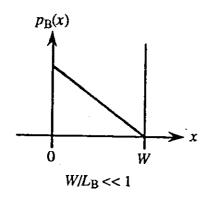
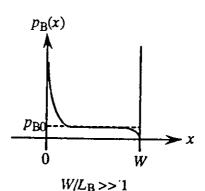
CHAPTER 11

11.1

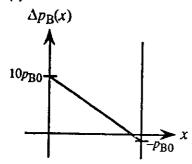
(a) The E, B, and C dopings are assumed to be nondegenerate and constant throughout a given region. The quasineutral widths of the emitter and collector are assumed to be much greater than the minority carrier diffusion lengths in the respective regions.

(b)

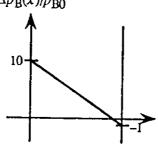




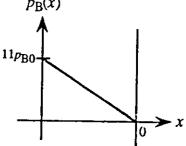
(c)



 $\Delta p_{\rm B}(x)/p_{\rm B0}$



 $p_{\rm B}(x)$



- (d) γ , $\alpha_{\rm T}$, and $\alpha_{\rm dc}$ are all typically slightly less than unity. $\beta_{\rm dc} \sim 100$ to 1000.
- (e) The desired equation is simply obtained by subtracting Eq. (11.47b) from Eq. (11.47a).

$$I_{\rm B} = I_{\rm E} - I_{\rm C} = (1 - \alpha_{\rm F}) I_{\rm F0} (e^{qV_{\rm EB}/kT} - 1) + (1 - \alpha_{\rm R}) I_{\rm R0} (e^{qV_{\rm CB}/kT} - 1)$$

(f) As noted in Eq. (11.46)

$$\alpha_F I_{F0} = \alpha_R I_{R0}$$

Thus

$$I_{R0} = \alpha_F I_{F0} / \alpha_R = (0.9944)(4.749 \times 10^{-15}) / (0.4286) = 1.102 \times 10^{-14} \text{ A}$$

- (g) The quasi-linear slope at low $V_{\rm EC}$ is caused by base width modulation. The sharp upward curvature when $V_{\rm EC} \to V_{\rm CE0}$ is caused by carrier multiplication and feedback.
- (h) Yes, it is possible. Even though $V_{\rm CB0}$ may be punch-through limited, the voltage where $M \to 1/\alpha_{\rm dc}$ may be less than the $V_{\rm CB0}$ due to punch-through.
- (i) "Current crowding" is caused by the voltage drop associated with the base current flowing laterally across the face of the emitter.
- (j) ... The drift-enhanced transport of carriers across the base decreases the transit time, thereby reduces recombination in the base, which ultimately increases α_T and the current gains.

...The built-in *E*-field and attendant decrease in the carrier transit times across the base lead to an improved high-frequency response.

- (k) A Gummel plot is a simultaneous semilog plot of I_B and I_C as a function of the input voltage V_{EB} .
- (1) The fall-off in β_{dc} at low I_C is due to an increasingly important I_{R-G} component in I_B as $I_C(V_{EB})$ is reduced. The fall-off in β_{dc} at high I_C levels is caused by high-level injection in the collector, current crowding, and/or series resistance.
- (m) No ... A totally poly-Si emitter would be essentially the same (function the same) as a totally crystalline-Si emitter.—The minority carrier distribution in a totally poly-Si emitter would be similar to Fig. 11.18(a). (Actually, a totally poly-Si emitter would undoubtedly lead to a poor E-B interface and a poorly operating BJT.)
- (n) In general, "heterojunction" refers to a junction between two dissimilar materials. With reference to the HBT, however, a "heterojunction" is understood to be a junction between two dissimilar semiconductors.
- (o) In a standard BJT the device is fabricated in a single semiconductor and $N_E >> N_B$. In HBTs the emitter is a wider band gap semiconductor and the base is the heaviest doped of the transistor regions ($N_E << N_B >> N_C$).

(a)
$$\gamma = \frac{I_{\rm Ep}}{I_{\rm Ep} + I_{\rm En}} = \frac{I_{\rm I}}{1.001I_{\rm I}} = 0.9990$$

(b)
$$\alpha_{\rm T} = \frac{I_{\rm Cp}}{I_{\rm Ep}} = \frac{0.999I_{\rm I}}{I_{\rm I}} = 0.9990$$

(c)
$$\alpha_{\rm dc} = \gamma \alpha_{\rm \Gamma} = 0.9980$$

$$\beta_{\rm dc} = \frac{\alpha_{\rm dc}}{1 - \alpha_{\rm dc}} = 499$$

(d) There are two possible solution approaches: Approach #1...

$$I_{\rm E} = I_{\rm Ep} + I_{\rm En} = 1.001I_{\rm I}$$

 $I_{\rm C} = I_{\rm Cp} + I_{\rm Cn} = 0.999I_{\rm I} + 10^{-6}I_{\rm I}$
 $I_{\rm B} = I_{\rm E} - I_{\rm C} = (1.999 \times 10^{-3})I_{\rm I}$

Approach #2...

$$I_{B1} = I_{En} = 0.001I_{I}$$

 $I_{B2} = I_{Ep} - I_{Cn} = 0.001I_{I}$
 $I_{B3} = -I_{Cn} = -(10^{-6})I_{I}$
 $I_{B} = I_{B1} + I_{B2} + I_{B3} = (1.999 \times 10^{-3})I_{I}$

Note: I_{Cn} was incorrectly shown on Fig. P11.2 as a negative current in the first printing.

(e) Yes. All currents on the figure are constant across the depletion regions. This implies (see the Depletion Region Considerations in Subsection 6.1.2) that recombination—generation is negligible in these regions.

11.3

Provided the direction of positive current flow for I_E , I_C , and I_B are as defined in Fig. 10.2(b), the equations in Subsection 11.1.1 and 11.1.2 can be appropriately modified for an npn BJT by simply making the following substitutions:

$$n_{\rm E} \rightarrow p_{\rm E}$$
 $I_{\rm Ep} \rightarrow I_{\rm En}; \ I_{\rm En} \rightarrow I_{\rm Ep}$
 $p_{\rm B} \rightarrow n_{\rm B}$ $I_{\rm Cp} \rightarrow I_{\rm Cn}; \ I_{\rm Cn} \rightarrow I_{\rm Cp}$
 $n_{\rm C} \rightarrow p_{\rm C}$ $V_{\rm EB} \rightarrow V_{\rm BE}; \ V_{\rm CB} \rightarrow V_{\rm BC}$

The desired sample computational results along with the generation program (m-file P_11_04.m), are reproduced below. Note: The assumed device area, $A = 10^{-4}$ cm², was omitted in the first printing.

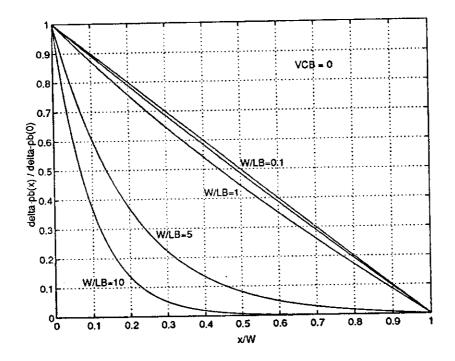
- (a) $\cosh(W/L_P) = 1.0018$ $sinh(W/L_P) = 5.9559 \times 10^{-2}$ $W/L_{\rm P} = 5.9524 \times 10^{-2}$
- (b) $I_{Ep} = 4.9473 \times 10^{-3} \text{ A}$ $I_{\rm En} = 7.2194 \times 10^{-6} \text{ A}$ $I_{\rm Cp} = 4.9386 \times 10^{-3} \, \text{A}$ $I_{\rm Cn} = 9.4373 \times 10^{-15} \text{ A}$
- (c) $I_E = 4.9545 \times 10^{-3} \text{ A}$ $I_{\rm C} = 4.9386 \times 10^{-3} \text{ A}$ $I_{\rm B} = 1.5971 \times 10^{-5} {\rm A}$
- (d) $I_{\rm B1} = 7.2194 \times 10^{-6} \text{ A}$ $I_{\rm B2} = 8.7515 \times 10^{-6} \text{ A}$ $I_{\rm B3} = -9.4373 \times 10^{-15} \text{ A}$
- (e) $\gamma = 0.99854$ $\alpha_{\rm T} = 0.99823$ $\alpha_{\rm dc} = 0.99678$ $\beta_{\rm dc} = 309.22$

MATLAB program script...

