6.1

(a) Majority carrier injection (diffusion) to the opposite side of the junction.

(b) Minority carriers wandering into the depletion region and being accelerated (drifting) to the opposite side of the junction.

(c) The reverse bias current is expected to be *small* because it arises from *minority carriers* which are few in number. The reverse bias current *saturates* because a small voltage all but eliminates majority carrier injection across the junction, and the remaining current due to minority carriers is independent of the applied voltage.

(d) generation and diffusion

(e) The primary reason is that $\mathcal{E} \neq 0$. Also, low level injection conditions seldom apply; $\partial n/\partial t|_{\text{thermal R-G}} \neq -\Delta n_p/\tau_n$ and $\partial p/\partial t|_{\text{thermal R-G}} \neq -\Delta p_n/\tau_p$.

(f) There is NO a priori justification.

(g) A diode where the contacts are far removed (several minority carrier diffusion lengths) from the edges of the depletion region.

(h) $np = n_i^2 e^{qV_A/kT}$ $...-x_p \leq x \leq x_n$

(i) I_0 is the extrapolated intercept of the straight-line region on the I(log scale) axis.

(j) True

6.2

The graphical ideal-device/Si-300K device comparison of $I–V$ characteristics is presented on the next page. (Also see Fig. 6.6, Fig. 6.17, and Exercise 6.9.) The causes of the deviations noted in the sketches are:
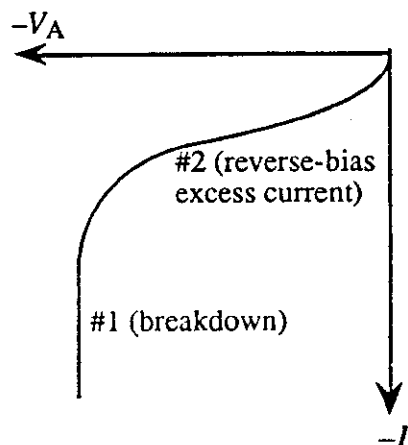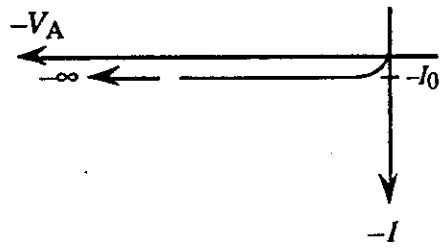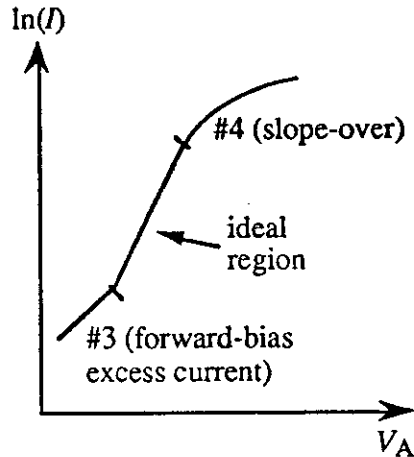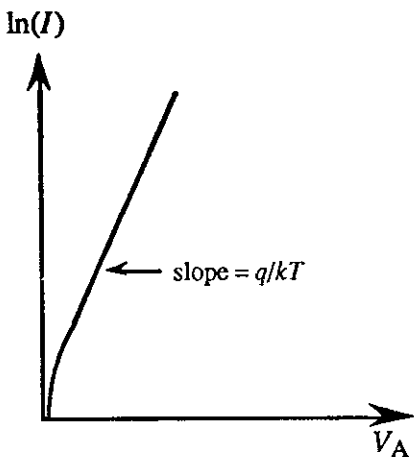
Causes

Deviation #1...Breakdown is caused by *avalanching* in the depletion region or tunneling through the depletion region (the *Zener process*). The latter is important if $V_{BR} \lesssim 4V$.

Deviation #2...Results from generation of carriers in the depletion region. (This was ignored in the ideal diode derivation.)

Deviation #3...Results from recombination of carriers in the depletion region.

Deviation #4...The slope-over on a log scale results from series resistance (bulk and/or contact resistance). An $\exp(qV_A/2kT)$ region at large forward biases, if observed, is due to high-level injection.

Graphical Comparison

slope = q/kT

#4 (slope-over)

ideal region

#3 (forward-bias excess current)

$-I_0$

$-I$

#2 (reverse-bias excess current)

#1 (breakdown)

$-I$

⇑
Ideal

⇑
Si, 300 K

(a)/(b)/(c) To first order, the sketches here should be essentailly identical to Fig. 6.1(a)–(c), except the Fermi level on the p-side is close to or actually in the valence band. Although it is perhaps unreasonable to expect from someone who has read only through Subsection 6.1.1 in the text, the majority carrier hole concentration on the p-side of the junction should also be visualized as much larger than the majority carrier electron concentration on the n-side of the junction. Likewise, the hole minority carrier concentration on the n-side is much greater than the electron minority carrier concentration on the p-side in a p^+-n diode. Consequently, one would expect the hole component of the current to dominate over the electron component of the current under both forward and reverse biases.—The hole current arrows in the part (b) and (c) diagrams should be significantly larger than the electron current arrows. Note that the qualitative conclusion here is consistent with the formula-based observations made in Subsection 6.1.4.

6.4

$$V_A > 0$$
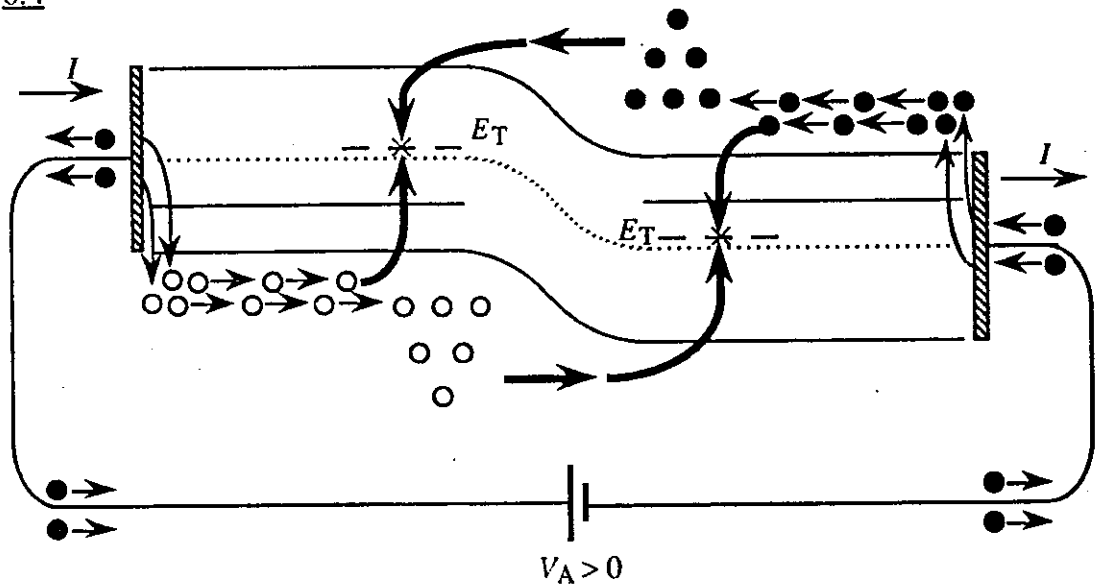
6.5

If $V_A \neq 0$, $V(x_n) = V_{bi} - V_A$ and Eq. (5.5) in the Subsection 5.1.4 derivation assumes the modified form

$$-\int_{-x_p}^{x_n} \mathcal{E}dx = V_{bi} - V_A$$

Continuing to assume $J_N = 0$ when $V_A \neq 0$ then leads to the modified Eq. (5.8) relationship

$$V_{bi} - V_A = \frac{kT}{q} \ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

or

$$\frac{n(x_n)}{n(-x_p)} = e^{q(V_{bi}-V_A)/kT} = \frac{N_A N_D}{n_i^2} e^{-qV_A/kT}$$

where the second form of the preceding equation is obtained by eliminating V_{bi} using Eq. (5.10). Finally, noting $n(x_n) = N_D$, simplifying, and solving for $n(-x_p)$ yields

$$n(-x_p) = \frac{n_i^2}{N_A} e^{qV_A/kT}$$

and since $n_0(-x_p) = n_i^2/N_A$,

$$\Delta n(-x_p) = \frac{n_i^2}{N_A}\left(e^{qV_A/kT} - 1\right) \qquad \text{...Eq. (6.15)}$$

6-4

6.6

The MATLAB Diary session leading to the desired result was copied to a wordprocessor, irrelvant entries eliminated, and the results condensed into boxed regions. The modified version is reproduced below. Please note that I at $V_A = -0.1V$ and $V_A = -50V$ are nearly identical, indicating the saturation current has all but reached the saturation value at only $V_A = -0.1V$. Also note the small size of the room temperature saturation current, $\sim 10^{-13}$ A, and the hugh increase in current at the elevated temperature.

