Corrections

Where		Mistake	Correction
P. 6	Eq. 2.1	Missing source	[1]
P. 6	Before Fig. 2.6	Missing test journal	See *Force Test Journal* Appendix A
P. 9	Before Eq. 3.5	"Equation 3.3 and 3.2 are combined,"	"Equation 3.2 is substituted into Equation 3.4,"
P. 9	Eq. 3.6	"[·]"	"[N]"
P. 10	Eq. 3.9	"[·]"	"[N]"
P. 10	Section 3.2, top	Missing source	"This section is based on [2]."
P. 10	Eq. 3.10	"[·]"	"[J]"
P. 10	Eq. 3.11	"[·]"	"[J]"
P. 10	1st 'where', line 1	"[·]"	"[N]"
P. 10	1st 'where', line 2	"[·]"	"[N]"
P. 10	Eq. 3.12	"[·]"	"[m]"
P. 10	Eq. 3.13	"[·]"	"[J]"
P. 10	Eq. 3.14	"[·]"	"[J]"
P. 10	2nd 'where', line 1	"[·]"	"[J]"
P. 11	Line 1	"[·]"	"[J]"
P. 11	Eq. 3.15	"[·]"	"[J]"
P. 11	Eq. 3.16	"[·]"	"[J]"
P. 11	Eq. 3.17	"[·]"	"[J]"
P. 11	Before Eq. 3.20	"[...] forces d'Alambert principle,"	"[...] forces, the Lagrange-d'Alembert Principle is used,"
P. 12	Line 1	"For yields [...]"	"For this system Equation 3.20 yields [...]"

Where	Mistake	Correction
P. 12 Eq. 3.21	"[·]"	"[N]"
P. 12 Section 3.3 top	Missing sources	"This section is based on [3] and [4]"
P. 12 Fig. 3.5 caption	"coulombViscous1"	"The green plot shows the contribution of Coulomb friction, while the blue shows that of viscous friction."
P. 12 Fig. 3.6 caption	"coulombViscous2"	"Coulomb and viscous friction combined."
P. 13 At "$k_{tanh} = 250$"	Missing source	[1]
P. 13 Fig. 3.7 caption	"tanhApprox"	"Plot of tanh for approximation of Coulomb friction with different values of k_{tanh}"
P. 14 Table 3.1	Missing parameters	See *Parameters* Appendix B.
P. 15 Top	Missing source	"This chapter is based on [5]"
P. 16 After Eq. 4.13	"where the output is [...]"	"where the input is [...]"
P. 17 After Eq. 4.22	"Both $\dot{x}_3$ and $\dot{x}_3$ [...]"	"Both $\dot{x}_3$ and $\dot{x}_4$ [...]"
P. 19 Poles	"$[\,-3 \;\; -5 \;\; -8\,]$"	"$[\,-5 \;\; -6 \;\; -9\,]$"
P. 19 Gain vector	"$\mathbf{k} = [\,-12.22 \;\; -8.05 \;\; -20.10\,]$"	"$\mathbf{k} = [\,-27.50 \;\; 13.34 \;\; 29.30\,]$"
P. 19 Fig. 4.1	Old design/values in figure	Updated figure in Appendix C Figure 2
P. 19 Fig. 4.2	Old design/values in figure	Updated figure in Appendix C Figure 3
P. 21 Fig. 4.3	Old design/values in figure	Updated figure in Appendix C Figure 4
P. 23 Eq. 5.1	"[·]"	"[N]"
P. 25 2nd paragraph	"[...] a little less than $\theta = \frac{\pi}{2}$."	"[...] $\theta = \frac{7\pi}{4}$"
P. 25 After Fig. 5.1	"[...] raised above $\frac{\pi}{2}$ [...]"	"[...] raised to $\frac{7\pi}{4}$ [...]"

Appendix

A | Force Test Journal

Date: Aug. 10, 2018

Purpose

The purpose of the test is to measure the force on the cart directly in relation to the input given by the microcontroller to ensure that the relation between the 12 bit input and the actual force output is accurate.

