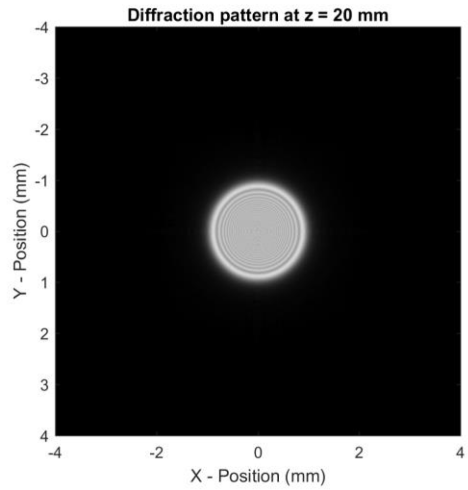
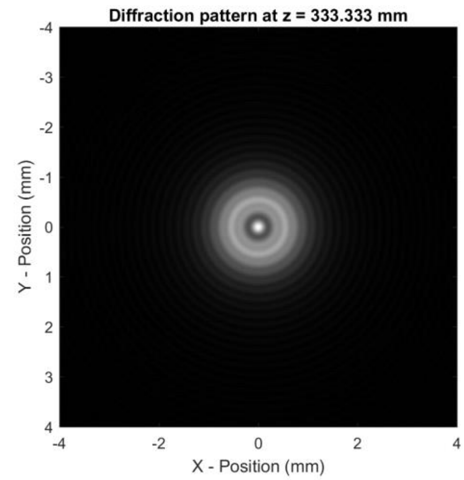
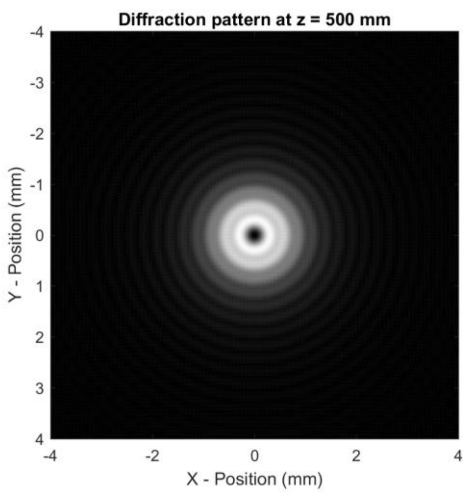
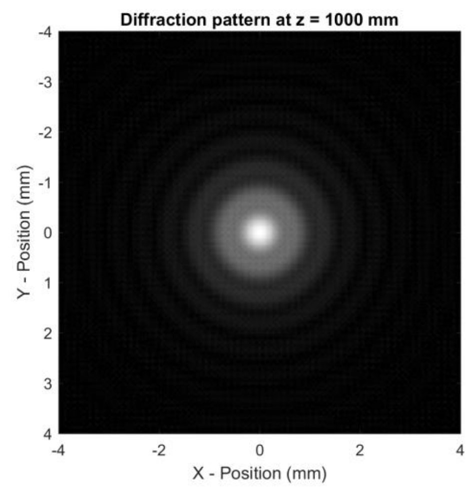
Cole Nielsen
Homework No. 6
EE 5621

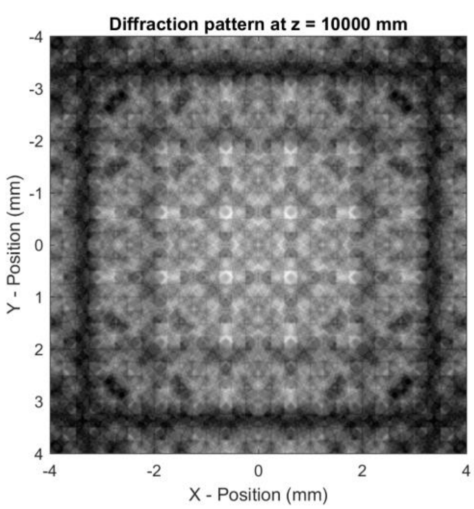
1- Fresnel Transfer Function Method, circular aperture of 2mm diameter (U^1, not intensity)