MATLAB Diary session...

```
%Given:  n+ -p step junction with
NA=1.0e15;
taun=1.0e-6;
A=1.0e-3;

%Universal constants
q=1.6e-19;
k=8.617e-5;

%(a) Currents at T=300K
T=300;
µn=1345;   %Value from Fig. 3.5(a)
DN=k*T*µn;
LN=sqrt(DN*taun);
ni=1.0e10;
I0=q*A*DN/LN*ni^2/NA;
VA=[-50 -0.1 0.1 0.5];
I=I0*(exp(VA./(k*T))-1)
```

```
I = -9.4345e-14 A    ...for VA=-50V
    -9.2374e-14 A    ...for VA=-0.1V
    4.4212e-12  A    ...for VA=0.1V
    2.3696e-05 A     ...for VA=0.5V
```
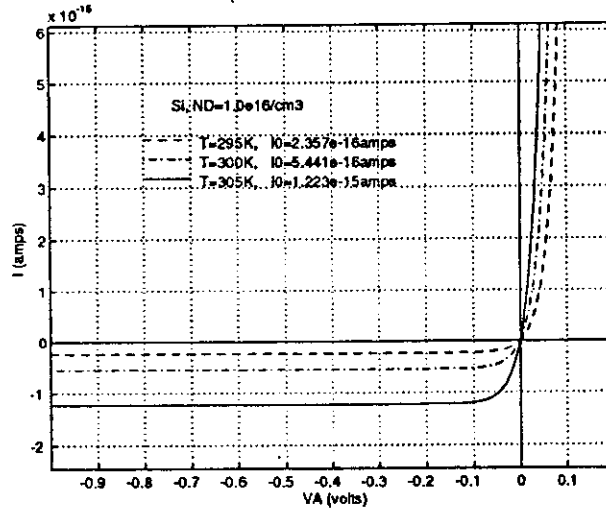
```
%(b) Currents at T=500K
ni=2.716e14;   %Value from table in Exercise 2.4
T=500;
µn=(1345)*(T/300)^(-2.3);   %See Fig. 3.7a (approx. value)
DN=k*T*µn;
LN=sqrt(DN*taun);
I0=q*A*DN/LN*ni^2/NA;
I=I0*(exp(VA./(k*T))-1)
```
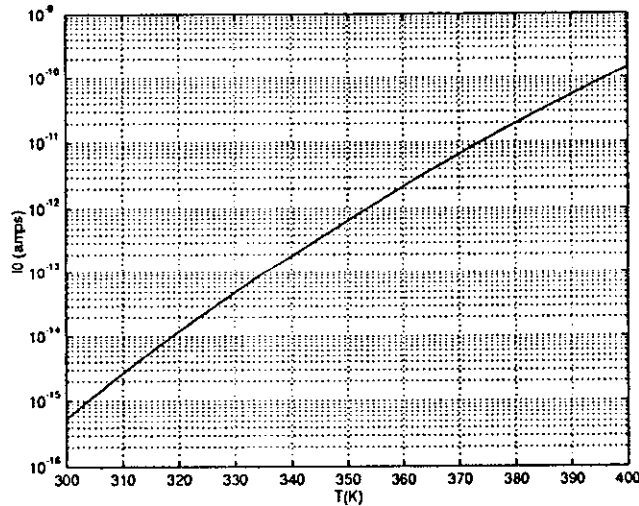
```
I = -4.9932e-05 A    ...for VA=-50V
    -4.5030e-05 A    ...for VA=-0.1V
    4.5866e-04 A     ...for VA=0.1V
    5.4745e+00 A     ...for VA=0.5V
```

6.7

(a) Sample ideal-diode I–V T-dependence plot

(b) I_0 versus T plot

(c) MATLAB m-files P_06_07a.m and P_06_07b.m, available on the Instructor's disk, were used to generate the part (a) and part (b) plots, respectively. Both plots verify the very strong temperature dependence of the ideal-diode I–V characteristics. Only a 5 K rise in temperature leads to a very significant increase in the part (a) current; the part (b) plot exhibits over a 10^5 increase in I_0 associated with a 100 K increase in temperature!

6.8

(a) Let us examine the minority carrier diffusion equation for hole. In general

$$\frac{\partial \Delta p_n}{\partial t} = D_P \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

For the steady state problem at hand $\partial \Delta p_n / \partial t = 0$. Also, $\partial^2 \Delta p_n / \partial x^2 = 0$ if one goes far from the junction on the n-side where the carrier perturbation introduced by the junction has decayed to zero. Thus

$$0 = -\frac{\Delta p_n(x \to \infty)}{\tau_p} + G_L$$

or

$$\Delta p_n(x \to \infty) = G_L \tau_p \qquad \Leftarrow \text{boundary condition}$$

(b) One simply parallels the ideal diode derivation to obtain the desired I–V_A relationship. Given a p^+-n junction, however, we need consider only the lightly doped n-side of the junction. Specifically, under steady state conditions and with x' as defined in Fig. 6.5(a), we must solve

$$0 = D_P \frac{d^2 \Delta p_n}{dx'^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

subject to the boundary conditions

$$\Delta p_n(x'=0) = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right)$$

$$\Delta p_n(x' \to \infty) = G_L \tau_p$$

The general solution is

$$\Delta p_n(x') = G_L \tau_p + A_1 e^{-x'/L_P} + A_2 e^{x'/L_P}$$

Because $\exp(x'/L_P) \to \infty$ as $x' \to \infty$, the only was the second boundary condition can be satisfied is for A_2 to be identically zero. With $A_2 = 0$, the application of the first boundary condition yields

$$\Delta p_n(x'=0) = G_L \tau_p + A_1 = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right)$$

or

$$A_1 = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right) - G_L \tau_p$$

and

$$\Delta p_n(x') = G_L \tau_p + \left[(n_i^2/N_D)\left(e^{qV_A/kT} - 1\right) - G_L \tau_p\right] e^{-x'/L_P}$$

The associated hole current density is then

$$J_P(x') = -qD_P \frac{d\Delta p_n}{dx'} = q\frac{D_P}{L_P}\left[(n_i^2/N_D)(e^{qV_A/kT}-1)-G_L\tau_p\right]e^{-x'/L_P}$$

and for a p^+-n diode

$$I = AJ = A[J_N(x=-x_p) + J_P(x=x_n)] \cong AJ_P(x'=0)$$

or

$$I = qA\frac{D_P}{L_P}\frac{n_i^2}{N_D}\left(e^{qV_A/kT}-1\right)-qA\frac{D_P\tau_p}{L_P}G_L$$

Finally noting $D_P\tau_p = L_P^2$, we conclude

$$I = I_0\left(e^{qV_A/kT}-1\right)+I_L$$

$$\text{where}\quad I_0 = qA\frac{D_P}{L_P}\frac{n_i^2}{N_D}$$

$$I_L = -qAL_PG_L$$

(c) In constructing the characteristics we note that the usual ideal-diode characteristic results if $G_L = 0$. A $G_L \neq 0$ characteristic is obtained by subtracting the same constant value from all I-values on the dark ($G_L = 0$) curve. Effectively, the entire dark I–V curve is simply translated downward by an amount equal to I_L. Since $I_L \propto G_L$, the downward translation increases in proportion to G_L. The characteristics are concluded to be of the form sketched below.
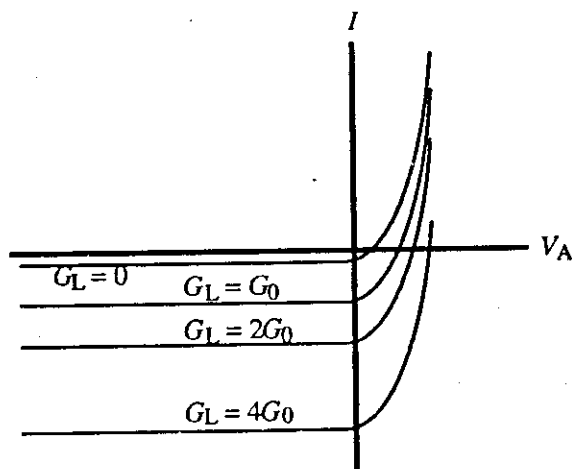
6.9
General nature of the *I–V* characteristics of a solar cell subject to illumination...
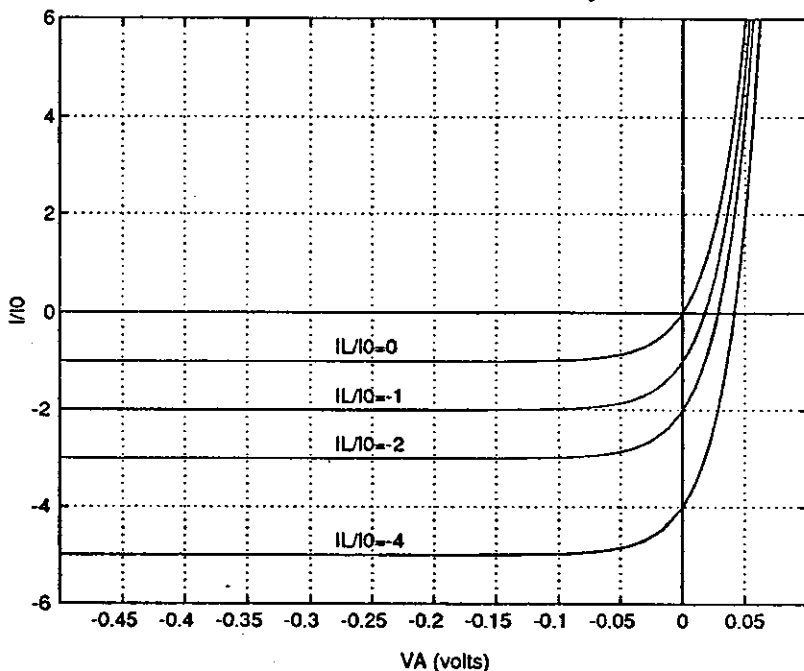
MATLAB program script used to generate the characteristics...

```
%Solar Cell Characteristics

%Intialization
clear;  close

%Constants
k=8.617e-5;  T=300;

%Computation Proper
VA=linspace(-0.5,0.1);
Il=exp(VA./(k*T))-1;  %I/I0 with IL/I0=0
I2=Il-1;  I3=Il-2;  I4=Il-4; ·
I=[Il; I2; I3; I4];

%Plotting Result
plot(VA,I)
axis([-0.5 0.1 -6 6]);  grid
xlabel('VA (volts)'); ylabel('I/I0');
text(-0.28,-0.7,'IL/I0=0');  text(-0.28,-1.7,'IL/I0=-1');
text(-0.28,-2.7,'IL/I0=-2');  text(-0.28,-4.7,'IL/I0=-4');
xx=[-0.5 0.1];  xy=[0 0];  yx=[0 0];  yy=[-6 6];
hold on
plot(xx,xy,'-w',yx,yy,'-w')
```