Setup

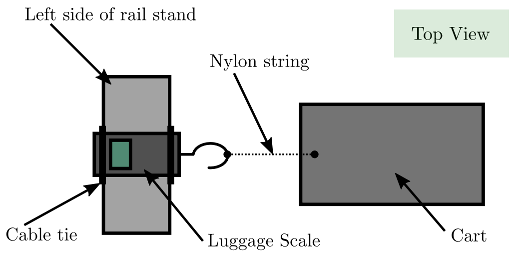

Figure 1: Setup diagram

List of Equipment

Instrument	**Type**
Cart pendulum	Setup at AAU Control Lab
Laptop	Asus UX31E
Luggage scale	B00ZWNGZFO ES-PS01
Cable ties for mounting the Luggage scale	
Nylon string for attaching the scale to the cart	

Procedure

1. Attach the luggage scale to the setup using the cable ties as seen in Figure 1.
2. Attach the cart to the luggage scale as seen in Figure 1.
3. Power up the setup.
4. Connect the Teensy in the setup through USB to the laptop.

5. Ready and compile the code giving a constant reference of 1.3 A (supposed) armature current.
6. Upload the code to the Teensy board.
7. Take reading of the the weight pulled by the cart.
8. repeat step 5 to 7 with 0.1 A increments for each measurement up to 4.5 A.

Results

[A]	**[kg]**
1.3	0.30
1.4	0.36
1.5	0.40
1.6	0.46
1.7	0.47
1.8	0.53
1.9	0.55
2.0	0.63

[A]	**[kg]**
2.1	0.66
2.2	0.71
2.3	0.73
2.4	0.75
2.5	0.78
2.6	0.80
2.7	0.82
2.8	0.84

[A]	**[kg]**
2.9	0.86
3.0	0.90
3.1	0.92
3.2	0.95
3.3	0.97
3.4	1.02
3.5	1.05
3.6	1.08

[A]	**[kg]**
3.7	1.11
3.8	1.16
3.9	1.17
4.0	1.20
4.1	1.22
4.2	1.26
4.3	1.28
4.4	1.32
4.5	1.34

To obtain the forces, F [N], all readings, [kg], are multiplied by the gravitational acceleration $g = 9.82\,\mathrm{m} \cdot \mathrm{s}^{-1}$.

B | Parameters

The cart friction parameters estimated by [1], are given by,

$$b_{c,c} = \begin{cases} 3.021 & \text{if} \quad \dot{x} > 0 \\ 2.746 & \text{if} \quad \dot{x} < 0 \end{cases}, \qquad b_{c,v} = \begin{cases} 1.937 & \text{if} \quad \dot{x} > 0 \\ 1.422 & \text{if} \quad \dot{x} < 0 \end{cases}, \qquad [\mathrm{N}]$$

however in this project the values are used for control design only and not for a Kalman, so it is considered adequate to average the values, which are as given in Table 1 below.

Parameter		**Notation**	**Quantity**	**Unit**
Pendulum Mass		m	0.201	kg
Cart Mass	*	M	5.273	kg
Rod Length		l	0.3235	m
Length of Rail		l_r	0.89	m
Pulley Radius		r	0.028	m
Cart Coulomb Friction	*	$b_{c,c}$	2.884	N
Cart Viscous Friction	*	$b_{c,v}$	1.680	$\mathrm{N \cdot m^{-1} \cdot s}$
Pendulum Coulomb Friction	*	$b_{p,c}$	0.004	$\mathrm{N \cdot m}$
Pendulum Viscous Friction	*	$b_{p,v}$	0.4×10^{-3}	$\mathrm{N \cdot m \cdot s}$

Table 1: '*' indicates that the parameter is estimated[1].

