CHAPTER 12

12.1

- (a) Under the quasistatic assumption the carriers and hence the device under analysis are assumed to respond to a time-vaying signal as if it were a d.c. bias. In the derivation of the generalized two-port model, one specifically equates the total time-varying terminal currents (i_B, i_C) to the d.c. currents that would exist under equivalent biasing conditions.
- (b) Two separate definitions are necessary because, contrary to the polarities assumed in the development of the generalized small-signal model, the I_B and I_C currents were previously taken to be positive flowing out of the base and the collector terminals in a pnp BJT. (As noted in Section 10.1, the direction of positive current was so chosen to avoid unnecessary complications, serious sign-related difficulties, in the physical description of current flow inside the BJT when operated in the standard amplifying mode.)
- (c) The Hybrid-Pi model gets its name from the π -like arrangement of circuit elements with "hybrid" (a combination of conductance and resistance) units.
- (d) Names (see the first paragraph in Subsection 12.1.2):

gm...transconductance

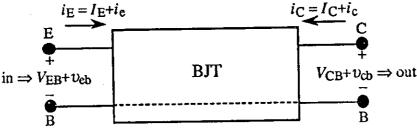
r_c...output resistance

 r_{π} ...input resistance

 $r_{\rm H}$...feedthrough resistance

- (e) The capacitors model the collector-base and emitter-base pn junction capacitances which cannot be neglected at higher frequencies.
- (f) The minority carrier concentration in the base continues to increase as pictured in plot (iii) of Fig. 12.4(d) until a maximum build-up consistent with the applied biases is attained. The base current varies as Q_B/τ_B and therefore also continues to increase toward a saturating maximum value. (In the quantitative analysis, i_B increases from $I_{CC}\tau_t$ at the start of saturation to a saturating value of $I_{BB}\tau_B$.) Once saturation biased, i_C remains essentially constant at $i_C = I_{CC} = V_{CC}/R_L$.
- (g) In words, the base transit time is the average time taken by minority carriers to diffuse across the quasineutral base. Mathematically (see Eq. 12.22), $\tau_l = W^2/2D_B$.
- (h) $\beta_{dc} = I_C/I_B = \tau_B/\tau_t$
- (i) An $i_B < 0$ aids the widthdrawal of stored charge from the quasineutral base, which in turn reduces both the storage delay time and the fall time.
- (j) A Schottky diode clamp is a circuit arrangement where a Schottky diode is connected between the collector and base of a BJT as pictured in Fig. 12.7(a). The Schottky diode conducts at a lower forward bias than a pn junction and therefore minimizes the forward (saturation-mode) bias that is applied to the BJT under turn-on conditions. This reduces the stored charge and speeds up the turn-off transient. (Also see Subsection 12.2.4.)

12.2 The BJT viewed as a two-port network and connected in the common-base configuration is pictured below.



Invoking the quasistatic assumption we can write

$$i_{\rm E}(V_{\rm EB} + \upsilon_{\rm eb}, V_{\rm CB} + \upsilon_{\rm cb}) \cong I_{\rm E}(V_{\rm EB} + \upsilon_{\rm eb}, V_{\rm CB} + \upsilon_{\rm cb}) = I_{\rm E}(V_{\rm EB}, V_{\rm CB}) + i_{\rm e}$$

$$i_{\rm C}(V_{\rm EB} + \upsilon_{\rm eb}, V_{\rm CB} + \upsilon_{\rm cb}) \cong I_{\rm C}(V_{\rm EB} + \upsilon_{\rm eb}, V_{\rm CB} + \upsilon_{\rm cb}) = I_{\rm C}(V_{\rm EB}, V_{\rm CB}) + i_{\rm c}$$
or
$$i_{\rm e} = I_{\rm E}(V_{\rm EB} + \upsilon_{\rm eb}, V_{\rm CB} + \upsilon_{\rm cb}) - I_{\rm E}(V_{\rm EB}, V_{\rm CB})$$

$$i_{\rm c} = I_{\rm C}(V_{\rm EB} + \upsilon_{\rm eb}, V_{\rm CB} + \upsilon_{\rm cb}) - I_{\rm C}(V_{\rm EB}, V_{\rm CB})$$