6.10

(a) | Reverse biased | — there is a deficit of minority carrier in the quasineutral region immediately adjacent to the depletion region.

(b) Low-level injection DOES prevail. As required for low-level injection

$$|\Delta n_p|_{max} \cong n_{p0} \ll p_p \qquad \text{... for } x \leq -x_p$$

$$|\Delta p_n|_{max} \cong p_{n0} \ll n_n \qquad \text{... for } x \geq x_n$$

(c) Since we have low level injection,

$$N_A \cong p_{p0} \cong p_p = 10^{14}/cm^3$$

$$N_D \cong n_{n0} \cong n_n = 10^{15}/cm^3$$

(d) Invoking the law of the junction,

$$n(-x_p)p(-x_p) = n_i^2 e^{qV_A/kT}$$

or

$$V_A = \frac{kT}{q} \ln\left[\frac{n(-x_p)p(-x_p)}{n_i^2}\right]$$

As deduced from Fig. P6.10,

$$n(-x_p) = 10^3/cm^3$$

$$p(-x_p) = 10^{14}/cm^3$$

and

$$n_i = \sqrt{n(\infty)p(\infty)} = \sqrt{n(-\infty)p(-\infty)} = \sqrt{10^{20}} = 10^{10}/cm^3$$

The foregoing manipulation to obtain n_i was necessary because the semiconductor used in fabricating the diode was not specified in the problem statement. Lastly, substituting into the V_A expression gives

$$V_A = (0.0259) \ln\left[\frac{(10^3)(10^{14})}{10^{20}}\right] = -0.18V$$

6.11

Under equilibrium conditions

$$p_{p0} = N_A = 10^{17}/cm^3 \qquad\qquad n_{n0} = N_D = 10^{16}/cm^3$$
$$n_{p0} = n_i^2/N_A = 10^3/cm^3 \qquad\quad p_{n0} = n_i^2/N_D = 10^4/cm^3$$

Invoking the depletion edge boundary conditions (Eqs. 6.15 and 6.18) gives

$$\Delta n_p(-x_p) = (n_i^2/N_A)\left(e^{qV_A/kT} - 1\right) = (10^3)e^{23.03} = 10^{13}/cm^3$$

$$\Delta p_n(x_n) = (n_i^2/N_D)\left(e^{qV_A/kT} - 1\right) = (10^4)e^{23.03} = 10^{14}/cm^3$$

Since the perturbed carrier concentrations in the quasineutral regions are greatest at the depletion region edges, and since $\Delta n_p(-x_p) \ll p_{p0}$ and $\Delta p_n(x_n) \ll n_{n0}$, we have low-level injection. This makes $p_p \cong p_{p0}$ everywhere on the p-side of the junction and $n_n \cong n_{n0}$ everywhere on the n-side of the junction.

Next invoking the ideal diode solution, we can write
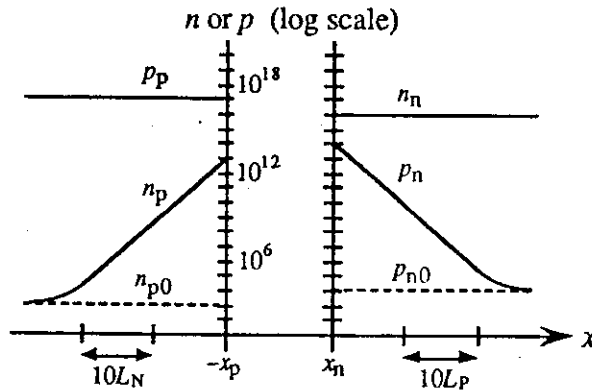
$$n_p = n_{p0} + \Delta n_p(-x_p)e^{-x''/L_N}$$

and

$$p_n = p_{n0} + \Delta p_n(x_n)e^{-x'/L_P}$$

Thus at points 0, 10, and 20 diffusion lengths from the depletion region edges we compute

x''/L_N or x'/L_P	$n_p(cm^{-3})$	$p_n(cm^{-3})$
0	10^{13}	10^{14}
10	4.54×10^8	4.54×10^9
20	2.16×10^4	2.16×10^5 (~5% n_{p0} or p_{n0} contribution)

The sketch constructed using the foregoing data is shown below.

6.12

(a) Reproduced below is the requested sample plot of the V_A versus T expected from the specified pn diode temperature sensor when the forward bias current through the diode is held constant at $I = 10^{-4}$ A. The plot was produced using MATLAB file P_06_12.m supplied on the Instructor's disk. Note the nearly linear dependence of V_A on the ambient temperature. A linear temperature dependence of the monitored parameter is of course highly desirable in producing a temperature sensor.
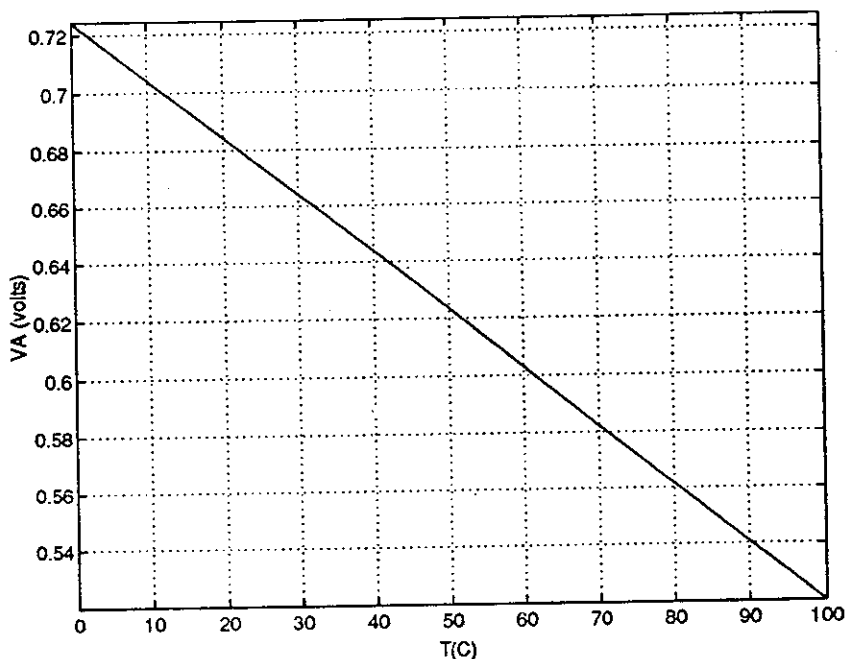
(b) After running the program with a given I setting, the Command window in MATLAB can be used to access the V_A values corresponding to $T = 0°C$ and $T = 100°C$. For the two currents cited in the problem one finds:

I	V_A (volts) at $T = 0°C$	V_A (volts) at $T=100°C$	$\Delta V_A/\Delta T$ (V/°C)
10^{-4}	0.724	0.520	2.04×10^{-3}
10^{-3}	0.779	0.594	1.85×10^{-3}

The computation results clearly indicate the sensitivity of the sensor is greater when $\boxed{I = 10^{-4} \text{ A}}$.

6.13

(a) As read from Fig. 6.11, $V_{BR} \cong 320$ V.

(b) $W = \left[\dfrac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_A)\right]^{1/2} = \left[\dfrac{2K_S\varepsilon_0}{qN_D}V_{BR}\right]^{1/2} = \left[\dfrac{(2)(11.8)(8.85\times10^{-14})(320)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2}$

$\quad = 2.04\times10^{-3}\text{cm} = \mathbf{20.4\mu m}$

(c) $|\mathcal{E}|_{max} = |\mathcal{E}(0)| = \dfrac{qN_D}{K_S\varepsilon_0}x_n = \dfrac{qN_D}{K_S\varepsilon_0}W = \dfrac{(1.6\times10^{-19})(10^{15})(2.04\times10^{-3})}{(11.8)(8.85\times10^{-14})}$

$\quad = \mathbf{3.13\times10^5 \; V/cm}$

6.14

(a) As the student hopefully discovers, the problem statement is very misleading. Although a possible solution approach, the modification of Exercise 6.2 to obtain the desired reverse-bais I/I_0 versus V_A plot is extremely inefficient. With the current normalized by I_0, one can write

$$I/I_0 = M(e^{qV_A/kT} - 1) \qquad ...V_A \leq 0$$

The only parameters required for the computation are m and N_D. Moreover, except for the normally inconsequential portion of the plot near $V_A = 0$, and the fact that the x-axis is not normalized to V_{BR}, the plot produced is *identical* to the $m = 6$ plot in Exercise 6.5.

