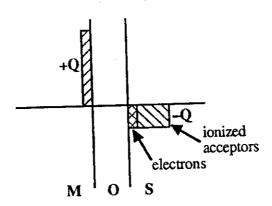
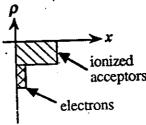
CHAPTER 16

| <u>16.1</u> | | | | |
|-------------|--------|------------------------------|---------------------|----------------------|
| <u>Part</u> | Doping | Biasing <u>Conditio</u> n | Energy Band Diagram | Block Charge Diagram |
| (a) | p | depletion | E_{F} | |
| (b) | n | flat band | $E_{\mathbf{F}}$ | |
| (c) | p | depl/inv transition | $E_{\mathbf{F}}$ | electrons |
| (d) | n a | accumulation | $E_{\mathbf{F}}$ | electrons |
| (e) | p | inversion | $E_{\mathbf{F}}$ | electrons M O S |



(b) The part (a) charge diagram is in agreement with the ρ/qN_A versus x plot in Fig. 16.8(c). To obtain the total charge in the semiconductor at each point one adds the separate block charges shown in part (a).



The spike near x = 0 in the Fig. 16.8(c) plot simply reflects the forming inversion layer of electrons at the surface. By definition, at the onset of inversion $n_{\text{surface}} = N_A$. Thus, at the special V_T bias point $\rho_S = -q(n_{\text{surface}} + N_A) = -2qN_A$, or $\rho/qN_A = -2$ at x = 0 at the onset of inversion.

(c) Since $\phi_F/(kT/q) = 12$, inverting Eq.(16.8a) yields

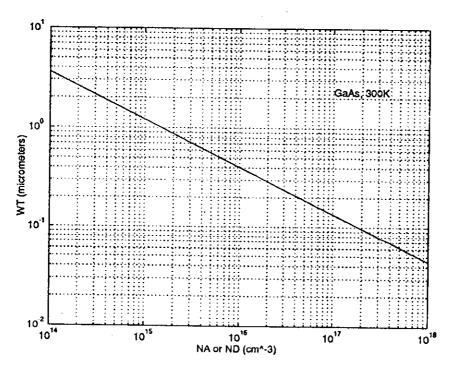
$$N_{\rm A} = n_{\rm i} e^{\phi_{\rm F}/(kT/q)} = 1.00 \times 10^{10} e^{12} = 1.63 \times 10^{15} / \text{cm}^3$$

$$W_{\rm T} = \left[\frac{2K_{\rm S}\varepsilon_0}{qN_{\rm A}}(2\phi_{\rm F})\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\times10^{-14})(24)(0.0259)}{(1.6\times10^{-19})(1.63\times10^{15})}\right]^{1/2} = 0.706\mu{\rm m}$$

(From Fig. 16.9 one also reads $W_T \cong 0.7 \mu \text{m.}$)

The above W_T is indeed consistent with the positioning of the end of the approximate charge distribution (the dashed-line distribution) in Fig. 16.8(c).

16.3 The required W_T versus doping plot appropriate for GaAs and the MATLAB program script that generated the plot are reproduced below.



MATLAB program script...

```
%WT versus NA or ND for GaAs at 300K
%Initialization
clear; close;
%Constants and parameters
q=1.6e-19;
e0=8.85e-14;
ni=2.25e6;
KS=12.85;
kT=0.0259;
NB=logspace(14,18); %NB = NA or ND
                                        %Plotting result
%WT calculation
                                        loglog(NB,WT); grid
ØF=kT.*log(NB./ni);
                                        xlabel('NA or ND (cm^-3)');
WT=sqrt(4*KS*e0.*øF./(q.*NB));
                                        ylabel('WT (micrometers)');
WT=(1.0e4).*WT; %WT in micrometers
                                        text (1.0e17, 2.3, 'GaAs, 300K')
```

(a)

$$\frac{\phi_{\rm F}}{kT/q} = -\ln{(N_{\rm D}/n_{\rm i})} = -\ln{\left(\frac{10^{15}}{10^{10}}\right)} = -11.51$$

$$\phi_{\rm F} = -11.51 \ (kT/q) = -(11.51)(0.0259) = -0.298 \rm V$$

(b) Using Eq.(16.16) with $N_A \rightarrow -N_D$,

$$W = W_{\rm T} = \left[\frac{2K_{\rm S}\varepsilon_0}{-qN_{\rm D}}(2\phi_{\rm F})\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\times10^{-14})(2)(0.298)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2} = 0.882\mu\text{m}$$

