Max Planck Institute
for Astrophysics

The diffuse Milky Way – Sharpening the picture with new inference techniques

Niels Oppermann

with
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MPA seminar, Garching, 2013-07-01

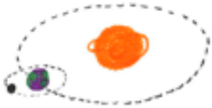
Galileo Galilei:
“The Milky Way is nothing else but a mass of innumerable stars planted together in clusters.”

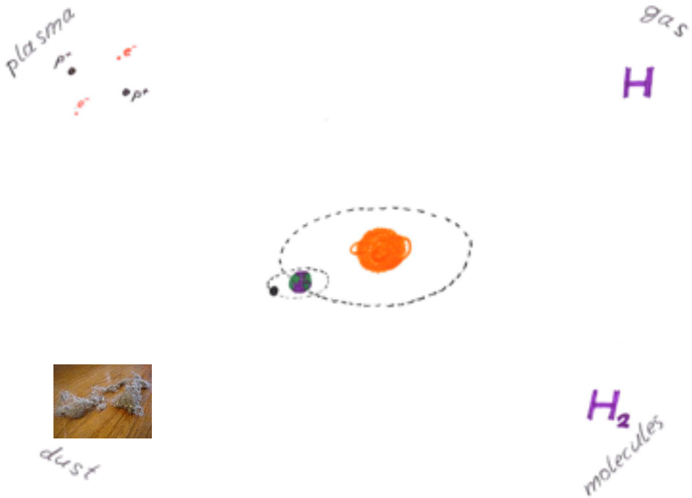
Image credit: B. Gilli/ESO

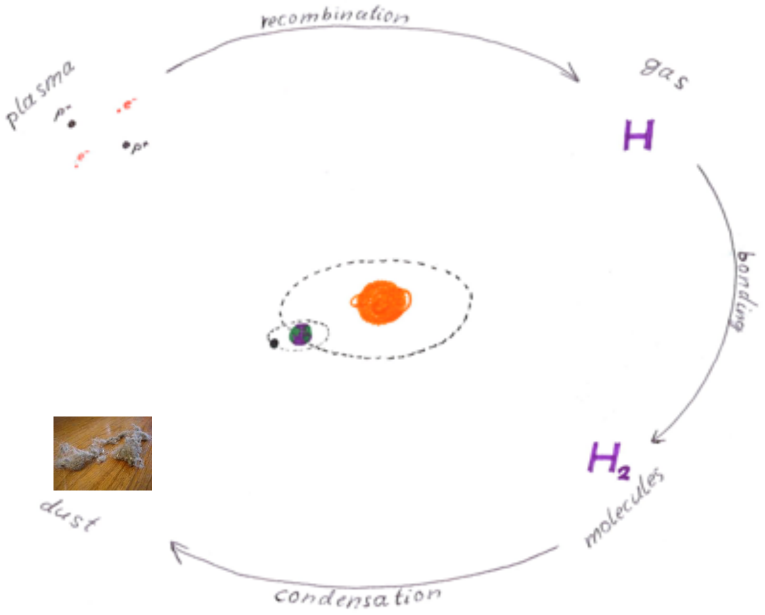
Galileo Galilei:
“The Milky Way is nothing else but a mass of innumerable stars planted together in clusters.”

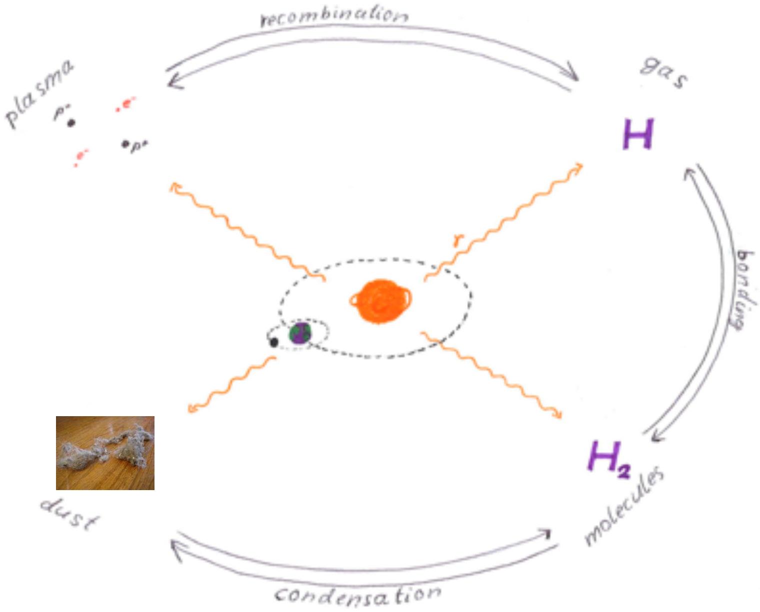
Nothing else?

