CHAPTER 17

17.1

(a) Carriers enter the channel at the source contact and leave the channel (or are "drained") at the drain contact.

(b) Channel...inversion layer beneath the MOS gate which electrically connects the source and drain.

(c) The portion of the characteristics where $V_D > V_{Dsat}$ for a given V_G (the approximately horizontal portion of the characteristics) is referred to as the saturation region of operation.

(d) The depletion–inversion transition point voltage and the threshold voltage are one and the same voltage.

(e) There is an additional carrier scattering mechanism in the surface channel of a MOSFET; namely, surface scattering. With increased scattering the mobility decreases.

(f) The square-law name arises from the fact that I_{Dsat} varies as the square of V_{Dsat} in this first-order formulation (see Eq. 17.22).

(g) The bulk-charge theory gets its name from the fact that source–to–drain variations in the depletion–layer or "bulk" charge are modeled correctly in the formulation.

(h) I_D versus V_G with V_D held constant.

(i)
$$g_d \equiv \left.\frac{\partial I_D}{\partial V_D}\right|_{V_G=\text{constant}} ; \qquad g_m \equiv \left.\frac{\partial I_D}{\partial V_G}\right|_{V_D=\text{constant}}$$

(j) The source and drain islands in a MOSFET supply the minority carriers required to obtain a low-frequency characteristic. Under inversion conditions minority carriers merely use the surface channel to flow laterally into and out of the MOS gate area in response to the applied ac signal.

17.2
(a)

$$\phi_F = \frac{kT}{q} \ln (N_A/n_i) = 0.0259 \ln(10^{15}/10^{10}) = 0.298\text{V}$$

$$V_T = 2\phi_F + \frac{K_S x_o}{K_O} \sqrt{\frac{4qN_A}{K_S \varepsilon_0} \phi_F} \qquad ...(17.1a)$$

$$= (2)(0.298) + \frac{(11.8)(5\times10^{-6})}{(3.9)} \left[\frac{(4)(1.6\times10^{-19})(10^{15})(0.298)}{(11.8)(8.85\times10^{-14})}\right]^{1/2}$$

$$\mathbf{V_T = 0.800\ V}$$

(b) In the square-law theory

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L} (V_G - V_T)^2 \qquad ...(17.22)$$

$$C_o = \frac{K_O \varepsilon_0}{x_o} = \frac{(3.9)(8.85\times10^{-14})}{(5\times10^{-6})} = 6.90 \times 10^{-8}\ \text{F/cm}^2$$

$$I_{Dsat} = \frac{(5\times10^{-3})(800)(6.9\times10^{-8})(2-0.8)^2}{(2)(5\times10^{-4})} = \mathbf{0.397mA}$$

(c) In the bulk-charge theory we must first determine V_{Dsat} using Eq.(17.29). We know ϕ_F and V_T from part (a), but must compute V_W before substituting into the V_{Dsat} expression.

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_A}(2\phi_F)\right]^{1/2} = \left[\frac{(2)(11.8)(8.85\times10^{-14})(2)(0.298)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2} = 0.882\mu\text{m}$$

$$V_W \equiv \frac{qN_A W_T}{C_o} = \frac{(1.6\times10^{-19})(10^{15})(8.82\times10^{-5})}{(6.90\times10^{-8})} = 0.205\text{V}$$

Noting that $V_G - V_T = 1.20\text{V}$, substituting into Eq.(17.29) then gives

$$V_{Dsat} = 1.20 - 0.205\left\{\left[\frac{(1.20)}{(2)(0.298)} + \left(1 + \frac{(0.205)}{(4)(0.298)}\right)^2\right]^{1/2} - \left[1 + \frac{(0.205)}{(4)(0.298)}\right]\right\}$$

or

$$V_{Dsat} = 1.06\text{V} \qquad \text{...smaller than } V_{Dsat} \text{ of square-law theory as expected}$$

Now

$$\frac{Z\bar{\mu}_n C_o}{L} = \frac{(5\times10^{-3})(800)(6.90\times10^{-8})}{(5\times10^{-4})} = 5.52\times10^{-4}\ \text{amps/V}^2$$

Finally, substituting into Eq.(17.28) gives I_{Dsat} if $V_D = V_{Dsat}$. Thus

$$I_{Dsat} = (5.52\times10^{-4})\left\{(1.20)(1.06) - \frac{(1.06)^2}{2} - \frac{4}{3}(0.205)(0.298)\left[\left(1+\frac{(1.06)}{(2)(0.298)}\right)^{3/2} - \left(1+\frac{(3)(1.06)}{(4)(0.298)}\right)\right]\right\}$$

$\mathbf{I_{Dsat} = 0.349mA}$ ⇐bulk charge result (smaller than the square-law result as expected)

(d) Clearly here the device is biased below pinch-off. From Table 17.1 we note that both the square-law and bulk-charge theories reduce to the same result if $V_D = 0$.

$$g_d = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T) = (5.52\times10^{-4})(2-0.8) = \mathbf{0.662mS}$$

(e) In the square-law theory, $V_{Dsat} = V_G - V_T$. Thus $V_{Dsat} = 1.20$V and $V_D = 2$V. Since $V_D > V_{Dsat}$, the device is saturation (above-pinch-off) biased, and from Table 17.1

$$g_m = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T) = \mathbf{0.662mS} \qquad \text{...same as } g_d \text{ of part (d)}$$

(f) In part (c) we calculated the bulk-charge $V_{Dsat} = 1.06$V. Since $V_D > V_{Dsat}$, the device is above-pinch-off biased, and from Table 17.1

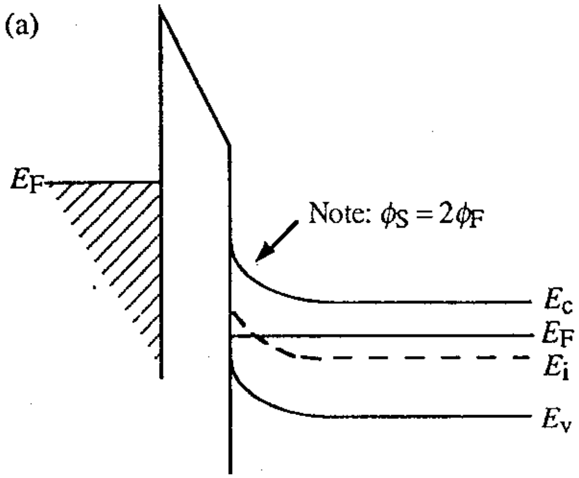
$$g_m = \frac{Z\bar{\mu}_n C_o}{L} V_{Dsat} = (5.52\times10^{-4})(1.06) = \mathbf{0.585mS}$$

(g) For the applied $V_G = 2$V, $V_{Dsat} = 1.20$V in the square-law theory and $V_{Dsat} = 1.06$V in the bulk-charge theory. Since in either case $V_D < V_{Dsat}$, we can utilize the second form of Eq.(17.37).

