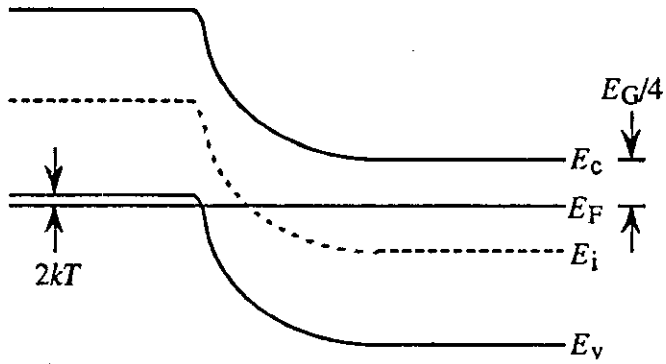
CHAPTER 5

5.1

(a) False
(b) True
(c) False
(d) True
(e) True
(f) False
(g) True
(h) True
(i) False
(j) True

5.2

(a)



(b) Employing Eq. (5.12),

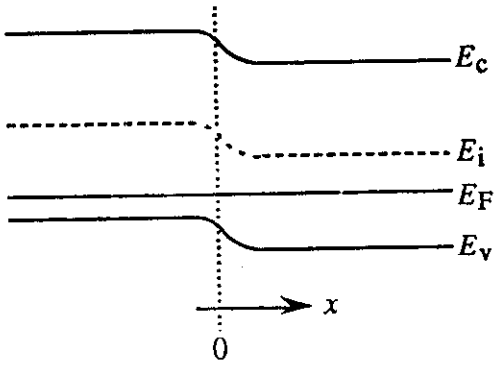
$$V_{bi} = \frac{1}{q}\left[(E_i - E_F)_{p\text{-side}} + (E_F - E_i)_{n\text{-side}}\right]$$

$$\cong \frac{1}{q}(E_G/2 + 2kT + E_G/4) = \frac{1}{q}(3E_G/4 + 2kT) = \frac{3}{4}(1.12) + 2(0.0259)$$

$$= \mathbf{0.89\ V}$$

5.3

(a) Because $N_{A1} > N_{A2}$ and $p = n_i \exp[(E_i - E_F)/kT] \cong N_A$ far from the junction, it follows that $(E_i - E_F)_{x<0} > (E_i - E_F)_{x>0}$. The energy band diagram must therefore be of the form



(b) Given that the dopings are nondegenerate, the same development leading to Eq. (5.10) can be employed, except

$$n(x_n) \cong n_i^2/N_{A2}$$

$$n(-x_p) \cong n_i^2/N_{A1}$$

which when substituted into Eq. (5.8) yields

$$\boxed{V_{bi} = \frac{kT}{q} \ln\left(\frac{N_{A1}}{N_{A2}}\right)}$$

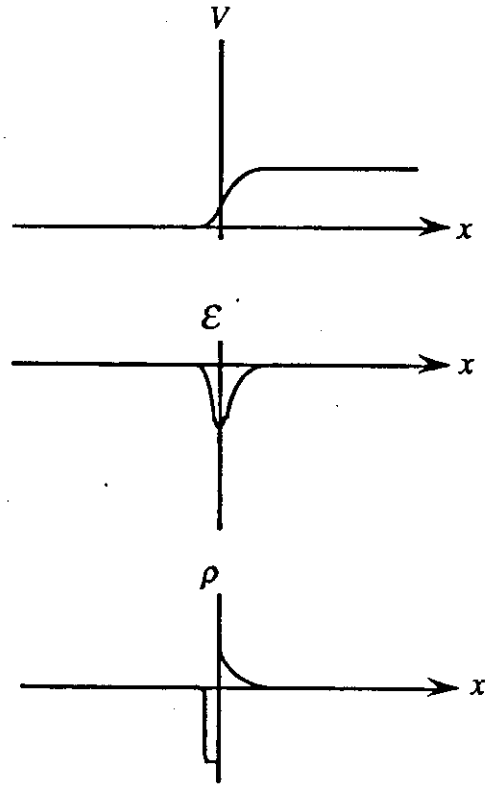
Alternatively, one can write

$$V_{bi} = \frac{1}{q}\left[E_i(-\infty) - E_i(+\infty)\right] = \frac{1}{q}\left[(E_i - E_F)_{p1\text{-side}} - (E_i - E_F)_{p2\text{-side}}\right]$$

$$= \frac{1}{q}\left[kT \ln(N_{A1}/n_i) - kT \ln(N_{A2}/n_i)\right] = \frac{kT}{q} \ln(N_{A1}/N_{A2})$$

Note that, as must be the case, $V_{bi} \to 0$ if $N_{A1} = N_{A2}$.

(c)



(It should be emphasized that the above are rough sketches. The exact functional dependencies cannot be deduced employing a graphical approach.)

(d) No. It is true that the minus charge shown on the $x < 0$ portion of the part (c) ρ-plot is caused by a depletion of holes on the higher-doped N_{A1}-side of the junction, leading to net charge associated with the ionized acceptors. The plus charge on the N_{A2}-side of the junction, however, cannot be attributed to ionized donors. There is no donor doping! Th only other source of a positive charge is holes.—There is a hole concentration on the n-sid of the junction in *excess* of N_{A2}. [The hole excess on the N_{A2}-side of the junction can actually be inferred from the energy band diagram in part (a).] Since $p > N_{A2}$ near the junction, we cannot invoke the depletion approximation.