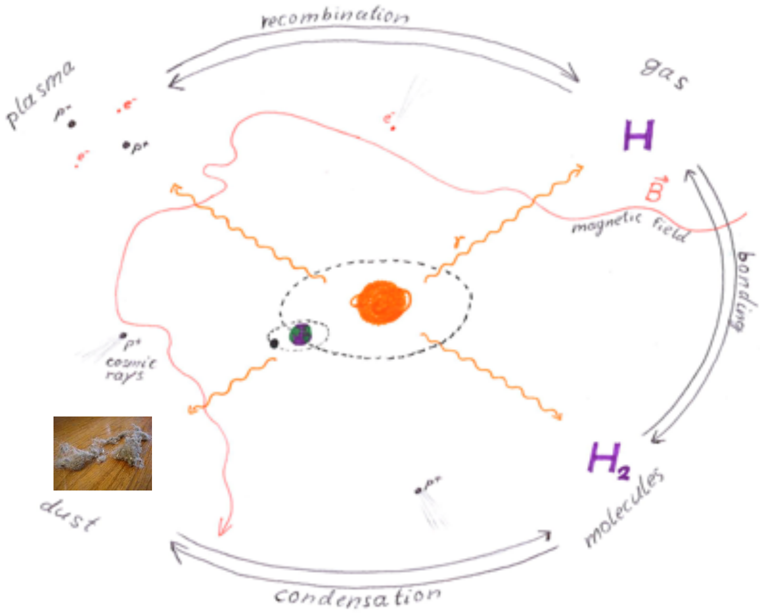
Image credit: B. Gilli/ESO

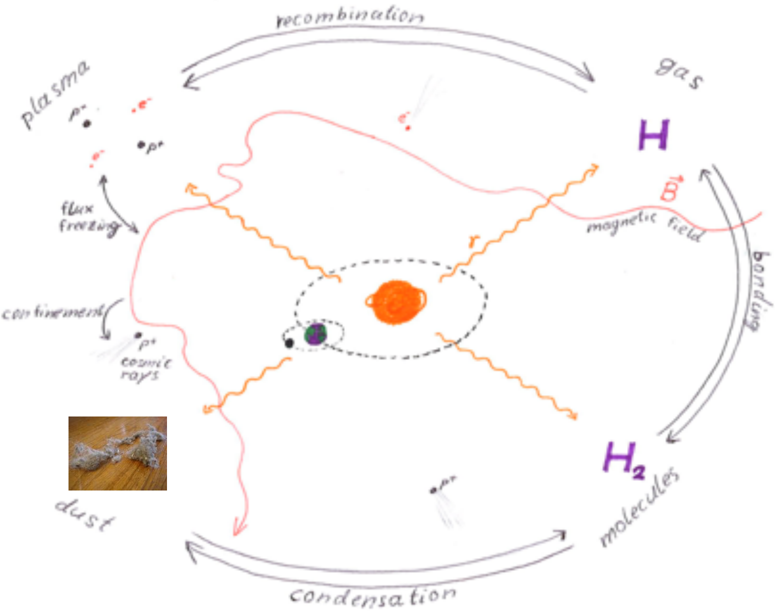



plasma
p+
e-
e-
p+
gas
H
dust
H2
molecules


recombination
plasma
gas
H
bonding
H2
molecules
condensation
dust


recombination
plasma
gas
H
bonding
H_2
molecules
dust
condensation


recombination
plasma
gas
H
$\vec{B}$
magnetic field
bonding
H_2
molecules
condensation
dust
p^+
e^-
γ
cosmic rays



recombination
plasma
gas
H
flux freezing
magnetic field
bonding
confinement
cosmic rays
H2
molecules
dust
condensation

