CHAPTER 15

15.1

(a) Field Effect...modulation of the semiconductor conductivity by an electric field applied normal to the surface of the semiconductor.

(b) Channel...nondepleted current carrying portion of the semiconductor "bar" between the source and drain in a J–FET.

(c) As viewed from the exterior of the device, the drain current flows *out-of* the drain contact in a p-channel device. Holes are the channel carriers in a p-channel device and by definition these must flow along the channel into the drain. The current has the same direction as the hole flow—from source to drain and out of the drain contact.

(d) Gradual channel approximation...In this approximation it is assumed the electrostatic variables in one direction (say the y-direction) change slowly compared to the rate of change of the electrostatic variables in a second direction (say the x-direction). The y-direction dependence is then neglected and the electrostatic variables computed using a pseudo-one-dimensional analysis at each point y.

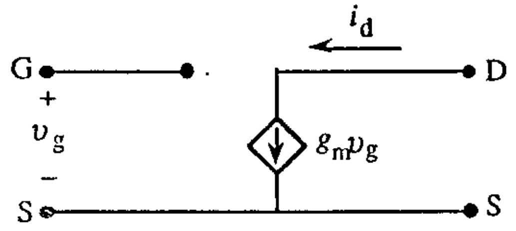
(e) Pinch-off...complete depletion of the channel region; touching of the top and bottom depletion regions in the symmetrical J–FET.

(f) As given by text Eqs. (15.18),

$$g_{\mathrm{d}} = \left.\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{D}}}\right|_{V_{\mathrm{G}}=\text{constant}} \qquad \text{...drain conductance}$$

$$g_{\mathrm{m}} = \left.\frac{\partial I_{\mathrm{D}}}{\partial V_{\mathrm{G}}}\right|_{V_{\mathrm{D}}=\text{constant}} \qquad \text{...transconductance}$$

(g) In a long channel J–FET, I_{D}(V_{G} held constant) $\cong$ constant for $V_{\mathrm{D}} > V_{\mathrm{Dsat}}$. Thus $g_{\mathrm{d}} \cong 0$ and the g_{d} conductance in Fig. 15.19(b) can be neglected in drawing the equivalent circuit.



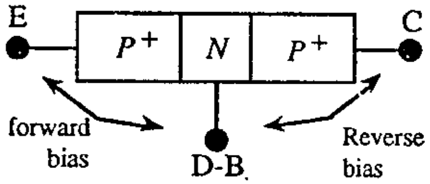
(h) MESFET...*me*tal semiconductor field effect transistor. D-...depletion mode; E-...enhancement mode.

(i) Once $|\mathcal{E}_y|$ exceeds ~10^4 V/cm, the carrier drift velocity is no longer proportional to the magnitude of the electric field as assumed in the long-channel analysis.

(j) In the two-region theory the carrier drift velocity is set equal to v_{sat} at all points in the channel between y_1 and the drain. y_1 is the point in the channel where $|\mathcal{E}_y|$ has increased to v_{sat}/(low-field mobility).

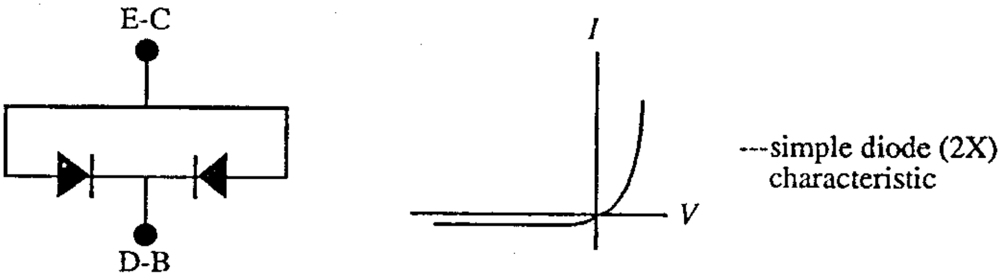
15.2

(a) If $d << L_P$, the two *pn* junctions will be interacting like in a BJT. Moreover, the biases are equivalent to active mode biasing in a BJT.



Obviously, we are being asked for the common base output characteristics (Fig. 10.4a or Fig. 11.4d) of a bipolar junction transistor.

(b) Since here $d >> L_P$, the two *pn* junctions do not interact, and we simply have two diodes in parallel.



(c) The biasing here is identical to that normally encountered in standard J-FET operation. The physical properties are also those of a J-FET. The desired characteristics are clearly just the I_D–V_D characteristics of the J-FET with $V_D \rightarrow V_{DB}$ and $V_G \rightarrow V_{EB}$.