(c) Evaluating Eq.(16.12) at x = 0 yields \mathcal{E}_S . Thus, with $N_A \rightarrow -N_D$ in Eq. (16.12),

$$\mathcal{E}_{S} = -\frac{qN_{D}}{K_{S}\varepsilon_{0}}W = -\frac{(1.6\times10^{-19})(10^{15})(0.882\times10^{-4})}{(11.8)(8.85\times10^{-14})} = -1.35\times10^{4} \text{ V/cm}$$

(d) Substituting into Eq.(16.26) gives

$$V_G = 2\phi_F + \frac{K_S}{K_O} x_o \mathcal{E}_S$$
 ... \mathcal{E}_S evaluated at $\phi_S = 2\phi_F$
= $-(2)(0.298) - \frac{(11.8)(10^{-5})(1.35 \times 10^4)}{3.9}$
= -1.00 V

Except for the doping type, the parameters used in this problem are identical to those assumed in constructing Fig. 16.10. Since $\phi_S = 2\phi_F$, the $|V_G|$ calculated in part (d) should correspond to the depletion/inversion transition point in Fig. 16.10. Indeed, in the figure $V_T \equiv 1V$.

(e) The MATLAB program script yielding a computer generated computation of ϕ_F , W, \mathcal{E}_S , and V_G is listed on the next page and included on the Instructor's disk as m-file P_16_04.m. Note that a normalized ϕ_S , $\phi_S/2\phi_F$, is taken to be one of the input variables. Also, donor dopings must be input as a negative concentration in running the program.

```
MATLAB program script...
```

```
%Autocalculation of ØF, W, ES and VG
%Initialization
clear; close;
format compact
%Constants and parameters
q=1.6e-19;
e0=8.85e-14;
ni=1.0e10;
KS=11.8:
KO=3.9:
kT=0.0259;
%Input variables
xo=input('Please input xo in cm, xo = ');
N=input('Please input NA or -ND in cm^-3, N = ');
r=input('Please input øS/2øF, øS/2øF = ');
NB=abs(N); s=N/NB;
%øF and WT calculation
ØF=s*kT*log(NB/ni)
\phi S = r * 2 * \phi F;
W0=\operatorname{sqrt}(2*KS*e0*\emptysetS/(q*N));
                               %WO in cm
W=(1.0e4)*W0 %W in micrometers
%Surface Electric Field (ES) calculation
ES=(q*N*W0)/(KS*e0)
%VG calculation
VG=ØS+KS*xo*ES/KO
```

16.5
(a) In general we can write

Also
$$V_G = \phi_S + \frac{K_S}{K_O} x_o \mathcal{E}_S$$
 ...Eq.(16.26)

$$\phi_S = \frac{kT}{q} U_S$$
 ...Eq.(B.2)

and Eq.(B.16) evaluated at the surface gives

$$\mathcal{E}_{S} = \widehat{U}_{S} \frac{kT}{q} \frac{F(U_{S}, U_{F})}{L_{D}}$$

Substituting the above ϕ_S and \mathcal{E}_S expressions into the general V_{G} - ϕ_S relationship yields the desired result;

$$V_{\rm G} = \frac{kT}{q} \left[U_{\rm S} + \widehat{U}_{\rm S} \frac{K_{\rm S} x_{\rm O}}{K_{\rm O} L_{\rm D}} F(U_{\rm S}, U_{\rm F}) \right]$$

(b) The required $V_{\rm G}$ versus $U_{\rm S}$ computation is performed by the MATLAB m-file listed below. Setting $x=0.1\mu{\rm m}$ and $N_{\rm D}=10^{15}/{\rm cm}^3$ yields a plot identical to Fig. 16.10 except the entire plot is reflected through the origin of coordinates.

MATLAB program script...

```
%VG versus US Calculation
%Initialization
       close
clear;
format compact
%Universal and System Constants
q=1.60e-19;
e0=8.85e-14;
kT=0.0259:
%Device and Material Constants
KS=11.8:
KO=3.9;
ni=1.00e10;
LD=sqrt((KS*e0*kT)/(2*q*ni));
s=input('Employ xo=1.0e-5cm and ND=1.0e15/cm3? 1-Yes, 2-No...');
  if s=1
  Net=1.0e15;
  xo=1.0e-5:
  else
  Net=input('Input the net semi doping in cm-3, ND-NA = ');
  xo=input('Input the oxide thickness in cm, xo = ');
  end
N=abs(Net); sign=-Net/N;
UF=sign*log(N/ni)
%Computation Proper
US=UF-21:1:UF+21;
S=US./abs(US);
F=sqrt(exp(UF).*(exp(-US)+US-1)+exp(-UF).*(exp(US)-US-1));
VG=kT*(US+S*(KS*xo)/(KO*LD).*F);
%Plot result
plot(US, VG); grid
   if s=1
   axis({-40, 10, -4, 4});
xlabel('US'); ylabel('VG (volts)')
```

