



Nonlinear Control Strategies

Robustness to Parameter Variation and Input Noise

by
Niels Skov Vestergaard

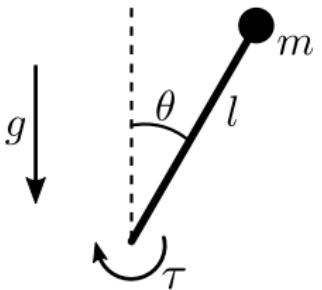
Agenda



- ▶ Introduction
 - Model
 - LQR
- ▶ Nonlinear Control Design
 - Feedback Linearization
 - Sliding Mode
 - Lyapunov Redesign
- ▶ Results
 - Robustness to Parameter Variation
 - Robustness to Input Noise

Introduction

The System



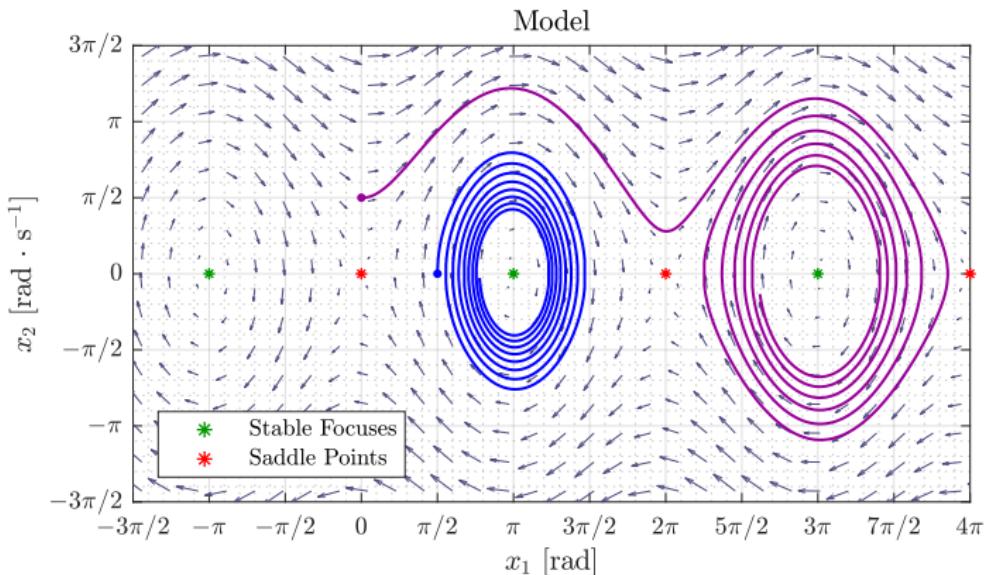
$$a = \frac{g}{l} , \quad b = \frac{k}{m} , \quad c = \frac{1}{ml^2} , \quad u = \tau , \quad \text{s.t.},$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = a \sin x_1 - bx_2 + cu$$