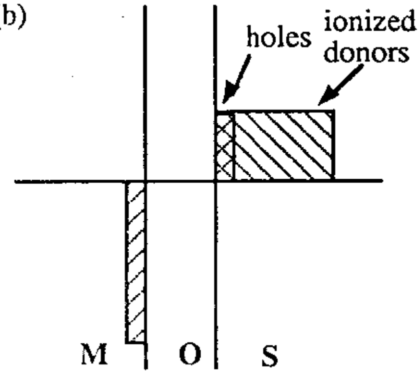
$$f_{max} = \frac{\bar{\mu}_n V_D}{2\pi L^2} = \frac{(800)(1)}{(2\pi)(5\times10^{-4})^2} = \mathbf{509MHz}$$

17.3

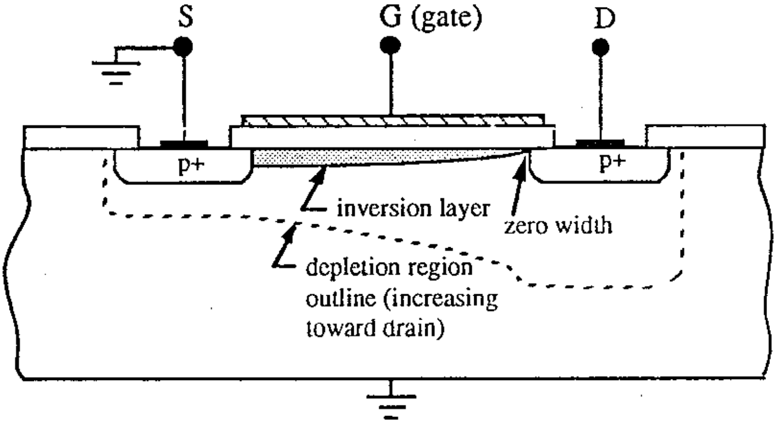
(a)


E_F
Note: $\phi_S = 2\phi_F$
E_c
E_F
E_i
E_v

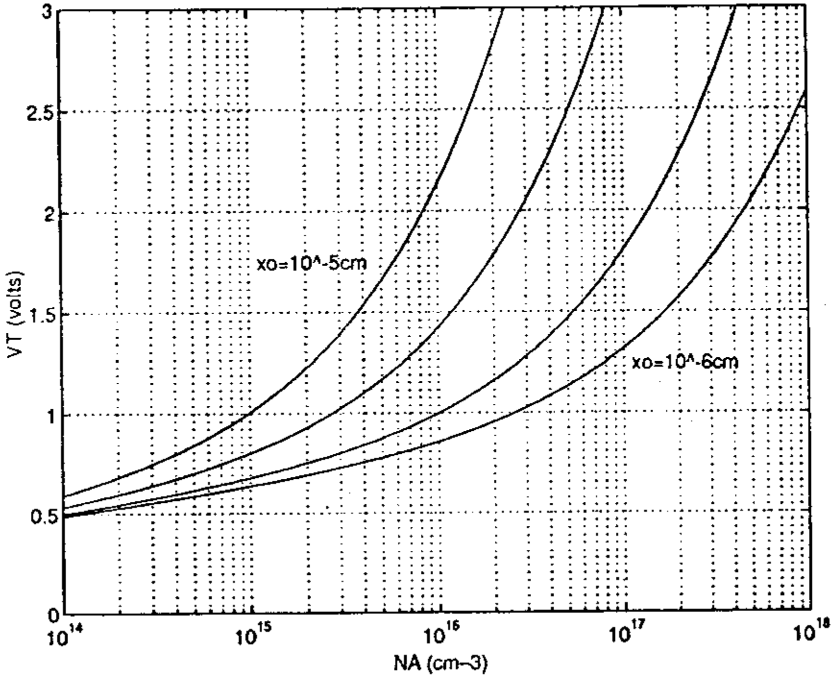
(b)


holes
ionized donors
M
O
S

(c)


S
G (gate)
D
p+
p+
inversion layer
zero width
depletion region outline (increasing toward drain)

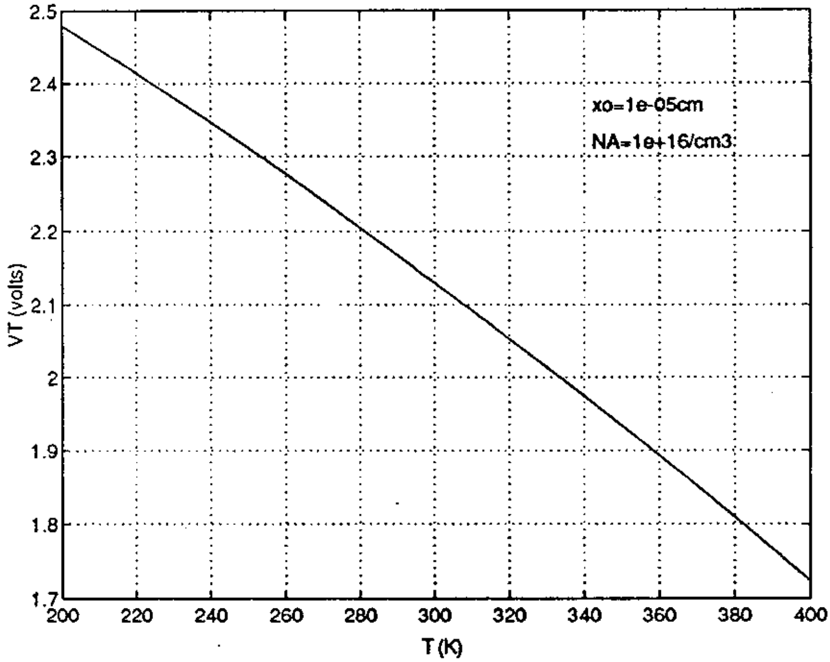
17.4



MATLAB program script...

```
%Problem 17.4...VT vs. NA with xo as a parameter
%Initialization
clear;  close
%Constants and Parameters
q=1.6e-19;   e0=8.85e-14;
kT=0.0259;   ni=1.0e10;
KS=11.8;     KO=3.9;
NA=logspace(14,18);
xo=[1.0e-6  2.0e-6  5.0e-6  1.0e-5];
%VT Computation
øF=kT.*log(NA./ni);
for i=1:4,
  xoo=xo(i);
  VT= 2 .*øF+((KS*xoo)/KO).*sqrt((4 .*q.*NA.*øF)./(KS*e0));
  semilogx(NA,VT);  axis([1.0e14,1.0e18,0,3])
  hold on
end
grid;  xlabel('NA (cm-3)');  ylabel('VT (volts)')
text(1.1e17,1.25,'xo=10^-6cm');  text(1.1e15,1.75,'xo=10^-5cm');
hold off
```

17.5



The threshold voltage is seen to decrease in an almost linear fashion with increasing T.