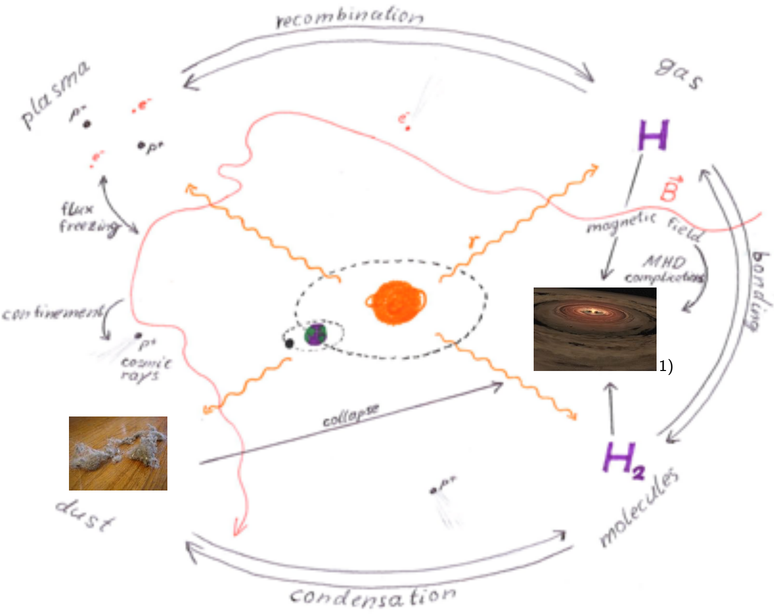
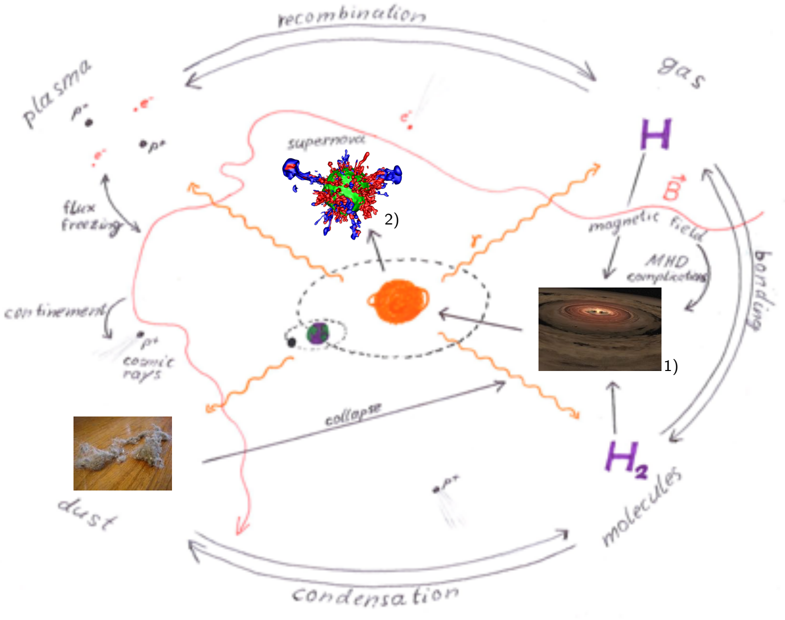


Image credits: 1) D. Darling



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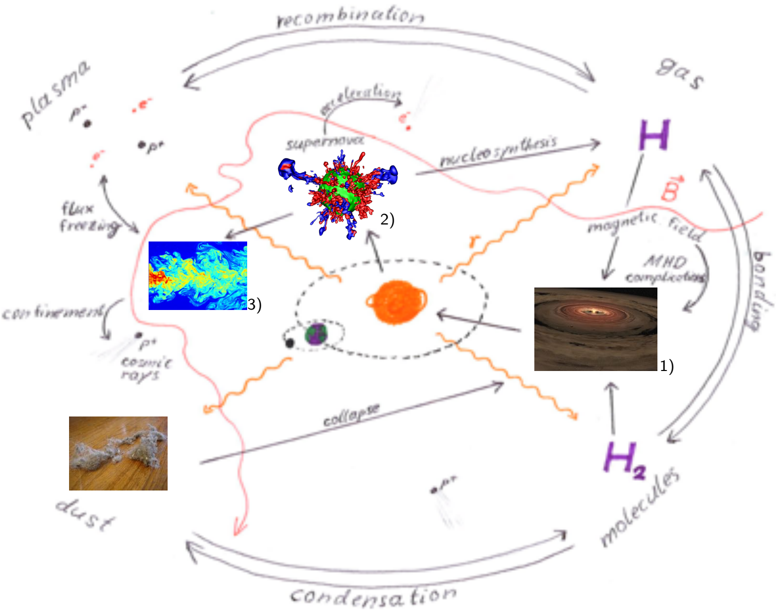