MATLAB program script...

```
%  Breakdown-included Reverse Bias I-VA Characteristics
%
%  This program assumes an ideal diode with the one modification
%  that avalanche multiplication is taken into account using:
%                   M=1/(1-(abs(VA)/VBR)^m).
%
%  Appropriate for Si diodes at 300K, VBR is calculated from
%                   VBR = 60*(ND/1e16)^(-3/4)
%Initialization
clear;  close

% Computational Constants
kT=0.0259;
ND=input('Input the n-side doping, ND = ');
m=input('Input the value of m (3≤m≤6), m = ');
```

```
% VBR, M calculation
VBR=60*(ND/1e16)^(-3/4);
VA1=linspace(-VBR+.1,-0.1,200);
VA2=linspace(-0.1,0,25);
VA=[VA1,VA2];
M=(1-abs(VA./VBR).^m).^(-1);
% I/I0 Calculation
Iratio=M.*(exp(VA./(kT))-1);

% Plot
plot(VA',Iratio')
axis([-1.1*VBR(1),0,-5,0]);  grid
xlabel('VA (volts)');  ylabel('I/I0')
text(-VBR,-0.3,'SILICON, T = 300K')
text(-VBR,-0.7,['ND=',num2str(ND(1)),'/cm3,   m=',num2str(m)])
```
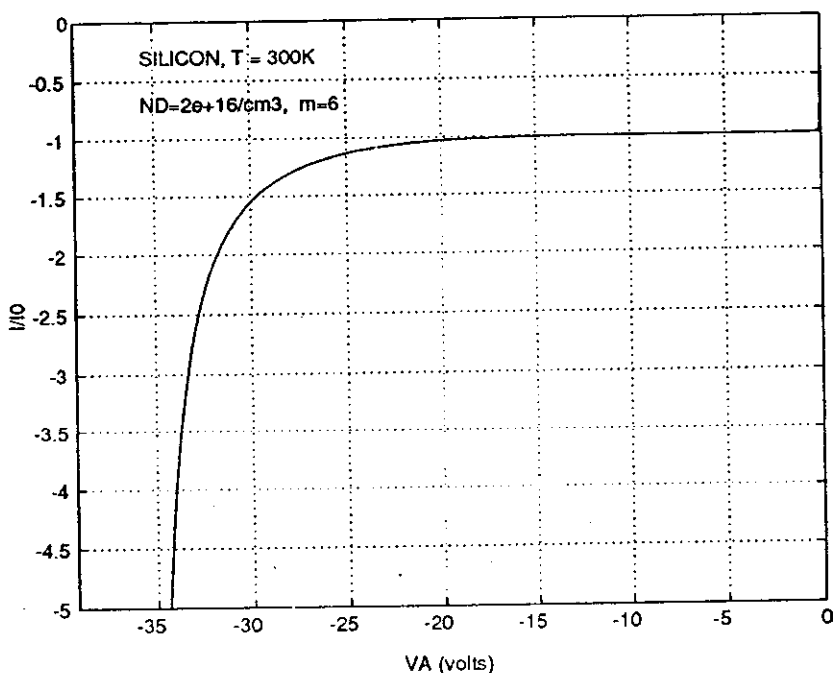

(b) Like in Exercise 6.5, the approach to breakdown becomes more gradual with decreasing values of m. Varying N_D has no effect on the general shape of the curve, although the breakdown voltage does of course increase with N_D.

If normalized properly, the τ_0 supplied in the statement of the problem is not required. Specifically,

$$\frac{I_{R\text{-}G}(V_A)}{|I_{R\text{-}G}(-V_{BR}/2)|} = -M\frac{W(V_A)}{W(-V_{BR}/2)} = -M\sqrt{\frac{V_{bi}-V_A}{V_{bi}+V_{BR}/2}} \quad \ldots \text{ reverse biases} > \text{few } kT/q$$

where, assuming a p^+-n junction,

$$V_{bi} \cong E_G/2q + (kT/q)\ln(N_D/n_i)$$

As might be suspected from the fact that the $I_{R\text{-}G}$ current typically dominates in reverse-biased Si diodes at room temperature, the sample computational plot reproduced below has a general shape very close to the experimental data presented in Fig. 6.10(b). The N_D employed in the sample computation was even chosen to yield a close approximation to the observed V_{BR}. Note, however, that the final approach to V_{BR} in the theoretical plot is more gradual than that observed experimentally.
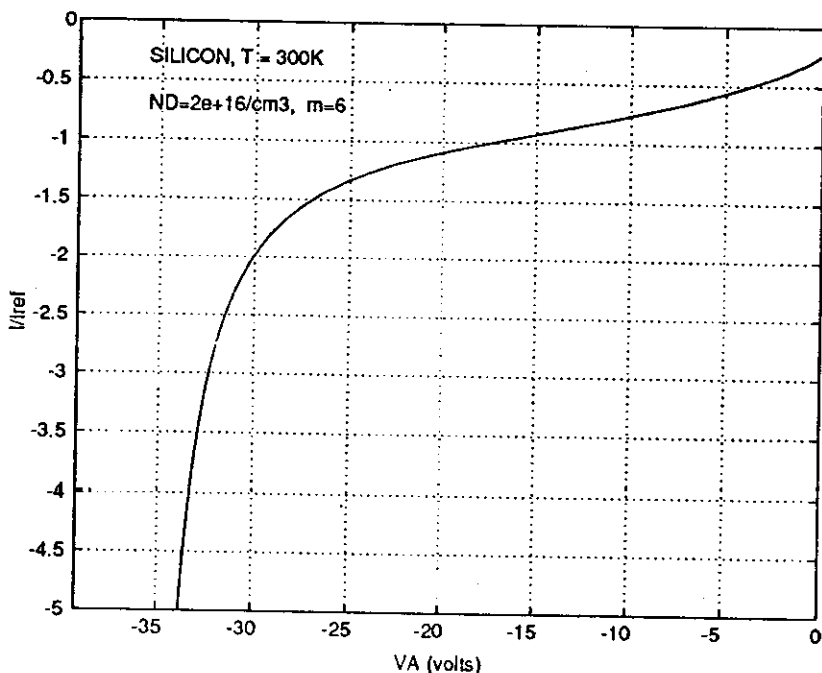
MATLAB program script (Problem 6.15)...

```
%  Breakdown-included Reverse Bias I-VA Characteristics
%                  Si, p+n, 300K
%  This program assumes Ireverse=IR-G
%  Avalanche multiplication is taken into account using:
%               M=1/(1-(abs(VA)/VBR)^m).
%
%  VBR is calculated from
%               VBR = 60*(ND/1e16)^(-3/4)
%Initialization
clear;   close

% Universal and System Constants
kT=0.0259;
EG=1.12;
ni=1.0e10;
ND=input('Input the n-side doping, ND = ');
m=input('Input the m value (3≤m≤6), m = ');

% VBR, M calculation
VBR=60*(ND/1e16)^(-3/4);
VA=linspace(-VBR+0.1,-0.1,200);
M=(1-abs(VA./VBR).^m).^(-1);

%Current Calculation [Iratio=IRG(VA)/|IRG(-VBR/2)|]
Vbi=EG/2+kT.*log(ND./ni);
Wratio=sqrt((Vbi-VA)./(Vbi+VBR/2));
Iratio=-M.*Wratio;

% Plot
plot(VA,Iratio)
axis([-1.1*VBR(1),0,-5,0]);   grid
xlabel ('VA (volts)');   ylabel ('I/Iref');
text(-VBR,-0.3,'SILICON, T = 300K')
text(-VBR,-0.7,['ND=',num2str(ND(1)),'/cm3,   m=',num2str(m)])
```

6.16

We know

$$I_{DIFF} = -I_0 = -qA \frac{n_i^2}{N_D} \frac{D_P}{L_P}$$

and

$$I_{R\text{-}G} = -qA \frac{n_i}{2\tau_0} W$$

Equating I_{DIFF} and $I_{R\text{-}G}$ yields the requirement

$$\frac{n_i}{N_D} \frac{D_P}{L_P} = \frac{W}{2\tau_0}$$

Assuming $\tau_0 = \tau_p$, as suggested in the problem statement, and solving for n_i, gives

$$n_i(T_D) = N_D \frac{L_P}{2D_P\tau_p} W = N_D \frac{W}{2L_P}$$

where T_D is the transition temperature above which the diffusion component of the current is expected to dominate. With $V_{bi} - V_A = V_{BR}/2$ and V_{BR} read from Fig. 6.11, we compute

$$W = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi} - V_A)\right]^{1/2} = \left[\frac{K_S\varepsilon_0}{qN_D} V_{BR}\right]^{1/2} = \left[\frac{(11.8)(8.85\times10^{-14})(55)}{(1.6\times10^{-19})(10^{16})}\right]^{1/2}$$

$$= 1.89\times10^{-4} \text{ cm}$$

and

$$n_i(T_D) = N_D \frac{W}{2L_P} = \frac{(10^{16})(1.89\times10^{-4})}{(2)(10^{-2})} \cong 10^{14}/\text{cm}^3$$

From the Fig. 2.20 plot of n_i versus T we conclude $\boxed{T_D \cong 470 \text{ K}}$.