MATLAB program script...

```
%Problem 17.5...VT vs. T

%Initialization
clear;  close

%Constants and Parameters
q=1.6e-19;  k=8.617e-5;
e0=8.85e-14;
KS=11.8;  KO=3.9;
T=linspace(200,400);
kT=k.*T;
xo=input('Input the oxide thickness in cm, xo = ');
NA=input('Input the Si doping in cm-3, NA = ');

%ni versus T
 %Constants
A=2.510e19;
Eex=0.0074;
 %Band Gap vs. T
EG0=1.17;
```

```
a=4.730e-4;
b=636;
EG=EG0-a.*(T.^2)./(T+b);
 %Effective mass ratio (mnr=mn*/m0, mpr=mp*/m0)
mnr=1.028 + (6.11e-4).*T - (3.09e-7).*T.^2;
mpr=0.610 + (7.83e-4).*T - (4.46e-7).*T.^2;
 %Computation of ni
ni=A.*((T./300).^(1.5)).*((mnr.*mpr).^(0.75)).*exp(-(EG-Eex)./(2
.*k.*T));
%VT Computation
øF=kT.*log(NA./ni);
VT= 2 .*øF+((KS*xo)/KO).*sqrt((4 .*q.*NA.*øF)./(KS*e0));
plot(T,VT); grid
xlabel('T (K)'); ylabel('VT (volts)')
text(342,2.37,['xo=',num2str(xo),'cm'])
text(342,2.32,['NA=',num2str(NA),'/cm3'])
```

17.6

Differentiating Eq.(17.17) with respect to V_D with V_G held constant yields

$$\left.\frac{\partial I_D}{\partial V_D}\right|_{V_G=\text{constant}} = \frac{Z\,\bar{\mu}_n C_o}{L}(V_G - V_T - V_D) \overset{\text{set}}{=} 0$$

Solving we obtain

$$V_G - V_T - V_{Dsat} = 0$$

or

$$\mathbf{V_{Dsat} = V_G - V_T}$$

17.7

$$J_P \cong J_{Py} \cong q\mu_p p \mathcal{E}_y \cong -q\mu_p p \frac{d\varphi}{dy} \qquad (17.7')$$

$$I_D = \int\!\!\int J_{Py}\, dx dz = Z\int_0^{x_c(y)} J_{Py}\, dx \qquad (17.8a')$$

(Note that I_D is defined to be positive flowing *out-of* the drain.)

$$I_D = \left(-Z\frac{d\phi}{dy}\right)\left(q\int_0^{x_c(y)} \mu_p(x,y)p(x,y)dx\right) \tag{17.8b'}$$

$$Q_P(y) = q\int_0^{x_c(y)} p(x,y)dx \tag{17.3'}$$

$$\bar{\mu}_p = \frac{q}{Q_P(y)}\int_0^{x_c(y)} \mu_p(x,y)p(x,y)dx \tag{17.4'}$$

$$I_D = -Z\bar{\mu}_p Q_P \frac{d\phi}{dy} \tag{17.9'}$$

$$\int_0^L I_D dy = I_D L = -Z\int_0^{V_D} \bar{\mu}_p Q_P \, d\phi \tag{17.10'}$$

$$I_D = -\frac{Z\bar{\mu}_p}{L}\int_0^{V_D} Q_P \, d\phi \qquad \text{(NOTE: } V_D \le 0\text{, which gives } I_D \text{ the correct sign.)} \tag{17.11'}$$

$$\Delta Q_{gate} = -\Delta Q_{semi} \cong -Q_P \tag{17.12'}$$

$$\Delta Q_{gate} = C_o(V_G - V_T) \tag{17.13'}$$

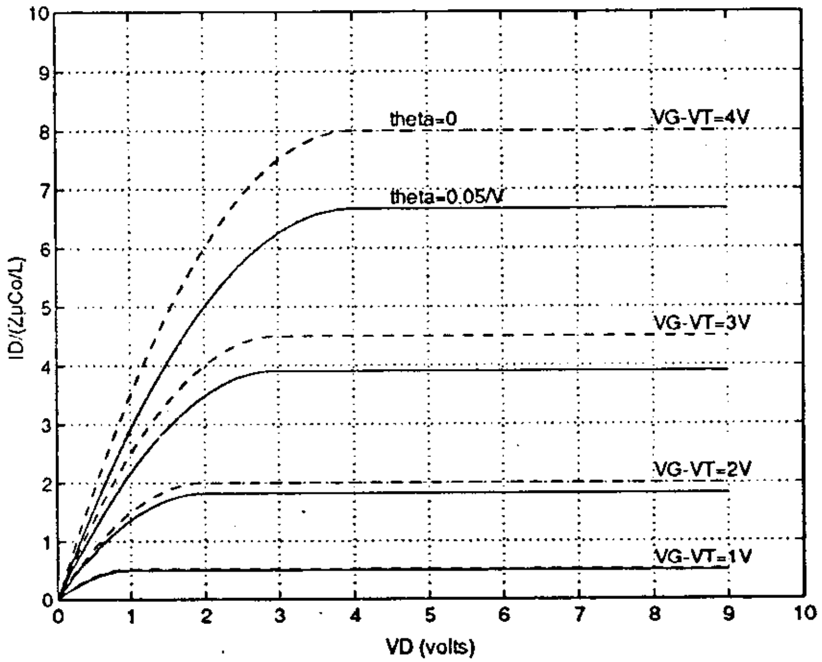
$$Q_P \cong -C_o(V_G - V_T) \tag{17.14'}$$

$$Q_P(y) \cong -C_o(V_G - V_T - \phi) \tag{17.16'}$$

$$I_D = \frac{Z\bar{\mu}_p C_o}{L}\left[(V_G - V_T)V_D - V_D^2/2\right] \qquad \begin{array}{l} \ldots 0 \ge V_D \ge V_{Dsat} \\ \ldots V_G \le V_T \end{array} \tag{17.17'}$$

Note that Eq.(17.17') is the same as the text Eq.(17.17) except $\bar{\mu}_n \rightarrow \bar{\mu}_p$ and there is a polarity reversal in the inequalities specifying the range of valid V_D and V_G values.

