3.1

(a) See Fig. 3.10(a).

(b) _____●_____ E_c

_____ E_v

(c) _____ E_c

_____ E_v

○⇕ $E_G/4$

(d) _____●_____ E_c

Light ⌇⌇➤

_____○_____ E_v

(e) _____●_____ E_c

Thermal energy ➤

_____○_____ E_v

(f) See Fig. 3.15(a).

(g) See Fig. 3.15(b).

(h) See Fig. 3.15(e)

<u>3.2</u>

(a) Given: $\upsilon_d = 10^3$ cm/sec

$\quad\quad\quad\quad\; \mathcal{E} = \Delta V/L = 2V/cm$

$\quad\quad\quad\quad\; \mu_p = \upsilon_d/\mathcal{E} = \mathbf{500\ cm^2/V\text{-}sec}$

(b) (i) Lattice scattering

(ii) Ionized impurity scattering

(See the *Relationship to Scattering* discussion in Subsection 3.1.3.)

(c) $\mu_{intrinsic}$ is **higher than** $\mu_{heavily\ doped}$

<u>Reason</u>: In intrinsic material the scattering is due exclusively to lattice scattering. In heavily doped materials, ionized impurity scattering is also important. The more scattering there is, the lower the mobility.

(d) Given N_{D1} and $N_{A2} \gg n_i$, we know from Eqs. (3.8) that

$$\rho = \frac{1}{q\mu_n N_{D1}} \quad \text{...}n\text{-type wafer 1;} \quad\quad \rho = \frac{1}{q\mu_p N_{A2}} \quad \text{...}p\text{-type wafer 2}$$

In most semiconductors including GaAs, μ_n is greater than μ_p for a given doping and system temperature. Since we are given $N_{D1} = N_{A2}$, taking the wafer temperatures to be the same, and with $\mu_n > \mu_p$, we conclude from the above equations that $\rho(\textbf{wafer 2}) > \rho(\textbf{wafer 1})$. Note that the conclusion here is consistent with Fig. 3.8(b).

(e) $D_N = (kT/q)\mu_n = (0.0259)(1300) = \mathbf{33.7\ cm^2/sec}$

(f) $\Delta p \ll n_0, \quad n \cong n_0 \quad\quad$ ...if n-type

$\quad \Delta n \ll p_0, \quad p \cong p_0 \quad\quad$ ...if p-type

(g) **R-G center**

(h) **Increase.** Per Eq. (3.33a), $\tau_p = 1/c_p N_T$. Since N_T decreased after processing, τ_p increased.

3.3

The MATLAB program script required to complete part (b) of Exercise 3.1 is listed below. Except for labeling and elimination of the $10^{15}/cm^3$ curves which lie almost on top of the $10^{14}/cm^3$ curves, the output is identical to the solid line portions of Fig. 3.7. (A slightly modified version of the program was in fact used to produce Fig. 3.7.)

MATLAB program script ...

```
%Mobility versus Temperature (Si)

%Initialization
clear;   close

T=logspace(log10(200),log10(500));

s=menu('Choose the carrier type','Electrons','Holes');

%Fit Parameters
NDref=1.3e17;   NAref=2.35e17;
μnmin=92;   μpmin=54.3;
μn0=1268;   μp0=406.9;
an=0.91;   ap=0.88;
TNref=2.4;  Tμmin=-0.57;
Tμn0=-2.33;  Tμp0=-2.23;
Ta=-0.146;

%Mobility Calculation
for i=14:18,
   N=10.^i;
   if s==1,
      %Electrons
      NDrefT=NDref*(T./300).^TNref;
      μnminT=μnmin.*(T./300).^Tμmin;
      μn0T=μn0.*(T./300).^Tμn0;
      anT=an.*(T./300).^Ta;
      μn=μnminT+μn0T./(1+(N./NDrefT).^anT);
      y=μn;
   else
      %Holes
      NArefT=NAref*(T./300).^TNref;
      μpminT=μpmin.*(T./300).^Tμmin;
      μp0T=μp0.*(T./300).^Tμp0;
      apT=ap.*(T./300).^Ta;
      μp=μpminT+μp0T./(1+(N./NArefT).^apT);
      y=μp;
   end
   %Plotting results
   if i==14,
```

```
    if s==1,
        %Electrons
        loglog(T,y); grid;
            axis([1e2,1e3,1e2,1e4]);
        xlabel('T (K)');
        ylabel('Electron mobility (cm2/V-sec)');
        text(220,3300,'ND=1.0e14');
        text(220,230,'ND=1.0e18');
        text(600,6500,'SILICON');
    else
        %Holes
        loglog(T,y); grid;
        axis([1e2,1e3,1e2,1e4]);
        xlabel('T (K)');
        ylabel('Hole mobility (cm2/V-sec)');
        text(205,1150,'NA=1.0e14');
        text(205,125,'NA=1.0e18');
        text(600,6500,'SILICON');
        end
    elseif i>=15,
        hold on
        loglog(T,y);
    else
    end
end
```

3.4

(a) (Optional reading assignment.)

(b) MATLAB program script ...

```
%Mobility vs. T(K) Data
%Determining the power-dependence exponent
clear;  close

%Input data and take log of data
T=150:10:290;
μn=[6757 5910 5216 4619 4209 3743 3306 2978 2675 2415 2185 ...
    1985 1805 1646 1501];
x=log(T);
y=log(μn);

%Perform least-squares fit to data
c=polyfit(x,y,1);
a=c(2);
b=-c(1);
```
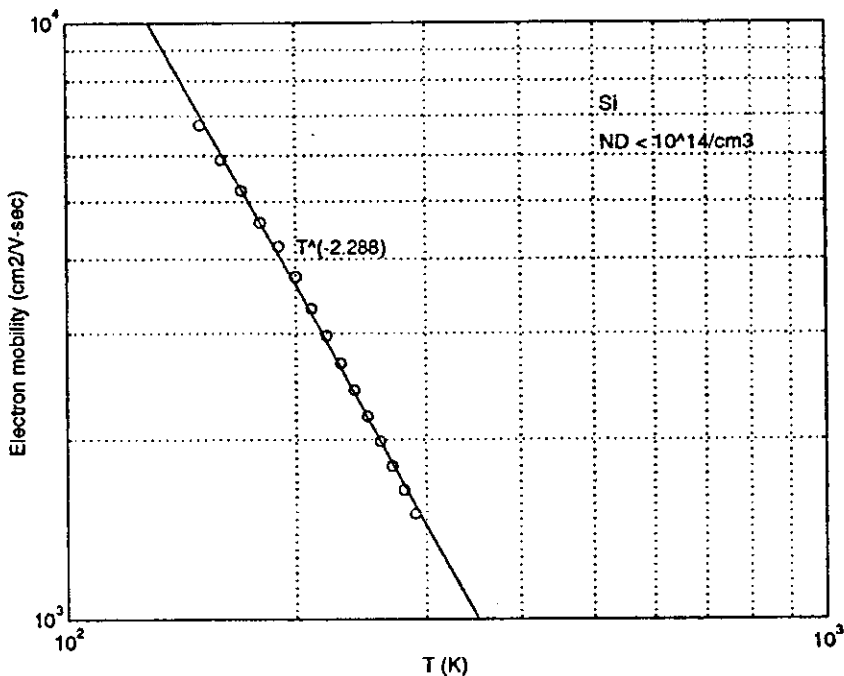
```
Tfit=120:10:350;
yfit=exp(a).*Tfit.^(-b);

%Plot data, draw fit line, note fit value
loglog(T,μn,'o',Tfit,yfit); grid
axis([1e2,1e3,1e3,1e4]);
xlabel('T (K)'); ylabel('Electron mobility (cm2/V-sec)');
text(200,4200,['T^(-',num2str(b),')']);
text(510,6300,'ND < 10^14/cm3');
text(510,7300,'Si');
```


(c) The agreement between experiment and the text plot is impressive. The experimentall.
determined **b** fit value is almost identical to the **b**-value noted on the text plot, 2.29 versu:
2.3. The magnitude of the measured mobility is also in close agreement with the
Fig. 3.7(a) plot. A lowly-doped sample was employed, of course, so that lattice scatterir
would dominate and a power-law dependence would be observed.

