PSF-corrected aperture photometry from multi-band / multi-survey data: deconvolve the photometry aperture by the reflected PSF

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Galaxy aperture photometry can have multiple goals. Here we focus on the problem of multi-band aperture photometry, where the challenge is to compensate for PSF differences between the different images of the galaxy.

Note the important distinction between the – quite separate – problems of obtaining colours (typically for photometric redshifts), and obtaining total fluxes. Obviously total fluxes in different bands can be combined to make colours for photometric redshifts, but photo-z can just as well be obtained from a smaller aperture on the galaxy, provided the aperture photometry measurements capture the same part of the galaxy in every band. Such an approach may well be more optimal, if the colours of the brighter parts of a galaxy can be meaured with greater SNR, and the central regions are redder and more 'photo-z friendly'.

Aperture photometry of a galaxy whose observed image is $O(x, y)$ can be written generally as

$$F_A = \int\int \mathrm{d}x\mathrm{d}y\, O(x, y)W_A(x, y) \tag{1}$$

where W_A is the aperture weight function (e.g. a top hat, or a Gaussian tapered weight). Only if $W_A = 1$ wherever $O > 0$ will F_A be equal to the total flux.

$O(x, y)$ is the convolution of the PSF and the true, pre-seeing image I:

$$O(x, y) = \int\int \mathrm{d}x'\mathrm{d}y' P(x - x', y - y')I(x', y') \equiv P \otimes I, \tag{2}$$

so we can write the aperture flux F_A in terms of the true image I, the aperture function W_A and the PSF P:

$$F_A = \int\int \mathrm{d}x\mathrm{d}y \int\int \mathrm{d}x'\mathrm{d}y' P(x - x', y - y')I(x', y')W_A(x, y). \tag{3}$$

Swapping $x \leftrightarrow x',\ y \leftrightarrow y'$ gives

$$F_A = \int\int \mathrm{d}x\mathrm{d}y\, I(x, y)\tilde{W}_A^P(x, y), \tag{4}$$

where

$$\tilde{W}_A^P(x, y) = \int\int \mathrm{d}x'\mathrm{d}y' P(x' - x, y' - y)W_A(x', y') \equiv \overline{P} \otimes W_A \tag{5}$$

where $\overline{P}(x, y) = P(-x, -y)$ is the point-reflected PSF. This proves that **the aperture flux measured with W_A on the observed image O (eq. 1) is equal to the aperture flux measured with the reflected PSF-convolved filter $\tilde{W}_A^P$ on the pre-seeing image I (eq. 4).**

This then provides the route to multiband photometry with exactly seeing-compensated apertures, by inverting the convolution in eq. 5:

1. Choose a pre-seeing aperture function $\tilde{W}_A$, to be applied to all bands (ensuring consistent aperture photometry)
2. For each image i to be photometered, calculate the post-seeing aperture function W_A^i as the *de*-convolution of $\tilde{W}_A$ by the reflected PSF $\overline{P}_i$ of that image, $W_A^i = \tilde{W}_A \otimes^{-1} \overline{P}_i$.
3. Calculate the aperture flux for each band using this deconvolved aperture function W_A^i.

In a situation where different images come from different surveys or data sets, the only external information that is required by each survey is the pre-seeing aperture function $\tilde{W}_A$ (as a function of sky coordinates). The calculation of F_A then further only requires knowledge of the observed image and its PSF. The only caveat is that this procedure will only work if the deconvolution $\tilde{W}_A \to W_A^i$ is possible and regular, i.e., that the pre-seeing aperture is not narrower than the PSF of any of the data being combined. It will also not work well with aperture functions with high frequency features such as the sharp edge of a top hat.

In KiDS we use a procedure such as this, GAaP, which operates on Gaussianised-PSF images and uses elliptical Gaussian aperture functions (Kuijken et al., 2015). This makes all (de)convolutions analytic. The Gaussianisation step requires large-image convolutions with variable kernels, and a large volume of intermediate image data products. But as the above shows there is no mathematical reason *per se* to include a Gaussianisation step. An advantage of the more direct formulation here is that the error analysis is simple, simply propagating the pixel noise through the aperture weights.

References

Kuijken, K., Heymans, C., Hildebrandt, H., et al. 2015, MNRAS, 454, 3500