%Sample Ideal-Transistor Computations...Problem 11.4 %Initialization clear; format compact format short e %Parameters and constants NC=1.0e15;NB=1.0e16; NE=1.0e18; $\mu C = 1345;$ μB=437; $\mu E = 263;$ DC=34.8; DB=11.3; DE=6.81:tauC=1.0e-6; tauB=1.0e-6; tauE=1.0e-7; LC=5.90e-3; LB=3.36e-3; LE=8.25e-4; WB=2.0e-4;

```
VEB=0.7;
 VCB=-5:
 W=WB:
 A=1.0e-4;
 q=1.6e-19:
 ni=1.0e10;
 kT=0.0259:
 %Computations
  %Part(a)
 c=cosh (W/LB)
 s=sinh(W/LB)
 r=W/LB
  %Part(b)
 nE0=ni^2/NE;
pB0=ni^2/NB;
nC0=ni^2/NC;
IEp=q*A*DB*pB0/LB*(c/s*(exp(VEB/kT)-1)-1/s*(exp(VCB/kT)-1))
IEn=q*A*DE/LE*nE0*(exp(VEB/kT)-1)
ICp=q*A*DB*pB0/LB*(1/s*(exp(VEB/kT)-1)-c/s*(exp(VCB/kT)-1))
ICn=-q*A*DC/LC*nC0*(exp(VCB/kT)-1)
 %Part(c)
IE=IEp+IEn
IC=ICp+ICn
IB=IE-IC
 %Part (d)
IB1=IEn
IB2=IEp-ICp
IB3=-ICn
 %Part(e)
gamma=IEp/(IEp+IEn)
aT=ICp/IEp
adc=gamma*aT
Bdc=adc/(1-adc)
```

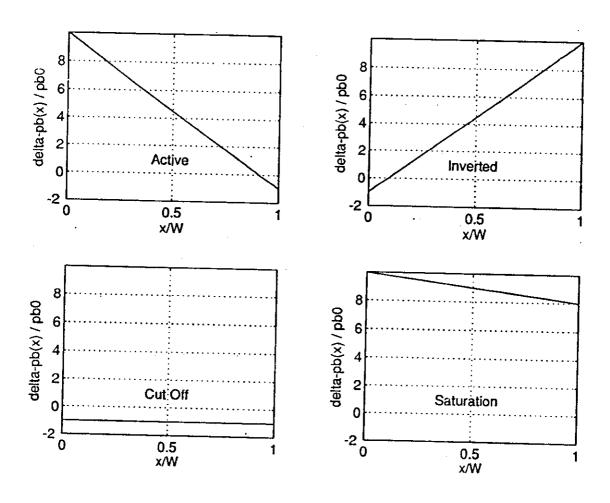
The plot displayed on the next page illustrates how the minority carrier distribution in the base of a BJT varies with W/L_B . The plot is consistent with the fact that we expect a linear distribution when $W/L_B << 1$ and an exponentially decaying distribution when $W/L_B >> 1$. It is perhaps somewhat surprising, however, that W/L_B values as large as unity yield a nearly linear distribution.



MATLAB program script...

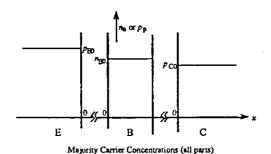
```
%Minority Carrier Distribution in the base of a BJT
%Variation with W/LB
%Initialization
clear; close
\Delta pb(x)/\Delta pb(0) vs. x/W (VCB=0) calculation
%Let r=W/LB, z=x/W, \Delta pbr=\Delta pb(x)/\Delta pb(0)
r=[10 5 1 0.5 0.1];
z=linspace(0,1);
for i=1:5,
   \Delta pbr(i,:)=sinh(r(i).*(1-z))./sinh(r(i));
   end
%Plotting result
plot(z,\Delta pbr);
axis([0 1 0 1]); grid
xlabel('x/W'); ylabel('delta-pb(x) / delta-pb(0)');
text(0.65, 0.85, VCB = 0);
text(0.5,0.52,'W/LB=0.1'); text(0.4,0.42,'W/LB=1');
text(0.27,0.28,'W/LB=5'); text(0.07,0.12,'W/LB=10');
```

11.6
"Demonstration plots" illustrating the general nature of the minority carrier distribution in a base of a BJT corresponding to the four biasing modes are displayed below. There also follows a listing of the MATLAB m-file used to produce the plots.

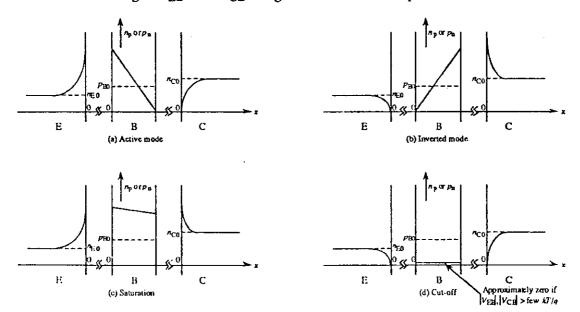


```
MATLAB program script...
%BJT Biasing Modes Illustration
%Initialization
clear; close
%Δpb(x)/pb0 vs. x/W calculation
%Let r=W/LB, z=x/W, and \Deltapbr=\Deltapb(x)/pb0
r=0.1:
z=linspace(0,1);
A=[10 -1 -1 10];
B=[-1\ 10\ -1\ 8];
\Delta pbr = A'*sinh(r*(1-z))/sinh(r) + B'*sinh(r*z)/sinh(r);
%Constructing Plots
  subplot (2,2,1); plot (z,\Delta pbr(1,:));
axis([0 1 -2 10]); grid
xlabel('x/W'); ylabel('delta-pb(x) / pb0');
text(0.4,1,'Active');
   subplot (2,2,2); plot (z,\Delta pbr(2,:));
axis([0 1 -2 10]); grid
xlabel('x/W'); ylabel('delta-pb(x) / pb0');
text(0.37,1,'Inverted');
   subplot (2,2,3); plot (z,\Delta pbr(3,:));
 axis([0 1 -2 10]); grid
xlabel('x/W'); ylabel('delta-pb(x) / pb0');
 text(0.39,1,'Cut Off');
   subplot (2,2,4); plot (z,\Delta pbr(4,:));
 axis([0 1 -2 10]); grid
 xlabel('x/W'); ylabel('delta-pb(x) / pb0');
 text(0.33,1,'Saturation');
```

Majority carriers...Assuming low-level injection, the majority carrier concentrations will be esentially unperturbed from their equilibrium values in all cases. Thus the majority carrier sketches will all be the same, with the solid carrier-distribution lines lying on top of the dashed equilibrium values in the three device regions.



Minority carriers...With $W << L_{\rm B}$, the minority carrier concentration will vary linearly with position in the base under all biasing conditions. In the emitter and collector regions the carrier concentrations will decay exponentially with distance from the edges of the respective E-B and C-B depletion regions. The carrier concentrations will be greater than the equilibrium values when the applied biases are positive and less than the equilibrium values when the applied biases are negative. The distributions are concluded to be of the general form pictured below. Note that $p_{\rm B}(x)$ in the base is approximately zero everywhere under cutoff biasing if $|V_{\rm EB}|$ and $|V_{\rm CB}|$ are greater than a few kT/q volts.



- (a) $V_{\rm EB} < 0$. The E-B junction is concluded to be reverse biased because both $\Delta n_{\rm E}/n_{\rm E0}$ and $\Delta p_{\rm B}/p_{\rm B0}$ are less than zero at the edges of the E-B depletion region.
- (b) $V_{\rm CB} > 0$. The C-B junction is concluded to be forward biased because both $\Delta p_{\rm B}/p_{\rm B0}$ and $\Delta n_{\rm C}/n_{\rm C0}$ are greater than zero at the edges of the C-B depletion region.
- (c) The boundary condition at the base edge of the C-B depletion region (Eq. 11.4b) requires

$$\Delta p_{\rm B}(W) \,=\, p_{\rm B0}(e^{qV_{\rm CB}/kT_{\rm -}}1)$$

Thus

$$\Delta p_{\rm B}(W)/p_{\rm B0} = e^{qV_{\rm CB}/kT} - 1 = 10$$

and assuming room temperature operation

$$V_{\rm CB} = (kT/q) \ln(11) = (0.0259) \ln(11) = 0.062 \text{ V}$$

(d) Inverted

(e) For an npn BJT, $n_E \rightarrow p_E$, $p_B \rightarrow n_B$, and $n_C \rightarrow p_C$. However, little else changes. The E-B junction would still be reverse biased, the C-B junction forward biased, and the BJT operated in the inverted mode with $V_{BC} = 0.062$ V. (Note that V_{BC} replaces V_{CB} .)