Introduction

The System

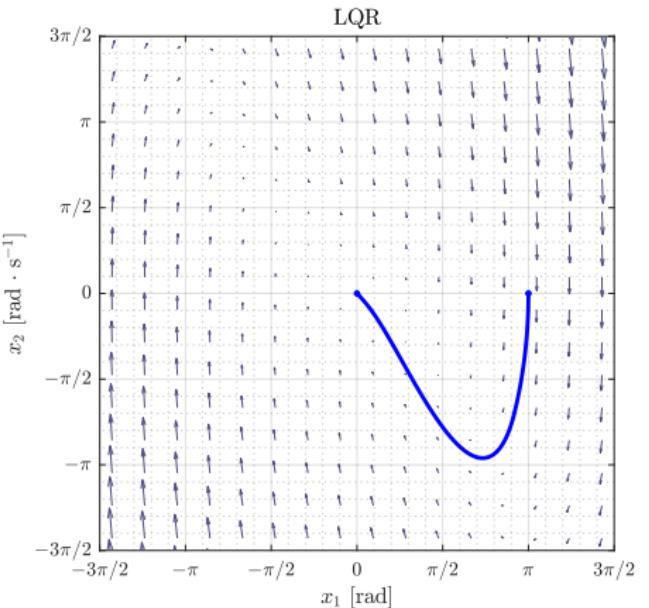


Introduction

Linear Control



4



Nonlinear Control Design

Feedback Linearization



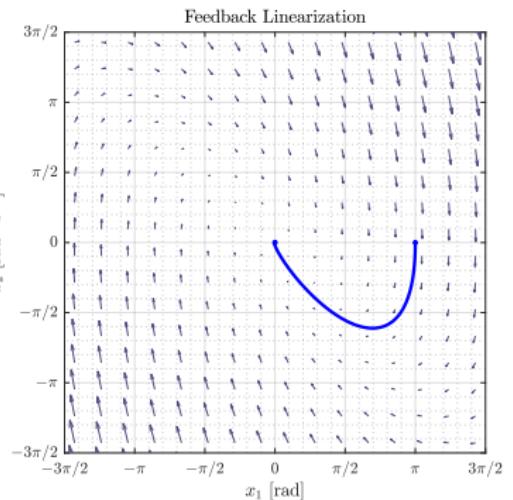
$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \hat{h}_1(x) + \hat{h}_2(x) + \hat{g}(x)u$$

$$u = \frac{1}{\hat{g}(x)}(-\hat{h}_1(x) + v)$$

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \hat{h}_2(x) + v$$



Nonlinear Control Design

Sliding Mode



$$s = a_1x_1 + x_2 = 0 \quad V = \frac{1}{2}s^2$$

$$\dot{V} \leq g(x)|s| \left| \frac{a_1x_2 + h(x)}{g(x)} \right| + g(x)su$$

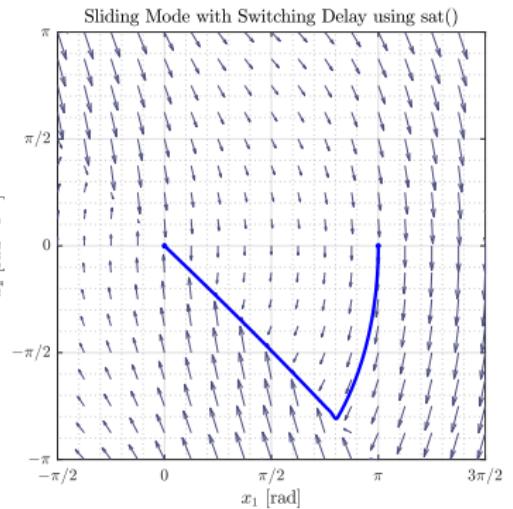
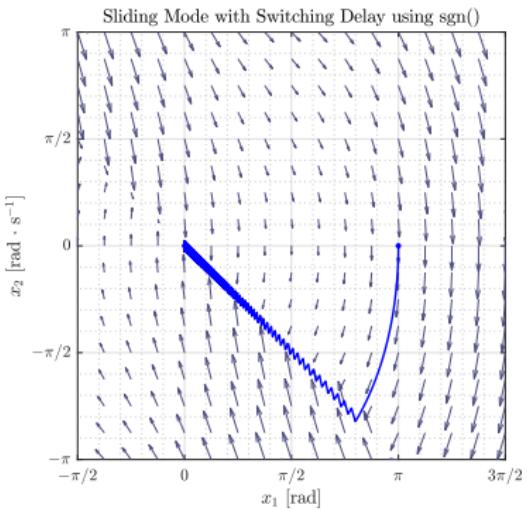
$$\dot{V} \leq g(x)|s| \left| \frac{a_1x_2 + h(x)}{g(x)} \right| - g(x)\text{sgn}(s)s \left| \frac{a_1x_2 + h(x)}{g(x)} \right|$$

$$u = -\text{sgn}(s)\varrho(x) \quad \text{where, } \varrho(x) \geq \left| \frac{a_1x_2 + h(x)}{g(x)} \right|$$

$$u = -\text{sgn}(s)\beta(x) \quad \text{where, } \beta(x) = \varrho(x) + \beta_0 \quad \text{and } \beta_0 > 0$$

