M.Sc. Thesis

Sound Zones with a Cost Function based on Human Hearing

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Abstract

Sound Zones with a Cost Function based on Human Hearing

THESIS

submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE

in

ELECTRICAL ENGINEERING

by

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Abstract

Acknowledgments

I would like to thank dr. J. Martínez-Castañeda, M. Bo Møller, PhD, dr.ir. R.C. Hendriks, and dr. P. Martínez-Nuevo for all their help and support during the project.

A special thanks to M. Bo Møller, PhD for often pushing me in the right direction through our numerous white board sessions. Secondly, I would also like to especially thank dr. J. Martínez-Castañedafor being a major help during, and outside of the project.

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Delft, The Netherlands
September 15, 2021

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Introduction $\quad$ 1

1.1 $\quad$ Preface: the Sound Zone Problem

Sound systems are used worldwide to fill rooms with enjoyable audio content. Problems arise however when multiple people in the same room want to enjoy different audio content at the same time.

For example, one person may want to enjoy a movie on the television, while others may want to listen to their music. If they are in the same room, their desires clash: neither person can fully enjoy their chosen activity without disturbing the other. In short, the interference of multiple audio sources can lead to a situation where both individual experiences are diminished.

In recent years, attempts have been made to solve this problem by controlling the spatial reproduction of sound in such a way that different areas in a room have distinct content, without interfering with each other. This is typically done by controlling the sound pressure created by an array of loudspeakers.

One class of algorithms that attempt to do so are sound zone algorithms [1]. Sound zone algorithms partition the space of the room into multiple so-called sound zones. Each sound zone is assigned different audio content. The sound zone algorithms decide how to use the loudspeakers of the sound system to reproduce said audio content in every zone. Using the principals of constructive and destructive interference, this is done in such a way that there is minimal interference between zones [1]. That is to say: the audio content of each zone is not audible in the others.

In the previously listed example, there would be one zone that would contain the audio of a movie and another zone that would contain music. An image depicting the situation is given in Figure 1.1. The sound zone algorithm determines how to best use the sound system to reproduce these two zones. In the ideal case, both people can now enjoy the full potential of their audio content, without bothering one another.

In practice however, the sound zone algorithm will not always do a perfect job [2]. The performance of algorithms depends on the environment and the available sound system. Depending on the chosen zones, number and position of the loudspeakers, and the room, the interference between zones can typically only be reduced by so much. As such, audio content of one zone is often still audible in other zones [2].

Improving sound zone algorithms is thus an active topic of research. One recent approach is to include a model of the human auditory system, which models how
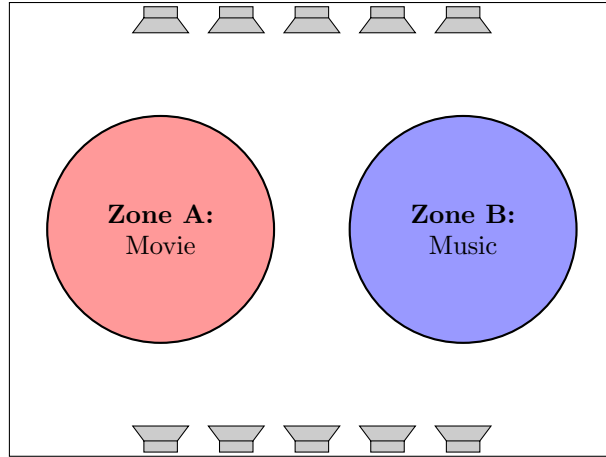
Figure 1.1: A room containing a sound system consisting of an array of loudspeakers and two zones. The goal of the sound zone algorithm is to control the sound system in such a way that the red zone contains the audio of a movie, and the blue zone contains the music.

sound is perceived by humans. Typically, sound zone algorithms use sound pressure [1], which is a physical quantity characterizing the sound. Sound pressure does not always accurately describe what is important for the perception of sound. As such, including a perceptual model may allow the algorithm to focus on reproducing and canceling the parts of the audio content that matter perceptually.

Early results show that a perceptual sound zone approach is promising. Recent work by Donley et al. explored including the absolute threshold of hearing, which models the lowest sound pressure humans can hear, into sound zone algorithms. This pursuit found an increased quality of the reproduced audio in the zones [3]. Other work by Lee et al. showed that including a perceptually-motivated weighting in the sound zone algorithm outperforms traditional algorithms [4, 5].

This work seeks to further explore this perceptual approach by proposing novel perceptual sound zone algorithms and exploring the benefits of including perceptual information.

1.2 Objectives and Organization

This section states the goals of the thesis and organization of the rest of this document.

As stated in the preface, the goal of the work described in this thesis is to investigate the construction and the benefits of including perceptual information in sound zone algorithms. To this end, work in this thesis seeks to answer two research questions:

- **RQ1:** *"How can auditory perceptual models be included in sound zone algorithms?"*

- **RQ2:** *"What are the benefits of including auditory perceptual models in sound zone algorithms?"*

The answers to these questions are given in Chapter 6. What follows is the approach that is taken to answer these research questions, alongside the structure of the rest of this document.

1.2.1 Creation of Perceptual Sound Zone Algorithms

The first research question RQ1,

"How can auditory perceptual models be included in sound zone algorithms?"

is answered in Chapter 2, Chapter 3, and Chapter 4. These chapters document the design of a perceptual sound zone algorithm. The chapters are structured as follows.

- First, in Chapter 2 a literature review is performed to determine which perceptual models are suitable for use in a sound zone algorithm. In this pursuit, one perceptual model is found and hypothesized to be the most promising, then selected and discussed in further detail.

- Next, in Chapter 3 motivates a perceptual sound zone approach. This is done by reviewing literature to determine which sound zone approaches exist and reflecting on the mathematical properties of the selected perceptual model. This results in the proposal of a perceptual sound zone framework.

- Finally, in Chapter 4 implements two perceptual sound zone algorithms using proposed perceptual approach.

1.2.2 Determining Benefits of Perceptual Sound Zone Algorithms

The second research question RQ2,

"What are the benefits of including auditory perceptual models in sound zone algorithms?"

is answered in Chapter 5. This is done by comparing the perceptual sound zone algorithms derived in answering RQ1 with a reference sound zone algorithm.

Perceptual Model Review and Implementation

2

In the field of psycho-acoustics significant research has been done in characterizing the auditory perception and time-frequency analysis capabilities of the human ear [6]. From this understanding, a number of perceptual models have been proposed which aim to model perception of auditory stimuli [7].

Perceptual models are proposed for various purposes. Objective audio quality measures, for example, are perceptual models which aim to predict the perceived quality of audio [8]. In another example, perceptual audio coding uses models of auditory perception to minimize the perceived artifacts when performing compression of audio [9].

In general, many perceptual models apply an analysis filter bank to obtain a time-frequency representation of the stimuli. Often, the of the human human ear is also taken into account at this stage [7, 10]. This representation is used to determine its perceptually relevant aspects [9].

One aspect often used in perceptual models are are the auditory masking properties of the input stimuli [9]. In general, auditory masking refers the effects one sound has on the perception of other sounds [6]. In simultaneous masking for example, one loud tone may overpower a tone of a similar frequency, rendering it inaudible [6].

Another aspect that is often used is the "threshold of hearing", which determines the minimum sound pressure level that can be perceived by a human [9].

Using these principals, one can define the "masking threshold" of an input stimuli. This determines the sound pressure level required for other stimuli to be audible to a human observer in presence of the input stimuli [6]. These principals are used in perceptual audio coding to make the encoding noise inaudible [9].

Chapter Structure

The goal of this chapter is to find a suitable perceptual model for the creation of a perceptual sound zone algorithm. This chapter is structured as follows.

- This chapter begins with a literature review into perceptual models is given in Section 2.1. The purpose of this review is to document candidates for the perceptual model that will be used in the perceptual sound zone algorithm. In addition to this, the reviewed models could also serve as potential candidates for use in the evaluation of the perceptual sound zone algorithm that will be proposed in Chapter 4.

- To perform the selection of a perceptual model from the candidates discussed in the literature review, criteria reflecting desirable properties for the model

for use in sound zones are be defined in Section 2.2. The criteria are then used to select a perceptual model.

- Afterwards, the selected perceptual model is discussed in more detail in Section 2.3 by stating implementation details and describing its behavior.

2.1 Review of Perceptual Models from Literature

This section documents a literature review into perceptual models of the human auditory system. This review is used in Section 2.2 to determine which perceptual model is most suitable for use in a perceptual sound zone algorithm.

Especially promising are models that attach some "score" or "rating" to the perceptual quality of input signals. These ratings can be used in algorithms to obtain an optimal rating through optimization. In addition to this, they can also be used to quantify the quality of results from later algorithms.

As such, the focus of the literature review is not on the latest findings in the field of psycho-acoustics, or models that most accurately emulate the behavior of the human ear, but rather on algorithms that quantify quality.

To this end, in this section, two categories of perceptual models are considered. First in Section 2.1.1, "objective measures" are discussed. These are models which attempt to predict the perceptual quality ratings from listening tests. Next, perceptual models from "audio coding" are discussed in Section 2.1.2. These models are used to quantify how audible the artifacts of audio compression are.

2.1.1 Review of Objective Measures

In order to objectively determine the perceived quality of audio one approach is to use listening tests. These are tests in which subjects are asked to rate a property (or properties) of a set of audio stimuli. One example where listening tests are used is for the evaluation of speech intelligibility of hearing aids [11]. Another example is determining which loudspeaker has higher perceived sound quality.

Performing listening tests is however often cumbersome due to the large amount of human labour involved. This motivates the use of objective quality measures, which attempt to predict the outcomes of these objective listening tests. This is very useful for algorithm developers, as they can get an indication of how well they are doing without having to perform a labour intensive listening test [11].

Note however that a objective quality measure does not replace a listening test: it can only be used to give an indication. Findings should always be confirmed with listening tests.

The objective measures that are considered in this review take a reference and degraded audio stimuli as inputs. Most of the discussed models take the following approach. First, input stimuli are converted to their so-called internal representations, which models how the human auditory system transforms the stimuli. Various features are then derived from this internal representation. The features are then mapped to a prediction of the results of a listening test.

These objective quality measures are promising for integration into sound zone algorithms as they summarize the quality of a signal into a single value, which can be potentially optimized for. It stands to reason that if an objective quality measure correlates with audio quality, optimizing over such a measure could improve

the sound quality of sound zone algorithms.

As such, this section explores various objective measures. This is done by considering three classes different objective measures, namely: measures that quantify the quality and intelligibility of speech audio and measures for the general quality of audio.

2.1.1.1 Review of Objective Speech Quality Measures

There have been a number of attempts to create objective measures to quantify the perceived quality of speech. In this section three objective speech quality measures are discussed. Namely the Perceptual Evaluation of Speech Quality (PESQ) [12] measure, Perceptual Objective Listening Quality Assessment (POLQA) [13]. measure, and Virtual Speech Quality Objective Listener (ViSQOL) [14, 15] measure.

- PESQ is a metric which attempts to determine the perceived quality of speech. It was standardized by the International Telecommunication Union (ITU-T) in 2001. PESQ is computed by first applying an auditory transform that maps the reference and degraded speech into a time-frequency representation that models the perceived loudness of the signals. From this internal representation, so-called symmetric and asymmetric disturbances are determined by computing differences between the time-frequency bins of the reference and degraded speech. A non-linear average is then taken to obtain the average disturbance per time bin. These averaged disturbances are then mapped to the outcomes of listening test outcomes through linear combination [12].

- POLQA is speech quality metric which was standardized by the International Telecommunication Union (ITU-T) in 2011. It was intended to be the successor of PESQ, with the improvement of having more accurate predictions on a wider range of distortions. POLQA works with a similar internal representation to PESQ, but computes distortion in a different way as to be capable of handing global temporal compression and expansions [13].

- ViSQOL is a metric developed in 2012 in a collaboration between Trinity College and Google. ViSQOL uses a different internal representation than PESQ and POLQA as it uses neurograms rather than loudness representations. Neurograms contain the neural firing activity of the auditory nerve in time-frequency bins, and NSIM determines how similar the firing patterns of two neurograms are. The neurograms are then compared by means of the Neurogram Similarly Index Measure (NSIM). This similarity is then related to the outcomes of listening tests through a laplacian fit [14], which is then used to make predictions.

In general, PESQ, POLQA and ViSQOL require many steps to compute and were found difficult to optimize for due to conditional branches within the algorithms and many non-differentiable steps such as clipping [12, 13, 14]. Some attempts have been made however to reformulate PESQ in order to make it more tractable for optimization by approximating the disturbances by other functions [16].

2.1.1.2 Review of Objective Speech Intelligibility Measures

Intelligibility of speech is defined as the percentage of words identified correctly given a degraded speech signal. Objective speech intelligibility metrics seek to predict this percentage. In this section, two of these metrics are discussed. Namely, the Short Time Objective Intelligibility (STOI) [11] measure and the Speech Intelligibility In Bits (SIIB) [17] measure.

- STOI was proposed by Taal et al. in 2011 as a speech intelligibility metric that could make accurate predictions for a speech signals that were degraded time-frequency weighted distortions.

  For its internal representation, it finds a time-frequency internal representation through filtering the input stimuli with a filter bank consisting of 1/3 octave bands, and then segmented the filter taps into short time frames. Silent bins that do not contain speech are removed, and clipping is applied to limit the effect of one severely degraded time-frequency bin. The average correlation coefficient between the time-frequency bins of the internal representation of the reference and degraded segments is then computed, and averaged over all bins to determine the intelligibility [11].

- SIIB was introduced by Van Kuyk et al. in 2017 as a speech intelligibility metric that could be motivated through information theory through the mutual information rate. As such, SIIB is given in bits

  The idea behind SIIB is that the intelligibility of speech is related to the information shared between intended and degraded speech. SIIB models how transformation of the reference speech signal to the degraded speech signal as a transmission channel. Among other aspects, this transmission channel includes a model of the human auditory system [17]. This communication channel is then used to compute the mutual information rate.

Both STOI and SIIB are difficult to optimize for directly.

In STOI, the removal of silent regions and the clipping operator are non-differentiable operations. Furthermore, the computation of the correlation coefficient is a non-convex function of the degraded speech [11].

SIIB is in general non-convex and non-differentiable as it uses the Karhunen-Loève transform and a K-nearest neighbor estimator to compute the mutual information rate. However, if the communication channel is approximated as gaussian, the mutual information can be computed in closed form, and SIIB becomes a differentiable measure [17].

2.1.1.3 Review of Objective Audio Quality Measures

The previous objective quality metrics are both intended for evaluating speech. In this section, a number of objective quality metrics are discussed that are designed instead for evaluating the perceived quality of general audio . Namely, the Perceptual Evaluation of Audio Quality (PEAQ) [18] and ViSQOLAudio [19]. The latter is an adapted version of the ViSQOL speech quality measures.

- PEAQ is a audio quality metric standardized by the International Telecommunication Union (ITU-T). PEAQ estimates a quality grade by first computing an internal representation of the reference and degraded audio signals. This results in a time-frequency representation of the input stimuli from which a number of perceptually relevant feature, referred to by PEAQ as Model Output Variables (MOVs), are extracted. An example of these MOVs are the loudness of the noise or the bandwidth of the input stimuli. These MOVs are then mapped to the final audio quality grade through a neural network [18].

- In 2015, it was found that with some adjustments ViSQOL could be used to determine audio quality, which resulted in a new metric ViSQOLAudio. Among the adjustments were the removal of the voice activity detector included in ViSQOL and the use of a larger bandwidth to cover the entire spectrum of hearing from 50 Hz to 20000 Hz, rather than just the bandwidth of speech [19].

PEAQ, and ViSQOLAudio are both difficult to optimize for. A number of the MOVs computed in PEAQ, such as the partial noise loudness, are non-differentiable [18]. As ViSQOLAudio is similar to ViSQOL with some small adjustments, it is similarly difficult to optimize for [19].

2.1.1.4 Review of Distraction Model

One especially promising objective measure is the distraction proposed by Francombe et al. in 2015 [20]. This measure was designed with the application of sound zones in mind.

The distraction was determined to be the keyword that best describes the perceptual experience of interfering audio programs. This was determined through an elicitation study performed also performed by Francombe et al. in 2014 [21]. This prompted the creation of the model.

To create the model, a listening test was performed where the participants were subjected to audio-on-audio interference. The subjects were played a target audio stimuli they were instructed to focus listening to. At the same time, an interferer audio stimuli was played to distract the participant from the target. The participants were given a scale between 0 and 100 on which they were asked to rate how distracting the interference was when listening to the target program, where a 100 indicates that the interferer "overpowered" the target audio [20].

The target-interferer stimuli pairs and corresponding ratings resulted in a dataset. This dataset was then used to fit a model which predicted the distraction given novel a target-interferer stimuli pair. The model consisted of taking a linear combination of 5 features that were computed from the stimuli [20].