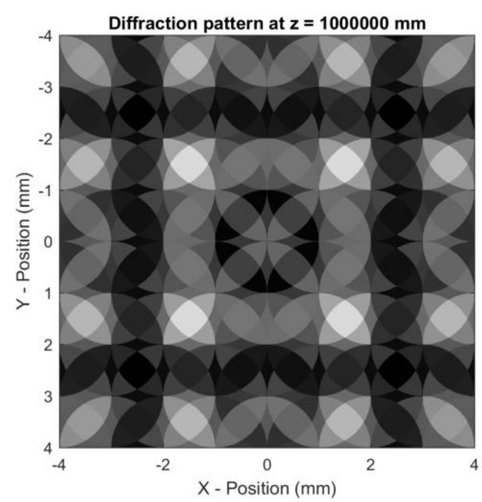




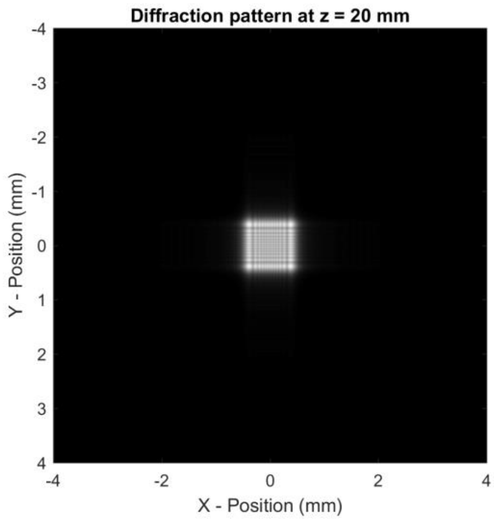


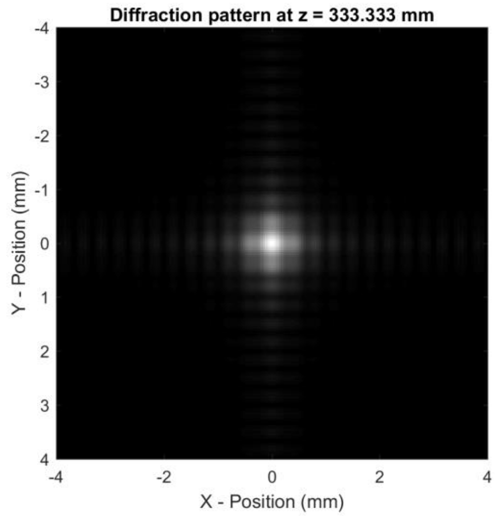


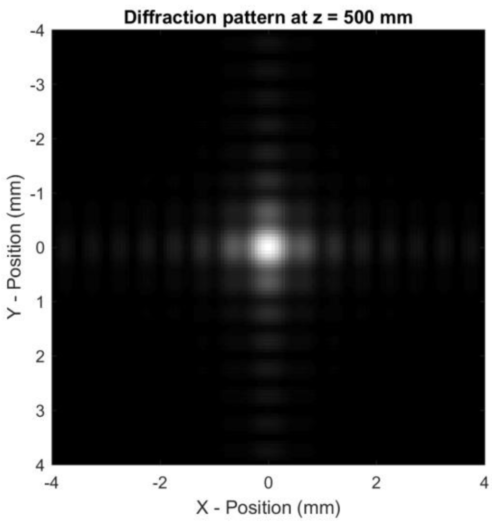


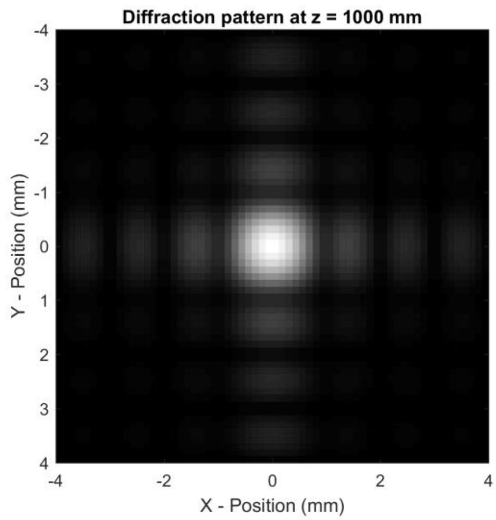


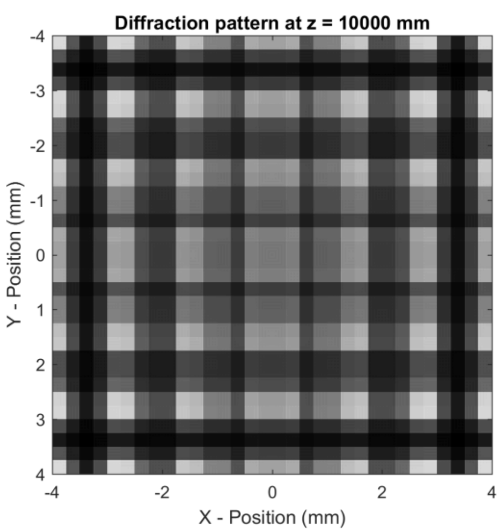
1- Fresnel Transfer Function Method, square aperture of 1mm side length (U^1, not intensity)