6.17

Let x_1 and x_2 be the ends of the d-region, with $d = x_2 - x_1$. Also let τ_{01} and τ_{02} be the generation lifetime outside and inside the d-region, respectively. Clearly, based on Eq. (6.42),

$$I_{R\text{-}G} = -qA \int_{-x_p}^{x_n} \frac{n_i}{\tau_p(n_1/n_i) + \tau_n(p_1/n_i)}\, dx$$

$$= -qA \left[\int_{-x_p}^{x_1} \frac{n_i}{2\tau_{01}}\, dx + \int_{x_1}^{x_2} \frac{n_i}{2\tau_{02}}\, dx + \int_{x_2}^{x_n} \frac{n_i}{2\tau_{01}}\, dx \right]$$

$$= -qA \left[\frac{n_i}{2\tau_{01}} (W-d) + \frac{n_i}{2\tau_{02}} d \right]$$

Since τ_n and τ_p are both proportional to $1/N_T$, it follows that $\tau_{02} = \tau_{01}/3$ and

$$\boxed{I_{R\text{-}G} = -\frac{qAn_i}{2\tau_{01}} (W + 2d)}$$

6.18

At the desired I-value, we infer from the statement of the problem that

$$V_A = V_J + IR_S = 1.1V_J$$

or

$$V_J = 10IR_S$$

Thus at the specified operational point (invoking Eq. 6.49),

$$I = I_0\, e^{qV_J/kT} = I_0\, e^{(10IR_S)/(kT/q)} = (10^{-14})\, e^{(10IR_S)/(0.0259)}$$

An iterative technique, employing the fzero function in MATLAB for example, must be used to numerically solve the transcendental equation for I. We find...

(a) If $R_S = 2\ \Omega$, $I = 37.5$ mA.

(b) If $R_S = 20\ \Omega$, $I = 3.44$ mA.

<u>6.19</u>

The lines drawn manually by the author through the straight-line regions (roughly $0.12V \leq V_A \leq 0.5V$ and $0.64V \leq V_A \leq 0.76V$) on the Fig. P6.19 plot were found to pass through the (V_A, I) points

$(0.20V, 10^{-10}A)$, $(0.92V, 1A)$ ... $0.64V \leq V_A \leq 0.76V$ region
$(0V, 10^{-9}A)$, $(1V, 2\times10^{-2}A)$ ... $0.12V \leq V_A \leq 0.5V$ region

Employing the relationships presented in Exercise 6.7, we conclude

$$n_1 = \frac{0.92 - 0.20}{(0.0259) \ln(1/10^{-10})} = 1.21$$

$$n_2 = \frac{1}{(0.0259) \ln(2\times10^{-2}/10^{-9})} = 2.30$$

$$I_{01} = I/e^{qV_A/n_1 kT}\big|_{V_A=0.2V} = (10^{-10})/e^{0.2/[(1.21)(0.0259)]} \cong 1.7\times10^{-13} \text{ A}$$

$$I_{02} = (\text{extrapolated } V_A=0 \text{ intercept}) \cong 10^{-9} \text{ A}$$

A MATLAB program, listed after the discussion of results and incorporating the polyfit function, was used to effect a least squares fit to the data in the straight-line regions. The fit yields

$\ln(I) = -29.68 + 32.45V_A$ ... $0.64V \leq V_A \leq 0.76V$
$n_1 = 1.19$
$I_{01} = 1.29\times10^{-13}$ A

$\ln(I) = -20.57 + 16.67V_A$ ... $0.12V \leq V_A \leq 0.5V$
$n_2 = 2.32$
$I_{02} = 1.16\times10^{-9}$ A

The agreement between the manual and least-square approaches is amazingly good. As a general rule, data which has minimal noise can be accurately analyzed using the manual approach. In either case, the n_1 and n_2 values came out a bit larger than expected. (The author believes the high value for n_2 was caused by a non-negligible shunt conductance.—See Prob. 6.21.) Note, however, that $I_{02} \gg I_{01}$ as expected.

MATLAB fit program script...

```
%Least squares fit for Problem 6.19.

%Initialization
clear;  format short e
format compact

%Data
s=menu('CHOOSE VOLTAGE REGION','0.64V≤VA≤0.76V','0.12V≤VA≤0.5V');
if s==1,
VA=0.64:0.02:0.76;
I=[1.359e-4,2.531e-4,4.852e-4,9.444e-4,1.841e-3,3.518e-3,6.433e-3];
else
VA=0.12:0.02:0.5;
I=[8.210e-9,1.183e-8,1.73e-8,2.449e-8,3.416e-8,4.764e-8,6.501e-8, ...
8.866e-8, 1.209e-7,1.666e-7,2.305e-7,3.201e-7,4.462e-7,6.285e-7, ...
8.845e-7,1.249e-6,1.776e-6,2.527e-6,3.615e-6,5.2e-6];
end

%Computation
y=log(I);
p=polyfit(VA,y,1);
y0=p(2)
slope=p(1)
nj=1/(0.0259*slope)
I0=exp(y0)
```

6.20

We take the slope-over in the I–V characteristics for $V_A \geq 0.8$V to be caused by the series resistance, R_S. In what follows, we determine the extrapolated "ideal" ($R_S=0$) voltage required to achieve a given current level by employing the least squares fit to the near-ideal data region established in Prob. 6.19. Specifically,

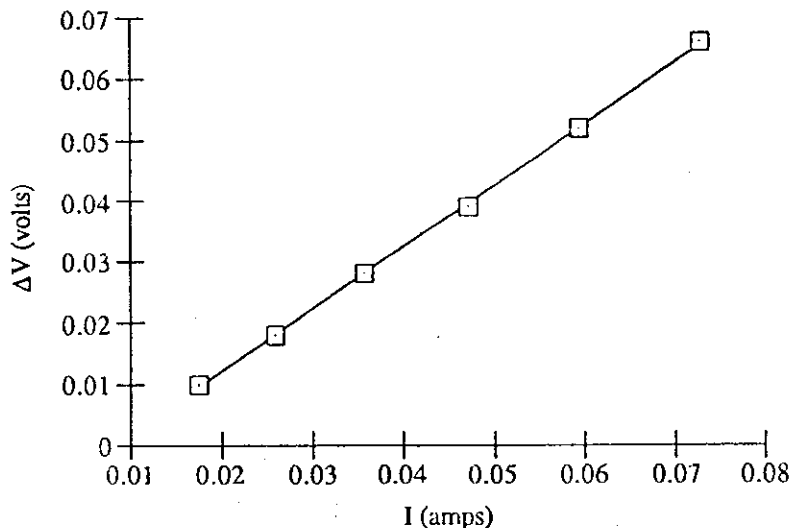$$I = I_{01}e^{qV_A/n_1kT} = (1.29\times10^{-13})e^{V_A/[(1.19)(0.0259)]}$$

or

$$V_A(\text{ideal}) = (1.19)(0.0259)\ln(I/1.29\times10^{-13})$$

One computes...

I (amps)	V_A(observed)	V_A(ideal)	ΔV
0.01752	0.800	0.790	0.010
0.02585	0.820	0.802	0.018
0.03579	0.840	0.812	0.028
0.04706	0.860	0.821	0.039
0.05941	0.880	0.828	0.052
0.07264	0.900	0.834	0.066

Plotting the data...

A least squares fit to the data yields $\Delta V = -0.0082 + 1.02I$, and $R_S = $ slope = **1.02 ohm.**

<u>6.21</u>

(a) This part of the problem was included to make sure that the student had the correct computational relationships. Obviously, the total current flowing through the device is equal to the current through the ideal representation plus the current through R_{SH}. Since V_J is the voltage drop across R_{SH}, we conclude

$$I = I_J + I_{SH} = I_J + V_J/R_{SH}$$

It is also obvious from the Fig. P6.21 circuit that the applied voltage is dropped in part across the pn junction and in part across R_S. Since I is the current flowing through R_S,

$$V_A = V_J + IR_S$$

(b)–(e) A single MATLAB program, listed below and included on the instructor's disk, was constructed to perform the required computations in parts (b)–(e) of the problem. Menus provided in the program allow the user to obtain the graphical solutions associated with a given part of the problem. Sample program output is included after the program listing.