3 - 5

3.5

(a) In an intrinsic semiconductor $n = p = n_i$ and

$$\rho = \frac{1}{q(\mu_n + \mu_p)n_i}$$

The $\rho_{intrinsic}$ values recorded in the following table were computed using the maximum mobility values deduced from Fig. 3.5 and the n_i values read from Fig. 2.20 or the tables adjacent to Fig. 2.20.

Semi	μ_n (cm²/V-sec)	μ_p (cm²/V-sec)	n_i (cm⁻³)	$\rho_{intrinsic}$ (Ω-cm)	ρ_{max} (Ω-cm)
Ge	~4000	~1900	2.5×10^{13}	42.4	45.3
Si	1358	461	10^{10}	3.44×10^5	3.95×10^5
GaAs	~8000	~400	2.25×10^6	3.31×10^8	7.76×10^8

(b) If the np product relationship is used to eliminate p in terms of n in the general resistivity relationship (Eq. 3.7), one obtains

$$\rho = \frac{1}{q(\mu_n n + \mu_p n_i^2/n)}$$

Following the standard procedure for determining the maximum of a function, we can find the maximum resistivity by differentiating ρ with respect to n and setting the result equal to zero. In performing the differentiation, we assume $d\mu_n/dn = 0$ and $d\mu_p/dn = 0$. This is reasonable since the maximum resistivity is expected to occur at very low carrier concentrations (low dopings) where the mobilities are essentially doping (and therefore n) independent. Proceeding as described,

$$\frac{d\rho}{dn} = -\frac{1}{q\left(\mu_n n + \mu_p n_i^2/n\right)^2}\left(\mu_n - \mu_p n_i^2/n^2\right) = 0$$

and at the resistivity maximum $\mu_n = \mu_p n_i^2/n^2$, or

$$n = \sqrt{\mu_p/\mu_n}\, n_i$$

Substituting the n value at the maximum back into the general resistivity expression then yields,

$$\rho_{max} = \frac{1}{q(\sqrt{\mu_n\mu_p}\,n_i + \sqrt{\mu_n\mu_p}\,n_i)} = \frac{1}{2q\sqrt{\mu_n\mu_p}\,n_i}$$

The ρ_{max} values computed from the foregoing expression are recorded in the part (a) table.

<u>3.6</u>

(a) $\rho = \dfrac{1}{q\mu_n N_D}$ ...Eq.(3.8a)

$= \dfrac{1}{(1.6 \times 10^{-19})(1248)(10^{16})} = 0.501$ **ohm-cm**   ...μ_n from Fig. 3.5(a)

$\rho \approx 0.5$ **ohm-cm**   ...by inspection from Fig. 3.8(a)

(b) Since $N_A = N_D$, $n = p = n_i = 10^{10}/\text{cm}^2$. Moreover, the total number of scattering centers is $N_D + N_A = 2 \times 10^{16}/\text{cm}^3$. Thus, from Fig. 3.5(a), $\mu_n = 1165$ cm^2/V-sec, $\mu_p = 419$ cm^2/V-sec, and

$\rho = \dfrac{1}{q(\mu_n + \mu_p)n_i} = \dfrac{1}{(1.6 \times 10^{-19})(1584)(10^{10})} = 3.95 \times 10^5$ **ohm-cm**

(c) Here $n = p = n_i = 10^{10}/\text{cm}^3$. With $N_A = 0$ and $N_D = 0$, one has the maximum possible carrier mobilities. From Fig. 3.5(a), $\mu_{nmax} = 1358$ cm^2/V-sec and $\mu_{pmax} = 461$ cm^2/V-sec.

$\rho = \dfrac{1}{q(\mu_n + \mu_p)n_i} = \dfrac{1}{(1.6 \times 10^{-19})(1819)(10^{10})} = 3.44 \times 10^5$ **ohm-cm**

Because of the lower mobilities in compensated material, ρ(part b) > ρ(part c).

(d) $R = \rho l/A$

$\rho = RA/l = (500)(10^{-2})/(1) = 5$ ohm-cm

Since the bar is n-type, we conclude from Fig. 3.8(a) that $N_D \cong 9 \times 10^{14}/\text{cm}^3$.

(e) For a sample where $N_D \gg n_i$, $\rho = 1/q\mu_n N_D$. Furthermore, since the sample is lightly doped, lattice scattering will dominate and μ_n will decrease with increasing T. Fig. 3.7a confirms the preceding observation. Thus, with $\rho \propto 1/\mu_n$, heating up the sample causes the resistivity to **increase**.

3.7

The problem is to produce a plot of the Si 300K resistivity versus doping. The required MATLAB program script and output plot are reproduced below. Formed employing the same computation relationships and parametric values, the MATLAB produced plot should be identical to Fig. 3.8 in the text.

MATLAB program script ...

```
%Resistivity versus Dopant Concentration (Si,300K)

%Initialization
clear;  close
q=1.6e-19;
```

```
%Fit Parameters
NDref=1.3e17;  NAref=2.35e17;
μnmin=92;   μpmin=54.3;
μn0=1268;   μp0=406.9;
an=0.91;    ap=0.88;

%Resistivity Calculation
N=logspace(13,20);
μn=μnmin+μn0./(1+(N/NDref).^an);
μp=μpmin+μp0./(1+(N/NAref).^ap);
rhon=(1)./(q.*μn.*N);
rhop=(1)./(q.*μp.*N);
  (continued at right)
```

```
%Plotting results
loglog(N,rhon,N,rhop); grid;
axis([1e13,1e20,1e-4,1e3]);
xlabel('NA or ND (cm-3)');
ylabel('Resistivity (ohm-cm)');

text(8.0e14,30,'p-type');
text(2.0e14,4,'n-type');
text(1.1e18,130,'Si, 300K');
```
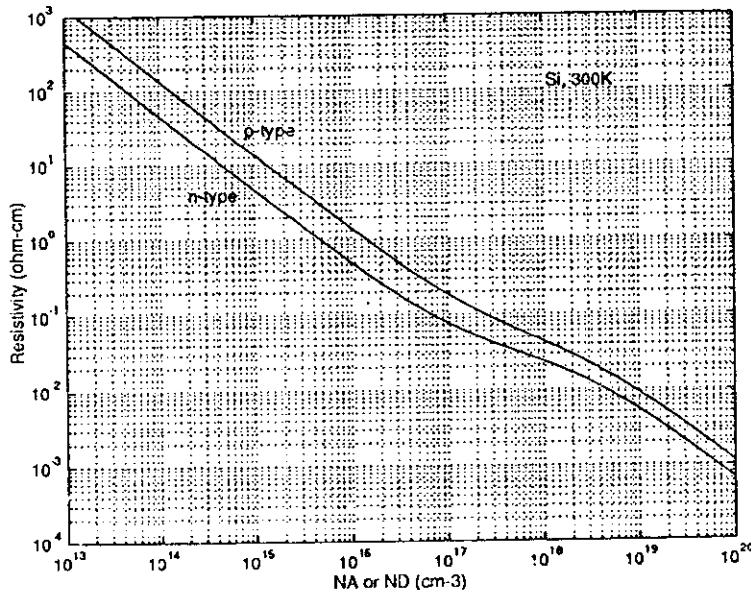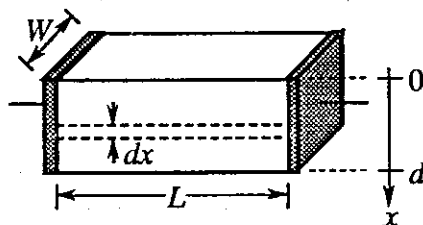
3.8

(a) Consider the bar-shaped resistor pictured below. Note that the x-coordinate is oriented vertically, with $x = 0$ positioned at the top of the bar. For a uniformly doped semiconductor bar, $R = 1/G = \rho L/A$, where R is the resistance, G the conductance, L the bar length, and A the bar cross-sectional area. In the given problem, however, N_D and therefore $\rho = 1/q\mu_n N_D$ are a function of x.