5.4

(a) $$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = (0.0259)\ln\left[\frac{(2\times10^{15})(10^{15})}{(10^{20})}\right] = \mathbf{0.614\ V}$$

(b) $$x_p = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}V_{bi}\right]^{1/2} = \mathbf{3.655\times10^{-5}\ cm}$$

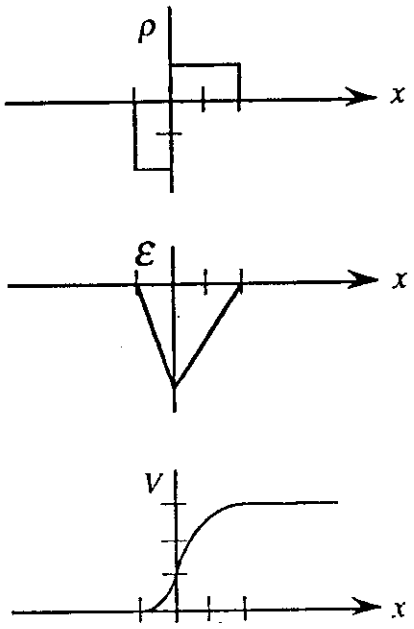
$$x_n = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_A}{N_D(N_A+N_D)}V_{bi}\right]^{1/2} = \mathbf{7.31\times10^{-5}\ cm}$$

$$W = x_n + x_p = \mathbf{1.10\times10^{-4}\ cm}$$

(c) $$\mathcal{E}(0) = -\frac{qN_D}{K_S\varepsilon_0}x_n = -\frac{(1.6\times10^{-19})(10^{15})(7.31\times10^{-5})}{(11.8)(8.85\times10^{-14})} = \mathbf{-1.12\times10^{4}\ V/cm}$$

(d) $$V(0) = \frac{qN_A}{2K_S\varepsilon_0}x_p^2 = \frac{(1.6\times10^{-19})(2\times10^{15})(3.655\times10^{-5})^2}{(2)(11.8)(8.85\times10^{-14})} = \mathbf{0.205\ V}$$

(e)



5.5

(a) $$V_{bi} = \frac{kT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right) = (0.0259)\ln\left[\frac{(10^{17})(10^{15})}{(10^{20})}\right] = \mathbf{0.716\ V}$$

(b) $$x_p = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}V_{bi}\right]^{1/2} = \mathbf{9.62\times10^{-7}\ cm}$$

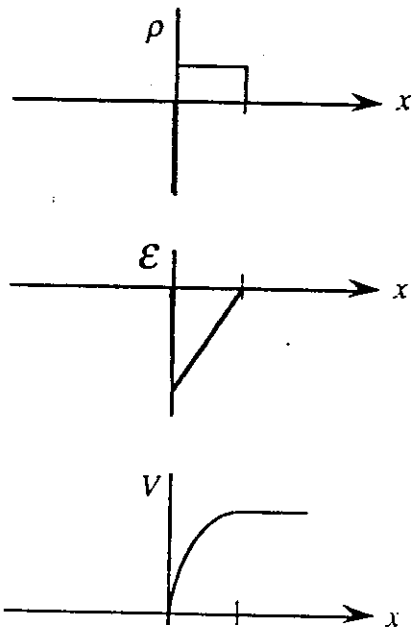
$$x_n = \left[\frac{2K_S\varepsilon_0}{q}\frac{N_A}{N_D(N_A+N_D)}V_{bi}\right]^{1/2} = \mathbf{9.62\times10^{-5}\ cm}$$

$$W = x_n + x_p = \mathbf{9.72\times10^{-5}\ cm}$$

(c) $$\mathcal{E}(0) = -\frac{qN_D}{K_S\varepsilon_0}x_n = -\frac{(1.6\times10^{-19})(10^{15})(9.62\times10^{-5})}{(11.8)(8.85\times10^{-14})} = \mathbf{-1.47\times10^{4}\ V/cm}$$

(d) $$V(0) = \frac{qN_A}{2K_S\varepsilon_0}x_p^2 = \frac{(1.6\times10^{-19})(10^{17})(9.62\times10^{-7})^2}{(2)(11.8)(8.85\times10^{-14})} = \mathbf{7.09\times10^{-3}\ V}$$

(e)



In Problem 5.4 the widths of the *n*- and *p*-sides of the depletion region and the corresponding variation of the electrostatic variables are comparable reflecting the fact that $N_A \sim N_D$. Here with $N_A >> N_D$, we find the depletion width and potential drop lie almost exclusively on the lowly doped *n*-side of the junction.

5.6

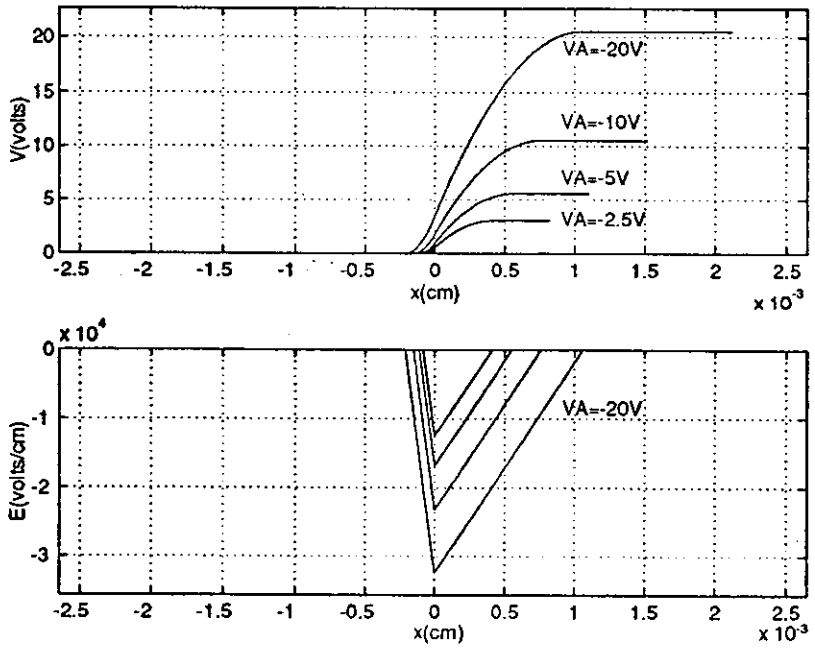
The MATLAB program listed below (and included on the Instructor's disk as m-file P_05_06.m) can be used to establish the expected answers for a given set of instructor specified parameters.