17.8



MATLAB program script...

```
%Problem 17.8...effective mobility per Eq. (17.5)
%ID-VD Characteristics /// Square-Law Theory

%Initialization
clear; close

%Let VGT = VG - VT;
for VGT=4:-1:1,
 %Primary Computation
  VD=linspace(0,VGT);
  ID0=VGT.*VD-VD.*VD./2;
  ID0sat=VGT*VGT/2;
  ID0=[ID0,ID0sat];
  ID1=(VGT.*VD-VD.*VD./2)./(1+0.05*VGT);
  ID1sat=(VGT*VGT/2)/(1+0.05*VGT);
  ID1=[ID1,ID1sat];
  VD=[VD,9];
 %Plotting and Labeling
  if VGT==4,
  plot(VD,ID0,'g--',VD,ID1,'r');  grid
  axis([0 10 0 10])
  xlabel('VD (volts)');  ylabel('ID/(ZμCo/L)')
  text(8,ID0sat+0.2,'VG-VT=4V')
  text(4.5,ID0sat+0.2,'theta=0')
  text(4.5,ID1sat+0.2,'theta=0.05/V')
  hold on
  else,
  plot(VD,ID0,'g--',VD,ID1,'r');
 %Labeling of VG-VT curves < 4
  if VGT==3,
  text(8,ID0sat+0.2,'VG-VT=3V');
  elseif VGT==2,
  text(8,ID0sat+0.2,'VG-VT=2V');
  else,
  text(8,ID0sat+0.2,'VG-VT=1V');
  end
  end
end
```

17.9

(a) From Fig. P17.9 we note in general that

$$V_G = V_D + V_B \qquad \text{or} \qquad V_D = V_G - V_B$$

In the square-law formulation $V_{Dsat} = V_G - V_T$. If $V_B = V_T/2$, then $V_D = V_G - V_T/2 > V_{Dsat}$ and the MOSFET is *always biased into saturation.* Noting $I_D = 0$ if $V_G < V_T$ or $V_D < V_T/2$, and using Eq.(17.22), we conclude

$$I_D = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2 = \frac{Z\bar{\mu}_n C_o}{2L}(V_D - V_T/2)^2 \qquad ...V_D > V_T/2$$

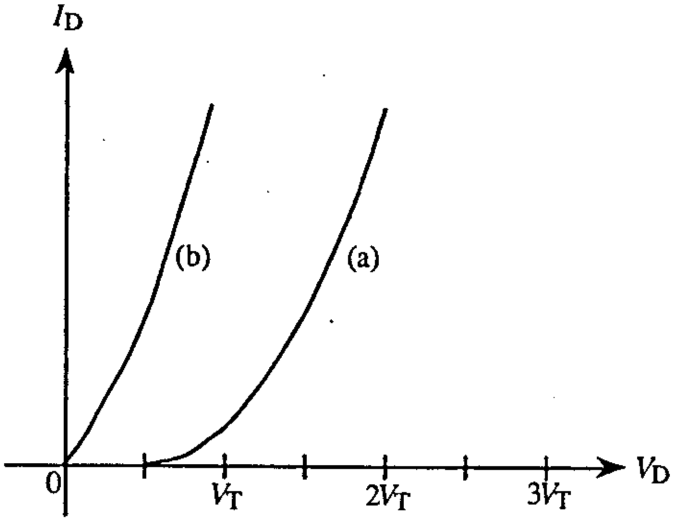
and

$$I_D = 0 \qquad ...V_D < V_T/2$$

(b) If $V_B = 2V_T$, then $V_D = V_G - 2V_T < V_G - V_T = V_{Dsat}$ and the MOSFET is always biased in the linear region of operation. The device turns on for $V_G > V_T$ or $V_D > -V_T$ and is therefore on for all $V_D \geq 0$. Using Eq.(17.17) we obtain

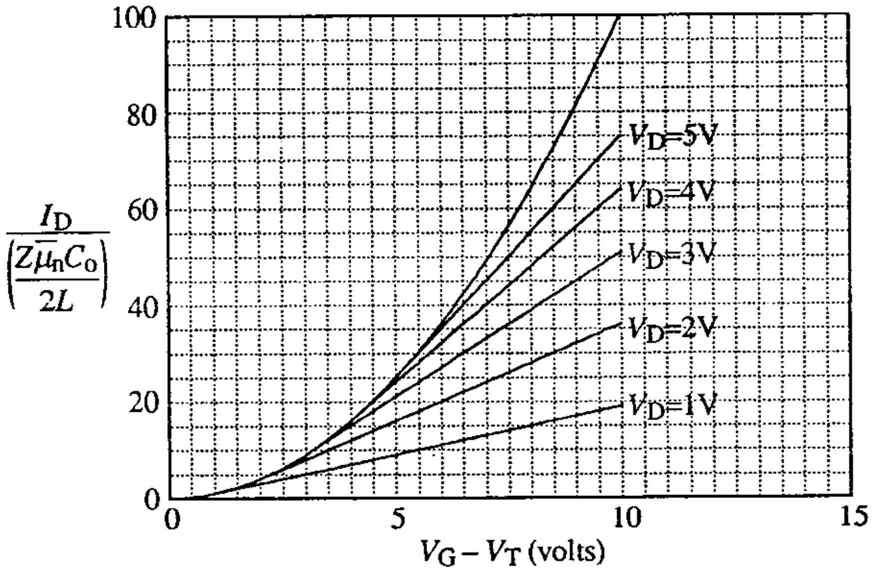
$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_D - V_D^2/2\right] = \frac{Z\bar{\mu}_n C_o}{L}\left[(V_D + V_T)V_D - V_D^2/2\right]$$

$$= \frac{Z\bar{\mu}_n C_o}{L}\left(V_D^2/2 + V_T V_D\right) = \frac{Z\bar{\mu}_n C_o}{2L}\left[(V_D + V_T)^2 - V_T^2\right] \qquad ...V_D \geq 0$$



Note that both curves have the same general shape; the part (b) curve is simply shifted to the left and displaced downward.

17.10



The above I_D versus $V_G–V_T$ characteristics were arrived at as follows:

(i) Suppose we systematically increase $V_G–V_T$ from zero with V_D held constant. Initially V_D is greater than V_G-V_T and the device is in saturation. (Use is being made of the square-law theory.) Thus initially

$$I_D = I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2$$

and we conclude I_D *varies as the square of* $V_G–V_T$ if $V_G–V_T < V_D$.

(ii) When $V_G–V_T$ becomes equal to V_D, the device moves into the linear region of operation. In the linear region

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}[(V_G–V_T)V_D - V_D^2/2]$$

and I_D varies linearly with V_G-V_T.

(iii) With increased V_D, one stays on the voltage-squared part of the curve for a longer and longer range of voltages. Once $V_G–V_T > V_D$, a linear region whose slope increases with increasing V_D is observed.

