

# **EE 3161**

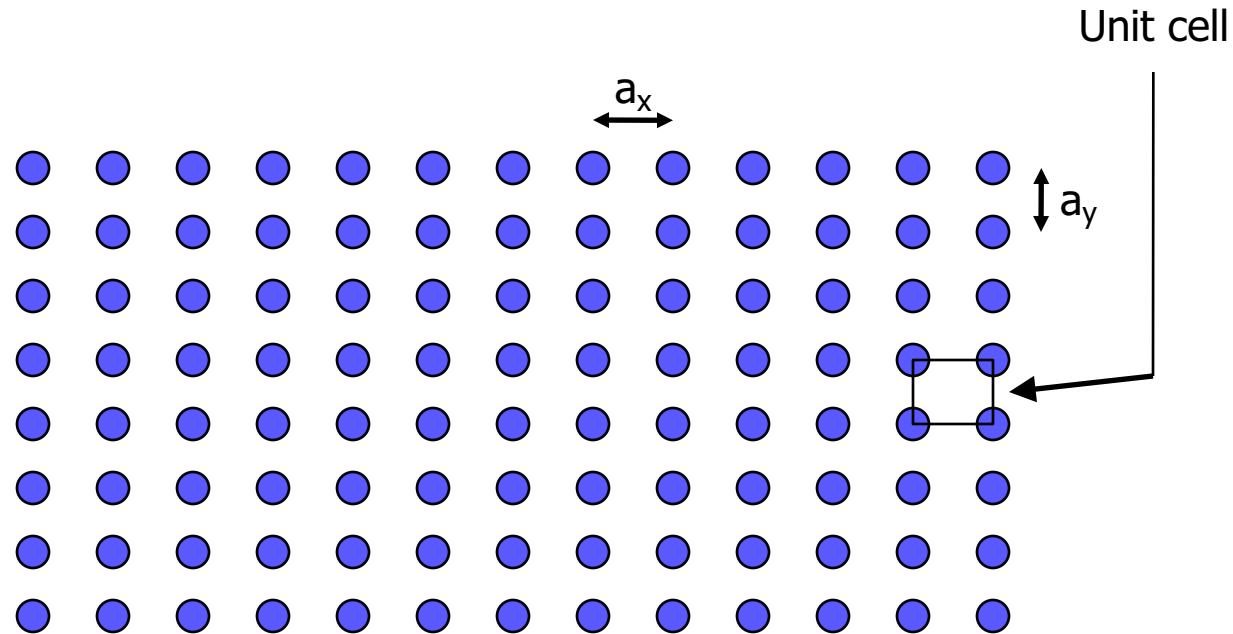
## **Semiconductor Devices**

**Professor Sarah Swisher**

# **Midterm #1 Review**

# Review

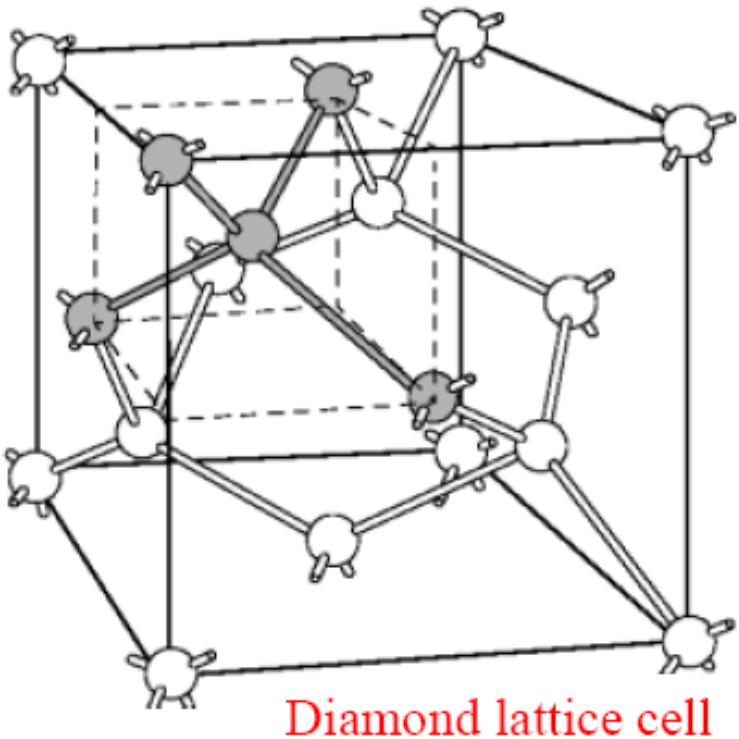
- **Unit cell and lattice constant:**



- The unit cell is the smallest division of a crystal, which, if repeated, could generate the whole crystal,
- The lattice constant is the translational distance of the unit cell. The lattice constant can vary for different directions.

# Review

- Diamond crystal lattice:



- Each atom in the diamond lattice is bonded to its four nearest neighbors.

# Review

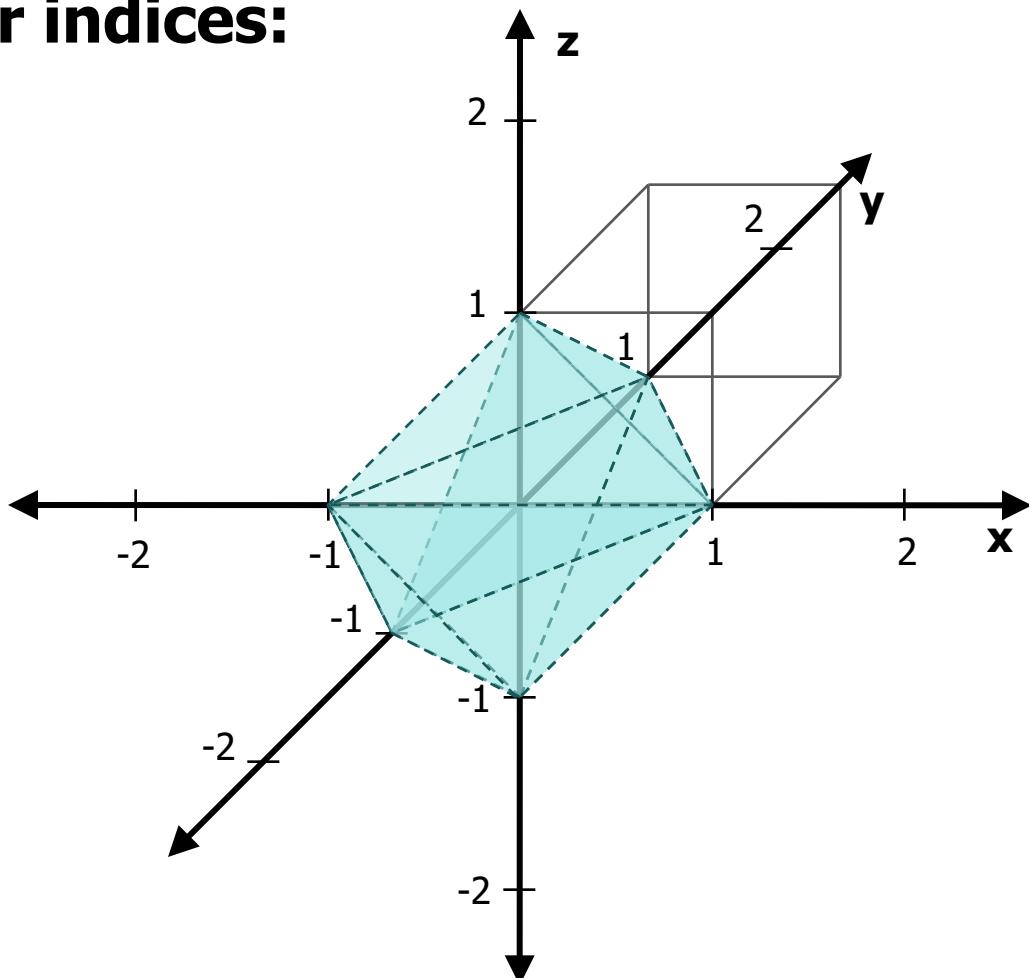
- **Miller indices definition:**

- **The Miller indices: a set of integers that identify a type of crystal plane, where the integers are the reciprocal of the intercepts of the plane on the crystallographic axes.**

# Review

- **Miller indices:**

$\{1\bar{1}1\}$

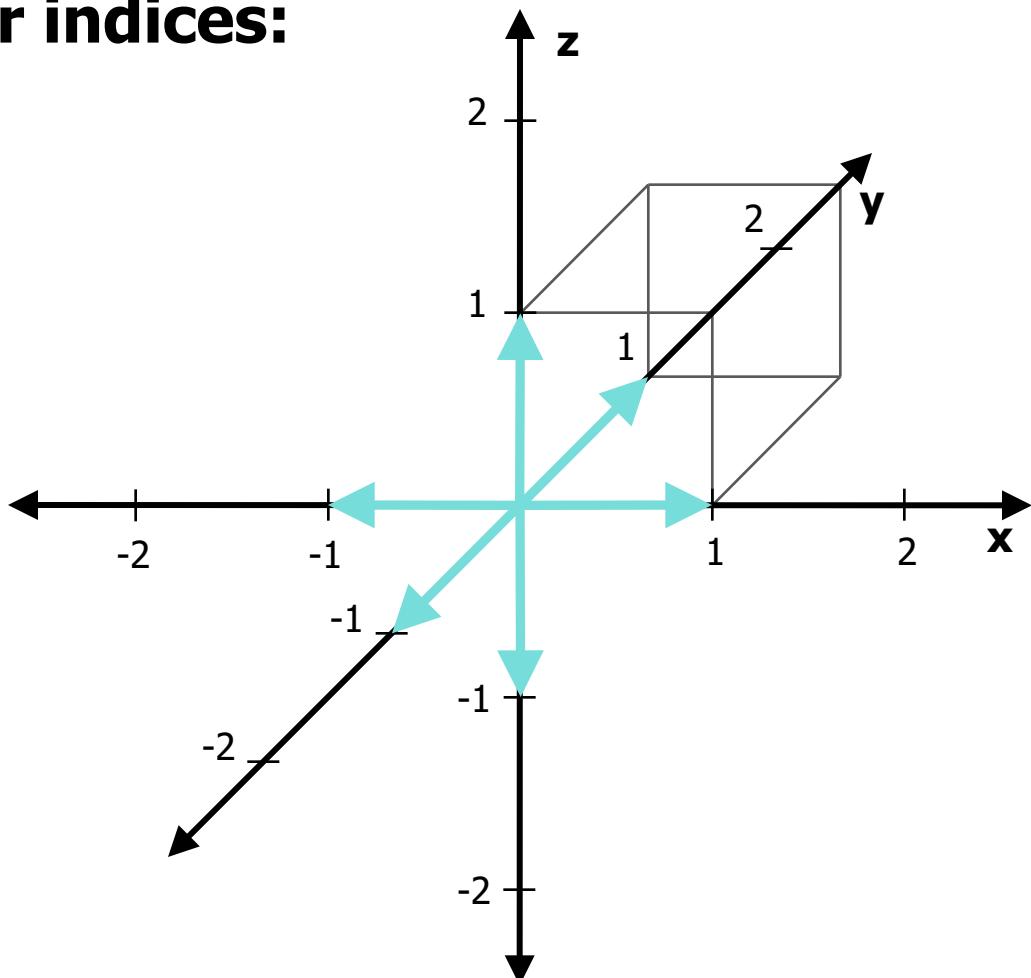


$(1\bar{1}1)$   
 $(\bar{1}11)$   
 $(1\bar{1}\bar{1})$   
 $(11\bar{1})$   
 $(\bar{1}\bar{1}1)$   
 $(111)$   
 $(\bar{1}1\bar{1})$   
 $(1\bar{1}\bar{1})$   
 $(\bar{1}\bar{1}\bar{1})$

- **A family of equivalent planes that are indistinguishable due to crystal symmetry are denoted using braces.**

# Review

- Miller indices:



$\langle 1\ 0\ 0 \rangle$

[1 0 0]  
[0 1 0]  
[0 0 1]  
[1 0 0]  
[0 1 0]  
[0 0 1]

- Same principle holds for crystal directions. Directions denoted by square brackets, and equivalent directions by triangular brackets.

# Review

- Periodic table of elements:

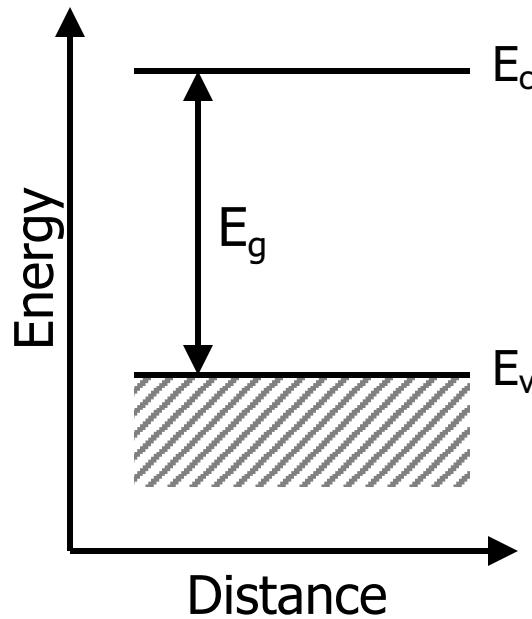
The image shows a standard periodic table of elements. Several groups of elements are highlighted with colored boxes: a green box covers groups IA and IIA; a blue box covers group IB; a red box covers groups IIIA, IVA, VA, and VIA; and a pink box covers groups IIIB through VIIIB. The table includes element symbols, atomic numbers, and group/period labels.

IA		IIA																			0	
1	H	3	4																		2	He
2	Li	11	12																		9	F
3	Na	Mg	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	10	Ne
4	K	Ca	Sc	Ti	Y	Cr	Mn	Fe	Co	Ni	Cu	Zn	Ga	Ge	As	Se	Br	Kr	17	18	Cl	Ar
5	Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn	Sb	Te	I	Xe	53	54	At	Rn
6	Cs	Ba	*La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg	Tl	Pb	Bi	Po	85	86	At	Rn		
7	Fr	Ra	+Ac	Rf	Ha	106	107	108	109	110	110											

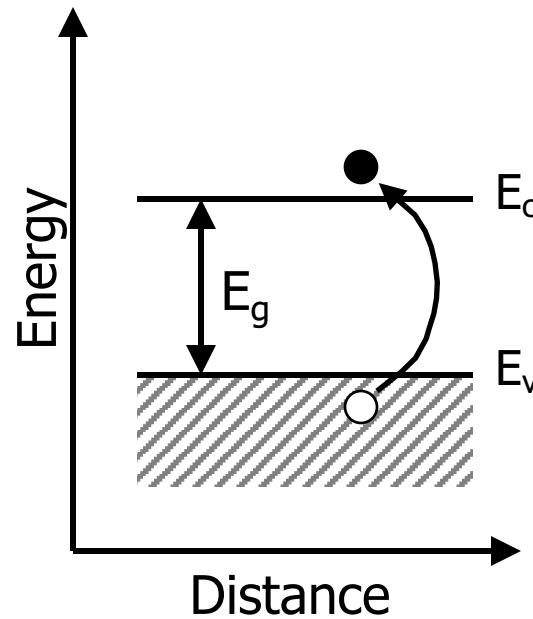
- Semiconductors include group-IV elements, III-V, II-VI and IV-VI compounds and their alloys.

# Review

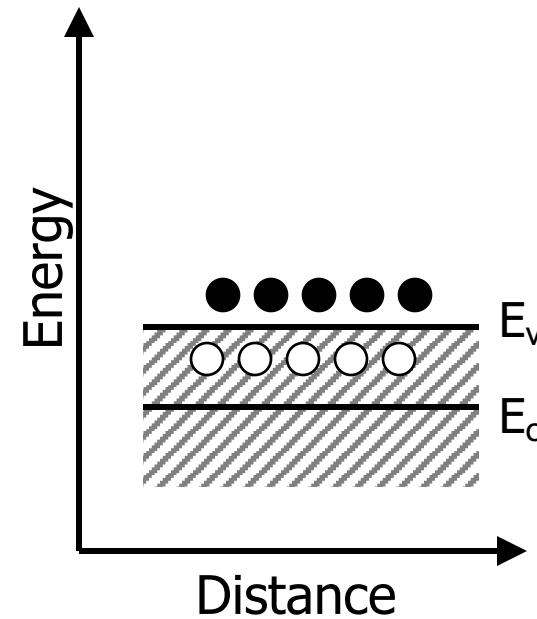
- What is a semiconductor?



Insulator



Semiconductor



Semi-metal / metal

- A semiconductor is a material with a well-defined band gap, usually with  $E_g \sim 0.2 - 3$  eV.

# Review

- Energy gaps of common semiconductors at 300 K:

	IIIA	IVA	VIA	VIA
5	6	7	8	
B	C	N	O	
13	14	15	16	
Al	Si	P	S	
30	31	32	33	34
Zn	Ga	Ge	As	Se
48	49	50	51	52
Cd	In	Sn	Sb	Te
80	81	82	83	84
Hg	Tl	Pb	Bi	Po

Material	$E_g$ (300 K)
Si	1.12 eV
Ge	0.66 eV
Sn	semi-metal
-----	-----
AlP	2.45 eV
GaAs	1.42 eV
InSb	0.18 eV
-----	-----
ZnSe	2.70 eV
CdTe	1.44 eV

- Band gap decreases with increasing atomic number,
- Band gap increases with increasing ionic bond component.

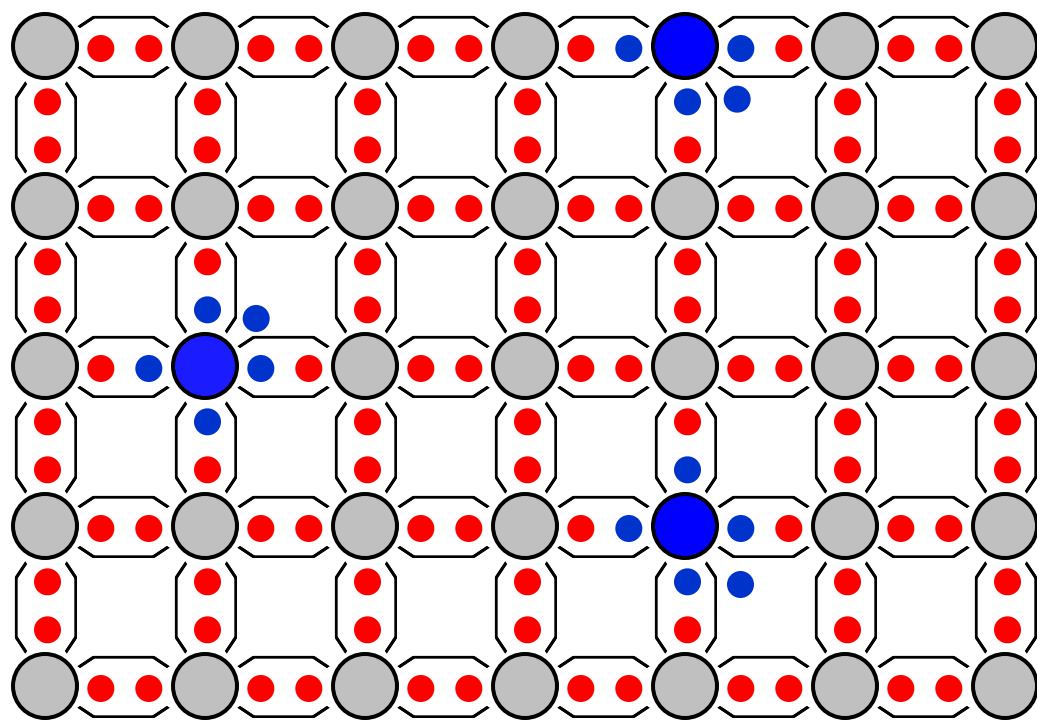
# Review

- **Effect of impurities:**

- Donors.