Nonlinear Control Design

Sliding Mode



Nonlinear Control Design

Sliding Mode



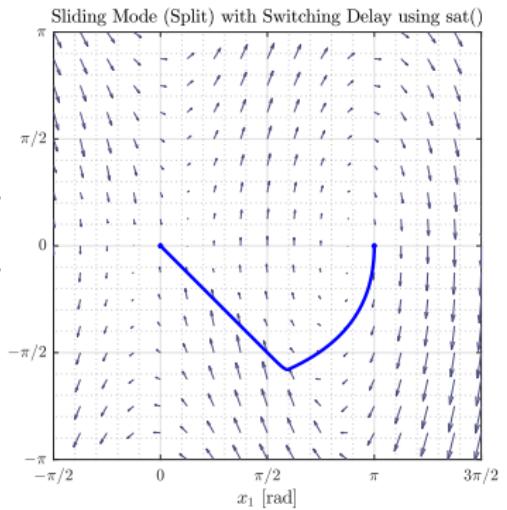
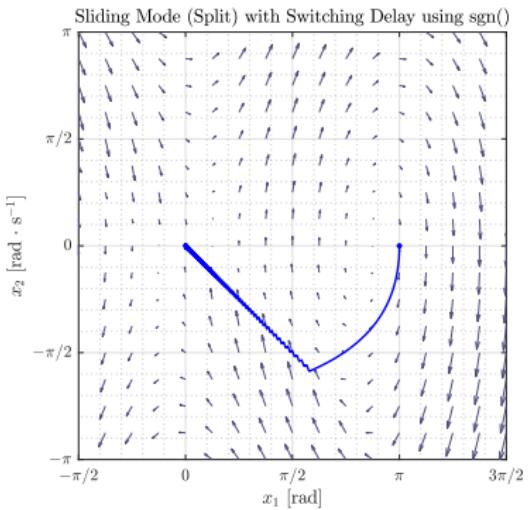
$$\dot{s} = a_1 x_2 + h(x) + g(x)u \quad , \quad \text{where } u = -\frac{a_1 x_2 + \hat{h}(x)}{\hat{g}(x)} + v$$

$$\dot{s} = \delta(x) + g(x)v \quad , \quad \text{where } \delta(x) = a_1(1 - \frac{g(x)}{\hat{g}(x)})x_2 + h(x) - \frac{g(x)}{\hat{g}(x)}\hat{h}(x)$$

$$v = -\beta(x)\operatorname{sgn}(s) \quad , \quad \text{where } \beta(x) \geq \varrho(x) + \beta_0 \quad , \quad \text{and } \varrho(x) \geq \left| \frac{\delta(x)}{\hat{g}(x)} \right|$$

Nonlinear Control Design

Sliding Mode



Nonlinear Control Design

Lyapunov Redesign



$$\delta = \frac{1}{\hat{g}(x)} \left[(h(x) - \hat{h}(x)) + (g(x) - \hat{g}(x))u \right] \quad \text{where,} \quad u = \psi(x) + v_1$$

$$\delta = \frac{1}{\hat{c}} \left(\frac{a\hat{c} - \hat{a}c}{\hat{c}} \sin x_1 + (\hat{b} - b)x_2 + \frac{c - \hat{c}}{\hat{c}}(k_1 x_1 + k_2 x_2) \right) + \frac{c - \hat{c}}{\hat{c}} v_1$$

$$|\delta| \leq \rho \|\mathbf{x}\|_2 + \kappa_0 |v_1| \quad \text{where,}$$

$$\rho = \frac{1}{\hat{c}} \left(\left| \frac{a\hat{c} - \hat{a}c}{\hat{c}} \right| + |\hat{b} - b| + \left| \frac{c - \hat{c}}{\hat{c}} \right| \sqrt{k_1^2 + k_2^2} \right) \quad \text{and} \quad \kappa_0 = \left| \frac{c - \hat{c}}{\hat{c}} \right|$$

Nonlinear Control Design

Lyapunov Redesign



$$v_1 = -\eta(t, x)\operatorname{sgn}(\omega)$$

$$\omega = \frac{\partial V}{\partial \mathbf{x}} G(\mathbf{x}) \quad \text{where,} \quad G(x) = [0 \ \hat{g}(x)]^\top$$

$$\eta(t, x) \geq \rho(t, x)/(1 - \kappa_0)$$

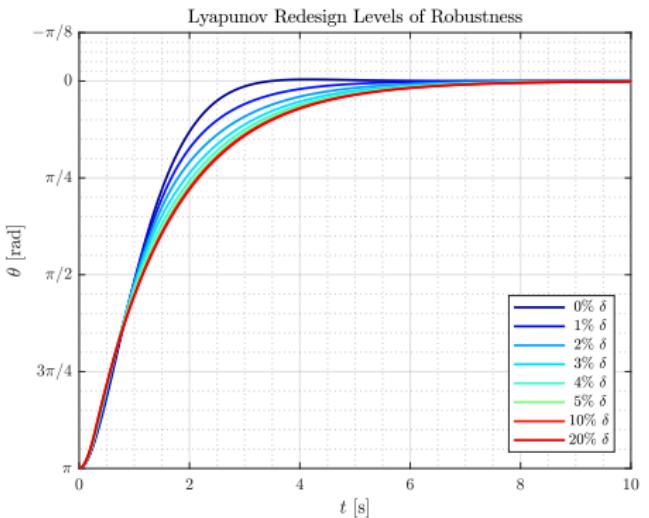
$$v_1 = \begin{cases} -\eta(t, x)\omega/\|\omega\|_2, & \text{if } \eta(t, x)\|\omega\|_2 \geq \varepsilon \\ -\eta^2(t, x)\omega/\varepsilon, & \text{if } \eta(t, x)\|\omega\|_2 < \varepsilon \ , \quad \eta(t, x) \geq \rho(t, x)/(1 - \kappa_0) \end{cases}$$