17.11

(a)

$$J_N = J_{Nr} = q\mu_n n\mathcal{E}_r = -q\mu_n n\frac{d\phi}{dr} \tag{17.7'}$$

If the z-direction points from the surface into the bulk,

$$I_D = -\int\int J_{Nr}\,dzd\theta = -\int_0^{z_c} 2\pi r J_{Nr}\,dz \qquad \text{...}z_c \text{ is the channel depth}$$

$$= \left(-2\pi r\frac{d\phi}{dr}\right)\left(-q\int_0^{z_c}\mu_n n\,dz\right) \tag{17.8'}$$

Since the second quantity enclosed in parentheses above is just $\bar{\mu}_n Q_N$, we can write

$$I_D = -2\pi r\bar{\mu}_n Q_N\frac{d\phi}{dr} \tag{17.9'}$$

Integrating over the r-width of the channel,

$$\int_{r_1}^{r_2}\frac{I_D}{2\pi r}dr = \frac{I_D}{2\pi}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{I_D}{2\pi}\ln(r_2/r_1) = -\bar{\mu}_n\int_0^{V_D} Q_N\,d\phi \tag{17.10'}$$

and

$$I_D = -\frac{2\pi}{\ln(r_2/r_1)}\bar{\mu}_n\int_0^{V_D} Q_N\,d\phi \tag{17.11'}$$

The change in geometry does not modify Eqs.(17.12) through (17.16). Thus

$$Q_N = -C_o(V_G - V_T - \phi)$$

and

$$\boxed{I_D = \frac{2\pi}{\ln(r_2/r_1)}\bar{\mu}_n C_o\left[(V_G - V_T)V_D - V_D^2/2\right]}$$

(b) Setting $r_2 = r_1 + L$, we can write

$$\ln(r_2/r_1) = \ln\left(\frac{r_1+L}{r_1}\right) = \ln(1 + L/r_1)$$

If $L/r_1 << 1$

$$\ln(1 + L/r_1) = (L/r_1) - \frac{1}{2}(L/r_1)^2 + \cdots \cong L/r_1$$

Thus

$$\frac{2\pi}{\ln(r_2/r_1)} \rightarrow \frac{2\pi r_1}{L} = \frac{Z}{L}$$

and one obtains the usual $I_D - V_D$ result.

17.12

(a) Utilizing the Eq. (17.22) square-law result,

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2$$

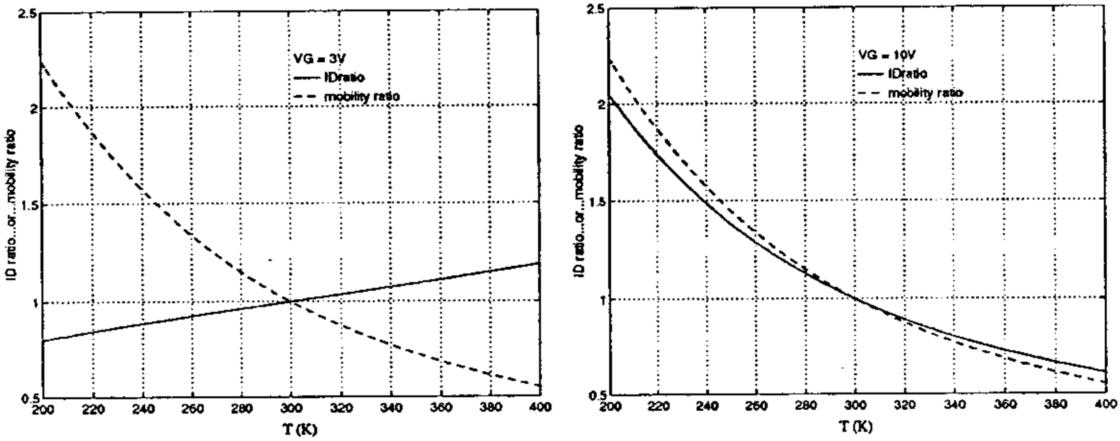
and

$$\frac{I_{Dsat}(T)}{I_{Dsat}(300K)} = \frac{\bar{\mu}_n(T)}{\bar{\mu}_n(300K)}\left[\frac{V_G - V_T(T)}{V_G - V_T(300K)}\right]^2$$

Assuming $\bar{\mu}_n$ has the same temperature dependence as μ_n (and neglecting any differences in the effective mobility as a function of temperature that may result from operating at slightly different $V_G - V_T$ points), we obtain the computational expression

$$\frac{I_{Dsat}(T)}{I_{Dsat}(300K)} = \frac{\mu_n(T)}{\mu_n(300K)}\left[\frac{V_G - V_T(T)}{V_G - V_T(300K)}\right]^2$$

(b)



The results here are rather interesting. If the device is V_G biased far above turn-on, then the $V_G - V_T$ term in the part (a) expression becomes approximately unity and the characteristics exhibit essentially the same temperature dependence as the mobility — generally decreasing with temperature. However, the threshold voltage change with temperature is sufficiently large that a totally different I_{Dsat} temperature-dependence is observed if the chosen V_G is only slightly greater than V_T. — The change in the degree of surface inversion becomes more important than the change in mobility.

It should be noted that in performing the computations the μ_n value in N_A-doped Si was assumed to be the same as that in equivalently N_D-doped Si.

MATLAB program script...

```
%Problem 17.12...IDsat(T)/IDsat(300K) vs. T
%Initialization
clear;  close

%Constants and Parameters
q=1.6e-19;
k=8.617e-5;
KS=11.8;
KO=3.9;
e0=8.85e-14;
T=linspace(200,400,101);  %Note:  T(51)=300K;
kT=k.*T;
xo=1.0e-5;
NA=1.0e16;
VG=input('Input gate voltage in volts, VG = ');
```

```
%ni versus T
 %Constants
A=2.510e19;
Eex=0.0074;
 %Band Gap vs. T
EG0=1.17;
a=4.730e-4;
b=636;
EG=EG0-a.*(T.^2)./(T+b);
 %Effective mass ratio (mnr=mn*/m0, mpr=mp*/m0)
mnr=1.028 + (6.11e-4).*T - (3.09e-7).*T.^2;
mpr=0.610 + (7.83e-4).*T - (4.46e-7).*T.^2;
 %Computation of ni
ni=A.*((T./300).^(1.5)).*((mnr.*mpr).^(0.75)).*exp(-(EG-Eex)./(2
.*k.*T));
%VT Computation
øF=kT.*log(NA./ni);
VT= 2 .*øF+((KS*xo)/KO).*sqrt((4 .*q.*NA.*øF)./(KS*e0));
%Mobility Computation
 %Fit Parameters
NDref=1.3e17;  TNref=2.4;
µnmin=92;      Tµmin=-0.57;
µn0=1268;      Tµn0=-2.33;
an=0.91;       Ta=-0.146;
 %Mobility Calculation
NDrefT=NDref*(T./300).^TNref;
µnminT=µnmin.*(T./300).^Tµmin;
µn0T=µn0.*(T./300).^Tµn0;
anT=an.*(T./300).^Ta;
µn=µnminT+µn0T./(1+(NA./NDrefT).^anT);
%IDsat Computation and Plot
 %IDratio=IDsat(T)/IDsat(300K) and µnratio=µn(T)/µn(300K);
µnratio=µn/µn(51);
IDratio=µnratio.*((VG-VT)/(VG-VT(51))).^2;
 %Plotting result
plot(T,IDratio,T,µnratio,'g--'); grid
xlabel('T (K)'); ylabel('ID ratio...or...mobility ratio')
 %Key
text(302,2.25,['VG = ',num2str(VG),'V'])
x=[302,312]; y1=[2.15,2.15]; y2=[2.05,2.05]
hold on; plot(x,y1,x,y2,'g--')
text(314,2.15,'IDratio')
text(314,2.05,'mobility ratio')
hold off
```