Next performing a Taylor series expansion of the first term on the right-hand side of the above equations, and keeping only first order terms, we obtain

$$I_{E}(V_{EB}+v_{eb},V_{CB}+v_{cb}) = I_{E}(V_{EB},V_{CB}) + \frac{\partial I_{E}}{\partial V_{EB}} v_{cb} + \frac{\partial I_{E}}{\partial V_{CB}} v_{cb} + \frac{\partial I_{E}}{\partial V_{CB}} v_{cb}$$

$$I_{C}(V_{EB}+v_{eb},V_{CB}+v_{cb}) = I_{C}(V_{EB},V_{CB}) + \frac{\partial I_{C}}{\partial V_{EB}} v_{cb} + \frac{\partial I_{C}}{\partial V_{CB}} v_{cb}$$

$$v_{cb} + \frac{\partial I_{C}}{\partial V_{CB}} v_{cb}$$

which when substituted into the preceding equations gives

$$i_{e} = \frac{\partial I_{E}}{\partial V_{EB}} |_{V_{CB}} v_{eb} + \frac{\partial I_{E}}{\partial V_{CB}} |_{V_{EB}} v_{cb}$$

$$i_{c} = \frac{\partial I_{C}}{\partial V_{EB}} |_{V_{CB}} v_{eb} + \frac{\partial I_{C}}{\partial V_{CB}} |_{V_{EB}} v_{cb}$$

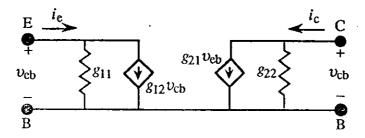
If the direction of positive current flow is as defined in Fig. 10.2 ($+I_E$ out and $+I_C$ in for an npn BJT, $+I_E$ in and $+I_C$ out for a pnp BJT), then introducing

yield the emitter and collector a.c. current node equations

$$i_{\rm e} = g_{11}v_{\rm eb} + g_{12}v_{\rm cb}$$

 $i_{\rm c} = g_{21}v_{\rm eb} + g_{22}v_{\rm cb}$

The low-frequency small-signal equivalent circuit characterizing the a.c. response of the BJT connected in the common base configuration is therefore concluded to be



From an inspection of Fig. 11.5(d), one concludes $I_C \cong 1.1$ mA at the specified operating point. Given the BJT is to be modeled using the simplified equivalent circuit of Fig. 12.2(a), and assuming T = 300 K, one computes (referring to Eqs. 12.9),

$$g_{\rm m} = \frac{qI_{\rm C}}{kT} = \frac{1.1 \times 10^{-3}}{0.0259} = 4.25 \times 10^{-2} \,\rm S$$
 $r_{\pi} = \frac{kT/q}{I_{\rm B}} = \frac{0.0259}{5 \times 10^{-6}} = 5.18 \times 10^{3} \,\Omega$

12.4

The node equations appropriate for the B and C terminals in the Hybrid-Pi model (Fig. 12.2b) assume the form

$$i_{b} = v_{be}/r_{\pi} + v_{bc}/r_{\mu}$$
$$i_{c} = g_{m}v_{be} + v_{cb}/r_{\mu} + v_{ce}/r_{o}$$

But $v_{bc} = -v_{cb} = v_{be} - v_{ce}$. Thus

$$i_{b} = v_{bc} \left(\frac{1}{r_{\pi}} + \frac{1}{r_{\mu}} \right) - v_{cc} \left(\frac{1}{r_{\mu}} \right)$$

$$i_{c} = v_{bc} \left(g_{m} - \frac{1}{r_{H}} \right) + v_{cc} \left(\frac{1}{r_{H}} + \frac{1}{r_{O}} \right)$$

A comparison of the preceding equations with text Eqs. (12.6) leads to the conclusion

$$g_{11} = \frac{1}{r_{\pi}} + \frac{1}{r_{\mu}}$$
 $g_{12} = -\frac{1}{r_{\mu}}$
 $g_{21} = g_{m} - \frac{1}{r_{u}}$ $g_{22} = \frac{1}{r_{u}} + \frac{1}{r_{0}}$

Clearly $r_{\mu} = -1/g_{12}$. Moreover, substituting $1/r_{\mu} = -g_{12}$ into the other three expressions allows us to solve for the remaining Hybrid-Pi parameters in terms of the generalized model parameters. Specifically,

$$r_{\pi} = 1/(g_{11} + g_{12})$$
 $r_{\mu} = -1/g_{12}$
 $g_{\text{m}} = g_{21} - g_{12}$ $r_{\text{O}} = 1/(g_{22} + g_{12})$

Although in a somewhat different order, the preceding are Eqs. (12.10).