Nonlinear Control Design

Lyapunov Redesign

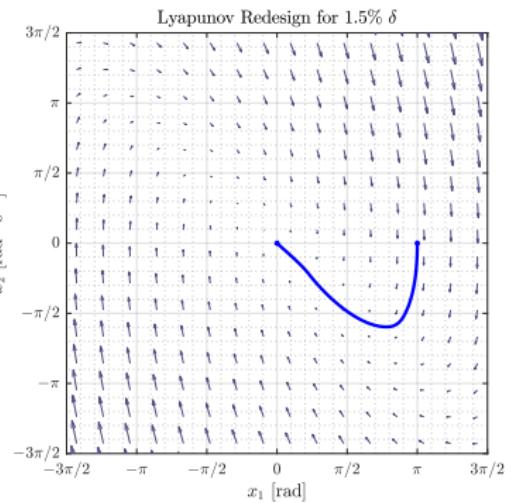
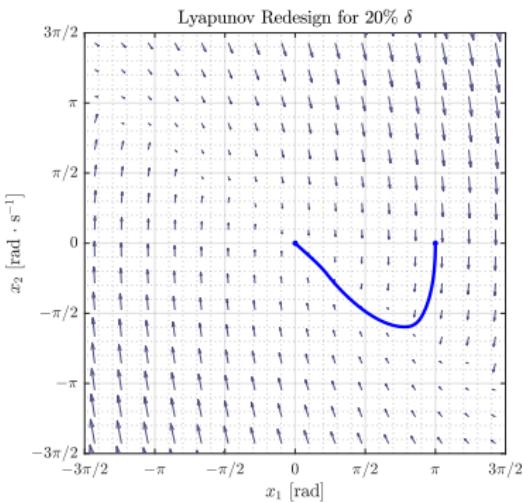


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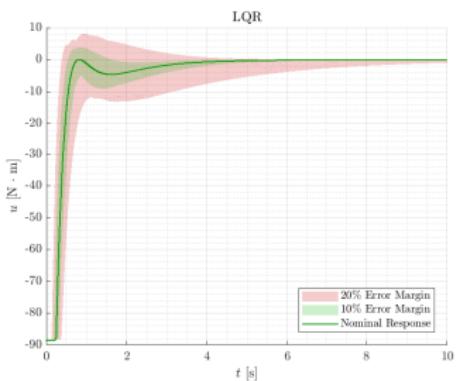
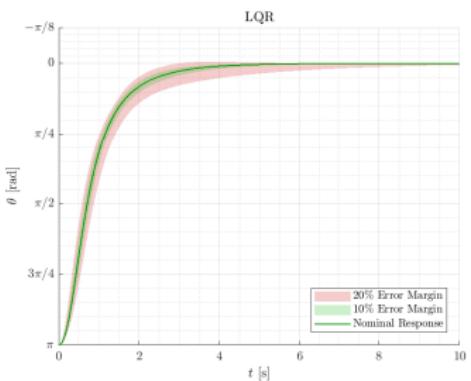
Nonlinear Control Design

Lyapunov Redesign



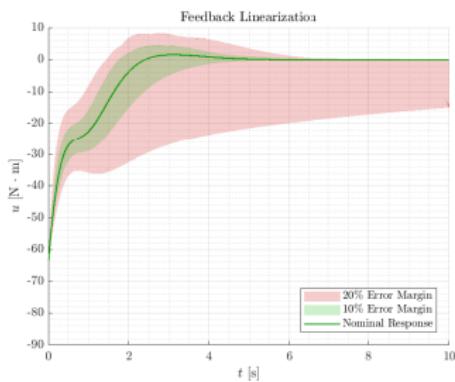
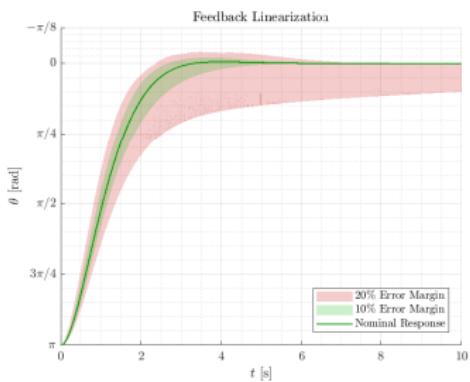
Results