MATLAB program script...

```
%Problem 5.6
%Initialization
clear;  format compact

%Constants and Parameters
q=1.6e-19;  e0=8.85e-14;
ni=1.0e10;  kT=0.0259;
KS=11.8;
NA=input('Input NA in cm-3, NA=');
ND=input('Input ND in cm-3, ND=');
VA=input('Input VA in volts, VA=');

%(a) Vbi...the built-in-voltage
format compact
Vbi=kT*log(NA*ND/ni^2)

%(b) xp, xn, and W
xp=sqrt(2*KS*e0/q*(Vbi-VA)*ND/(NA*(NA+ND)))
xn=sqrt(2*KS*e0/q*(Vbi-VA)*NA/(ND*(NA+ND)))
W=xn+xp

%(c) Electric field at x = 0
E0=-q*ND/(KS*e0)*xn

%(d) Electrostatic potential at x = 0
V0=q*NA/(2*KS*e0)*xp^2
fprintf('UNITS...\nxp(cm), xn(cm), W(cm)')
fprintf('\nVbi(volts), E0(V/cm), V0(volts)\n')
```

5.7

(a)/(b)/(c) Constructed in accordance with suggestions provided in the problem statement, the P_05_07.m program yielding the desired results, along with sample output, are displayed below. The program produces a plot of 1 to 4 simultaneous $\mathcal{E}$ and V versus x curves corresponding to different applied voltages. In addition, a listing of relevant parameters and computational constants is sent to the Command window.



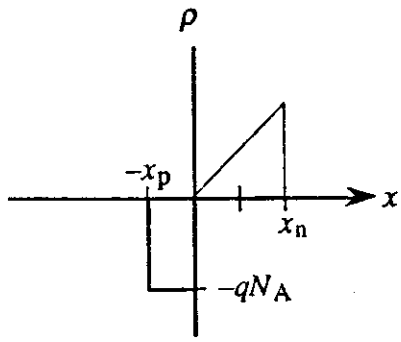
COMPUTATIONAL RESULTS
NA =1.0000e+15
ND =2.0000e+14
VA =-20
Vbi =0.5547
xn =0.0011
xp =2.1147e-04
W =0.0013
E0 =-3.2400e+04
V0 =3.4258

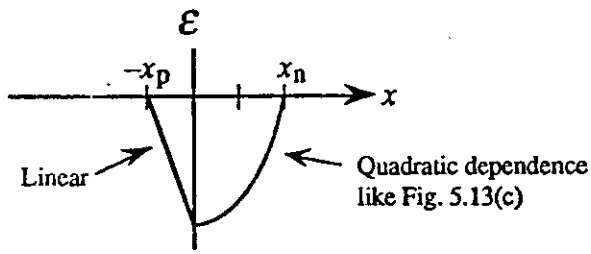
MATLAB program script...

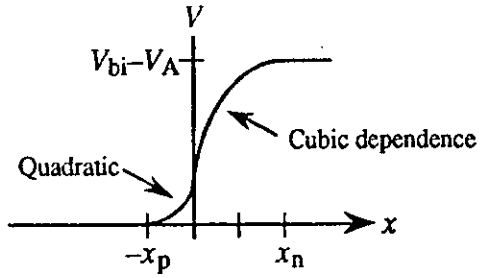
```
%This program computes and plots the ELECTROSTATIC VARIABLES (V,E).
%It also provides an output of key computational parameters.
%A Si step junction maintained at 300K is assumed.
%The solution is based on the depletion approximation.
fprintf('\n\nELECTROSTATIC VARIABLES COMPUTATION (step,Si,300K)');

%Initialization
  clear; close
  format compact

%Constants
  q=1.6e-19;
  k=8.617e-5;
  e0=8.85e-14;
%Parameters
  ni=1.0e10;
  kT=0.0259;
  KS=11.8;
%Variables
  NA=input('Input the p-side doping, NA = ');
  ND=input('Input the n-side doping, ND = ');
  s=input('Specify the number of curves to compute (1-4), ');
  VA=input('Specify the minimum applied bias, VA = ');

%Computation Proper
for ii=1:s
   %Built-in Voltage
     Vbi=kT*log(NA*ND/ni^2);
   %xn, xp, and W
     X=(2*KS*e0)*(Vbi-VA)/q;
     xn=sqrt(X*NA/(ND*(NA+ND)));
     xp=sqrt(X*ND/(NA*(NA+ND)));
     W=xn+xp;
   %p-side electrostatic variables
     x=linspace(-xp,0);
     Vp=(q*NA/(2*KS*e0))*(xp+x).^2;
     Ep=-(q*NA/(KS*e0))*(xp+x);
   %n-side electrostatic variables
     xx=linspace(0,xn);
     Vn=Vbi-VA-(q*ND/(2*KS*e0))*(xn-xx).^2;
     En=-(q*ND/(KS*e0))*(xn-xx);
     xxx=[xx,2*xn];
     Vn=[Vn,Vn(100)];

%Plotting results
   if ii==1,
      Emax=1.1*En(1);
      Vmax=1.1*Vn(100);
      xmax=2.5*max(xn,xp);
    else
   end
   subplot(2,1,1), plot(x,Vp,xxx,Vn);
     axis([-xmax,xmax,0,Vmax]); grid on
     xlabel('x(cm)'); ylabel('V(volts)');
     hold on
   subplot(2,1,2), plot(x,Ep,xx,En);
     axis([-xmax,xmax,Emax,0]); grid on
     xlabel('x(cm)'); ylabel('E(volts/cm)');
     hold on
   %Print out numerical results
   fprintf('\n\nCOMPUTATIONAL RESULTS\n')
   NA, ND, VA, Vbi, xn, xp, W, E0=En(1),
   V0=Vn(1)
   %Recycle
   VA=VA/2;
end
hold off
```