Computations were first performed to determine the $V_{\rm EB}$ values required to obtain an $I_{\rm C}=1$ mA with and without accounting for base width modulation. These $V_{\rm EB}$ voltages were then incorporated directly into the final program (P_12_05.m on the Instructor's disk). In the MATLAB program, the user is asked whether he/she wishes to input $V_{\rm EB}$ and $V_{\rm EC}$ or to use the preset values. The small incremental voltage deviations from the d.c. voltage values used in approximating the partial derivatives appearing in Eqs. (12.5) were varied until a factor of two change in the incremental values led to no change to five significant places in the computed g_{ij} parameters. The g_{ij} parameters were in turn used to compute the Hybrid-Pi parameters employing Eqs. (12.10).

Sample results with and without accounting for base width modulation are tabulated below. In both cases there is at most a third-place difference between the $g_{\rm m}$ and r_{π} computed from first principles and the $g_{\rm m}$ and r_{π} computed using Eqs. (12.9). As expected, g_{12} and g_{22} are approximately zero when base width modulation is assumed to be negligible, and therefore r_0 and r_{μ} become infinite. Finite values are obtained for r_0 and r_{μ} when base width modulation is included. Note that base width modulation has little effect on $g_{\rm m}$ but leads to a significant increase in r_{π} . An increase in $\beta_{\rm dc} \equiv g_{\rm m} r_{\pi}$ is of course

expected when base width modulation is included.

```
No base-width modulation
```

```
g_{\rm m} = 3.8685 \times 10^{-2} \,\text{S}
r_{\rm o} = \infty
r_{\pi} = 4.5960 \times 10^{3} \,\Omega
r_{\mu} = \infty
g_{\rm m} = 3.8612 \times 10^{-2} \,\text{S} ...using Eq. (12.9)
r_{\pi} = 4.6047 \times 10^{3} \,\Omega
```

 $I_{\rm C} = 1.0000 \, \text{mA}$

With base-width modulation included

```
g_{\rm m} = 3.8510 \times 10^{-2} \, {\rm S}
r_{\rm o} = 1.4932 \times 10^{5} \, {\rm \Omega}
r_{\pi} = 5.9530 \times 10^{3} \, {\rm \Omega}
r_{\mu} = 7.5141 \times 10^{7} \, {\rm \Omega}
g_{\rm m} = 3.8611 \times 10^{-2} \, {\rm S} ...using Eq. (12.9)
r_{\pi} = 5.9761 \times 10^{3} \, {\rm \Omega}
```

 $V_{EB} = 0.66961 \text{ V}$ $V_{EC} = 10 \text{ V}$ $I_{C} = 1.0000 \text{ mA}$

 $V_{\rm EB} = 0.67416 \text{ V}$

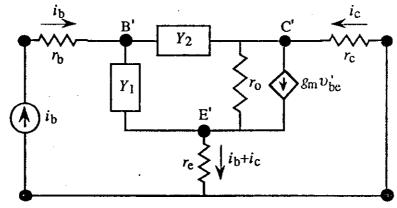
 $V_{EC} = 10 \text{ V}$

```
MATLAB program script...
%Computation of the Hybrid Pi Parameters (Problem 12.5)
%Initialization
clear: close
format compact; format short e
bw=input('Include base-width modulation? 1-Yes, 2-No...');
s=input('Manually input VEB and VEC? 1-Yes, 2-No...');
%Input Eber-Moll Parameters
BJT0
%Voltages used in Calculation
VbiE=kT*log(NE*NB/ni^2);
VbiC=kT*log(NC*NB/ni^2);
if s=1,
VEB0=input('Input VEB in volts, VEB = ');
VECO=input('Input VEC in volts, VEC = ');
else
VEC0=1.0
if bw==1. VEB0=0.669606
          VEB0=0.674162
else
end: end
%iB and iC Calculations
VEB=VEB0;
VEC=VECO:
iB=[];
iC=[];
for i=1:5,
  if bw=1,
  VCB=VEB-VEC:
  BJTmod
  else
  end
IB0=(1-aF).*IF0+(1-aR).*IR0;
IB1=(1-aF).*IF0+(1-aR).*IR0.*exp(-VEC/kT);
IB=(IB1.*exp(VEB/kT)-IB0);
IC=((aF.*IF0-IR0.*exp(-VEC/kT)).*(IB+IB0)./IB1+IR0-aF.*IR0);
%Reset Voltages
if i==1, VEB=VEBO-0.0001;
                           else:
                                  end
if i==2, VEB=VEBO+0.0001;
                           else:
                                  end
if i==3, VEB=VEBO; VEC= VECO-0.01;
                                     else;
if i==4, VEC=VECO+0.01; else:
iB=[iB, IB];
iC=[iC,IC];
```