Computing said features could however not be performed in real time. The reason for this was that as the original distraction model is too computationally complex [22]. To this end, in 2017, Rämö et al. proposed a version of the distraction model that could be run in real-time. This was done by approximating the features of the original distraction model by computationally less complex alternatives. The

resulting real-time distraction model was found to be less precise, but could be run in 0.04% of the time of the original distraction model [22].

On face value, the real-time distraction model seems promising to optimize over. However, while easy to compute, the model is non-differentiable as the model uses piecewise functions and non-convex due to taking the logarithm of the square of the input signals. In addition to this, the model also performs operations that are difficult to express mathematically, such as counting the number of short-time blocks that exceed a certain threshold [22].

2.1.2 Review of Perceptual Models used in Audio Coding

The second class of perceptual models that are considered are the perceptual models used in audio coding. Audio coding algorithms attempt to find an low-bitrate representation of an audio input signal, which is a form of lossy compression. As such, audio coding algorithms typically introduces errors in doing so, which can be a detriment to the listening experience.

To minimize the impact of these errors, many audio coding algorithms use a perceptual model to quantify how disturbing the introduced distortions are. The perceptual model is used to introduce encoding errors in such a way that the audio output signal is perceptually indistinguishable from the audio input signal [10]. This model typically takes form of a distortion function which determines how audible the difference between a reference input audio signal and a distorted output audio signal is. This function can used to for example encode an input audio signal such that it has minimal distortion for a specified bitrate.

The perceptual models used in audio coding are promising for integration into a sound zone algorithm, as they are often mathematically tractable. As stated, these perceptual models typically take the form of some sort of distortion function that quantifies how perceptually disturbing the introduced artifacts are. One approach for example could be to define sound zone algorithms that minimize a distortion function.

As such this section explores three perceptual models from audio coding.

2.1.2.1 Review of Perceptual Models from ISO MPEG Standard

The ISO/IEC 11172-3 standard specifies a coded representation for audio files [23], and a corresponding decoder. An encoder said representation is not part of the standard. This is done deliberately, to allow for future improvements to the encoder, without having to change the standard [24].

The standard does however provide a number of examples of possible encoders, with increasing complexity. Alongside these example encoders, two psycho-acoustical models are included for use during the encoding process.

The psycho-acoustical models work by subdividing the input audio signal into frequency bands which correspond to the frequency bands in the human auditory

system. The model then determines how much quantization noise can be added separately per band without the noise becoming audible. As such, the model assumes that the distortion signal is noise-like [7], which is usually the case for quantization noise for audio coders.

The output of the psycho-acoustical model is thus the amount of noise that can be added per band. In the case of audio coding, this can then be used to control quantization noise. Note that this perceptual model does not come in the form of the earlier described distortion function. This technique has however been used for various signal processing purposes, such as audio watermarking [10]. As such, examples exist from which optimization schemes could be inspired.

2.1.2.2   Review of Par Detectability Measure

In 2005, van der Par et al. proposed a novel perceptual model designed for use in audio coding [7]. The model defines a distortion measure which determines the "detectability" of a distortion signal in presence of masking signal. That is to say, the function quantifies the degree to which a human is to detect a distortion signal. For audio coding purposes, this distortion signal is error introduced due to the audio compression.

Similarly to the ISO MPEG perceptual models, the Par detectability typically operates on short-time segments, typically in the order of 20 to 200 milliseconds. The proposed method however differentiates itself from the previously discussed ISO MPEG models in three ways.

Firstly, the paper uses newer findings from psycho-acoustic literature, namely spectral integration. In spectral integration, the masking effects from neighboring bands are taken into account when computing the masking effects. The psycho-acoustical models defined in the ISO MPEG standard does not do this as it effectively works independently per band [10].

Secondly, it assumes that the distortion signal is sinusoidal, rather than noise-like. As such, it is more effective in hiding sinusoidal distortion.

Thirdly and finally, the perceptual model is described as a distortion function which quantifies how detectable a disturbance stimuli is. The proposed distortion measure can be expressed as a squared L2-norm.

This mathematical tractability makes for easy integration into existing least-square problems. As such, the Par model has been used in many signal processing applications, examples ranging from speech enhancement to removing perceptually irrelevant sinusoidal components [25, 26].

2.1.2.3   Review of Taal Detectability Measure

A paper from 2012 by Taal et al. proposed a novel perceptual model [10] which introduces a alternative definition to the detectability defined in the Par model.

In contrast to the Par detectability, the Taal detectability measure takes temporal characteristics of a signal into account. The inclusion of temporal information allows

for the suppression of "pre-echoes", which is an artifact that the Par model suffers from. The "pre-echoes" artifacts arises from the assumption that the masking effects of the masking signal are stationary across time. As a result, audio coding algorithms may assume that audio content is masked while it is not, which could results in quantization noise not being masked.

In contrast to other temporal perceptual models, the Taal Detectability has a relatively low computational complexity. In addition to this, it can also be expressed as a squared L2-norm. The computational demand was however shown to be higher than the Par Detectability [10], especially for longer time segments.

2.2 Selection of Perceptual Model for Perceptual Sound Zone Algorithms

In Section 2.1 a literature review discussing various perceptual models is given. This section determines which of these perceptual models from the review are suitable for use in a perceptual sound zone algorithm.

First, Section 2.2.1 discusses desirable properties of the perceptual model for use in a perceptual sound zone algorithms. In Section 2.2.2 these requirements are used to evaluate the models reviewed in Section 2.1

2.2.1 Desirable Properties of Perceptual Model for use in Perceptual Sound Zone Algorithms

As is shown in Chapter 3, many sound zone algorithms are posed as optimization problems. As such, it desirable for the perceptual model to be intergratable in optimization problems.

The goal optimization problems is typically to minimize or maximize a cost function, which is done by leveraging the (sub)differential of the function. Algorithms which contain conditional branching or complex operations cannot readily be integrated into cost functions, and are therefore less promising.

In addition to this, sound zone algorithms can often posed as convex optimization problems. This is a sub-class of optimization problems that guarantees that the optimizer is globally unique, rather than there being many local optima [27]. In addition to this, there are many efficient solvers available for convex optimization problems. Therefore, it is also preferable that the perceptual models can preserve convexity when integrated into cost functions.

2.2.2 Evaluating Reviewed Perceptual Models for use in Perceptual Sound Zone Algorithm

All objective audio measures discussed in Section 2.1.1 were found to be difficult to optimize. As discussed, all models showed a degree of non-differentiability and non-convexity in their computation. As such, they are difficult to integrate into convex optimization problems and will not be used in the perceptual sound zone algorithm. However, as the objective audio measures predict the outcomes of listening tests, they will prove useful in the evaluation of the results.

From all three discussed perceptual models from audio coding, the perceptual models proposed by the ISO MPEG standard were found to be the least promising. As stated in Section 2.1.2, this is due to the fact that these models do not define a cost function which can be optimized over: instead, only the noise that can be added per auditory band is determined.

As such, the decision is between the Par and Taal detectability. As stated in Section 2.1.2, both models can be expressed through a squared L2-norm, which is a

convex function [27]. For this reason, it both the Par and Taal detectability are promising for the creation of a perceptual sound zone algorithm.

In contrast to the Par model, the Taal detectability takes into account temporal properties of the input signal. This is beneficial, as it will lead to a more accurate description of the masking properties of the input signals.

However, it has been shown to be at the cost of computational complexity. The Taal detectability has been shown to be take at least 2 times as much time to compute as the Par detectability, with this disparity seemingly growing as a function of input signal length [10].

In addition to this, the Taal model operates in the time-domain, whereas the Par model operates in the frequency-domain [7, 10]. Frequency-domain sound zone approaches have been shown to be less demanding computationally than time-domain approaches [28].

For the reasons given above, lower computational complexity, the Par detectability is used in the perceptual sound zone algorithm. Exploring the possibilities of the Taal detectability is left to future work and not further explored in this work.

2.3 Discussion and Implementation of the Par Detectability Measure

In Section 2.2, it was determined that the Par detectability measure is the most suited model for the perceptual sound zone algorithm of all perceptual models considered in the literature review given in Section 2.1. In this section, in order to give the reader a greater understanding of the model, the Par detectability measure is considered in greater detail.

This section is organized as follows. First, Section 2.3.1 gives a high-level description of the Par detectability measure, providing an intuitive understanding and introducing some of the notation that is used. Next, the steps to computing the detectability are described in Section 2.3.2. Finally, Section 2.3.3 rewrites the detectability as a squared L2-norm.

2.3.1 High-Level Description of the Par Detectability Measure

In this section, a high-level description of the Par detectability measure is given. This is done to give the reader a basic understanding of the model before going into greater detail.

The Par detectability maps two input sequences to a positive real value, i.e. $D : (\mathbb{R}^{N_x}, \mathbb{R}^{N_x}) \mapsto \mathbb{R}^+$. The two input sequences are the masking signal $x[n] \in \mathbb{R}^{N_x}$ and the disturbance signal $\varepsilon[n] \in \mathbb{R}^{N_x}$. The detectability of these two sequences is denoted as $D(x[n], \varepsilon[n])$.

Imagine a human that is listening to both the masking signal $x[n]$ and the disturbance signal $\varepsilon[n]$ at the same time. The detectability $D(x[n], \varepsilon[n])$ can be understood as how easily a human listener can detect the disturbance signal $\varepsilon[n]$ in presence of the masking signal $x[n]$. The signal $x[n]$ is referred to as the masking signal because its masking properties are model to determine how well it masks the disturbance signal $\varepsilon[n]$.

For this interpretation to be accurate, the signals $x[n]$ and $\varepsilon[n]$ should be short-time signals. The paper uses a signal length of to 20 to 200 milliseconds. This is important, as the model assumes that the psycho-acoustical properties of $x[n]$ and $\varepsilon[n]$ are stationary.

The measure is normalized in such a way that the detectability $D(x[n], \varepsilon[n])$ is equal to 1 when the disturbance signal $\varepsilon[n]$ is "just noticeable" in presence of masking signal $x[n]$. That is to say: if the detectability is 1, the disturbance is on the verge of being noticeable and not noticeable.

The detectability $D(x[n], \varepsilon[n])$ can also attain a value larger than 1. The larger values of the detectability correspond with an increased perceived presence of the disturbance signal $\varepsilon[n]$.

2.3.2   Computation Details of the Par Detectability

This section explores calculating the Par detectability. The first thing to note about the Par detectability is that it is computed using the frequency domain representations of its inputs [7]. To this end, let $X[k]$ and $\mathcal{E}[k]$ denote the frequency domain representations of the masking signal $x[n]$ and the disturbance signal $\varepsilon[n]$ respectively.

After determining the frequency domain representations, the Par detectability computes an internal representation of the input signals $X[k]$ and $\mathcal{E}[k]$. This internal representation models how the input stimuli appear to the human auditory system. For the Par detectability measure, this is modeled by filtering the input stimuli.

Two subsequent filters are applied. The first filter models how parts of the ear filter the incoming sound with an outer- and middle-ear filter $H_{\mathrm{om}}[k]$. Next, a 4th order Gammatone filter bank is applied, modeling the frequency-place transform that occurs in basilar membrane inside of the ear [7].

The Gammatone filter bank consists of N_g filters. The frequency domain representation of each individual filter is denoted by $\Gamma_i[k]$, for $1 \leq i \leq N_g$. The filters in the filter bank $\Gamma_i[k]$ have a bandwidth given by the equivalent rectangular bandwidth (ERB) and center frequencies given by the corresponding equivalent rectangular bandwidth number scale (ERBS). Expressions for the gammatone filters $\Gamma_i[k]$ are provided by the original paper [7].

After filtering, the power per Gammatone filter tap is computed. Let M_i and S_i denote the output power of the ith filter tap for the masking signal $X[k]$ and the disturbance signal $\mathcal{E}[k]$ respectively. This output power can be understood as the amount of power perceived per frequency band of the human ear. The relationship between the input quantities and the output power of the filter taps can be given as follows:

$$M_i = \frac{1}{N_x} \sum_{k=0}^{N_x-1} |H_{\mathrm{om}}[k]|^2 \, |\Gamma_i[k]|^2 \, |X[k]|^2 \tag{2.1}$$

$$S_i = \frac{1}{N_x} \sum_{k=0}^{N_x-1} |H_{\mathrm{om}}[k]|^2 \, |\Gamma_i[k]|^2 \, |\mathcal{E}[k]|^2 \tag{2.2}$$

The output powers can then be used to define the within-channel detectability D_i per filter tap i. This can be thought of the detectability per frequency band of the human ear, and is defined as follows:

$$D_i = \frac{N_x S_i}{N_x M_i + C_a} \tag{2.3}$$

Here, C_a is a calibration constant that ensures that the absolute threshold of hearing is predicted correctly. This can be understood by considering the case where no masking signal $x[n]$ is present, in which case $M_i = 0$ for all i. If not for the calibration constant C_a, the detectability of any non-zero disturbance $\varepsilon[n]$ would

17

be infinite. In order to take the frequency-dependence of the threshold of hearing into account, the previously described outer- and middle ear filters are defined as the inverse of the threshold of hearing [7].

The total detectability $D(x[n], \varepsilon[n])$ can then be computed as the scaled sum of all within channel detectabilities. It is defined as follows:

$$D(x[n], \varepsilon[n]) = C_s L_{\text{eff}} \sum_{i=0}^{N_g} D_i \tag{2.4}$$

$$= C_s L_{\text{eff}} \sum_{i=0}^{N_g} \frac{\sum_{k=0}^{N_x-1} |H_{\text{om}}[k]|^2 |\Gamma_i[k]|^2 |\mathcal{E}[k]|^2}{\sum_{k=0}^{N_x-1} |H_{\text{om}}[k]|^2 |\Gamma_i[k]|^2 |X[k]|^2 + C_a} \tag{2.5}$$

Here, C_s is a calibration constant chosen such that a just noticeable disturbance signal results in a detectability of $D(x[n], \varepsilon[n]) = 1$. The constant L_{eff} is the integration time of the human auditory system. It is chosen equal to the segment length of $x[n]$ and $\varepsilon[n]$ in milliseconds.

In order to further understand detectability, consider the behavior of the expression of the detectability $D(x[n], \varepsilon[n])$ above. Imagine that the spectrum of the masking signal is much larger than the disturbance signal, i.e. $X[k] \gg \mathcal{E}[k]$ for all frequency bins k. In this case, the detectability of $\varepsilon[n]$ will be small due to the masking of the masking signal $x[n]$ or due to the threshold of hearing (determined by the calibration constant C_a).

Conversely consider the case that the spectrum of the masking signal is much smaller than the disturbance signal, i.e. $X[k] \ll \mathcal{E}[k]$ for all frequency bins k. In this case, the resulting detectability is determined greatly by the calibration coefficient C_a:

- If the total energy of the filtered disturbance signal is much larger than the calibration constant $S_i \gg C_a$ for all i, the detectability becomes large. This models the case that the disturbance signal is large relative to the threshold of hearing.

- Alternatively, if $S_i \ll C_a$ for all i, the disturbance signal is inaudible due to the threshold of hearing and the detectability will be low accordingly.

This concludes the analysis of the Par model. The determination of the calibration constants C_a and C_s is discussed in Appendix A.

2.3.3   Least-Squares Formulation of the Par Detectability

This section will rewrite the previously introduced detectability into a least-squares representation [10]. This representation is more mathematically tractable than Equation (2.5) and thus will allow for easier integration into existing sound zone algorithms.

To obtain this expression, the sum of squares will be expressed as a L2-norm.

Consider the following rewrite of the detectability given in Equation (2.5):

$$D(x[n], \varepsilon[n]) = C_s L_{\text{eff}} \sum_{i=0}^{N_g} \frac{\sum_{k=0}^{N_x-1} |H_{\text{om}}[k]|^2 |\Gamma_i[k]|^2 |\mathcal{E}[k]|^2}{\sum_{k=0}^{N_x-1} |H_{\text{om}}[k]|^2 |\Gamma_i[k]|^2 |X[k]|^2 + C_a}$$

$$= \sum_{i=0}^{N_g} \left(\frac{C_s L_{\text{eff}}}{||H_{\text{om}}[k]\Gamma_i[k]X[k]||_2^2 + C_a} \right) \sum_{k=0}^{N_x-1} |H_{\text{om}}[k]|^2 |\Gamma_i[k]|^2 |\mathcal{E}[k]|^2$$

$$= \sum_{k=0}^{N_x-1} \left(\sum_{i=0}^{N_g} \frac{C_s L_{\text{eff}} |\Gamma_i[k]|^2}{||H_{\text{om}}[k]\Gamma_i[k]X[k]||_2^2 + C_a} \right) |H_{\text{om}}[k]|^2 |\mathcal{E}[k]|^2$$

$$= \sum_{k=0}^{N_x-1} |W_x[k]|^2 |\mathcal{E}[k]|^2$$

$$= ||W_x[k]\mathcal{E}[k]||_2^2$$

The rewrite above introduced perceptual weighting $W_x[k] \in \mathbb{R}^{N_x}$ informed by the auditory masking effects of the masking signal $x[n]$. The entries of the perceptual weighting can be understood as the importance of those frequencies for the total detectability. The perceptual weighting $W_x[k]$ is defined as follows:

$$W_x[k] = \left(\sqrt{\sum_{i=0}^{N_g} \frac{C_s L_{\text{eff}} |\Gamma_i[k]|^2}{||H_{\text{om}}[k]\Gamma_i[k]X[k]||_2^2 + C_a}} \right) |H_{\text{om}}[k]| \qquad (2.6)$$

Note from this formulation that the perceptual weighting is only a function of the masking signal $x[n]$.

