mathrm{x}2} & \mathbf{0}_{3\mathrm{x}2} & \mathbf{0}_{3\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}3} & \mathbf{W}_\mathrm{I} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{W}_\mathrm{wc} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{W}_\mathrm{wave} & \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{W}_\mathrm{noise} \\ \mathbf{0}_{2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} & \mathbf{0}_{2\mathrm{x}2} \end{bmatrix} \qquad \mathbf{D}_{12} = \begin{bmatrix} \mathbf{0}_{2\mathrm{x}3} \\ \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}2} \\ \mathbf{W}_\mathrm{u} \end{bmatrix}$$

- Feedback gain

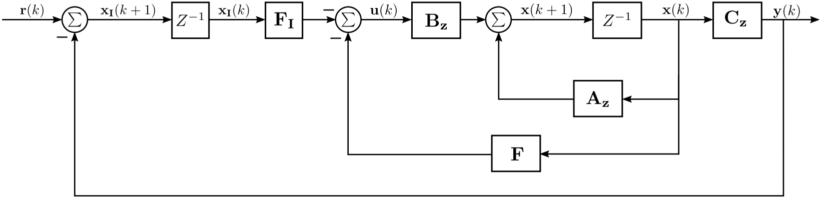
$$\mathbf{X}_\infty = Ric \begin{bmatrix} \mathbf{A}_1 & \gamma^{-2}\mathbf{B}_1\mathbf{B}_1^{\mathrm{T}} - \mathbf{B}_2\mathbf{B}_2^{\mathrm{T}} \\ -\mathbf{C}_1^{\mathrm{T}}\mathbf{C}_1 & -\mathbf{A}_1^{\mathrm{T}} \end{bmatrix}$$

$$\mathbf{F}_\infty = -\mathbf{B}_2^{\mathrm{T}}\mathbf{X}_\infty$$

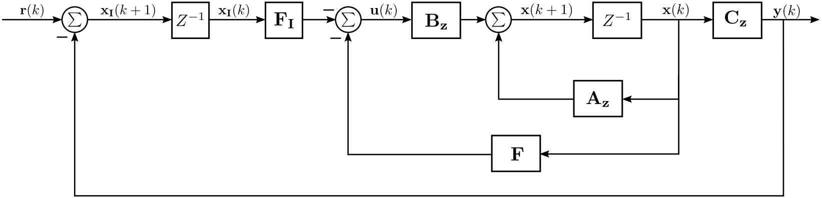
Agenda

Inner Controller

Linear Quadratic Controller Design



Inner Controller
Linear Quadratic Controller Design



$$\begin{bmatrix} \mathbf{x}(k+1) \\ \mathbf{x}_{\mathrm{I}}(k+1) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{\mathrm{z},3\mathrm{x}3} & \mathbf{0}_{3\mathrm{x}2} \\ -\mathbf{C}_{\mathrm{z},2\mathrm{x}3} & \mathbf{I}_{2\mathrm{x}2} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_{\mathrm{I}}(k) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_{\mathrm{z},3\mathrm{x}2} \\ \mathbf{0}_{2\mathrm{x}2} \end{bmatrix} \mathbf{u}(k) + \begin{bmatrix} \mathbf{0}_{3\mathrm{x}2} \\ \mathbf{I}_{2\mathrm{x}2} \end{bmatrix} \mathbf{r}(k)$$

$$\mathbf{y}(k) = \begin{bmatrix} \mathbf{C}_{\mathrm{z},2\mathrm{x}3} & \mathbf{0}_{2\mathrm{x}2} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_{\mathrm{I}}(k) \end{bmatrix}$$

- Discrete cost function

$$\mathcal{J}_{\mathrm{z}} = \sum_{k=0}^{\infty} \mathbf{x}^{\mathrm{T}}(k)\mathbf{Q}_{\mathrm{z}}\mathbf{x}(k) + \mathbf{u}^{\mathrm{T}}(k)\mathbf{R}_{\mathrm{z}}\mathbf{u}(k)$$

- Continuous cost function

$$\mathcal{J} = \int_0^\infty \mathbf{x}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)\;dt$$