(a) Eq.(16.28) may be viewed as a quadratic equation with $\sqrt{\phi_S}$ as the variable.

$$(\sqrt{\phi_{\rm S}})^2 + \frac{K_{\rm S}}{K_{\rm O}} x_{\rm O} \sqrt{\frac{2qN_{\rm A}}{K_{\rm S}\varepsilon_{\rm O}}} \sqrt{\phi_{\rm S}} - V_{\rm G} = 0$$

Introducing

$$b \equiv \frac{K_{\rm S}}{K_{\rm O}} x_{\rm O} \sqrt{\frac{2qN_{\rm A}}{K_{\rm S}\varepsilon_{\rm O}}}$$

and choosing the (+) root solution so that $\sqrt{\phi_S} > 0$, one obtains,

$$\sqrt{\phi_S} = -\frac{b}{2} + \left[\left(\frac{b}{2} \right)^2 + V_G \right]^{1/2} = \frac{b}{2} \left[1 + \frac{V_G}{(b/2)^2} \right]^{1/2} - 1$$

or

$$\sqrt{\phi_{\rm S}} = \left(\frac{K_{\rm S}}{K_{\rm O}} x_{\rm O} \sqrt{\frac{qN_{\rm A}}{2K_{\rm S}\varepsilon_{\rm O}}}\right) \left[1 + \frac{V_{\rm G}}{(b/2)^2}\right]^{1/2} - 1$$

Substituting the $\sqrt{\phi_S}$ result into Eq.(16.15) then yields

$$W = \frac{K_{S}x_{O}}{K_{O}} \left[\sqrt{1 + \frac{V_{G}}{V_{\delta}}} - 1 \right]$$

if

$$V_{\delta} = \left(\frac{b}{2}\right)^2 = \frac{q}{2} \frac{K_{S} x_0^2}{K_{O}^2 \varepsilon_0} N_{\Lambda}$$

We have indeed obtained the text result.

(b) (i)
$$\phi_{\rm F} = -\frac{kT}{q} \ln(N_{\rm D}/n_{\rm i}) = -(0.0259) \ln\left(\frac{10^{15}}{10^{10}}\right) = -0.298 \text{V}$$

$$W_{\rm T} = \left[\frac{2K_{\rm S}\varepsilon_0}{-qN_{\rm D}}(2\phi_{\rm F})\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\times10^{-14})(2)(0.298)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2} = \mathbf{0.882}\mu{\rm m}$$

(From Fig. 16.9 with $N_D = 10^{15}/\text{cm}^3$ one would estimate $W_T = 0.9 \mu\text{m}$.)

(ii) From Eq.(16.34d),

$$\frac{C}{C_{\rm O}} = \frac{1}{1 + \frac{K_{\rm O}W_{\rm T}}{K_{\rm S}x_{\rm o}}} \qquad \dots \text{inv}(\omega \to \infty)$$

$$= \frac{1}{1 + \frac{(3.9)(0.882)}{(11.8)(0.1)}} = \mathbf{0.255}$$

(iii) Some care must be exercised in working this part of the problem. An acceptable approach is to proceed as in Problem 16.4, first calculating \mathcal{E}_S using Eq.(16.12) and then substituting into Eq.(16.26). In fact, the parameters are the same as in Prob. 16.4 and thus the expected answer is $V_T = -1.00V$. Alternatively, one might consider substituting into Eq.(16.28) directly; $V_G = V_T$ when $\phi_S = 2\phi_F$. However, Eq.(16.28) is only valid for p-type devices and simply changing N_A to $-N_D$ will not yield the correct V_T . [For an n-type device the "+" between the two right-hand terms in Eq.(16.28) is replaced with a "-" sign.] Nevertheless, Eq.(16.28) can be used if we first act as if the doping was p-type, and then just change the sign of the result noting the voltage symmetry between ideal n- and p-type devices.

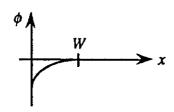
$$V_{\rm T} = -\left[(2\phi_{\rm F}) + \frac{K_{\rm S}}{K_{\rm O}} x_{\rm O} \sqrt{\frac{2qN_{\rm A}}{K_{\rm S}\varepsilon_{\rm O}}} (2\phi_{\rm F}) \right] \qquad (\phi_{\rm F} > 0)$$

$$= -\left\{ (2)(0.298) + \frac{(11.8)(10^{-5})}{3.9} \left[\frac{(2)(1.6 \times 10^{-19})(10^{15})(2)(0.298)}{(11.8)(8.85 \times 10^{-14})} \right]^{1/2} \right\}$$

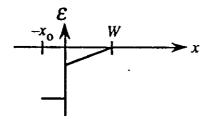
$$= -1.00 \text{ V} \leftarrow \text{expected result}$$

(iv) The parameters used in this problem are identical to those assumed in constructing Fig. 16.13. Thus the part (ii) C/C_0 value should correspond to the high-frequency inversion value on the figure and the $V_{\rm T}$ calculated in part (iii) should be the depletion/inversion transition voltage shown in the figure. This is indeed the case.