<u>15.3</u>
(a) Following the *Hint* one obtains,

$$\int_0^y I_D dy' = I_D y = 2qZ\mu_n N_D a \int_0^{V(y)} [1 - W(V')/a]dV'$$

$$y = \frac{2qZ\mu_n N_D a}{I_D}\int_0^{V(y)} [1 - W/a]dV'$$

$$= \frac{2qZ\mu_n N_D a}{I_D}\left\{V - \frac{2}{3}(V_{bi}-V_P)\left[\left(\frac{V+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]\right\}$$

Note that, given the parallel development, setting $V_D \rightarrow V$ inside the Eq. (15.9) braces yields the foregoing integration result. Eliminating I_D using Eq. (15.9) then yields

$$\frac{y}{L} = \frac{V - \frac{2}{3}(V_{bi}-V_P)\left[\left(\frac{V+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]}{V_D - \frac{2}{3}(V_{bi}-V_P)\left[\left(\frac{V_D+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]} \qquad \Leftarrow \text{Answer}$$

(b) If $V_G = 0$, $V_D = 5V$, $V_{bi} = 1V$ and $V_P = -8V$,

$$\frac{y}{L} = \frac{V - (2/9)\left[(V+1)^{3/2} - 1\right]}{5 - (2/9)(6^{3/2} - 1)}$$

and

V	y/L
1	0.303
2	0.546
3	0.738
4	0.888

The above data was used in constructing Fig. 15.11(c).

15.4

Differentiating Eq.(15.9) with respect to V_D with V_G held constant yields

$$\left.\frac{\partial I_D}{\partial V_D}\right|_{V_G=\text{constant}} = \frac{2qZ\mu_n N_D a}{L}\left[1-\left(\frac{V_D+V_{bi}-V_G}{V_{bi}-V_P}\right)^{1/2}\right] \; set= \; 0$$

Solving we obtain

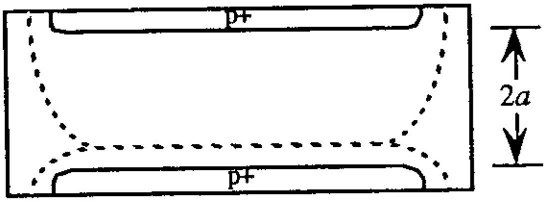
$$\left(\frac{V_{Dsat}+V_{bi}-V_G}{V_{bi}-V_P}\right)^{1/2} = 1$$

or

$$\boxed{V_{Dsat} = V_G - V_P}$$

15.5

(a)



NOTE: The bottom depletion width is the same as at equilibrium; the top depletion width is greater than a.

(b) We can state

$$2a = 2\left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_P)\right]^{1/2} = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_{PT})\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}V_{bi}\right]^{1/2}$$

$\Uparrow$ normal situation; $\Uparrow$ top gate depletion width; $\Uparrow$ bottom gate depletion width

Thus

$$2(V_{bi}-V_P)^{1/2} = (V_{bi}-V_{PT})^{1/2} + V_{bi}^{1/2}$$

or

$$V_{PT} = V_{bi} - \left[2(V_{bi}-V_P)^{1/2} - V_{bi}^{1/2}\right]^2$$

Given $V_{bi} = 1\text{V}$, $V_P = -8\text{V}$, one obtains

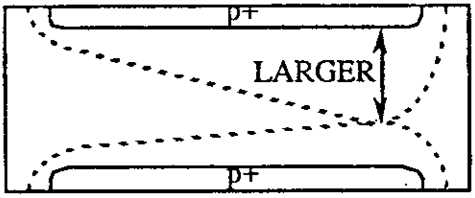
$$V_{PT} = 1 - [2\sqrt{9} - 1]^2$$

or

$$\mathbf{V_{PT} = -24V}$$

The above answer is clearly consistent with part (a). The top depletion width needs to be wider than when the two gates are tied together, thereby necessitating a larger applied $|V_G|$.

(c)



Assumes $V_{PT} < V_{GT} < 0$.

NOTE: Although the bottom $V_{GB} = 0$, the bottom depletion width still contributes to the constriction of the channel.