III A	IV A	V A
5 B	6 C	7 N
13 Al	14 Si	15 P
31 Ga	32 Ge	33 As
49 In	50 Sn	51 Sb

- Silicon ion
- Valence electron
- Donor



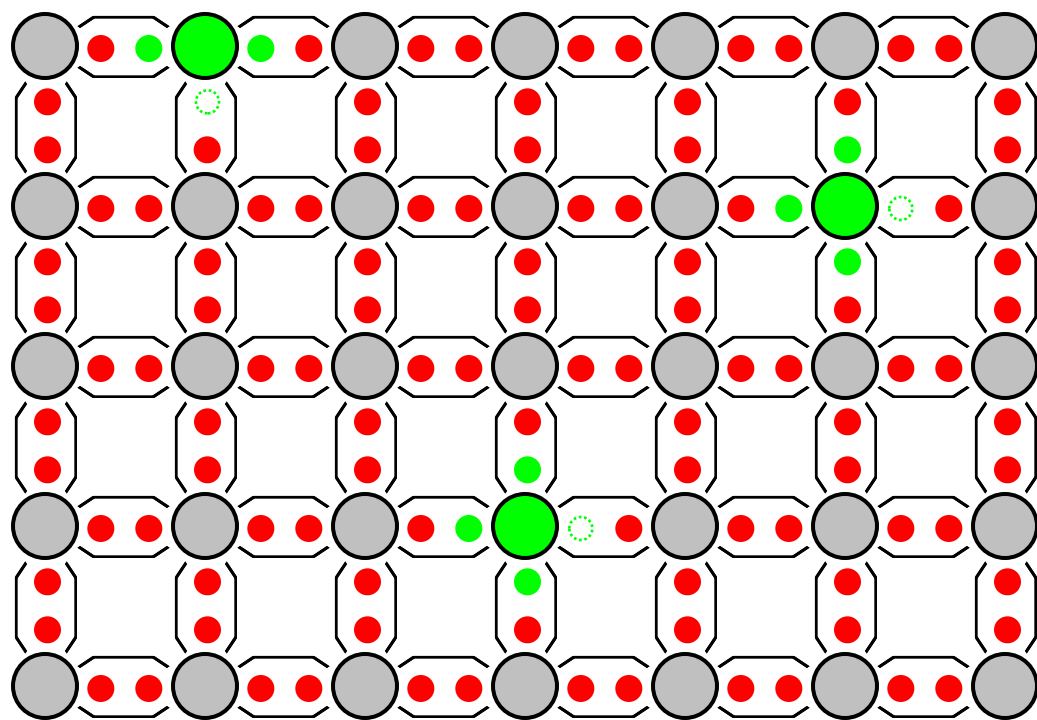
- **Donor impurities have one more electron than required for bonding → excess electrons in the conduction band.**

# Review

- **Effect of impurities:**
  - Acceptors.

III A	IV A	V A
5 B	6 C	7 N
13 Al	14 Si	15 P
31 Ga	32 Ge	33 As
49 In	50 Sn	51 Sb

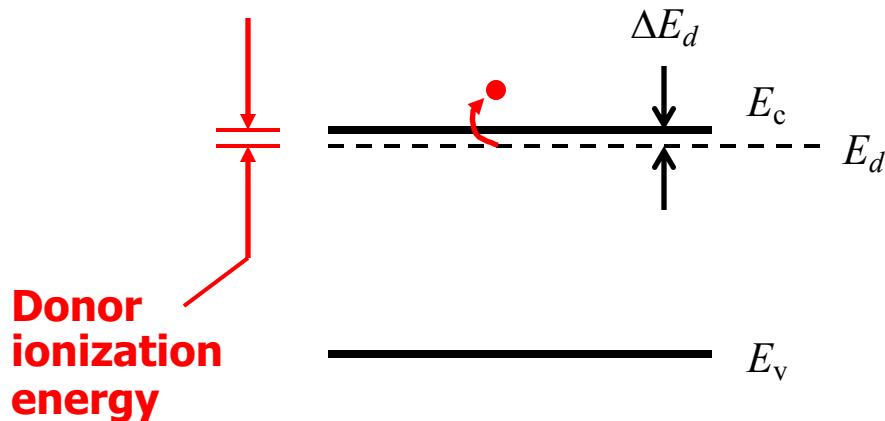
- Silicon ion
- Valence electron
- Acceptor



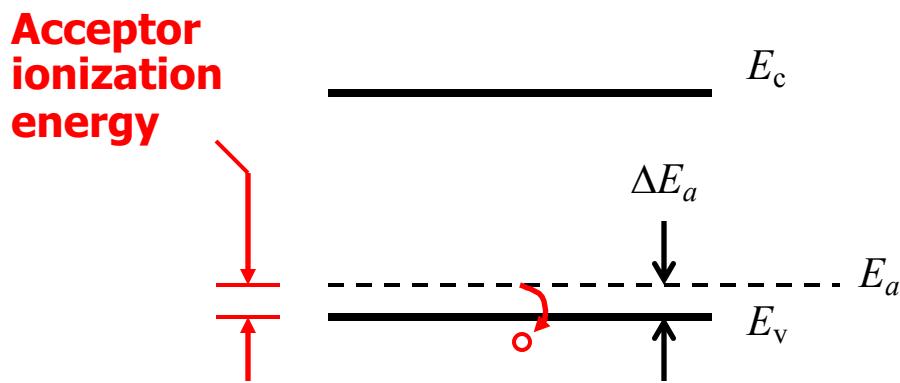
- **Acceptor impurities have one less electron than required for bonding → excess holes in the valence band.**

# Review

- **Impurity binding energy:**
  - Binding energy for common dopants in silicon.



<b>Donors</b>	
Dopant	$\Delta E_d$ (meV)
Sb	39
P	45
As	54



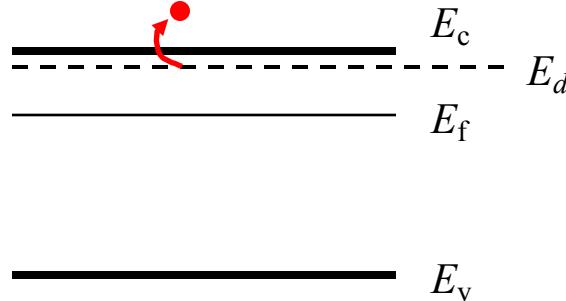
<b>Acceptors</b>	
Dopant	$\Delta E_a$ (meV)
B	45
Al	67
Ga	72
In	160

# Review

- Dopant occupation probabilities:

$$N_d^+ = N_d \cdot \left[ 1 - \frac{1}{1 + \frac{1}{2} e^{(E_d - E_f)/kT}} \right]$$

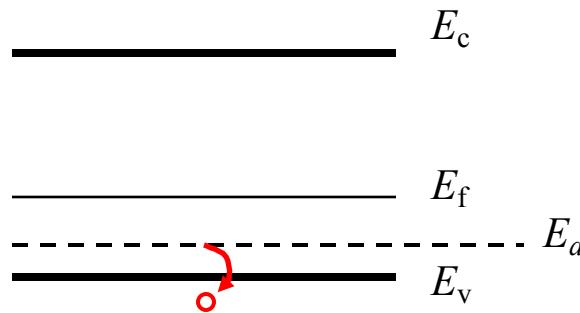
**$N_d$  = donor concentration**



Donor ionization energy

$$N_a^- = N_a \cdot \left[ 1 - \frac{1}{1 + \frac{1}{4} e^{(E_f - E_a)/kT}} \right]$$

**$N_a$  = acceptor concentration**



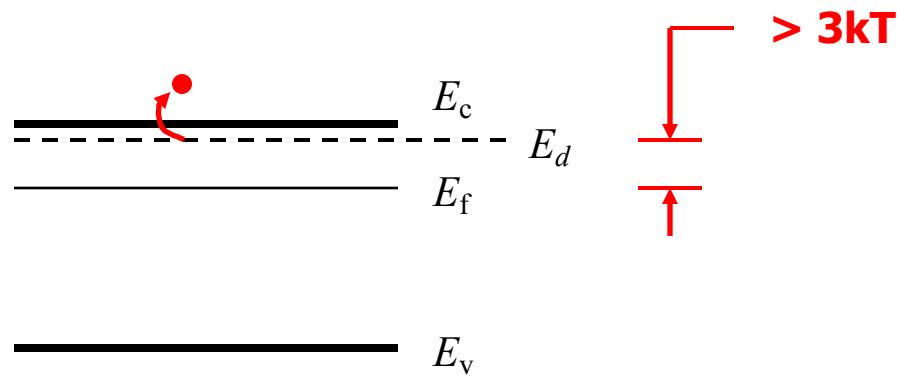
Acceptor ionization energy

# Review

- **Dopant occupation probabilities:**

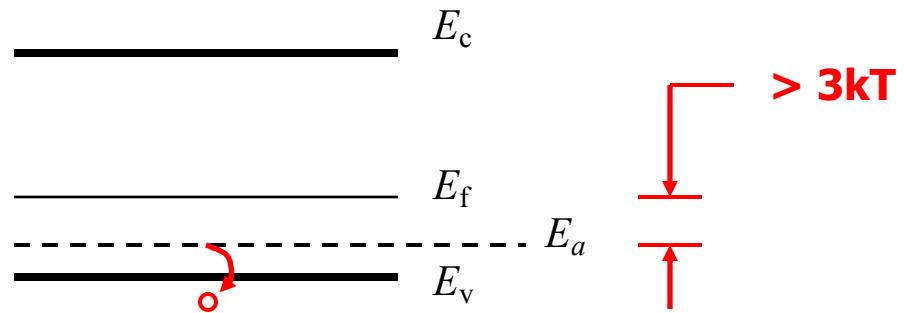
$$N_d^+ \approx N_d$$

when  $E_d - E_f > 3kT$



$$N_a^- \approx N_a$$

when  $E_f - E_a > 3kT$



- **Normally assume that dopant impurities are fully ionized.**

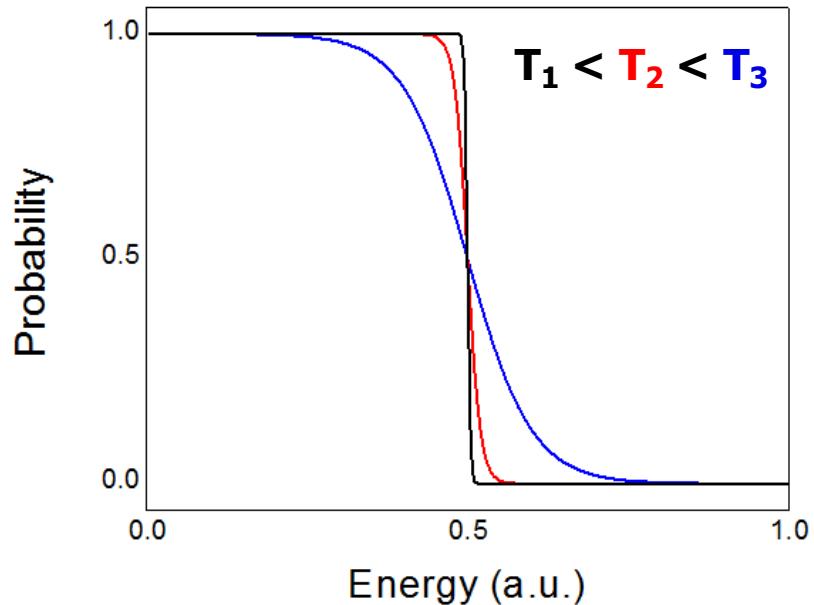
# Review

- **Fermi-Dirac distribution function:**

- Electrons obey Fermi-Dirac statistics:

$$f(E) = \frac{1}{1 + e^{(E-E_f)/kT}}$$

↑  
Fermi function  
↑  
Fermi energy  
↑  
Thermal energy



- **Fermi-Dirac distribution function:** the occupation probability of an electronic state. Has values from 0 to 1,
- **Fermi energy:** the energy at which the occupation probability is equal to 0.5.

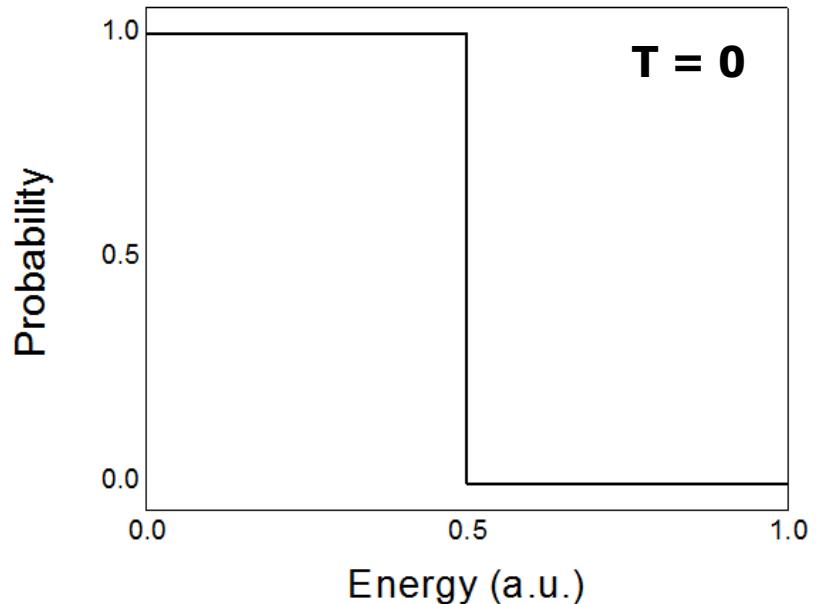
# Review

- **Fermi-Dirac distribution function:**

- Electrons obey Fermi-Dirac statistics:

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

↑  
**Fermi function**  
↑  
**Fermi energy**  
↑  
**Thermal energy**



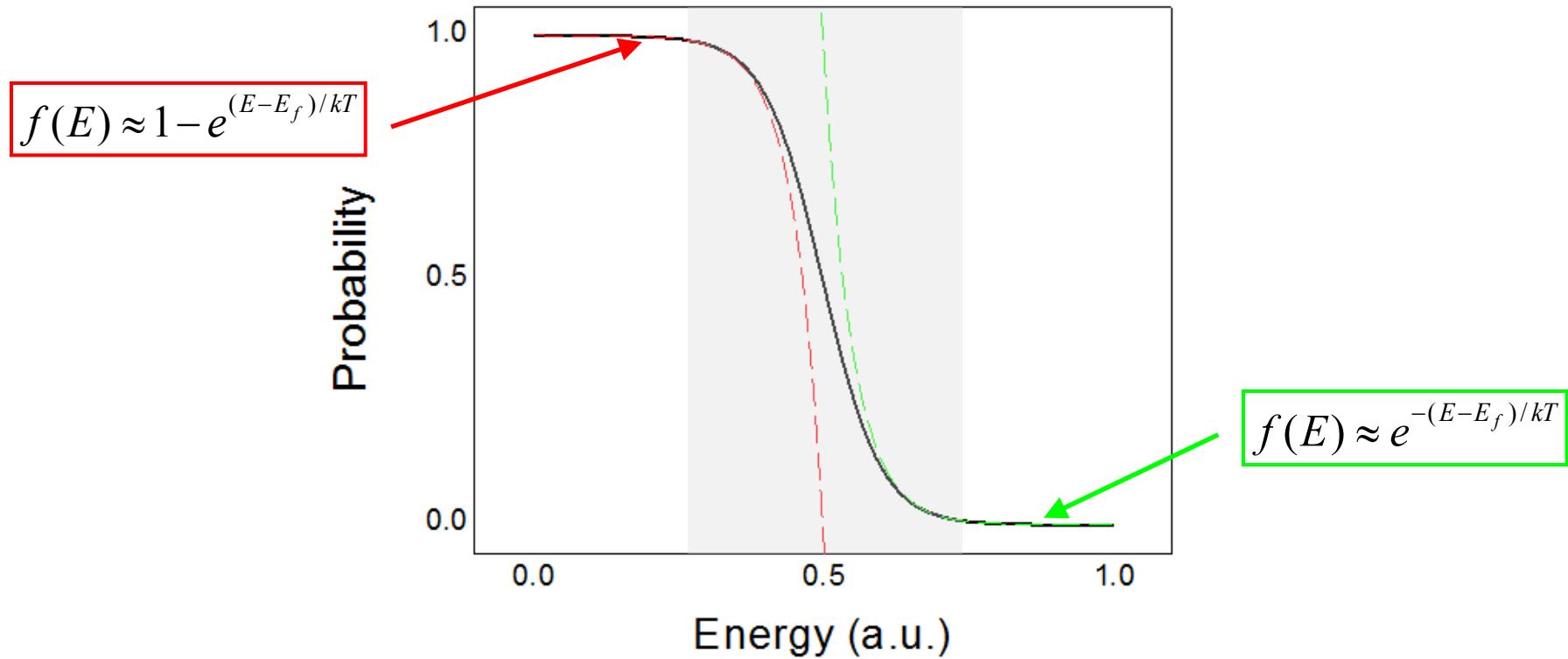
- At zero temperature,  $f(E)$  is an inverse step function:

for  $E < E_f$ ,  $f(E) = 1$

for  $E > E_f$ ,  $f(E) = 0$ .

# Review

- **Fermi-Dirac distribution function:**
  - Boltzmann approximation:



- When the energy is far from the Fermi energy, the Fermi-Dirac distribution function can be approximated as an exponential.
- This is commonly referred to as the Boltzmann approximation.