Note also that the resulting detectability $D(x[n], \varepsilon[n])$ is a convex function of the disturbance signal $\varepsilon[n]$. The frequency-domain representation $\mathcal{E}[k]$ is related to the time-domain representation $\varepsilon[n]$ through the DFT, which is a linear operator. The perceptual weighting of $\mathcal{E}[k]$ performed by $W[k]$ is also a linear operation. As such, $W[k]\mathcal{E}[k]$ is an affine function of $\varepsilon[n]$.

In order to gain a deeper understanding of the behavior of the perceptual weighting $W_x[k]$, consider Figure 2.1. The figure relates the auditory masking threshold and the corresponding perceptual weighting when a 1000 Hz tone at 70 dB SPL is used as masking signal $x[n]$. Recall that the masking threshold is minimal sound pressure level that is required for an additional stimuli to be audible in presence of the masking signal [6]. In addition to this, the threshold of hearing is also depicted to highlight the additional masking that occurs due to the masking signal.

As can be seen, the masking threshold peaks at 52 dB SPL at 1000 Hz. This implies that an additional tone must be at least 52 dB SPL in order to be audible. As depicted, this results in a low perceptual weighting implying that a disturbance at this frequency is less detectable. Note also that the low and higher frequencies are also weighted lower due to the threshold of hearing.

Masking Threshold for a 1 KHz sine at 70 dB SPL

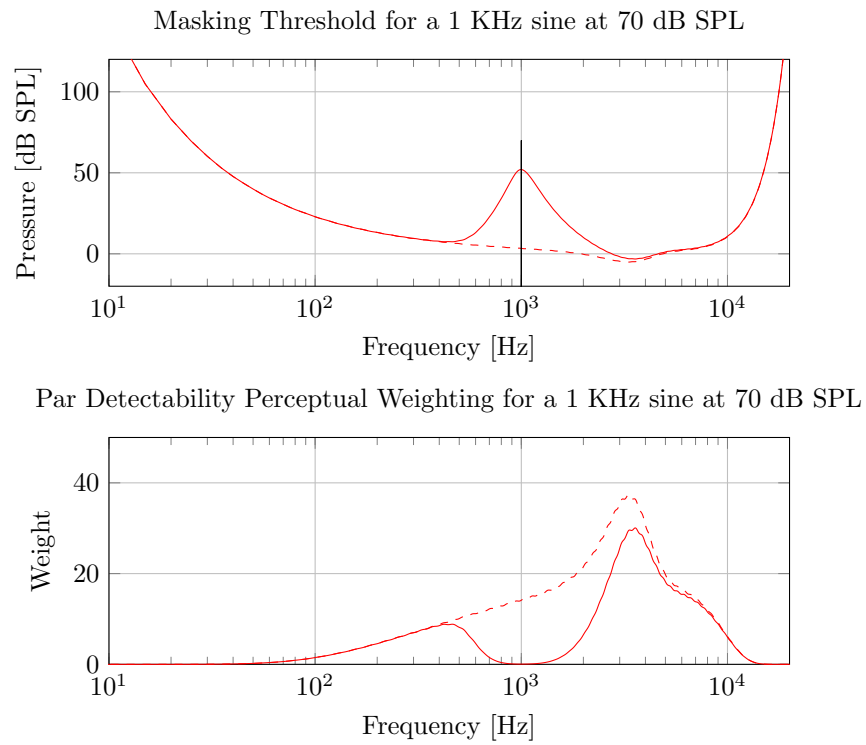Par Detectability Perceptual Weighting for a 1 KHz sine at 70 dB SPL

Figure 2.1: Depiction of the masking threshold and corresponding perceptual weighting function for a 1000 Hz tone with an amplitude of 70 dB SPL. The threshold of hearing is also depicted.

3 Sound Zone Approach Review and Implementation

As mentioned in the introduction, the problem that sound zones seeks to solve is the reproduction of multiple types of audio content in the same room with minimal interference. This way, multiple people can enjoy different audio content without disturbing one another.

This section seeks to build on this description in order to provide the understanding necessary for the rest of this work.

Controlling the spatial distribution of sound is done by calculating the audio the loudspeakers must produce to approximate the desired sound field in the given space. The space inside the enclosure is divided up into multiple zones. Each zone is assigned target sound pressure that we would like to have reproduced inside of it. This target sound pressure could be any kind of audio content, for example music, the sound of a movie, or speech.

To understand this principal, consider the example given by Figure 3.1. The loudspeakers array that is present in the room is to be controlled by the sound zone algorithm in such a way that the desired content is reproduced in each zone. As mentioned, this is to be done in a way that results in minimal interference, e.g. it is undesirable to be able to hear content B when inside the red zone.
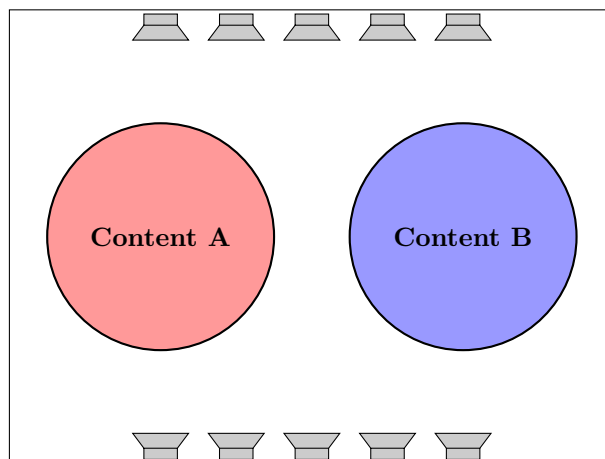
Figure 3.1: A birds-eye view of a room is depicted. It is divided into two zones: a red zone and a blue zone. Each zone is assigned different content: content A and content B respectively. In the northern and southern parts of the room, a loudspeaker array is mounted on the walls.

There are various approaches to solving the sound zone problem. An important concept that is used frequently is that "bright zones" and "dark zones". These are explained next.
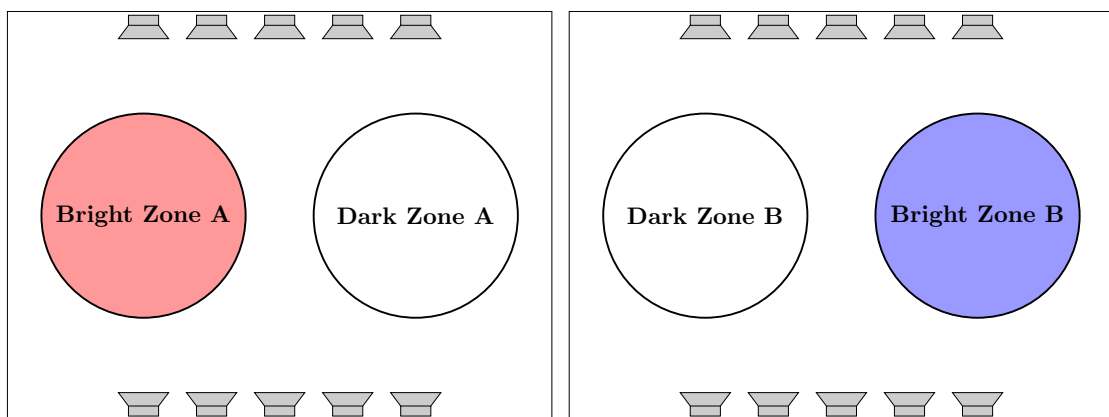
Figure 3.2: A birds-eye view of a room is given twice, each depicting two different sound zone problems. Combining the solutions to both subproblems results in a reproduction of sound with minimal interference between zones.

Sound zone problems are typically decomposed into a separate subproblem for every zone. Each one of these subproblems considers only two zones: one bright zone and one dark zone. The goal of each subproblems is to reproduce a specified target sound pressure in the bright zone while restricting the sound pressure in the dark zones.

The combination of both subproblems provides a solution to the sound zone problem. To ease the understanding of this concept, consider an example of this decomposition is given in Figure 3.2.

Here, a decomposition of the example given in Figure 3.1 into two bright-dark zone pairs. For the first problem, the goal is to reproduce content A in bright zone A while minimizing the amount of sound pressure in dark zone A. Similarly for the second problem: reproduce content B in bright zone B while minimizing the amount of sound pressure in dark zone B. Combining the two solutions results in a solution with content reproduced in both zones with minimal interference between zones.

Chapter Structure

The goal of the rest of this chapter is to motivate the proposal of a perceptual sound zone approach that uses the Par distortion detectability introduced in Chapter 2.

- This chapter begins in Section 3.1 with the presentation of a mathematical framework that can be used to describe the sound zone problem.

- This mathematical framework is then used to describe the two main sound zone approaches, "Pressure Matching" and "Acoustic Contrast Control", in Section 3.2.

- Finally, Section 3.3 motivates the proposed perceptual sound zone approach. This is done by reflecting on the mathematical properties of the Par detectabil-

ity and the sound zone approaches review in Section 3.2.

Figure 3.3: A birds-eye view of a room $\mathcal{R} \subset \mathbb{R}^3$ containing the zones $\mathcal{A} \subset \mathcal{R}$ and $\mathcal{B} \subset \mathcal{R}$ depicted in red and blue respectively. The room contains $N_L = 8$ loudspeakers, which are denoted by the red dots in the corners of the room.
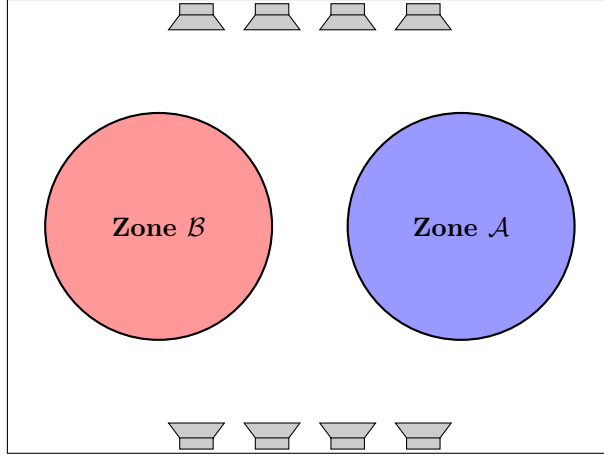
3.1 Sound Zone Problem Data Model

In the previous section, a high-level description of the sound zone problem is given. In this section a mathematical framework for a room containing sound zones will be introduced. This framework will be used later in the derivation of the sound zone algorithms in Section 3.2.

The contents of this section are as follows. First, Section 3.1.1 develops a spatial description of a room containing two zones and a loudspeaker array. Then, Section 3.1.2 defines the objective of the sound zone algorithm formally as realizing a desired target sound pressure at discrete points in the room.

The relationship between the sound pressure in the room and loudspeaker input signals is then be given in Section 3.1.3, completing the mathematical framework. This is then used in Section 3.1.4 to select a suitable target sound pressure which is used in the remainder of this thesis.

3.1.1 Room Topology

A room $\mathcal{R}$ can be modeled as a closed subset of three dimensional space, $\mathcal{R} \subset \mathbb{R}^3$. The two non-overlapping zones $\mathcal{A}$ and $\mathcal{B}$ are contained within the room $\mathcal{R}$, i.e. $\mathcal{A} \subset \mathcal{R}$ and $\mathcal{B} \subset \mathcal{R}$ where $\mathcal{A} \cap \mathcal{B} = \emptyset$. That is, there is no intersection between zones. In general, the room can contain any number of zones, but this thesis focusses on the two zone case. In addition to the zones, the room $\mathcal{R}$ also contains N_L loudspeakers, which are modeled as point sources. An example of a possible room, loudspeakers and pair of zones are visualized in Figure 3.3.

Sound zone algorithm aim to use the sound pressure generated by the loudspeakers to realise a specified target sound pressure in the space described by zones $\mathcal{A}$ and $\mathcal{B}$. This is to be done in such a way that there is minimal interference between zones, meaning that target sound pressure intended for one zone should not be audible
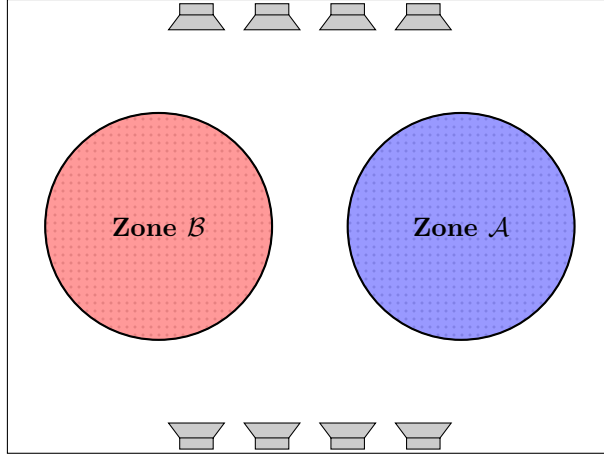
Figure 3.4: The previously introduced room $\mathcal{R}$ with zones $\mathcal{A}$ and $\mathcal{B}$ discretized.

in the other zones. Thus, allowing for multiple distinct audio experiences in the room.

The sound field generated by loudspeakers can be controlled by specifying their input signals. As such, the goal of the sound zone algorithm is finding loudspeaker input signals in such a way that specified target sound pressure is attained.

The rest of this section will focus on formalizing this notion mathematically.

3.1.2   Defining Target Pressure

As mentioned, the goal of the sound zone algorithm is to realize a specified target sound pressure in the different zones $\mathcal{A}$ and $\mathcal{B}$ in the room $\mathcal{R}$.

Currently, the zones are given as continuous regions in space. However, most sound zone approaches will instead discretize the zones by sampling the continuous zones $\mathcal{A}$ and $\mathcal{B}$ into so-called control points. The sound pressure is then controlled only in these control points.

Thus, we discretize zones $\mathcal{A}$ and $\mathcal{B}$ into a total of N_a and N_b control points respectively. Let A and B denote the sets of the resulting control points points contained within zones $\mathcal{A}$ and $\mathcal{B}$ respectively.

Now let $t^{(m)}[n]$ denote the target sound pressure at control point m in either A or B, i.e. $m \in A \cup B$. Our goal is thus to realize $t^m[n]$ in all control points $m \in A \cup B$ using the loudspeakers present in the room.

3.1.3   Realizing Sound Pressure through the Loudspeakers

The sound pressure produced by the loudspeakers can be controlled by specifying their input signals. Let $x^{(l)}[n] \in \mathbb{R}^{N_x}$ denote the loudspeaker input signal of length N_x for the lth loudspeaker. For now, it is assumed that the loudspeaker input signals are of finite length. In a later part of the thesis, a short-time formulation is given that supports infinite length sequences.

As such, the goal of the sound zone algorithm is to find loudspeaker inputs $x^{(l)}[n]$ such that the target sound pressure $t^{(m)}[n]$ is realized for all $m \in A \cup B$.

In order to do so, a relationship must be established between the loudspeaker inputs $x^{(l)}[n]$ and the resulting sound pressure at control points $m \in A \cup B$. This relationship can be established by using a linear model based on room impulse responses (RIRs) $h^{(l,m)}[n] \in \mathbb{R}^{N_h}$ [29].

The RIRs $h^{(l,m)}[n]$ determine the sound pressure at control point m due to playing loudspeaker signal $x^{(l)}[n]$ from loudspeaker l. Mathematically, let $p^{(l,m)}[n] \in \mathbb{R}^{N_x+N_h-1}$ represent said sound pressure. It can be defined as follows [1]:

$$p^{(l,m)}[n] = \left(h^{(l,m)} * x^{(l)}\right)[n], \tag{3.1}$$

Here, the $*$ operator is used to denote linear convolution. The achieved sound pressure $p^{(l,m)}[n]$ only considers the contribution of loudspeaker l at reproduction point m. Let $p^{(m)}[n] \in \mathbb{R}^{N_x+N_h-1}$ denote the total sound pressure due to all N_L loudspeakers, which can be expressed as the sum over all contributions $p^{(l,m)}[n]$ as follows:

$$p^{(m)}[n] = \sum_{l=0}^{N_L-1} \left(h^{(l,m)} * x^{(l)}\right)[n]. \tag{3.2}$$

With the data model completed, the goal of the sound zone algorithm can be restated formally. Namely, to find the loudspeaker input signals $x^{(l)}[n]$ such that the achieved sound pressure $p^{(m)}[n]$ attains the target sound pressure $t^{(m)}[n]$ for all control points $m \in A \cup B$.