Let us conceptually break the bar up into small dx sections. Within a dx section, the doping is approximately constant and the conductance of a small section (dG) is given by

$$dG = d(1/R) = \frac{Wdx}{\rho L} = \frac{W}{L} q\mu_n N_D \, dx$$

The dx sections are in parallel and therefore the conductance of the sections simply add to yield the total conductance.

$$G = \Sigma dG = \int_0^d \frac{W}{L} q\mu_n N_D \, dx \quad \text{or} \quad R = 1/G = \frac{L/W}{q\displaystyle\int_0^d \mu_n N_D \, dx}$$

(b) The required R versus N_{D0} plot and MATLAB program script are reproduced below. Note that, consistent with the presented plot and given $L = W$, $R \to 1/qd\mu_n N_{DB} = 9.21\times10^4$ ohms if $N_{D0} \to 0$.

MATLAB program script ...

```
%IC Resistor, resistance calculation
%Rintg, a function subprogram, is a run-time requirement

%Initialization
clear;   close
global ND0 i

%Constants and Resistor Parameters
q=1.6e-19;
ND0=logspace(14,18);
d=5.0e-4;    %d in cm
```

```
%Resistance Calculation
%   A change of variable to x'=x/d was made in
%   evaluating the R integral
j=length(ND0);
for i=1:j
R(i)=(1)./(q*d.*quad8('Rintg',0,1));
end

%Plotting result
loglog(ND0,R);   grid
xlabel('ND0 (cm-3)');   ylabel('R (ohms)')
-------------------
function [y] = Rintg(x)
global ND0 i
%ND Calculation
  NDB=1.0e14;
  ND=NDB+ND0(i).*exp((-5).*x);    %Note that a*d=5
%Mobility Calculation
  μnmin=92;   μn0=1268;
  NDref=1.3e17;   an=0.91;
  μn=μnmin+μn0./(1+(ND./NDref).^an);
%Integrand
  y=μn.*ND;
```


3 - 10
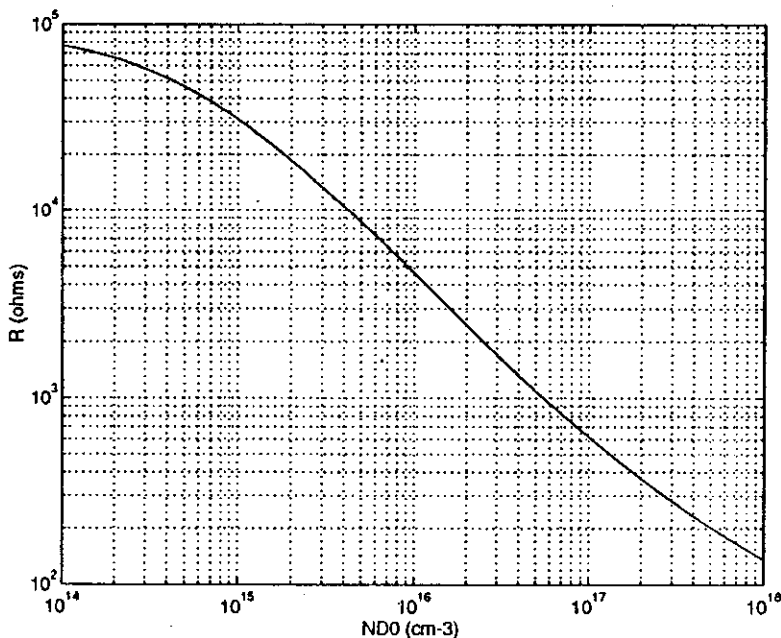
3.9

(a) It is a relatively simple matter to modify the Problem 3.3 program to obtain ρ versus T plots. The only required computational changes involve the ρ computation itself. This is accomplished by replacing the y-lines in the program with $y = 1/q\mu_n N$ or $y = 1/q\mu_p N$ as appropriate. Cosmetic changes to the comments, labels, and label positions are obviously necessary. The T=logspace assignment is best replaced with a T=linspace assignment, the loglog calls must be replaced by semilogy calls, and the old axis command must also be deleted or revised. The modified program yielding the ρ versus T plots reproduced below is included on the instructor's disk as program P_03_09.m.

At the higher temperatures n_i becomes comparable to the smaller N_D and N_A values. Specifically, $n_i = 2.72 \times 10^{14}/cm^3$ at $T = 500$ K. To be correct, ρ must be computed using Eq. (3.7), with the carrier concentrations calculated using Eqs. (2.29). The use of Eqs. (2.29) in turn requires that n_i be supplementally calculated as a function of T. The corrections noted here, corrections which primarily affect only the N_D and $N_A = 10^{14}/cm^3$ curves, would require a significant modification of the original program.

(b) The agreement with the plots cited in the semiconductor literature is excellent over the entire range of mutually plotted temperatures and dopings!
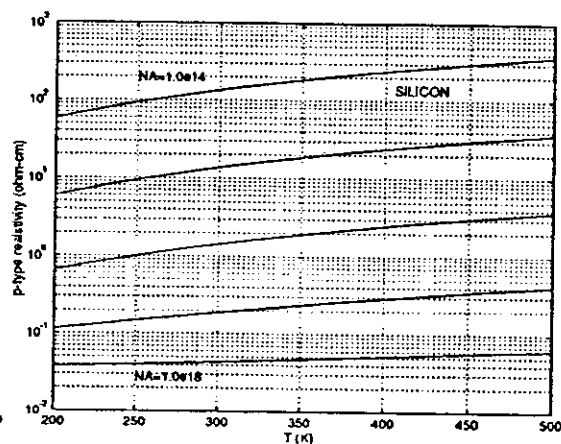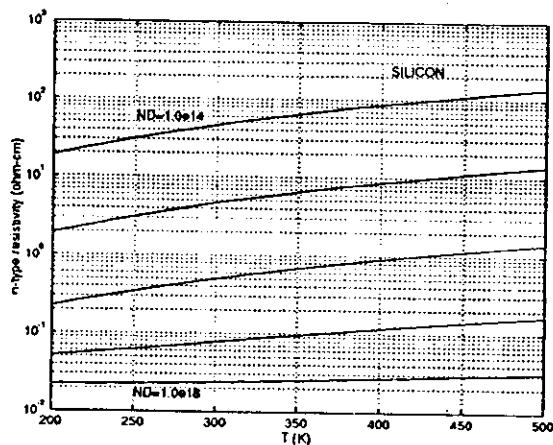
3.10

(This is clearly an open-ended problem with several alternative solution approaches and many possible solutions. What follows is only one possible approach and solution.)

(a) The choice of sensor dimensions is totally arbitrary. A relatively small cross section would be advisable to permit a more rapid response to temperature changes. (One might consider sawing up a wafer to fabricate the sensor.) To be specific, let us assume the sensor is **bar-like**, with length $l = 1$ **cm** and cross-sectional area $A = $ **1mm×1mm** $= 10^{-2}$**cm^2**. Considering the resistance of the sensor, we know

$$R = \rho l/A \quad \text{or} \quad \rho = (A/l)R$$

where the resistance is restricted to

$$1\,\Omega \leq R \leq 1000\,\Omega$$

But we have chosen

$$A/l = 10^{-2}\text{ cm}$$

We therefore require

$$10^{-2}\,\Omega\text{-cm} \leq \rho \leq 10\,\Omega\text{-cm}$$

at all measurement temperatures.

The doping must be chosen such the ρ falls within the cited range over the operating temperature range ($-30°C \leq T \leq 40°C$). At room temperature and taking the Si to be n-type, reference to Fig. 3.8(a) indicates that we must have

$$4.5 \times 10^{14}/\text{cm}^3 \leq N_D \leq 4.5 \times 10^{18}/\text{cm}^3$$

Anticipating changes in the allowed N_D range at higher and lower temperatures, let us choose N_D in the middle of the above noted range, say $N_D = 10^{16}/$**cm^3**, and check to see if the ρ requirement is met at the temperature extremes. Referring to Fig. 3.7(a), we conclude

at $T = -30°C = 243$ K, $\mu_n \sim 1800$ cm^2/V-sec and $\rho = 0.347\,\Omega$-cm

at $T = 40°C = 313$ K, $\mu_n \sim 1100$ cm^2/V-sec and $\rho = 0.568\,\Omega$-cm

The computed extrema temperature values fall comfortably within the allowable ρ-range. Previously boldfaced items are then the acceptable design solution.