5.8

Essentially, the solution is just a superposition of the step junction solution for $x < 0$ and the linearly graded solution for $x > 0$.



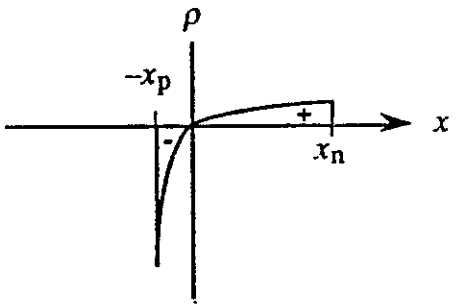




5.9

(a) The carrier concentrations are assumed to be negligible compared to the net doping concentration in a region $-x_p \le x \le x_n$ straddling the metallurgical junction. The charge density outside the depletion region is taken to be identically zero.

(b) We must have $\rho = q(N_D - N_A)$ for $-x_p \le x \le x_n$ and the total (+) charge must equal the total (–) charge.



(c) Since based on the depletion approximation

$$\rho = \begin{cases} qN_0\,(1 - e^{-\alpha x}) & \ldots -x_p \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$

substitution into Poisson's equation gives

$$\frac{d\mathcal{E}}{dx} = \begin{cases} \dfrac{qN_0}{K_S\varepsilon_0}\,(1 - e^{-\alpha x}) & \ldots -x_p \le x \le x_n \\ 0 & \ldots x \le -x_p \text{ and } x \ge x_n \end{cases}$$

Separating variables and integrating from the $-x_p$ depletion region edge where $\mathcal{E} = 0$ to an arbitrary point x in the depletion region, one obtains

$$\mathcal{E}(x) = \frac{qN_0}{K_S\varepsilon_0}\int_{-x_p}^{x}(1 - e^{-\alpha x'})dx' = \frac{qN_0}{K_S\varepsilon_0}\left(x' + \frac{e^{-\alpha x'}}{\alpha}\right)\Bigg|_{-x_p}^{x} = \frac{qN_0}{K_S\varepsilon_0}\left(x + \frac{e^{-\alpha x}}{\alpha} + x_p - \frac{e^{\alpha x_p}}{\alpha}\right)$$

or

$$\mathcal{E}(x) = \frac{qN_0}{K_S\varepsilon_0}\left[(x + x_p) + \frac{1}{\alpha}\left(e^{-\alpha x} - e^{\alpha x_p}\right)\right] \qquad \ldots -x_p \le x \le x_n$$

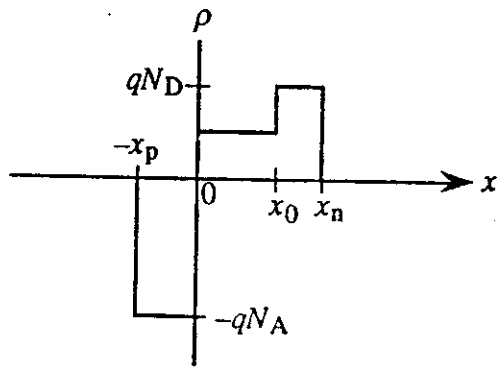
5.10

(a) Given that $x_n > x_0$ for all applied biases of interest, then surely $x_n > x_0$ under equilibrium ($V_A = 0$) conditions. Now, the derivation of the Eq. (5.8) V_{bi} relationship is valid for an arbitrary nondegenerate doping profile. Moreover, $n(-x_p) = n_i^2/N_A$, and wit $x_n > x_0$, $n(x_n) = N_D$ for the given junction profile, just like in a standard step junction. Assuming the p- and $x_n > x_0$ n-region dopings to be nondegenerate, the standard Eq. (5.10) result therefore applies,

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

(This problem points out that only the dopings at the depletion region edges are relevant i determining V_{bi}.)

(b) Since $\rho = q(N_D - N_A)$ for $-x_p \le x \le x_n$ and zero elsewhere under the depletion approximation, we conclude



(c) Invoking the depletion approximation gives

$$\rho = \begin{cases} 0 & \ldots x < -x_p \\ -qN_A & \ldots -x_p \le x \le 0 \\ qN_D/2 & \ldots 0 \le x \le x_0 \\ qN_D & \ldots x_0 \le x \le x_n \\ 0 & \ldots x > x_n \end{cases}$$

Substituting into Poisson's equation then yields

$$\frac{d\mathcal{E}}{dx} = \begin{cases} -qN_A/K_S\varepsilon_0 & \ldots -x_p \le x \le 0 \\ qN_D/2K_S\varepsilon_0 & \ldots 0 \le x \le x_0 \\ qN_D/K_S\varepsilon_0 & \ldots x_0 \le x \le x_n \end{cases}$$