3.1.4   Choice of Target Pressure

The target sound pressure $t^{(m)}[n]$ describes the desired content for a specific control point m. So far, the choice of target sound pressure $t^{(m)}[n]$ has been kept general. In this section, a choice to properly define the target pressure is given and motivated.

Assume that the users of the sound zone system have selected desired playback audio signals $s_{\mathcal{A}}[n] \in \mathbb{R}^{N_x}$ and $s_{\mathcal{B}}[n] \in \mathbb{R}^{N_x}$ that they wish to hear in zone $\mathcal{A}$ and $\mathcal{B}$ respectively. In order to accommodate the wishes of the user, the target sound pressure is chosen as follows:

$$\begin{aligned}
t^{(m)}[n] &= \sum_{l=0}^{N_L} \left(h^{(l,m)} * s_{\mathcal{A}}\right)[n] \qquad \forall\, m \in A, \\
t^{(m)}[n] &= \sum_{l=0}^{N_L} \left(h^{(l,m)} * s_{\mathcal{B}}\right)[n] \qquad \forall\, m \in B.
\end{aligned} \tag{3.3}$$

This choice for the target pressure can be understood as the sound pressure that arises in a certain zone when using the loudspeaker array to play only the desired audio in that zone. For example, when in zone $m \in A$, the target sound pressure

is set equal to the sound pressure corresponding to what arises when playing only $s_{\mathcal{A}}[n]$ from the loudspeaker array.

The motivation for choosing this target is that it is physically attainable in each zone separately with the given loudspeakers, their positions, and the room acoustics.

3.2 Review of Sound Zone Approaches

The two main approaches in sound zone literature are pressure matching (PM) and acoustic contrast control (ACC). From these two approaches, pressure matching is used in the perceptual sound zone algorithm proposed in Chapter 4. This section introduce and describes both approaches using the previously derived data model with the goal of sketching their mathematical properties. This is then later used in Section 3.3 to determine which one is suitable for integration with the Par detectability discussed in Chapter 2.

Classically, the sound zone problem is divided up into subproblems as described in the introduction of this chapter. The resulting loudspeaker input signals $x^{(l)}[n]$ are determined for a single bright-dark zone pair: the loudspeaker input signals are found such that the target audio is achieved in the bright zone, while leakage is minimized in the dark zone. If a solution for multiple zones is desired, then multiple problems must be solved independently and their resulting loudspeaker input signals combined [1].

There is another approach however. In a multi-zone approach, the loudspeaker input signals are instead determined jointly for all zones, rather than decomposing into bright-dark zone pairs. This is the approach that is taken in this thesis, as it was found to be more general is simpler to present. For simplicity, but without loss of generality, this thesis limits the number of zones to two. The approach is however generalizable to any multiplicity of zones.

In a two zone multi-zone approach, the loudspeaker input signals $x^{(l)}[n]$ are decomposed into two parts as follows:

$$x^{(l)}[n] = x_{\mathcal{A}}^{(l)}[n] + x_{\mathcal{B}}^{(l)}[n]. \tag{3.4}$$

Here, $x_{\mathcal{A}}^{(l)}[n]$ and $x_{\mathcal{B}}^{(l)}[n]$ are the parts of the loudspeaker input signal responsible for reproducing the target sound pressure in zone $\mathcal{A}$ and $\mathcal{B}$ respectively.

Through this decomposition, it is possible to consider the sound pressure that arises at a specified control point due to the separate loudspeaker input signals:

$$p_{\mathcal{Z}}^{(m)}[n] = \sum_{l=0}^{N_L} \left(h^{(l,m)} * x_{\mathcal{Z}}^{(l)} \right)[n], \tag{3.5}$$

where $\mathcal{Z} \in (\mathcal{A}, \mathcal{B})$ represents either zones. Here, $p_{\mathcal{A}}^{(m)}[n]$ and $p_{\mathcal{B}}^{(m)}[n]$ can be understood to be the pressure that arises due to playing loudspeaker input signals $x_{\mathcal{A}}^{(l)}[n]$ and $x_{\mathcal{B}}^{(l)}[n]$ respectively . The total achieved sound pressure at control point m is then given by the addition of the two achieved sound pressures:

$$p^{(m)}[n] = p_{\mathcal{A}}^{(m)}[n] + p_{\mathcal{B}}^{(m)}[n]. \tag{3.6}$$

This decomposition is used to describe a multi-zone variant of both a pressure matching approach in Section 3.2.1, and acoustic contrast control approach in Section 3.2.2.

3.2.1 Pressure Matching

The "Pressure Matching" (PM) is widely used in the literature to solve the sound zone problem [1, 2]. In this section, a "Multi-Zone Pressure-Matching" (MZ-PM) algorithm is derived using the derived data model.

In pressure matching approaches, one attempts to design suitable loudspeaker input signals in such a way that the resulting sound pressure in the zone matches the specified target sound pressure for that zone, while simultaneously minimizing the sound pressure that results in other zones as to minimize the interference or crosstalk between zones [1, 30].

Formally to choose $x_{\mathcal{A}}^{(l)}[n]$ such that the resulting achieved pressure $p_{\mathcal{A}}^{(m)}[n]$ attains the target sound pressure $t^{(m)}[n]$ in all control points $m \in A$. At the same time however, $p_{\mathcal{A}}^{(m)}[n]$ should result in minimal sound pressure in all control points $m \in B$. Any sound pressure resulting from $x_{\mathcal{A}}^{(l)}[n]$ in zone $\mathcal{B}$ can be understood as leakage, or cross-talk between the zones.

Similar arguments can be given for $x_{\mathcal{B}}^{(l)}[n]$.

An optimization problem that achieves this goal is formulated as follows:

$$
\begin{aligned}
\underset{x_{\mathcal{A}}^{(l)}[n],\, x_{\mathcal{B}}^{(l)}[n]\,\forall l}{\arg\min} \quad & \sum_{m \in A} \left\| p_{\mathcal{A}}^{(m)}[n] - t^{(m)}[n] \right\|_2^2 + \sum_{m \in A} \left\| p_{\mathcal{B}}^{(m)}[n] \right\|_2^2 + \\
& \sum_{m \in B} \left\| p_{\mathcal{B}}^{(m)}[n] - t^{(m)}[n] \right\|_2^2 + \sum_{m \in B} \left\| p_{\mathcal{A}}^{(m)}[n] \right\|_2^2,
\end{aligned}
\tag{3.7}
$$

where the $|| \cdot ||_2^2$ operator denotes the squared L^2-norm.

To further understand the optimization problem, consider the following definitions:

$$
\mathrm{RE}_{\mathcal{Z}}^{(m)} = \left\| p_{\mathcal{Z}}^{(m)}[n] - t^{(m)}[n] \right\|_2^2 \qquad \forall\, m \in Z,
\tag{3.8}
$$

$$
\mathrm{LE}_{\mathcal{Z}}^{(m)} = \left\| p_{\mathcal{Z}}^{(m)}[n] \right\|_2^2 \qquad\qquad \forall\, m \notin Z.
\tag{3.9}
$$

Here, $\mathrm{RE}_{\mathcal{Z}}^{(m)}$ is the reproduction error for zone $\mathcal{Z} \in (\mathcal{A}, \mathcal{B})$ for control point $m \in Z$. This error corresponds to how well the achieved sound pressure $p_{\mathcal{Z}}^{(m)}[n]$ matches the target sound pressure $t^{(m)}[n]$ for a control point in the bright zone Z.

$\mathrm{LE}_{\mathcal{Z}}^{(m)}$ is the leakage error in zone $\mathcal{Z} \in (\mathcal{A}, \mathcal{B})$ for control point $m \notin Z$. This can be understood as the sound pressure that "leaks" into control point m in zones other than $\mathcal{Z}$ when attempting to reproduce the target sound pressure $t^{(m)}[n]$ in zone $\mathcal{Z}$. This can be also be understood as the "interference" or "cross-talk" between zones.

Using these definition we can rewrite the optimization problem:

$$
\underset{x_{\mathcal{A}}^{(l)}[n],\, x_{\mathcal{B}}^{(l)}[n]\,\forall l}{\arg\min} \quad \sum_{m \in A} \mathrm{RE}_{\mathcal{A}}^{(m)} + \sum_{m \in B} \mathrm{LE}_{\mathcal{A}}^{(m)} + \sum_{m \in B} \mathrm{RE}_{\mathcal{B}}^{(m)} + \sum_{m \in A} \mathrm{LE}_{\mathcal{B}}^{(m)}.
\tag{3.10}
$$

From this it becomes clear that this approach results in trade-off between minimizing the reproduction errors $\text{RE}_{\mathcal{Z}}^{(m)}$ and leakage errors $\text{LE}_{\mathcal{Z}}^{(m)}$.

Some pressure matching approaches attempt to control this trade-off by introducing weights for the different error terms, or by adding constraints. Choosing constraints can however be challenging as the squared L2 pressure error does not always correlate well with how the error is perceived.

3.2.2 Acoustic Contrast Control

"Acoustic Contrast Control" is another widely used sound zone approach from literature. The acoustic contrast control (ACC) approach to sound zones attempts to maximize the acoustic contrast between the bright zone and the dark zone. Acoustic contrast is defined as the ratio of the total sound energy of the bright zone and the dark zone. Essentially, the goal is to maximize the difference in sound pressure level between the bright and dark zones.

In this section, a "Multi-Zone Acoustic Contrast Control" (MZ-ACC) algorithm will be described. As the previously described data model is in the time domain, this approach will take inspiration from a time-domain approach found in literature known as the broadband acoustic contrast control (BACC) approach [31, 32, 2].

In contrast to the MZ-PM approach, the MZ-ACC approach does not optimize directly over the loudspeaker input signals $x_{\mathcal{A}}^{(l)}[n]$ and $x_{\mathcal{B}}^{(l)}[n]$. Instead, it indirectly controls the loudspeaker input signals by optimizing over FIR filter coefficients $w_{\mathcal{A}}^{(l)}[n] \in \mathbb{R}^{N_w}$ and $w_{\mathcal{B}}^{(l)}[n] \in \mathbb{R}^{N_w}$. These filters are applied to the desired playback signals $s_{\mathcal{A}}^{(l)}$ and $s_{\mathcal{B}}^{(l)}$ respectively to form the final loudspeaker input signals.

This relationship between the loudspeaker input signals and the filter coefficients is thus given as follows:

$$x_{\mathcal{Z}}^{(l)}[n] = \left(w_{\mathcal{Z}}^{(l)} * s_{\mathcal{Z}} \right)[n]. \tag{3.11}$$

This definition also relates the filter coefficients to the resulting sound pressure through Section 3.2.

As mentioned, the goal of the ACC approach is to maximize the acoustic contrast between bright and dark zones, which was defined as the ratio between the sound energy in the bright and dark zones. The total sound energy in a zone will be defined as the sum of squares of the sound pressure in a control point. As such, the acoustic contrast $\text{AC}_{\mathcal{Z}}$ for a zone $\mathcal{Z}$ can be defined as follows:

$$\text{AC}_{\mathcal{Z}} = \frac{\sum_{m \in Z} \left\| p_{\mathcal{Z}}^{(m)}[n] \right\|_2^2}{\sum_{m \notin Z} \left\| p_{\mathcal{Z}}^{(m)}[n] \right\|_2^2}. \tag{3.12}$$

In an ACC approach the goal is to maximize the total acoustic contrast. Thus, consider the following optimization problem:

$$\underset{w_{\mathcal{A}}^{(l)}[n], w_{\mathcal{B}}^{(l)}[n] \, \forall l}{\arg\max} \quad \text{AC}_{\mathcal{A}} + \text{AC}_{\mathcal{B}}. \tag{3.13}$$

As mentioned, the optimization is performed over the loudspeaker filter coefficients rather than over the loudspeaker input signals.

Acoustic contrast control is discussed mainly for completion as an alternative to the pressure matching approach. As is discussed in Section 3.3, a pressure matching is used for the perceptual sound zone algorithm.

3.3 Proposed Perceptual Sound Zone Approach

In Chapter 2 it was determined that the Par detectability is the perceptual model most suited for use in a perceptual sound zone algorithm. Previously, in Section 3.2, two different sound zone techniques were discussed pressure matching (PM), and acoustic contrast control (ACC).

In this section proposes how the Par detectability measure can be combined with the previously discussed sound zone approaches. This is done by first reflecting on the mathematical properties of the Par detectability measure in Section 3.3.1. Next, Section 3.3.2 introduces the proposed perceptual sound zone approach.

3.3.1 Analysis of Mathematical Properties of Proposed Least Squares formulation of Par Detectability Measure

This section will reflect on the mathematical properties of the Par detectability.

Recall from Section 2.3 that the detectability $D(x[n], \varepsilon[n])$ quantifies how detectable a disturbance $\varepsilon[n] \in \mathbb{R}^{N_x}$ is in presence of a masking signal $x[n] \in \mathbb{R}^{N_x}$. The Par detectability assumes that the time-scale of its inputs are short, in the order of 20 to 200 ms.

The detectability is computed using the frequency domain representation of its input signals. To this end, $X[k]$ and $\mathcal{E}[k]$ were introduced to denote the frequency domain representations of $x[n]$ and $\varepsilon[n]$ respectively.

In Section 2.3.3 it was found that the detectability could be expressed in least-squares fashion as follows:

$$D(x[n], \varepsilon[n]) = ||W_x[k]\mathcal{E}[k]||_2^2 \tag{3.14}$$

Here, perceptual weighting $W_x[k]$ models the psycho-acoustical masking effects of the masking signal $x[n]$. The weights are applied to the frequency bins of the disturbance signal $\mathcal{E}[k]$, and the sum of squares is used determine the final detectability rating.

It was noted in Section 2.3.3 that the Par detectability measure is a convex function of the disturbance signal $\mathcal{E}[k]$ when the masking signal is held constant. As such, one approach is to specify a sound zone algorithm which leverages the optimization over this disturbance signal in some way. This will be done by adopting a model for the disturbance $\mathcal{E}[k]$. For example, the disturbance could model the sound pressure error.

In summary, the detectability has the following properties that must be taken into account:

1. It is computed in the short-time frequency-domain.

2. It is convex when optimizing over the disturbance signal $\mathcal{E}[k]$.

3.3.2 Introducing Proposed Perceptual Sound Zone Approach

In Section 3.2 two different sound zone approaches were discussed: pressure matching (PM) and acoustic contrast control (ACC). Currently, neither approach operates in the frequency domain on short-time scales. However, it has been shown in literature that formulations of both ACC and PM exist that satisfy this condition [33].

Either ACC or PM can be used. Through inspection, it was found that the Par detectability measure can be used to formulate a PM approach directly by modeling the errors minimized by PM as disturbance signals. As such, in this work the focus will be on the PM approach.

To see this, recall in the discussion of PM given in Section 3.2 that PM was shown to minimize the sum of the reproduction error in the bright zone $\text{RE}_{\mathcal{Z}}^{(m)}$ and the leakage to the dark zone $\text{LE}_{\mathcal{Z}}^{(m)}$. The definitions of $\text{RE}_{\mathcal{Z}}^{(m)}$ and $\text{LE}_{\mathcal{Z}}^{(m)}$ are given by Equation (3.8) and Equation (3.9) respectively. Here, the reproduction error $\text{RE}_{\mathcal{Z}}^{(m)}$ can be understood as the total error between the achieved and target sound pressure in the bright zone of zone $\mathcal{Z}$. The leakage $\text{LE}_{\mathcal{Z}}^{(m)}$ can be understood as the sound pressure that arises in zones other than $\mathcal{Z}$, essentially "leaking" into other zones, when reproducing the target in the bright zone.

Both can be thus be considered as "errors" that we wish to minimize. Viewing it this way, one choice for model these errors as the disturbance signals $\varepsilon[n]$. To this end, consider the reproduction error detectability $\text{RED}_{\mathcal{Z}}^{(m)}$ and the leakage error detectability $\text{LED}_{\mathcal{Z}}^{(m)}$. Fom now, ignore to the fact that the quantities for target and achieved sound pressure defined so far do not satisfy the short-time requirements. The perceptual errors are defined as:

$$\text{RED}_{\mathcal{Z}}^{(m)} = D(t^{(m)}[n], p_{\mathcal{Z}}^{(m)}[n] - t^{(m)}[n]) \qquad \forall\, m \in Z \qquad (3.15)$$

$$\text{LED}_{\mathcal{Z}}^{(m)} = D(t^{(m)}[n], p_{\mathcal{Z}}^{(m)}[n]) \qquad \forall\, m \notin Z \qquad (3.16)$$

The reproduction error detectability $\text{RED}_{\mathcal{Z}}^{(m)}$ is determined by the detectability of the sound pressure difference between the achieved sound pressure $p_{\mathcal{Z}}^{(m)}[n]$ and the target sound pressure $t^{(m)}[n]$ for the control point m. This sound pressure difference can be understood as the "error" in sound pressure.