(d) When $V_D = V_{Dsat}$, $W_T + W_B \rightarrow 2a$ and $V(L) = V_{Dsat}$. Also

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi} + V - V_{GT})\right]^{1/2} \qquad \text{...top depletion width}$$

$$W_B = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi} + V - V_{GB})\right]^{1/2} \qquad \text{...bottom depletion width}$$

Since in the problem at hand $V_{GB} = 0$, we obtain at pinch-off

$$2a = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}+V_{Dsat}-V_{GT})\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}+V_{Dsat})\right]^{1/2}$$

But from part (b)...

$$2a = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_{PT})\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}V_{bi}\right]^{1/2}$$

So finally, cancelling the $2K_S\varepsilon_0/qN_D$ factor everywhere,

$$\boxed{(V_{bi}-V_{PT})^{1/2} + V_{bi}^{1/2} = (V_{bi}+ V_{Dsat}-V_{GT})^{1/2} + (V_{bi}+V_{Dsat})^{1/2}}$$

(e) From the part (c) answer, one can tell by inspection that V_{Dsat} for $V_{GB} = 0$ operation will be *greater than* V_{Dsat} for $V_{GB} = V_{GT}$ operation. The top side depletion width needs to be wider, in turn necessitating more current flow and a higher V_{Dsat} at the pinch-off point. (Alternative) Using the parameters of part (b), if $V_{bi} = 1\text{V}$, $V_P = -8\text{V}$ and $V_{PT} = -24\text{V}$, one concludes $V_{Dsat} = V_G - V_P = 6\text{V}$ for $V_{GB} = V_{GT} = -2\text{V}$ operation and $V_{Dsat} \cong 7\text{V}$ from the part (d) result if $V_{GT} = -2\text{V}$. Again V_{Dsat} ($V_{GB} = 0$ operation) is greater than V_{Dsat} ($V_{GB} = V_{GT}$ operation). Note that the two V_{Dsat}'s are equal if $V_{GT} = 0$.

(f) Since the top and bottom depletion widths are not equal, the symmetry of the structure is destroyed and one must start by revising Eq.(15.3).

$$I_D = -Z\int_{W_T(y)}^{2a-W_B(y)} J_{Ny}dx = Z\int_{W_T(y)}^{2a-W_B(y)} \left(q\mu_n N_D\frac{dV}{dy}\right)dx = qZ\mu_n N_D\frac{dV}{dy}[2a-W_B(y)-W_T(y)]$$

or

$$I_D = 2qZ\mu_n N_D a\frac{dV}{dy}\left[1-\frac{W_T+W_B}{2a}\right]$$

Integrating next over the length of the channel yields,

$$I_D = \frac{2qZ\mu_n N_D a}{L}\int_0^{V_D}\left[1-\frac{W_T+W_B}{2a}\right]dV \qquad \text{...revised Eq.(15.5)}$$

Using the W_T, W_B, and $2a$ expressions presented in part (d), one obtains

$$\frac{W_T+W_B}{2a} = \frac{(V_{bi}+V-V_{GT})^{1/2}+(V_{bi}+V)^{1/2}}{(V_{bi}-V_{PT})^{1/2}+V_{bi}^{1/2}} \qquad \text{...}V_{GB}=0$$

and

$$I_D = \frac{2qZ\mu_n N_D a}{L}\int_0^{V_D}\left[1-\frac{(V_{bi}+V-V_{GT})^{1/2}+(V_{bi}+V)^{1/2}}{(V_{bi}-V_{PT})^{1/2}+V_{bi}^{1/2}}\right]dV$$

Performing the integration gives the desired solution

$$I_D = \frac{2qZ\mu_n N_D a}{L}\left[V_D-\frac{2}{3}\frac{(V_{bi}+V_D-V_{GT})^{3/2}+(V_{bi}+V_D)^{3/2}-(V_{bi}-V_{GT})^{3/2}-V_{bi}^{3/2}}{(V_{bi}-V_{PT})^{1/2}+V_{bi}^{1/2}}\right]$$

15.6

(a) The general W-relationship for one-sided power-law profiles was noted to be (Eq. 7.6)

$$W = \left[\frac{(m+2)K_S\varepsilon_0}{qb}(V_{bi}-V_A)\right]^{1/(m+2)}$$

For a linearly graded junction $m = 1$ and $b = N_0/a$, or

$$\boxed{W = \left[\frac{3K_S\varepsilon_0 a}{qN_0}(V_{bi}-V_A)\right]^{1/3}}$$

(b) It should be noted first of all that

$$n \cong N_D - N_A = N_0\frac{x}{a}$$ ...in the nondepleted left-hand side of the channel ($W \le x \le a$)

giving

$$J_N = J_{Ny} = q\mu_n N_0 \frac{x}{a}\mathcal{E}_y = -q\mu_n N_0 \frac{x}{a}\frac{dV}{dy}$$ ...in the left-hand portion of the conducting channel