# Review

- Density-of-states effective mass for common semiconductors:

	III A	IV A	V A	VI A
	5	6	7	8
	B	C	N	O
	13	14	15	16
IB	Al	Si	P	S
	30	31	32	33
	Zn	Ga	Ge	As
	34			Se
	48	49	50	51
	Cd	In	Sn	Sb
				Te
	80	81	82	83
	Hg	Tl	Pb	Bi
				Po

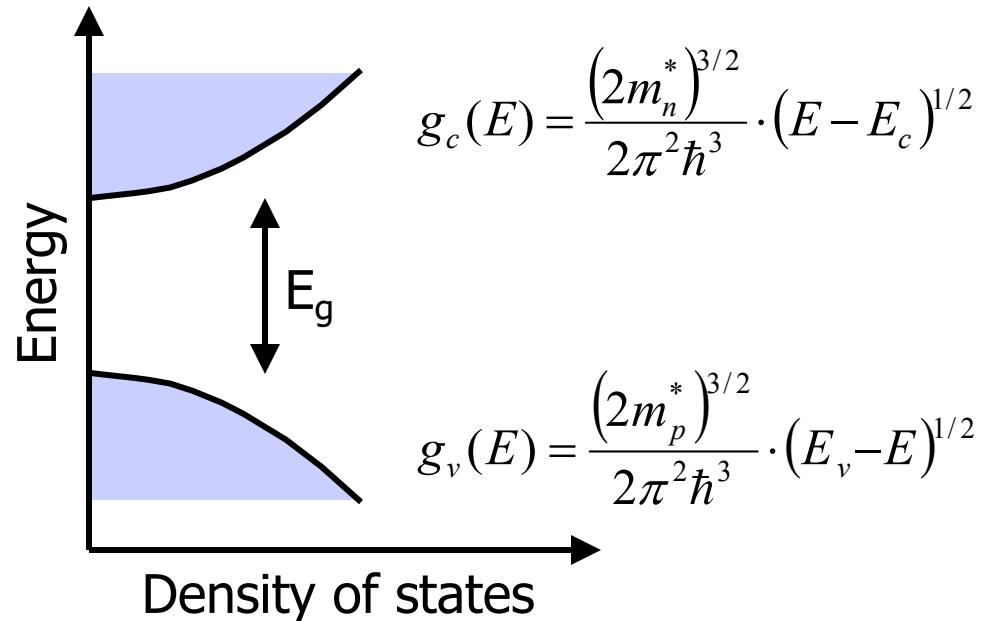
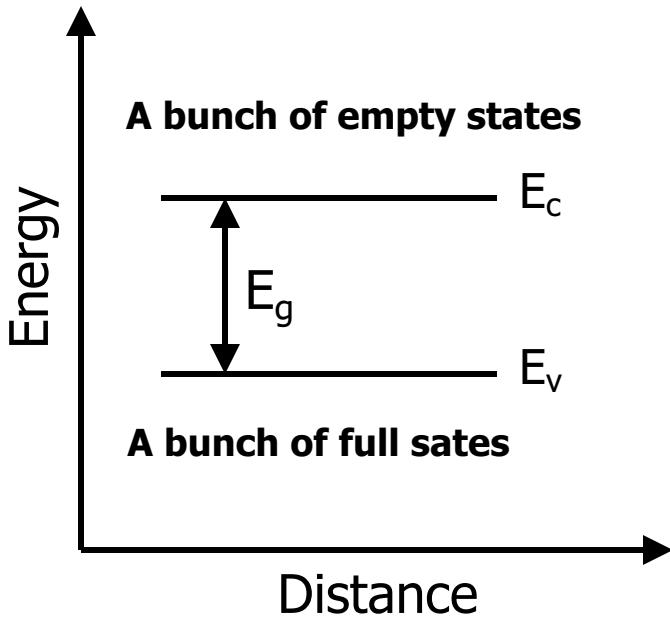
Material	$m^*_n/m_0$	$m^*_p/m_0$
Si	1.18	0.81
Ge	0.55	0.36
GaAs	0.066	0.52
InSb	0.014	0.43
ZnSe	0.21	0.60
CdTe	0.11	0.40

Free electron rest mass  $m_0 = 9.11 \times 10^{-31} \text{ kg}$

- The effective mass is often less than the free electron mass! Electrons seem lighter in semiconductors!!
- Holes usually “heavier” than electrons.

# Review

- **Density of states:**
  - Number of states per unit energy.



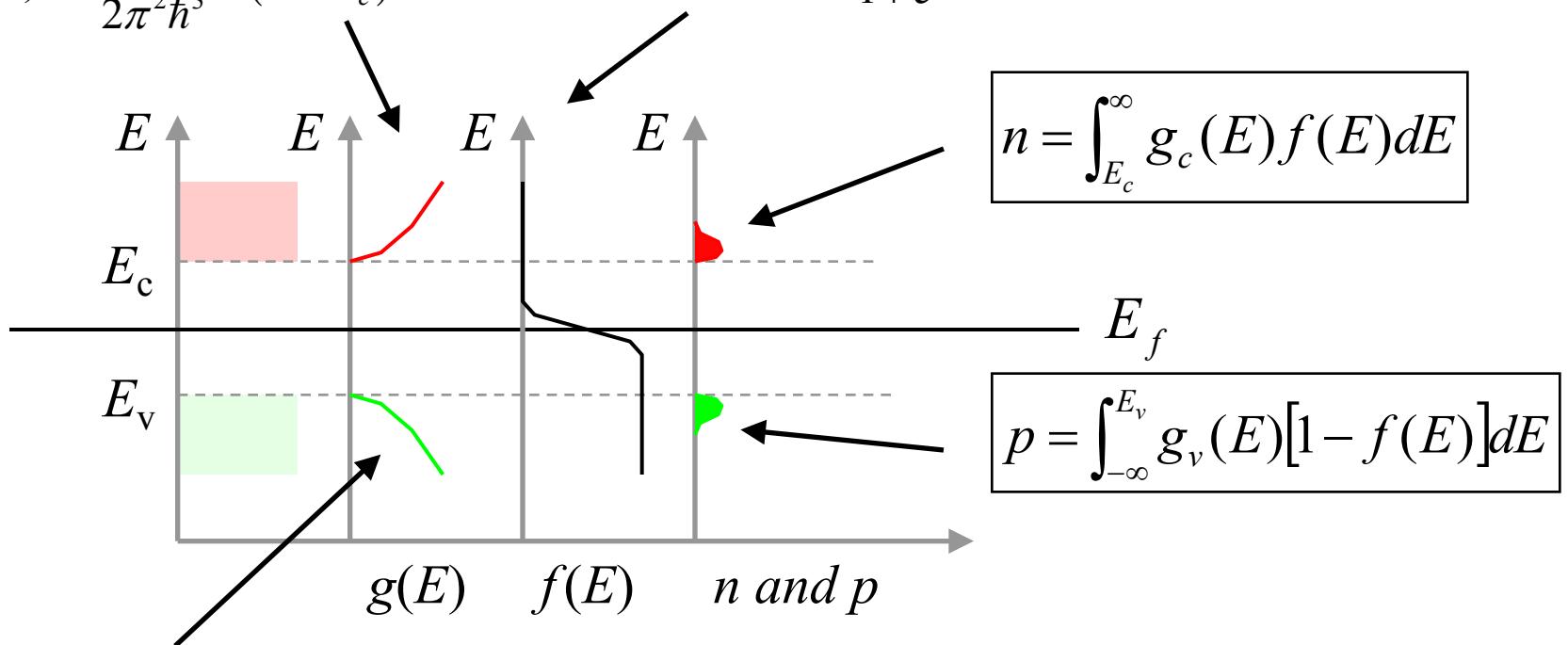
- **$g_c(E)$  and  $g_v(E)$  increase as the square root of energy away from the band edge,**
- **$g_c(E)$  and  $g_v(E)$  depend upon the effective mass and valley degeneracy.**

# Review

- Calculating the carrier concentration:

$$g_c(E) = \frac{(2m_n^*)^{3/2}}{2\pi^2\hbar^3} \cdot (E - E_c)^{1/2}$$

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$



$$g_v(E) = \frac{(2m_p^*)^{3/2}}{2\pi^2\hbar^3} \cdot (E_v - E)^{1/2}$$

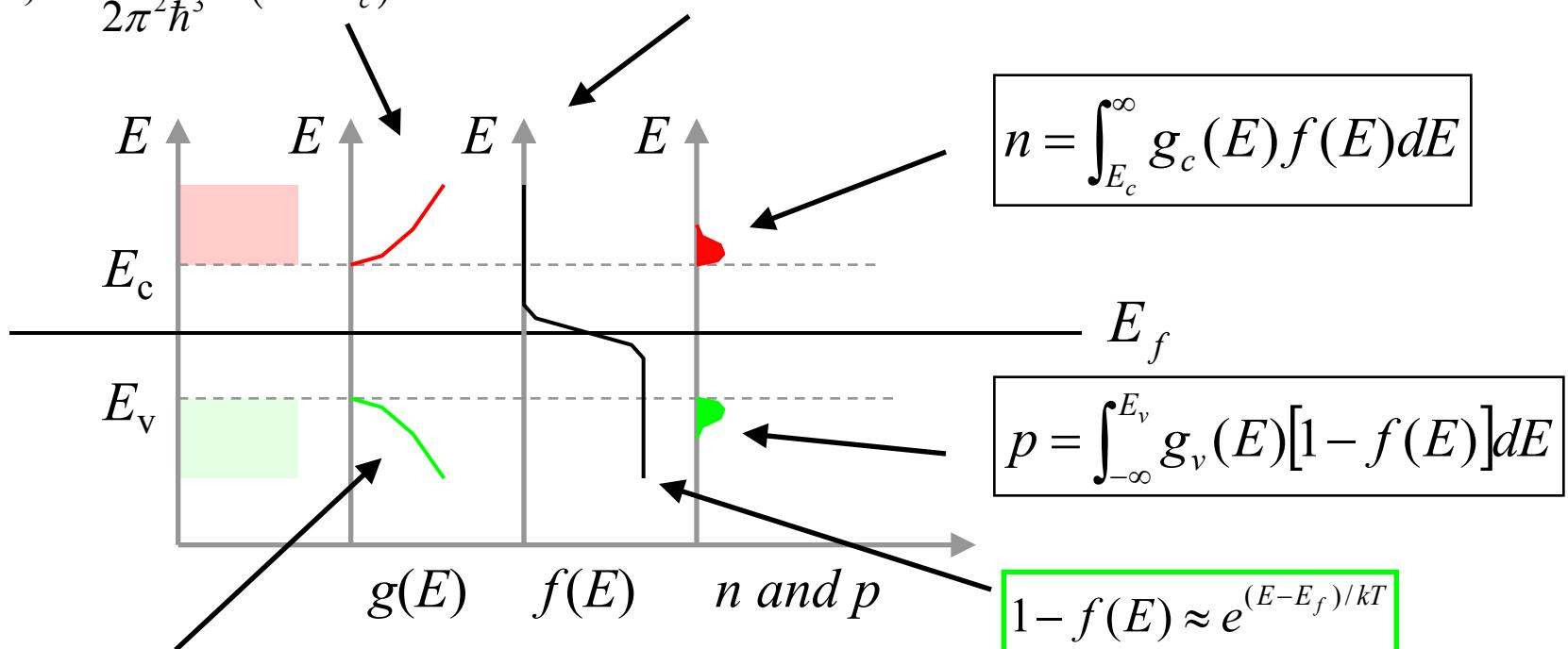
- Difficult to solve analytically.

# Review

- Calculating the carrier concentration:
  - Use Boltzmann approximation.

$$g_c(E) = \frac{(2m_n^*)^{3/2}}{2\pi^2\hbar^3} \cdot (E - E_c)^{1/2}$$

$$f(E) \approx e^{-(E-E_f)/kT}$$

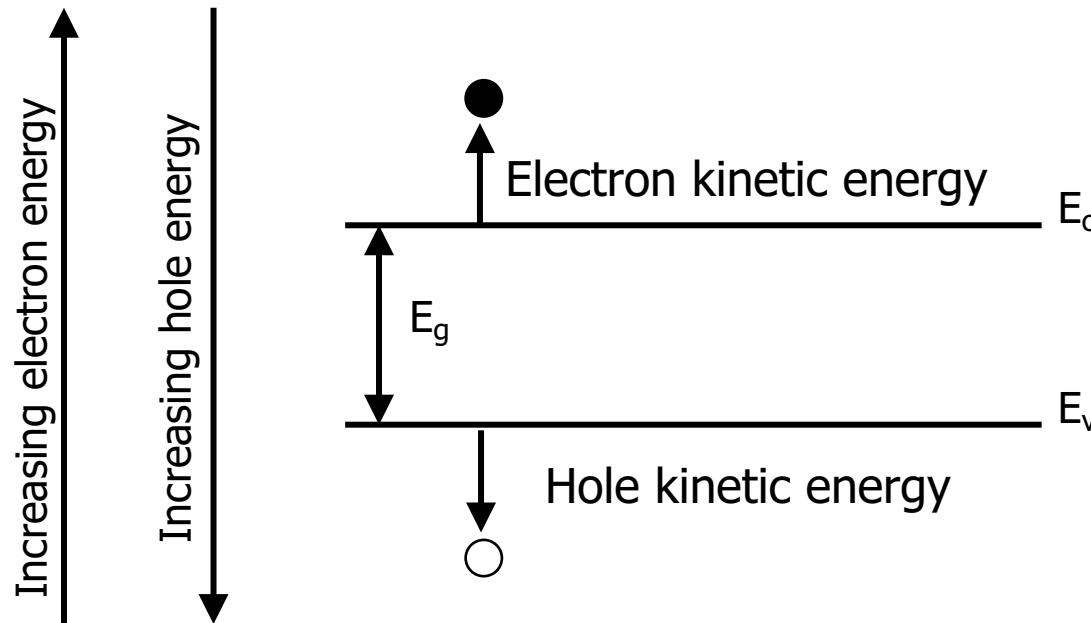


$$g_v(E) = \frac{(2m_p^*)^{3/2}}{2\pi^2\hbar^3} \cdot (E_v - E)^{1/2}$$

- Easy to solve analytically.

# Review

- Qualitative picture so far:

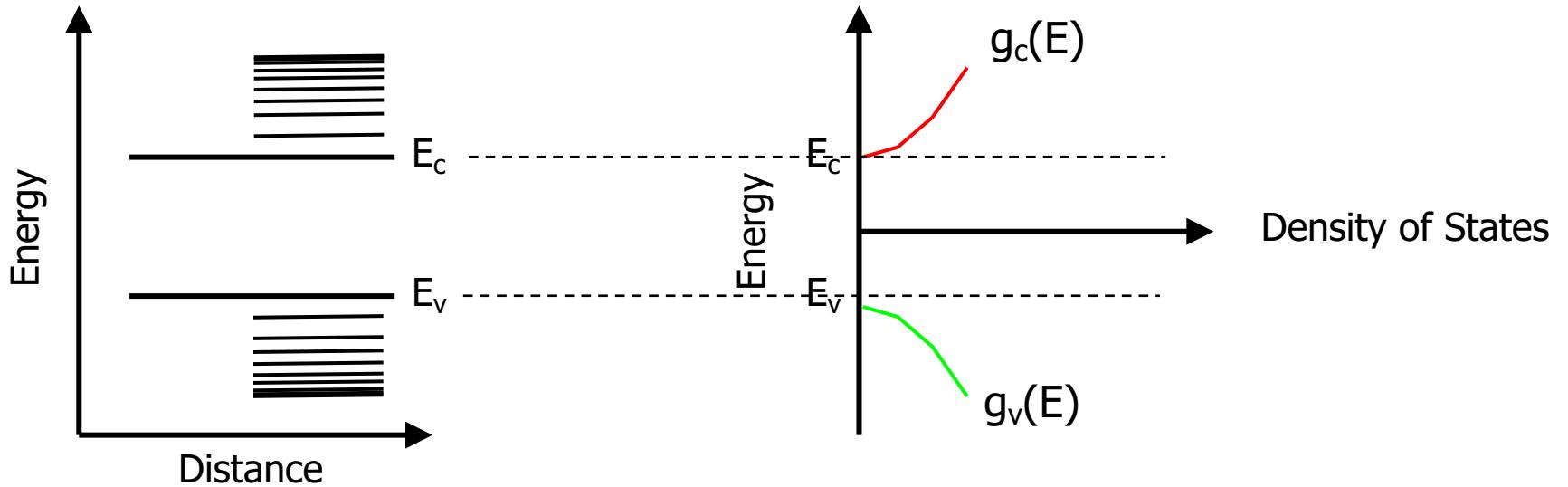


**Electrons and holes both tend to be in their lowest energy positions:**

- Electrons “roll downhill” in energy band diagrams
- Holes “float up” like bubbles in water

# Review

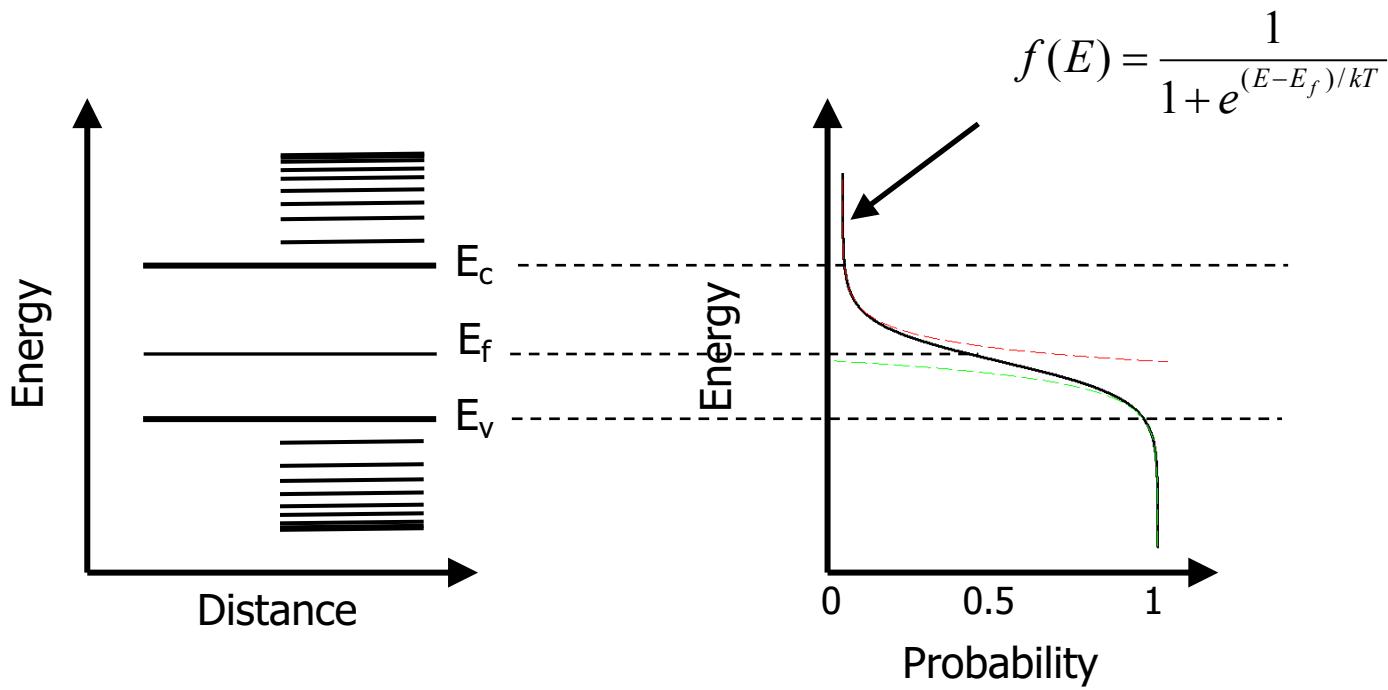
- Qualitative picture so far:



$$g_c(E) = \frac{\text{number of states in } \Delta E}{\Delta E * \text{volume}} \quad \left( \frac{\#}{eV \cdot cm^3} \right)$$

# Review

- Qualitative picture so far:



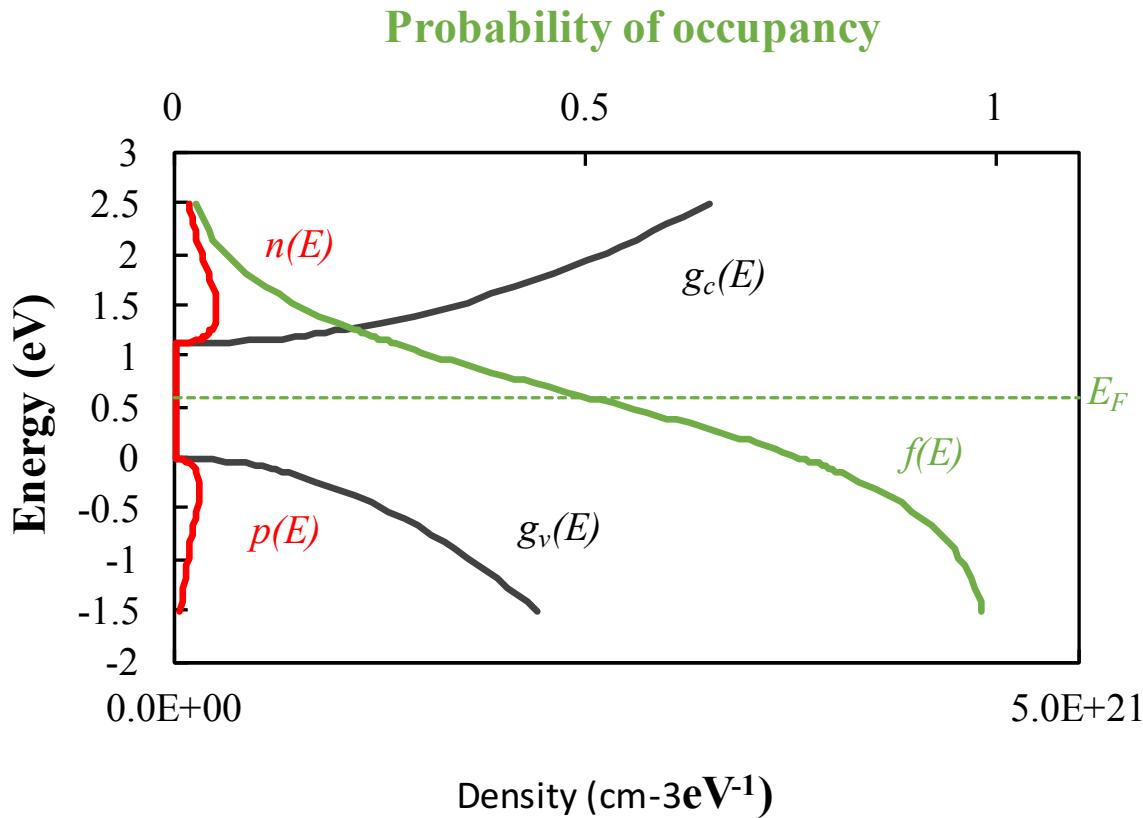
$f(E)$  = probability that an available state at energy  $E$  contains an electron