17.13

With R_S and R_D taken into account, the channel voltages at $y=0$ and $y=L$ become $V(0) = I_D R_S$ and $V(L) = V_D - I_D R_D$. Inserting the revised voltage limits into Eq. (17.10), and likewise modifying Eq. (17.11), we obtain

$$I_D = -\frac{Z\bar{\mu}_n}{L}\int_{I_D R_S}^{V_D - I_D R_D} Q_N \, d\phi \tag{17.11'}$$

where in the square-law theory

$$Q_N = -C_o(V_G - V_T - \phi) \tag{17.16}$$

Substituting the Eq. (17.16) expression for Q_N into Eq. (17.11') and integrating yields

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left\{(V_G - V_T)[V_D - I_D(R_S+R_D)] - \frac{(V_D - I_D R_D)^2}{2} + \frac{(I_D R_S)^2}{2}\right\}$$

or

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left\{(V_G - I_D R_S - V_T)[V_D - I_D(R_S+R_D)] - \frac{[V_D - I_D(R_S+R_D)]^2}{2}\right\} \tag{17.17'}$$

Turning next to the modification of Eq. (17.21), we note that when $V_D = V_{Dsat}$, $Q_N(L) = 0$, $\phi(L) = V_{Dsat} - I_{Dsat}R_D$, and from Eq. (17.16),

$$0 = -C_o[V_G - V_T - (V_{Dsat} - I_{Dsat}R_D)]$$

or

$$V_{Dsat} - I_{Dsat}R_D = V_G - V_T \tag{17.21'}$$

Finally, setting $V_D = V_{Dsat}$ and $I_D = I_{Dsat}$ in Eq. (17.17'), and simplifying the result using Eq. (17.21'), one obtains

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - I_{Dsat}R_S - V_T)^2 \tag{17.22'}$$

Note that replacing V_D by $V_D - I_D(R_S+R_D)$ and V_G by $V_G - I_D R_S$ in Eqs. (17.17), (17.21), and (17.22) does indeed yield Eqs. (17.17'), (17.21'), and (17.22').

17.14

Following the text suggestion, we set $Q_N(L) = 0$ in Eq.(17.27) when $V(L) = V_D \to V_{Dsat}$.

$$Q_N(L) = -C_o\left[V_G - V_T - V_{Dsat} - V_W\left(\sqrt{1+V_{Dsat}/2\phi_F} - 1\right)\right] \overset{\text{set}}{=} 0$$

or

$$V_G - V_T - V_{Dsat} - V_W\left(\sqrt{1 + V_{Dsat}/2\phi_F} - 1\right) = 0$$

Manipulating the preceding into a form which can be solved for V_{Dsat} we note

$$V_G - V_T + V_W - V_{Dsat} = V_W\sqrt{1 + V_{Dsat}/2\phi_F}$$

Squaring

$$V_{Dsat}^2 - 2(V_G - V_T + V_W)V_{Dsat} + (V_G - V_T + V_W)^2 = V_W^2 + V_W^2 V_{Dsat}/2\phi_F$$

or

$$V_{Dsat}^2 - \left[\frac{V_W^2}{2\phi_F} + 2(V_G - V_T + V_W)\right]V_{Dsat} + (V_G - V_T + V_W)^2 - V_W^2 = 0$$

Solving the quadratic equation gives

$$V_{Dsat} = \frac{V_W^2}{4\phi_F} + (V_G - V_T + V_W) \pm \left\{\left[\frac{V_W^2}{4\phi_F} + (V_G - V_T + V_W)\right]^2 - (V_G - V_T + V_W)^2 + V_W^2\right\}^{1/2}$$

$$= V_G - V_T + V_W\left(1 + \frac{V_W}{4\phi_F}\right) \pm \left[\left(\frac{V_W^2}{4\phi_F}\right)^2 + 2\frac{V_W^2}{4\phi_F}(V_G - V_T + V_W) + V_W^2\right]^{1/2}$$

$$= V_G - V_T + V_W\left(1 + \frac{V_W}{4\phi_F}\right) \pm V_W\left[\left(\frac{V_W}{4\phi_F}\right)^2 + 2\frac{V_W}{4\phi_F} + \frac{V_G - V_T}{2\phi_F}\right]^{1/2}$$

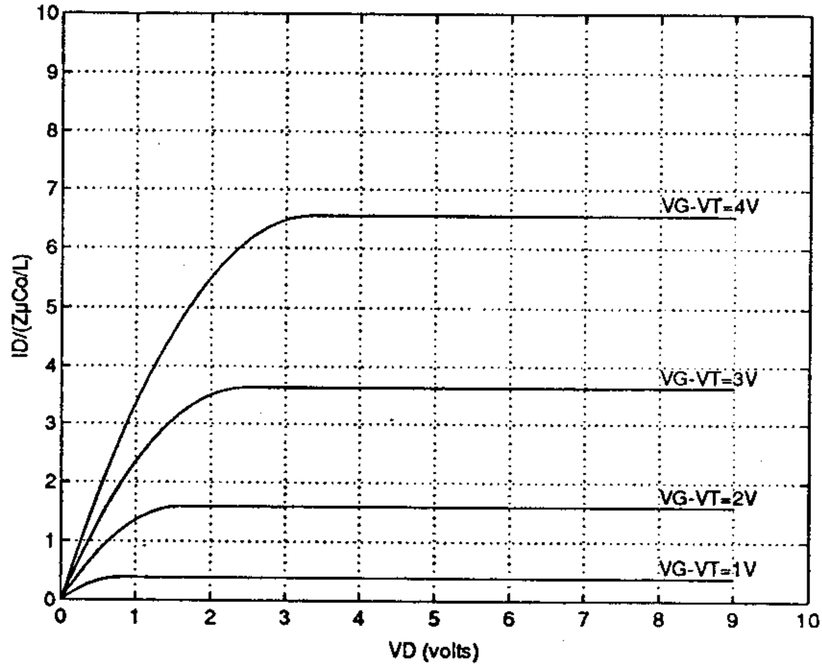
$$= V_G - V_T + V_W\left(1 + \frac{V_W}{4\phi_F}\right) \pm V_W\left[\frac{V_G - V_T}{2\phi_F} + \left(1 + \frac{V_W}{4\phi_F}\right)^2\right]^{1/2}$$