Neglecting the μ_n doping dependence, we can write

$$I_D = 2Z\int_{W(y)}^{a}\left(q\mu_n N_0 \frac{x}{a}\frac{dV}{dy}\right)dx = 2qZ\mu_n\frac{N_0}{a}\frac{dV}{dy}\int_W^a x dx = qZ\mu_n\frac{N_0}{a}\frac{dV}{dy}\left(a^2-W^2\right)$$

or

$$I_D = qZ\mu_n N_0 a\frac{dV}{dy}\left[1-\left(\frac{W}{a}\right)^2\right]$$ ...revised form of Eq.(15.3b)

(The "2" appears in front of the first integral above because equal contributions are obtained from the left- and right-hand sides of the channel.) Integrating over the length of the channel then yields,

$$I_D = \frac{qZ\mu_n N_0 a}{L}\int_0^{V_D}\left[1-\left(\frac{W}{a}\right)^2\right]dV$$

But from part (a),

$$W = \left[\frac{3K_S\varepsilon_0 a}{qN_0}(V_{bi} + V - V_G)\right]^{1/3} \qquad \text{where } V_A = V_G - V$$

and

$$a = \left[\frac{3K_S\varepsilon_0 a}{qN_0}(V_{bi} - V_P)\right]^{1/3}$$

so

$$\frac{W}{a} = \left(\frac{V_{bi} + V - V_G}{V_{bi} - V_P}\right)^{1/3}$$

Substituting into the I_D equation,

$$I_D = \frac{qZ\mu_n N_0 a}{L}\int_0^{V_D}\left[1 - \left(\frac{V_{bi} + V - V_G}{V_{bi} - V_P}\right)^{2/3}\right]dV$$

and after integrating

$$\boxed{I_D = \frac{qZ\mu_n N_0 a}{L}\left\{V_D - \frac{3}{5}(V_{bi}-V_P)\left[\left(\frac{V_D+V_{bi}-V_G}{V_{bi}-V_P}\right)^{5/3} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{5/3}\right]\right\}}$$

15.7

Noting

$$I_{D0} \equiv I_{Dsat}\big|_{V_G=0 \text{ and } R_S=R_D=0} = G_0\left\{-V_P - \frac{2}{3}(V_{bi}-V_P)\left[1-\left(\frac{V_{bi}}{V_{bi}-V_P}\right)^{3/2}\right]\right\}$$

and introducing

$$V_{ref} = -V_P - \frac{2}{3}(V_{bi}-V_P)\left[1-\left(\frac{V_{bi}}{V_{bi}-V_P}\right)^{3/2}\right]$$

gives

$$I_{D0} = G_0 V_{ref}$$

Using the results from Exercise 15.3 we can then write:

•For $V_D \le V_{Dsat}$

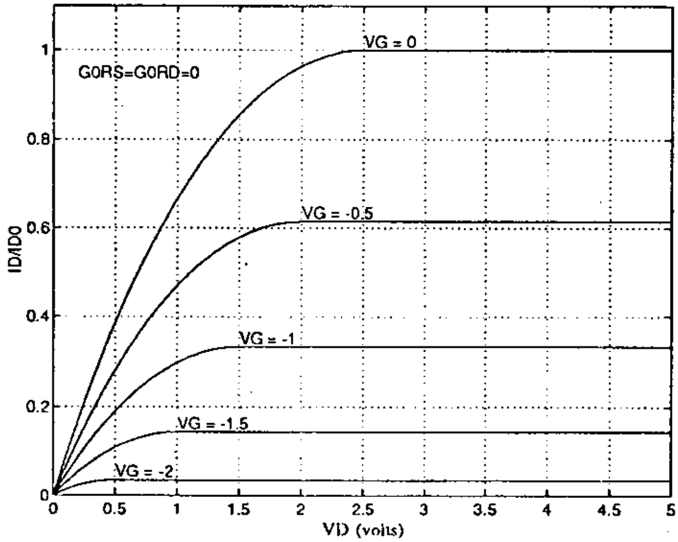
$$\frac{I_D}{I_{D0}} = \frac{V_D}{V_{ref}} - \frac{I_D}{I_{D0}} G_0(R_S+R_D)$$