Separating variables and integrating from the depletion region edges where $\mathcal{E} = 0$ to arbitrary points in the p-region and $x_n > x_0$ n-region yield the same relationships and results as in the standard step-junction analysis. To obtain the $0 \le x \le x_0$ solution, we can either integrate from $x = 0$ where $\mathcal{E}$ is known from the p-region solution to an arbitrary point in the $0 \le x \le x_0$ region, or we can start the integration at $x = x_n$ where $\mathcal{E}$ is known from the $x_n > x_0$ n-region solution and integrate backward into the lighter-doped n-region. Taking the former approach we can write

$$\int_{\mathcal{E}(0)}^{\mathcal{E}(x)} d\mathcal{E} = \frac{qN_D}{2K_S\varepsilon_0}\int_0^x dx'$$

or

$$\mathcal{E}(x) = \mathcal{E}(0) + \frac{qN_D}{2K_S\varepsilon_0}x = \frac{-qN_A}{K_S\varepsilon_0}x_p + \frac{qN_D}{2K_S\varepsilon_0}x \qquad \ldots 0 \le x \le x_0$$

If the integration is performed from $x = x_n$ backward into the lighter doped n-region, one obtains the equivalent result

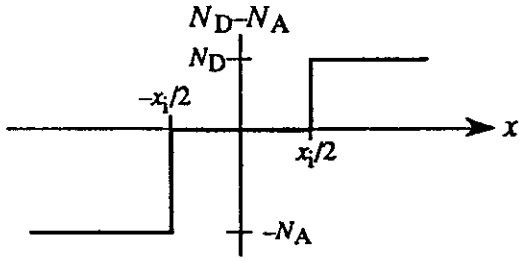
$$\mathcal{E}(x) = -\frac{qN_D}{K_S\varepsilon_0}\left(x_n - \frac{x_0}{2} - \frac{x}{2}\right) \qquad \ldots 0 \le x \le x_0$$

Thus the final result is

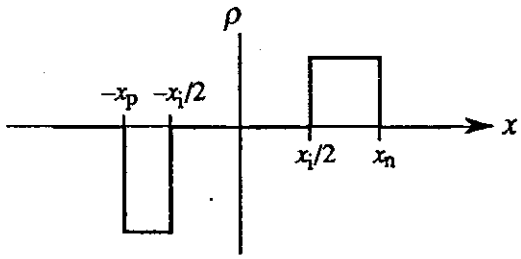
$$\mathcal{E}(x) = \begin{cases} -(qN_A/K_S\varepsilon_0)(x_p+x) & \ldots -x_p \le x \le 0 \\ -(q/K_S\varepsilon_0)(N_A x_p - N_D x/2) & \ldots 0 \le x \le x_0 \\ \quad \text{or} & \\ -(qN_D/K_S\varepsilon_0)(x_n - x_0/2 - x/2) & \ldots 0 \le x \le x_0 \\ -(qN_D/K_S\varepsilon_0)(x_n - x) & \ldots x_0 \le x \le x_n \end{cases}$$

5.11

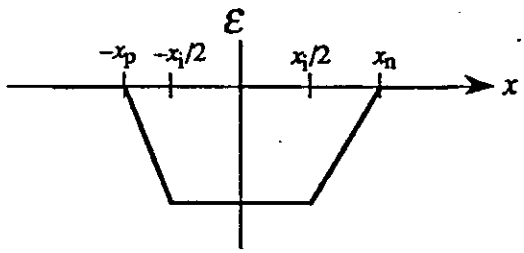
(a)



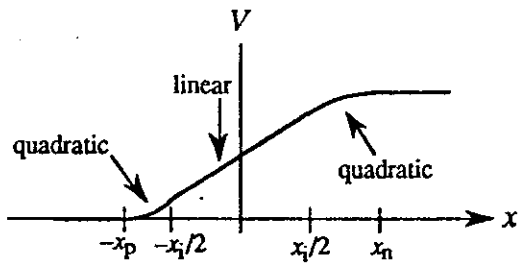
N_A assumed greater than N_D

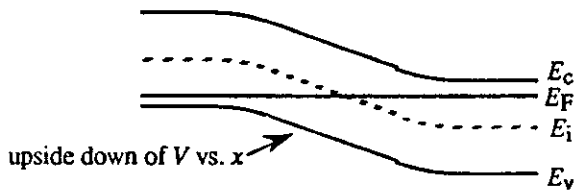


(+ and – areas must be the same size.)



(Since $\rho = 0$ in the i-region, the electric field is constant in that region. One expects a step-junction type solution outside the i-region.)





(b) The derivation of the Eq. (5.8) V_{bi} relationship is valid for an arbitrary doping profile. Moreover, $n(x_n) = N_D$ and $n(-x_p) = n_i^2/N_A$ for the p-i-n diode just like a step-junction pn diode. Assuming the p- and n- region dopings to be nondegenerate, the standard Eq.(5.10) result therefore applies and

$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

(This problem points out that only the dopings at the depletion region edges are relevant in determining V_{bi}.)