(a) ϕ has the same shape as the "upside down" of the bands.



(b) \mathcal{E} is proportional to the slope of the bands. Also, as emphasized in a footnote on p. 581, $\mathcal{E}_{ox} \cong 3\mathcal{E}_{S}$ in an ideal MOS-C.

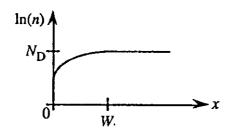


(c) Yes. Inside the semiconductor E_F is position independent.

(d) Noting

$$n = n_{\rm i} e^{(E_{\rm F} - E_{\rm i})/kT}$$

we conclude



(e) Since $E_F = E_i$ at the Si-SiO₂ interface, $n!_{x=0} = n_i = 10^{10} / \text{cm}^3$.

(f)
$$N_{\rm D} = n_{\rm bulk} = n_{\rm i} e^{[E_{\rm FS} - E_{\rm i}({\rm bulk})]/kT} = (10^{10})e^{0.29/0.0259} = 7.29 \times 10^{14}/{\rm cm}^3$$

(g) $\phi_S = (1/q)[E_i(bulk) - E_i(surface)] = -0.29 \text{ V}$

(h) Some care must be exercised in completing this part of the problem. Simply employing Eq. (16.28) with N_A replaced by $-N_D$ yields an incorrect result because $\mathcal{E}_S < 0$ when an *n*-bulk MOS-C is depletion biased. Specifically, for an *n*-bulk device

$$\mathcal{E}_{S} = -\left[\frac{2qN_{D}}{K_{S}\varepsilon_{0}}(-\phi_{S})\right]^{1/2}$$

and

$$V_{\rm G} = \phi_{\rm S} - \frac{K_{\rm S} x_{\rm O}}{K_{\rm O}} \sqrt{\frac{2qN_{\rm D}}{K_{\rm S} \varepsilon_{\rm O}}} (-\phi_{\rm S})$$

Thus here

$$V_{\rm G} = -0.29 - \frac{(11.8)(2\times10^{-5})}{(3.9)} \left[\frac{(2)(1.6\times10^{-19})(7.29\times10^{14})}{(11.8)(8.85\times10^{-14})} (0.29) \right]^{1/2}$$

or

$$V_{\rm G} = -0.78 \text{ V}$$

(i)
$$V_G = \Delta \phi_{ox} + \phi_S$$

 $\Delta \phi_{ox} = V_G - \phi_S = -0.78 + 0.29 = -0.49 \text{ V}$

(j)
$$V_{\delta} = -\frac{q}{2} \frac{K_{\rm S} x_{\rm O}^2}{K_{\rm O}^2 \epsilon_0} N_{\rm D} = -\frac{(1.6 \times 10^{-19})}{2} \frac{(11.8)(2 \times 10^{-5})^2 (7.29 \times 10^{14})}{(3.9)^2 (8.85 \times 10^{-14})} = 0.2045 \text{ V}$$

$$\frac{C}{C_{\rm O}} = \frac{1}{\sqrt{1 + V_{\rm O}/V_{\rm S}}} \approx \frac{1}{\sqrt{1 + 0.78/0.20}} = 0.45$$

(Eqs. 16.15 and 16.34b may alternatively be used to compute C/C_{O} .)

16.8

Inversion ... e, 4

Depletion ... c, 3

Flat band ... b, 1

 $V_{\rm G} = V_{\rm T}$... d, 2

Accumulation ... a, 5

(a) Yes. The Fermi level inside the semiconductor is position independent.