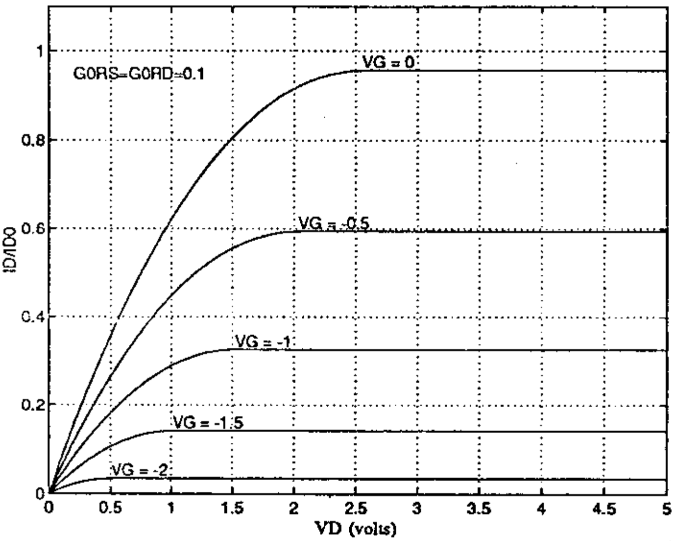
$$-\frac{2}{3}\left(\frac{V_{bi}-V_P}{V_{ref}}\right)\left[\left(\frac{V_D-\frac{I_D}{I_{D0}}G_0R_DV_{ref}+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}-\left(\frac{\frac{I_D}{I_{D0}}G_0R_SV_{ref}+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]$$

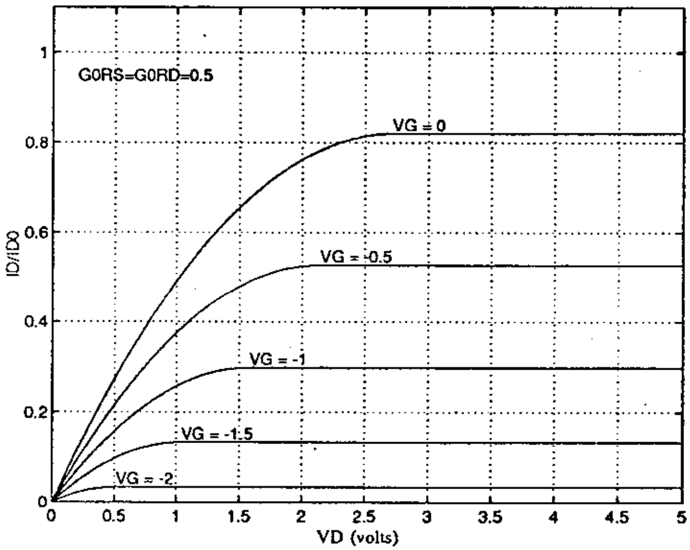
•For $V_D \ge V_{Dsat}$

$$\frac{I_{Dsat}}{I_{D0}} = \frac{V_G-V_P}{V_{ref}} - \frac{I_{Dsat}}{I_{D0}}G_0R_S - \frac{2}{3}\left(\frac{V_{bi}-V_P}{V_{ref}}\right)\left[1-\left(\frac{\frac{I_{Dsat}}{I_{D0}}G_0R_SV_{ref}+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]$$

The foregoing relationships can be iterated using the fzero function in MATLAB to determine I_D/I_{D0} or I_{Dsat}/I_{D0} as a function of V_D with V_G held constant at preselected values. Running the P_05_07.m file on the Instructor's disk yields the results reproduced below and on the next page. With $G_0R_S = G_0R_D = 0$, one obtains the same characteristics as those displayed in Fig. 15.16. Although the characteristics retain their same general shape when $G_0R_S = G_0R_D > 0$, an increase in the series resistances causes a significant decrease in I_{Dsat} and a slight increase in V_{Dsat}.




G0RS=G0RD=0.1
VG = 0
VG = -0.5
VG = -1
VG = -1.5
VG = -2
ID/ID0
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
VD (volts)


G0RS=G0RD=0.5
VG = 0
VG = -0.5
VG = -1
VG = -1.5
VG = -2
ID/ID0
0
0.2
0.4
0.6
0.8
1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
VD (volts)

15.8

(a) Since the gate is shorted to the source, $V_G = 0$. Also, $I = I_D$ and $V = V_D$. Thus, referring to Eqs.(15.9), (15.12), and (15.13),

$$G = \frac{I_D}{V_D} = G_0\left\{1 - \frac{2}{3}\left(\frac{V_{bi}-V_P}{V_D}\right)\left[\left(\frac{V_D+V_{bi}}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}}{V_{bi}-V_P}\right)^{3/2}\right]\right\} \quad ...0 \le V_D \le V_{Dsat} = -V_P$$

and

$$G_{sat} = \frac{I_{Dsat}}{V_D} = G_0\left\{\frac{-V_P}{V_D} - \frac{2}{3}\left(\frac{V_{bi}-V_P}{V_D}\right)\left[1 - \left(\frac{V_{bi}}{V_{bi}-V_P}\right)^{3/2}\right]\right\} \quad ...V_D \ge V_{Dsat} = -V_P$$