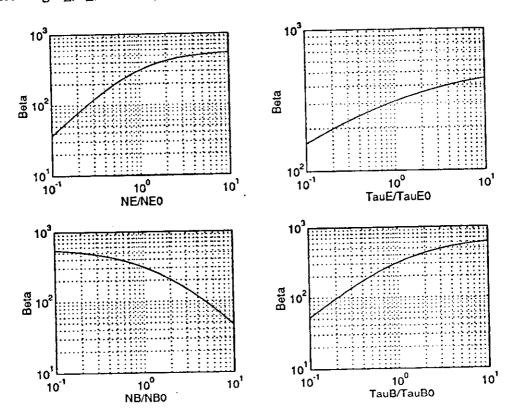
<u>11.9</u>

Change	Effect on γ	Effect on α_{Γ}	Effect on β_{dc}
Increase W _B	Decreases	Decreases	Decreases
Increase τ _B	No Effect (if W< <lb)< td=""><td>Increases</td><td>Increases</td></lb)<>	Increases	Increases
Increase N _B	Decreases	Decreases	Decreases
Increase τ _E	Increases	No Effect	Increases
Increase N _E	Increases	Essentially No Effect	Increases

Explanations:

- (1) In all cases, the effect on β_{dc} is determined by noting the effect on γ and α_T . Specifically, β_{dc} exhibits the same modification as α_{dc} , and $\alpha_{dc} = \gamma \alpha_T$. Thus, if the $\gamma \alpha_T$ product decreases, then β_{dc} decreases; if the $\gamma \alpha_T$ product increases, β_{dc} increases.
- (2) Assuming increasing W_B likewise increases W, the effect of increasing W_B on γ and α_T is obvious by inspecting Eqs. (11.41) and (11.42), respectively.
- (3) Since $L_{\rm B} = (D_{\rm B}\tau_{\rm B})^{1/2}$, increasing $\tau_{\rm B}$ increases $L_{\rm B}$, which in turn increases $\alpha_{\rm T}$. In a typical BJT, $W << L_{\rm B}$ and from Eq. (11.41) we conclude increasing $\tau_{\rm B}$ has no effect on γ . However, if W is allowed to be arbitrary, then per Eq. (11.31) increasing $\tau_{\rm B}$ increases $L_{\rm B}$, which decreases the sinh/cosh term and thereby increases γ .
- (4) Inspecting Eq. (11.41) it is obvious γ decreases if $N_{\rm B}$ increases. Noting that the mobility decreases if $N_{\rm B}$ increases and given $L_{\rm B} = (D_{\rm B}\tau_{\rm B})^{1/2} = [(kT/q)\mu_{\rm B}\tau_{\rm B}]^{1/2}$, it follows that $\alpha_{\rm T}$ also decreases if $N_{\rm B}$ increases.
- (5) Since $L_{\rm E} = (D_{\rm E}\tau_{\rm E})^{1/2}$, an increase in $\tau_{\rm E}$ leads to an increase in $L_{\rm E}$, which in turn leads to an increase in γ as is obvious from Eq. (11.41). The $\tau_{\rm E}$ parameter has no effect on transport through the base and $\alpha_{\rm T}$.
- (6) It is obvious from an inspection of Eqs. (11.41) and (11.42) that γ increases with increasing $N_{\rm E}$ and that $N_{\rm E}$ does not explicitly affect $\alpha_{\rm T}$. Changing $N_{\rm E}$ will slightly change the $V_{\rm bi}$ across the emitter-base junction and thereby affect the emitter-base depletion width. The net effect on $\alpha_{\rm T}$, however, should typically be small.

11.10 The requested plots and the generating program are reproduced below. The graphical results here are indeed consistent with the answers to Problem 11.9.— β_{dc} increases with increasing N_{E} , τ_{E} , and τ_{B} . β_{dc} decreases with increasing N_{B} .



MATLAB program script...

%Problem 11.10...Beta Calculation %Initialization clear; close %Reference Values and Constants NEO=1.0e18; TauEO=1.0e-7; NBO=1.0e16; TauBO=1.0e-6; WB=2.0e-4; kT=0.0259;

```
W=WB;
%Mobility Fit Parameters
NDref=1.3e17; NAref=2.35e17;
μnmin=92; μpmin=54.3;
\mun0=1268; \mup0=406.9;
an=0.91; ap=0.88;
%Beta vs. NE/NEO
  %Base Parameters
    NB=NB0:
    \muB=\mupmin+\mup0./(1+(NB./NAref).^ap);
    DB=kT.*µB;
    TauB=TauB0;
    LB=sqrt (DB. *TauB);
  %Emitter Parameters
    NE=logspace(log10(0.1*NE0),log10(10*NE0));
    \mu E = \mu n \min + \mu n 0. / (1 + (NE./NDref).^an);
    DE=kT.*µE;
    TauE=TauE0:
    LE=sqrt (DE.*TauE);
  %Beta Calculation
    Y=1 ./((DE./DB).*(NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
    X=NE./NEO;
  %Plotting
    subplot(2,2,1); loglog(X,Y); grid
    xlabel('NE/NEO'); ylabel('Beta')
%Beta vs. TauE/TauE0
  %Revised Emitter Parameters
    NE=NE0:
    \mu E = \mu n min + \mu n0. / (1 + (NE./NDref).^an);
    DE=kT.*\mu E;
    TauE=logspace(log10(0.1*TauE0),log10(10*TauE0));
    LE=sqrt (DE.*TauE);
  %Beta Calculation
    Y=1 ./((DE./DB).*(NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
    X=TauE/TauE0;
  %Plotting
    subplot(2,2,2); loglog(X,Y); grid
    xlabel('TauE/TauE0'); ylabel('Beta')
%Beta vs. NB/NB0
  %Emitter Parameters
  %NE,\muE, and DE same as prior calculation
    TauE=TauE0;
    LE=sqrt (DE.*TauE);
```

```
%Base Parameters
   NB=logspace(log10(0.1*NB0),log10(10*NB0));
   \muB=\mupmin+\mup0./(1+(NB./NAref).^ap);
   DB=kT.*µB;
   TauB=TauB0;
   LB=sqrt (DB. *TauB);
 %Beta Calculation
   Y=1 ./((DE./DB).*(NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
   X=NB/NB0;
 %Plotting
    subplot(2,2,3); loglog(X,Y); grid
   xlabel('NB/NB0'); ylabel('Beta')
%Beta vs. TauB/TauB0
  %Revised Base Parameters
    NB=NB0:
    \muB=\mupmin+\mup0./(1+(NB./NAref).^ap);
    DB=kT.*µB;
    TauB=logspace(log10(0.1*TauB0),log10(10*TauB0));
    LB=sqrt (DB.*TauB);
  %Beta Calculation
    Y=1 ./((DE./DB).*(NB./NE).*(W./LE)+(0.5).*(W./LB).^2);
    X=TauB/TauB0;
  %Plotting
    subplot(2,2,4); loglog(X,Y); grid
    xlabel('TauB/TauB0'); ylabel('Beta')
```

(a) Like in the standard ideal-transistor analysis, the general solution for $\Delta n_{\rm E}(x^{"})$ is

$$\Delta n_{\rm E}(x'') = A_1 e^{-x''/L_{\rm E}} + A_2 e^{x''/L_{\rm E}}$$

However, because of the finite width of the emitter, $A_2 \neq 0$. Rather

$$\Delta n_{\rm E}(x"=0) = n_{\rm E0}(e^{qV_{\rm EB}/kT} - 1) = A_1 + A_2$$

 $\Delta n_{\rm E}(x"=W_{\rm E}) = 0 = A_1e^{-W_{\rm E}/L_{\rm E}} + A_2e^{W_{\rm E}/L_{\rm E}}$

Solving for A₁ and A₂ yields

$$A_1 = \frac{\Delta n_{\rm E}(0) \, e^{W_{\rm E}/L_{\rm E}}}{e^{W_{\rm E}/L_{\rm E}} - e^{-W_{\rm E}/L_{\rm E}}}$$
 and $A_2 = -\frac{\Delta n_{\rm E}(0) \, e^{-W_{\rm E}/L_{\rm E}}}{e^{W_{\rm E}/L_{\rm E}} - e^{-W_{\rm E}/L_{\rm E}}}$