Robustness to Parameter Variation



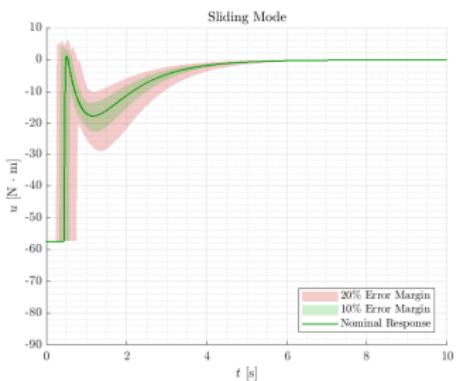
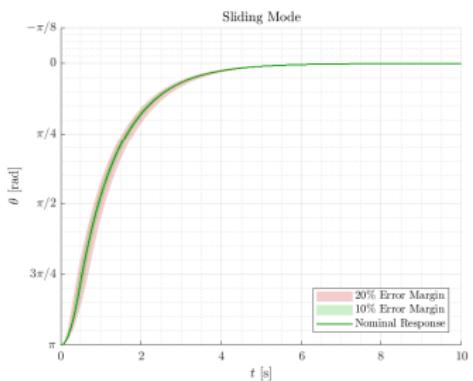
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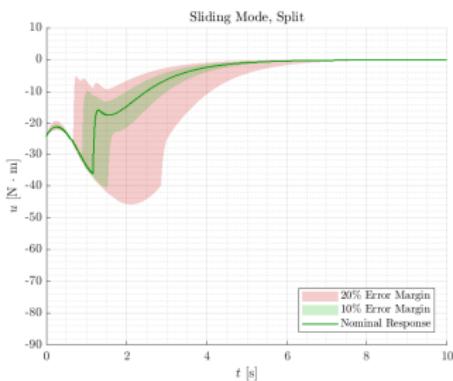
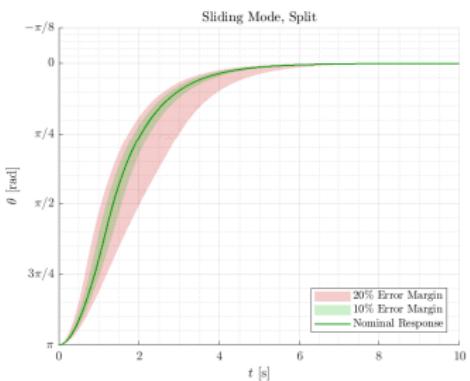
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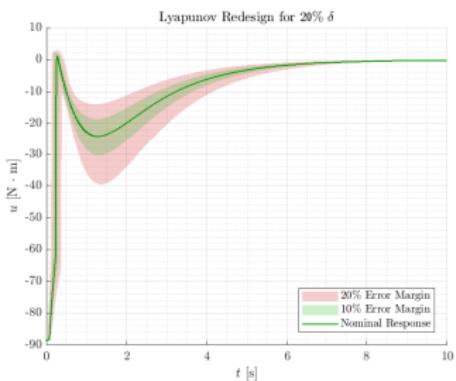
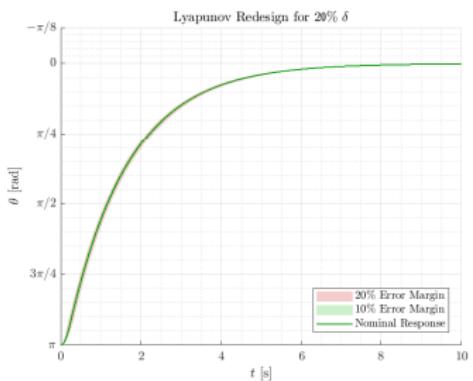
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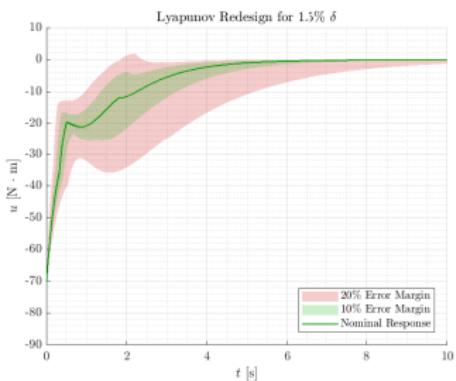
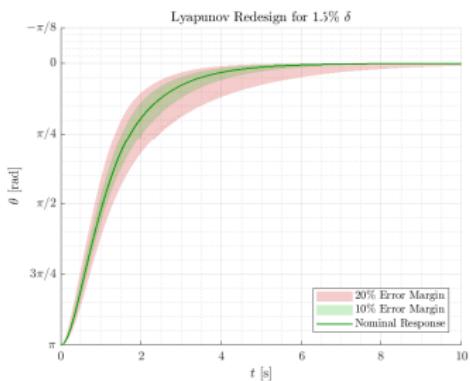