Note that if the (+) root is chosen $V_{Dsat}(\text{+root}) > V_G - V_T$. Choosing the (–) root on the other hand yields $V_{Dsat} < (V_G - V_T)$. As discussed in the text, in the bulk-charge formulation, part of the change in the gate charge goes into balancing changes in the depletion-layer charge. Thus, there is less inversion-layer charge at a given V_G relative to the square-law formulation. Consequently V_{Dsat} occurs at a lower voltage than $V_G - V_T$. We choose the (–) root and slightly rearrange our result to finally obtain Eq.(17.29).

$$V_{Dsat} = V_G - V_T - V_W\left\{\left[\frac{V_G - V_T}{2\phi_F} + \left(1 + \frac{V_W}{4\phi_F}\right)^2\right]^{1/2} - \left(1 + \frac{V_W}{4\phi_F}\right)\right\}$$

17.15

The required computer program and a sample output is reproduced below.



MATLAB program script...

```
%ID-VD Characteristics /// Bulk-Charge Theory

%Initialization
clear; close

%Constants and Parameters
q=1.60e-19;
e0=8.85e-14;
kT=0.0259;
ni=1.0e10;
KS=11.8;
KO=3.9;
NA=input('input the doping in cm-3,  NA = ');
xo=input('input the oxide thickness in cm,  xo = ');

%Computed Parameters
øF=kT*log(NA/ni);
WT=sqrt((4*KS*e0*øF)/(q*NA));
Co=KO*e0/xo;
VW=q*NA*WT/Co;
```

17.16 (Solution not supplied.)

17.17 (Solution merely involves straightforward mathematical manipulations.)

17.18

(a) Given $V_D = 0$, then $\phi(y) = 0$ and

$$Q_N(\text{all } y) = -C_o(V_G - V_T) = -\frac{K_O\varepsilon_0}{x_o}(V_G - V_T) = -\frac{(3.9)(8.85\times10^{-14})(2)}{5\times10^{-6}}$$

$$= \mathbf{-1.38\times10^{-7} coul/cm^2}$$

(b) $$g_d|_{V_D=0} = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T) \quad \text{...making use of Table 17.1}$$

$$= -\frac{Z\bar{\mu}_n Q_N}{L} = \frac{(70\times10^{-4})(550)(1.38\times10^{-7})}{7\times10^{-4}}$$

$$= \mathbf{7.59\times10^{-4} S}$$

17.19

$|V_T|$ is the same in ideal p-channel and n-channel MOSFETs with the same x_o and bulk doping concentration. Thus, with the devices also equivalently biased, one concludes from Table 17.1 that the same g_m's will result if

$$\frac{Z_p}{L_p}\bar{\mu}_p = \frac{Z_n}{L_n}\bar{\mu}_n$$

where the subscripts indicate the channel type. This same conclusion is reached whether one uses the square-law theory or bulk-charge theory and whether the devices are biased below or above pinch-off.

Next, examining the first form of Eq.(17.37), we again quite generally conclude that C_O(p-channel) must equal C_O(n-channel) for the f_{max} values to be the same. Since $C_O = K_O\varepsilon_0 ZL/x_o$, we therefore require

$$Z_pL_p = Z_nL_n$$

```
%ID-VD Computation and Plot
for VGT=4:-1:1,    %VGT = VG - VT;
 %Compuation
  A=VGT/(2*øF);  B=1+VW/(4*øF);
  VDsat=VGT-VW*(sqrt(A+B^2)-B);
  VD=linspace(0,VDsat);
  ID1=VGT.*VD-VD.*VD/2;
  VDF=VD./(2*øF);
  ID2=(4/3)*VW*øF.*((1+VDF).^1.5-(1+1.5*VDF));
  ID=ID1-ID2;
  IDsat1=VGT.*VDsat-VDsat.*VDsat/2;
  VDFsat=VDsat./(2*øF);
  IDsat2=(4/3)*VW*øF.*((1+VDFsat).^1.5-(1+1.5.*VDFsat));
  IDsat=IDsat1-IDsat2;
  VD=[VD,9];
  ID=[ID,IDsat];
 %Plotting and Primary Labeling
  if VGT==4,
  plot(VD,ID);  grid;
  axis([0 10 0 10]);
  xlabel('VD (volts)');  ylabel('ID/(ZµCo/L)');
  text(8,IDsat+0.2,'VG-VT=4V');
  hold on
  else,
  plot(VD,ID);
 %The following 'if' labels VG-VT curves < 4
  if VGT==3,
  text(8,IDsat+0.2,'VG-VT=3V');
  elseif VGT==2,
  text(8,IDsat+0.2,'VG-VT=2V');
  else,
  text(8,IDsat+0.2,'VG-VT=1V');
  end
  end
 end
 hold off
```

Substituting Z_p from the first relationship into the second relationship and simplifying, one obtains

$$L_p = \sqrt{\frac{\overline{\mu}_p}{\overline{\mu}_n}}\, L_n$$

and

$$Z_p = \frac{Z_n L_n}{L_p} = \sqrt{\frac{\overline{\mu}_n}{\overline{\mu}_p}}\, Z_n$$

The mobilities deduced from Fig. 3.5a yield $\overline{\mu}_n = \mu_n/2 = 673$ cm^2/V-sec and $\overline{\mu}_p = \mu_p/2 = 229$ cm^2/V-sec. Thus the required *p*-channel device dimensions are

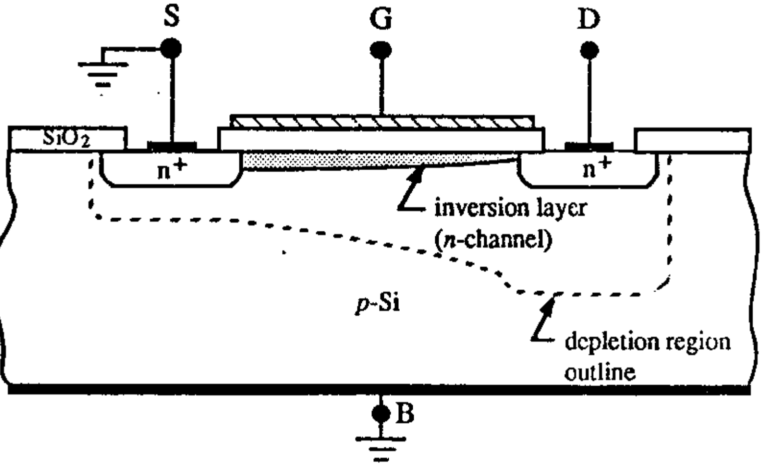
$$Z = \sqrt{673/229} \times 50 = \mathbf{85.7\mu m}$$

and

$$L = \sqrt{229/673} \times 5 = \mathbf{2.92\mu m}$$

17.20

(a) Since the applied V_D is greater than zero, we infer the given MOSFET is an *n*-channel device. Also, at point (1) the MOSFET is biased below saturation. Thus the channel narrows near the drain but is not pinched-off.