Substituting back into the general solution then gives

$$\Delta n_{\rm E}(x'') = \Delta n_{\rm E}(0) \left[\frac{e^{(W_{\rm E}-x'')/L_{\rm E}} - e^{-(W_{\rm E}-x'')/L_{\rm E}}}{e^{W_{\rm E}/L_{\rm E}} - e^{-W_{\rm E}/L_{\rm E}}} \right]$$

or

$$\Delta n_{\rm E}(x") = n_{\rm E0} \frac{\sinh[(W_{\rm E}-x")/L_{\rm E}]}{\sinh(W_{\rm E}/L_{\rm E})} \left(e^{qV_{\rm EB}/kT} - 1\right)$$

Applying Eq. (11.7), we obtain the revised I_{En} expression

$$I_{\rm En} = qA \frac{D_{\rm E}}{L_{\rm E}} n_{\rm E0} \frac{\cosh(W_{\rm E}/L_{\rm E})}{\sinh(W_{\rm E}/L_{\rm E})} \left(e^{qV_{\rm EB}/kT} - 1\right)$$

(b) Relative to the standard result (Eq. 11.20), the finite width of the emitter leads to the modification

$$n_{\rm E0} \rightarrow n_{\rm E0} \frac{\cosh(W_{\rm E}/L_{\rm E})}{\sinh(W_{\rm E}/L_{\rm E})}$$

or equivalently, since $n_{\rm E0} = n_{\rm i}^2/N_{\rm E}$,

$$\frac{1}{N_{\rm E}} \rightarrow \frac{1}{N_{\rm E}} \frac{\cosh(W_{\rm E}/L_{\rm E})}{\sinh(W_{\rm E}/L_{\rm E})}$$

Revised expressions for the performance parameters analogous to Eqs. (11.31)–(11.34) are therefore obtained by simply making the above noted substitution. Specifically,

$$\left(\frac{D_{\rm E}}{D_{\rm B}}\frac{L_{\rm B}}{L_{\rm E}}\frac{N_{\rm B}}{N_{\rm E}}\right) \to \left(\frac{D_{\rm E}}{D_{\rm B}}\frac{L_{\rm B}}{L_{\rm E}}\frac{N_{\rm B}}{N_{\rm E}}\right) \frac{\cosh(W_{\rm E}/L_{\rm E})}{\sinh(W_{\rm E}/L_{\rm E})}$$

in the γ , $\alpha_{\rm dc}$, and $\beta_{\rm dc}$ expressions. $\alpha_{\rm T}$ remains unchanged.

(c) With $W_E/L_E << 1$,

$$\frac{\cosh(W_{\rm E}/L_{\rm E})}{\sinh(W_{\rm E}/L_{\rm E})} \cong \frac{L_{\rm E}}{W_{\rm E}}$$

and $L_{\rm E}$ in the Eq. (11.41)–(11.44) expressions is to first order simply replaced by $W_{\rm E}$. This yields, for example,

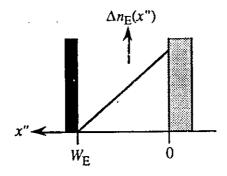
$$\gamma = \frac{1}{1 + \frac{D_E}{D_B} \frac{N_B}{N_E} \frac{W}{W_E}}$$

A similar modification applies to α_{dc} and β_{dc} . Being independent of L_E , the α_T expression remains unchanged.

- (d) A systematic decrease in W_E obviously causes a monotonic decrease in γ and a corresponding decrease in β_{dc} . Reducing the width of the emitter has the negative effect of degrading the gain of the BJT (which is the point of the problem).
- (e) If $W_E \ll L_E$ the part (a) result simplifies to

$$\Delta n_{\rm E}(x'') = \Delta n_{\rm E}(0) \left(1 - \frac{x''}{W_{\rm E}}\right)$$

or one obtains a linear distribution analogous to the situation in the base. Moreover, since the emitter is forward biased under active mode biasing, $\Delta n_E(0) > 0$. Thus we conclude



"Diode" Configuration (a)...

(a) Note that $I = I_E$, $V_{EB} = V_A$, and $I_C = 0$ for the given circuit configuration. Since $I_C = 0$, it follows from Eq. (11.47b) that

$$\alpha_{\rm F}I_{\rm F0} (e^{qV_{\rm EB}/kT_{\rm -}}1) = I_{\rm R0} (e^{qV_{\rm CB}/kT_{\rm -}}1)$$

OF

$$(e^{qV_{CB}/kT}-1) = (\alpha_F I_{FO}/I_{RO})(e^{qV_{EB}/kT}-1)$$

and

$$I = I_{\rm E} = I_{\rm F0} (e^{qV_{\rm EB}/kT} - 1) - (\alpha_{\rm R}I_{\rm R0})(\alpha_{\rm F}I_{\rm F0}/I_{\rm R0})(e^{qV_{\rm EB}/kT} - 1)$$

Therefore

$$I = (1 - \alpha_R \alpha_F) I_{F0} (e^{qV_A/kT_-} 1)$$

(b)
$$\Delta p_{\rm B}(0)/p_{\rm B0} = (e^{qV_{\rm EB}/kT}-1) = (e^{qV_{\rm A}/kT}-1)$$

 $\Delta p_{\rm B}(W)/p_{\rm B0} = (e^{qV_{\rm CB}/kT}-1) = (\alpha_{\rm F}I_{\rm F0}/I_{\rm R0})(e^{qV_{\rm A}/kT}-1)$

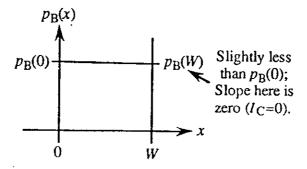
(c) Simplifying

$$\Delta p_{\rm B}(0)/p_{\rm B0} = (e^{qV_{\rm A}/kT_{\rm -}}1)$$

$$\Delta p_{\rm B}(W)/p_{\rm BO} = \alpha \left(e^{qV_{\rm A}/kT}-1\right)$$

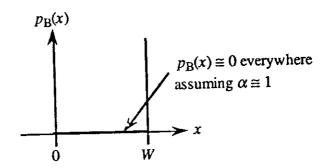
(d) If $V_A \gg kT/q$,

$$p_{\rm B}(0) = p_{\rm B0}e^{qV_{\rm A}/kT}$$
 and $p_{\rm B}(W) = \alpha p_{\rm B0}e^{qV_{\rm A}/kT}$



(e) If
$$-V_A >> kT/q$$
,

$$p_{\rm B}(0) \cong 0$$
 and $p_{\rm B}(W) \cong (1-\alpha)p_{\rm B0} \cong 0$



"Diode" Configuration (b)...

(a) For the given circuit configuration, $V_{\rm EB} = V_{\rm A}$, $V_{\rm CB} = 0$, and $I = I_{\rm E}$. It therefore follows from Eq. (11.47a) that

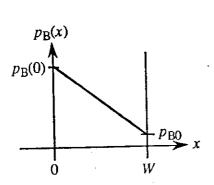
$$I = I_{\text{F0}} \left(e^{qV_{\text{A}}/kT} - 1 \right)$$

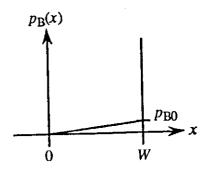
(b)
$$\Delta p_{\rm B}(0)/p_{\rm B0} = (e^{qV_{\rm EB}/kT}-1) = (e^{qV_{\rm A}/kT}-1)$$

$$\Delta p_{\rm B}(W)/p_{\rm B0} = (e^{qV_{\rm CB}/kT}-1) = 0$$

(c) There is no further simplification of the part (b) result.

(d)





"Diode" Configuration (c)...