$E_f$  = Fermi energy

$f(E_f) = 0.5$

# Review

- Qualitative picture so far:



$$n = \int_{E_c}^{\infty} g_c(E) f(E) dE$$

$n(E) \ll g_c(E)$ : mostly empty states in the conduction band

$$p = \int_{-\infty}^{E_v} g_v(E) [1 - f(E)] dE$$

$p(E) \ll g_v(E)$ : mostly filled states in the valence band

# Review

- Qualitative picture so far:

**“Equilibrium”: the unperturbed state of a system**

- No external voltages, magnetic fields, etc.
- No other perturbing forces acting on the semiconductor
- Invariant with time
- The “rest” condition

# Review

- **Calculating the carrier concentration:**
  - Relations for electron and hole carrier densities.

$$n = N_c \cdot e^{(E_f - E_c)/kT} \quad \text{where } N_c = 2 \cdot \left( \frac{m_n^* kT}{2\pi\hbar^2} \right)^{3/2}$$

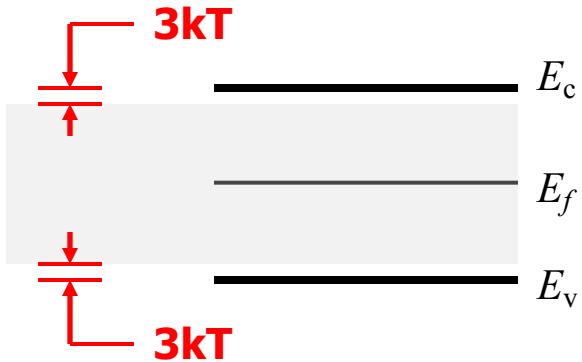
- **N<sub>c</sub> is called the effective conduction-band density of states.**
- 

$$p = N_v \cdot e^{(E_v - E_f)/kT} \quad \text{where } N_v = 2 \cdot \left( \frac{m_p^* kT}{2\pi\hbar^2} \right)^{3/2}$$

- **N<sub>v</sub> is called the effective valence-band density of states.**

# Review

- **Calculating the carrier concentration:**
  - Carrier density for electrons and holes:



$$n = N_c \cdot e^{(E_f - E_c)/kT}$$

Electrons

$$p = N_v \cdot e^{(E_v - E_f)/kT}$$

Holes

- **Expression valid for both intrinsic and extrinsic (doped) semiconductors, provided that:**

$$E_v + 3kT < E_f < E_c - 3kT$$

- A semiconductor that satisfies the above condition is considered to be non-degenerate.

# Review

- **Intrinsic carrier concentration:**
  - Define intrinsic carrier concentration,  $n_i$ :

$$n_i^2 \equiv n \cdot p$$

$$n_i^2 = np = N_v N_c \cdot e^{(E_f - E_c)/kT} \cdot e^{(E_v - E_f)/kT}$$

$$n_i = \sqrt{N_c N_v} \cdot e^{-E_g/2kT}$$

- **Intrinsic carrier concentration depends upon the band gap, temperature, and density of states.**

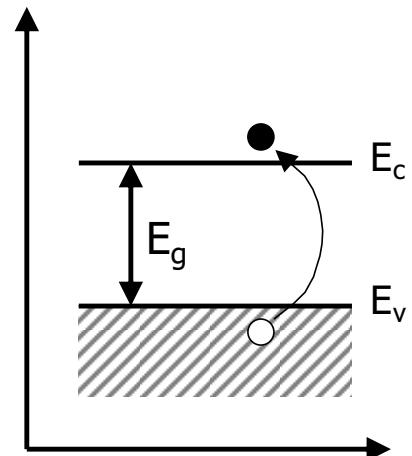
# Review

- Intrinsic concentration for common semiconductors:

	III A	IV A	V A	VI A
5	B	C	N	O
13	Al	Si	P	S
30	Zn	Ga	Ge	As
31	30	31	32	33
32	31	32	Ge	As
33	32	33	P	S
34	33	34	35	Se
48	Cd	In	Sn	Sb
49	48	49	50	51
50	49	50	In	Sb
51	50	51	Sn	Te
52	51	52	Sb	Te
80	Hg	Tl	Pb	Bi
81	80	81	82	83
82	81	82	Pb	Bi
83	82	83	84	Po
84	83	84	85	

$T = 300 \text{ K}$

Material	$E_g \text{ (eV)}$	$n_i \text{ (cm}^{-3}\text{)}$
Si	1.12	$1.0 \times 10^{10}$
Ge	0.66	$2.5 \times 10^{13}$
InSb	0.18	$2.0 \times 10^{16}$

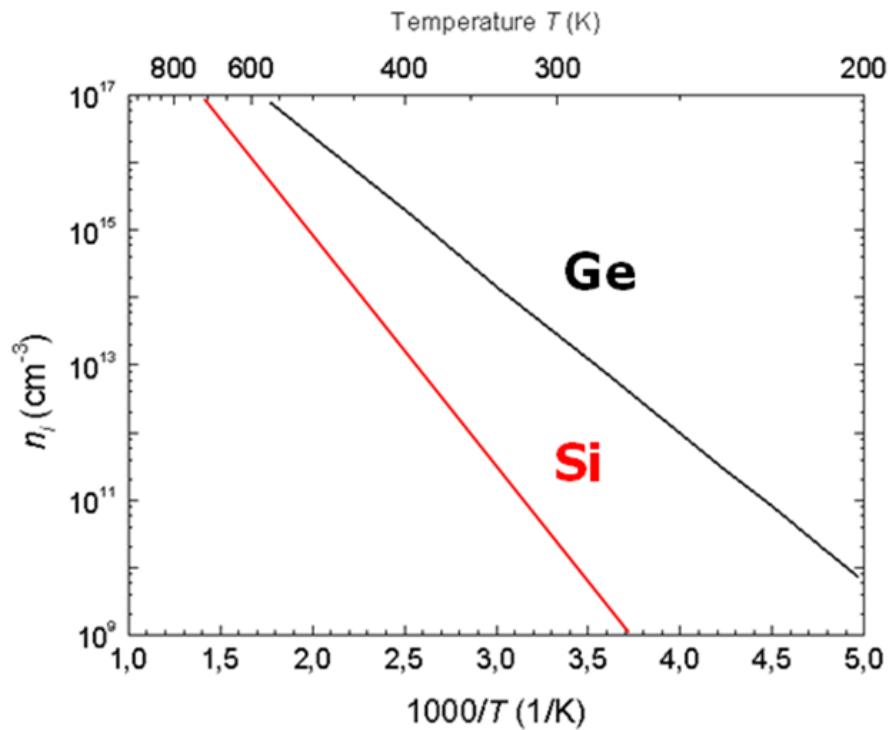


- $n_i$  increases exponentially with decreasing band gap.

# Review

- Intrinsic concentration for common semiconductors:

	IIIA	IVIA	VIA	VIIA
5	B	C	N	O
13	Al	Si	P	S
30	Zn	Ga	Ge	As
31				Se
48	Cd	In	Sn	Sb
49				Te
80	Hg	Tl	Pb	Bi
81				Po



- $n_i$  increases exponentially with increasing temperature.

# Review

- **Carrier concentration in doped semiconductors:**
  - Charge neutrality condition:

$$(p + N_d^+) - (n + N_a^-) = 0$$

The diagram shows the charge neutrality equation  $(p + N_d^+) - (n + N_a^-) = 0$ . Red annotations explain the terms: 'Holes' points to  $p$ , 'Ionized donors' points to  $N_d^+$ , 'Electrons' points to  $n$ , and 'Ionized acceptors' points to  $N_a^-$ . A red '+' sign is above  $N_d^+$  and a red '-' sign is above  $N_a^-$ .

- **Charge neutrality condition states that a bulk semiconductor in equilibrium is charge neutral, i.e., the net charge concentration is zero.**

# Review

- **Carrier concentration in doped semiconductors:**
  - Law of mass action:

$$n_i^2 = n \cdot p$$

- **Law of mass action states that the product of the electron and hole concentration in a non-degenerate semiconductor is a constant equal to the intrinsic carrier concentration.**

# Review

- **Carrier concentration in doped semiconductors:**

$$n = \frac{(N_d - N_a)}{2} + \sqrt{\left(\frac{N_d - N_a}{2}\right)^2 + n_i^2} \quad N_d > N_a$$

$$p = \frac{(N_a - N_d)}{2} + \sqrt{\left(\frac{N_a - N_d}{2}\right)^2 + n_i^2} \quad N_a > N_d$$

- **General solutions for electron and hole concentrations.**

# Review

- **Carrier concentration in doped semiconductors:**
  - N-type doped semiconductor:

$$n \approx N_d$$

- **Majority carriers**

$$N_d \gg n_i, N_d \gg N_a$$

$$p \approx \frac{n_i^2}{N_d}$$

- **Minority carriers**

- **In n-type semiconductors electrons are majority carriers and holes are minority carriers.**

# Review

- **Carrier concentration in doped semiconductors:**
  - P-type doped semiconductor:

$$n \approx \frac{n_i^2}{N_a}$$

- **Minority carriers**

$$N_a \gg n_i, N_a \gg N_d$$

$$p \approx N_a$$

- **Majority carriers**

- **In p-type semiconductors holes are majority carriers and electrons are minority carriers.**

# Review

- **Determining the Fermi level:**
  - Intrinsic semiconductor:

$$E_i = \frac{(E_v + E_c)}{2} + \frac{kT}{2} \cdot \ln\left(\frac{N_v}{N_c}\right)$$

- The Fermi level in an intrinsic semiconductor is called the **intrinsic Fermi level,  $E_i$ .**
- The intrinsic Fermi level is near mid-gap, but can be higher or lower depending upon the density of states.

# Review

- **Determining the Fermi level:**
  - Intrinsic semiconductor:

$$E_i = \frac{(E_v + E_c)}{2} + \frac{3kT}{4} \cdot \ln\left(\frac{m_p^*}{m_n^*}\right)$$

- The intrinsic Fermi level can also be expressed in terms of the electron and hole density-of-states effective masses.

# Review

- **Determining the Fermi level:**
  - Alternative expression for carrier densities:

$$n = n_i \cdot e^{(E_f - E_i)/kT}$$

$$p = n_i \cdot e^{(E_i - E_f)/kT}$$

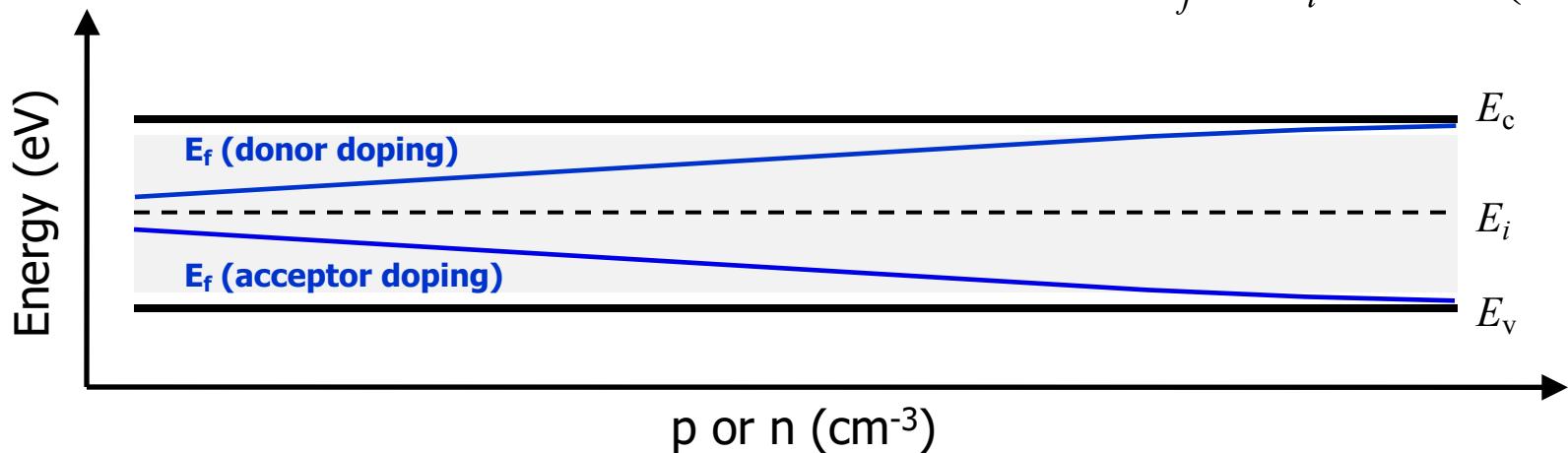
● Valid for non-degenerate semiconductors.

# Review

- **Determining the Fermi level:**
  - Doping dependence of Fermi level

$$E_f = E_c - kT \ln(N_c / n)$$

$$E_f = E_i + kT \ln(n / n_i)$$



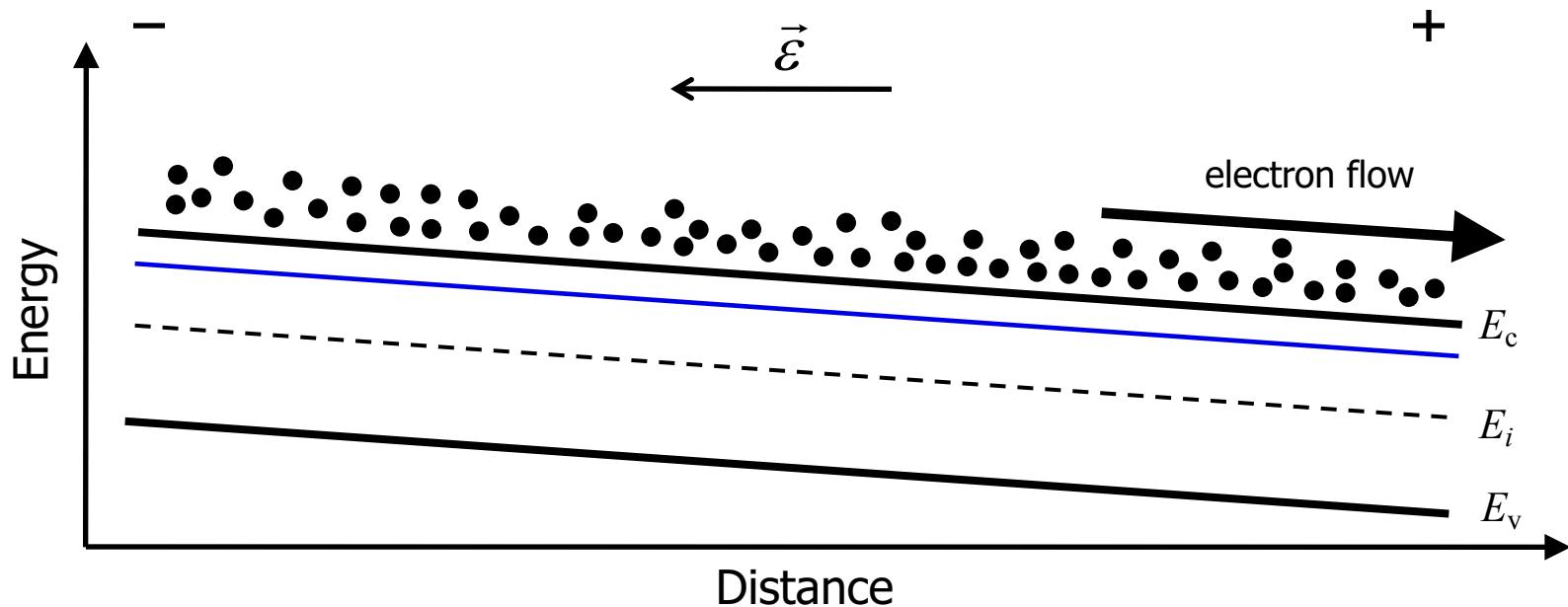
$$E_f = E_v + kT \ln(N_v / p)$$

$$E_f = E_i - kT \ln(p / n_i)$$

- **Fermi level moves away from mid-gap, toward the band edges with increasing doping.**

# Review

- **Drift:**
  - What does the band diagram look like?

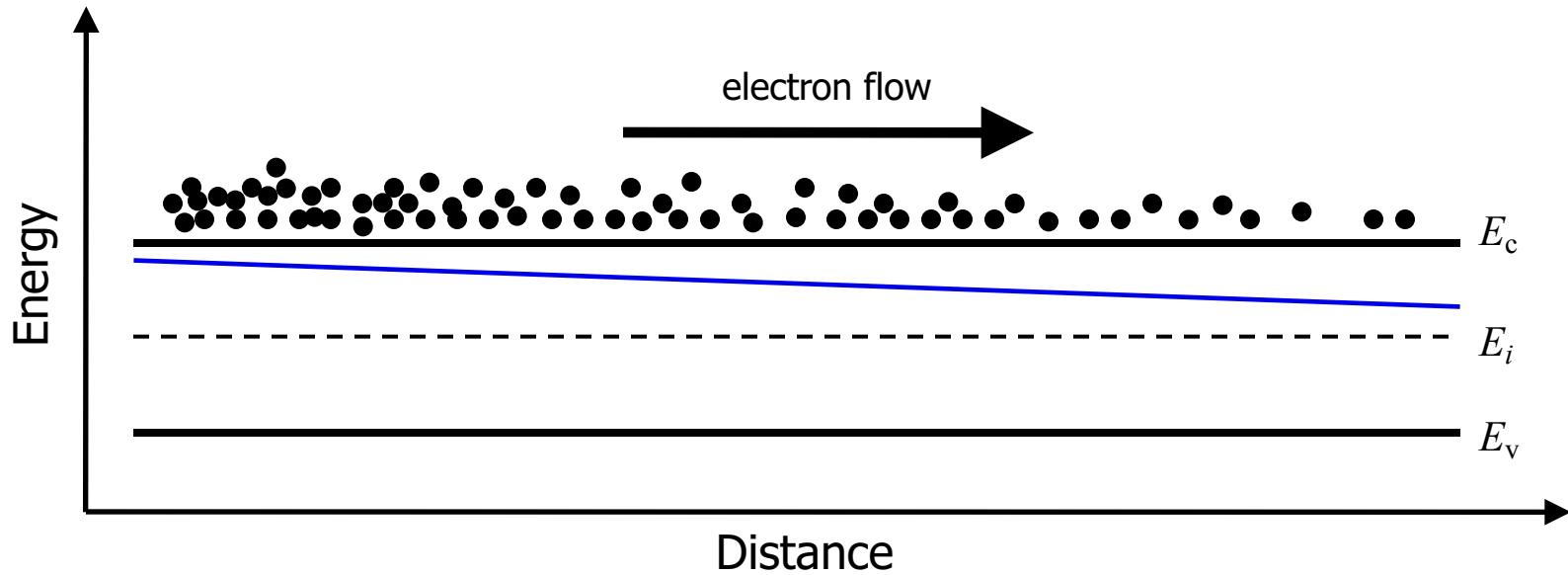


● Electrons flow “down hill” on an energy band diagram.