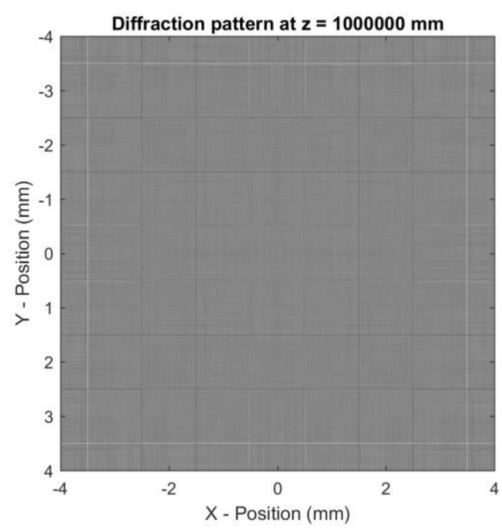




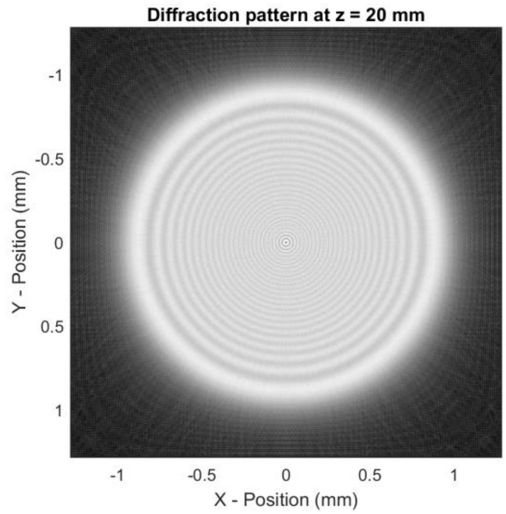


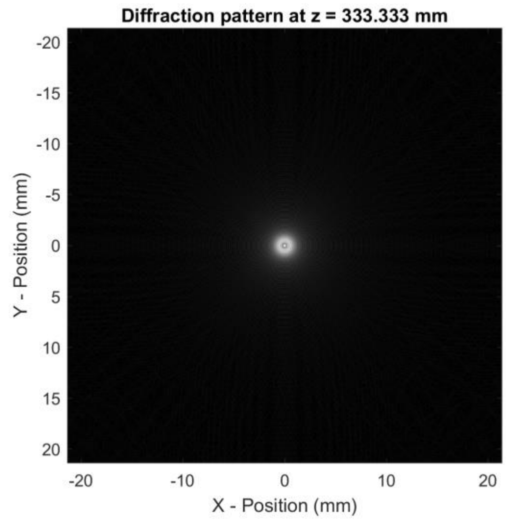


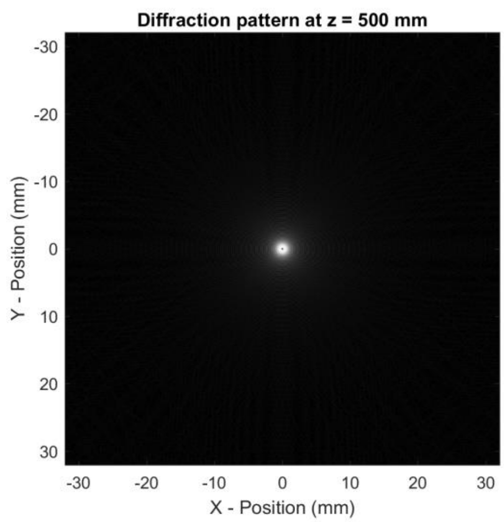


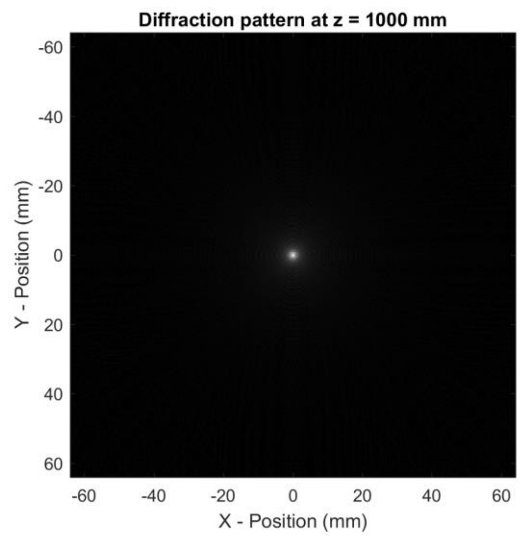


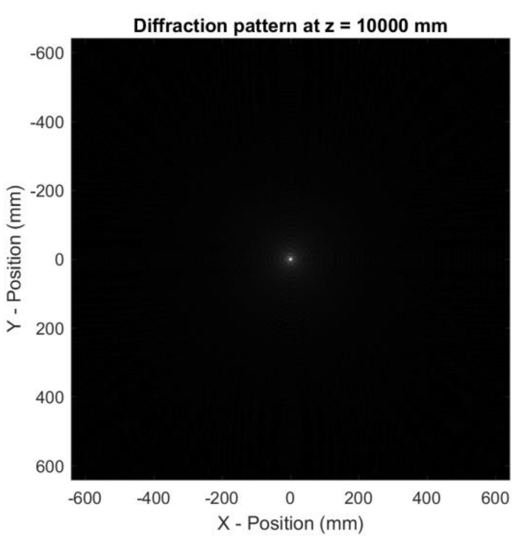
2- Fourier Transfer method - circular aperture of 2mm diameter (U^0.5, not intensity)