(b) In the square-law theory $V_{Dsat} = V_G - V_T$. Thus

$$V_G = V_{Dsat} + V_T = \mathbf{6V}$$

(c) The point (2) bias corresponds to the pinch-off point. At the pinch-off point, and based on the square-law theory, the charge in the MOSFET channel goes to zero at the drain.

$$Q_N(L) = \mathbf{0}$$

(d) With $V_D = 4V$ and $V_G - V_T = 3V$, $V_D > V_{Dsat} = V_G - V_T$. Consequently, for the readjusted gate voltage, the MOSFET is being operated in the saturation region. Since $I_{Dsat} \propto (V_G - V_T)^2$, it follows that

$$\frac{I_{Dsat1}}{I_{Dsat2}} = \left(\frac{V_{G1} - V_T}{V_{G2} - V_T}\right)^2$$

Here identifying the desired $I_D = I_{Dsat1}$ ($V_{G1} - V_T = 3V$) and $I_{Dsat2} = 10^{-3}$A ($V_{G2} - V_T = 5V$) from the given characteristics, we conclude

$$I_D = (10^{-3})\left(\frac{3}{5}\right)^2 = \mathbf{3.6\times10^{-4}\ A}$$

(e) By definition $g_d = \partial I_D/\partial V_D$ with V_G held constant. Inspecting the given characteristic, we conclude $\partial I_D/\partial V_D = 0$ at bias point (3) and $g_d = 0$. Alternatively, in the saturation region of operation, I_{Dsat} is not a function of V_D. Consequently $\partial I_{Dsat}/\partial V_D = 0$ and $g_d = \mathbf{0}$.

(f) At bias point (3) the MOSFET is in the above pinch-off region of operation and from Table 17.1

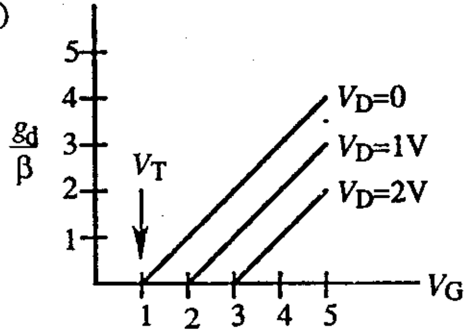
$$g_m = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T) = \frac{2I_{Dsat}}{V_G - V_T} = \frac{(2)(10^{-3})}{5} = \mathbf{4\times10^{-4}\ S}$$

(g) For an *n*-channel (*p*-bulk) MOSFET, one expects a low-frequency MOS-C type characteristic similar to that displayed in Fig. 17.13(b).

17.21

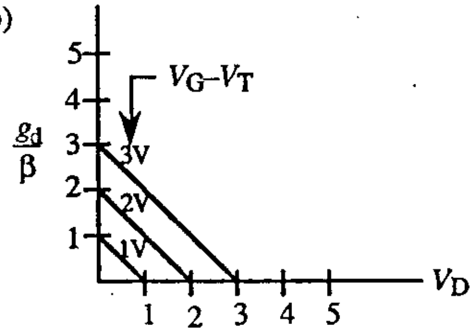
Making use of Table 17.1 and defining $\boxed{\beta = Z\,\overline{\mu}_n C_o/L}$, we conclude:

(a)



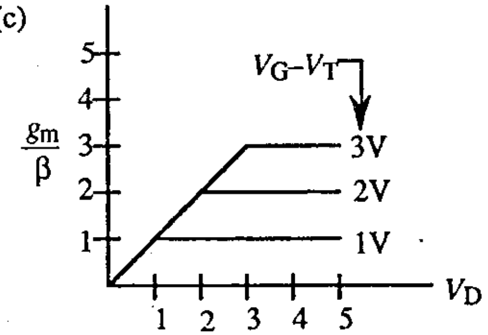
$$\frac{g_d}{\beta} = \begin{cases} 0 & \ldots V_G - V_T \le V_D \\ V_G - V_T - V_D & \ldots V_G - V_T \ge V_D \end{cases}$$

(b)



$$\frac{g_d}{\beta} = \begin{cases} V_G - V_T - V_D & \ldots V_D \le V_G - V_T \\ 0 & \ldots V_D \ge V_G - V_T \end{cases}$$

(c)



$$\frac{g_m}{\beta} = \begin{cases} V_D & \ldots V_D \le V_G - V_T \\ V_G - V_T & \ldots V_D \ge V_G - V_T \end{cases}$$

17.22

There is of course no set answer to this question. The answer, however, should include some of the following points:

Externally the J–FET and MOSFET yield similar electrical characteristics and even appear similar physically, with the terminal leads being designated the source, drain, and gate.

The gate voltage in both devices determines the maximum conductance of the internal channel. However, there are major differences in the nature of the conducting channel and the substructure used to modulate the channel conductance. The J–FET channel is a narrow piece of bulk material; in the basic transistor configuration the MOSFET channel is an inversion layer which is created by the applied gate voltage. Manipulation of the channel conductance is accomplished by reverse biasing a *pn* junction in the J–FET; the MOSFET channel conductance is modulated by the bias applied to the MOS structure.

The first and even second-order quantitative analyses leading to the dc characteristics are quite similar for the two devices. Nonetheless, there are two complicating factors which enter the MOSFET analysis, factors which are not present in the J–FET analysis. First of all, carriers in a surface channel experience motion-impeding collisions with the Si surface which lower the mobility of the carriers and necessitate the introduction of an effective carrier mobility. Secondly, the carrier concentration and current density in the surface channel are strong functions of position, dropping off rapidly as one proceeds into the semiconductor bulk. For the device structure analyzed in the text, the carrier concentration and current density are of course constant across the J–FET channel.

Both first order theories give rise to an I_{Dsat} which varies (or varies approximately) as the square of the gate voltage. The general ac response and first order equivalent circuits for the two devices are identical.