(c) Invoking the depletion approximation, we can write

$$\rho = \begin{cases} 0 & \ldots x < -x_p \\ -qN_A & \ldots -x_p \le x \le -x_i/2 \\ 0 & \ldots -x_i/2 \le x \le x_i/2 \\ qN_D & \ldots x_i/2 \le x \le x_n \\ 0 & \ldots x > x_n \end{cases}$$

Substitution into Poisson's equation yields

$$\frac{d\mathcal{E}}{dx} = \begin{cases} -qN_A/K_S\varepsilon_0 & \ldots -x_p \le x \le -x_i/2 \\ 0 & \ldots -x_i/2 \le x \le x_i/2 \\ qN_D/K_S\varepsilon_0 & \ldots x_i/2 \le x \le x_n \end{cases}$$

Separating variables and integrating from the depletion region edges where $\mathcal{E} = 0$ to arbitrary points in the n- and p-regions yield the same relationships and results as in the step junction analysis. In the i-region, $\mathcal{E}$ = constant = $\mathcal{E}(-x_i/2)$. Thus we conclude

$$\mathcal{E}(x) = \begin{cases} -(qN_A/K_S\varepsilon_0)(x_p+x) & \ldots -x_p \le x \le -x_i/2 \\ -(qN_A/K_S\varepsilon_0)(x_p-x_i/2) & \ldots -x_i/2 \le x \le x_i/2 \\ -(qN_D/K_S\varepsilon_0)(x_n-x) & \ldots x_i/2 \le x \le x_n \end{cases}$$

Setting $\mathcal{E}(x) = -dV/dx$, separating variables, and integrating from the depletion region edges to arbitrary points in the n- and p-regions again yields the same relationships and results as in the step junction analysis. Introducing $\mathcal{E}(-x_i/2) \equiv \mathcal{E}_i$ and $V(-x_i/2) \equiv V_i$, we note that in the i-region $dV/dx = -\mathcal{E}_i$ and

$$\int_{V_i}^{V(x)} dV = -\mathcal{E}_i \int_{-x_i/2}^{x} dx'$$

or

$$V(x) = V_i - \mathcal{E}_i(x+x_i/2)$$

Thus

$$V(x) = \begin{cases} (qN_A/2K_S\varepsilon_0)(x_p+x)^2 & \ldots -x_p \le x \le -x_i/2 \\ (qN_A/2K_S\varepsilon_0)[(x_p-x_i/2)(x_p+x_i/2+2x)] & \ldots -x_i/2 \le x \le x_i/2 \\ V_{bi}-V_A-(qN_D/2K_S\varepsilon_0)(x_n-x)^2 & \ldots x_i/2 \le x \le x_n \end{cases}$$

To determine x_n and x_p we require $\mathcal{E}(x)$ and $V(x)$ to be continuous at $x = x_i/2$, or

$$N_A(x_p-x_i/2) = N_D(x_n-x_i/2)$$

and

$$(qN_A/2K_S\varepsilon_0)[(x_p-x_i/2)(x_p+3x_i/2)] = V_{bi}-V_A-(qN_D/2K_S\varepsilon_0)(x_n-x_i/2)^2$$

Solving for $x_n-x_i/2$ from the first equation directly above, substituting into the second equation, and rearranging, gives

$$(x_p-x_i/2)^2 + \frac{2N_Dx_i}{N_A+N_D}(x_p-x_i/2) - \frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}(V_{bi}-V_A) = 0$$

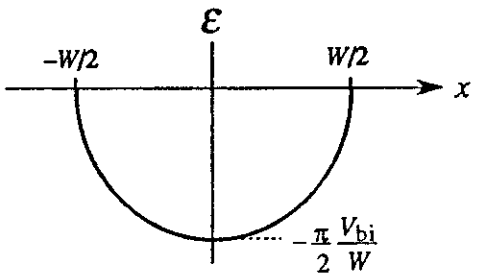
Finally, solving the quadratic equation yields

$$x_p-\frac{x_i}{2} = \frac{N_D}{N_A}\left(x_n-\frac{x_i}{2}\right) = \frac{N_Dx_i}{N_A+N_D} + \left[\left(\frac{N_Dx_i}{N_A+N_D}\right)^2 + \frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}(V_{bi}-V_A)\right]^{1/2}$$

where the (+) root has been chosen because $x_p-x_i/2$ must be greater than zero. Note that the result here reduces to Eq. (5.34) if $x_i \to 0$.

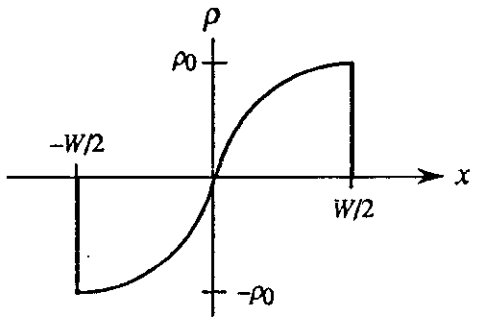
5.12

(a) $$\mathcal{E}(x) = -\frac{dV}{dx} = -\frac{\pi}{2}\frac{V_{bi}}{W}\cos\left(\frac{\pi x}{W}\right) \qquad \ldots -\frac{W}{2} \le x \le \frac{W}{2}$$



(b) Poisson's equation states $d\mathcal{E}/dx = \rho/K_S\varepsilon_0$. Thus

$$\rho(x) = K_S\varepsilon_0\frac{d\mathcal{E}}{dx} = \left(\frac{\pi^2}{2}\right)\left(\frac{K_S\varepsilon_0 V_{bi}}{W^2}\right)\sin\left(\frac{\pi x}{W}\right) \equiv \rho_0\sin\left(\frac{\pi x}{W}\right) \qquad \ldots -\frac{W}{2} \le x \le \frac{W}{2}$$