Likewise (utilizing Table 15.1),

$$g = \frac{dI_D}{dV_D} = g_d|_{V_G=0} = G_0\left[1 - \left(\frac{V_D+V_{bi}}{V_{bi}-V_P}\right)^{1/2}\right] \quad ...0 \le V_D \le V_{Dsat} = -V_P$$

and

$$g_{sat} = \frac{dI_{Dsat}|_{V_G=0}}{dV_D} = 0 \quad ...V_D \ge V_{Dsat} = -V_P$$

(b) With $V_D = V_{Dsat}/2 = -V_P/2$

$$R = \frac{1}{G} = \frac{1}{G_0\left\{1 - \frac{4}{3}\left(\frac{V_{bi}-V_P}{-V_P}\right)\left[\left(\frac{V_{bi}-V_P/2}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}}{V_{bi}-V_P}\right)^{3/2}\right]\right\}}$$

$$r = \frac{1}{g} = \frac{1}{G_0\left[1 - \left(\frac{V_{bi}-V_P/2}{V_{bi}-V_P}\right)^{1/2}\right]}$$

$$G_0 = \frac{2qZ\mu_n N_D a}{L} = 2\,(1.6\times10^{-19})(1248)(10^{16})(5\times10^{-5}) = 2.00\times10^{-4}\ \text{S}$$

$$R = \frac{1}{(2\times10^{-4})\left\{1-\left(\frac{4}{3}\right)\left(\frac{3}{2}\right)\left[\left(\frac{2}{3}\right)^{3/2}-\left(\frac{1}{3}\right)^{3/2}\right]\right\}} = 16.9\ \text{k}\Omega$$

$$r = \frac{1}{(2\times10^{-4})\left[1-\left(\frac{2}{3}\right)^{1/2}\right]} = 27.3\ \text{k}\Omega$$

15.9

(a) The same development as presented in Section 17.3.2 can be followed with the replacement of C_O with C_G.

(b) At maximum (whether one considers below or above pinch-off biasing), one can write

$$g_m \le G_0 = \frac{2qZ\mu_n N_D a}{L}$$

Also, in general,

$$C_G = 2\int_0^L \frac{K_S \varepsilon_0 Z}{W}\,dy$$

Since $a \ge W(y)$

$$C_G \ge 2\int_0^L \frac{K_S \varepsilon_0 Z}{a}\,dy = \frac{2K_S \varepsilon_0 ZL}{a}$$

If g_m is replaced by something greater than or equal to itself, and C_G is replaced by something less than or equal to itself, then it follows that

$$f_{max} = \frac{g_m}{2\pi C_G} \le \frac{2qZ\mu_n N_D a}{2\pi L} \bullet \frac{a}{2K_S \varepsilon_0 ZL} = \frac{q\mu_n N_D a^2}{2\pi K_S \varepsilon_0 L^2}$$

(c)

$$f_{max}(\text{limit}) = \frac{q\mu_n N_D a^2}{2\pi K_S \varepsilon_0 L^2} = \frac{(1.6\times 10^{-19})(1248)(10^{16})(5\times 10^{-5})^2}{2\pi\,(11.8)(8.85\times 10^{-14})(5\times 10^{-4})^2}$$

$$= \mathbf{3.04\ \ GHz}$$

15.10

(a)/(b) With the device saturation biased and $V_G = 0$, we conclude from Table 15.1 that

$$g_m = G_0\left[1 - \left(\frac{V_{bi}}{V_{bi} - V_P}\right)^{1/2}\right]$$

where

$$G_0 \equiv \frac{2qZ\mu_n N_D a}{L}$$

The only parameter in G_0 which is temperature dependent is μ_n. Thus

$$\frac{g_m(T)}{g_m(300K)} = \left(\frac{\mu_n(T)}{\mu_n(300K)}\right)\left(\frac{1-[V_{bi}(T)/(V_{bi}-V_P)]^{1/2}}{1-[V_{bi}(300K)/(V_{bi}-V_P)]^{1/2}}\right)$$