# Review

- **Diffusion:**

- What does the band diagram look like?

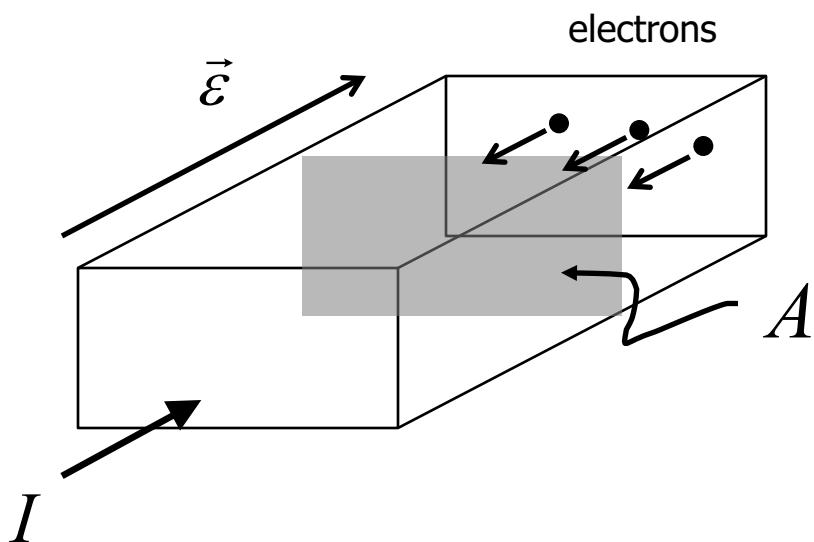


- **Electrons diffusion from regions of high concentration to regions of low concentration**

# Review

- **Drift current:**

- How much current is carried by carriers in a semiconductor?



$$I = J \cdot A$$

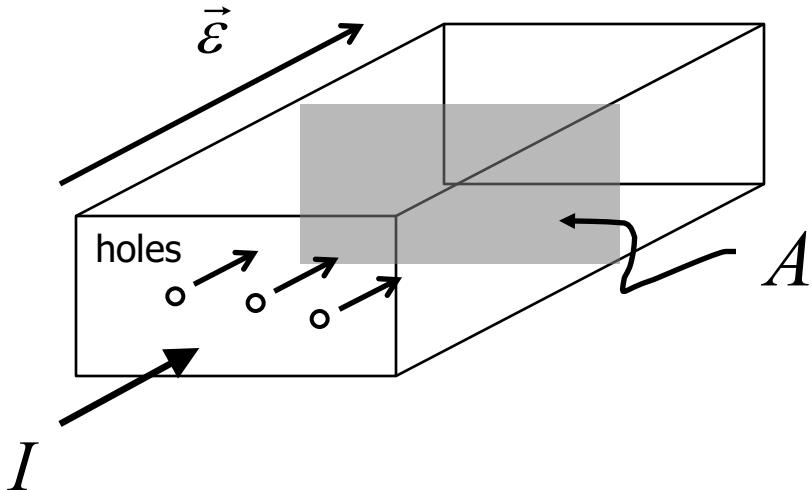
Annotations for the equation:

- Current: Points to the arrow  $I$ .
- Current density: Points to the arrow  $J$ .
- Area: Points to the label  $A$ .
- Current density: Points to the label  $J_{n|drift}$ .
- Carrier charge: Points to the label  $-qn$ .
- Drift velocity: Points to the label  $\vec{v}_d$ .
- Carrier concentration: Points to the label  $n$ .

# Review

- **Drift current:**

- How much current is carried by carriers in a semiconductor?



$$I = J \cdot A$$

Annotations for the equation:

- Current: Points to the symbol  $I$ .
- Current density: Points to the symbol  $J$ .
- Area: Points to the symbol  $A$ .
- Current density: Points to the symbol  $\vec{J}_{p|drift}$ .
- Carrier charge: Points to the symbol  $qp$ .
- Drift velocity: Points to the symbol  $\vec{v}_d$ .
- Carrier concentration: Points to the symbol  $n$ .

# Review

- **Drift:**

- Concept of mobility:

## Electrons

$$\vec{v}_d = -\mu_n \cdot \vec{\mathcal{E}}$$

$$\mu_n = \frac{q\langle\tau_n\rangle}{m_n^*}$$

## Holes

$$\vec{v}_d = \mu_p \cdot \vec{\mathcal{E}}$$

$$\mu_p = \frac{q\langle\tau_p\rangle}{m_p^*}$$

- Mobility is defined as the ratio of velocity to electric field,
- Mobility increases with decreasing effective mass and longer times between scattering events.

# Review

- Room-temperature bulk mobilities for common semiconductors:

	IIIA	IVA	VIA	VIA	
	5 B	6 C	7 N	8 O	
IB	13 Al	14 Si	15 P	16 S	
	30 Zn	31 Ga	32 Ge	33 As	34 Se
	48 Cd	49 In	50 Sn	51 Sb	52 Te
	80 Hg	81 Tl	82 Pb	83 Bi	84 Po

Material	$\mu_n$	$\mu_p$
Si	1400	450
Ge	3900	1900
GaAs	8500	400
GaSb	3000	1000
InAs	40000	500
InSb	77000	850

Units = [cm/s] / [V/cm] = cm<sup>2</sup>/Vs

- Mobility tends to be higher in materials with lighter effective mass,
- Mobility tends to be higher for electrons than holes.

# Review

- **Drift:**

- Ohm's law:

$$\vec{J}_{drift} = \sigma \vec{\mathcal{E}}$$

**Ohm's Law**

$$\sigma = q(p\mu_p + n\mu_n)$$

**Conductivity**

$$\rho = \frac{1}{\sigma} = \frac{1}{q(p\mu_p + n\mu_n)}$$

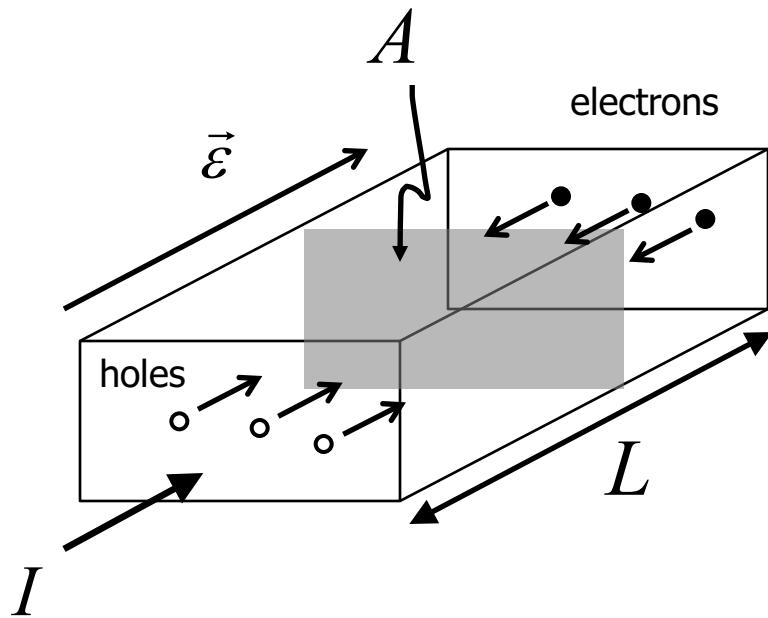
**Resistivity**

# Review

- **Drift:**

- Resistance:

$$\rho = \frac{1}{\sigma} = \frac{1}{q(p\mu_p + n\mu_n)}$$



$$R = \rho \frac{L}{A}$$

# Review

- **Diffusion:**

- Diffusion current density:

$$\vec{J}_{diff} = -D \cdot \nabla \rho$$

$$J_{diff} = -D \cdot \frac{d\rho}{dx}$$

In one-dimension.

$$J_{diff} = qD_n \cdot \frac{dn}{dx} - qD_p \cdot \frac{dp}{dx}$$

Electron and hole contributions.

# Review

- **Diffusion:**

- Einstein relation:

$$\frac{D_n}{\mu_n} = \frac{kT}{q}$$

- Relates diffusion coefficient to mobility.
- Valid for non-degenerate semiconductors.

- “D over mu” = “kT over q”

# Review

- **Diffusion:**

- Total current is sum of drift and diffusion components:

$$\vec{J}_{tot} = \vec{J}_{diff} + \vec{J}_{drift}$$

$$\vec{J}_{tot} = q \left[ D_n \frac{dn}{dx} - D_p \frac{dp}{dx} + (n\mu_n + p\mu_p) \vec{\mathcal{E}}_x \right]$$

# Review

- **Generation/Recombination**

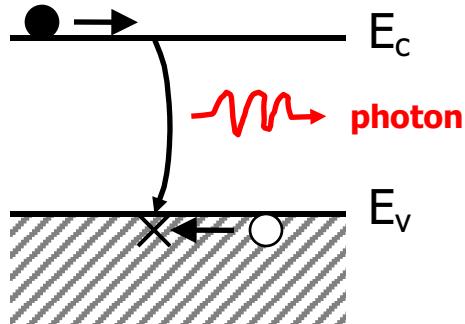
- Generation is a process whereby electrons and holes are created.

- Recombination is a process whereby electrons and holes are annihilated or destroyed.

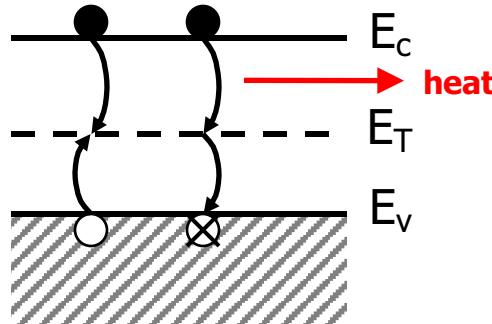
# Review

- **Generation / Recombination**

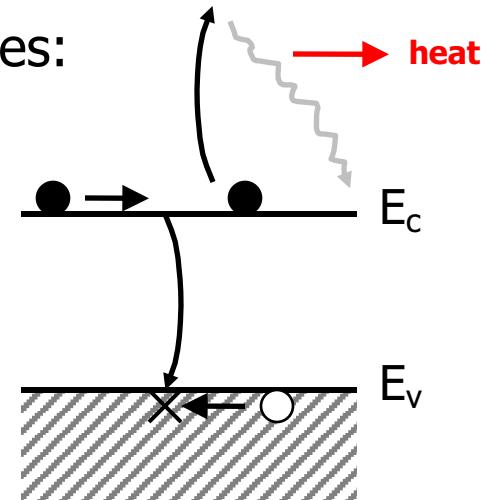
- Different generation/recombination processes:



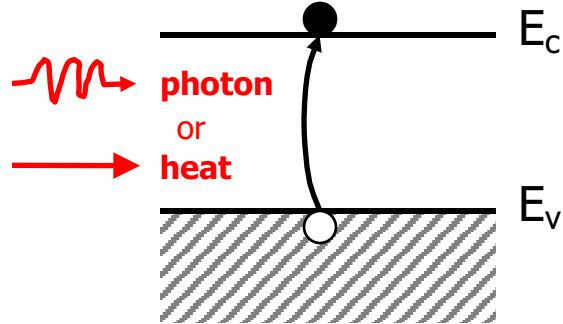
Band to band  
recombination



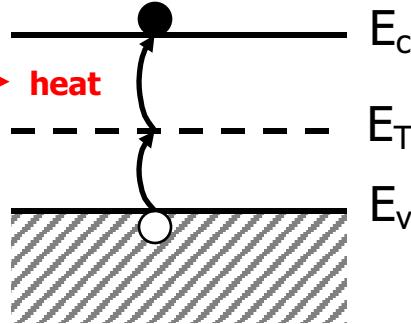
R-G center  
recombination



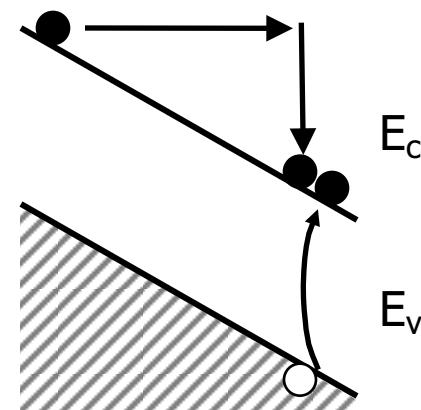
Auger  
recombination



Band to band  
generation



R-G center  
generation



Impact ionization

# Review

- **Generation/Recombination**
  - Low-level injection:

N-type

$$n \approx n_0$$

$$\Delta p \ll n_0$$

P-type

$$\Delta n \ll p_0$$

$$p \approx p_0$$

- **Conditions for low-level injection.**

# Review

- **Generation/Recombination**

- Rate equations:

## N-type

$$\frac{\partial p}{\partial t} \Big|_{R-G} = - \frac{\Delta p}{\tau_p}$$

$$\tau_p = 1/c_p N_t$$

## P-type

$$\frac{\partial n}{\partial t} \Big|_{R-G} = - \frac{\Delta n}{\tau_n}$$

$$\tau_n = 1/c_n N_t$$

- **Minority carrier lifetime is proportional to the trap density.**

# Review

- **Equations of state:**

- Continuity equations:

$$\frac{\partial n}{\partial t} = -\frac{1}{q} \nabla \bullet \vec{J}_N + \frac{\partial n}{\partial t} \Big|_{thermal R-G} + \frac{\partial n}{\partial t} \Big|_{other}$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \bullet \vec{J}_P + \frac{\partial p}{\partial t} \Big|_{thermal R-G} + \frac{\partial p}{\partial t} \Big|_{other}$$

- **Time rate of change in carrier concentration equal to divergence in current plus additional R-G rate.**

# Review

- **Equations of state:**
  - Minority carrier diffusion equations:

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

- **The above equations can be utilized to solve numerous problems necessary for semiconductor device analysis.**

# Review

- **Equations of state:**
  - Definition of diffusion lengths:

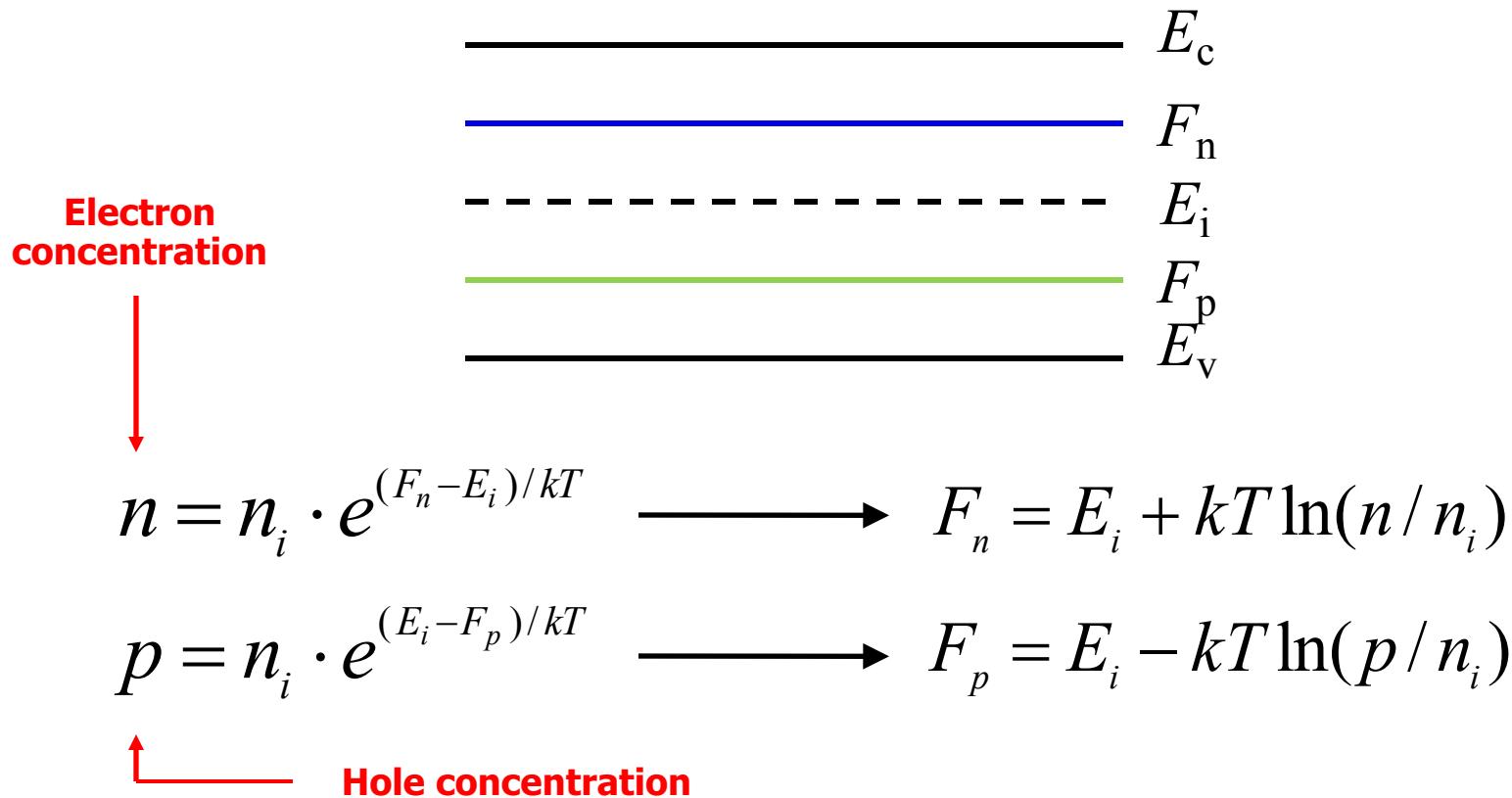
$$L_n \equiv \sqrt{D_n \tau_n}$$

$$L_p \equiv \sqrt{D_p \tau_p}$$

- Diffusion length can be defined as the square root of the minority diffusion coefficient times the minority carrier lifetime.

# Review

- **Quasi Fermi levels:**
  - Revisiting the Fermi level:



- In general, need to define quasi-Fermi levels.