- Continuous cost function

$$\mathcal{J} = \int_0^\infty \mathbf{x}^\mathrm{T}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^\mathrm{T}(t)\mathbf{R}\mathbf{u}(t)\ dt$$

$$Q = diag\left(\frac{1}{{\psi_\mathrm{max}}^2}\ ,\ \frac{1}{{\dot{\psi}_\mathrm{max}}^2}\ ,\ \frac{1}{{\dot{x}_{b,\mathrm{max}}}^2}\ ,\ \frac{1}{{x_{\mathrm{I},\psi,\mathrm{max}}}^2}\ ,\ \frac{1}{{x_{\mathrm{I},\dot{x}_b,\mathrm{max}}}^2}\right)$$

- Continuous cost function

$$\mathcal{J} = \int_0^\infty \mathbf{x}^\mathrm{T}(t)\mathbf{Q}\mathbf{x}(t) + \mathbf{u}^\mathrm{T}(t)\mathbf{R}\mathbf{u}(t)\, dt$$

$$Q = diag\left(\frac{1}{\psi_{\mathrm{max}}{}^2}, \frac{1}{\dot{\psi}_{\mathrm{max}}{}^2}, \frac{1}{\dot{x}_{b,\mathrm{max}}{}^2}, \frac{1}{x_{\mathrm{I},\psi,\mathrm{max}}{}^2}, \frac{1}{x_{\mathrm{I},\dot{x}_b,\mathrm{max}}{}^2}\right)$$

$$R = diag\left(\frac{1}{F_{1_{\mathrm{max}}}{}^2}, \frac{1}{F_{2_{\mathrm{max}}}{}^2}\right)$$

- Discretize the cost function to get $\mathbf{Q}_z$ and $\mathbf{R}_z$
- Solve the discrete-time algebraic Riccati equation
- Get the feedback gains

$$\mathbf{u}(k) = -\begin{bmatrix} \mathbf{F} & \mathbf{F}_{\mathrm{I}} \end{bmatrix} \begin{bmatrix} \mathbf{x}(k) \\ \mathbf{x}_{\mathrm{I}}(k) \end{bmatrix}$$

Simulation of LQR and $\mathcal{H}_\infty$ design

- Disturbances from wind and current
  - $\pm$1.5 N along $\dot{x}_b$
  - $\pm$1.5 N·m around z_b
- The waves are sinusoidal, with frequency varying between 0-10 Hz

Simulation of LQR and $\mathcal{H}_\infty$ design

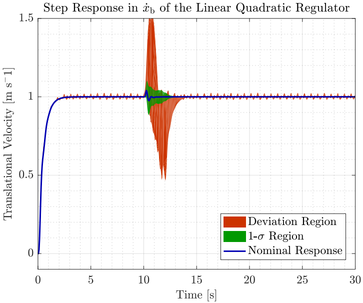
- Disturbances from wind and current
  - ±1.5 N along $\dot{x}_b$
  - ±1.5 N·m around z_b
- The waves are sinusoidal, with frequency varying between 0-10 Hz
- The parameters are varied $\pm20\%$
  - Mass, m
  - Moment of inertia, I_z, around z_b
  - The damping coefficients d_x and d_ψ
  - The lengths l_1 and l_2

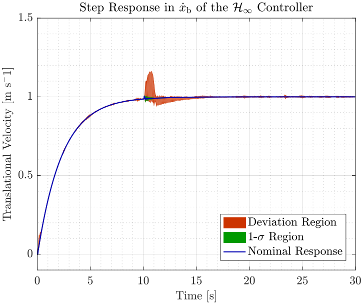
Simulation of LQR and $\mathcal{H}_\infty$ design

- Disturbances from wind and current
  - ± 1.5 N along $\dot{x}_b$
  - ± 1.5 N·m around z_b
- The waves are sinusoidal, with frequency varying between 0-10 Hz
- The parameters are varied $\pm 20\%$
  - Mass, m
  - Moment of inertia, I_z, around z_b
  - The damping coefficients d_x and d_ψ
  - The lengths l_1 and l_2
- Monte Carlo simulations with 1000 realizations