The target sound pressure is used as the masking signal, in doing so the masking properties of the target sound pressure $t^{(m)}[n]$ is used. Note that this is an approximation: in reality, it cannot be assumed that the sound algorithm exactly attains the target sound pressure. In the ideal case, the masking properties of the total achieved sound pressure would be used instead.

However, this quantity depends on the loudspeaker input signals over which the optimization is performed, and is thus not available a priori. One approach would be to include the masking signal in the optimization, but this would violate convexity, as stated in Section 2.3.3. As such, by the masking effects of the achieved pressure are modeled by those of the target sound pressure.

The $\mathrm{RED}_{\mathcal{Z}}^{(m)}$ models how detectable the deviation from target sound pressure is One interpretation of minimizing $\mathrm{RED}_{\mathcal{Z}}^{(m)}$ is finding the sound pressure difference which is least detectable.

Similarly, the leakage error detectability $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ is determined by how detectable the sound pressure of zone $\mathcal{Z}$ is in other zones. Note as $m \notin \mathcal{Z}$, this definition considers all points outside of the bright zone for $\mathcal{Z}$. Here, the masking signal is again chosen as the target sound pressure of the control point in question. $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ models how detectable the leakage of zone $\mathcal{Z}$ is in presence of the intended target sound pressure. Minimizing $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ could thus be understood as finding the leakage that is minimally-detectable in presence of the target sound pressure.

These perceptually modified errors are found to be a promising way of combining pressure matching and the Par detectability, and are used to state perceptual sound zone algorithms in Chapter 4. The short-time frequency-domain quantities required for the definitions of $\mathrm{RED}_{\mathcal{Z}}^{(m)}$ and $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ are introduced in this chapter.

Using the ACC approach to formulate perceptual sound zone algorithms is not explored further in this work, but is found to be promising future work.

Implementation of Proposed Perceptual Sound Zone Algorithms

4

In Chapter 2 the Par detectability is selected as the most promising perceptual model for use in a perceptual sound zone algorithm. Next, Chapter 3 various sound zone algorithms are discussed, ultimately leading to the proposal of a sound zone approach in Section 3.3 that uses the Par detectability measure in order to create sound zones.

This chapter uses the proposed perceptual sound zone approach to propose and implement two perceptual sound zone algorithms.

Chapter Structure

This chapter is structured as follows.

- First, Section 4.1 discusses the reformulation of the time-domain pressure matching approach given in Section 3.2 to a short-time frequency-domain pressure-matching approach. This is necessary for the perceptual approach discussed in Section 3.3 to be implementable.

- Next, Section 4.2 discusses using the approach proposed in Section 3.3 to formulate two perceptual sound zone algorithms.

4.1 Proposed Short-Time Frequency-Domain Reformulation of the Pressure Matching Approach

In Section 3.3 a perceptual sound zone approach is proposed based on the pressure matching approach discussed in Section 3.2. In this approach, the Par detectability measure is used to quantify the perceptual cost of sound pressure errors. In doing so, Section 3.2 introduces the reproduction error detectability $\text{RED}_{\mathcal{Z}}^{(m)}$ and the leakage error detectability $\text{LED}_{\mathcal{Z}}^{(m)}$ per control point m introduced conceptually, but without defining them.

As noted, the original pressure matching approach from Section 3.2 operates on full-length input sequences in the time-domain. The detectability however operates on short-time segments of 20 to 200 milliseconds in the frequency-domain. As such, in order to define the reproduction error detectability and the leakage error detectability, this section proposes a short-time frequency-domain pressure matching approach.

First in Section 4.1.1 the existing pressure matching approach is reformulated to operate on short-time segments through a "block-based" approach. Next, Section 4.1.2 adapts the short-time pressure matching algorithm to operate in the frequency domain.

4.1.1 Proposed Block-Based Pressure-Matching Algorithm

In order to operate on short-time segments, all quantities introduced in the data model from Section 3.1 are converted to their short-time equivalent representations. This is done by expressing quantities using overlapping blocks of samples of the quantities.

Here, the blocks are each of size N_w and $N_w - H$ samples. The constant H denotes the hop size, which is the number of samples between each successive block.

First, the short-time equivalent representations of the desired playback signal $s_{\mathcal{Z}}[n]$ and the loudspeaker input signals $x_{\mathcal{Z}}^{(l)}[n]$ for zone $\mathcal{Z}$ and loudspeaker l are discussed.

In order to do so, $s_{\mathcal{Z}}[n]$ and $x_{\mathcal{Z}}^{(l)}[n]$ will be split up into multiple overlapping blocks by using shifted windows $w[n - kH]$.

The window $w[n] \in \mathbb{R}^H$ is a non-causal window with support $-N_w + 1 \leq n \leq 0$. Here, $w[n]$ is chosen such that it complies with the Constant Overlap Add (COLA) condition for a given hop size H. The COLA condition requires that the the sum of all H-shifted windows add to unity for all samples n. It is given as follows:

$$\sum_{k=-\infty}^{\infty} w[n - kH] = 1 \quad \forall n \tag{4.1}$$

Using the windows as defined above, consider the following representation of $s_{\mathcal{Z}}[n]$,

$$s_{\mathcal{Z}}[n] = s_{\mathcal{Z}}[n] \sum_{k=-\infty}^{\infty} w[n - kH]$$

$$= \sum_{k=-\infty}^{\infty} \tilde{s}_{\mathcal{Z},k}[n] w[n - kH] \tag{4.2}$$

and of $x_{\mathcal{Z}}^{(l)}[n]$,

$$x_{\mathcal{Z}}^{(l)}[n] = x_{\mathcal{Z}}^{(l)}[n] \sum_{k=-\infty}^{\infty} w[n - kH]$$

$$= \sum_{k=-\infty}^{\infty} \tilde{x}_{\mathcal{Z},k}^{(l)}[n] w[n - kH] \tag{4.3}$$

Here, $\tilde{s}_{\mathcal{Z},k}[n]$ and $\tilde{x}_{\mathcal{Z},k}^{(l)}[n]$ represent the content of the kth blocks of the playback signal $s_{\mathcal{Z}}[n]$ and loudspeaker input signals $x_{\mathcal{Z}}^{(l)}[n]$.

As such, $\tilde{s}_{\mathcal{Z},k}[n] = s_{\mathcal{Z}}[n]$ and $\tilde{x}_{\mathcal{Z},k}^{(l)}[n] = s_{\mathcal{Z}}[n]$ for $-N_w + 1 + kH \leq n \leq kH$ and zero for all other samples n. One interpretation is that the windows decimate the signal, into segments of size N_w, which, due to the COLA condition, can be reconstructed perfectly.

One way of interpreting the equations above is as a projection of $s_{\mathcal{Z}}[n]$ and $x_{\mathcal{Z}}^{(l)}[n]$ on a basis of frames spanned by shifted overlapping windows $w[n - kH]$. Here, $\tilde{s}_{\mathcal{Z},k}[n]$ and $\tilde{x}_{\mathcal{Z},k}^{(l)}[n]$ can be thought of as the coefficients for the basis functions resulting from the projection.

Let $\tilde{s}_{\mathcal{Z}}[n, \mu]$ and $\tilde{x}_{\mathcal{Z}}^{(l)}[n, \mu]$ represent the desired playback signal and the loudspeaker input signals with the contributions up to and including the μth block. This can be expressed as follows:

$$\tilde{s}_{\mathcal{Z}}[n, \mu] = \sum_{k=-\infty}^{\mu} \tilde{s}_{\mathcal{Z},k}[n] w[n - kH] \tag{4.4}$$

$$\tilde{x}_{\mathcal{Z}}^{(l)}[n, \mu] = \sum_{k=-\infty}^{\mu} \tilde{x}_{\mathcal{Z},k}^{(l)}[n] w[n - kH] \tag{4.5}$$

This form will converge to the real desired playback signal as $\mu \to \infty$. As such, $\tilde{s}_{\mathcal{Z}}[n, \infty] = s_{\mathcal{Z}}[n]$ and $\tilde{x}_{\mathcal{Z}}^{(l)}[n, \infty] = x_{\mathcal{Z}}^{(l)}[n]$.

This representation is beneficial, as it can be used to show that the $\tilde{x}_{\mathcal{Z}}^{(l)}[n, \mu]$ can be computed recursively:

$$\tilde{x}_{\mathcal{Z}}^{(l)}[n, \mu] = \tilde{x}_{\mathcal{Z},\mu}^{(l)}[n] w[n - \mu H] + \sum_{k=-\infty}^{\mu-1} \tilde{x}_{\mathcal{Z},k}^{(l)}[n] w[n - kH] \tag{4.6}$$

$$= \tilde{x}_{\mathcal{Z},\mu}^{(l)}[n] w[n - \mu H] + \tilde{x}_{\mathcal{Z}}^{(l)}[n, \mu - 1] \tag{4.7}$$

As the newest block depends on the previous blocks, this representation shows that $x_{\mathcal{Z}}^{(l)}[n]$ can be computed block-by-block.

With the block-based equivalents of the desired playback signal $\tilde{s}_{\mathcal{Z}}[n,\mu]$ and the loudspeaker input signals $\tilde{s}_{\mathcal{Z}}[n,\mu]$ defined, the block-based equivalents of the target and achieved sound pressure $\tilde{t}_{\mathcal{Z}}[n,\mu]$ and $\tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu]$ can be computed:

- The block-based target sound pressure $\tilde{t}^{(m)}[n,\mu]$ can be defined by simply sub-stituting the definition for the block-based desired playback signal $\tilde{s}_{\mathcal{Z}}[n,\mu]$ into the definition of the target pressure given by Equation (3.3):

$$
\begin{aligned}
\tilde{t}^{(m)}[n,\mu] &= \sum_{l=0}^{N_L-1} \left(h^{(l,m)} * \tilde{s}_{\mathcal{Z}}[\mu] \right)[n] \\
&= \sum_{l=0}^{N_L-1} \sum_{k=-\infty}^{\mu} \left(h^{(l,m)} * \tilde{s}_{\mathcal{Z},k} w_k \right)[n] \qquad (4.8) \\
&= \sum_{l=0}^{N_L-1} \left(h^{(l,m)} * \tilde{s}_{\mathcal{Z},\mu} w_\mu \right)[n] + \tilde{t}^{(m)}[n,\mu-1]
\end{aligned}
$$

Here, $w_k[n]$ is defined to be equal to $w[n-kH]$ and is introduced for notational convenience. The definition above holds for all points $m \in Z$, i.e. the points contained in zone $\mathcal{Z}$.

As can be seen, the block based target sound pressure for the block μ can be computed recursively by adding the contribution of the newest block $\tilde{s}_{\mathcal{Z},\mu}[n]$ the target sound pressure of the previous block.

- The block-based resulting sound pressure $\tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu]$ can be defined by sim-ply substituting the definition for the block-based loudspeaker input signals $\tilde{x}_{\mathcal{Z}}^{(l)}[n,\mu]$ into the definition of the resulting pressure given by Equation (3.2). This results in the following:

$$
\begin{aligned}
\tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu] &= \sum_{l=0}^{N_L-1} \left(h^{(l,m)} * \tilde{x}_{\mathcal{Z}}^{(l)}[\mu] \right)[n] \\
&= \sum_{l=0}^{N_L-1} \sum_{k=-\infty}^{\mu} \left(h^{(l,m)} * \tilde{x}_{\mathcal{Z},k}^{(l)} w_k \right)[n] \qquad (4.9) \\
&= \sum_{l=0}^{N_L-1} \left(h^{(l,m)} * \tilde{x}_{\mathcal{Z},\mu}^{(l)} w_\mu \right)[n] + \tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu-1]
\end{aligned}
$$

The definition above again holds for all points $m \in Z$.

As can be seen, the block based resulting sound pressure for the block μ can also be computed recursively.

With this, all quantities required for the block-based formulation of the pressure matching approach are defined.

It is shown that all quantities can be computed recursively. This is used in the block-based pressure matching approach by computing the blocks of the loudspeaker input signal $x_{\mathcal{Z}}^{(l)}[n]$ one by one.

As such, the kth loudspeaker input signal coefficient $\tilde{x}_{\mathcal{Z},\mu}^{(l)}[n]$ is computed such that the resulting resulting sound pressure $\tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu]$ best matches the target sound pressure $\tilde{t}^{(m)}[n,\mu]$.

Note that in this approach only newest loudspeaker coefficients $\tilde{x}_{\mathcal{Z},\mu}^{(l)}$ are being controlled. Thus, the previous coefficients $\tilde{x}_{\mathcal{Z},k}^{(l)}[n]$ for $-\infty \leq k \leq \mu-1$ are held fixed.

The block-based optimization problem can be found by simply replacing all quantities in the previously derived optimization problem with their block-based counterparts. The problem is given as follows:

$$
\underset{\tilde{x}_{\mathcal{A},\mu}^{(l)}[n],\,\tilde{x}_{\mathcal{B},\mu}^{(l)}[n]\,\forall l}{\arg\min} \quad \sum_{m\in A}\left|\left|\tilde{p}_{\mathcal{A}}^{(m)}[n,\mu]-\tilde{t}_{\mu}^{(m)}[n,\mu]\right|\right|_2^2 + \sum_{m\in A}\left|\left|\tilde{p}_{\mathcal{B}}^{(m)}[n,\mu]\right|\right|_2^2 +
$$
$$
\sum_{m\in B}\left|\left|\tilde{p}_{\mathcal{B}}^{(m)}[n,\mu]-\tilde{t}_{\mu}^{(m)}[n,\mu]\right|\right|_2^2 + \sum_{m\in B}\left|\left|\tilde{p}_{\mathcal{A}}^{(m)}[n,\mu]\right|\right|_2^2
$$

(4.10)

Note that this problem implicitly contains the target sound pressure and resulting sound pressure of the previous blocks $-\infty \leq k \leq \mu-1$ due to the aforementioned recursive definitions. As a result, the history of what has been transmitted by the loudspeaker previously is included in the optimization.

This is beneficial, as due to overlap, this allows for block μ to potentially improve the results of previous blocks. However, the loudspeaker input signals of the current block μ, can only effect so many previous blocks (depending on the overlap). As such, in order to reduce the complexity of the optimization without effecting the results, one may choose to truncate the number of previous blocks $-\infty \leq k \leq \mu-1$ in the history of the optimization.

The problem above is solved recursively for all loudspeaker input signal coefficients $\tilde{x}_{\mathcal{A},\mu}^{(l)}[n]$ and $\tilde{x}_{\mathcal{B},\mu}^{(l)}[n]$. The final loudspeaker input signals $x_{\mathcal{Z}}^{(l)}[n]$ can then be found by means of Equation (4.3).

4.1.2 Proposed Block-Based Frequency-Domain Pressure-Matching Algorithm

This section will adjust the block-based data model equivalent frequency domain formulation in order to propose a block-based frequency-domain pressure-matching algorithm. This is done by first introducing a transformation relating the time and frequency domain quantities.

A suitable transform is the discrete Fourier transform (DFT). However, it is important to take a number of precautions before applying the DFT directly. As shown in Equation (3.2) the computation of the sound pressures used in the optimiza-

tion problem introduced previously involves taking the linear convolution of the loudspeaker input signals with the room impulse responses.

Time domain circular convolution can be computed in the frequency domain through the Hadamard product. Time domain circular convolution coincides with time domain linear convolution only if the two operands are zero-padded sufficiently. To be specific, both operands need be zero-padded to the length of the resulting linear convolution.

As such, the frequency domain transform requires this zero padding to be built in. The convolutions described in the previous chapter are between the window coefficients of size N_w and the room impulse responses of size N_h. Thus, the both must be zero padded to convolution length $N_w + N_h - 1$ before going to the frequency domain.

Let $x[n]$ and $X[k]$ denote the time- and frequency-domain representations of an arbitrary sequence. A suitable transform is given by the following $N_w + N_h - 1$ point DFT:

$$X[k] = \sum_{n=0}^{N_w+N_h-2} x[n] \exp\left(\frac{-j2\pi kn}{N_w + N_h - 1}\right) \tag{4.11}$$

Converting the previously introduced block-based pressure matching to a frequency domain equivalent version essentially involves converting the sound pressures $\tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu]$ and $\tilde{t}^{(m)}[n,\mu]$ to their frequency domain counterparts, which are denoted by $\tilde{P}_{\mathcal{Z},\mu}^{(m)}[k]$ and $\tilde{T}_\mu^{(m)}[k]$ respectively.