MATLAB program script...

```
%GENERALIZED DIODE I-V CHARACTERISTICS
%Initialization
clear;  close
p=menu('CHOOSE THE TYPE OF PLOTS','Semilog','Linear');
%Model Parameters
s=menu('CHOOSE PARAMETER INPUT', 'Basic Parameter List', ...
'I01 and I02 computed', 'Arbitrary Model Parameters');
I01=1.0e-13;  I02=1.0e-9;
n1=1;         n2=2;
RS=1;         RSH=1.0e12;
if s==2, I00;
elseif s==3,
AMP=input('Input in order inside [ ], I01,I02,n1,n2,RS,RSH...');
I01=AMP(1);   I02=AMP(2);
n1=AMP(3);    n2=AMP(4);
RS=AMP(5);    RSH=AMP(6);
else
end
jj=menu('CHOOSE THE PARAMETER TO BE VARIED', ...
'None','RSH','RS','I02','I01');
if jj==1, j=1;
else  j=4;
end
for ii=1:j,
    %Computation Proper
    VJ=linspace(0.01,1.00);
    kT=0.0259;
```

```
I=I01*(exp(VJ/(n1*kT))-1)+I02*(exp(VJ/(n2*kT))-1)+VJ/RSH;
VA=VJ+I*RS;

%Plotting Result
  if p==1,
  semilogy(VA,I)
  axis([0, 1, 1.0e-10, 1.0e-2]);   grid on
xlabel('VA (volts)');   ylabel('I (amps)')
hold on
  else
plot(VA,I);
axis([0,1,0,1.0e-2]);   grid on
xlabel('VA (volts)');   ylabel('I (amps)')
hold on
end

%Resetting the variable parameter
if jj==2,       RSH=RSH*1.0e-3;
elseif jj==3,   RS=RS*10;
else
if jj==4,       I02=I02*1.0e-1;
elseif jj==5,   I01=I01*1.0e1;
else
end
end
end
```

Subprogram I00 (Required for part d)

```
%IO1 and IO2 Calculation from first principles
%Constants and Parameters
q=1.6e-19;
e0=8.85e-14;
kT=0.0259;
ni=1.0e10;
KS=11.8;
EG=1.12;

%Parameters
A=1.0e-2;
NA=1.0e15;
μn=1345;
tau=1.0e-6;

%Computed Constants
Vbi=EG/2+kT*log(NA/ni);
VA0=Vbi/4;
W0=sqrt(2*KS*e0/(q*NA)*(Vbi-VA0));

%Desired Quantities
I01=q*A*(ni^2/NA)*sqrt(kT*μn/tau)
I02=(q*A*ni*W0/tau)*(kT/(Vbi-VA0))
```
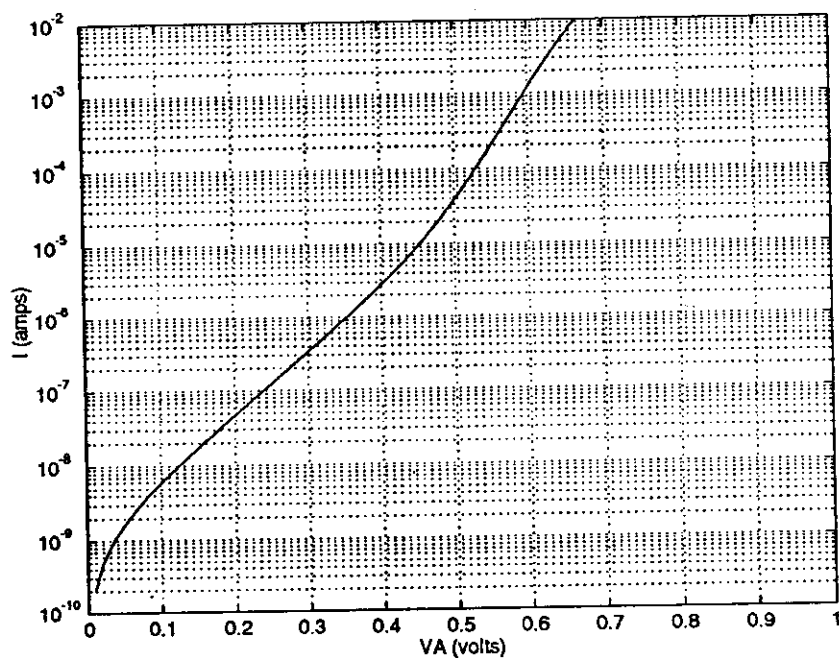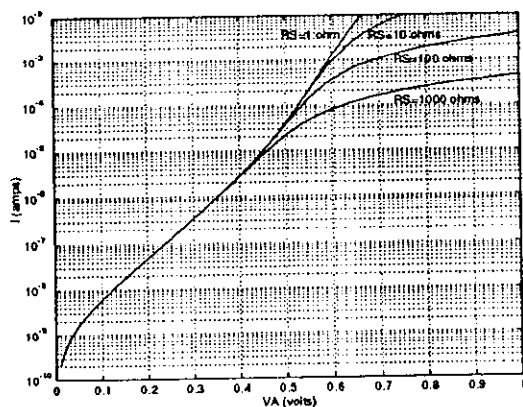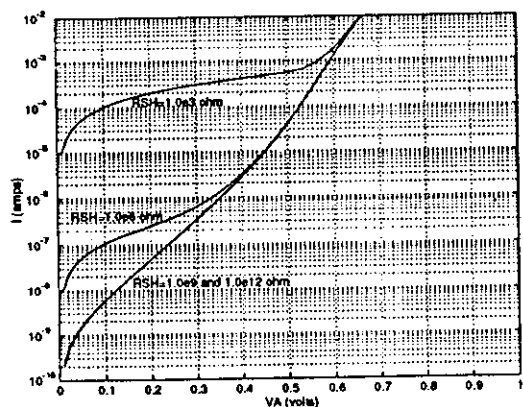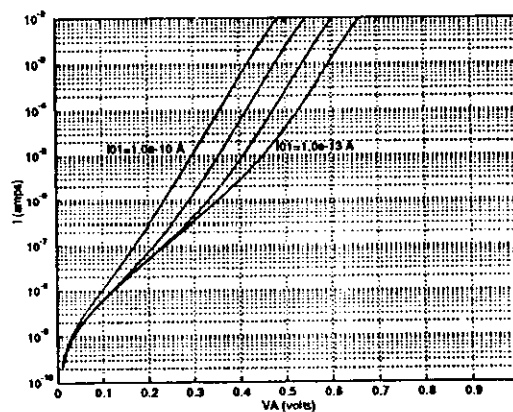
(b) ----------------

(c) ----------------

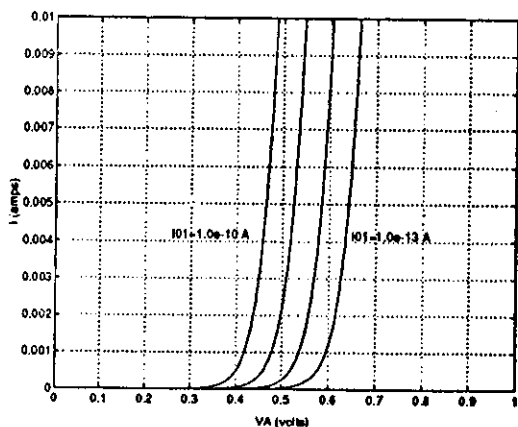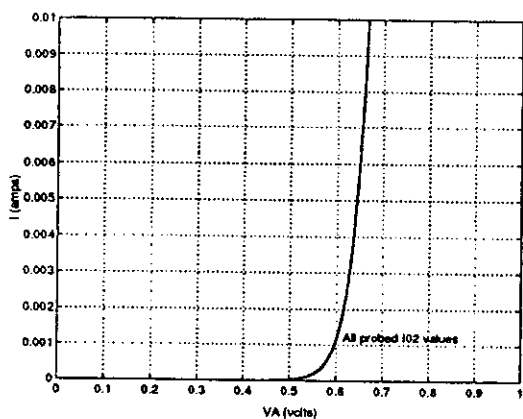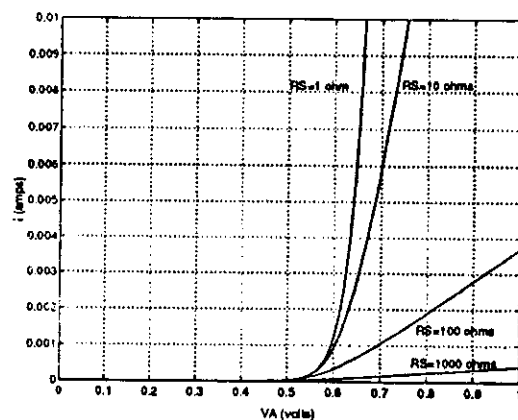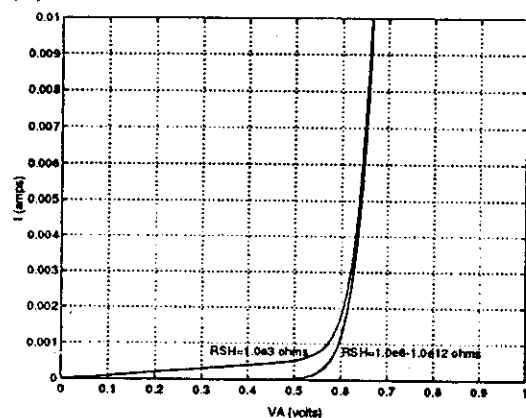*(c) (Continued)*
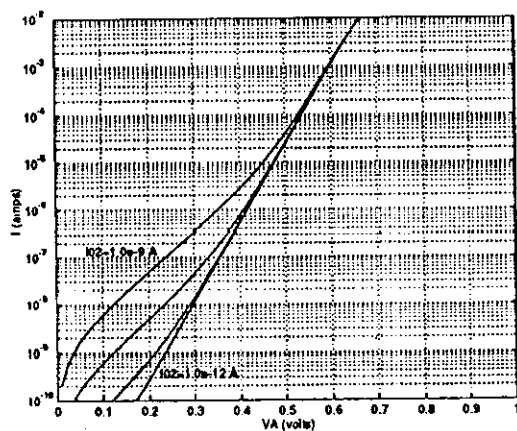
(d) -----------------
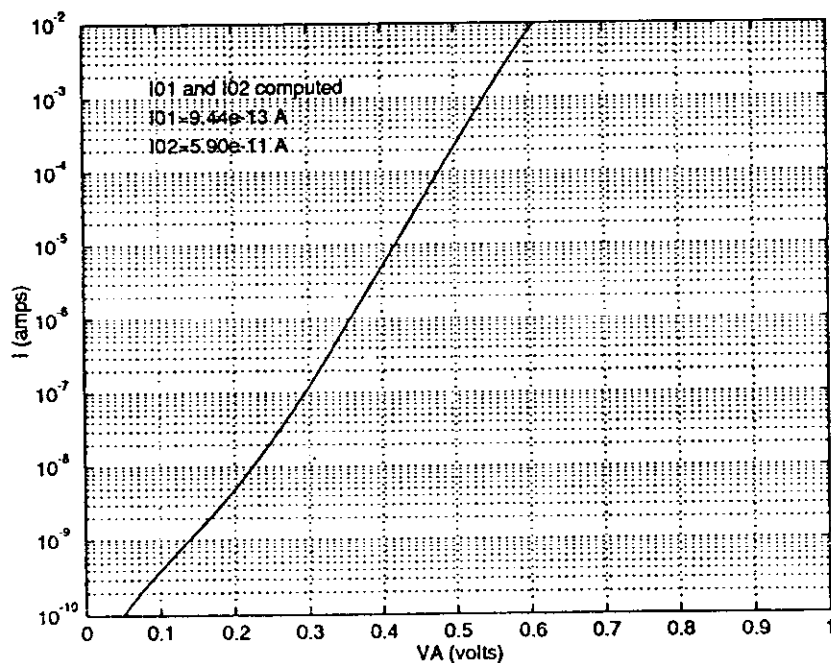
(e) ---------------