with

$$V_{bi} = (kT/q)\ln\left(N_A N_D/n_i^2\right)$$

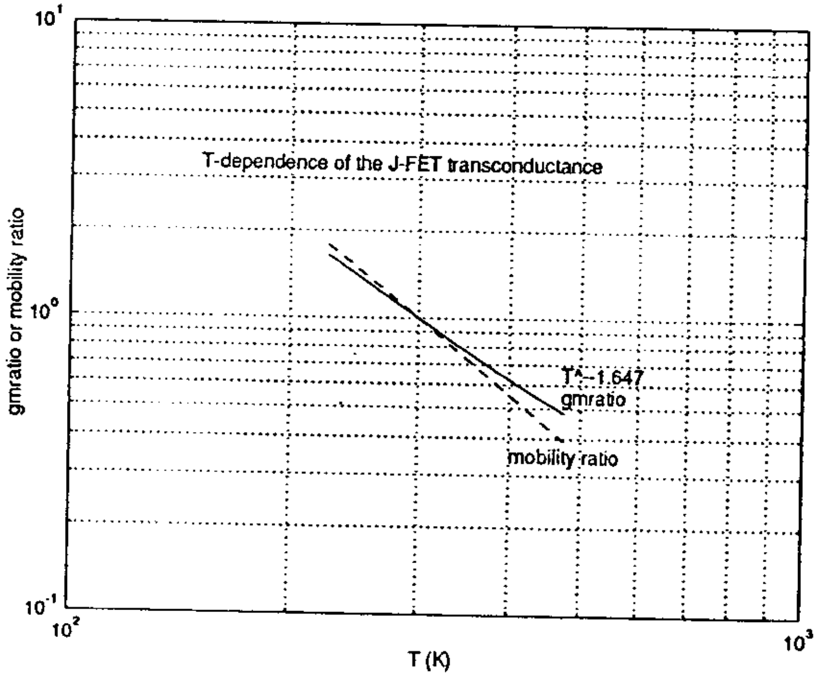
and

$$a = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_P)\right]^{1/2}$$

or

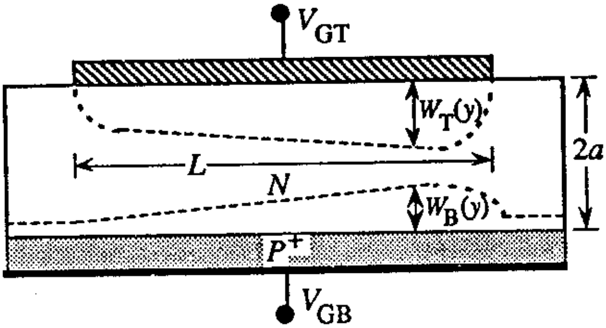
$$(V_{bi}-V_P)|_T = (V_{bi}-V_P)|_{300K} = (qN_D a^2)/(2K_S\varepsilon_0)$$

The required computations for both part (a) and part (b) are performed by file P_15_10.m on the Instructor's disk. The μ_n vs. T dependence was established employing the empirical-fit relationships found in Exercise 3.1 and programmed in file P_03_03.m. The n_i vs. T dependence was computed following the procedure outlined in Exercise 2.4(a). The resultant $g_m(T)/g_m(300K)$ and $\mu_n(T)/\mu_n(300K)$ plots reproduced in the following figure clearly exhibit a power-law type dependence, with a least squares fit yielding $g_m(T)/g_m(300K) = (T/300)^{-1.647}$. The variation of the transconductance with temperature is seen to arise primarily from the variation of the carrier mobility with temperature.



15.11

The device subject to analysis is pictured below



In the two region model the long-channel theory can be employed for drain biases below saturation. Paralleling the solution to Problem 15.5(f), let $W_T(y)$ be the top gate (MS) depletion width and $W_B(y)$ the bottom gate (p^+-n) depletion width. In general

$$I_D = -Z\int_{W_T(y)}^{2a-W_B(y)} J_{Ny}dx = Z\int_{W_T}^{2a-W_B} \left(q\mu_n N_D \frac{dV}{dy}\right) dx = qZ\mu_n N_D \frac{dV}{dy}[2a-W_B-W_T]$$

or

$$I_D = 2qZ\mu_n N_D a \frac{dV}{dy}\left(1-\frac{W_T+W_B}{2a}\right)$$

Integrating next over the length of the channel yields,

$$I_D = \frac{2qZ\mu_n N_D a}{L}\int_0^{V_D}\left(1-\frac{W_T+W_B}{2a}\right)dV$$

Now

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{biT}+V-V_{GT})\right]^{1/2} \qquad \text{...top depletion width}$$

$$W_B = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{biB}+V-V_{GB})\right]^{1/2} \qquad \text{...bottom depletion width}$$

and, given total depletion of the channel occurs when $V_{GT} = V_P$ and $V_D = V_{GB} = 0$,

$$2a = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{biT}-V_P)\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}V_{biB}\right]^{1/2}$$

Thus

$$\frac{W_T+W_B}{2a} = \frac{(V_{biT}+V-V_{GT})^{1/2} + (V_{biB}+V-V_{GB})^{1/2}}{(V_{biT}-V_P)^{1/2} + V_{biB}^{1/2}}$$