# Review

- **Quasi Fermi levels:**
  - Defining current in terms of quasi-Fermi levels:

$$\vec{J}_n = \mu_n n \nabla F_n$$

$$\vec{J}_p = \mu_p p \nabla F_p$$

- **A gradient in quasi-Fermi-level inside a semiconductor is an indicator that current is flowing.**

# Review

- **Basic electrostatic equations:**

$$\frac{\partial \mathcal{E}_x}{\partial x} = + \frac{\rho}{\kappa_S \epsilon_0}$$

$$\mathcal{E}_x(0^-) = \mathcal{E}_x(0^+)$$

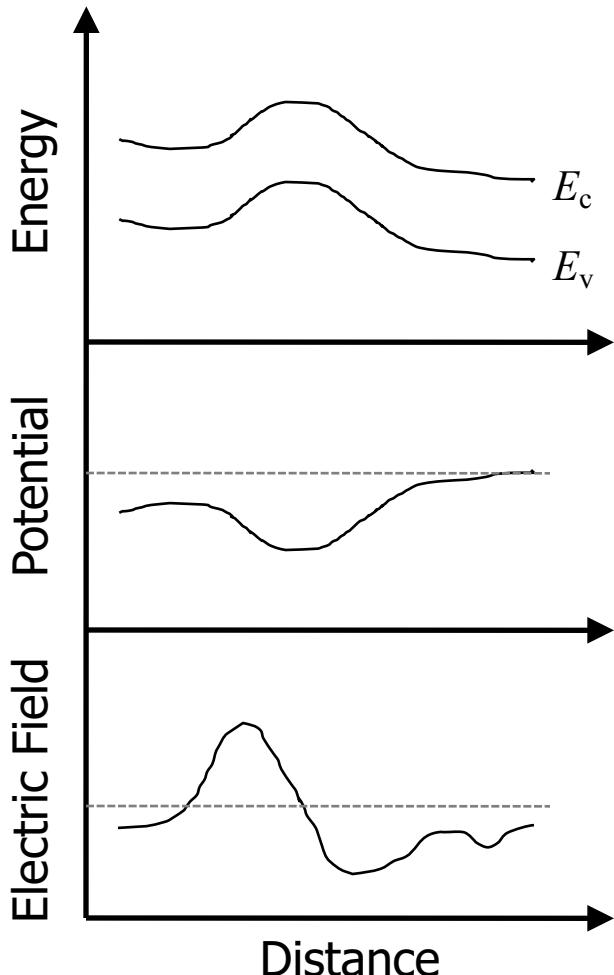
$$\mathcal{E}_x = - \frac{\partial V}{\partial x}$$

$$E_i = -qV + E_{ref}$$

- Poisson's equation using electric field.
- Continuity of normal electric flux density.
- Definition of electric field .
- Relation between voltage and energy.

# Review

- Basic electrostatic equations:



$$V = -\frac{1}{q} (E_c - E_{ref})$$

$$\mathcal{E}_x = \frac{1}{q} \cdot \frac{dE_c}{dx} = \frac{1}{q} \cdot \frac{dE_v}{dx} = \frac{1}{q} \cdot \frac{dE_i}{dx}$$

# Review

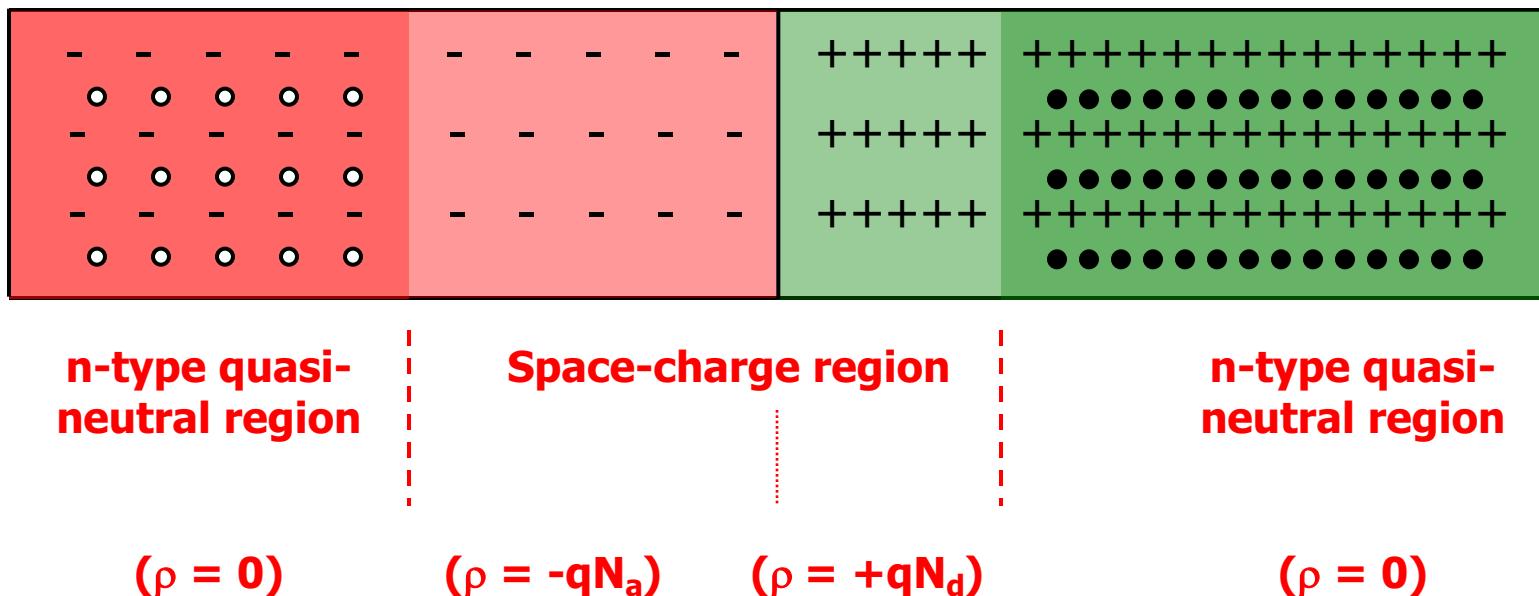
- Depletion approximation:

- Assume:

(1) There is an abrupt boundary between the depletion region and the quasi-neutral region,

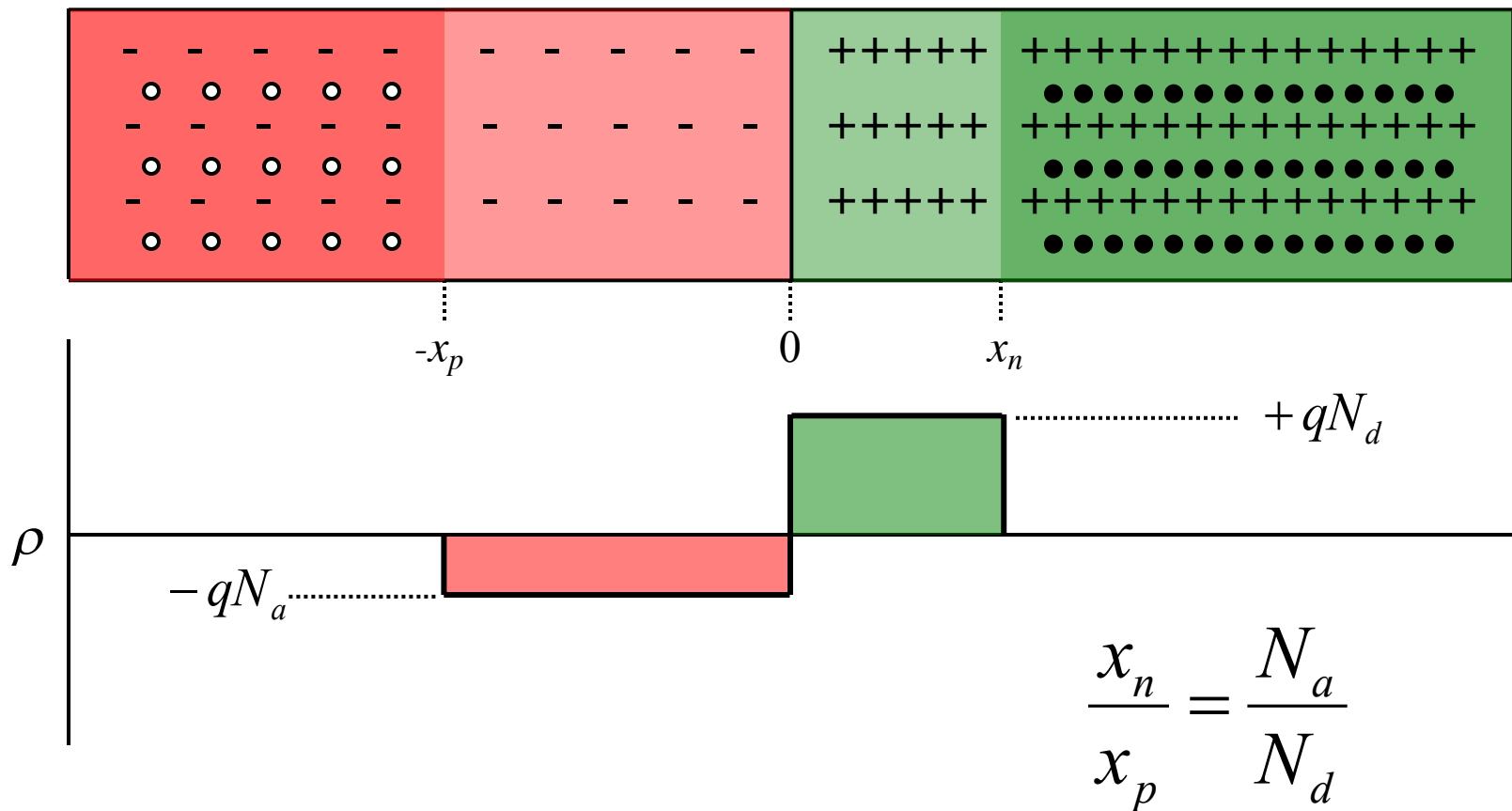


(2) There are no free carriers in the depletion region.



# Review

- **Band profiles and built-in voltage:**
  - Depletion layer width on either side of the junction is proportional to the doping density on the other side.

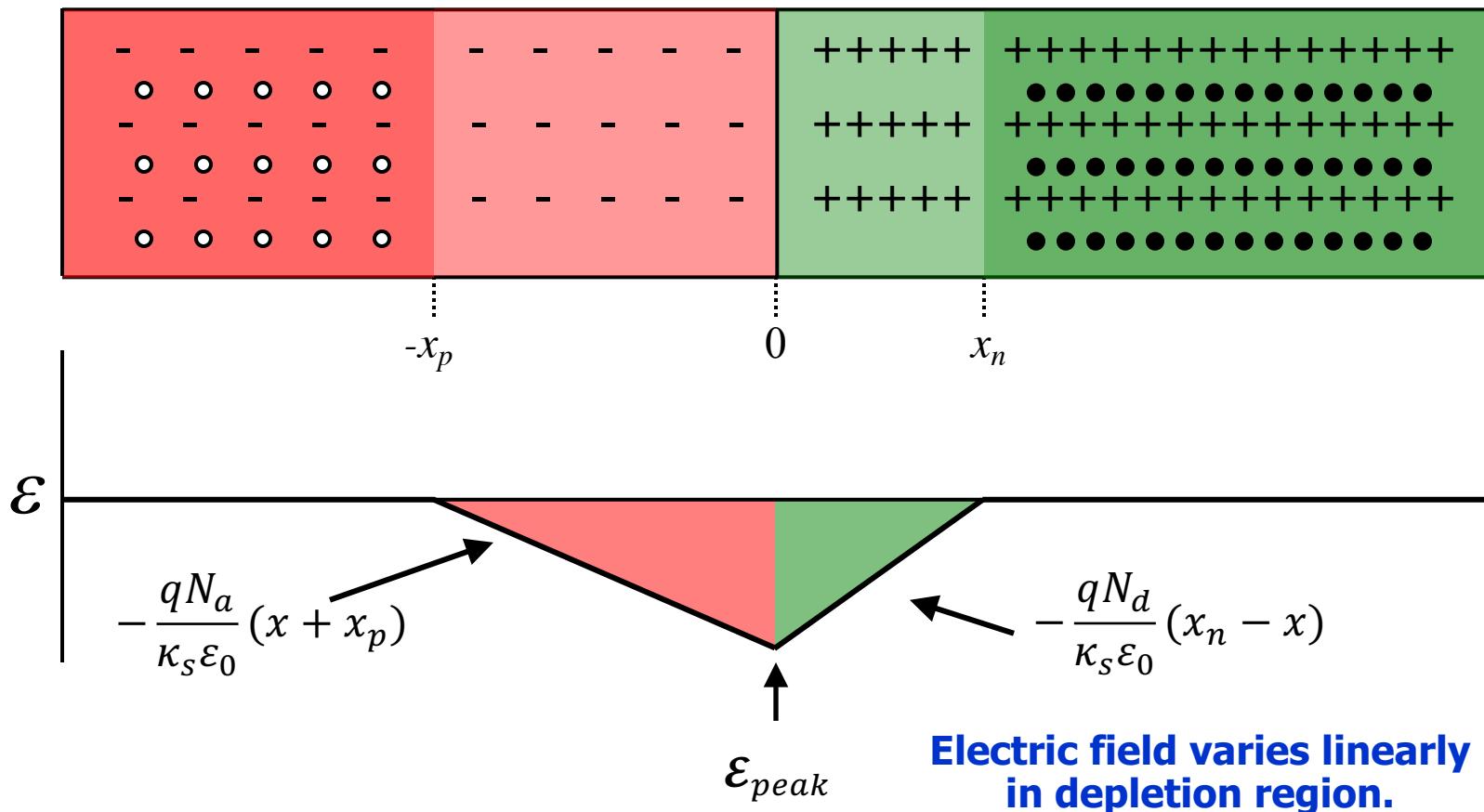


# Review

- **Band profiles and built-in voltage:**

- Peak electric field occurs at junction interface.

$$\mathcal{E}_{peak} = \sqrt{\frac{2qV_{bi}}{\kappa_S \epsilon_0} \cdot \left( \frac{N_a N_d}{N_a + N_d} \right)}$$

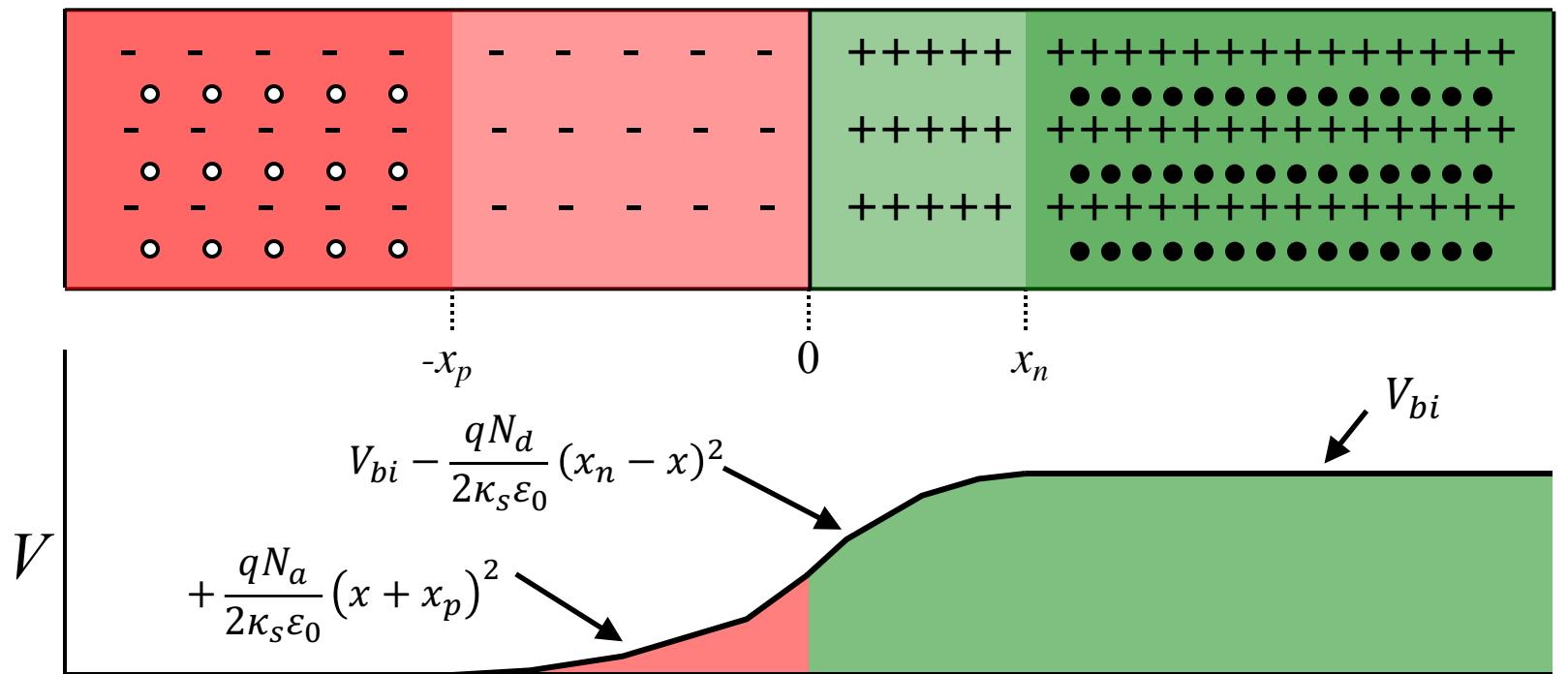


# Review

- **Band profiles and built-in voltage:**

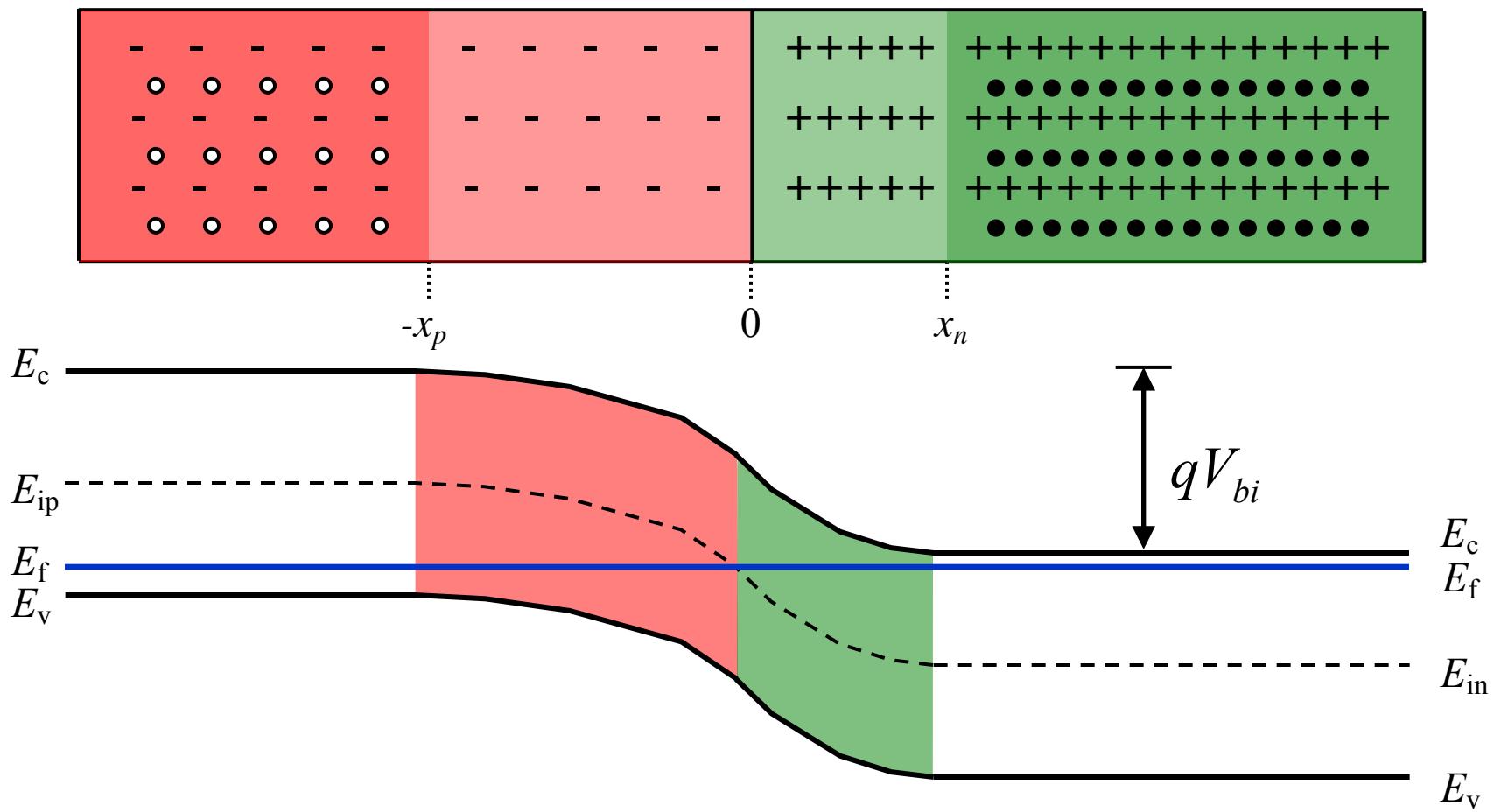
$$V_{bi} = \frac{kT}{q} \cdot \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

- Built-in voltage depends logarithmically on doping density.