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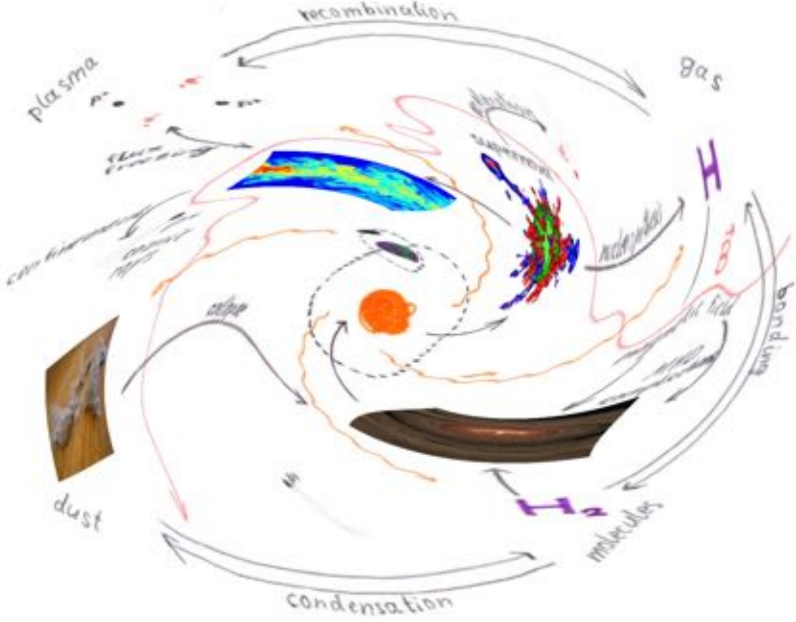

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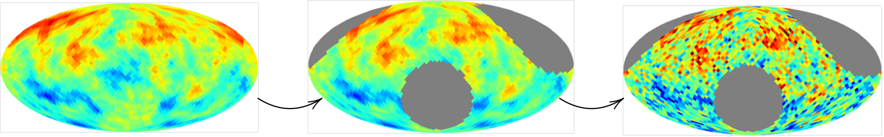
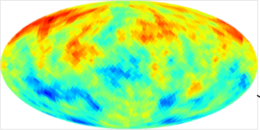
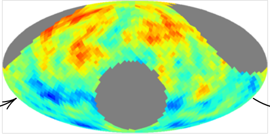
Signal inference

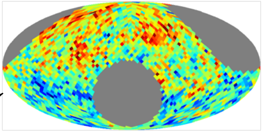


signal s | signal response $R(s)$ | data d

Signal inference



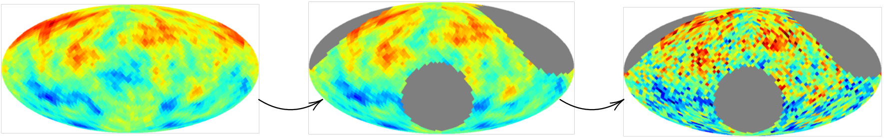




signal s | signal response $R(s)$ | data d

???

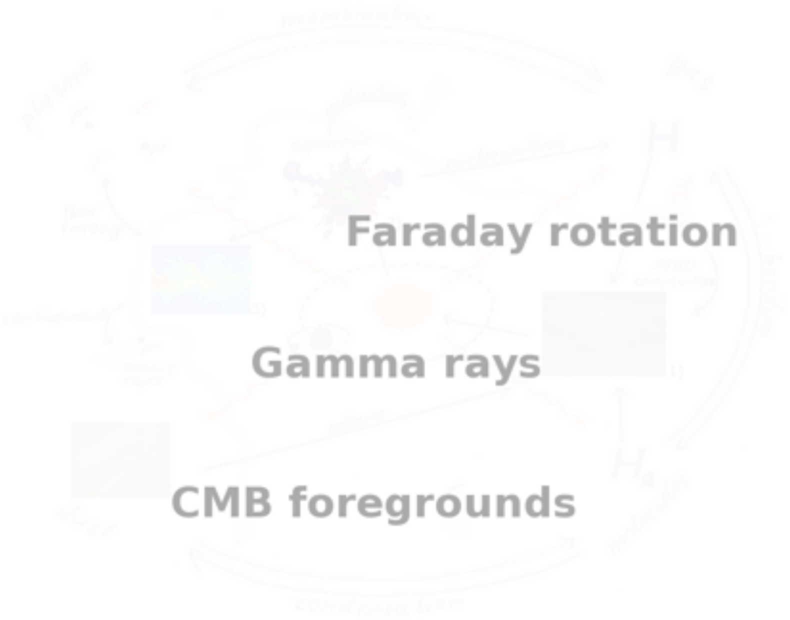
Signal inference



signal s | signal response $R(s)$ | data d

???