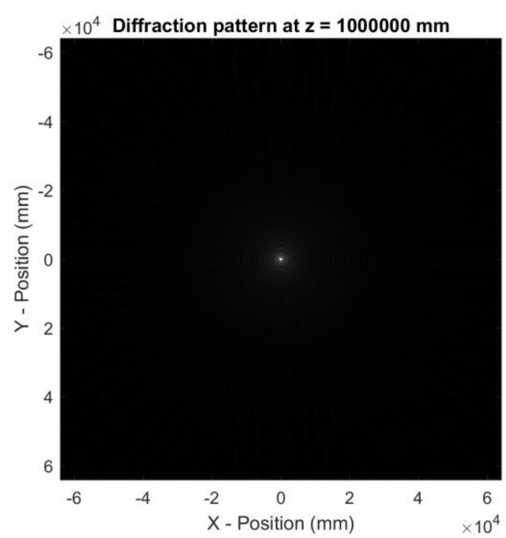




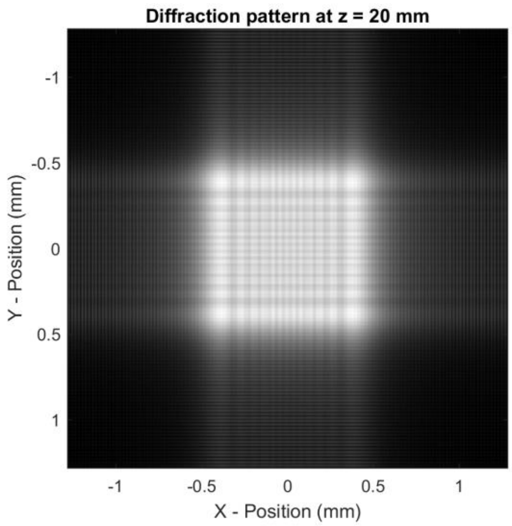


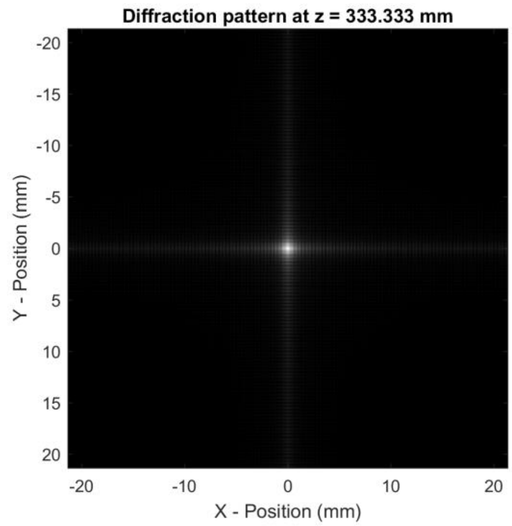


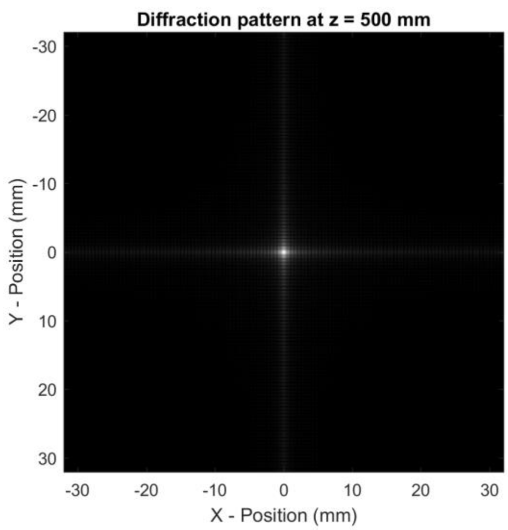