Comparison: The same results presented employing semilog plots in part (c) and linear plots in part (e) provide a decidedly different view of how a parameter affects the observed characteristics. The effect of varying I_{02} over three orders of magnitude is not observable on the linear plot (which emphasizes the higher current levels) but is clearly visible on the semilog plot. On the other hand, the effect of the series resistance is much more dramatic when viewed on a linear scale. Although the semilog plots tend to be more informative, the comparative results here suggest that it would be worthwhile to observe the forward bias *I–V* curves on both a semilog and linear scale when evaluating the characteristics.

(f) Part (b) confirms the general viability of the computational approach, yielding results that correspond very closely to experimentally observed characteristics. Part (d) confirms that a "first principle" computation of the I_0's yields similar results. Parts (c) and (e) illustrate how common nonidealities affect the observed forward bias *I–V* characteristics. Since an examination of the *I–V* characteristics is often made to deduce information about an exploratory or defective device, the information provided has significant practical utility.

<u>6.22</u>

We make the following observations:

(1) Δp_n must= 0 for $x_b \leq x \leq x_c$ because $\tau_p = 0$. This yields the boundary condition $\Delta p_n = 0$ at $x = x_b$.

(2) Because we have a p^+-n diode, we need only deal with the n-side of the junction in establishing an expression for I. Moreover, the depletion width is all but totally on the n-side of the junction given a p^+-n diode (i.e., $x_n \cong W$).

(3) Because $\tau_p = \infty$ for $0 \leq x \leq x_b$, there will no I_{R-G} current. ($\tau_p = \infty$ implies that there are no R–G centers.) Thus, we need only develop an expression for the diffusion current flowing in the diode.

Let us proceed with the derivation. Since we are interested in the static state, $\partial \Delta p_n / \partial t = 0$. Also, $G_L = 0$ (no light) and $\Delta p_n / \tau_p \to 0$ because $\tau_p = \infty$. Thus the minority carrier diffusion equation reduces to the form

$$d^2 \Delta p_n / dx^2 = 0 \qquad \dots W \leq x \leq x_b$$

subject to the boundary conditions

$$\Delta p_n(x_b) = 0 \quad \text{and} \quad \Delta p_n(W) = (n_i^2/N_D)(e^{qV_A/kT} - 1)$$

The general (narrow-base type) solution is

$$\Delta p_n(x) = A_1 + A_2 x$$

Applying the boundary conditions

$$0 = A_1 + A_2 x_b \quad \text{and} \quad \Delta p_n(W) = A_1 + A_2 W$$

giving

$$\Delta p_n(W) = -A_2(x_b - W) \quad \text{or} \quad A_2 = -\Delta p_n(W)/(x_b - W)$$

and

$$A_1 = -A_2 x_b = \Delta p_n(W) \frac{x_b}{x_b - W}$$

Thus

$$\Delta p_n(x) = \Delta p_n(W) \left(\frac{x_b - x}{x_b - W} \right) = \frac{n_i^2}{N_D} \left(\frac{x_b - x}{x_b - W} \right) (e^{qV_A/kT} - 1) \qquad \dots W \leq x \leq x_b$$

$$J_P \cong -qD_P \frac{d\Delta p_n}{dx} = q \frac{n_i^2}{N_D} \left(\frac{D_P}{x_b - W} \right) (e^{qV_A/kT} - 1)$$

and

$$\boxed{I \cong A J_P = qA \frac{n_i^2}{N_D} \left(\frac{D_P}{x_b - W} \right) (e^{qV_A/kT} - 1)}$$

6-27

The foregoing result is identical to the limiting-case narrow-base result discussed in Subsection 6.3.2. x_b-W is equivalent to the x_c' introduced in the cited Subsection. Note that there would be no advantage to translating the computational origin of coordinates to the depletion region edge in this problem. Also note that since $\Delta p_n(x)$ is a linear function of x, J_P is constant throughout the narrow-base ($W \le x \le x_b$) region.

6.23

Referring to the solution to Problem 6.22, we again need consider only I_{DIFF} because $I_{R-G} = 0$ given $\tau_p = \infty$ in the depletion region. Likewise, the same general solution as in Prob. 6.22 applies in the $W \le x \le x_b$ region.

$$\Delta p_n(x) = A_1 + A_2 x$$

Although $\Delta p_n(x_b) \ne 0$ in the present problem, we still have

$$\Delta p_n(W) = (n_i^2/N_D)(e^{qV_A/kT} - 1) = A_1 + A_2 W$$

which allows us to write

$$\Delta p_n(x) = \Delta p_n(W) + A_2(x-W) \qquad ... W \le x \le x_b$$

Introducing $x' = x - x_b$, the solution in the $x' > 0$ region will have the usual wide-base form since $x_c - x_b >> L_P$. Specifically,

$$\Delta p_n(x') = Be^{-x'/L_P} \qquad ... x' \ge 0 \text{ or } x \ge x_b$$

Both Δp_n and $J_P \propto d\Delta p_n/dx$ must be continuous at $x = x_b$. The continuity requirements yield

$$\Delta p_n(W) + A_2(x_b-W) = B$$

$$A_2 = -B/L_P$$

Solving for the remaining solution constants gives

$$A_2 = -\frac{\Delta p_n(W)}{[L_P + (x_b-W)]}$$

$$B = \frac{\Delta p_n(W) L_P}{[L_P + (x_b-W)]}$$

Therefore

$$\Delta p_n(x) = \Delta p_n(W)\left[1 - \frac{x-W}{L_P + (x_b-W)}\right] \qquad \dots\ W \le x \le x_b$$

$$\Delta p_n(x') = \frac{\Delta p_n(W)\,L_P}{[L_P + (x_b-W)]}\, e^{-x'/L_P} \qquad \dots\ x' > 0$$

and

$$J_P(x) \cong -qD_P\frac{d\Delta p_n}{dx} = q\,\frac{n_i^2}{N_D}\left(\frac{D_P}{L_P + (x_b-W)}\right)(e^{qV_A/kT} - 1) \qquad \dots\ W' \le x \le x_b$$

$$J_P(x') \cong -qD_P\frac{d\Delta p_n}{dx'} = q\,\frac{n_i^2}{N_D}\left(\frac{D_P}{L_P + (x_b-W)}\right)(e^{qV_A/kT} - 1)e^{-x'/L_P} \qquad \dots\ x' > 0$$

Finally,
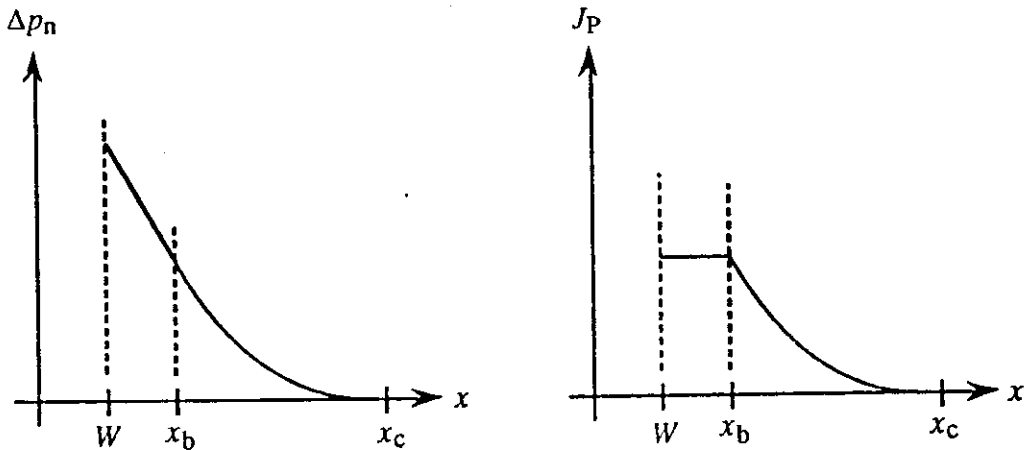
$$I = AJ_P(x=W) = AJ_P(x'=0)$$

or

$$\boxed{I = qA\,\frac{n_i^2}{N_D}\left(\frac{D_P}{L_P + (x_b-W)}\right)(e^{qV_A/kT} - 1)}$$

Note that Δp_n and J_P exhibit the positional dependence sketched below.