(b)
$$R = \frac{\rho l}{A} = \frac{1}{q\mu_n N_D}\frac{l}{A}$$

$$\frac{dR}{dT} = \frac{1}{qN_D}\frac{l}{A}\frac{d}{dT}\left(\frac{1}{\mu_n}\right) = -\frac{l}{A}\frac{1}{q\mu_n^2 N_D}\frac{d\mu_n}{dT}$$

For maximum sensitivity it is preferable to use the lowest possible doping. As can be deduced from Fig. 3.7(a), both $|d\mu_n/dT|$ and $1/q\mu_n^2 N_D$ decrease with increasing doping concentrations.

(c) Technically, the temperature sensor can be used at any temperature where

$$10^{-2}\ \Omega\text{-cm} \le \rho = 1/q\mu_n N_D \le 10\ \Omega\text{-cm}$$

or

$$62.5\ \text{cm}^2/\text{V-sec} \le \mu_n \le 6.25 \times 10^4\ \text{cm}^2/\text{V-sec} \qquad (N_D = 10^{16}/\text{cm}^3)$$

Inspecting Fig. 3.7(a), the temperature range of operation is at least $200K \le T \le 500K$. At temperatures approaching 100K and lower, the mobility is expected to decrease with decreasing T. The sensor would still work but R would begin to increase with decreasing T. More than likely the contacts to the device would fail before the lower mobility limit is reached at very high T.

(d) Computation of the sensor R versus T were made using the μ_n versus T fit relationship given in Exercise 3.1.

MATLAB program script ...

```
%R vs. T of Temperature Sensor (Problem 3.10d)

%Initialization
clear;   close

%Constants, parameters, and independent variable
q=1.6e-19;
ND=1.0e16;
l=1;       %l=length of bar-like sensor
A=0.01;   %A=cross-sectional area
TC=linspace(-30,40);
T=273.18+TC;

%Electron mobility computation
  %Fit Parameters
  NDref=1.3e17;  an=0.91;
  µnmin=92;   µn0=1268;
  TNref=2.4;   Ta=-0.146;
  Tµn0=-2.33;   Tµmin=-0.57;
  %Computation
  NDrefT=NDref*(T./300).^TNref;
  µnminT=µnmin.*(T./300).^Tµmin;
  µn0T=µn0.*(T./300).^Tµn0;
  anT=an.*(T./300).^Ta;
  µn=µnminT+µn0T./(1+(ND./NDrefT).^anT);

%R Computation
R=(l/A)./(q.*µn.*ND);
```
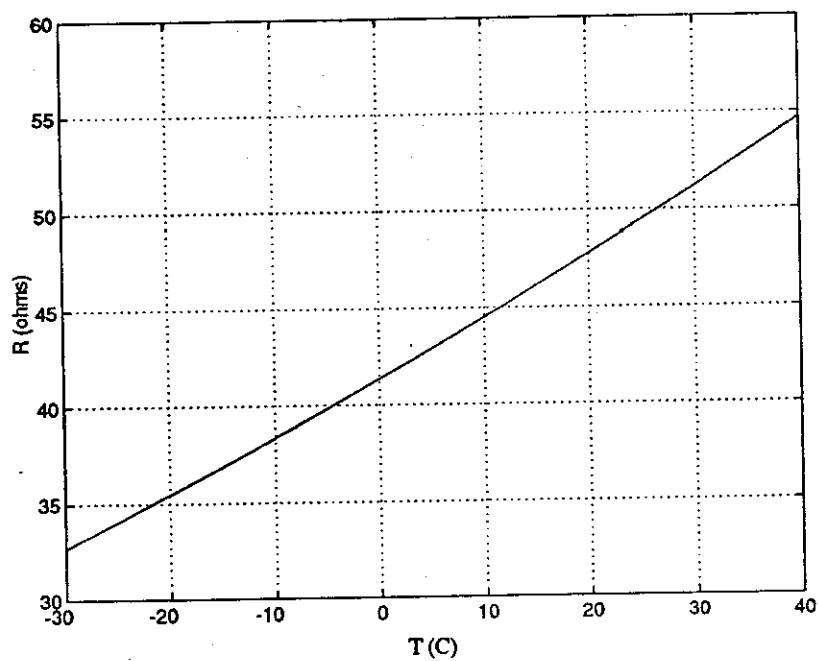
```
%Plot
plot(TC,R); grid
axis([-30,40,30,60])
xlabel('T (C)');  ylabel('R (ohms)')
```

3.11

In Problems 2.7 and 2.8 it is noted that the peak in the electron distribution versus energy inside the conduction band occurs at $E_c + kT/2$. Moreover, as explained in Subsection 3.1.5, $E - E_c$ is interpreted to be the kinetic energy (KE) of electrons in the conduction band. Thus

$$KE_{peak} = \frac{1}{2} m^* v_{peak}^2 = E_{peak} - E_c = kT/2$$

Setting $m^* = m_0$, we obtain

$$v_{peak} = \sqrt{\frac{kT}{m_0}} = \left[\frac{(0.0259)(1.6 \times 10^{-19})}{9.11 \times 10^{-31}} \right]^{1/2} = 6.75 \times 10^4 \text{ m/sec} = \mathbf{6.75 \times 10^6 \text{ cm/sec}}$$

3.12

The brief explanation how one arrives at a given answer, an explanation applicable to all the energy band diagrams, is given immediately below. Sketches indicating the *general form* of the expected answers follow the explanations.

(a) $\boxed{\text{Yes}}$ for all cases. The semiconductor is concluded to be in equilibrium because the Fermi level has the same energy value (it is constant) as a function of position.

(b) V vs. x has the same functional form as the "upside down" of E_c (or E_i or E_v). The sketches that follow were constructed taking the arbitrary reference voltage to be $V = 0$ at $x = 0$.

(c) $\mathcal{E}$ vs. x is determined by simply noting the slope of the energy bands as a function of position.

(d) For electrons, $PE = E_c - E_F$ and $KE = E - E_c$; for holes $PE = E_F - E_v$ and $KE = E_v - E$.

(e) The general carrier concentration variation with position can be deduced by noting $E_F - E_i$ vs. x. Under equilibrium conditions, $n = n_i \exp[(E_F - E_i)/kT]$ and $p = n_i \exp[(E_i - E_F)/kT]$ if the semiconductor is nondegenerate.

(f) Since $J_{N|drift} = q\mu_n n \mathcal{E}$, the general variation of $J_{N|drift}$ with position can be deduced by conceptually forming the product of the $\mathcal{E}$ vs. x dependence sketched in part (c) and the n vs. x dependence sketched in part (e). Under equilibrium conditions, $J_N = J_{N|drift} + J_{N|diff} = 0$. Thus $J_{N|diff} = -J_{N|drift}$.