(b)...
$$\phi_{\rm F} = (1/q)[E_{\rm i}({\rm bulk}) - E_{\rm F}] = 0.3 \text{ V}$$

(c)...
$$\phi_S = (1/q)[E_i(bulk) - E_i(surface)] = \phi_F = 0.3 \text{ V}$$

(d)...
$$E_F(\text{metal}) - E_F(\text{semi}) = -qV_G$$
 ... Eq. (2.1)
 $V_G = (1/q)[E_F(\text{semi}) - E_F(\text{metal})] = 0.6 \text{ V}$

(e) Based on the delta-depletion approximation,

$$V_{\rm G} = \phi_{\rm S} + \frac{K_{\rm S} x_{\rm O}}{K_{\rm O}} \sqrt{\frac{2qN_{\rm A}}{K_{\rm S} \varepsilon_{\rm O}}} \phi_{\rm S} \qquad \Leftarrow \text{Eq.}(16.28)$$

where from prior parts of the problem $V_G = 0.6$ V and $\phi_S = 0.3$ V. Also,

$$\phi_{\rm F} = (kT/q) \ln(N_{\rm A}/n_{\rm i})$$

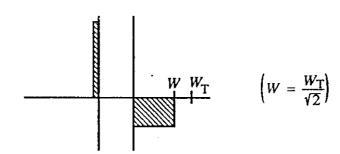
or

$$N_{\rm A} = n_{\rm i} e^{\phi_{\rm F}/(kT/q)} = (10^{10})e^{0.3/0.0259} = 1.073 \times 10^{15}/\text{cm}^3$$

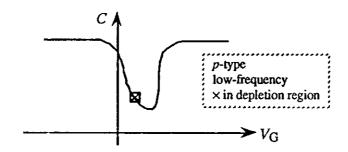
Thus

$$x_{\rm O} = \frac{V_{\rm G} - \phi_{\rm S}}{\frac{K_{\rm S}}{K_{\rm O}} \sqrt{\frac{2qN_{\rm A}}{K_{\rm S}\varepsilon_{\rm 0}}} \phi_{\rm S}} = \frac{0.6 - 0.3}{\left(\frac{11.8}{3.9}\right) \left[\frac{(2)(1.6 \times 10^{-19})(1.073 \times 10^{15})(0.3)}{(11.8)(8.85 \times 10^{-14})}\right]^{1/2}} = 0.10 \mu \rm m$$

(f)



(g)



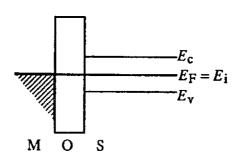
(h) Expressions (i) and (iv) are clearly wrong because they do not apply to depletion. Employing Eq. (16.28), we conclude $V_T = 1V$ and $V_G = 0.6V_T$. Thus referring to Eq. (16.37), expression (ii) is close but not the correct expression. Finally, noting that at the specified bias point,

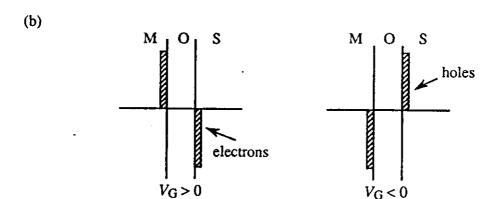
$$\phi_S = \phi_F$$
 and $W = \left[\frac{2K_S \varepsilon_0}{qN_A} \phi_F\right]^{1/2} = W_T / \sqrt{2}$

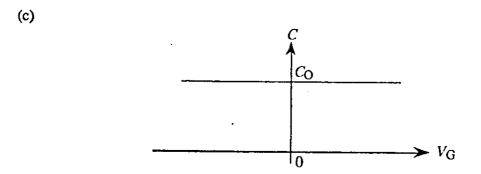
we conclude

$$C = \frac{C_{\rm O}}{1 + \frac{K_{\rm O}W}{K_{\rm S}x_{\rm O}}} = \frac{C_{\rm O}}{1 + \frac{K_{\rm O}W_{\rm T}}{\sqrt{2}K_{\rm S}x_{\rm O}}} \iff \boxed{\text{Expression (iii)}}$$

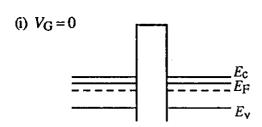


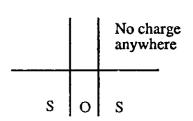


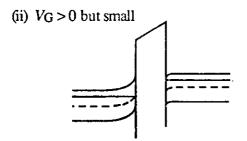


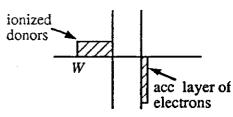


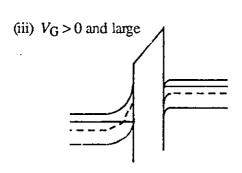
Justification: When $V_G > 0$, electrons pile-up in the Si immediately adjacent to the oxide giving rise to a low-frequency $C = C_O$. Similarly, when $V_G < 0$, holes pile-up in the Si immediately adjacent to the oxide giving rise to a low-frequency $C = C_O$. (Actually, $C \cong C_O$, but in the delta-depletion formulation the carrier layers are taken to be δ -functions at the Si surface.) Note that, within the framework of the delta-depletion formulation, there is no "depletion" or depletion-like region inside the given device.