end

```
%Compute Generalized Two-Port Model Parameters
g11=(iB(3)-iB(2))/0.0002;
g12=(iB(5)-iB(4))/0.02;
g21=(iC(3)-iC(2))/0.0002;
g22=(iC(5)-iC(4))/0.02;
fprintf('\nHybrid-Pi Model Parameters\n')
gm=g21-g12
if g22+g12==0
                ro=inf
else
                ro=1/(g22+g12)
end
rpi=1/(q11+q12)
if g12==0, rmu=inf
else,
           rmu=-1/g12
end
fprintf('\ngm and rpi computed using Eqs.(12.9)\n')
gm=iC(1)/0.0259
rpi=0.0259/iB(1)
```

 $\frac{12.6}{\text{(a)}}$ The high-frequency equivalent circuit of Fig. 12.2(c) with $v_{ce} = 0$ can be manipulated into the form



where

$$Y_1 = \frac{1}{r_{\pi}} + j\omega C_{\rm cb}$$

$$Y_2 = \frac{1}{r_{\mu}} + j\omega C_{cb}$$

Combining node and loop analysis we note

$$i_b = Y_1 v_{be}' - Y_2 v_{cb}'$$
 (1)

$$i_{\rm c} = g_{\rm m} v_{\rm be}' + Y_2 v_{\rm cb}' + v_{\rm ce}'/r_{\rm o}$$
 (2)

$$i_{c}r_{c} + v_{ce'} + (i_{b} + i_{c})r_{e} = 0$$
 (3)

$$v_{be'} - v_{ce'} + v_{cb'} = 0$$
 (4)

Eq. (4) is used to eliminate v_{cb} in Eqs. (1) and (2). Eqs. (1) and (2) are then combined to eliminate v_{be} . Next Eqs. (3) is used to eliminate v_{ce} . Finally, the i_c/i_b ration is formed giving

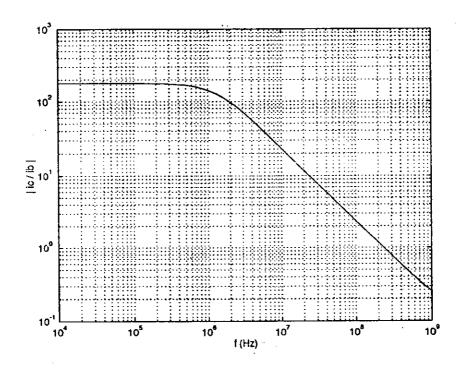
$$\frac{i_{c}}{i_{b}} = \frac{\left(Y_{2} + \frac{1}{r_{o}}\right)r_{e} + \left(\frac{g_{m} - Y_{2}}{Y_{1} + Y_{2}}\right)(Y_{2}r_{e} - 1)}{\left(\frac{Y_{2} - g_{m}}{Y_{1} + Y_{2}}\right)(r_{e} + r_{c})Y_{2} - \left(Y_{2} + \frac{1}{r_{o}}\right)(r_{e} + r_{c}) - 1}$$

(b) With $r_e = 0$ as given in the list of parameters, the i_c/i_b ratio simplifies to

$$\frac{i_{c}}{i_{b}} = \frac{\left(\frac{Y_{2} - g_{m}}{Y_{1} + Y_{2}}\right)}{\left(\frac{Y_{2} - g_{m}}{Y_{1} + Y_{2}}\right) r_{c} Y_{2} - \left(Y_{2} + \frac{1}{r_{0}}\right) r_{c} - 1}$$