Diagram (a) **Diagram (b)** **Diagram (c)**
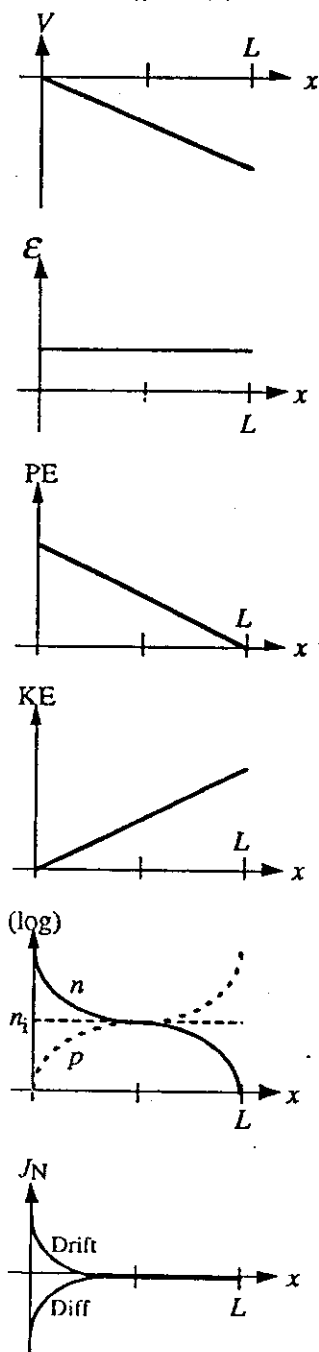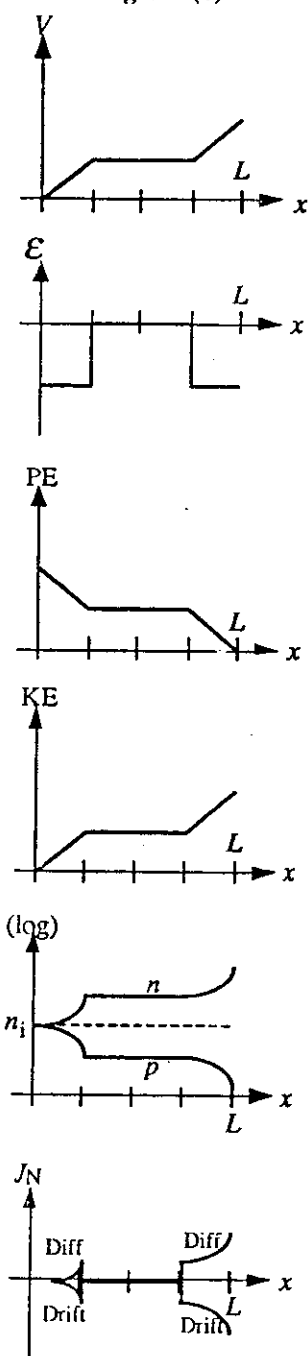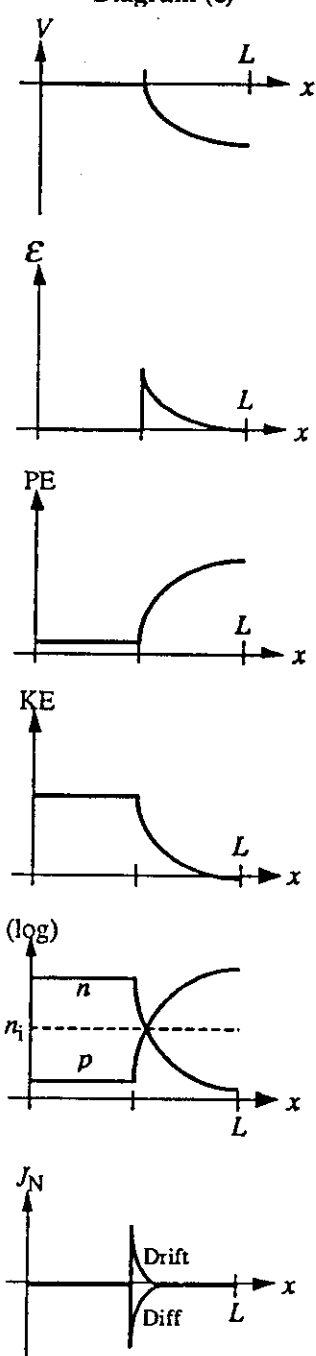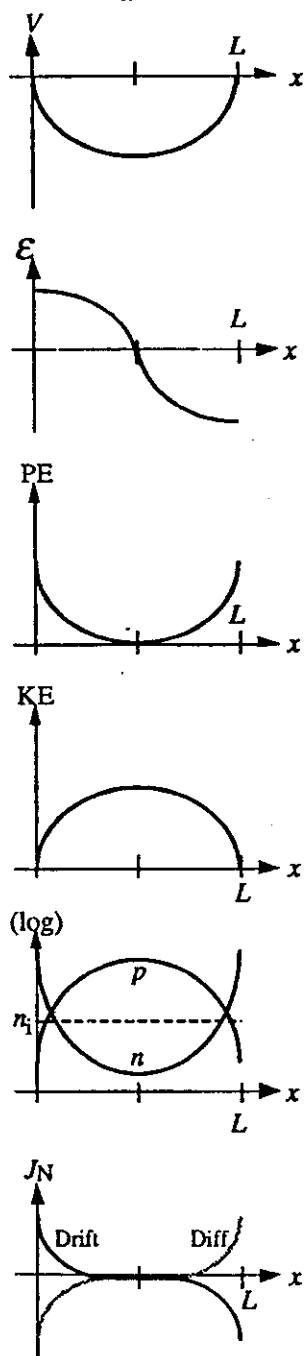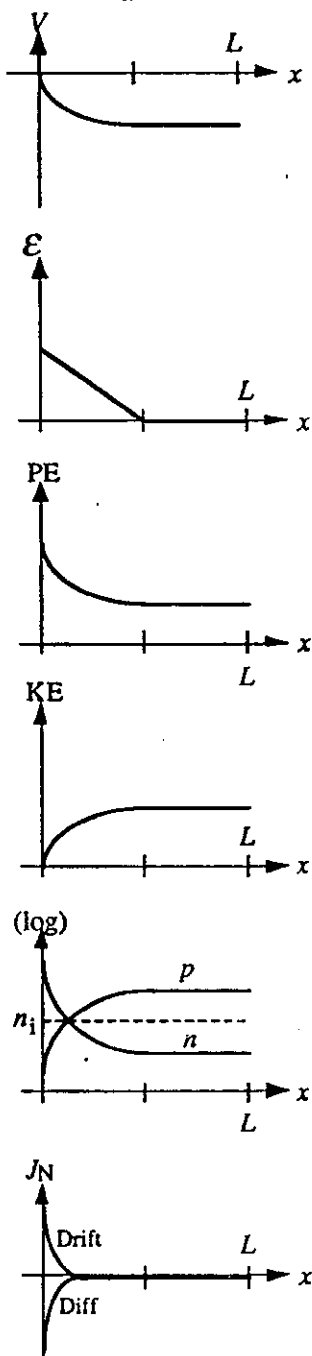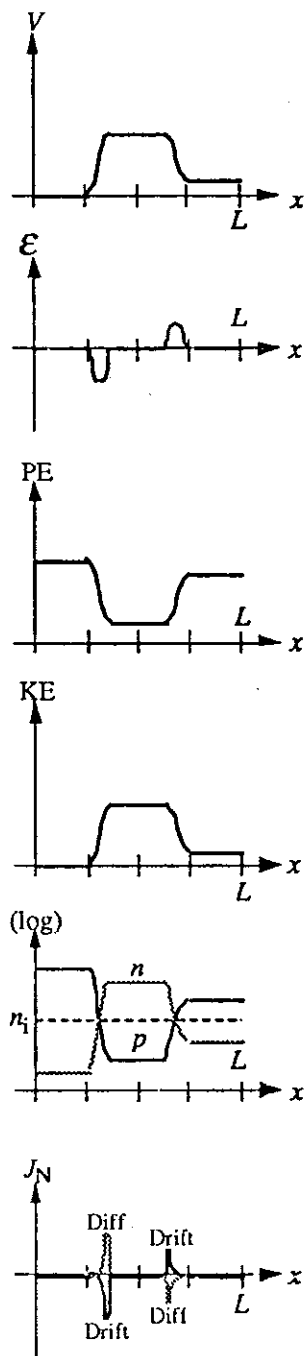
3.13

(a) (i) We know E_F = constant on the diagram since equilibrium conditions prevail.

(ii)........ $p(x) = n_i e^{(a-x)/b} = n_i e^{(E_i - E_F)/kT}$

Therefore

$E_i - E_F = kT(a-x)/b$ ...a linear function of x

Moreover

at $x = 0$, $E_i - E_F = kTa/b = 18kT = 0.466$ eV

at $x = L$, $E_i - E_F = kT(a-L)/b = 10kT = 0.259$ eV

(iii) $E_G(\text{Si}) = 1.12$ eV at room temperature and $E_i \cong (E_c + E_v)/2$.