$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\,\mathcal{P}(s)}{\mathcal{P}(d)}$$



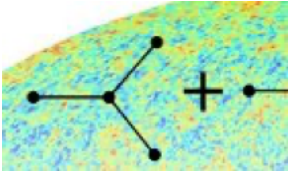
Faraday rotation

Gamma rays

CMB foregrounds

Image credits: 1) D. Darling; 2) N.J. Hammer/MPA; 3) C. Fukushima/TUDelft

Information Field Theory



Numerical IFT for python

NIFTY

http://www.mpa-garching.mpg.de/ift/

http://www.mpa-garching.mpg.de/ift/nifty/

Faraday rotation

Gamma rays

CMB foregrounds

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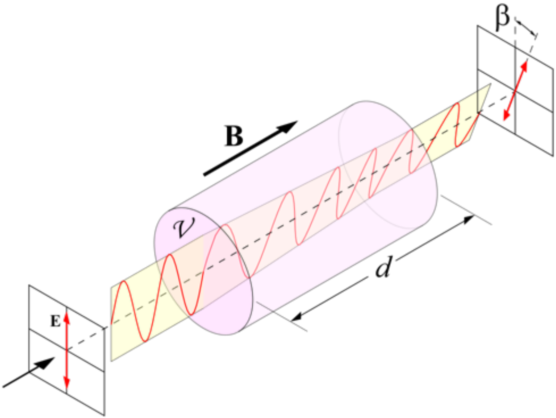
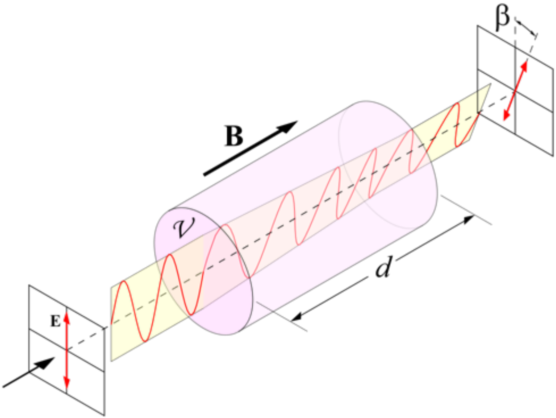


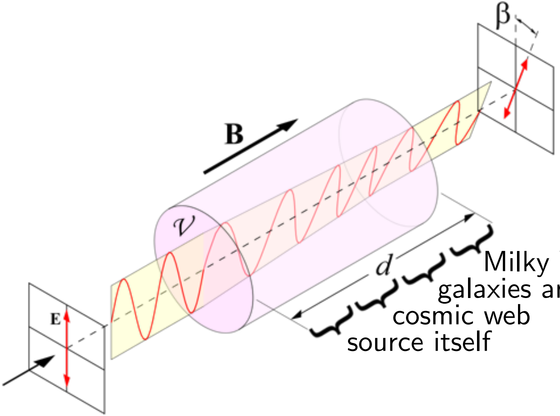
$$\mathrm{d}\beta \propto \lambda^2 n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

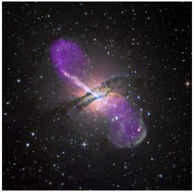
$$\Rightarrow \quad \beta \propto \lambda^2 \int_{r_{\mathrm{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$



$$\text{Faraday depth: } \phi \propto \int_{r_{\mathrm{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

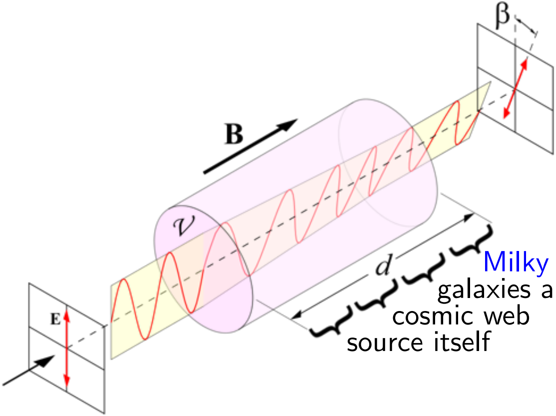
$$\beta = \phi \lambda^2$$

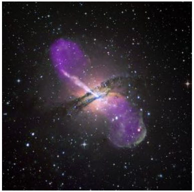




Faraday depth: $\phi \propto \int_{r_{\mathrm{source}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$