Using the MATLAB program to compute $|i_C/i_b|$ versus frequency, one determines an $f_T = 235$ MHz. Data sheets list the f_T of the 2N3906 pnp BJT to be approximately 200 MHz. (It should be noted that the Electronics Workbench software program was used to determine the d.c. operating point that produced an $I_C = 1$ mA. The series resistances listed in the problem statement were those quoted by the EW program. Zero-bias capacitance values employed in computing the Hybrid-Pi parameters were also extracted from the Electronics Workbench program.)

A plot of $|i_0/i_0|$ versus frequency, and the MATLAB m-file constructed to generate the plot and determine f_T , are reproduced on the next page.



MATLAB program script...

```
%Problem 12.6...fT determination
```

```
%Initialization
clear; close
%Parameters
gm=3.86e-2;
rpi=4.65e3;
ro=2.00e4;
rmu=3.59e6;
Ceb=23.6e-12;
Ccb=2.32e-12;
rb=10;
rc=2.8;
re=0;
```

```
%|ic/ib| vs. frequency
f=logspace(4,9,200);
w=2.*pi.*f;
Y1=1/rpi+j.*w.*Ceb;
Y2=1/rmu+j.*w.*Ccb;
R=(Y2-gm)./(Y1+Y2);
Den=R.*rc.*Y2 - (Y2+1/ro).*rc - 1;
beta=abs(R./Den); %beta=|ic/ib|
%Plot
loglog(f,beta); grid
xlabel('f (Hz)'); ylabel('| ic / ib |')
```

12.7 The Eqs. (6.68)/(6.69) solution for the I_{DIFF} flowing in a narrow base diode is

$$I_{\text{DIFF}} = qA \frac{D_{\text{P}}}{L_{\text{P}}} \frac{n_{\text{i}}^2}{N_{\text{D}}} \frac{\cosh(x_{\text{c}}'/L_{\text{P}})}{\sinh(x_{\text{c}}'/L_{\text{P}})} \left(e^{qV_{\text{A}}/kT} - 1\right)$$

For application to a BJT we make the symbol replacements.. $D_P \rightarrow D_B$, $L_P \rightarrow L_B$, $N_D \rightarrow$ $N_{\rm B}, x_{\rm c}^{\rm r} \to W$, and $V_{\rm A} \to V_{\rm EB}$. Then

$$I_{\text{DIFF}} = qA \frac{D_{\text{B}}}{L_{\text{B}}} \frac{n_{\text{i}}^2}{N_{\text{B}}} \frac{\cosh(W/L_{\text{B}})}{\sinh(W/L_{\text{B}})} \left(e^{qV_{\text{EB}}/kT} - 1\right)$$

Since W/LB << 1 in a standard transistor, the cosh/sinh factor can be expanded as noted in the problem statement to obtain

$$\frac{\cosh(W/L_{\rm B})}{\sinh(W/L_{\rm B})} \cong \frac{L_{\rm B}}{W} \left[1 + \frac{1}{3} \left(\frac{W}{L_{\rm B}} \right)^2 \right] \qquad \dots W/L_{\rm B} << 1$$

and

$$I_{\text{DIFF}} \cong \left(qA \frac{D_{\text{B}}}{W} \frac{n_{\text{i}}^2}{N_{\text{B}}}\right) \left[1 + \frac{1}{3} \left(\frac{W}{L_{\text{B}}}\right)^2\right] \left(e^{qV_{\text{EB}}/kT} - 1\right)$$

Introducing the substitutions cited in Subsection 7.3.2, that is,

$$\left(\frac{W}{L_{\rm B}}\right)^2 = \frac{W^2}{D_{\rm B}\tau_{\rm B}} \Rightarrow \frac{W^2}{D_{\rm B}\tau_{\rm B}} (1+j\omega\tau_{\rm B})$$

and

$$(e^{qV_{\text{EB}}/kT} - 1) \Rightarrow (qv_{\text{eb}}/kT)e^{qV_{\text{EB}}/kT}$$

yields the corresponding a.c. relationship

$$i_{\text{diff}} = \left(qA \frac{D_{\text{B}}}{W} \frac{n_{\text{i}}^2}{N_{\text{B}}}\right) \left(1 + \frac{1}{3} \frac{W^2}{D_{\text{B}} \tau_{\text{B}}} + j\omega \frac{W^2}{3D_{\text{B}}}\right) \left(\frac{q v_{\text{eb}}}{kT}\right) e^{qV_{\text{EB}}/kT}$$