Using the above information one concludes...
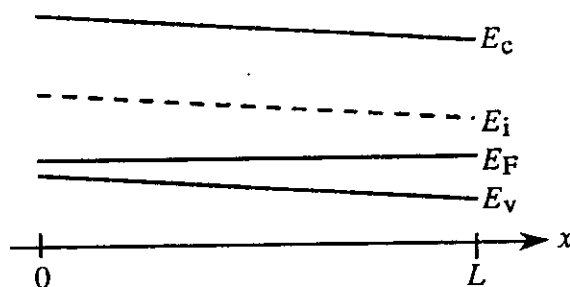
(b)
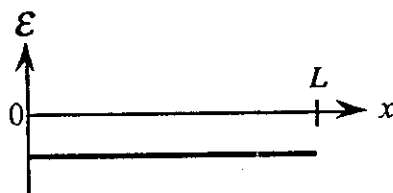
The above $\mathcal{E}$ versus x plot can be deduced by inspection from the slope of the energy band diagram. Quantitatively,

$$\mathcal{E} = \frac{1}{q}\frac{dE_i}{dx} = -\frac{kT/q}{b} = -\frac{0.0259}{10^{-5}} = -2.59 \times 10^3 \text{ V/cm}$$

(c) The direction of the electric field is such as to accelerate minority carrier electrons in the $+x$ direction. The built-in electric field should indeed assist in the transport of minority carrier electrons from $x = 0$ to $x = L$.

3.14

(a) Since $D = (kT/q)\mu$ and kT/q is not a function of the doping, a sketch of the D_N and D_P variation with N_A or N_D must have the same identical shape as the μ_n and μ_p plots in Fig. 3.5(a).

(b) MATLAB program script ...

```
%Diffusion Coefficient versus Dopant Concentration (Si,300K)

%Initialization
clear;  close
k=8.617e-5;
T=300;

%Fit Parameters
NDref=1.3e17;  NAref=2.35e17;
µnmin=92;  µpmin=54.3;
µn0=1268;  µp0=406.9;
an=0.91;  ap=0.88;

%Diffusion Coefficient Calculation
N=logspace(14,18);
µn=µnmin+µn0./(1+(N/NDref).^an);
µp=µpmin+µp0./(1+(N/NAref).^ap);
DN=k.*T.*µn;
DP=k.*T.*µp;

%Plotting results
loglog(N,DN,N,DP); grid;
axis([1e14,1e18,1,1e2]);
xlabel('NA or ND (cm-3)');
ylabel('Diffusion Coefficient (cm2/sec)');
text(1.0e15,38,'Electrons');
text(1.0e15,13,'Holes');
text(1.0e17,45,'Si,300K');
```

Diffusion Coefficient (cm2/sec) vs. NA or ND (cm-3)

Si,300K

Electrons

Holes
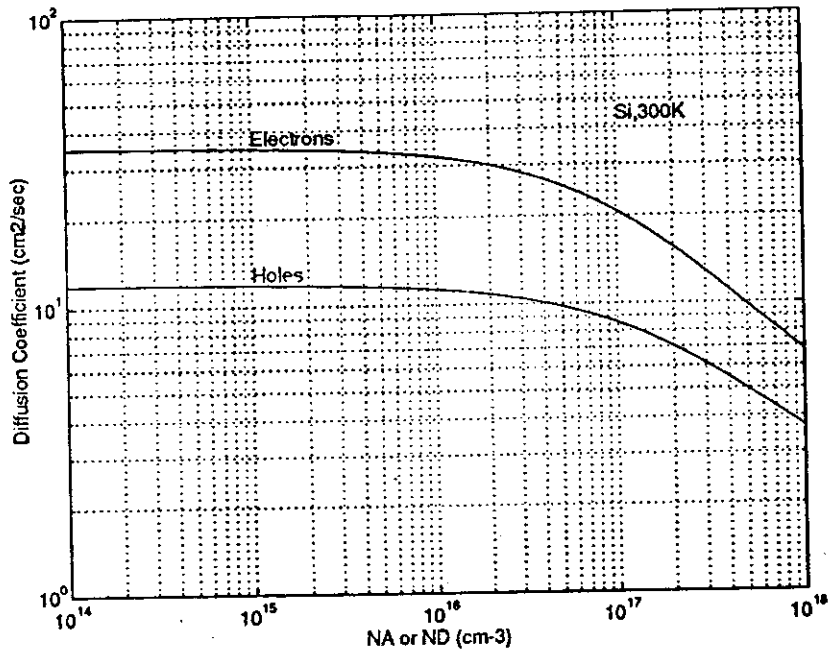
(c) The Einstein relationship used to convert mobility data to diffusion coefficient data assumes a modified form if the semiconductor is degenerate. At room temperature, Si becomes degenerate for doping concentrations in excess of roughly $10^{18}/cm^3$.

3 - 20

3.15

By definition

$$n = n_0 + \Delta n$$
$$p = p_0 + \Delta p$$

which when substituted into Eq. (3.35) gives

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal}\\\text{R-G}}} = \left.\frac{\partial n}{\partial t}\right|_{\substack{\text{i-thermal}\\\text{R-G}}} = \frac{n_i^2 - (n_0+\Delta n)(p_0+\Delta p)}{\tau_p(n_0+\Delta n+n_1) + \tau_n(p_0+\Delta p+p_1)}$$

$$= \frac{n_i^2 - n_0 p_0 - n_0\Delta p - p_0\Delta p - \Delta p^2}{\tau_p(n_0+\Delta p+n_i) + \tau_n(p_0+\Delta p+n_i)}$$

In the latter form of the preceding equation we have set $n_1 = p_1 = n_i$ and $\Delta n = \Delta p$ in accordance with the assumptions in the problem statement. The above equation is valid for both n- and p-type materials. To be specific, let us assume the semiconductor is n-type.

Examining the numerator of the simplified expression we note
$n_0 p_0 = n_i^2$; the n_i^2 and $-n_0 p_0$ terms cancel
$n_0\Delta p \gg p_0\Delta p$ ... since $n_0 \gg p_0$ in an n-type material
$n_0\Delta p \gg \Delta p^2$ ... since $\Delta p \ll n_0$ under low-level injection
All but the $n_0\Delta p$ term may be neglected.

Examining the denominator of the simplified expression we note
$\tau_p(n_0+\Delta p+n_i) \cong \tau_p n_0$ ... $n_0 \gg \Delta p$ and $n_0 \gg n_i$
$\tau_p n_0 \gg \tau_n(p_0+\Delta p+n_i)$ ... since $\tau_p \sim \tau_n$, and $n_0 \gg p_0+\Delta p+n_i$
All but the $\tau_p n_0$ term may be neglected.

We therefore conclude

$$\left.\frac{\partial p}{\partial t}\right|_{\substack{\text{i-thermal}\\\text{R-G}}} = -\frac{n_0\Delta p}{\tau_p n_0} = -\frac{\Delta p}{\tau_p} \quad ... \ n\text{-type material}$$

The reduced relationship is indeed just the special-case Eq. (3.34a).

3.16

(a) One assumes the minority carrier drift current is negligible compared to the diffusion current in deriving the equation; i.e., diffusion is taken to be the dominant mode of minority carrier transport — hence the name DIFFUSION equation.

(b) The equation is only valid for minority carriers.

(c) The recombination–generation term appearing in the equation, namely $-\Delta n_p/\tau_n$, is valid only under low-level injection conditions.

3.17

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L \qquad \text{...Hole minority carrier diff. eq.}$$

$$\frac{\partial \Delta p_n}{\partial t} = 0 \qquad \text{...steady state}$$

$$\frac{\Delta p_n}{\tau_p} \Rightarrow 0 \qquad \text{...R-G neglected}$$

$$G_L = 0 \qquad \text{...no light}$$

Thus

$$\frac{d^2 \Delta p_n}{dx^2} = 0$$

$$\Delta p_n(x) = A + Bx \qquad \text{...general solution}$$

$$\Delta p_n(0) = \Delta p_{n0} = A \qquad \text{...boundary condition no. 1}$$

$$\Delta p_n(L) = 0 = A + BL \qquad \text{...boundary condition no. 2}$$

$$\text{or} \quad B = -A/L = -\Delta p_{n0}/L$$

So finally

$$\boxed{\Delta p_n(x) = \Delta p_{n0}(1 - x/L) \qquad \text{...}0 \leq x \leq L}$$

3.18

(a) $\Delta n = n - n_0 = - n_0$ $(n_0 = n_i^2/N_A = 10^{20}/10^{16} = 10^4/\text{cm}^3)$

(b) $\boxed{\text{Generation}}$. There is a deficit of carriers at $t = 0^+$ and therefore the R–G process operates to eliminate the deficit by adding (generating) carriers.