Inner Controller

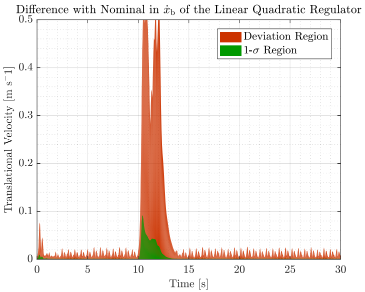
Comparison of the Controllers

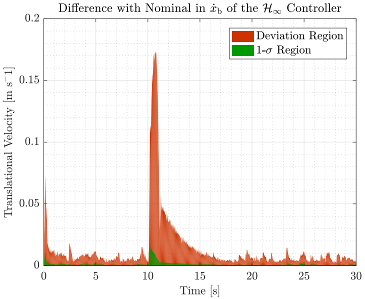




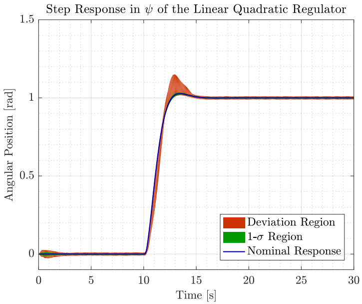
- LQR gives a faster response
- $\mathcal{H}_\infty$ controller is more robust to disturbances
- The disturbance at 10 s is due to the change in reference in ψ

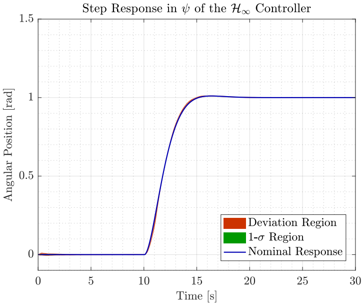
Inner Controller
Comparison of the Controllers





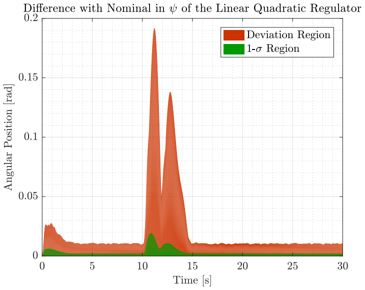
Inner Controller
Comparison of the Controllers

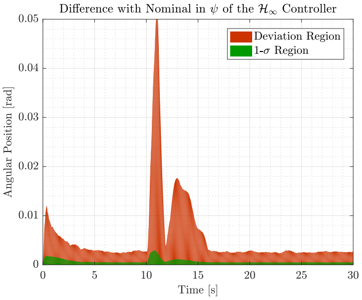




- LQR gives a faster response
- $\mathcal{H}_\infty$ controller is more robust to disturbances and it is faster than when tracking $\dot{x}_\mathrm{b}$

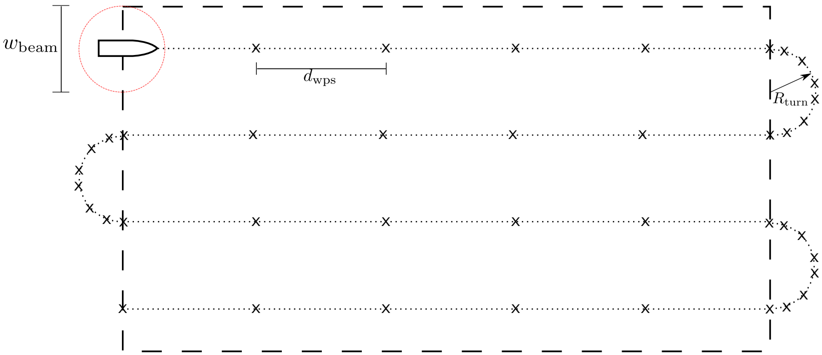
Inner Controller
Comparison of the Controllers





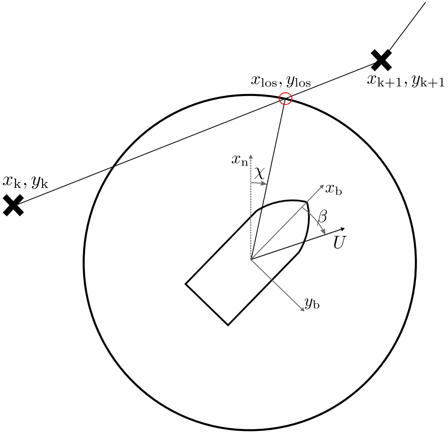
Agenda

- Introduction
- System Description
- Model
- Control Approach
- Sensor Fusion
- Inner Controller
- **Outer Controller**
  - **Path Generation Algorithm**
  - **Path Following Algorithm**
- **Results**
  - **Controller Results**
  - **Implementation Results**
- **Conclusion**