# Review

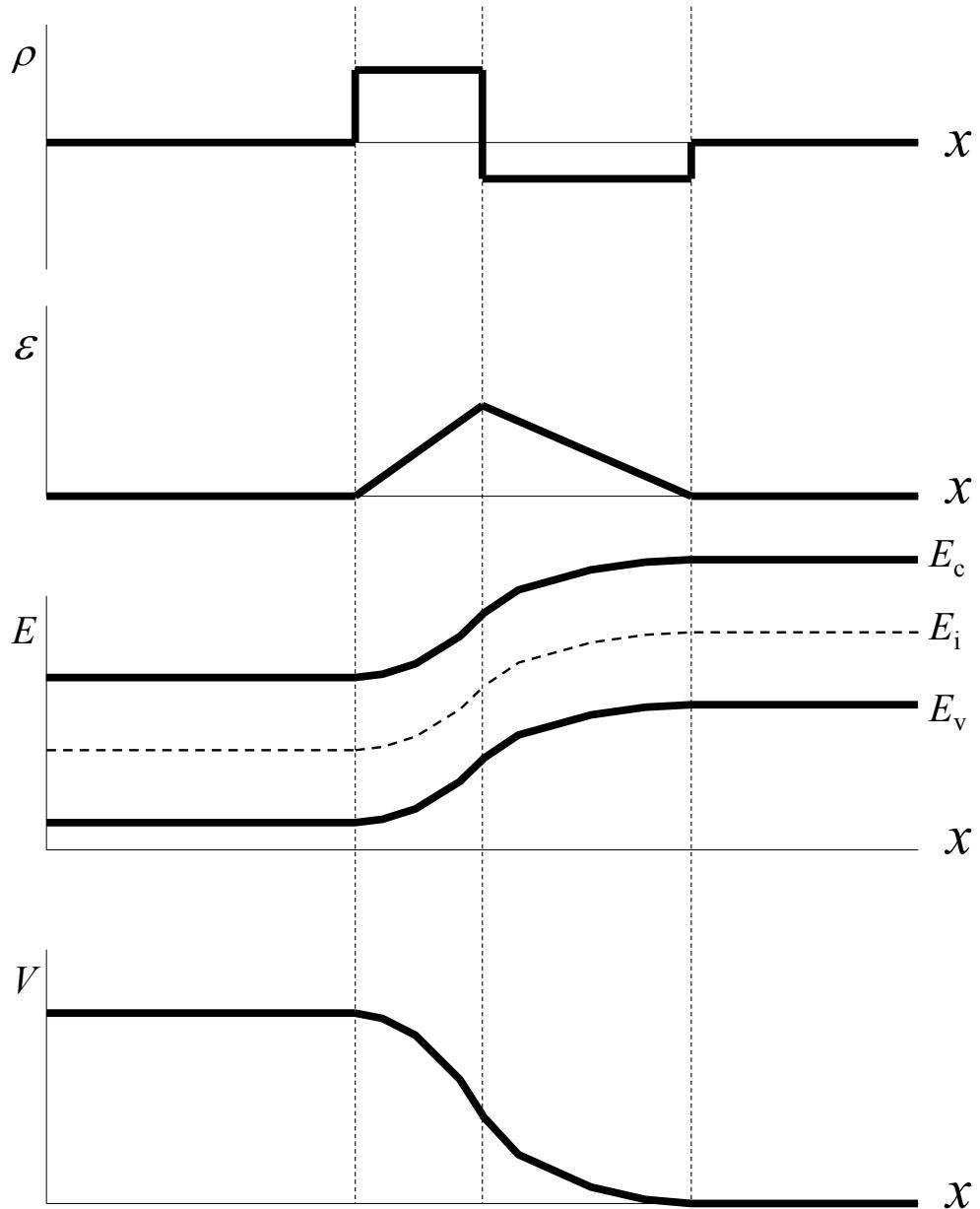
- PN junction band profile and built-in voltage:
  - Energy bands vary quadratically in depletion region,  $qV_{bi}$  is on the order of the energy gap,  $E_g$ .



# Review

- **Technique:**

- 1) **Draw the charge density vs. distance profile.**
- 2) **Integrate charge density in positive x-direction to get electric field.**
- 3) **Integrate electric field in positive x-direction to get any energy band.**
- 4) **Add other energy bands.**
- 5) **Flip any energy band upside down to get electrostatic potential.**



# Review

- Depletion widths on either side of PN junction:

$$x_p = \sqrt{\frac{2\kappa_S \epsilon_0 V_{bi}}{qN_a} \cdot \frac{N_d}{N_d + N_a}}$$

$$x_n = \sqrt{\frac{2\kappa_S \epsilon_0 V_{bi}}{qN_d} \cdot \frac{N_a}{N_d + N_a}}$$

$$\frac{x_n}{x_p} = \frac{N_a}{N_d}$$

# Review

- **Total depletion layer width equal to sum of depletion layer widths on either side of the junction:**

$$W = x_n + x_p$$

$$W = \sqrt{\frac{2\kappa_s \epsilon_0 V_{bi}}{q} \cdot \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}$$

# Review

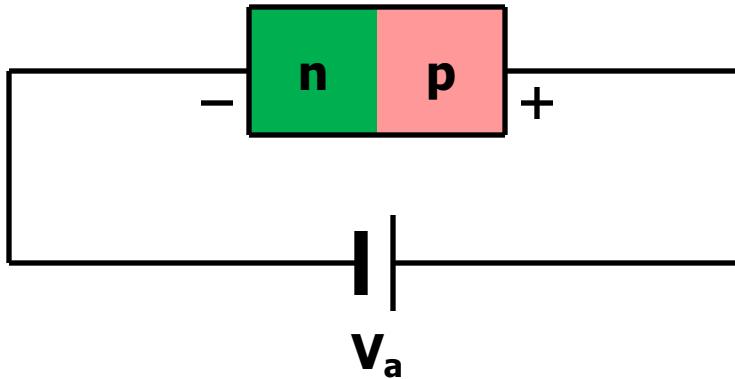
- **Effect of bias voltage (no current):**

$$W = \sqrt{\frac{2\kappa_S \epsilon_0 (V_{bi} - V_a)}{q} \cdot \left( \frac{1}{N_a} + \frac{1}{N_d} \right)}$$

- **Depletion width decreases for positive bias voltages and increases for negative bias voltages.**

# Review

- PN junction I-V characteristics:

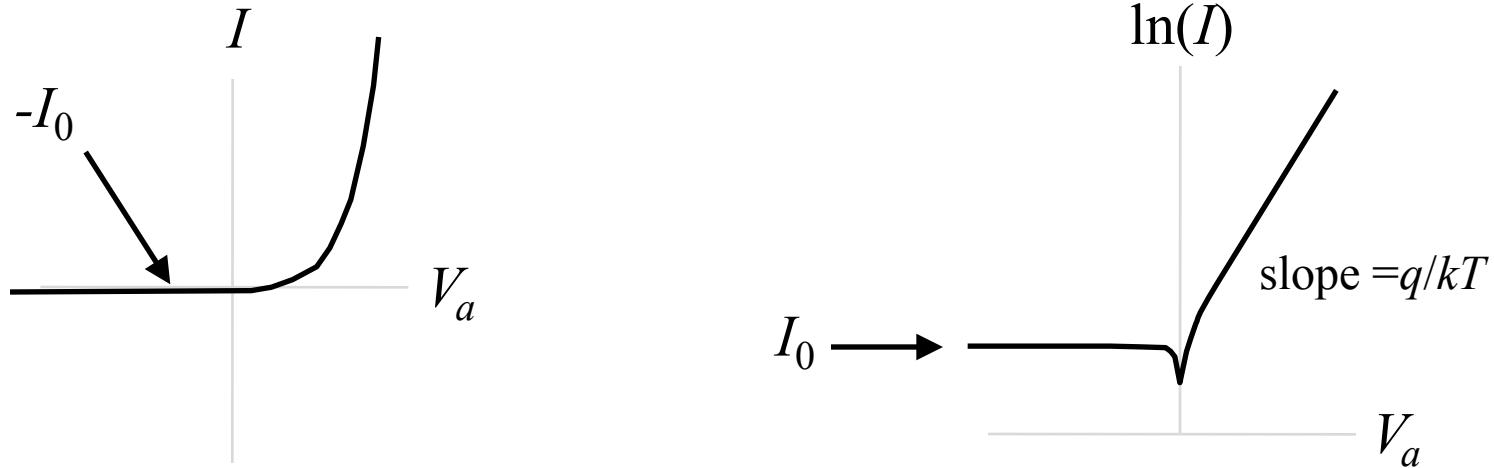


$$I = I_0 \cdot \left( e^{qV_a/kT} - 1 \right)$$

- The current in a PN diode is equal to an exponential for  $V_a > 0$ , and is constant for  $V_a < 0$ .

# Review

- PN junction I-V characteristics:



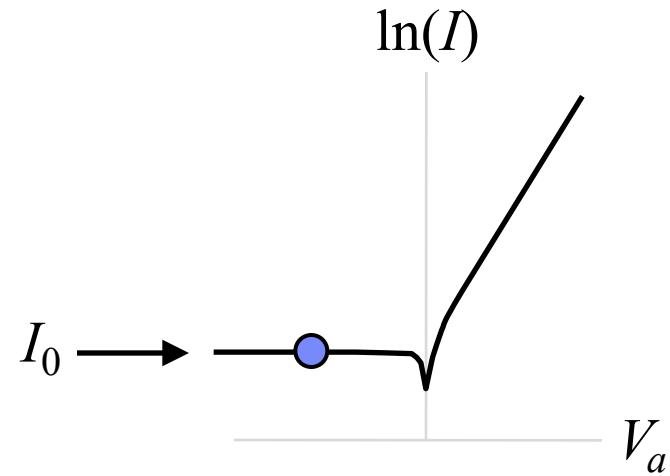
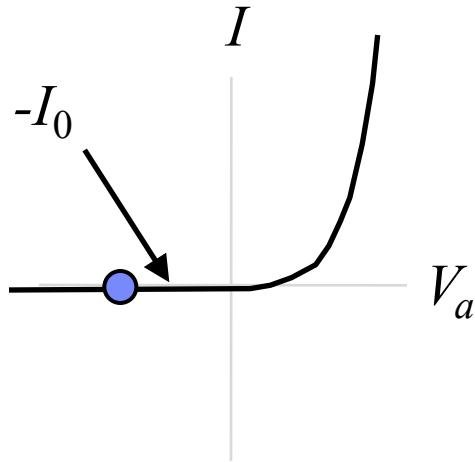
$$I = I_0 \cdot \left( e^{qV_a/kT} - 1 \right)$$

# Review

- PN diode regions of operation:

- Reverse bias:

$$I = I_0 \cdot \left( e^{qV_a/kT} - 1 \right)$$



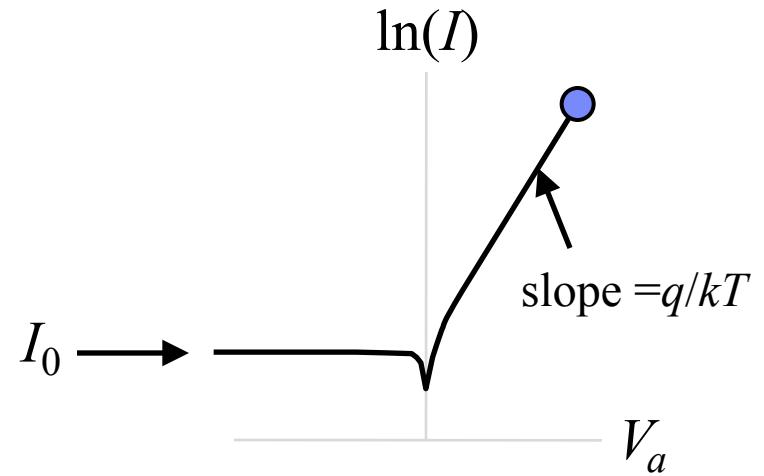
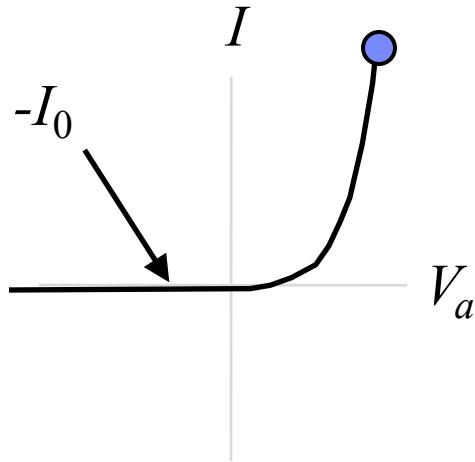
$$I \approx -I_0$$

# Review

- PN diode regions of operation:

- Forward bias:

$$I = I_0 \cdot \left( e^{qV_a/kT} - 1 \right)$$



$$I \approx I_0 \cdot e^{qV_a/kT}$$

# Review

- **Law of the junction:**

$$n_p(-x_p) = \frac{n_i^2}{N_a} e^{(qV_f/kT)}$$

$$p_n(x_n) = \frac{n_i^2}{N_d} e^{(qV_f/kT)}$$

# Review

- **Excess minority carrier concentrations at junction boundaries:**

$$\Delta n_p(-x_p) = \frac{n_i^2}{N_a} (e^{qV_f/kT} - 1)$$

$$\Delta p_n(x_n) = \frac{n_i^2}{N_d} (e^{qV_f/kT} - 1)$$

# Review

- **Law of the junction:**

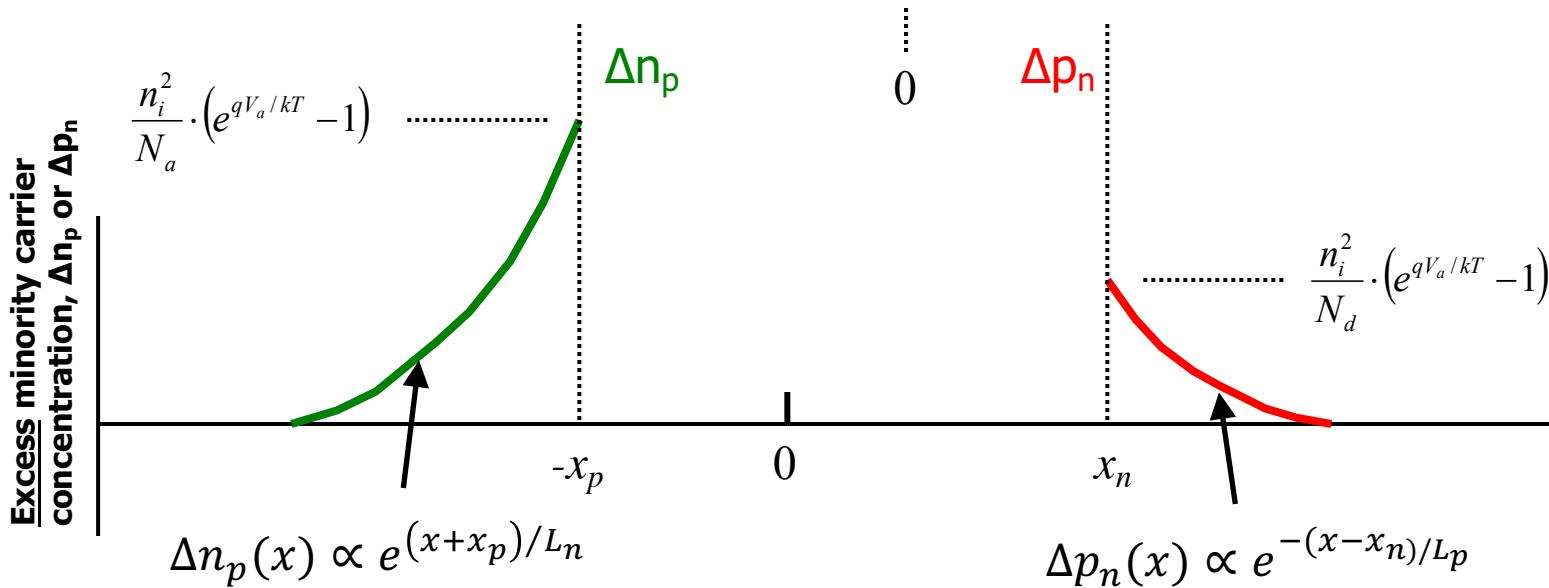
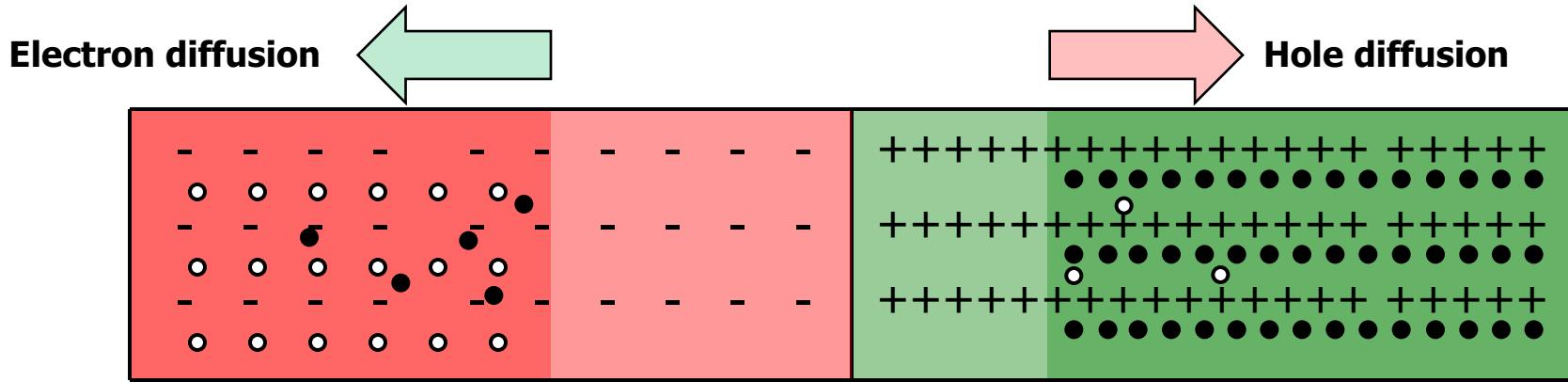
$$pn = n_i^2 e^{+qV_a/kT}$$

- **Valid anywhere within the depletion region of a pn junction,**
- **Assumes no generation / recombination within depletion region.**