(c) $\boxed{\text{Yes}}$. $|\Delta n| = n_0 = 10^4/\text{cm}^3 \ll p_0 = N_A = 10^{16}/\text{cm}^3$

(d)........ $\dfrac{\partial \Delta n_p}{\partial t} = D_N \dfrac{\partial^2 \Delta n_p}{\partial x^2} - \dfrac{\Delta n_p}{\tau_n} + G_L$ ... minority carrier diff. eq.

$G_L = 0$ since light is not affecting the wafer and the $D_N(\partial^2 \Delta n_p/\partial x^2)$ term is zero because there is no concentration gradient. Thus

$\dfrac{d \Delta n_p}{dt} = - \dfrac{\Delta n_p}{\tau_n}$ ...simplified equation

$\Delta n_p(0) = - n_0$ ...boundary condition

$\Delta n_p(t) = A e^{-t/\tau_n}$ ...general solution

$- n_0 = A$ ...applying b.c.

$\boxed{\Delta n_p(t) = -n_0 e^{-t/\tau_n}}$

3.19

Since G_{L0} is applied for a time $t \gg \tau_n$, steady state conditions will prevail prior to $t = 0$. Analogous to Sample Problem No. 1 discussed in the text, we can therefore state that

$$\Delta n_p(0) = G_{L0}\tau_n$$

Note that since the light intensity is reduced at $t = 0$, $\Delta n_p(0) = \Delta n_{p|max}$, and $\Delta n_{p|max} = G_{L0}\tau_n = 10^{16} \times 10^{-6} = 10^{10}/cm^3 \ll p_0 \cong N_A = 10^{14}/cm^3$. *Low level injection* conditions clearly prevail at all times. Also, all other conditions needed for use of the minority carrier diffusion equation have been met. Thus, we need to solve

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

which, for the problem at hand, simplifies to

$$\frac{d \Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_n} + \frac{G_{L0}}{2} \qquad \ldots t > 0$$

This equation is subject to the boundary condition,

$$\Delta n_p(0) = G_{L0}\tau_n$$

One obtains

$$\Delta n_p(t) = \frac{G_{L0}\tau_n}{2} + A e^{-t/\tau_n} \qquad \ldots \text{general solution}$$

and applying the boundary condition,

$$G_{L0}\tau_n = \frac{G_{L0}\tau_n}{2} + A \quad \Rightarrow \quad A = \frac{G_{L0}\tau_n}{2}$$

$$\boxed{\Delta n_p(t) = \frac{G_{L0}\tau_n}{2}\left(1 + e^{-t/\tau_n}\right)}$$

3.20

Since $\Delta n_p(x) \ll p_0$, we have low level injection. All conditions for use of the minority carrier diffusion equation are satisfied expect possibly $\mathcal{E} = 0$. However, we do not expect a small non-uniform distribution of minority carriers to give rise to a significant $\mathcal{E}$-field. Thus, assuming $\mathcal{E} \cong 0$, we can write in general

$$\frac{\partial \Delta n_p}{\partial t} = D_N \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

Simplifying...

$$\frac{\partial \Delta n_p}{\partial t} = 0 \qquad \text{...steady state}$$

$$0 = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L \qquad \text{...equation to be solved}$$

Solving...

$$\Delta n_p(x) = G_L \tau_n + A e^{-x/L_N} + B e^{x/L_N} \qquad (L_N = \sqrt{D_N \tau_n})$$

Boundary conditions...

(1) $\Delta n_p(0) = 0$ ...given

(2) $\Delta n_p(\infty) = G_L \tau_n$

The second boundary condition follows from the fact that far from $x = 0$ the concentration gradient caused by the extraction of carriers at $x = 0$ must vanish. If $D_N(d^2 \Delta n_p/dx^2) \to 0$. $\Delta n_p = G_L \tau_n$ as deduced from the simplified minority carrier diffusion equation. Thus

$$B = 0 \qquad \text{...applying boundary condition (2)}$$

$$0 = G_L \tau_n + A \qquad \text{...applying boundary condition (1)}$$

$$\text{or} \quad A = -G_L \tau_n$$

and

$$\boxed{\Delta n_p(x) = G_L \tau_n \left(1 - e^{-x/L_N}\right)}$$

3.21

(a) With a γN_D excess of holes being created at the bar ends, some of the excess will move into the bar interior via diffusion. Recombination will systematically reduce the excess as it moves into the bar. A distribution of excess minority carriers symmetrical about the center of the bar is expected because both ends of the bar are being perturbed in an identical manner. The expected general form of the $\Delta p_n(x)$ solution is therefore as sketched below.
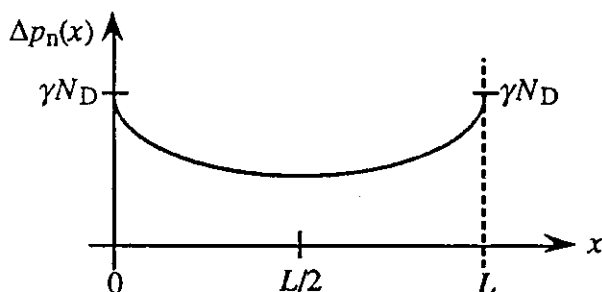
(b) Yes . As is obvious from the qualitative sketch,

$$\Delta p_{n|max} = \gamma N_D = 10^{-3} N_D$$

but $\quad n_0 \cong N_D \quad (N_D \gg n_i)$

therefore $\quad \Delta p_{n|max} \ll n_0 \quad$ as required for low-level injection

(c) Inside the bar $G_L = 0$ and steady state conditions prevail making $\partial \Delta p_n / \partial t = 0$. Thus the minority carrier diffusion equation simplifies to

$$0 = D_P \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p}$$

(d) Referring to solution no. 1 in Table 3.2, the general solution to the part (c) differential equation is

$$\Delta p_n(x) = A e^{-x/L_P} + B e^{x/L_P} \qquad \ldots L_P = \sqrt{D_P \tau_p}$$

The boundary conditions are

$$\Delta p_n(0) = \gamma N_D \quad \text{and} \quad \Delta p_n(L) = \gamma N_D$$

(e)………. $\quad J_P = q\mu_p p \mathcal{E} - q D_P \dfrac{dp}{dx} = -q D_P \dfrac{dp}{dx} = -q D_P \dfrac{d\Delta p_n}{dx}$

The first simplification above follows from the fact that $\mathcal{E} \cong 0$ and the drift current is assumed to be negligible compared with the diffusion current. This is consistent with the

prior use of the minority carrier diffusion equation. Secondly, $dp/dx = d\Delta p_n/dx$ because $p = p_0 + \Delta p$ and $dp_0/dx = 0$. Now,

$$\left.\frac{d\Delta p_n}{dx}\right|_{x=0} = \left.\left[-\frac{A}{L_P}e^{-x/L_P} + \frac{B}{L_P}e^{x/L_P}\right]\right|_{x=0} = \frac{B-A}{L_P}$$

Thus

$$\left.J_P\right|_{x=0} = -qD_P\left.\frac{d\Delta p_n}{dx}\right|_{x=0} = -q\frac{D_P}{L_P}(B-A)$$

3.22

(a) Far to the negative side of $x = 0$ there will be no perturbation and

$$p(-\infty) = p_0 = n_i^2/N_D = 10^{20}/10^{18} = 10^2/cm^3$$

(b) We expect a concentration gradient near $x = 0$ due to diffusion, but as $x \to \infty$ the gradient should vanish. Thus, far from $x = 0$ the carriers generated by the light must just balance the carriers being annihilated by thermal R–G under steady state conditions; i.e.,

$$G_L = \Delta p_n(\infty)/\tau_p$$

or

$$\Delta p_n(\infty) = G_L\tau_p = (10^{15})(10^{-6}) = 10^9/cm^3$$

and

$$p(\infty) = p_0 + \Delta p_n(\infty) \cong \Delta p_n(\infty) = 10^9/cm^3$$

(c) $\boxed{Yes}$. The largest Δp_n will occur at $x = \infty$ and

$$\Delta p_n(\infty) = 10^9/cm^3 \ll n_0 = N_D = 10^{18}/cm^3$$

(d) For $x < 0$ ...