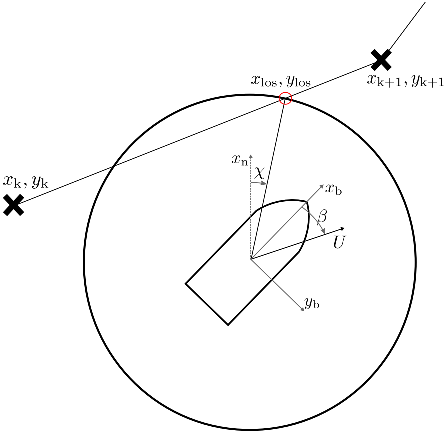


w_{beam}
d_{wps}
R_{turn}

Outer Controller
Path Following Algorithm



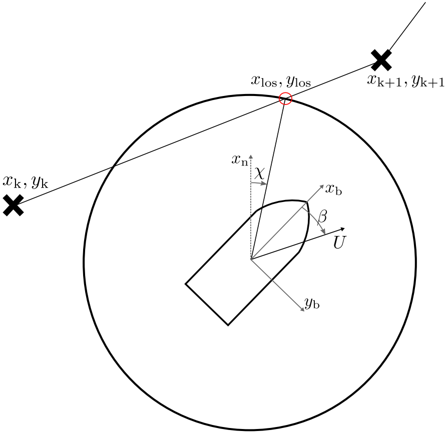
Outer Controller
Path Following Algorithm



$$\chi = \arctan\left(\frac{y_{\mathrm{LOS}} - y_{\mathrm{n}}}{x_{\mathrm{LOS}} - x_{\mathrm{n}}}\right)$$

Outer Controller

Path Following Algorithm

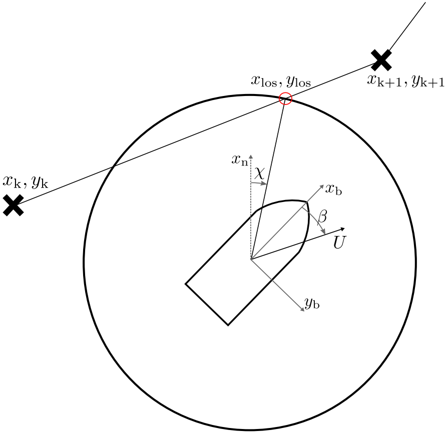


$$\chi = \arctan\left(\frac{y_{\text{LOS}} - y_{\text{n}}}{x_{\text{LOS}} - x_{\text{n}}}\right)$$

$$\beta = \arctan\left(\frac{\dot{y}_{\text{b}}}{\dot{x}_{\text{b}}}\right)$$

Outer Controller

Path Following Algorithm



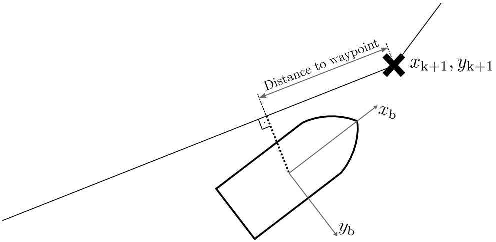
$$\chi = \arctan\left(\frac{y_{\mathrm{LOS}} - y_{\mathrm{n}}}{x_{\mathrm{LOS}} - x_{\mathrm{n}}}\right)$$

$$\beta = \arctan\left(\frac{\dot{y}_{\mathrm{b}}}{\dot{x}_{\mathrm{b}}}\right)$$

$$\psi_{\mathrm{ref}} = \chi - \beta$$

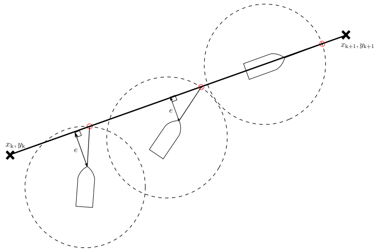
Outer Controller
Path Following Algorithm