# Review

- **Minority carrier injection profile:**

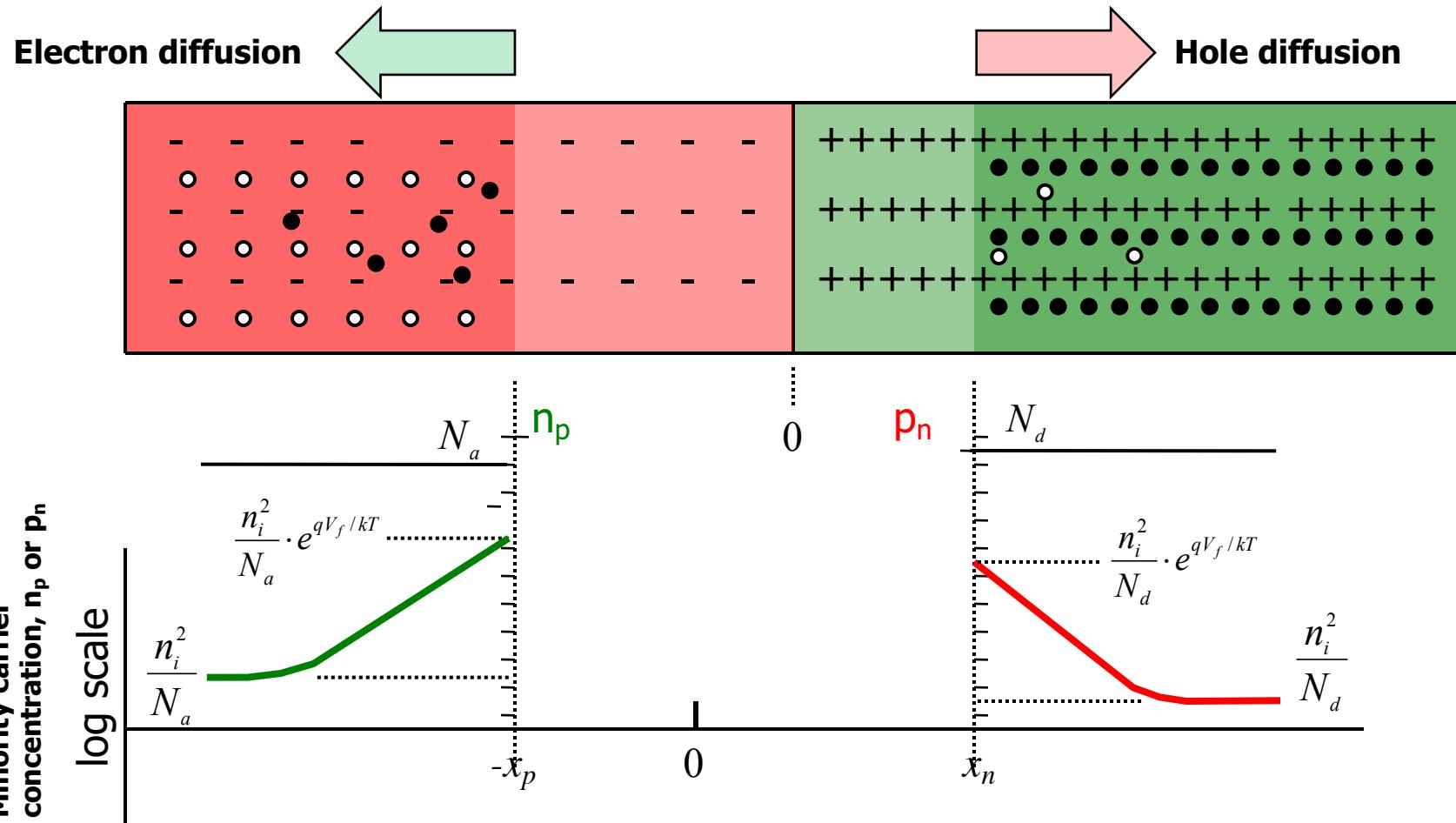
- PN junction under forward bias:



# Review

- Minority carrier injection profile:

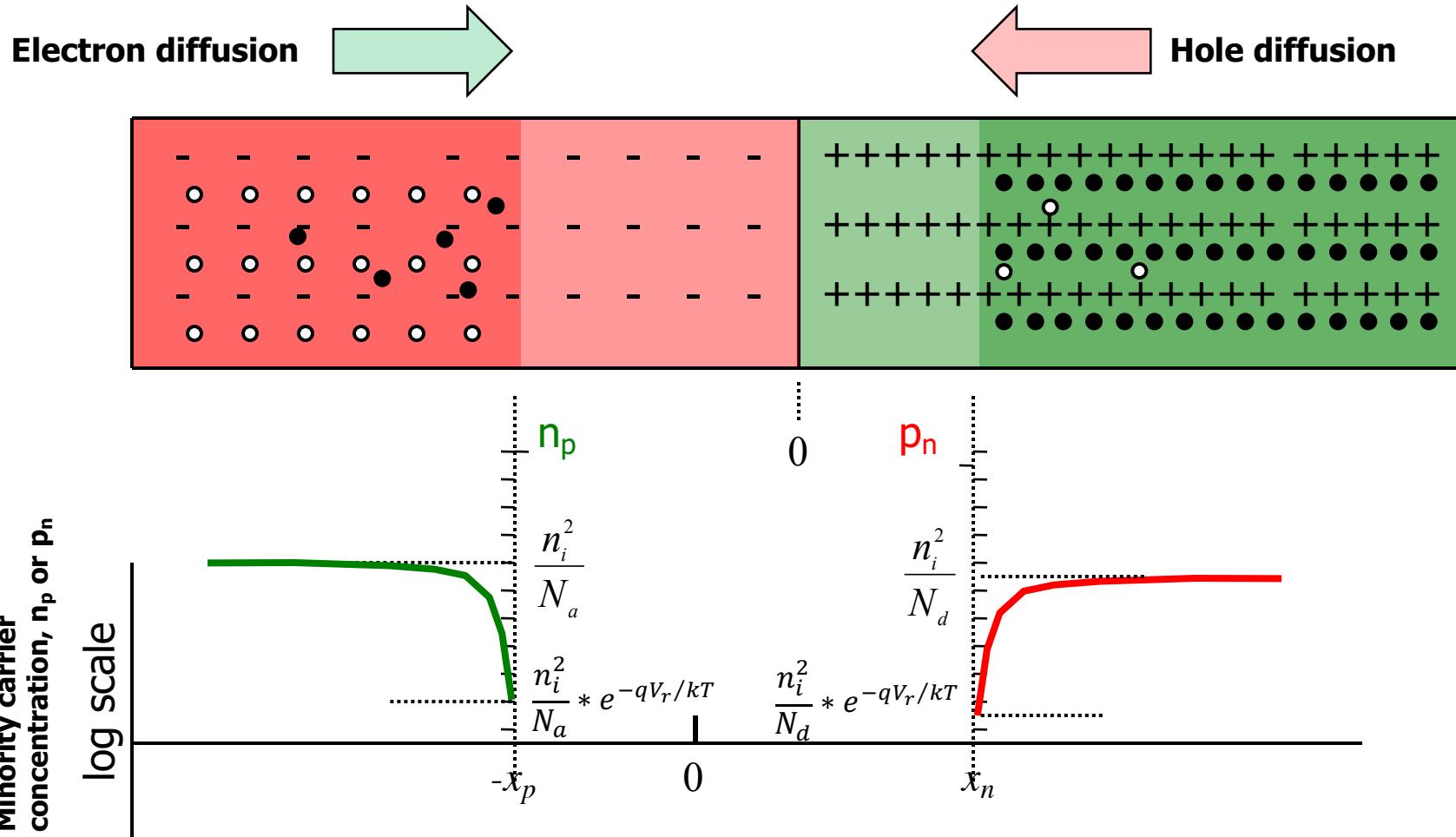
- PN junction under forward bias:



# Review

- **Minority carrier injection profile:**

- PN junction under **reverse bias**:



# Review

- Ideal diode equation:

$$J = J_0 \cdot \left( e^{qV_a / kT} - 1 \right)$$

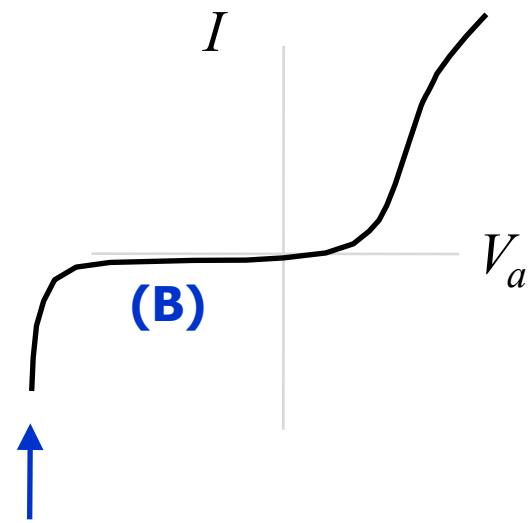
$$J_0 = q \cdot n_i^2 \cdot \left( \frac{D_p}{L_p} \cdot \frac{1}{N_d} + \frac{D_n}{L_n} \cdot \frac{1}{N_a} \right)$$

- Reverse leakage current density proportional to intrinsic carrier concentration squared and inverse of lower of the two doping densities.

# Review

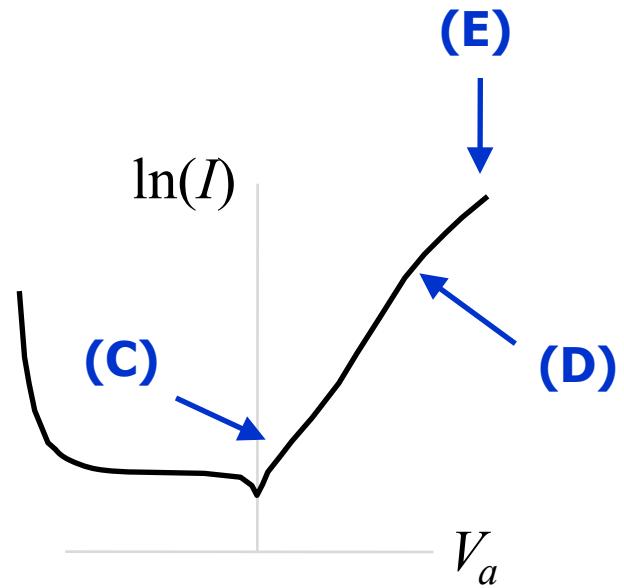
- Overview of non-ideal diode effects:

Realistic



(A)

- (A) Breakdown
- (B) Thermal generation
- (C) Thermal recombination
- (D) High-level injection
- (E) Series resistance



(C)

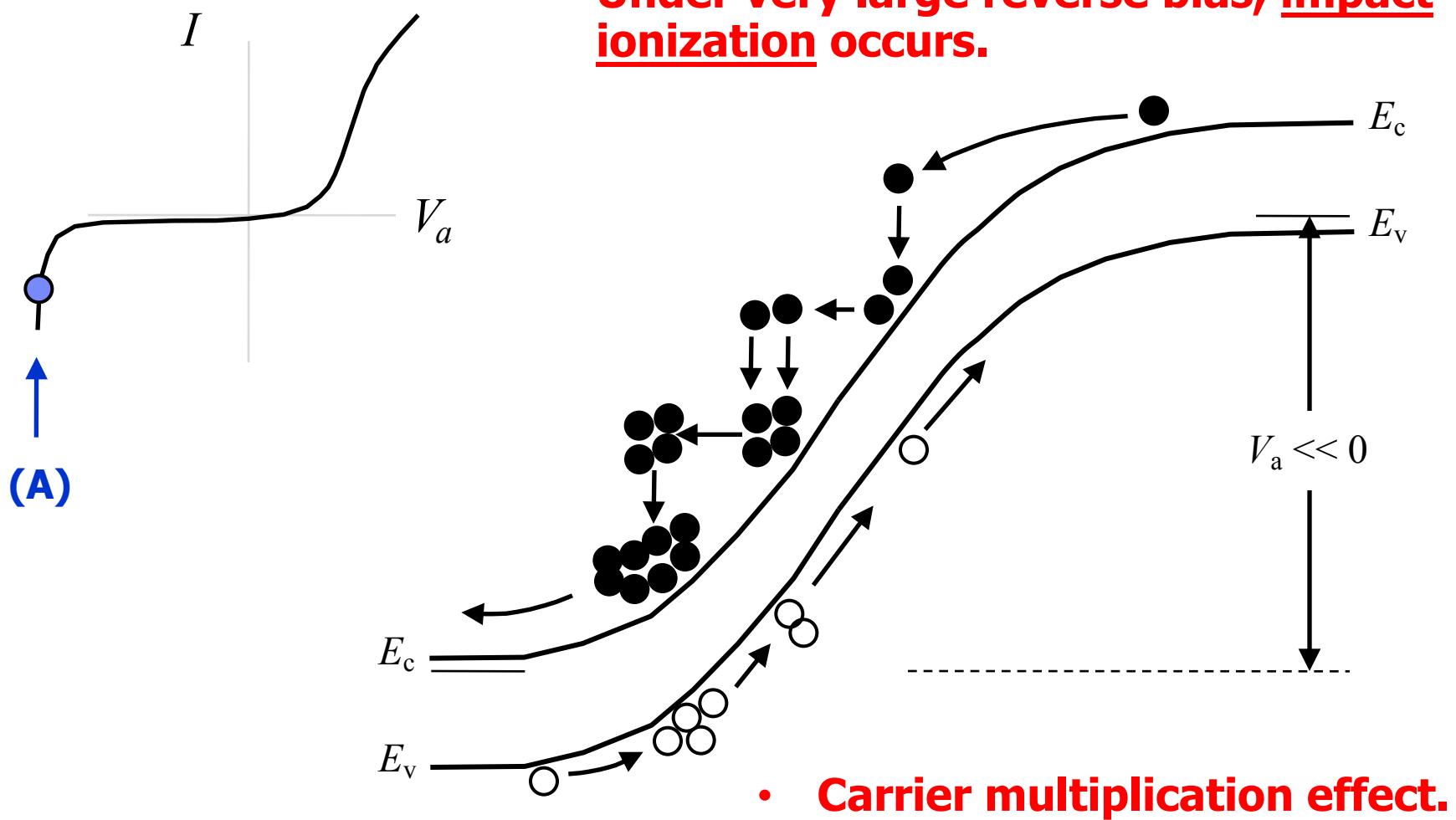
(D)

# Review

- **(A) Avalanche breakdown:**

- Occurs at very large negative bias voltages:

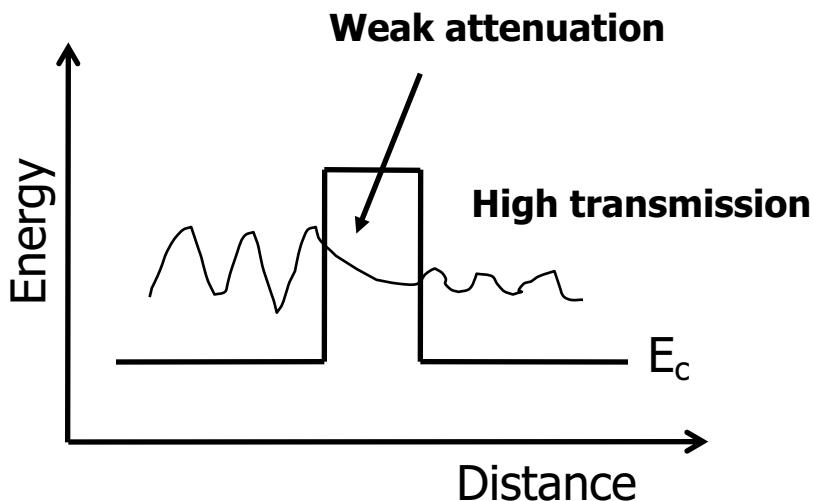
- Under very large reverse bias, impact ionization occurs.



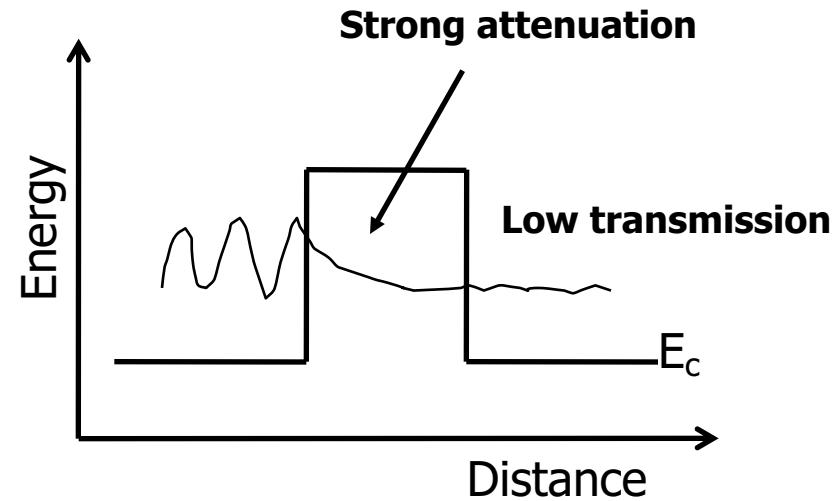
# Review

- (A) Electron wave penetration through a barrier:

Narrow barrier



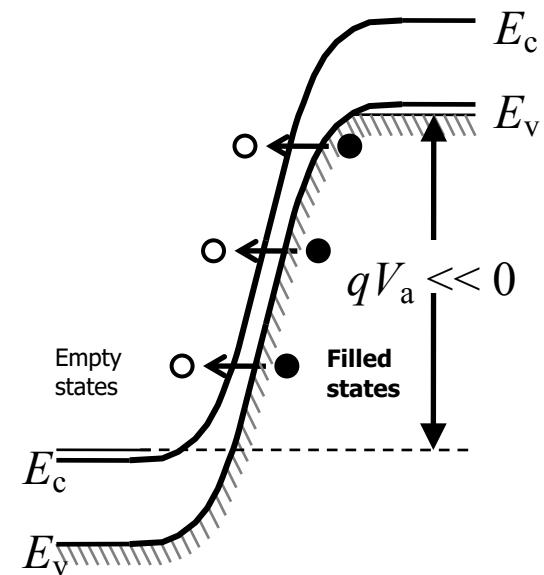
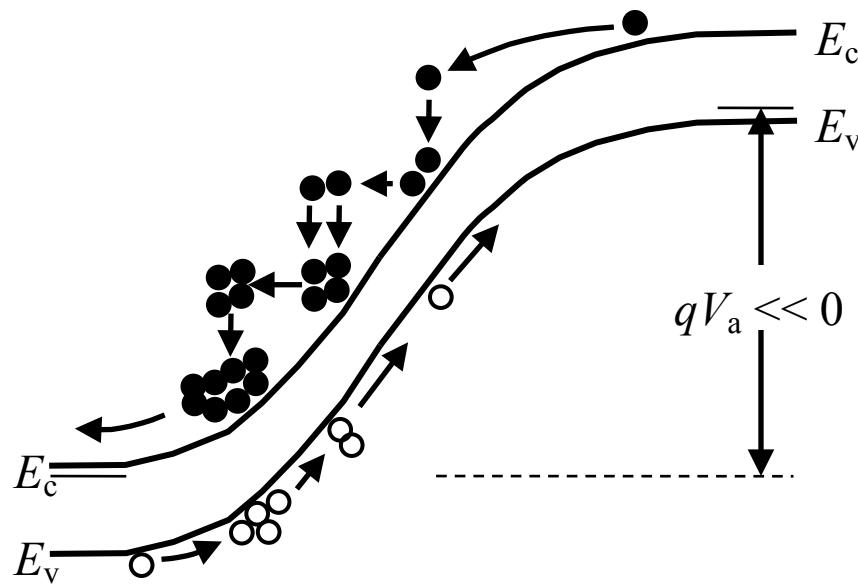
Wide barrier



- **Tunneling current is highest when energy barrier is narrow.**

# Review

- **(A) Breakdown comparison:**
  - Comparison between Zener and avalanche breakdown:



- Both require high electric fields, but avalanching requires high voltages and large depletion width in order to allow multiplication effect to build up.
- Zener breakdown requires very small depletion widths to allow tunneling to occur → usually requires degenerate doping levels.