Finally, by definition,

$$Y_{\rm D} = G_{\rm D} + j\omega C_{\rm D} = i_{\rm diff}/v_{\rm eb}$$

and therefore

$$C_{\rm D} = \left(\frac{W^2/3D_{\rm B}}{kT/q}\right) \left(qA \frac{D_{\rm B}}{W} \frac{n_{\rm i}^2}{N_{\rm B}}\right) e^{qV_{\rm EB}/kT} = \frac{2}{3} \left(\frac{\tau_{\rm t}}{kT/q}\right) \left(qA \frac{D_{\rm B}}{W} \frac{n_{\rm i}^2}{N_{\rm B}}\right) e^{qV_{\rm EB}/kT}$$

The pictured "on" point in Fig. 12.3(b) lies right on the $I_B = V_S/R_S$ line. Therefore $I_{BB} \equiv V_S/R_S = 30\mu A$.

Inspecting the plot we find $I_{CC} \cong V_{CC}/R_L = 5.0 \text{ mA}$.

We know $\beta_{dc} = I_C/I_B = \tau_B/\tau_t$. Although base width modulation clearly causes β_{dc} to vary somewhat depending on the d.c. operating point, it is reasonable to employ a median value in obtaining the desired estimate. Specifically, using the point where the load line crosses the $I_B = 15 \,\mu\text{A}$ characteristic, we obtain

$$\frac{\tau_{\rm B}}{\tau_{\rm t}} = \frac{I_{\rm C}}{I_{\rm B}} \cong \frac{(0.624)(V_{\rm CC}/R_{\rm L})}{I_{\rm B}} = \frac{(0.624)(5 \times 10^{-3})}{15 \times 10^{-6}} = 208$$

Thus

$$\frac{I_{\rm CC}\tau_{\rm t}}{I_{\rm BB}\tau_{\rm B}} \approx \frac{(5\times10^{-3})}{(30\times10^{-6})(208)} = 0.80$$

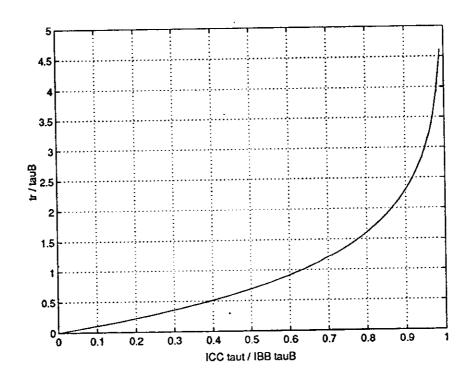
(a)/(b) The required plots and the generating MATLAB m-file are reproduced below. The computational relationships used in producing the plots were

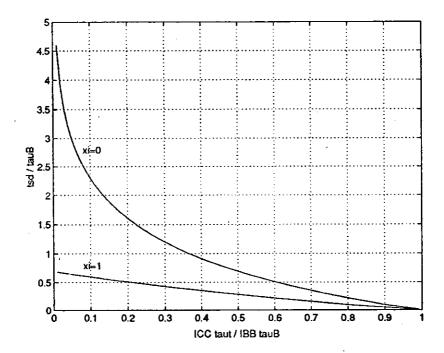
$$\frac{t_{\rm f}}{\tau_{\rm B}} = \ln\left(\frac{1}{1-x}\right)$$

$$\frac{t_{\rm sd}}{\tau_{\rm B}} = \begin{cases} \ln\left(\frac{1}{x}\right) & \dots \text{ if } \xi = 0\\ \ln\left(\frac{2}{1+x}\right) & \dots \text{ if } \xi = 1 \end{cases}$$