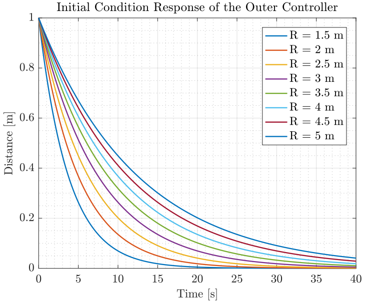
- Change active waypoints



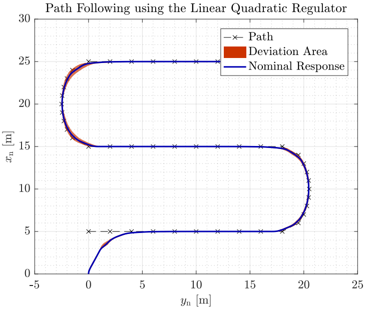
Outer Controller
Path Following Algorithm

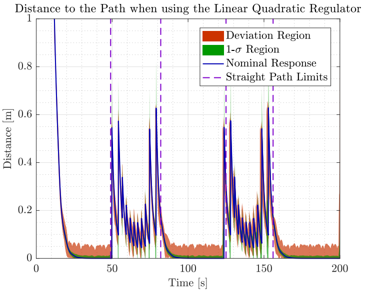
- Convergence to the path



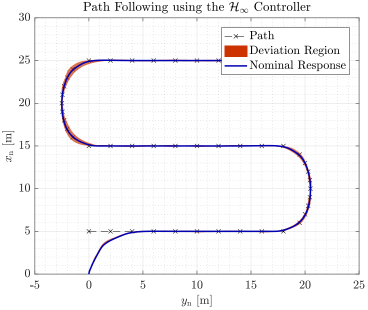


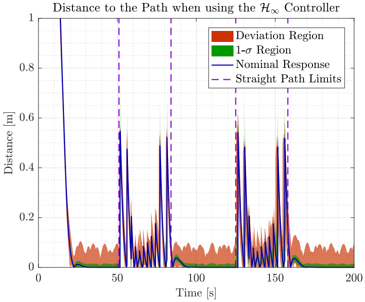
- LQR as inner controller



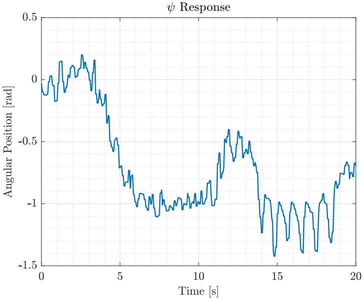


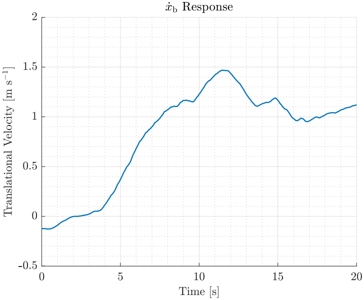
- Robust controller as inner controller



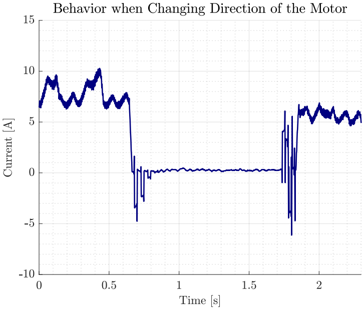


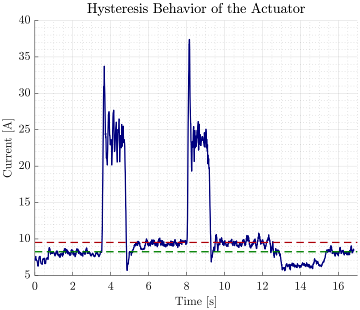
- Inner controller test





- Actuator tests





Conclusion

- A dynamic model of the system has been derived
- The estimator has been tuned and tested through simulation
- The controller has also been analyzed through simulations that include disturbances, noise and varying parameters
- The simulated results have not been fully replicated in the real vessel, but they show a promising behavior of the control system

Precision Control of an Autonomous Surface Vessel

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