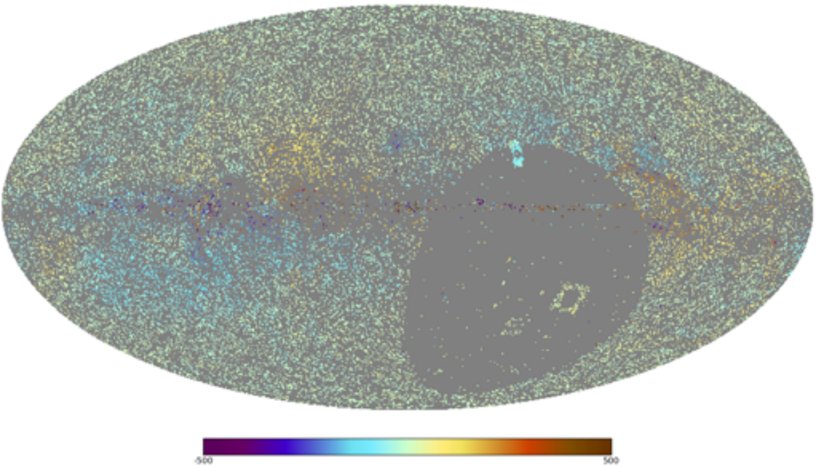
$$\beta = \phi \lambda^2$$



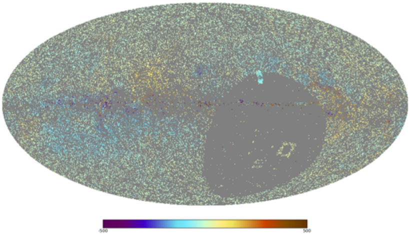


Galactic Faraday depth:

$$\phi \propto \int_{r_{\mathrm{Milky Way}}}^{0} n_{\mathrm{e}}(\vec{x}) B_r(\vec{x}) \mathrm{d}r$$

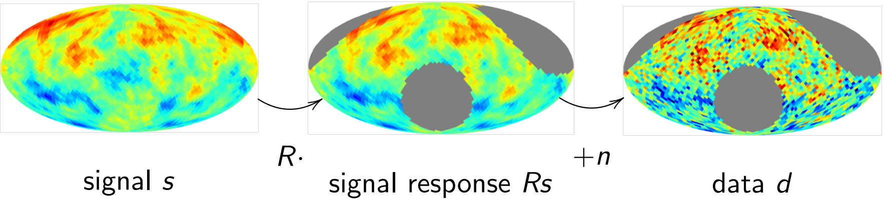


41 330 data points

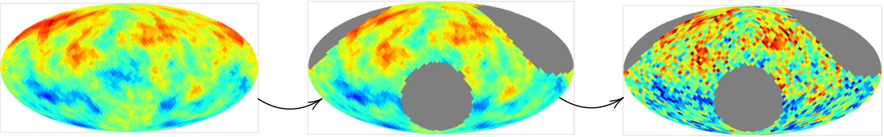


Challenges

- Regions without data
- Uncertain error bars:
  - complicated observations
  - $n\pi$-ambiguity
  - extragalactic contributions unknown



$$\boxed{d = Rs + n}$$



$R\cdot$ $\quad$ $+n$

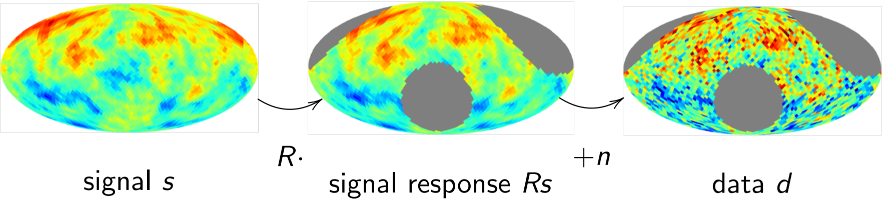
signal s $\qquad$ signal response Rs $\qquad$ data d

$$\boxed{d = Rs + n}$$

$$\mathcal{P}(s) = \mathcal{G}(s, S)$$

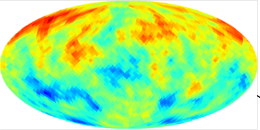
$$\mathcal{P}(n) = \mathcal{G}(n, N)$$

$$\mathcal{G}(s, S) = \frac{1}{|2\pi S|^{1/2}} \exp\left[\frac{1}{2} s^\dagger S^{-1} s\right]$$