Substituting the depletion width relationship into the I_D expression and performing the integration finally yields the desired computational relationship.

$$I_D = G_0\left[V_D - \frac{2}{3}\,\frac{(V_{biT}+V_D-V_{GT})^{3/2} + (V_{biB}+V_D-V_{GB})^{3/2} - (V_{biT}-V_{GT})^{3/2} - (V_{biB}-V_{GB})^{3/2}}{(V_{biT}-V_P)^{1/2} + V_{biB}^{1/2}}\right]$$

15.12

Setting $\mu_0 \rightarrow -\mu_n$ and $\mathcal{E} \rightarrow \mathcal{E}_y = -dV/dy$ in Eq. (15.21), and replacing μ_n in Eq. (15.2) with the resulting $\mu(\mathcal{E})$ expression, one obtains

$$J_{Ny} = -q\left(\frac{\mu_n}{1+\dfrac{\mu_n}{v_{sat}}\dfrac{dV}{dy}}\right)N_D\frac{dV}{dy} \qquad (15.2')$$

and

$$I_D = 2qZ\left(\frac{\mu_n}{1+\dfrac{\mu_n}{v_{sat}}\dfrac{dV}{dy}}\right)N_D a\frac{dV}{dy}\left(1-\frac{W}{a}\right) \qquad (15.3b')$$

or

$$I_D\left(1+\frac{\mu_n}{v_{sat}}\frac{dV}{dy}\right) = 2qZ\mu_n N_D a\frac{dV}{dy}\left(1-\frac{W}{a}\right)$$

Integrating over the length of the channel and remembering I_D is independent of y, we obtain

$$I_D\left[\int_0^L dy + \frac{\mu_n}{v_{sat}}\int_0^{V_D} dV\right] = 2qZ\mu_n N_D a\int_0^{V_D}\left(1-\frac{W}{a}\right)dV$$

or

$$I_D = \frac{2qZ\mu_n N_D a}{L\left(1+\dfrac{\mu_n}{v_{sat}}\dfrac{V_D}{L}\right)}\int_0^{V_D}\left(1-\frac{W}{a}\right)dV = \frac{I_D(\text{long-channel})}{1+\dfrac{\mu_n}{v_{sat}}\dfrac{V_D}{L}}$$

15.13

Since $dV/dy = -\mathcal{E}_y$, differentiating both sides of the Problem 15.3(a) result with respect to y yields

$$\frac{1}{L} = \frac{-\mathcal{E}_y + \mathcal{E}_y\left(\frac{V+V_{bi}-V_G}{V_{bi}-V_P}\right)^{1/2}}{V_D - \frac{2}{3}(V_{bi}-V_P)\left[\left(\frac{V_D+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]}$$

Next solving for $\mathcal{E}_y$ gives

$$\mathcal{E}_y L = \frac{V_D - \frac{2}{3}(V_{bi}-V_P)\left[\left(\frac{V_D+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]}{\left(\frac{V+V_{bi}-V_G}{V_{bi}-V_P}\right)^{1/2} - 1}$$

$\mathcal{E}_y = \mathcal{E}_{sat}$ when $V(L) = V_D = V_{Dsat}$. Thus, substituting into the preceding equation

$$\mathcal{E}_{sat} L = \frac{V_{Dsat} - \frac{2}{3}(V_{bi}-V_P)\left[\left(\frac{V_{Dsat}+V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2} - \left(\frac{V_{bi}-V_G}{V_{bi}-V_P}\right)^{3/2}\right]}{\left(\frac{V_{Dsat}+V_{bi}-V_G}{V_{bi}-V_P}\right)^{1/2} - 1} \tag{15.26}$$

15.14

(a) The MATLAB m-file P_15_14.m found on the Instructor's disk was constructed to calculate and plot the FET I_D–V_D characteristics predicted by the two region model. Characteristics numerically identical to those in Fig. 15.23 are obtained when the short channel parameters noted in the figure caption are input into the program. This is not too surprising since a version of the file was employed in constructing Fig. 15.23.

(b) An FET with a channel length of $L = 100\mu m$ qualifies as a long-channel device. With $L = 100\mu m$ the computed characteristics are indeed identical to those of the long-channel characteristics pictured in Fig. 15.16.

(c) Per the definition in the problem statement, the long-channel theory begins to "fail" when $\mathcal{E}_{sat}L = -5.575$**V**. Although there are a number of approaches that could be employed, the author obtained this result by simply monitoring the command window output of I_{Dsat}/I_{D0} (V_G=0) as a function of L with $\mathcal{E}_{sat}$ held constant at -10^4 V/cm.