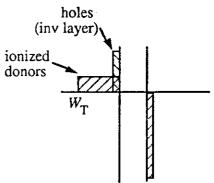






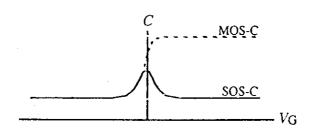






- (iv) $V_G < 0$ but small (ii) answer with semiconductor regions interchanged.
- (v) $V_G < 0$ and large (iii) answer with semiconductor regions interchanged.

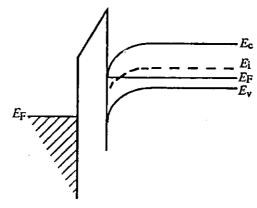
(c)



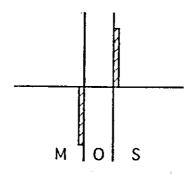
- (a) Curves a and b are standard low- and high-frequency C-V curves that result when the semiconductor component of the MOS-C is in equilibrium under d.c. biasing conditions. Curve c is a nonequilibrium deep-depletion characteristic.
- (b) In accumulation $C \rightarrow C_0 = K_0 \epsilon_0 A_0 / x_0$. Since both devices exhibit the same capacitance in accumulation, the two devices have the same oxide thickness. With x_0 being the same, the lower capacitance of device b in inversion indicates this device has a lower doping. (W_T increases with decreasing doping, thereby giving rise to a smaller capacitance; also see Fig. 16.14b.)

16.13

- (a) p-type ...For p-type devices accumulation (C_{max}) occurs for negative V_G and inversion (C_{min}) occurs at positive V_G . The exact opposite is true for n-type devices.
- (b) At point (2) the p-type MOS-C is far into inversion. Thus



(c) At point (1) the MOS-C is clearly deep into accumulation.



(d) From Fig. P16.13, $C_{\text{max}} = 100 \text{pF}$. However,

$$C_{\text{max}} = C_{\text{O}} = \frac{K_{\text{O}} \varepsilon_{0} A_{\text{G}}}{x_{\text{O}}}$$

$$x_0 = \frac{K_0 \varepsilon_0 A_G}{C_{\text{max}}} = \frac{(3.9)(8.85 \times 10^{-14})(3 \times 10^{-3})}{(10^{-10})} = 0.104 \mu \text{m}$$

(e) In the delta-depletion formulation

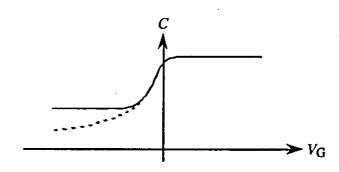
$$C = \frac{C_{\text{O}}}{1 + \frac{K_{\text{O}}W_{\text{T}}}{K_{\text{S}}x_{\text{O}}}} \quad \text{inv } (\omega \to \infty)$$
 (16.34d)

Thus

$$W_{\rm T} = \frac{K_{\rm S} x_{\rm O}}{K_{\rm O}} \left(\frac{C_{\rm O}}{C} - 1 \right) = \frac{(11.8)(0.104)}{(3.9)} \left(\frac{100}{20} - 1 \right) = 1.26 \mu {\rm m}$$

Employing Fig. 16.9, we conclude $N_A = 5 \times 10^{14}/\text{cm}^3$.

16.14



(b)
$$C_{\text{MAX}} = C_{\text{O}} = \frac{K_{\text{O}} \varepsilon_{\text{O}} A_{\text{G}}}{x_{\text{O}}} = \frac{(3.9)(8.85 \times 10^{-14})(10^{-3})}{10^{-5}} = 34.5 \text{ pF}$$

(c)
$$\phi_{\rm F} = -(kT/q) \ln(N_{\rm D}/n_{\rm i}) = -0.0259 \ln(2 \times 10^{15}/10^{10}) = -0.316$$

$$W_{\rm T} = \left[\frac{2K_{\rm S}\varepsilon_0}{qN_{\rm D}}(-2\phi_{\rm F})\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\times10^{-14})(2)(0.316)}{(1.6\times10^{-19})(2\times10^{15})}\right]^{1/2} = 6.42\times10^{-5} \,\rm cm$$

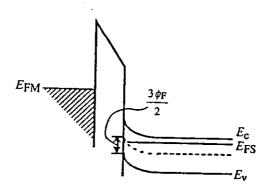
$$C_{\text{MIN}} = \frac{C_{\text{O}}}{1 + \frac{K_{\text{O}}W_{\text{T}}}{K_{\text{S}}x_{\text{O}}}} = \frac{34.5}{1 + \frac{(3.9)(6.42 \times 10^{-5})}{(11.8)(10^{-5})}} = 11.1 \text{ pF}$$

