Project description - "Self Induced Resonance" in interacting compartments

(Dated: April 23, 2024)

Brusselator model

We simulate the system:

$$\dot{x} = a\Omega - (b+d)x + \frac{c}{\Omega^2}x^2y \tag{1}$$

$$\dot{y} = bx - \frac{c}{\Omega^2}x^2y \tag{2}$$

Choose parameters: $a = 2, b = 5 \pm 1, c = 1, d = 1$. The parameter Ω is a determinant for the considered volume the reaction takes place in.

- Calculate the fixed points of this system (like it is performed in the Strogatz book).
- Derive the Jacobian matrix, and check that the prediction of frequency fits the simulated one (just use standard Euler integration here).
- What happens as the parameter Ω is changed? Does this make sense based on the eigenvalues of the Jacobian?
- Add an oscillatory term to the equation for $\dot{x} = a\Omega - (b+d)x + \frac{c}{\Omega^2}x^2y + A\cos(\omega t)$. Choose $b = 4.5$ and see if you can obtain a resonance curve for the system by changing ω.

Stochastic simulation

Now we want to simulate the system, by adding stochasticity. Choose $\Omega = 100$.
Simulate the system using the Gillespie algorithm.

- Make an estimation of the amplitude and the frequency of the simulated traces. What are these for b = 4.5 or b= 5.5?
- How does the amplitude change as you change the parameter Ω for b = 4.5 and 5.5 respectively? You should find that it plays a big role for $b = 4.5$ but not much for $b = 5.5$. Think about why this make sense?

Interacting systems

Let us now consider two compartments Ω_1 and Ω_2 with the constant that $\Omega_1 + \Omega_2 = \Omega$ Consider the differential equations:

$$\dot{x_i} = a\Omega_i - (b+d)x_i + \frac{c}{\Omega_i^2}x_i^2y_i \tag{3}$$

$$\dot{y_i} = bx_i - \frac{c}{\Omega_i^2}x_i^2y_i \tag{4}$$

- Use the ideal gass equation to verify that $\Omega_i = \frac{x_i+y_i}{\sum_j^2 x_j+y_j}$
- Quantify the dynamics of this simulated system for $b = 4.5$, and Compare this to the system for a fixed value of Ω.
- Compare the values for the stochastic and deterministic simulation.