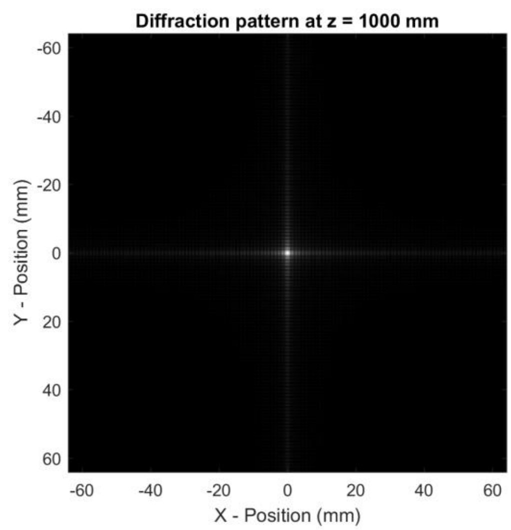


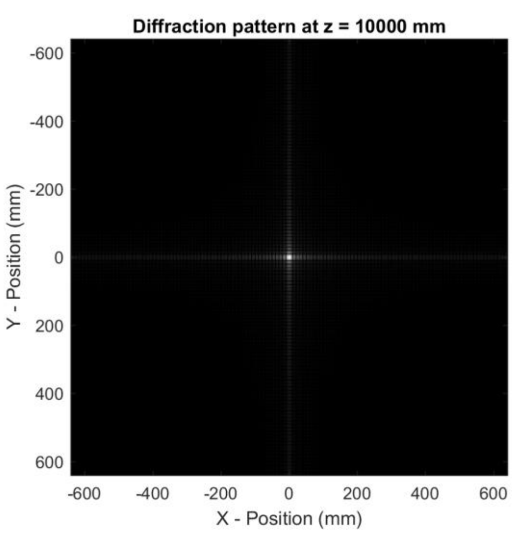
2- Fourier Transfer method - square aperture of 1mm length (U^0.5, not intensity)