This results in the following expressions.

$$\tilde{T}^{(m)}[k,\mu] = \tilde{T}^{(m)}[k,\mu-1] + \sum_{l=0}^{N_L} H^{(l,m)}[k]\tilde{S}_{\mathcal{Z},\mu}[k], \tag{4.12}$$

$$\tilde{P}_{\mathcal{Z}}^{(m)}[k,\mu] = \tilde{P}_{\mathcal{Z}}^{(m)}[k,\mu-1] + \sum_{l=0}^{N_L} H^{(l,m)}[k]\tilde{X}_{\mathcal{Z},\mu}^{(l)}[k]. \tag{4.13}$$

Here, $H^{(l,m)}[k] \in \mathbb{C}^{N_w+N_h-1}$ is the transformed version of the room impulse responses.

Furthermore, $\tilde{S}_{\mathcal{Z},\mu}[k] \in \mathbb{C}^{N_w+N_h-1}$ and $\tilde{X}_{\mathcal{Z},\mu}^{(l)}[k] \in \mathbb{C}^{N_w+N_h-1}$ are the frequency domain versions of the desired playback signal and the loudspeaker input signal, which are defined as follows:

$$\tilde{S}_{\mathcal{Z},\mu}[k] = \sum_{n=0}^{N_w+N_h-2} \tilde{s}_{\mathcal{Z},\mu}[n]w[n-\mu H] \exp\left(\frac{-j2\pi kn}{N_w + N_h - 1}\right) \tag{4.14}$$

$$\tilde{X}_{\mathcal{Z},\mu}^{(l)}[k] = \sum_{n=0}^{N_w+N_h-2} \tilde{x}_{\mathcal{Z},\mu}^{(l)}[n]w[n-\mu H] \exp\left(\frac{-j2\pi kn}{N_w + N_h - 1}\right) \tag{4.15}$$

Note that the window is implicitly included in the transformed quantities. This is done for ease of notation.

Using the previously derived quantities, it is possible express the frequency domain version of the block-based pressure matching approach as follows:

$$
\begin{aligned}
\underset{\tilde{x}_{\mathcal{A},\mu}^{(l)}[n],\, \tilde{x}_{\mathcal{B},\mu}^{(l)}[n]\,\forall l}{\arg\min} \quad & \sum_{m\in A} \left\lVert \tilde{P}_{\mathcal{A}}^{(m)}[k,\mu] - \tilde{T}^{(m)}[k,\mu] \right\rVert_2^2 + \sum_{m\in A} \left\lVert \tilde{P}_{\mathcal{B}}^{(m)}[k,\mu] \right\rVert_2^2 + \\
& \sum_{m\in B} \left\lVert \tilde{P}_{\mathcal{B}}^{(m)}[k,\mu] - \tilde{T}^{(m)}[k,\mu] \right\rVert_2^2 + \sum_{m\in B} \left\lVert \tilde{P}_{\mathcal{A}}^{(m)}[k,\mu] \right\rVert_2^2
\end{aligned}
\tag{4.16}
$$

Note also how the optimization is still performed over the time domain signal. This was done to constrain the loudspeaker input signal coefficient to size N_w, as that is an assumption made by the frame-based processing.

In principal, this introduces more complexity than solving directly over the frequency domain loudspeaker input coefficient $\tilde{X}_{\mathcal{Z},\mu}^{(l)}[k]$. This however introduces issues as it requires the truncation of the time-domain version to the first N_w samples.

Naively truncating N_w this way introduced artifacts. In experiments in which this approach is attempted the time-domain representation of the resulting frequency-domain loudspeaker input signals results in significant energy contained in the last $N_h - 1$ samples. If truncated, a significant portion of the signal energy would be disregarded, which serves as a possible explanation for the artifacts.

However, due to the computational benefits, formulating a frequency domain approach that minimizes the impact of or prevents these artifacts is found to be promising future work.

4.1.3 Proposed Short-Time Frequency-Domain Pressure-Matching in Vector Notation

To conclude ?? a vector notation alternative representation of the short-time frequency-domain pressure-matching approach is given, as vector notation is typically preferred for the understand and implementation of signal processing algorithms.

To this end, let $\tilde{\mathbf{t}}_\mu^{(m)} \in \mathbb{C}^{N_w+N_h-1}$ and $\tilde{\mathbf{p}}_\mu^{(m)} \in \mathbb{C}^{N_w+N_h-1}$ denote vectors that contain the target and achieved sound pressures $\tilde{T}^{(m)}[k,\mu]$ and $\tilde{P}^{(m)}[k,\mu]$ respectively. Their definitions are given as follows:

$$
\tilde{\mathbf{t}}_\mu^{(m)} = \tilde{\mathbf{t}}_{\mu-1}^{(m)} + \sum_{l=0}^{N_L} \mathbf{H}^{(l,m)} \tilde{\mathbf{s}}_{\mathcal{Z},\mu},
\tag{4.17}
$$

$$
\tilde{\mathbf{p}}_\mu^{(m)} = \tilde{\mathbf{p}}_{\mu-1}^{(m)} + \sum_{l=0}^{N_L} \mathbf{H}^{(l,m)} \tilde{\mathbf{x}}_{\mathcal{Z},\mu}^{(l)}.
\tag{4.18}
$$

Here, $\tilde{\mathbf{s}}_{\mathcal{Z},\mu} \in \mathbb{C}^{N_w+N_h-1}$ and $\tilde{\mathbf{x}}_{\mathcal{Z},\mu}^{(l)} \in \mathbb{C}^{N_w+N_h-1}$ are vectors containing the frequency versions of the playback and loudspeaker input signals $\tilde{S}_{\mathcal{Z},\mu}[k]$ and $\tilde{X}_{\mathcal{Z},\mu}^{(l)}[k]$ respectively. The matrix $\mathbf{H}^{(l,m)} \in \mathbb{C}^{N_w+N_h-1 \times N_w+N_h-1}$ contains the the elements of $H^{(l,m)}[k]$ on its diagonal.

From this, a vector based variant of the the short-time frequency-domain pressure matching optimization problem originally given in Equation (4.16) can be given as follows:

$$\underset{\tilde{x}^{(l)}_{\mathcal{A},\mu}[n],\,\tilde{x}^{(l)}_{\mathcal{B},\mu}[n]\,\forall l}{\arg\min} \quad \sum_{m \in A} \left\| \tilde{\mathbf{p}}^{(m)}_{\mathcal{A}}[\mu] - \tilde{\mathbf{t}}^{(m)}[\mu] \right\|_2^2 + \sum_{m \in A} \left\| \tilde{\mathbf{p}}^{(m)}_{\mathcal{B}}[\mu] \right\|_2^2 + \\ \sum_{m \in B} \left\| \tilde{\mathbf{p}}^{(m)}_{\mathcal{B}}[\mu] - \tilde{\mathbf{t}}^{(m)}[\mu] \right\|_2^2 + \sum_{m \in B} \left\| \tilde{\mathbf{p}}^{(m)}_{\mathcal{A}}[\mu] \right\|_2^2 \tag{4.19}$$

4.2 Proposal of Proposed Perceptual Pressure Matching Algorithms

Previously, Section 3.3 noted that pressure matching approach can be formulated using the detectability. In this section, this approach is used to propose two sound zone algorithms.

In the proposed approach, rather than optimizing the sum reproduction errors $\mathrm{RE}_{\mathcal{Z}}^{(m)}$ and leakage errors $\mathrm{LE}_{\mathcal{Z}}^{(m)}$, it is proposed to instead use the reproduction error detectability $\mathrm{RED}_{\mathcal{Z}}^{(m)}$ and the leakage error detectability $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ respectively. These can be understood to be perceptual alternatives to $\mathrm{RE}_{\mathcal{Z}}^{(m)}$ and $\mathrm{LE}_{\mathcal{Z}}^{(m)}$.

In Section 3.3 the description of reproduction error detectability $\mathrm{RED}_{\mathcal{Z}}^{(m)}$ and the leakage error detectability $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ is given in terms of full-length input sequences. As noted, this is inaccurate as the detectability operate on short-time scale, and is only done this way in order to clearly convey the concept.

Henceforth, using the concepts introduced in Section 4.1, let $\mathrm{RED}_{\mathcal{Z}}^{(m)}[\mu]$ and $\mathrm{LED}_{\mathcal{Z}}^{(m)}[\mu]$ denote the reproduction error detectability and leakage error detectability corresponding to block μ in control point m.

In order to define these error detectability quantities, recall that detectability is defined as follows:

$$D(x[n],\ \varepsilon[n]) = ||W_x[k]\mathcal{E}[k]||_2^2 \tag{4.20}$$

Using this definition alongside the short-time frequency domain definitions given in Section 4.1, the definition of the reproduction error detectability $\mathrm{RED}_{\mathcal{Z}}^{(m)}$ and the leakage error detectability $\mathrm{LED}_{\mathcal{Z}}^{(m)}$ can be given as follows:

$$\begin{aligned}
\mathrm{RED}_{\mathcal{Z}}^{(m)}[\mu] &= D(\tilde{t}^{(m)}[n,\mu],\ \tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu] - \tilde{t}^{(m)}[n,\mu]) \\
&= \left|\left| W_{\tilde{t}^{(m)}[\mu]}[k]\left(\tilde{P}_{\mathcal{Z}}^{(m)}[k,\mu] - \tilde{T}^{(m)}[k,\mu] \right)\right|\right|_2^2
\end{aligned} \tag{4.21}$$

$$\begin{aligned}
\mathrm{LED}_{\mathcal{Z}}^{(m)}[\mu] &= D(\tilde{t}^{(m)}[n,\mu],\ \tilde{p}_{\mathcal{Z}}^{(m)}[n,\mu]) \\
&= \left|\left| W_{\tilde{t}^{(m)}[\mu]}[k]\left(\tilde{P}_{\mathcal{Z}}^{(m)}[k,\mu] \right)\right|\right|_2^2
\end{aligned} \tag{4.22}$$

Here, $W_{\tilde{t}^{(m)}[\mu]}[k]$ can be understood as the perceptual weighting informed by the masking properties of the frequency domain target $\tilde{t}^{(m)}[n,\mu]$.

What follows is the proposal of two perceptual sound zone algorithms using the proposed error detectabilities.

4.2.1 Proposed Unconstrained Perceptual Pressure Matching Algorithm

This section proposes an algorithm which minimizes the detectability of the total error. This is similar to the pressure matching approach introduced in Section 3.2, in which the total error is minimized.

Consider the following optimization problem:

$$\underset{\tilde{x}_{\mathcal{A}}^{(l)}[n,\mu], \tilde{x}_{\mathcal{B}}^{(l)}[n,\mu]\,\forall\,l}{\arg\min} \quad \begin{aligned} & \sum_{m \in A} \mathrm{RED}_{\mathcal{A}}^{(m)}[\mu] + \sum_{m \in B} \mathrm{LED}_{\mathcal{A}}^{(m)}[\mu] + \\ & \sum_{m \in B} \mathrm{RED}_{\mathcal{B}}^{(m)}[\mu] + \sum_{m \in A} \mathrm{LED}_{\mathcal{B}}^{(m)}[\mu] \end{aligned} \tag{4.23}$$

The total detectability of the reproduction errors and the leakage errors is minimized by optimizing over the block-based representations of the loudspeaker input signals $\tilde{x}_{\mathcal{A}}^{(l)}[n,\mu]$ and $\tilde{x}_{\mathcal{B}}^{(l)}[n,\mu]$.

4.2.2 Proposed Constrained Perceptual Pressure Matching Algorithm

The previously discussed approach minimizes over the total detectability. In this section, a perceptual sound zone algorithm is proposed that introduces constraints to the problem.

The motivation for this approach is that is hypothesized that the Par distortion detectability has a consistent perceptual interpretation. As mentioned in Section 2.3, a detectability of 1 will consistently imply "just noticeable" [7].

This makes choosing constraints for detectability easier than typical non-perceptual pressure matching approaches. These approaches typically directly constrain the sound pressure energy, for which it is difficult to determine constraints [34] as the sound pressure energy does not have a consistent perceptual interpretation.

This motivates the proposal of a perceptually constrained sound pressure approach. In this approach, the reproduction error detectability will be constrained, while the leakage error detectability will be minimized in the cost function. To this end, the following optimization problem is defined:

$$\begin{aligned} \underset{\tilde{x}_{\mathcal{A}}^{(l)}[n,\mu], \tilde{x}_{\mathcal{B}}^{(l)}[n,\mu]\,\forall\,l}{\arg\min} \quad & \sum_{m \in B} \mathrm{LED}_{\mathcal{A}}^{(m)}[\mu] + \sum_{m \in A} \mathrm{LED}_{\mathcal{B}}^{(m)}[\mu] \\ \text{subject to} \quad & \mathrm{RED}_{\mathcal{A}}^{(m)}[\mu] \leq D_0 \quad \forall\, m \in A \\ & \mathrm{RED}_{\mathcal{B}}^{(m)}[\mu] \leq D_0 \quad \forall\, m \in B \end{aligned} \tag{4.24}$$

Here, D_0 is the maximum allowed detectability of the reproduction error per control point m. The intended effect of this constraint is limiting how detectable the deviation of the achieved sound pressure is from the specified target. Effectively, this allows for the controlling of the quality of the achieved sound pressure per control point m. To the knowledge of the authors, this is a novelty.

It is also hypothesized that the constraint D_0 allows for more control over the trade-off between reproduction error detectability and leakage error detectability.

It should be noted that constraining the leakage error detectability is also a valid choice. A possible optimization problem that achieves this end is defined below:

$$\underset{\tilde{x}_{\mathcal{A}}^{(l)}[n,\mu],\, \tilde{x}_{\mathcal{B}}^{(l)}[n,\mu]\, \forall\, l}{\arg\min} \quad \sum_{m \in B} \mathrm{RED}_{\mathcal{A}}^{(m)}[\mu] + \sum_{m \in A} \mathrm{RED}_{\mathcal{B}}^{(m)}[\mu]$$

$$\text{subject to} \quad \mathrm{LED}_{\mathcal{B}}^{(m)}[\mu] \leq D_0 \quad \forall\, m \in A$$

$$\mathrm{LED}_{\mathcal{A}}^{(m)}[\mu] \leq D_0 \quad \forall\, m \in B$$

(4.25)

Due to time constraints, this problem is not pursued any further. Constraining the detectability of the leakage error is however found to be promising future work.

Results and Analysis of Proposed Perceptual Sound Zone Algorithms

5

In the preceding Chapter 4, two perceptual sound zone algorithms are proposed based on the proposed perceptual sound zone framework given in Section 3.3. This chapter performs an evaluation in order to determine the benefits, if any, of including perceptual information in sound zone algorithms in the proposed fashion.

This is done by comparing the proposed algorithms with a reference algorithm.

Chapter Structure

In order to achieve this goal, this chapter is structured as follows:

- This chapter begins with a description of the evaluation methodology in Section 5.1, describing the experiments and the metrics used to evaluate them.

- Section 5.2 then presents and discusses the results of the described experiments. The proposed algorithms are compared to a reference algorithm through various perceptual metrics in order to determine their relative performance.
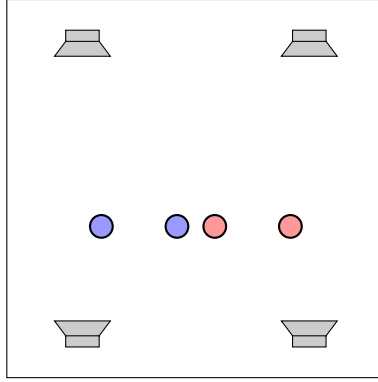
Figure 5.1: The square room setup used in the simulations for the evaluation of the algorithms. Two zones each consisting of two control points are depicted in blue and red. Four loudspeakers are placed in the corners of the room.

5.1 Evaluation Methodology

This section discusses the general approach for evaluation of the perceptual sound zone algorithms proposed in Section 4.2.1 and Section 4.2.2. All algorithms will be evaluated by means of simulation. As all derived algorithms were found to be computationally intensive, many of the parameter considerations for the simulations are chosen such that the computational load is kept feasible.

For the evaluation, the constrained and unconstrained perceptual pressure matching are compared to the short-time frequency pressure matching approach given in Section 4.1 by Equation (4.16). As all approaches are pressure matching approaches, doing so allows for a comparison between a perceptual and a non-perceptual sound zone algorithm.

Section 5.1 is structured as follows. First, Section 5.1.1 discusses and motivate the configuration used in the simulations.

5.1.1 Simulation Configuration

All sound zone algorithms are evaluated in a simulated square room of 5 by 5 meters, with a ceiling height of 3.4 meters. Two zones each consisting of two control points are in the middle of the room. In order to obtain a sufficiently challenging problem, the zones are placed in close proximity, which the two closest points 0.5 meters apart. This is important, as a trivial problem results makes it difficult to highlight the differences in performance.