(d) By definition, if $V_G = V_T$... $\phi_S = 2\phi_F = -0.632 \text{ V}$

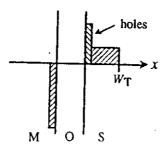
(e)
$$V_{\rm T} = (2\phi_{\rm F}) - \frac{K_{\rm S}x_{\rm O}}{K_{\rm O}} \sqrt{\frac{2qN_{\rm D}}{K_{\rm S}\varepsilon_{\rm O}}} (-2\phi_{\rm F})$$
 (Also see Prob. 16.7h.)

$$= -2(0.316) - \frac{(11.8)(10^{-5})}{(3.9)} \left[\frac{(2)(1.6 \times 10^{-19})(2 \times 10^{15})(0.632)}{(11.8)(8.85 \times 10^{-14})} \right]^{1/2} = 1.23 \text{ V}$$

(f)



(g) $|\phi_S| = |5\phi_F/2| > |2\phi_F|$ and the MOS-C is therefore inversion biased with $W = W_T$.

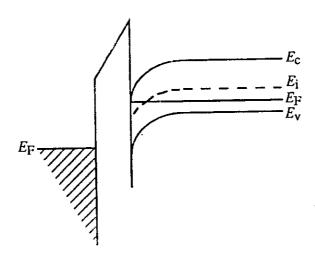


(h) Under the specified operating conditions, the MOS-C is expected to exhibit a total deep-depletion characteristic exemplified by the dashed line in the part (a) answer.

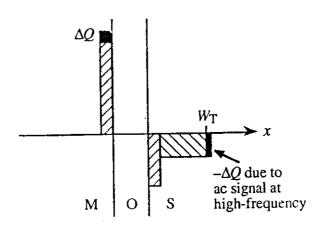
(a) p-type ...There is an inversion-layer of negative charge — electrons — shown in the block charge diagram. The semiconductor must therefore be p-type. (Also, the depletion-region charge is negative or clearly due to acceptor ions.)

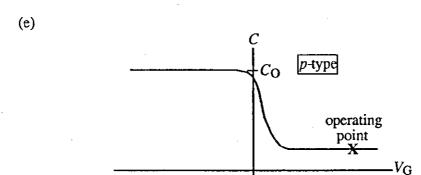
(b) Inversion biased ... As noted in part (a), there is an inversion layer with $n_S > N_A$ shown on the diagram.

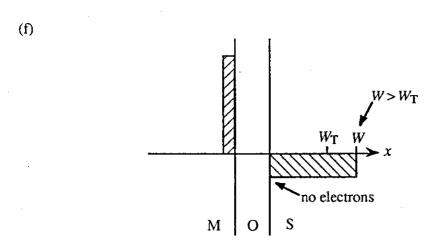
(c)



(d)







NOTE: Because the added depletion-layer charge is farther from the surface than the inversion layer charge, there is NOT a one-to-one correspondence between the two charges. Also, the charge on the metal will be slightly different than under equilibrium inversion conditions at the same $V_{\rm G}$ bias.

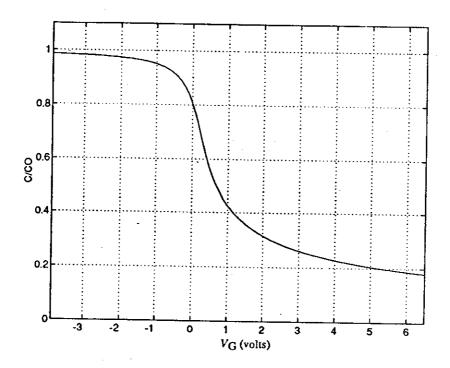
<u> 16.16</u>

The MATLAB program script yielding deep-depletion p-type MOS-C C-V characteristics and a sample plot $(x_0 = 0.2 \mu \text{m}, N_A = 7.8 \times 10^{14} / \text{cm}^3)$ are reproduced below. Note that the sample plot has been extended to $V_G = 5V_T$ (as opposed to stopping at $V_G = 3V_T$ per the directions in the first printing of the book). If the sample C-V curve is converted to an n-type characteristic AND translated approximately 2V along the voltage axis in the negative direction, the sample plot becomes a very good match to the experimental total-deep-depletion data displayed in Fig. 16.17.