(a)/(b) For the given configuration, $V_{\rm EB} - V_{\rm CB} = V_{\rm A}$ and $I = I_{\rm E} = I_{\rm C}$. Equating the Ebers-Moll relationships for $I_{\rm E}$ and $I_{\rm C}$ yields

$$(1-\alpha_{\rm F}) I_{\rm F0} (e^{qV_{\rm EB}/kT}-1) = -(1-\alpha_{\rm R}) I_{\rm R0} (e^{qV_{\rm CB}/kT}-1)$$

or

$$(e^{qV_{\text{EB}}/kT}-1) = -\xi (e^{qV_{\text{CB}}/kT}-1)$$
 ... $\xi = \frac{(1-\alpha_{\text{R}})I_{\text{R0}}}{(1-\alpha_{\text{F}})I_{\text{F0}}}$

Next noting

$$(e^{qV_{\text{EB}}/kT}-1) = (e^{qV_{\text{A}}/kT}e^{qV_{\text{CB}}/kT}-1) = e^{qV_{\text{A}}/kT}(e^{qV_{\text{CB}}/kT}-1) + (e^{qV_{\text{A}}/kT}-1)$$

Thus

$$(e^{qV_A/kT} + \xi)(e^{qV_{CB}/kT} - 1) = -(e^{qV_A/kT} - 1)$$

or

$$(e^{qV_{\text{CB}}/kT}-1) = -\frac{(e^{qV_{\text{A}}/kT}-1)}{(e^{qV_{\text{A}}/kT}+\xi)} = \frac{\Delta p_{\text{B}}(W)}{p_{\text{B0}}}$$

and

$$(e^{qV_{\text{EB}}/kT}-1) = \frac{\xi(e^{qV_{\text{A}}/kT}-1)}{(e^{qV_{\text{A}}/kT}+\xi)} = \frac{\Delta p_{\text{B}}(0)}{p_{\text{B0}}}$$

Finally, substituting into Eq. (11.47a) and simplifying

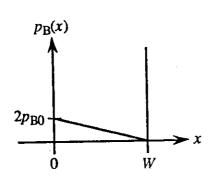
$$I = \frac{(1 - \alpha_{\rm R} \alpha_{\rm F}) I_{\rm R0} I_{\rm F0} (e^{qV_{\rm A}/kT} - 1)}{(1 - \alpha_{\rm F}) I_{\rm F0} e^{qV_{\rm A}/kT} + (1 - \alpha_{\rm R}) I_{\rm R0}}$$

(c) Under the given simplifications, $\xi \rightarrow 1$ and

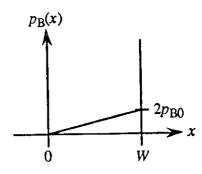
$$\frac{\Delta p_{\rm B}(0)}{p_{\rm B0}} = -\frac{\Delta p_{\rm B}(W)}{p_{\rm B0}} = \frac{e^{\,qV_{\rm A}/kT} - 1}{e^{\,qV_{\rm A}/kT} + 1}$$

(d)/(e) If $V_A >> kT/q$, $\Delta p_B(0)/p_{B0} = -\Delta p_B(W)/p_{B0} = 1$ or $p_B(0) = 2p_{B0}$ and $p_B(W) = 0$. If $-V_A >> kT/q$, $\Delta p_B(0)/p_{B0} = -\Delta p_B(W)/p_{B0} = -1$ or $p_B(0) = 0$ and $p_B(W) = 2p_{B0}$.

(d)



(e)



"Diode" Configuration (d)...

(a) For the given connection, $I = I_{\rm B}$ and $V_{\rm EB} = V_{\rm CB} = V_{\rm A}$. In general, employing Eqs. (11.47a) and (11.47b),

Thus

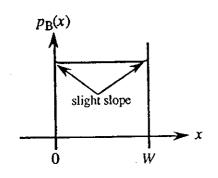
$$I_{\rm B} = I_{\rm E} - I_{\rm C} = (1 - \alpha_{\rm F}) I_{\rm F0} (e^{qV_{\rm EB}/kT} - 1) + (1 - \alpha_{\rm R}) I_{\rm R0} (e^{qV_{\rm CB}/kT} - 1)$$

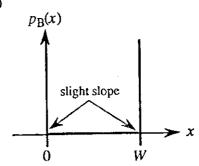
$$I = [(1 - \alpha_{\rm F})I_{\rm F0} + (1 - \alpha_{\rm R})I_{\rm R0}](e^{qV_{\rm A}/kT} - 1)$$

(b)
$$\Delta p_{\rm B}(0)/p_{\rm B0} = \Delta p_{\rm B}(W)/p_{\rm B0} = (e^{qV_{\rm A}/kT}-1)$$

(c) There is no further simplification of the part (b) result.

(d)





"Diode" Configuration (e)...

(a) Here $I = -I_C$, $V_{CB} = V_A$, and $I_E = 0$. Since $I_E = 0$, it follows from Eq. (11.47a) that

$$I_{\text{F0}} (e^{qV_{\text{EB}}/kT} - 1) = \alpha_{\text{R}} I_{\text{R0}} (e^{qV_{\text{CB}}/kT} - 1)$$

or

$$(e^{qV_{EB}/kT}-1) = (\alpha_R I_{R0}/I_{F0})(e^{qV_{CB}/kT}-1)$$

and

$$I = -I_{\rm C} = -(\alpha_{\rm F}I_{\rm F0})(\alpha_{\rm R}I_{\rm R0}/I_{\rm F0})(e^{qV_{\rm CB}/kT}-1) + I_{\rm R0}(e^{qV_{\rm CB}/kT}-1)$$

Therefore

$$I = (1 - \alpha_{\rm R} \alpha_{\rm F}) I_{\rm R0} (e^{qV_{\rm A}/kT} - 1)$$

(b)
$$\Delta p_{\rm B}(0)/p_{\rm B0} = (e^{qV_{\rm EB}/kT}-1) = (\alpha_{\rm R}I_{\rm R0}/I_{\rm F0}) (e^{qV_{\rm A}/kT}-1)$$

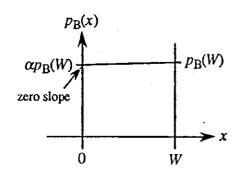
 $\Delta p_{\rm B}(W)/p_{\rm B0} = (e^{qV_{\rm CB}/kT}-1) = (e^{qV_{\rm A}/kT}-1)$

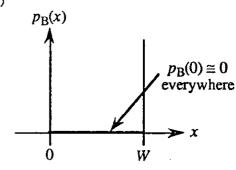
(c) Simplifying

$$\Delta p_{\rm B}(0)/p_{\rm B0} = \alpha \left(e^{qV_{\rm A}/kT} - 1\right)$$

$$\Delta p_{\rm B}(W)/p_{\rm B0} \,=\, (e^{qV_{\rm A}/kT}\!\!-1)$$

(d)





"Diode" Configuration (f)...

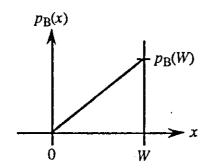
(a) Here $V_{\rm EB}=0$, $V_{\rm CB}=V_{\rm A}$, and $I=-I_{\rm C}$. It therefore follows from Eq. (11.47b) that

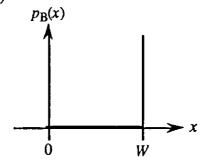
$$I = I_{R0} \left(e^{qV_A/kT} - 1 \right)$$

(b)
$$\Delta p_{\rm B}(0)/p_{\rm B0} = (e^{qV_{\rm EB}/kT_{-}}1) = 0$$
 $\Delta p_{\rm B}(W)/p_{\rm B0} = (e^{qV_{\rm CB}/kT_{-}}1) = (e^{qV_{\rm A}/kT_{-}}1)$

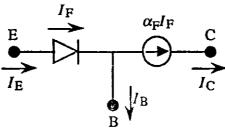
(c) There is no further simplification of the part (b) result.

(d)





(a) Since $V_{\rm CB}=0$ at the active-mode/saturation-mode boundary, it follows that $I_{\rm R}=0$. The two Ebers-Moll circuit elements involving $I_{\rm R}$ therefore vanish yielding the following simplified circuit.