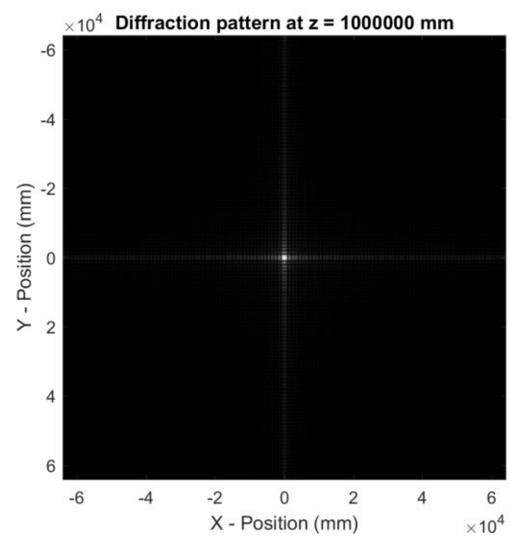












3- Sampling requirement: When using a quadratic phase exponential, the Fresnel number (dependent on spatial system parameters) is useful for determining the effective bandwidth of an aperture:

$$N_F = \frac{(\ell/2)^2}{\lambda z}$$

In particular when using quadratic phase exponentials it is important to note that when NF $>$ 0.25, a systems bandwidth is limited by the phase exponential rather than the finite aperture. For these simulations, l $=$ 1mm (assume the square aperture case), lambda $=$ 1 micrometer. This yields the result that NF $=$ 0.25 at 1 meter, and decreases for z $>$ 1 meter. We should expect that the bandwidth is limited by the aperture at the distances greater than 1m. Seeing the results from the simulation this appears to hold some truth as the results for $>$1m appear somewhat erroneous. The minimum sampling rate (Goodman pg. 139), if defined in terms of number of points in the aperture (call this K), is K $=$ 4NF if NF$>$0.25. For NF$<$0.25, K must be chosen such that the equivalent bandwidth prevents aliasing. Bandwidth can be stated in terms of M/l, where M is the number of points in the aperture that meets the bandwidth requirement. Therefore to prevent aliasing, K$>$M to meet sampling criteria. If Bx $=$ length/(lambda*z), it is trivial to find a value for K which satisfies the requirements. Note: the results given by this method may not be useful for NF$<$0.25 as seen by the simulations, as the simulation domain becomes very small compared the diffraction pattern. For the convolution method, sampling in the aperture must be K$>$M^2/4NF, which must similarly satisfy NF$>$0.25. It should be noted also for NF$<$0.25 (distance of greater than 1m in the simulation), the Fresnel convolution method will towards being a Fourier Transform of the aperture distribution which perhaps negates the usage of Fresnel diffraction equation. Rather, simpler methods such as Fraunhofer diffraction equation will yield essentially the same result.

The transfer function method is preferable when trying to calculate a diffraction pattern of fixed domain (being equal to the size of the source object domain), allowing for more detailed viewing of the central diffraction pattern lobes. At far diffraction distances however, this method runs into issues with sampling/aliasing as the diffraction patterns become mostly spurious (NF$<$0.25). Therefore the transfer function method is recommended for near field application. Sampling issues for this can be resolved by using a higher resolution source image (i.e. increase sampling frequency and number of samples). The Fourier transform method is preferable for more far-field calculations, as the image domain is scaled in accordance to the distance, yielding a pattern with many diffraction lobes at all distances. The size of the diffraction pattern shrinks as distance increases relative to the domain of the image. It is foreseeable that at great distances using this method that the diffraction pattern will become small enough relative to the computed image that it will essentially disappear. Therefore it is necessary to either increase the size of the array with this method with zero padding or use a more detailed source image. Generally this method is preferred at greater distance (far field) compared to the first method, as it always allows for viewing of multiple diffraction pattern lobes at all distances.

Also, on a side note, the transfer function method was computationally faster to calculate. This may imply that it may be better to use this method as larger matrices can be used allowing for greater resolution than the Fourier transform method.