MATLAB program script...

```
%p-type Deep Depletion MOS-C C-V Characteristics
%Initialization and Input
clear: close
format compact
NA=input('Please input the bulk doping in /cm3, NA=');
xo=input('Please input the oxide thickness in cm, xo=');
%Constants and Parameters
e0=8.85e-14:
\alpha=1.6e-19;
k=8.617e-5;
KS=11.8;
KO=3.9;
ni=1.0e10:
T=300:
kT=k*T;
%Computed Constants
UF=log(NA/ni);
LD=sqrt((kT*KS*e0)/(2*q*ni));
%C-V Computation for US < UF ( or VG < VI)
US=UF-21:0.5:UF;
F = sqrt(exp(UF).*(exp(-US)+US-1)+exp(-UF).*(exp(US)-US-1));
VG1=kT*(US+(US./abs(US)).*(KS*xo)/(KO*LD).*F);
DENOM1=\exp(UF).*(1-\exp(-US))+\exp(-UF).*(\exp(US)-1);
W1=(US./abs(US)).*LD.*(2*F)./DENOM1;
c1=1.0./(1+(KO*W1)./(KS*xo));
%C-V Computation for US > UF (or VI < VG < 5VT)
FI=sqrt(exp(UF).*(exp(-UF)+UF-1)+exp(-UF).*(exp(UF)-UF-1));
VI=kT*(UF+(KS*xo)/(KO*LD).*FI);
VT=2*øF+(KS*xo/KO)*sqrt((4*q*NA*øF)/(KS*e0));
Vdelta=(g/2)*(KS*xo^2*NA)/(KO^2*e0);
VG2=VI+0.1:0.1:5*VT;
c2=1./sqrt(1+VG2./Vdelta);
```

```
%Combining and Plotting results
c=[c1,c2];
VG=[VG1,VG2];
plot(VG,c); grid
axis([-3*VT,5*VT,0, 1.1])
xlabel('VG (volts)'); ylabel('C/CO')
```



(a)
$$C_{\rm O} = \frac{K_{\rm O} \varepsilon_0 A_{\rm G}}{x_{\rm o}} \implies x_{\rm o} = \frac{K_{\rm O} \varepsilon_0 A_{\rm G}}{C_{\rm O}}$$
$$x_{\rm o} = \frac{(3.9)(8.85 \times 10^{-14})(4.75 \times 10^{-3})}{82 \times 10^{-12}} = 0.200 \mu \text{m}$$

(b) From Fig. 16.17, $C/C_O(inv) \cong 0.39$.

$$C(\text{inv}) = \frac{C_{\text{O}}}{1 + \frac{K_{\text{O}}W_{\text{eff}}(\text{inv})}{K_{\text{S}}x_{\text{o}}}}$$

$$W_{\text{eff}}(\text{inv}) = \frac{K_S x_O}{K_O} \left[\frac{C_O}{C(\text{inv})} - 1 \right] = \frac{(11.8)(0.2)}{(3.9)} \left(\frac{1}{0.39} - 1 \right)$$
$$= 0.946 \mu \text{m}$$

and

$$\frac{W_{\rm eff}(\rm inv)}{L_{\rm D}} = \frac{9.46 \times 10^{-5}}{2.91 \times 10^{-3}} = 3.25 \times 10^{-2}$$

(c) If $W_{\rm eff}({\rm inv})$ is equated to $W_{\rm T}$, one estimates from Fig. 16.9 that $N_{\rm D} \approx 8.5 \times 10^{14}/{\rm cm}^3$ or $U_{\rm F} = -{\rm ln}(N_{\rm D}/n_{\rm i}) = -{\rm ln}(8.5 \times 10^{14}/1.00 \times 10^{10}) = -11.35$. Substituting $U_{\rm F} = -11.35$ into the expression for $W_{\rm eff}/L_{\rm D}$ one computes $W_{\rm eff}/L_{\rm D} = 3.374 \times 10^{-2}$. $W_{\rm eff}/L_{\rm D}$ is too large implying $N_{\rm D}$ and $|U_{\rm F}|$ are somewhat larger. Trying $U_{\rm F} = -11.45$ yields $W_{\rm eff}/L_{\rm D} = 3.223 \times 10^{-2}$; trying $U_{\rm F} = -11.40$ yields $W_{\rm eff}/L_{\rm D} = 3.298 \times 10^{-2}$. Clearly $U_{\rm F}$ is bracketed between -11.40 and -11.45. Subsequent calculations give $W_{\rm eff}/L_{\rm D} = 3.268 \times 10^{-2}$, 3.253×10^{-2} , 3.238×10^{-2} when $U_{\rm F} = -11.42$, -11.43, and -11.44, respectively. The best value appears to be

$$U_{\rm F} = -11.43$$
 and $N_{\rm D} = n_{\rm i}e^{-U_{\rm F}} = (10^{10})e^{11.43} = 9.20 \times 10^{14}/{\rm cm}^3$

NOTE: We actually pushed the calculation here beyond the accuracy of the data to illustrate the procedure.