(b) As deduced from the part (a) circuit,

$$I_{\rm B} = I_{\rm E} - I_{\rm C} = I_{\rm F} - \alpha_{\rm F} I_{\rm F} = (1 - \alpha_{\rm F}) I_{\rm F0} (e^{qV_{\rm EB}/kT} - 1)$$

but

$$V_{\rm EC} = V_{\rm EB} - V_{\rm CB} = V_{\rm EB} \qquad (V_{\rm CB} = 0)$$

Thus

$$I_{\rm B} = (1-\alpha_{\rm F}) I_{\rm F0} (e^{qV_{\rm EC}/kT} - 1)$$

and

$$V_{\rm EC} = \frac{kT}{q} \ln \left[1 + \frac{I_{\rm B}}{(1 - \alpha_{\rm F})I_{\rm F0}} \right]$$

11,14

(a) Solving Eq. (11.47a) for $\exp(qV_{EB}/kT) - 1$, one obtains

$$e^{qV_{EB}/kT} - 1 = \frac{I_E + \alpha_R I_{R0} (e^{qV_{CB}/kT} - 1)}{I_{F0}}$$

Substituting the $\exp(qV_{\rm EB}/kT) - 1$ expression into Eq. (11.47b) then yields

$$I_{C} = \alpha_{F}I_{E} + \alpha_{F}\alpha_{R}I_{R0}(e^{qV_{CB}/kT} - 1) - I_{R0}(e^{qV_{CB}/kT} - 1)$$
or
$$I_{C} = \alpha_{F}I_{E} - (1 - \alpha_{F}\alpha_{R})I_{R0}(e^{qV_{CB}/kT} - 1) \qquad \Leftarrow \text{desired equation}$$

(b) Making use of Eqs. (11.47a) and (11.47b), we can write

$$I_{\rm B} = I_{\rm E} - I_{\rm C} = (1 - \alpha_{\rm F})I_{\rm F0}(e^{qV_{\rm EB}/kT} - 1) + (1 - \alpha_{\rm R})I_{\rm R0}(e^{qV_{\rm CB}/kT} - 1)$$

Now

$$V_{\text{CB}} = V_{\text{CE}} + V_{\text{EB}} = V_{\text{EB}} - V_{\text{EC}}$$

and therefore

 $I_{\rm B} = (1 - \alpha_{\rm F})I_{\rm F0}e^{qV_{\rm EB}/kT} - (1 - \alpha_{\rm F})I_{\rm F0} + (1 - \alpha_{\rm R})I_{\rm R0}e^{qV_{\rm EB}/kT}e^{-qV_{\rm EC}/kT} - (1 - \alpha_{\rm R})I_{\rm R0}$ or rearranging to obtain the desired equation

$$I_{\rm B} = [(1-\alpha_{\rm F})I_{\rm F0} + (1-\alpha_{\rm R})I_{\rm R0}e^{-qV_{\rm EC}/kT}]e^{qV_{\rm EB}/kT} - [(1-\alpha_{\rm F})I_{\rm F0} + (1-\alpha_{\rm R})I_{\rm R0}]$$

(c) Eq. (11.47b) can be rewritten

$$I_{\rm C} = \alpha_{\rm F} I_{\rm F0} e^{qV_{\rm EB}/kT} - \alpha_{\rm F} I_{\rm F0} - I_{\rm R0} e^{qV_{\rm EB}/kT} e^{-qV_{\rm EC}/kT} + I_{\rm R0}$$
$$= (\alpha_{\rm F} I_{\rm F0} - I_{\rm R0} e^{-qV_{\rm EC}/kT}) e^{qV_{\rm EB}/kT} + I_{\rm R0} - \alpha_{\rm F} I_{\rm F0}$$

Referring to the I_B solution in part (b), we note

$$e^{qV_{EB}/kT} = \frac{I_{B} + (1-\alpha_{F})I_{F0} + (1-\alpha_{R})I_{R0}}{(1-\alpha_{F})I_{F0} + (1-\alpha_{R})I_{R0}e^{-qV_{EC}/kT}}$$

which when substituted into the IC expression yields

$$I_{\rm C} = \frac{(\alpha_{\rm F} I_{\rm F0} - I_{\rm R0} e^{-qV_{\rm EC}/kT})[I_{\rm B} + (1-\alpha_{\rm F})I_{\rm F0} + (1-\alpha_{\rm R})I_{\rm R0}]}{(1-\alpha_{\rm F})I_{\rm F0} + (1-\alpha_{\rm R})I_{\rm R0} e^{-qV_{\rm EC}/kT}} + I_{\rm R0} - \alpha_{\rm F} I_{\rm F0}$$

11,15

(a) As recorded in Exercise 11.7, the general relationship for the common emitter input characteristic is

$$I_{\rm B} = [(1-\alpha_{\rm F})I_{\rm F0} + (1-\alpha_{\rm R})I_{\rm R0}e^{-qV_{\rm EC}/kT}]e^{qV_{\rm EB}/kT} - [(1-\alpha_{\rm F})I_{\rm F0} + (1-\alpha_{\rm R})I_{\rm R0}]$$

For a routine bias of $V_{\rm EC}$ > few kT/q volts, the term involving $\exp(-qV_{\rm EC})$ will be negligible. Moreover, the second group of terms is typically small and may be completely ignored for $V_{\rm EB}$ biases normally encountered in practice. Thus the $I_{\rm B}$ expression reduces to

$$I_{\rm B} \cong (1-\alpha_{\rm F})I_{\rm F0}e^{qV_{\rm EB}/kT}$$

Using Eqs. (11.45a) and (11.46) to display the explicit parametric dependencies, one obtains

$$I_{\rm B} \cong qA \left[\frac{D_{\rm E}}{L_{\rm E}} n_{\rm E0} + \frac{D_{\rm B}}{L_{\rm B}} p_{\rm B0} \frac{\cosh(W/L_{\rm B}) - 1}{\sinh(W/L_{\rm B})} \right] e^{qV_{\rm EB}/kT}$$

In a standard transistor $W \ll L_B$ and

$$\frac{\cosh(W/L_{\rm B})-1}{\sinh(W/L_{\rm B})} \equiv \frac{(1/2)(W/L_{\rm B})^2}{W/L_{\rm B}} = W/2L_{\rm B}$$

and

$$I_{\rm B} = qA \left(\frac{D_{\rm E}}{L_{\rm E}} n_{\rm E0} + \frac{1}{2} \frac{D_{\rm B}}{L_{\rm B}} \frac{W}{L_{\rm B}} p_{\rm B0} \right) e^{qV_{\rm EB}/kT}$$

or

$$I_{\rm B} = qA \left(\frac{D_{\rm E}}{L_{\rm E}} \frac{n_{\rm i}^2}{N_{\rm E}} + \frac{1}{2} \frac{W}{L_{\rm B}} \frac{D_{\rm B}}{L_{\rm B}} \frac{n_{\rm i}^2}{N_{\rm B}} \right) e^{qV_{\rm EB}/kT}$$

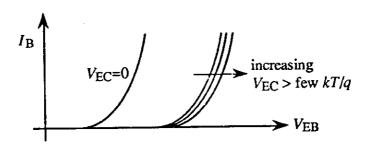
If $D_E N_B W/D_B N_E L_E >> (1/2)(W/L_B)^2$, then the first term inside the parentheses in the above expression will be much greater than the second term. We conclude for the specification

$$I_{\rm B} \cong qA \left(\frac{D_{\rm E}}{L_{\rm E}} \frac{n_{\rm i}^2}{N_{\rm E}}\right) e^{qV_{\rm EB}/kT}$$

There is no W-dependence in the above expression. Therefore the common emitter input characteristics are expected to be relatively insensitive to base width modulation if the gain is limited by the emitter efficiency.

:

(b) If the gain is not limited by the emitter efficiency, then the second term in the simplified I_B expression developed in part (a) must be retained. This second term decreases with decreasing W, and I_B is therefore predicted to decrease with increasing V_{EC} . For the specified situation, the expected form of the common emitter input characteristics are as sketched below.



 $\frac{11.16}{\text{If }V_{\text{EB}}} = 0.7 \text{ V}$ instead of zero as assumed in Exercise 11.8, then x_{nEB} will be reduced and V_{CB} is expected to increase. Specifically, recalculating x_{nEB} and V_{CB} ,

$$x_{\text{nEB}} = \left[\frac{2K_{\text{S}}\varepsilon_{0}}{q} \frac{N_{\text{E}}}{N_{\text{B}}(N_{\text{E}}+N_{\text{B}})} (V_{\text{bi}(\text{EB}}) - V_{\text{EB}}) \right]^{1/2}$$

$$= \left[\frac{(2)(11.8)(8.85 \times 10^{-14})(10^{18})(0.835 - 0.7)}{(1.6 \times 10^{-19})(10^{16})(1.01 \times 10^{18})} \right]^{1/2} = 1.32 \times 10^{-5} \text{ cm}$$

$$V_{\text{CB}} = V_{\text{bi(CB)}} - \frac{(W_{\text{B}} - x_{\text{nEB}})^{2}}{\frac{2K_{\text{S}}\varepsilon_{0}}{q} \frac{N_{\text{C}}}{N_{\text{B}}(N_{\text{C}} + N_{\text{B}})}} \dots \text{at punch-through}$$

$$= 0.656 - \frac{(2 \times 10^{-4} - 1.32 \times 10^{-5})^{2}}{\left[\frac{(2)(11.8)(8.85 \times 10^{-14})(10^{15})}{(1.6 \times 10^{-19})(10^{16})(1.1 \times 10^{16})}\right]} = -293 \text{ V}$$

(a) Transistor A. Referring to Eq. (11.41),

$$\gamma = \frac{1}{1 + \frac{D_{\rm E}}{D_{\rm B}} \frac{N_{\rm B}}{N_{\rm E}} \frac{W}{L_{\rm E}}}$$

In Transistor A, $N_E >> N_B$ and $\gamma \to 1$. In Transistor B, $N_E < N_B$ and γ is expected to be considerably less than unity.