The room contains 4 loudspeakers placed in the corners of the room at a height of 1.2 meters. Omnidirectional loudspeakers which radiate energy equally in all directions are used [29]. An image depicting the entire setup is given in Figure 5.1.

In order to synthesize room impulse responses used to relate the loudspeaker inputs and the resulting sound pressure in the control points, the image method [35] is used. The implementation by E. Habets of the image method is used [29].

For reasons of computational complexity, the simulated room impulse responses is limited to reverberation times of a maximum of 200 milliseconds. The definition of the reverberation time required for the intensity of reflections to reduce 60 dB relative to the direct path sound pressure [29]. To put the reverberation time used in the experiment into context, it has been found in a investigation into the reverberation time of furnished rooms that similarly sized furnished rooms have an average maximal reverberation time of 720 milliseconds [36].

To define the target sound pressure, content must be selected for the two zones. For this evaluation, speech content is used. The motivation for this is that the objective speech quality measures described in Chapter 2 are found to be more robust than the discussed general audio quality measures. In addition to this, it was found that many general audio quality measures have little to no free and openly available implementations.

The speech signals are down sampled to 8000 Hz. The motivation for this is that it is computationally intensive to run the algorithms at higher sampling rate.

In total, a dataset of 4 loudness-matched speech signals is used for evaluation. For each experiment, one speech signal is assigned to each zone. All possible combinations of the speech signals are formed, resulting in a total of 12 possible configurations.

5.1.2 Objective Measures for Result Evaluation

This section will motivate the evaluation criteria used to evaluate the results of the experiment. The perceptual measures PESQ, STOI, SIIB, and Distraction which are discussed in the the a literature review from Section 2.1 are used in the evaluation.

In addition to these perceptual measures, two traditional physical measures, namely the "mean square error" (MSE) and the "acoustic contrast" (AC) in terms of sound pressure are used in the evaluation of sound zone algorithms [?] are also used.

The motivation for including physical measures is to show in Section 5.2 that, while typically outperforming in terms of perceptual measures, the perceptual sound zone algorithms do not outperform the traditional reference sound zone algorithm.

Before discussing the evaluation measures, a brief summary is given of the outputs and inputs of the sound zone algorithm.

- **Target Sound Pressure** $t^{(m)}$:
  The desired sound pressure per control point m, defined by the speech signals corresponding to the zone $\mathcal{Z}$ the control point is in.

- **Achieved Sound Pressure** $p_{\mathcal{Z}}^{(m)}$:
  The sound pressure achieved by the algorithm at control point control point m for a zone $\mathcal{Z}$. This sound pressure has two different interpretations, depending on the control point under consideration.

  - **Achieved Bright Zone Sound Pressure for Zone** $\mathcal{Z}$:

When $m \in Z$, the achieved sound pressure represents how well the target sound pressure intended for zone $\mathcal{Z}$, namely $t^{(m)}$, is attained in control point.

- **Achieved Dark Zone Sound Pressure for Zone $\mathcal{Z}$:**
  When $m \notin Z$, the achieved sound pressure represents the sound pressure into other zones due to reproducing the bright zone in points $m \in Z$. It can be understood as the leakage.

- **Total Achieved Sound Pressure $\sum_{\mathcal{Z}} p_{\mathcal{Z}}^{(m)}$:**
  The sound pressure that results in a control point m due to contributions of all zones $\mathcal{Z}$. This represents the sound pressure that a user of the sound zone system would experience.

For more information on these quantities, the reader is referred to Section 3.1, where they are introduced. What follows is a description of various categories of evaluation measures.

5.1.2.1 Perceptual Quality Measures

One of the metrics that is used is the Perceptual Evaluation of Speech Quality (PESQ). As described in Section 2.1.1, PESQ is a metric which grades the quality of a degraded speech signal with respect to a reference speech signal. The resulting quality grade will be between 0 and 5, where 5 is the highest obtainable grade.

Another set of metrics that will be used are two speech intelligibility metrics: the Short-Time Objective Intelligibility (STOI) and the Speech Intelligibility in Bits (SIIB). STOI provides an intelligibility score between 0 and 1, 1 being the highest. SIIB instead scores the intelligibility with an information rate given in bits/s, lower-bounded by 0 bits/s. A maximum intelligibility corresponds to a rate of about 150 bits/s [17].

All perceptual measures evaluate the quality of a "degraded" input stimuli with respect to a "reference" stimuli. Using the previously introduced quantities, the measures are to evaluate sound zone performance as follows:

- **PESQ, STOI and SIIB of the Total Achieved Sound Pressure with respect to the Target Sound Pressure:**
  Corresponds to the quality / intelligibility of the achieved sound pressure, including interference, and will be referred to as the **"total PESQ"**, **"total STOI"** and the **"total SIIB"** per control point m.

- **PESQ, STOI and SIIB of the Achieved Bright Zone Sound Pressure with respect to the Target Sound Pressure:**
  Corresponds to the quality / intelligibility of the achieved sound pressure sans interference. This quantity is referred to as the **"bright zone PESQ"**, **"bright zone STOI"** and **"bright zone SIIB"** per control point m for zone $\mathcal{Z}$.

5.1.2.2 Perceptual Interference Measure

Another metric that will be used for evaluation is the Distraction model also introduced in Section 2.1.1. This model grades how distracting an interferer is in presence of target audio. The grade uses a scale from 0 to 100, where 100 is considered maximally distracting.

The distraction will be used as follows:

- **Distraction of the achieved dark Zone Sound Pressure with respect to the Achieved Bright Zone Sound Pressure:**
  This quantifies how distracting the dark zone sound pressures are when listening to the bright zone sound pressure per control point m. This will simply be referred to as the **"Distraction"** per control point m.

5.1.2.3 Physical Measures

One physical measure is the acoustic contrast (AC) between the achieved bright zone sound pressure and the achieved dark zone sound pressure can be used as a non-perceptual measure of interference. Initially introduced in Section 3.2, the acoustic contrast between two time-domain sequences $x[n] \in \mathbb{R}^N$ and $y[n] \in \mathbb{R}^N$ is given as follows:

$$\text{AC}(x,y) = \frac{||x[n]||_2^2}{||y[n]||_2^2} \tag{5.1}$$

Another non-perceptual metric that can be used to evaluate the quality of the result is the mean square error (MSE). The MSE between two time-domain sequences $x[n]$ and $y[n]$ is given as:

$$\text{MSE}(x,y) = \frac{1}{N} ||y[n] - x[n]||_2^2 \tag{5.2}$$

These physical measures are used to evaluate sound zone performance as follows:

- **MSE of the Total Achieved Sound Pressure with respect to the Target Sound Pressure:**
  Describes the MSE between the target and achieved sound pressure, and will be referred to as the **"total MSE**.

- **MSE of the Achieved Bright Zone Sound Pressure with respect to the Target Sound Pressure:**
  This is the MSE between the target and the achieved sound pressure sans interference, and will henceforth referred to as the **"bright zone MSE"** per control point m.

- **Acoustic contrast between the Achieved Bright Zone Sound Pressure and the Achieved Dark Zone Sound Pressure:**
  Quantifies the ratio of the acoustic potential energy of the bright zone and of the dark zone. From here on referred to as the **"Acoustic Contrast"** per control point m.
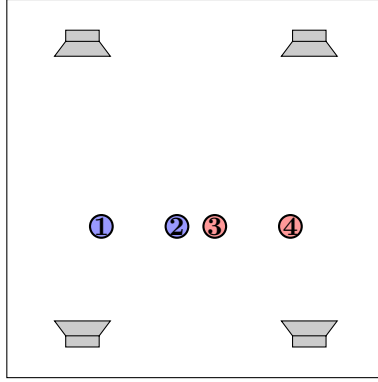
Figure 5.2: The square room used in Figure 5.1 labeled with numbers corresponding to the various control points.

5.2 Evaluation of Proposed Algorithms

In Section 4.2 two perceptual sound zone algorithms are proposed. First, an unconstrained perceptual pressure matching approach where the detectability of the sound pressure errors is minimized. Secondly, a constrained perceptual pressure matching approach which leverages the fact that the detectability has a consistent perceptual interpretation to constrain the detectability of the reproduction error.

This section will evaluate the results of the performed experiments for both proposed approaches. To this end, Table 5.1 the unconstrained perceptual pressure matching approach is evaluated, and in Section 5.2.2 the constrained perceptual pressure matching approach is evaluated.

In order to effectively describe various points in the room, a version of the room given by Figure 5.1 with the various control points numbered is given in Figure 5.2.

5.2.1 Evaluating Unconstrained Perceptual Pressure Matching

In this section, the unconstrained perceptual pressure matching algorithm will be evaluated in accordance to the approach discussed in Section 5.1. This is done by first evaluating the results of the simulations of the various setups quantitatively through the proposed measures. From this, a conclusions are drawn which are then motivated qualitatively by reasoning about algorithm behavior through waveforms.

Quantitative Analysis of Simulation Results

In order to quantify the performance of the unconstrained perceptual pressure matching approach, the various measures introduced in Section 5.1 are determined for all 12 simulations. The measures are averaged over all simulations and over all control points. For comparison purposes, the reference algorithm given in Section 4.1 is simulated in an identical fashion, and the measures are also evaluated and averaged for all simulations.

Measure		Unconstrained Perceptual	Reference
Distraction	Mean	7.83	12.69
	Standard Deviation	6.19	11.29
Acoustic Contrast	Mean	22.03	50.41
	Standard Deviation	5.96	29.02
Total PESQ	Mean	3.15	2.61
	Standard Deviation	0.27	0.28
Bright Zone PESQ	Mean	3.35	4.11
	Standard Deviation	0.29	0.17
Total STOI	Mean	0.94	0.94
	Standard Deviation	0.01	0.02
Bright Zone STOI	Mean	0.95	0.99
	Standard Deviation	0.01	0.01
Total SIIB	Mean	978.21	784.96
	Standard Deviation	69.20	164.45
Bright Zone SIIB	Mean	1125.50	1201.76
	Standard Deviation	66.18	90.99
Total MSE	Mean	0.115%	0.016%
	Standard Deviation	0.037%	0.007%
Bright Zone MSE	Mean	0.107%	0.008%
	Standard Deviation	0.036%	0.004%

Table 5.1: Summary of results for the evaluation of the unconstrained perceptual pressure matching approach, using the evaluation metrics defined in Section 5.1.

The results of this experiment is summarized in Table 5.1. This table depicts the mean and standard deviation of the measures taken over all 4 control points and all 12 experiments. As discussed in Section 2.1, it is important to note that the perceptual measures can only be used as an indication, and further listening tests must be performed to draw any real conclusions.

From the results in the table, the following observations are made and conclusions are drawn:

1. The perceptual approach outperforms the reference in all perceptual measures evaluating the total experience: total PESQ, STOI, and SIIB all attain higher values for the perceptual approach. Note that these measures evaluate the total experience, taking in consideration all sound pressure per control point including interference.

   This implies that the perceptual approach may result in an overall better perceptual experience.

2. The perceptual approach outperforms the reference in terms of the perceptual distraction measure, Implying that the interference in the perceptual approach may be less distracting

3. The reference approach outperforms the perceptual approach in perceptual measures evaluating the bright-zone quantities: bright zone PESQ, STOI and SIIB are all higher for the reference approach. Note that these measures are sans interference: they only evaluate how well the achieved sound pressure attains the target, ignoring interference.

This implies that, disregarding interference, the reference approximates the target more effectively perceptually speaking. However, in Item 1 it is concluded that adding the interferer results in a lower overall experience.

This implies that, although the reference algorithm approximates the target better perceptually, the interference that it introduces a sufficient disturbance to be outperformed by the reference.

4. The reference approach outperforms the perceptual approach for all physical measures: total and bright zone MSE and acoustic contrast. This is to be expected, as the reference approach optimizes the MSE directly.

    Interestingly, although the total MSE is lower, the reference is outperformed in terms of all total perceptual measures, as discussed in Item 1.

    In addition to this, the acoustic contrast between intended and interfering sound pressure for the reference approach is over twice as large than the perceptual approach. Nevertheless, the perceptual approach less distracting according to the distraction model as discussed in Item 2.

    These results imply that MSE or AC may not be optimal measures to use in the evaluation of the perceptual experience of sound zones.

In summary, from the observations above it is concluded that the perceptual sound zone algorithm may outperform the reference sound zone algorithm in terms of perceptual experience. This seems to be due to the perceptually disturbing interference introduced by the reference algorithm, as, when disregarding noise, the reference algorithm has a better reproduction of the target perceptually speaking. The distraction ratings also indicate that the noise introduced by the reference algorithm is more distracting.

In addition to this, these results also imply that the physical metrics MSE and AC do not correlate well with the perceptual measures, as the physical metrics predict the reference to outperform the perceptual approach.

Analyzing Algorithm Behavior

In the preceding section the reference and perceptual algorithms are compared quantitatively. Results indicate that the perceptual algorithm outperforms the reference in terms of the total experience. This section considers the behavior of the algorithm in an attempt to explain these results.

To this end, consider Figure 5.3. This figure contains a plot of the wave forms of the target sound pressure and achieved dark-zone sound pressure for both the perceptual and non-perceptual variants of pressure matching for control point $m = 2$ (see: Figure 5.2) from the experiments.

The selected control point is in zone $\mathcal{A}$. As explained in Section 5.1, the achieved dark-zone sound pressure can be understood as the interference due to the other zone, zone $\mathcal{B}$.
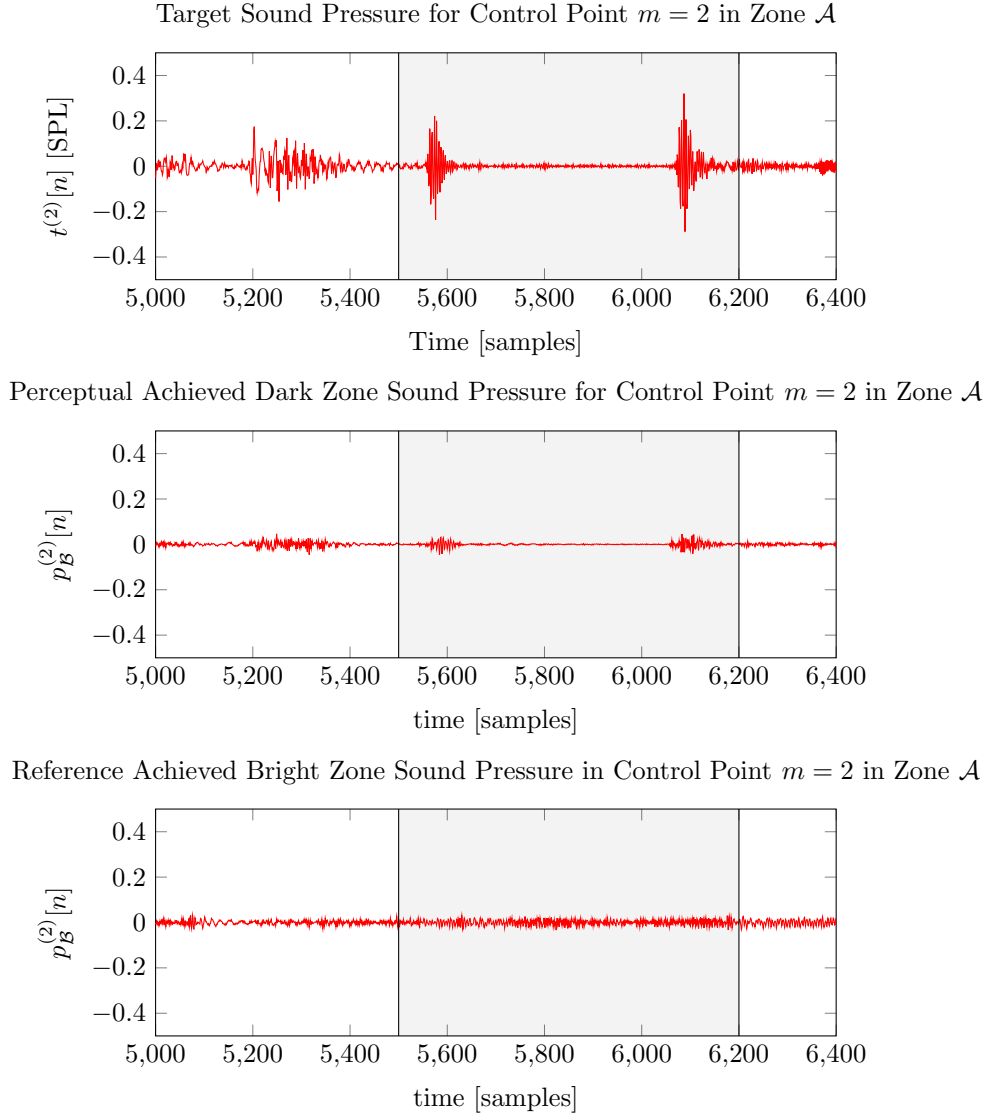
Figure 5.3: Depiction of the wave forms of the target sound pressure and achieved dark zone sound pressure for the unconstrained perceptual pressure matching approach and the reference pressure matching approach.