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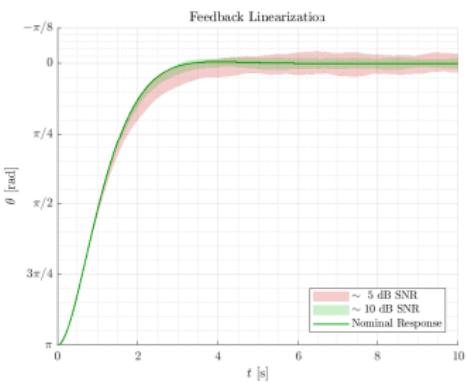
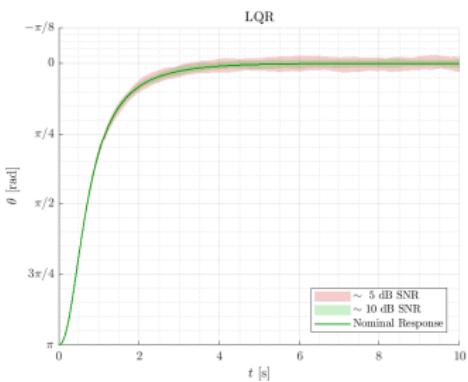
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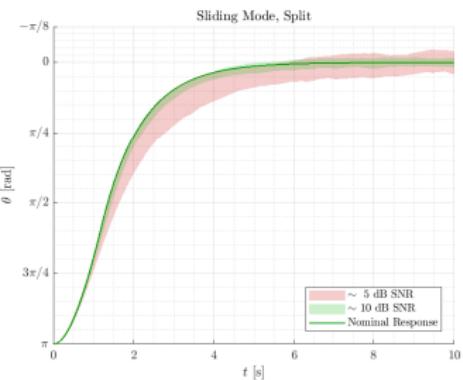
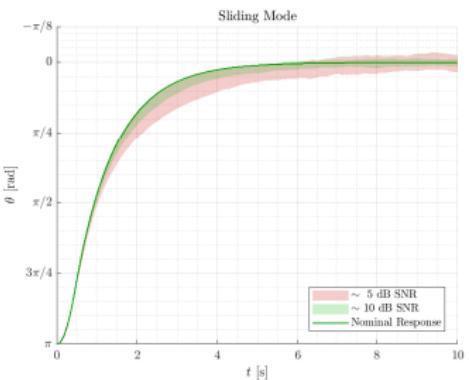
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Robustness to Input Noise



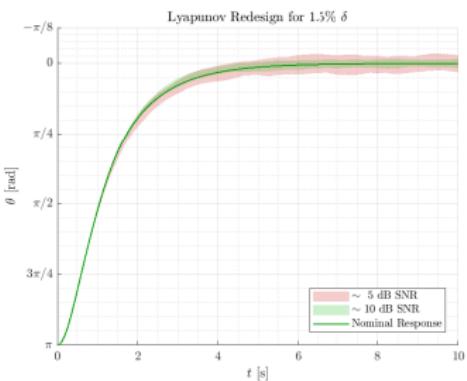
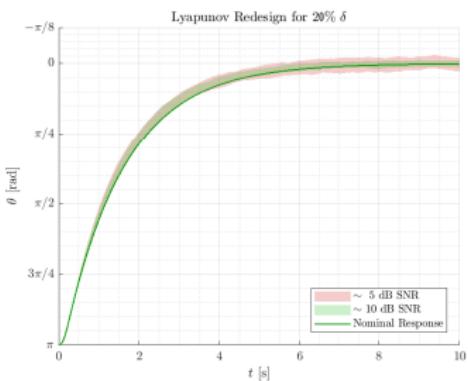
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