One might alternatively argue that $I_{\rm Ep} >> I_{\rm En}$ in Transistor A, while $I_{\rm En} > I_{\rm Ep}$ in Transistor B. Since $\gamma = I_{\rm Ep}/(I_{\rm Ep} + I_{\rm En})$ in a pnp transistor, Transistor A clearly has the greater emitter injection efficiency.

- (b) Under active mode biasing $V_{\rm EB} > 0$ and $V_{\rm CB} < 0$. Considering the more important reverse-bias collector-base junction, there is very little incursion of the depletion region into the base in Transistor A. For Transistor B, however, most of the depletion region lies in the base because $N_{\rm C} >> N_{\rm B}$. Thus Transistor B will be more sensitive to base width modulation.
- (c) Transistor A. $V_{\rm CB0}$ is approximately equal to $V_{\rm BR}$ of the C-B junction if the BJT is limited by avalanche breakdown. $V_{\rm BR}$ in turn is roughly inversely proportional to the doping on the lightly-doped side of the pn junction. In Transistor A, the collector is the lower doped with $N_{\rm C} = 10^{14}/{\rm cm}^3$; in Transistor B, the base has the lighter doping, $N_{\rm B} = 10^{15}/{\rm cm}^3$. Since $N_{\rm C}$ of Transistor A is less than $N_{\rm B}$ of Transistor B, Transistor A will exhibit the larger $V_{\rm CB0}$.

11.18
The following "data" was obtained by running the BJTplus program (E_11_10.m) with base width modulation included (but excluding carrier multiplication).

I _R (μA)	$V_{\mathrm{EC}}(V)$	I _C (mA)
<i>I</i> _B (μA) 2.5 2.5	40.9 101.2	0.807 1.089
5.0	40.9	1.614
5.0	101.2	2.177
7.5	40.9	2.421
7.5	101.2	3.266
10.0	40.9	3.228
10.0	101.2	4.354

Seeking to fit the data with the relationship

$$I_{\rm C} = B(V_{\rm EC} - V_{\rm E})I_{\rm B}$$

one computes

$$B = \frac{I_{\rm C2} - I_{\rm C1}}{I_{\rm R}(V_{\rm EC2} - V_{\rm EC1})}$$

and

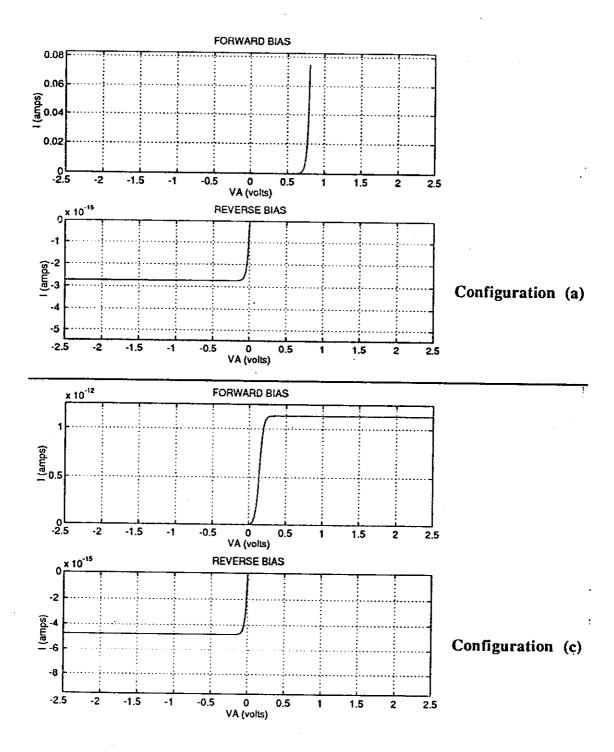
$$V_{\varepsilon} = V_{\text{EC2}} - \frac{I_{\text{C2}}}{B I_{\text{B}}}$$

The same answer, B = 1.87/V and $V_E = -132V$ is obtained employing all four data sets; i.e., for the given device

$$I_{\rm C} = (1.87I_{\rm B})(V_{\rm EC}+132)$$
 ... $V_{\rm EC}$ in volts; $I_{\rm B}$, $I_{\rm C}$ in amps

<u>11.19</u>

(a) Basically, one obtains the same "ideal diode" type characteristic for all of the "diode" connections EXCEPT for the floating-base (c)-connection. The (c)-connection exhibits a reverse-bias type characteristic under both reverse and forward biasing. Representative print-outs of the *I-V* characteristics generated by the P_11_19a.m program (included on the Instructor's disk) are reproduced on the next page.



(b) Generation of the *I-V* characteristics with base-width modulation and carrier multiplication included is considerably more challenging than initially envisioned by the author. This is especially true of the floating terminal connections (connections a, c, e). For one, it is necessary to associate separate carrier multiplication factors with the E-B and C-B junctions. If $V_{\rm EB0}$ and $V_{\rm CB0}$ are the avalanche breakdown voltages of the E-B and C-B junctions respectively, then $MF = 1/[1-(iV_{\rm A}|V_{\rm EB0})^m]$ is associated with carrier multiplication in the E-B junction under reverse bias conditions and $MR = 1/[1-(iV_{\rm A}|V_{\rm CB0})^m]$ is associated with carrier multiplication in the C-B junction. In the computations, carrier multiplication is included by replacing $I_{\rm F0}$ by $MF^*I_{\rm F0}$ and $I_{\rm R0}$ by $MR^*I_{\rm R0}$. (Note, howver, that $\alpha_{\rm F}I_{\rm F0}$ and $\alpha_{\rm R}I_{\rm R0}$ are NOT multiplied by the M-factor.) Since base width modulation and carrier multiplication has little effect on the $V_{\rm A} > 0$ characteristics (except for the (c)-connection), only the reverse bias characteristics are included in the revised computations (except for the (c)-connection).

The voltages required for the computations are straightforward in cases where two of

the terminals are shorted together. Specifically, in these cases one has

case (b) ...
$$V_{EB} = V_A$$
; $V_{CB} = 0$
case (d) ... $V_{EB} = V_A$; $V_{CB} = V_A$
case (f) ... $V_{EB} = 0$; $V_{CB} = V_A$

In connections (a) and (e) the voltage across the open junction floats up or down to assure that no current flows across this junction. The voltage across the open junction can be determined from the part (b) answers to Problem 11.12. Specifically, we find

case (a) ...
$$V_{EB} = V_A$$
; $V_{CB} = (kT/q) \ln[1 + (\alpha_F I_{F0}/I_{R0})(e^{qV_A/kT} - 1)]$
case (e) ... $V_{EB} = (kT/q) \ln[1 + (\alpha_R I_{R0}/I_{F0})(e^{qV_A/kT} - 1)]$; $V_{CB} = V_A$

The floating voltages pose somewhat of a computational problem. The Ebers-Moll parameters are needed to compute the floating voltage, and the computation of the Ebers-Moll parameters in turn depend on the floating voltage when base-width modulation is included. This is handled in the calculations by first computing the Ebers-Moll parameters ignoring base-width modulation, computing the floating voltage, redoing the Ebers-Moll parameters computation with base-width modulation included and employing the first-guess floating voltage, and then repeating the iteration procedure if necessary. It was found that only two complete iterations (two floating voltage and two base-width included computations of the Ebers-Moll parameters) was necessary.

As a final point relative to the applied voltages, please note that the avalanche breakdown in the E-B junction limits the maximum applied voltage in cases (a), (b) and (d). Avalanche breakdown in the C-B junction limits the maximum applied voltage in cases (e) and (f). With $N_B > N_C$, the breakdown voltage of the E-B junction is of course lower than

the breakdown voltage of the C-B junction.