Consider the highlighted region for the highlighted region for the perceptual algorithm. From this it can be seen that the magnitude of the interference is correlated to the magnitude of the target sound pressure for zone $\mathcal{A}$. Contrast this to the highlighted region for the reference algorithm, where the interference is a at a relatively constant level.

This may explain why, while having lower overall contrast, the perceptual approach outperforms the reference approach in terms of distraction and overall perceptual experience. When determining the interference for control point $m = 2$, the perceptual algorithm takes the target sound pressure for $\mathcal{A}$ into account. Effectively, when the target sound pressure is relatively loud, more interference is allowed as it is masked to a degree by the target sound pressure.

In doing so, the interference is less perceptually disturbing, which serves as a possible explanation to the results given in Table 5.1

5.2.2 Evaluating Constrained Perceptual Pressure Matching

Quantifying Algorithm Performance

Analyzing Algorithm Behavior

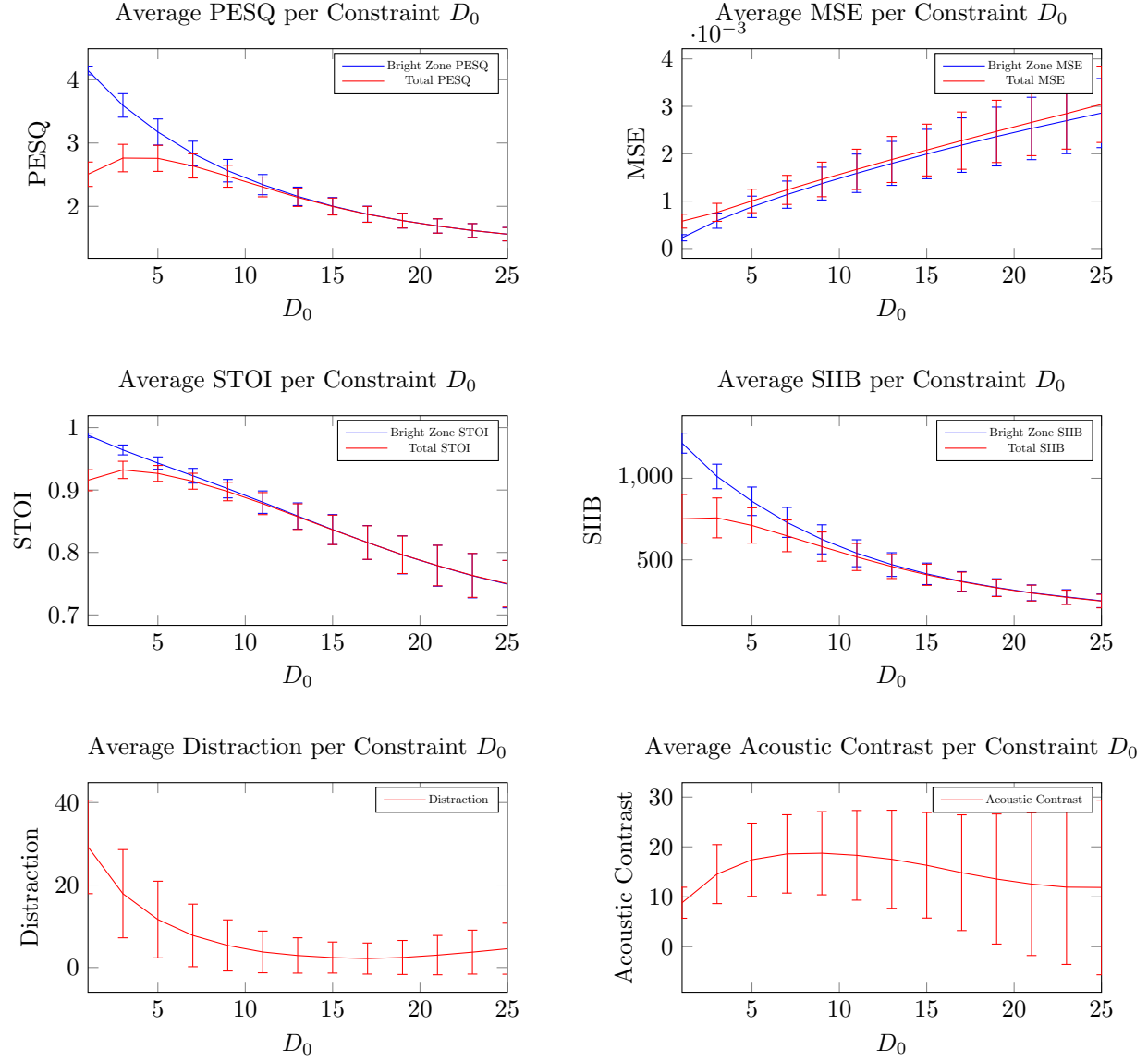Figure 5.4: Plots depicting stuff.
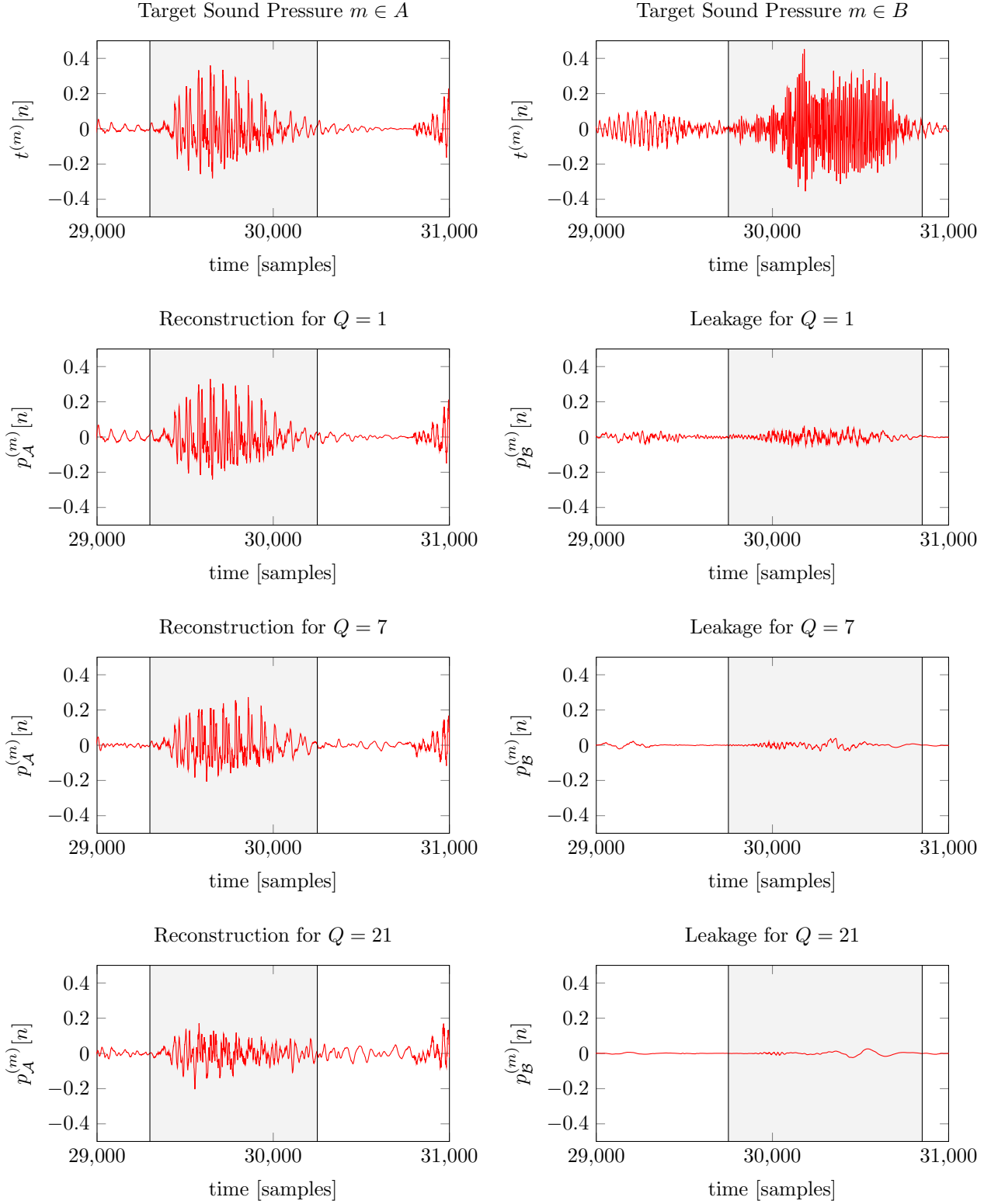
Figure 5.5: Plots depicting stuff.

Conclusion

<div align="right" style="font-size:3em">**6**</div>

This work seeks to explore the implementation and possible benefits of including perceptual information in sound zone algorithms. To this end, the introduction posed two research questions.

"How can auditory perceptual models be included in sound zone algorithms?"

In this work it is shown that a perceptual sound zone algorithm can be stated directly using perceptual models. Specifically, the Par detectability distortion can be used to find a perceptual variant of the popular pressure-matching approach from literature. This results in a sound zone algorithm that minimizes a cost function with a perceptual interpretation.

"What are the benefits of including auditory perceptual models in sound zone algorithms?"

Current findings suggest that the benefits of including perceptual models are twofold:

- The perceptual sound zone algorithm outperforms the reference

- The use of perceptual constraints.

The following work is found to be promising:

- Reformulating the short-time frequency-domain pressure-matching defined in Section 4.1 such that it allows for optimization over the frequency-domain representation of the loudspeaker input signals while limiting or entirely preventing the resulting artifacts in the resulting time-domain sequence.

  This is found to be promising as the resulting computational complexity of the algorithm could potentially be much lower due to the removal of the transformation between the time-domain and frequency-domain representation of the loudspeaker input signals.

- Reformulating the constrained perceptual pressure matching algorithm given in Section 4.2.2 to instead constrain the leakage error detectability.

  This approach is promising as it allows for constraints to be placed on the detectability sound zone leakage which have a consistent interpretation. This approach can then be compared to traditional pressure matching approaches which often also constrain sound zone leakage [1].

- Evaluation of the performance difference between perceptual and non-perceptual sound zone approaches. In Section 5.2, the perceptual sound zone

algorithms are shown to outperform the traditional non-perceptual reference in terms of various objective measures which are introduced in Section 2.1.

However, as discussed in Section 2.1, these objective measures can only be used to give an indication of performance. To objectively determine if the perceptual approach does indeed outperform traditional approaches, a listening test must be conducted.

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Calibration of the the Par Detectability Measure

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A calibration is necessary for the Par detectability to provide the correct output. In the previous section, it was shown that the constants C_a and C_s are used to this end. A correct calibration of the Par detectability must satisfy the following:

1. The just noticeable disturbance signal must result in a detectability of 1.

2. The threshold of hearing takes effect appropriately.

Both the concepts of just noticeable distortion and the threshold of hearing require knowledge of the sound pressure level of the stimuli. Thus, before determining the calibration coefficients, it is important to first discuss the relationship between the input signals $x[n]$ and $\varepsilon[n]$ and reproduced sound pressure level. This is the topic of Appendix A.1. Afterwards, Appendix A.2 discusses the determination of the coefficients C_a and C_s.

A.1 Relating Digital Representation and Sound Pressure Level

One difficulty of taking the threshold of hearing into account is that it is typically given in terms of sound pressure level (SPL), measured in dB. The one-sided spectrum of the threshold of hearing in dB SPL can be approximated by the following function [6]:

$$T_q(f) = 3.64 \left(\frac{f}{1000} \right)^{-0.8} + 0.001 \left(\frac{f}{1000} \right)^4 - 6.5 \exp\left[-0.6 \left(\frac{f}{1000} - 3.3 \right)^2 \right] \quad \text{(A.1)}$$

The signals $x[n]$ and $\varepsilon[n]$ are however given digital representation of audio.

For example, they might be given in a pulse code modulated (PCM) format within which they attain integer values between -32768 and 32767.

As such, to meaningfully integrate the threshold of quiet, the digital representation and the sound pressure levels must be related. This relationship can be modeled as follows:

$$X_{\mathrm{dB}}(f) = 10 \log_{10}(|X(f)|^2) + O_{\mathrm{dB}} \quad \text{(A.2)}$$

Here, $X_{\mathrm{dB}}(f)$ is the dB SPL representation of a given spectrum $X(f)$. Furthermore, O_{dB} is an offset to ensure the digital representation corresponds to the correct sound pressure level. In order to use this relationship to determine the appropriate digital equivalent of the threshold in quiet, a definition of the offset O_{dB} must be determined.

One way of determining the offset O_{dB} is by relating the sound pressure level and the digital representation of a full-scale sinusoid. A full-scale sinusoid is a sinusoid that has an amplitude of the maximum value that can be attained in the digital representation.

In our previous example, one way of doing so would be to state that a full-scale sinusoid with amplitude 32767 corresponds to e.g. a sound pressure level of 100 dB SPL. The interpretation of this is that playing a full-scale sinusoid will result in a sound pressure of 100 dB SPL when played from the sound system.

To do so, let the digital representation of the full-scale sinusoid be modeled by a sinusoid with amplitude A and frequency f_0. Consider the one-sided fourier representation of the digital representation of this full-scale sinusoid:

$$\mathcal{F}\left\{A\cos\left(2\pi f_0 t\right)\right\} = A\delta\left(f - f_0\right) \tag{A.3}$$

It is assumed that playing the digital representation of this sinusoid results in a sound pressure level of A_{dB} dB SPL. Substituting these definitions into Equation (A.2) results in the following definition for O_{dB}:

$$O_{dB} = 10\log_{10}\left(|A|^2\right) - A_{dB} \tag{A.4}$$

The offset fully defines the relationship between digital representation and sound pressure level, and allows for the conversion of the threshold of hearing to digital representation.

A.2 Determing Calibration Constants

There are various ways of calibrating this model, but this section will discuss the method of calibrating that is given in the original paper [7]. The given approach is to find the two unknowns C_a and C_s by solving a system of two equations that model the previously stated calibration requirements.

The first requirement is that a just noticeable disturbance signal must result in a detectability of 1. From perceptual literature it is known that a sinusoidal disturbance signal at a given frequency f_0 is just noticeable in presence of an in-phase sinusoidal masking signal that is 18 dB SPL louder [7]. To model this, consider the following masking and disturbance signals.

$$x_{JND}[n] = A_{70}\cos\left(2\pi f_0 n/f_s\right) \tag{A.5}$$
$$\varepsilon_{JND}[n] = A_{52}\cos\left(2\pi f_0 n/f_s\right) \tag{A.6}$$

Here, $x_{JND}[n]$ is a sinusoid with an amplitude A_{70}, which corresponds to 70 dB SPL. Furthermore, $\varepsilon_{JND}[n]$ is a sinusoid with an amplitude A_{52}, which is 18 dB SPL less. Note that the amplitudes are both given in digital representation, not sound pressure level representation. The digital representation amplitudes are found through Equation (A.2).

Thus, $\varepsilon_{JND}[n]$ must be just noticeable in presence of $x_{JND}[n]$. This can be expressed as follows:

$$D(x_{JND}[n], \varepsilon_{JND}[n]) = 1 \tag{A.7}$$

This expression forms the first equation in the system of equations that can be solved to calibrate the Par detectability.

The second requirement is that the threshold of hearing must be included correctly. The threshold of hearing defines the sound pressure levels that are the verge between audible and inaudible sound as a function of frequency. To this end, consider the following masking and disturbance signals:

$$x_{\mathrm{THR}}[n] = 0 \tag{A.8}$$

$$\varepsilon_{\mathrm{THR}}[n] = A_{\mathrm{tq}} \cos\left(2\pi f_0 n / f_s\right) \tag{A.9}$$

Here the masking signal $x_{\mathrm{THR}}[n]$ is zero. The disturbance signal is a sinusoid of frequency f_0 with amplitude A_{tq}, which is chosen such that it attains the threshold of quiet at f_0, i.e. $T_q(f_0)$.

As the threshold of quiet is the verge between audible and inaudible sound, it is assumed that a disturbance signal in presence of no masking signal that has an amplitude equal to the threshold of quiet is just noticeable. Recall that for just noticeable stimuli the detectability must be equal to 1. This allows us to specify the second equation in the system of equations:

$$D(0, \varepsilon_{\mathrm{THR}}[n]) = 1 \tag{A.10}$$

The system of equations defined by Equation (A.7) and Equation (A.10) can be solved through the bisection method. To see how this is done, the reader is referred to the original paper [7].