(c) Under the depletion approximation

$$\rho = q(N_D - N_A) \qquad \ldots -W/2 \le x \le W/2$$

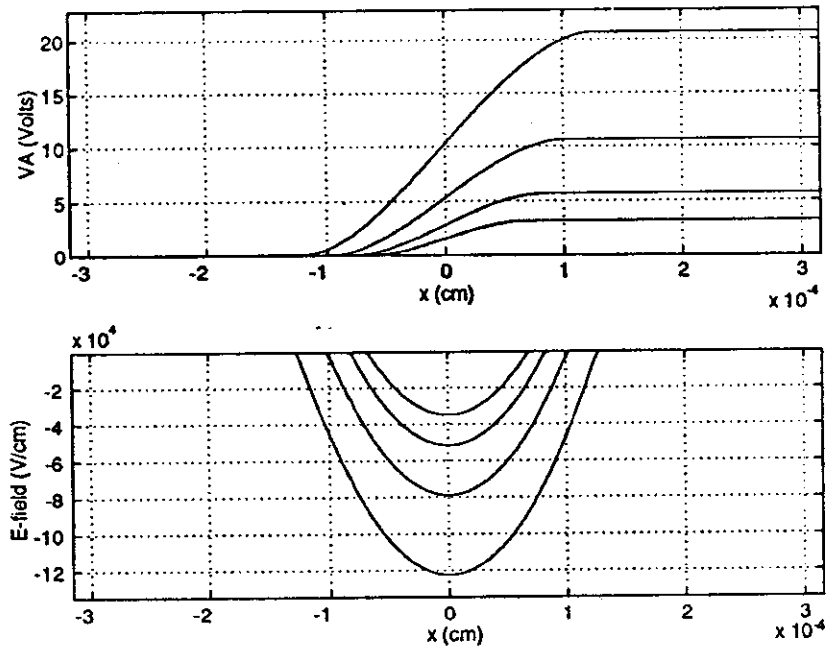
Thus

$$N_D - N_A = \rho/q \qquad \ldots -W/2 \le x \le W/2$$

The N_D–N_A plot is identical in shape to the ρ-plot, with the limiting y-axis values modified to $-\rho_0/q$ and ρ_0/q.

5.13

(a)/(b)/(c) Sample linearly graded junction results, $\mathcal{E}$ versus x and V versus x plots plus an output list of relevant parameters and computational constants, are displayed below. These results were generated by the MATLAB program P_05_13.m listed on the following pages. Note that Vbi.m, a subprogram used in the iteration procedure to determine the built-in voltage, must be present at runtime. We might metion that, having been developed later, the program presented here is somewhat more sophisticated and efficient than the step-junction computational program constructed as a solution to Problem 5.7.



COMPUTATIONAL RESULTS

a =1.0000e+20

Vbi =6.6725e-01

VA =-2.0000e+01	-1.0000e+01	-5.0000e+00	-2.5000e+00
W =2.5286e-04	2.0283e-04	1.6428e-04	1.3531e-04
E0 =-1.2260e+05	-7.8888e+04	-5.1748e+04	-3.5110e+04
V0 =2.0667e+01	1.0667e+01	5.6672e+00	3.1672e+00

MATLAB program script...

```
%A Si LINEARLY-GRADED junction maintained at 300K is assumed.
%The ELECTROSTATIC VARIABLES (V,E) are computed and plotted as
%a function of position; the program also provides an output
%of key computational parameters.
%The solution is based on the depletion approximation.
%The function file Vbi.m is required for the program to run.

%User Input
%  a is the dopant gradient constant in cm(-4)
%  VA is the applied voltage in volts

% Initialization
clear;  close
format short e
format compact
global aa e0 ni qq KS k T

% Constants
T=300;              % Temperature in Kelvin
k=8.617e-5;         % Boltzmann constant
e0=8.85e-14;        % permittivity of free space
ni=1e10;            % intrinsic concentration in Si at 300K
q=1.602e-19;        % electronic charge
KS=11.8;            % Dielectric constant of Si at 300K

%User input
disp('');
disp('Linearly Graded Junction Electrostatics');
a=input('Please enter a (cm-4), a=');
VA0=input('Please enter VA (volts), VA=');

%Iterate to determine Vbi
aa=a; %This is necessary because the fzero function uses a and q
qq=q;
Vbi=fzero('Vbi',1);

%Computational constants
for n = 1:4,
   VA(n)=VA0/2^(n-1);
   W(n)=(12*KS*e0*(Vbi-VA(n))/(q*a))^(1/3);
   xn(n)=W(n)/2;
   xp(n)=W(n)/2;
end

%Computation proper
xMAX=2.5*max(xn);
x=[linspace(-xMAX,0) linspace(0,xMAX)];
for n=1:4,
   Ex(n,:)=(x>=-xn(n)&x<=xn(n)).*q*a.*(x.^2-xn(n)^2)/(2*KS*e0);
   Vmax(n)=Vbi-VA(n);
```

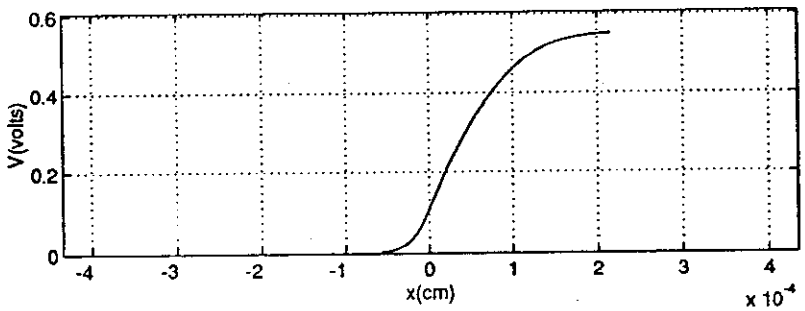
```
   Vx(n,:)=(x>=-xn(n)&x<=xn(n)).*q*a/(6*KS*e0).*(2*(xn(n))^3 ...
   +3*(xn(n))^2.*x- x.^3)+(x>xn(n))*Vmax(n);
end

% plot V vs x
subplot(2,1,1),  plot(x,Vx);  grid on
axis([-xMAX,xMAX,0,1.1*max(Vmax)])
xlabel('x (cm)');  ylabel('VA (Volts)')

% plot E-field vs x
Exx=[Ex(1,:) Ex(2,:) Ex(3,:) Ex(4,:)];
Emin=1.1*min(Exx);
subplot(2,1,2),  plot(x,Ex);  grid on
axis([-xMAX xMAX Emin 0])
xlabel('x (cm)');  ylabel('E-field (V/cm)')

pause

%Print out numerical results to Command window
fprintf('\n\nCOMPUTATIONAL RESULTS\n')
a, Vbi, VA, W, E0=Ex(:,100)', V0=Vx(:,200)'
```