where

 $x = I_{\rm CC} \tau / I_{\rm BB} \tau_{\rm B}$





MATLAB program script...

```
%Rise and Storage-Delay Time plots (Prob. 12.9)
%Initialization
clear:
        close
%Rise time computation
x=linspace(0.01, 0.99);
rise=log(1./(1-x));
                     %rise=tr/tauB
plot(x,rise); grid
xlabel('ICC taut / IBB tauB'); ylabel('tr / tauB')
pause
%Storage-Delay Time computation
                       %delay0=tsd/tauB, xi=0
delay0=log(1./x);
delay1=log(2./(1+x));
                       %delay1=tsd/tauB, xi=1
plot(x,delay0,x,delay1);
                          grid
xlabel('ICC taut / IBB tauB');
                               ylabel('tsd / tauB')
text(0.08,2.8,'xi=0'); text(0.08,0.8,'xi=1')
```

(a) Let t_1 be the time when $i_C = 0.9I_{CC}$ and t_2 the time when $i_C = 0.1I_{CC}$. Making use of Eq. (12.31b), we can then write

$$i_{\rm C}(t_1) = 0.9I_{\rm CC} = I_{\rm BB} \frac{\tau_{\rm B}}{\tau_{\rm t}} [(1+\xi)e^{-t_1/\tau_{\rm B}} - \xi]$$

$$i_{\rm C}(t_2) = 0.1I_{\rm CC} = I_{\rm BB} \frac{\tau_{\rm B}}{\tau_{\rm t}} [(1+\xi)e^{-t_2/\tau_{\rm B}} - \xi]$$

Solving for the t's yields

$$t_1 = \tau_B \ln \left(\frac{1 + \xi}{0.9 I_{CC} \tau_t / I_{BB} \tau_B + \xi} \right)$$

$$t_2 = \tau_{\rm B} \ln \left(\frac{1 + \xi}{0.1 I_{\rm CC} \tau_{\rm V} / I_{\rm BB} \tau_{\rm B} + \xi} \right)$$

and per the measurements-based definition

$$t_{\rm f} = t_2 - t_1 = \tau_{\rm B} \ln \left(\frac{0.9 I_{\rm CC} \tau_{\rm f} / I_{\rm BB} \tau_{\rm B} + \xi}{0.1 I_{\rm CC} \tau_{\rm f} / I_{\rm BB} \tau_{\rm B} + \xi} \right) = \tau_{\rm B} \ln \left(\frac{0.9 x + \xi}{0.1 x + \xi} \right)$$

where $x = I_{CC}\tau_t/I_{BB}\tau_B$

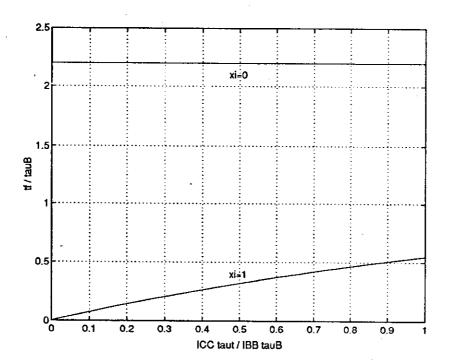
(b) With $\xi = 0$ and $\xi = 1$, the part (a) relationship simplifies to

$$\frac{t_{\rm f}}{\tau_{\rm B}} = \begin{cases} \ln 9 & \text{... if } \xi = 0\\ \ln(\frac{0.9x + 1}{0.1x + 1}) & \text{... if } \xi = 1 \end{cases}.$$

The requested t_f/t_B versus x plot is displayed on the next page along with the script of the MATLAB m-file used to generate the plot.

Consistent with the analysis in Subsection 12.2.3, the plotted fall times decrease when $\xi > 0$. This occurs because an $i_B < 0$ aids the withdrawal of charge from the quasineutral base. If the x-ratio increases either due to an increase in I_{CC} or a decrease in I_{BB} , the charge storage is enhanced relative to the charge removal capability of the base

current. Thus, the t_f/τ_B ratio for the $\xi = 1$ curve increases with increasing x. When $\xi = 0$, the charge removal from the base occurs only by recombination and the fall-time collector current assumes the simple form, $i_C = A\exp(-t/\tau_B)$. Since t_f is always evaluated employing the same relative i_C values, $i_C(t_1)/i_C(t_2) = \text{constant} = \exp(t_f/\tau_B)$, and t_f/τ_B is seen to be a constant independent of x.



MATLAB program script...