The (c)-connection poses the most serious computational problems. It is the only connection where both negative and positive values of V_A must be considered. Also, with the base floating, there is a built-in feedback effect as described in Subsection 11.2.4. One

therefore expects the reverse and forward limiting voltages to be somewhat less than the respective avalanche breakdown voltages. Finally, the junction voltages are coupled, with $V_{\rm EB} - V_{\rm CB} = V_{\rm A}$. Note from the (c)-connection analysis in Problem 11.12 that

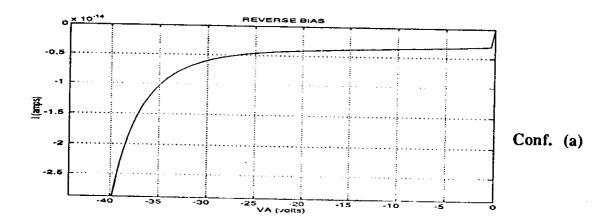
$$(e^{qV_{EB}/kT}-1) = (e^{qV_{CB}/kT}e^{qV_{A}/kT}-1) = -\frac{(1-\alpha_{R})I_{R0}}{(1-\alpha_{F})I_{F0}}(e^{qV_{CB}/kT}-1)$$

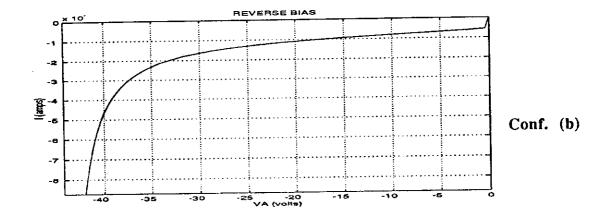
and upon solving for V_{CB} ,

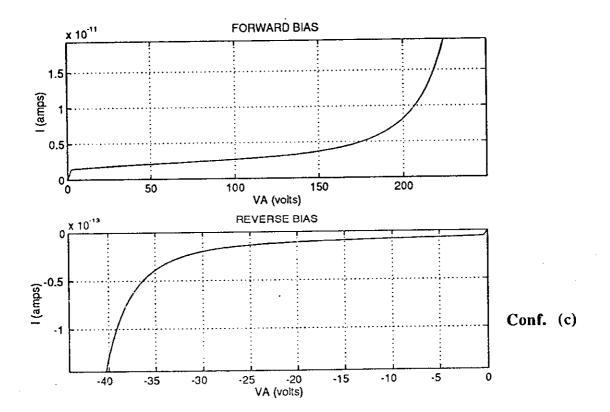
$$V_{\text{CB}} = \frac{kT}{q} \ln \left[\frac{1 + \frac{(1 - \alpha_{\text{R}})I_{\text{R0}}}{(1 - \alpha_{\text{F}})I_{\text{F0}}}}{e^{qV_{\text{A}}/kT} + \frac{(1 - \alpha_{\text{R}})I_{\text{R0}}}{(1 - \alpha_{\text{F}})I_{\text{F0}}}} \right]$$

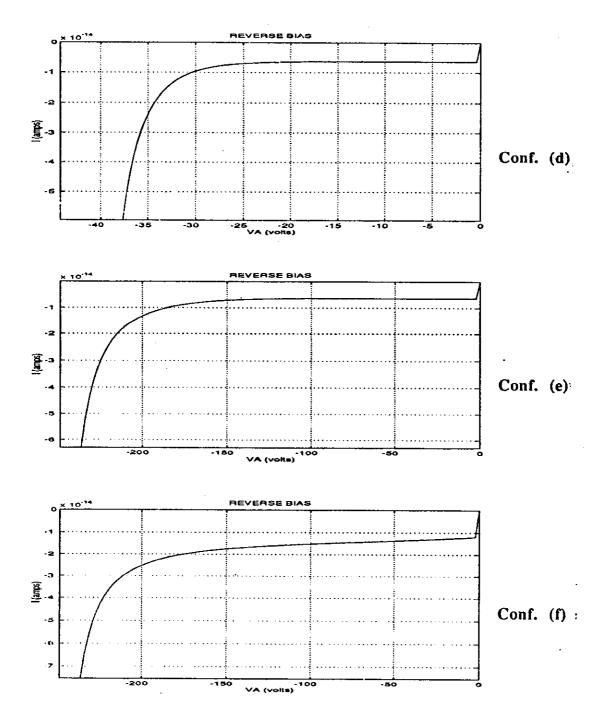
Again, it is necessary to perform an iteration to determine the junction voltages and the Ebers-Moll parameters. Some additional care must be taken in handling $\exp(qV_A/kT)$ for forward biases. Unless the V_{CB} expression is modified, the exponential will exceed the largest value handled by the MATLAB program. When this happens, the exponential is artificially set equal to ∞ , leading to erroneous results.

Print-outs of the *I-V* characteristics generated by the P_11_19b.m program (included on the Instructor's disk) are displayed below.

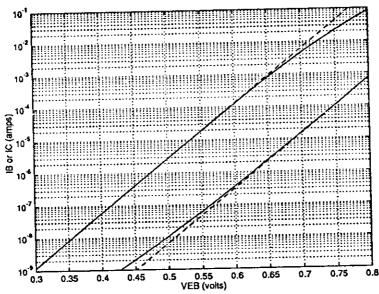


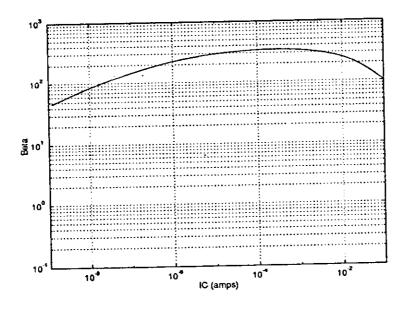






11.20
(a)/(b) The Gummel and gain plots produced per the instructions in the problem statement are displayed below. The plot-generating MATLAB program is listed after the plots. Although hardly perfect, the gain plot is a credible match to the experimental data presented in Fig. 11.16.





```
MATLAB program script...
%Gummel and Gain Plots...Problem 11.20
% (Note: subprogram BJTO is a runtime requirement.)
%Intialization
clear; close
%Input Constant and Material Parameters
BJT0
n2=1.5:
I02=1.0e-14;
VHi=0.75:
%Calculation of IB and IC
VEB=linspace(0.3, 0.85);
IBideal=IF0.*(1-aF).*(exp(VEB./kT)-1);
IRG=102.*(exp(VEB/(n2*kT))-1);
IB=IBideal+IRG:
ICideal=aF.*IR0.*(exp(VEB./kT)-1);
IC=ICideal./(1+exp((VEB-VHi)./(2*kT)));
%Gummel Plot
semilogy(VEB, IBideal, ':w', VEB, ICideal, ':w');
axis([.3, .8, 1.0e-9, 1.0e-1])
hold on
semilogy (VEB, IB, '-m', VEB, IC, '-c'); grid;
xlabel('VEB (volts)'); ylabel('IB or IC (amps)')
hold off
pause
%Gain Plot
Beta=IC./IB;
loglog(IC, Beta)
axis([1.0e-9,1.0e-1,1.0e-1,1.0e3]); grid
xlabel('IC (amps)'); ylabel('Beta')
```

The primary cause of the increase in β_{dc} with increasing V_{EC} is base-width modulation. As noted in the text, increasing V_{EC} increases the width of the C-B depletion region in the base, which in turn decreases W and hence increases β_{dc} . The observed I_{C} of course increases with β_{dc} per Eq. (11.50).

Supplemental Observation #1

The same device was used to produce Figs. 11.5(d) and P11.21. The BJTplus program results displayed in Exercise 11.10 provide a reasonable match to the Fig. 11.5(d) data, and should therefore be applicable to the Fig. P11.21 data. In Prob. 11.18 we found the linear region of the Exercise 11.10 characteristics could be accurately fit by the relationship

$$I_{\rm C} = (1.87I_{\rm B})(V_{\rm EC}+132)$$
 ... $V_{\rm EC}$ in volts; $I_{\rm B}$, $I_{\rm C}$ in amps

or for the given device,

$$\beta_{\rm dc} = I_{\rm C}/I_{\rm B} = 1.87(V_{\rm EC}+132)$$

One computes β_{dc} = 256, 266, and 275 if respectively V_{EC} = 5V, 10V, and 15V. The cited β_{dc} fit is only good of course for I_C values in the low mA range, but does exhibit the correct general dependence.

Supplemental Observation #2

The MATLAB program developed for Prob. 11.20 can be readily revised to produce a plot similar to Fig. P11.21. The plot one obtains is reproduced below. The revised program is included on the Instructor's disk as m-file P_11_21.m. Note that the predicted variation is somewhat smaller than that observed experimentally.