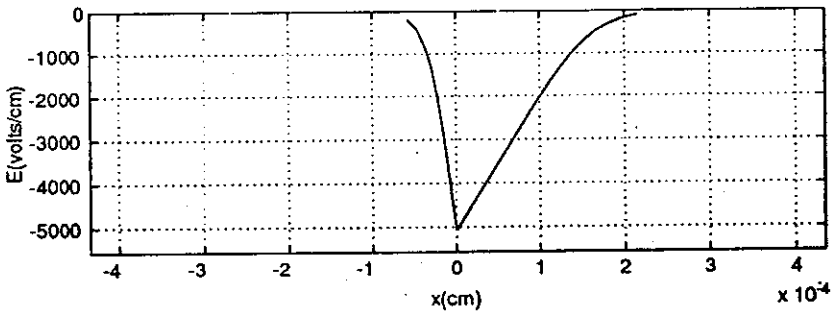
Subprogram Vbi.m

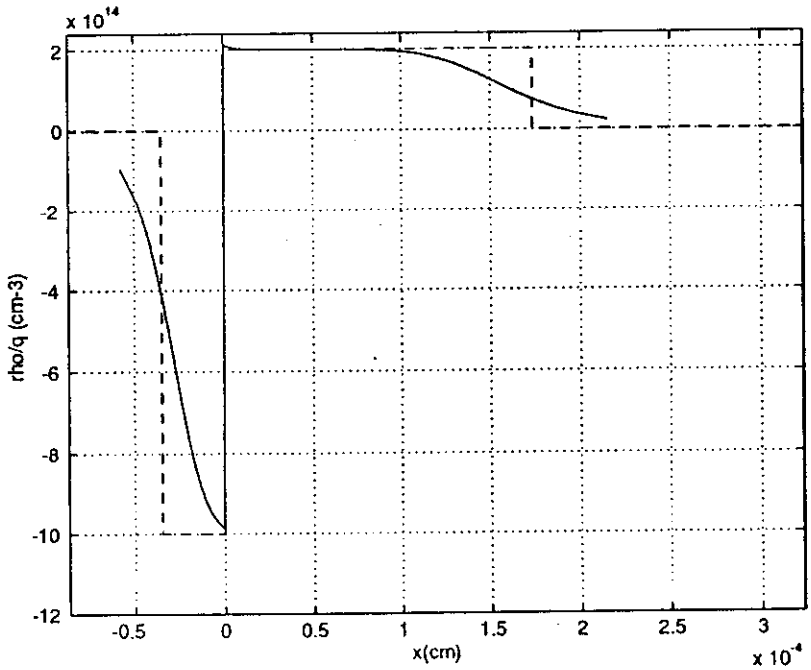
```
function y = f(Vbi);
global aa e0 ni qq KS k T
q=qq; a=aa;
y = Vbi-2*k*T*log(a*(12*KS*e0*Vbi/(q*a))^(1/3)/(2*ni));
```

5.14

(a)/(b) The exact solution $\mathcal{E}$, V, and ρ versus x computations for both parts (a) and (b) are performed by the m-file P_05_14.m on the Instructor's disk. The program may take several minutes to execute. Sample output is displayed on the next page. When the exact $\mathcal{E}$ and V vs. x solutions for a step junction are compared with the approximate solutions based on the depletion approximation, one finds the two sets of solutions are very similar in the vicinity of the metallurgical junction but deviate significantly as one approaches the semiconductor bulk. Whereas the $\mathcal{E}$ and V results are pretty much as expected, the exact ρ/q vs. x plot may come as somewhat of a surprise. Under equilibrium conditions, the true charge density profile is very crudely modeled by the depletion approximation, especially on the heavily doped side of the junction. Also, if examined carefully, the charge density plot exhibits a slight peak near $x = 0$ on the lightly doped side of the junction. The cited feature reflects an *excess of minority carriers* or "inversion layer" on the lightly doped side of the junction. This spill-over of carriers from the other side of the junction becomes more pronounced for highly asymmetrical junctions and leads to the computed difference in the $x = 0$ $\mathcal{E}$-field. We should note that the "inversion layer" disappears and the depletion approximation dramatically improves with reverse biasing.


V(volts)
0.6
0.4
0.2
0
-4
-3
-2
-1
0
1
2
3
4
x(cm)
x 10^-4


E(volts/cm)
0
-1000
-2000
-3000
-4000
-5000
-4
-3
-2
-1
0
1
2
3
4
x(cm)
x 10^-4


x 10^14
rho/q (cm-3)
2
0
-2
-4
-6
-8
-10
-12
-0.5
0
0.5
1
1.5
2
2.5
3
x(cm)
x 10^-4