C | Sliding Mode Design Figures

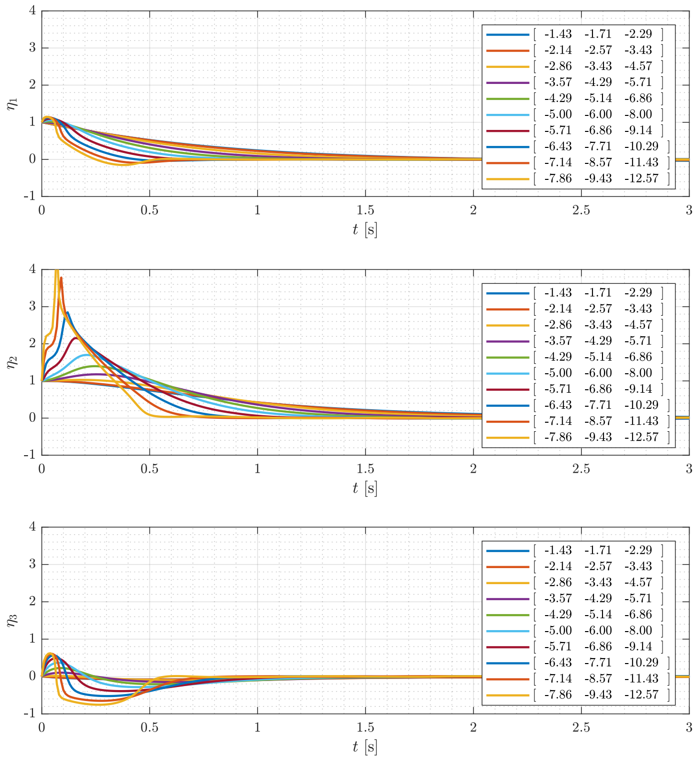

Figure 2: Comparison of some different choices of pole placements for the reduced order system. The chosen pole placement is $[\,-5 \;\; -6 \;\; -9\,]$ resulting in $\mathbf{k} = [\,-27.50 \;\; 13.34 \;\; 29.30\,]$.

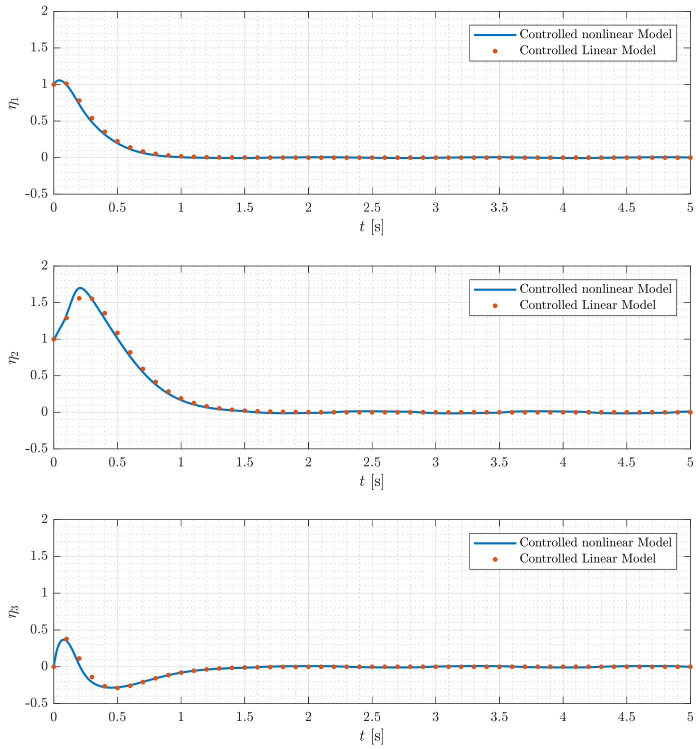

Figure 3: Result of the chosen pole placement for the reduced order system with comparison between linearized and non-linear plant using lsim and ode45 respectively.

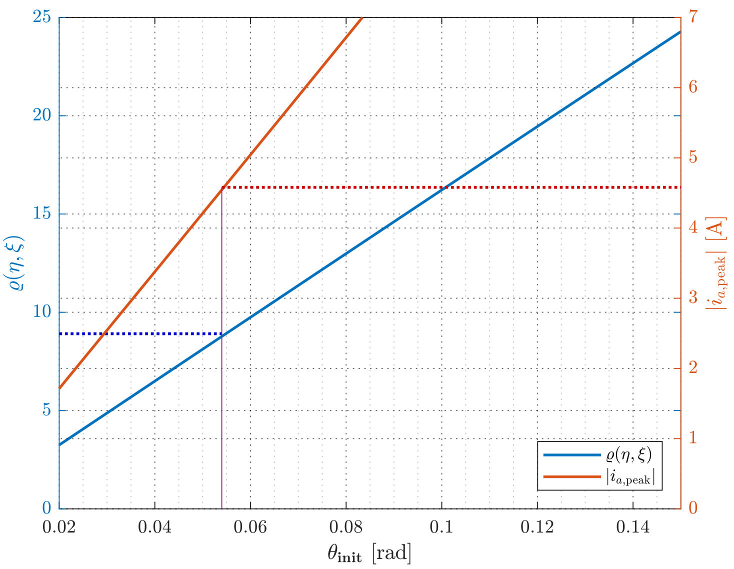

Figure 4: $\dot{\theta} = 0$ and the horizontal line marks $\theta_{\max} = 0.0540\,\text{rad}$ dictated by the current limitation of the motor, $i_{a,\max} = 4.58\,\text{A}$, and thereby indicating the maximum gain, $\varrho(\boldsymbol{\eta}, \xi) = 8.9095$, with $\beta_0 = 0.1$, allowing some margin for the supposed operational region.

Bibliography

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[5] Hassan K. Khalil. *Nonlinear Systems*. 3rd ed. 2015.