$$0 = D_P\frac{d^2\Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p}$$

$$\Delta p_n(x) = Ae^{-x/L_P} + Be^{x/L_P}$$

$$\Delta p_n(-\infty) = 0 \quad \Rightarrow \quad A = 0$$

$$\Delta p_n(x) = Be^{x/L_P} \quad \text{and} \quad \frac{d\Delta p_n(x)}{dx} = \frac{B}{L_P}e^{x/L_P}$$

For $x > 0$ ...

$$0 = D_P \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

$$\Delta p_n(x) = G_L \tau_p + A' e^{-x/L_P} + B' e^{x/L_P}$$

$$\Delta p_n(\infty) = G_L \tau_p \quad \Rightarrow \quad B' = 0$$

$$\Delta p_n(x) = G_L \tau_p + A' e^{-x/L_P} \quad \text{and} \quad \frac{d \Delta p_n(x)}{dx} = -\frac{A'}{L_P} e^{-x/L_P}$$

At $x = 0$ ...

$$B = G_L \tau_p + A' \qquad \text{... continuity of } \Delta p_n(x)$$

$$B/L_P = -A'/L_P \qquad \text{... continuity of } d\Delta p_n(x)/dx$$

or $\quad B = -A' = G_L \tau_p/2$

and

$$\Delta p_n(x) = \begin{cases} \frac{1}{2} G_L \tau_p e^{x/L_P} & \text{... } x \leq 0 \\ G_L \tau_p \left(1 - \frac{1}{2} e^{-x/L_P} \right) & \text{... } x \geq 0 \end{cases}$$

3.23

(a) The snake-like pattern provides a relatively large area for light absorption while making the film length/width ratio, and hence the dark resistance, as large as possible.

(b) $\qquad R = \rho l/A$

with $\rho \cong 1/q\mu_n N_D$ and $A = $ (width)(thickness) $= wt$

or $\qquad R = \dfrac{l}{q\mu_n N_D wt} = \dfrac{3}{(1.6 \times 10^{-19})(100)(10^{13})(3 \times 10^{-2})(5 \times 10^{-4})}$

$\qquad = \mathbf{1.25 \times 10^9 \ ohms}$

(c) Under low-level injection conditions, one could assert $G_L = \Delta p/\tau_p$. However, with the light-on resistance much less than the dark resistance, it is clear that one actually has high level ($n \gg n_0$) injection. The usual relationships cannot be used.

3.24

(a) $n_0 = n_i e^{(E_F - E_i)/kT} = (10^{10})e^{0.3/0.0259} = \mathbf{1.07 \times 10^{15}/cm^3}$

       $p_0 = n_i e^{(E_i - E_F)/kT} = (10^{10})e^{-0.3/0.0259} = \mathbf{9.32 \times 10^4/cm^3}$

(b) $n = n_i e^{(F_N - E_i)/kT} = (10^{10})e^{0.318/0.0259} = \mathbf{2.15 \times 10^{15}/cm^3}$

       $p = n_i e^{(E_i - F_P)/kT} = (10^{10})e^{0.3/0.0259} = \mathbf{1.07 \times 10^{15}/cm^3}$

(c) $N_D \cong n_0 = \mathbf{1.07 \times 10^{15}/cm^3}$

(d) $\boxed{\text{No}}$ Due to illumination, $\Delta p \approx n_0$ and n differs significantly from n_0. For low level injection one must have $\Delta p << n_0$ and $n \cong n_0$.

(e) $\rho_{before} \cong \dfrac{1}{q\mu_n N_D} = \dfrac{1}{(1.6 \times 10^{-19})(1345)(1.07 \times 10^{15})} = \mathbf{4.34 \ ohm\text{-}cm}$

       $\rho_{after} = \dfrac{1}{q(\mu_n n + \mu_p p)} = \dfrac{1}{(1.6 \times 10^{-19})[(1345)(2.15 \times 10^{15}) + (458)(1.07 \times 10^{15})]}$

       $= \mathbf{1.85 \ ohm\text{-}cm}$

3.25

(1) Since $\mathcal{E} \cong 0$, E_c, E_i, and E_v will be position independent.

(2) Given $n \cong N_D$, it follows that $F_N \cong E_F$ and

$$E_F - E_i \cong kT \ln(N_D/n_i) = (0.0259)\ln(10^{15}/10^{10}) = 0.298 \text{ eV}$$

(3) $p = p_0 + \Delta p_n = n_i^2/N_D + n_i(1 - x/L) = n_i e^{(E_i - F_P)/kT}$

and
$$p_0 = n_i^2/N_D = 10^5/\text{cm}^3$$

$$\Delta p_n = n_i(1 - x/L) = (10^{10})(1 - x/L)$$

Clearly

at $x = 0$, $p \cong n_i$ and $F_P = E_i$

at $x = L$, $p = p_0$ and $F_P \doteq E_F$

For most x (except values near $x = L$), $\Delta p_n \gg p_0$
and
$$e^{(E_i - F_P)/kT} \cong 1 - x/L$$

$E_i - F_P = kT \ln(1 - x/L)$ $\Rightarrow$ $F_P - E_i = 0.0027\text{eV}$ if $x/L = 0.1$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $F_P - E_i = 0.018\text{eV}$ if $x/L = 0.5$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $F_P - E_i = 0.060\text{eV}$ if $x/L = 0.9$

From the calculations note that F_P stays fairly close to E_i until $x \cong L$.

Putting all the above information together, one concludes,
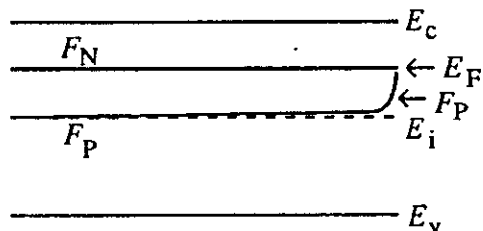
<u>3.26</u>

(a) In constructing the plot, all energies are referenced to E_v and normalized by kT. Specifically,

$$(F_N - E_v)/kT = E_G/2kT + \ln(N_D/n_i)$$

$$(F_P - E_v)/kT = E_G/2kT - \ln\left[p_0/n_i + (\Delta p_{n0}/n_i)e^{-x/L_P}\right]$$

Because the independent variable is taken to be x/L_P, it is not necessary to compute L_P. The τ_p given in the problem statement is not required in performing the computations.

MATLAB program script ...

```
%Quasi-Fermi level plot

%Initialization
clear;  close

%Input parameters
EG=1.12;
kT=0.0259;
ni=1.0e10;
ND=input('Input ND in cm-3, ND = ');
Δpn0=input('Input Δpn0 in cm-3, Δpn0 = ');

%Calculations
p0=ni^2/ND;
zmax=log(100*Δpn0/p0);
z=linspace(0,zmax);   %z=x/LP
%All energies referenced to Ev and normalized by kT
  Ec=EG/kT;
  FN=EG/(2*kT)+log(ND/ni);
  FP=EG/(2*kT)-log(p0/ni+(Δpn0/ni).*exp(-z));

%Plotting results
plot(z,FP,'r');
axis([0,zmax,-5,45])
xlabel('x/LP');  ylabel('(E-Ev)/kT')
hold on
x=[0,zmax];
y1=[Ec,Ec];       y2=[FN,FN];  plot(x,y1,x,y2,'b')
y3=[Ec/2,Ec/2];   y4=[0,0];    plot(x,y3,'--',x,y4,'y')
zput=zmax+0.2;
text(zput,43.2,'Ec');  text(zput,21.6,'Ei');  text(zput,0,'Ev')
text(zput/2,FN+1,'FN');  text(zput/2,FP(50)-1,'FP')
```

(b) The program output with $N_D = 10^{15}/\text{cm}^3$ and $\Delta p_{n0} = 10^{10}/\text{cm}^3$ that is displayed below compares favorably with the sketch in Exercise 3.5. The sketch shows a greater x-length of the band diagram. Also, in producing the text sketch, the graphic artist incorrectly added some curvature to the near $x = 0$ part of the F_P line.