(a) $V_{bi} \cong 0.92$ V   for a p^+-n junction ... See Fig. E5.1.

(b) $V_{BR} \cong 55$ V   given $N_D = 10^{16}/cm^3$ ... See Fig. 6.11.

(c)

$$W \cong x_n = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_A)\right]^{1/2} \stackrel{set}{=} x_b$$

$$x_b^2 = \frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_A)$$

$$V_A = V_{bi} - \frac{qN_Dx_b^2}{2K_S\varepsilon_0} = 0.92 - \frac{(1.6\times10^{-19})(10^{16})(2\times10^{-4})^2}{(2)(11.8)(8.85\times10^{-14})}$$

$$= -29.7 \text{ V}$$

(d) Per Eq. (3.33a), $\tau_p \propto 1/N_T$. Thus, $\tau_{p1}/\tau_{p2} = N_{T2}/N_{T1} = 100$.

(e) There are two solution regions.

Bias region #1...29.7V $\leq |V_A| < V_{BR}$ $(x_n \geq x_b)$

If $x_n \geq x_b$, the quasineutral n-region is totally confined to the $N_T = N_{T2}$ or $\tau_p = \tau_{p2}$ portion of the n-side. The situation is identical to that for a standard "ideal" diode and we can just write down the usual p^+-n diode result.

$$\boxed{I_{DIFF} = -qA\frac{D_P}{L_{P2}}\frac{n_i^2}{N_D}}$$ ... $L_{P2} = \sqrt{D_P\tau_{p2}}$

Bias region #2...0 $\leq |V_A| \leq 29.7V$ $(x_n < x_b)$

If $x_n < x_b$, then there are two spatial n-regions, $x_n \leq x \leq x_b$ and $x \geq x_b$, as pictured below, that must be handled separately. Note the definition of the x' and x'' coordinates.
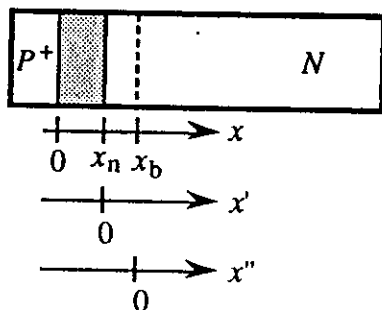
	For $x_n \leq x \leq x_b$	*For* $x > x_b$

Solve $\Rightarrow$ $\quad 0 = \dfrac{d^2\Delta p_n}{dx'^2}$ $\qquad\qquad 0 = D_P\dfrac{d^2\Delta p_n}{dx''^2} - \dfrac{\Delta p_n}{\tau_{p2}}$

(The recombination term is neglected because $L_{P1} \gg x_b$.)

Gen. sol. $\Rightarrow$ $\quad \Delta p_{n1}(x') = A_1 + A_2 x'$ $\qquad \Delta p_{n2}(x'') = B_1 e^{-x''/L_{P2}} + B_2 e^{x''/L_{P2}}$

(Narrow-base diode sol.) $\qquad\qquad$ (Wide-base diode sol. $L_{P2} = \sqrt{D_P\tau_{p2}}$)

B.C. $\Rightarrow$ $\quad \Delta p_{n1}(0) = \dfrac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)$ $\quad \Delta p_{n2}(\infty) = 0$

Applying the available boundary conditions yields

$$\Delta p_{n1}(x') = \Delta p_{n1}(0) + A_2 x' \quad \ldots\, 0 \leq x' \leq x_b - x_n$$

$$\Delta p_{n2}(x'') = B_1 e^{-x''/L_{P2}} \qquad \ldots\, x'' \geq 0$$

Both Δp_n and $d\Delta p_n/dx$ must be continuous at $x = x_b$. This requires...

$$\Delta p_{n1}(x'=x_b - x_n) = \Delta p_{n2}(x''=0)$$

$$\left.\dfrac{d\Delta p_{n1}}{dx'}\right|_{x'=x_b - x_n} = \left.\dfrac{d\Delta p_{n2}}{dx''}\right|_{x''=0}$$

or

$$\Delta p_n(0) + A_2(x_b - x_n) = B_1$$

$$A_2 = -B_1/L_{P2}$$

Solving for the constants...

$$\Delta p_{n1}(0) = B_1 + B_1\left(\dfrac{x_b - x_n}{L_{P2}}\right) \quad \text{or} \quad B_1 = \dfrac{\Delta p_{n1}(0)\, L_{P2}}{L_{P2} + (x_b - x_n)}$$

and

$$A_2 = -\dfrac{\Delta p_{n1}(0)}{L_{P2} + (x_b - x_n)}$$

Substituting back into the $\Delta p_n(x')$ solution then yields

$$\Delta p_{n1}(x') = \Delta p_{n1}(0)\left[1 - \frac{x'}{L_{P2} + (x_b - x_n)}\right] \qquad \dots 0 \le x' \le x_b - x_n$$

and finally

$$\boxed{I_{DIFF} = -qAD_P\left.\frac{d\Delta p_{n1}}{dx'}\right|_{x'=0} = qA\left(\frac{D_P}{L_{P2} + (x_b - x_n)}\right)\frac{n_i^2}{N_D}\left(e^{qV_A/kT} - 1\right)}$$

The above solution is rather interesting in that the reverse bias I_{DIFF} does not saturate. Because $x_b - x_n$ decreases with increasing reverse bias and vanishes at the end of the biasing region (at $V_A = -29.7V$), I_{DIFF} systematically increases in magnitude with increasing reverse bias. The general form of the predicted reverse bias characteristics, taking into account both biasing regions, is sketched below.
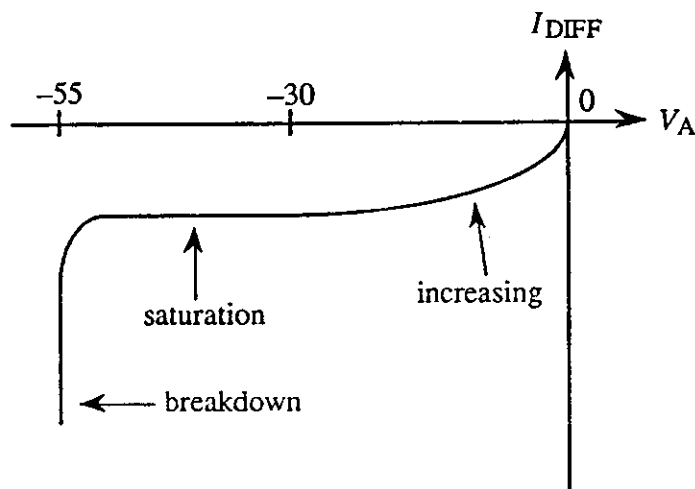
(f) Here again there are two solution regions corresponding to whether $x_n < x_b$ or $x_n > x_b$.

Bias region #1...$0 \le |V_A| \le 29.7V$ $(x_n < x_b)$

Here the depletion region lies totally in the denuded zone and

$$I_{R\text{-}G} = -qA\int_{-x_p \cong 0}^{x_n \cong W} \frac{n_i}{2\tau_{p1}}\,dx = -qA\frac{n_i}{2\tau_{p1}}W$$

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Bias region #2...29.7V $\leq |V_A| < V_{BR}$ $(x_n \geq x_b)$

Here the depletion region is partly in the denuded zone and partly in the region of the semiconductor with a high R–G center concentration. Nevertheless, the R–G current is readily obtained by summing the contributions from the two spatial regions.

$$I_{R\text{-}G} = -qA\left[\int_0^{x_b} \frac{n_i}{2\tau_{p1}}\,dx + \int_{x_b}^{x_n \cong W} \frac{n_i}{2\tau_{p2}}\,dx\right]$$

$$= -qA\left[\frac{n_i}{2\tau_{p1}}x_b + \frac{n_i}{2\tau_{p2}}(W-x_b)\right]$$

Thus we conclude

increases with W

more rapid increase because $\tau_{p2} = \tau_{p1}/100$

breakdown

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