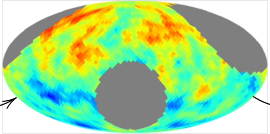
$$d = Rs + n$$

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$



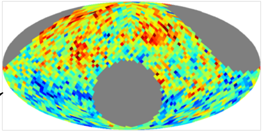
signal s

$R\cdot$



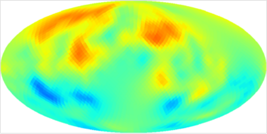
signal response Rs

$+n$



data d

$\downarrow \; DR^{\dagger}N^{-1}\cdot$



Wiener Filter

$$d = Rs + n$$

$$m = \int \mathcal{D}s \; s \; \mathcal{P}(s|d)$$

$$m = Dj, \;\; \text{where} \quad \begin{array}{l} j = R^{\dagger}N^{-1}d \\ D = \left(S^{-1} + R^{\dagger}N^{-1}R\right)^{-1} \end{array}$$

$$S(\hat{n}, \hat{n}') = \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s)$$

$$\Rightarrow S_{(\ell m),(\ell' m')} = \int \mathcal{D}s \; s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s)$$

$$\begin{aligned}
S(\hat{n}, \hat{n}') &= \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s) \\
&= S(\hat{n} \cdot \hat{n}') \\
\Rightarrow S_{(\ell m),(\ell' m')} &= \int \mathcal{D}s \; s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s) \\
&= \delta_{\ell\ell'} \delta_{mm'} C_\ell
\end{aligned}$$

$\hookrightarrow$ angular power spectrum

$$
\begin{aligned}
S(\hat{n}, \hat{n}') &= \int \mathcal{D}s \; s(\hat{n}) s(\hat{n}') \mathcal{P}(s) \\
&= S(\hat{n} \cdot \hat{n}') \\
\Rightarrow S_{(\ell m),(\ell' m')} &= \int \mathcal{D}s \; s_{\ell m} s^*_{\ell' m'} \mathcal{P}(s) \\
&= \delta_{\ell \ell'} \delta_{m m'} C_\ell
\end{aligned}
$$

$\hookrightarrow$ angular power spectrum

$$
N_{ij} = \delta_{ij} \sigma_i^2 \eta_i
$$

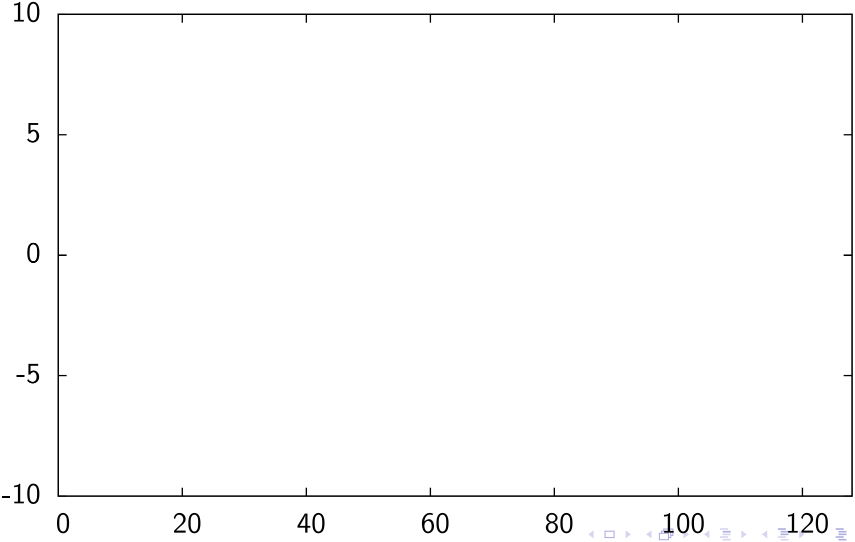
$\hookrightarrow$ error bar correction factors

(uncorrelated noise)

1D example

Assumptions:

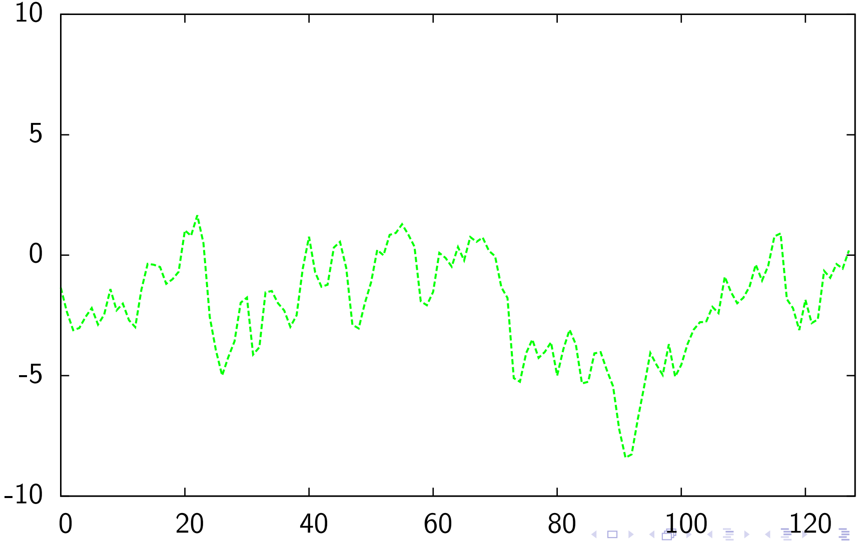
-
-



1D example

Assumptions:

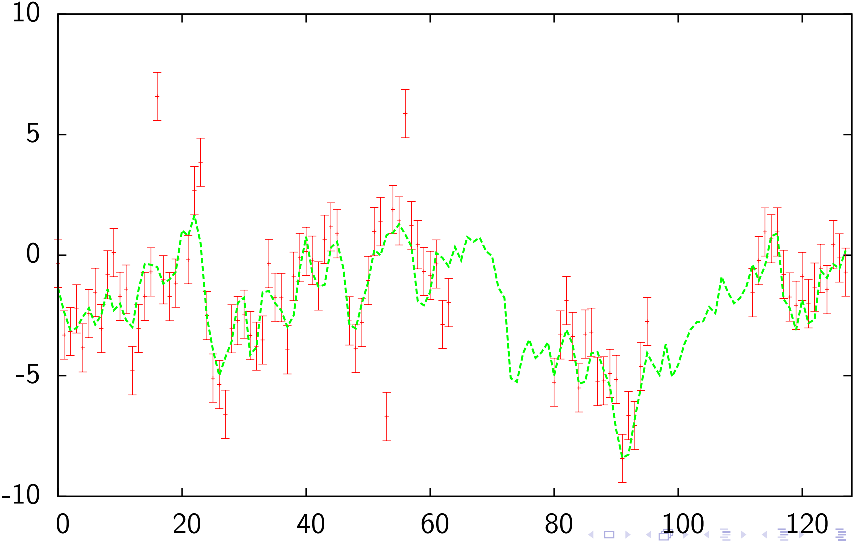
- signal field statistically homogeneous Gaussian random field
-



1D example

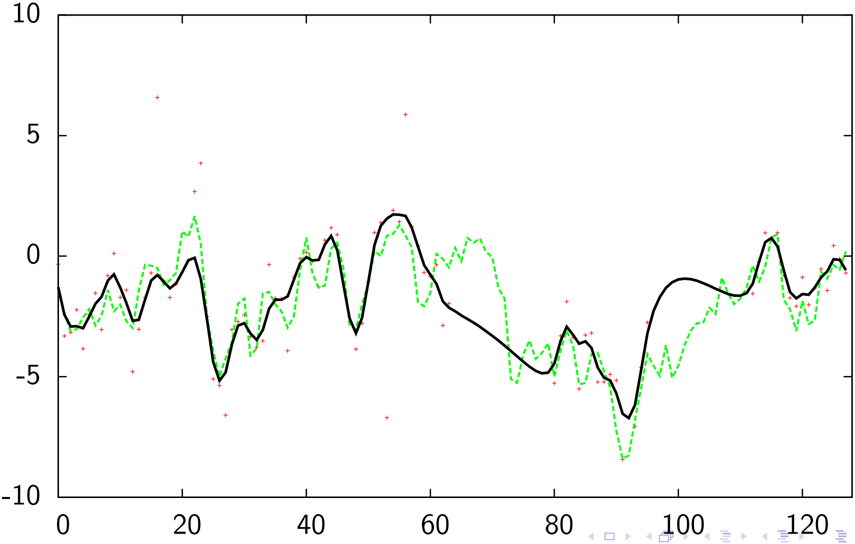
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



1D example

- Reconstruct (iteratively):
  signal, power spectrum, noise variance



1D example

- Reconstruct (iteratively):
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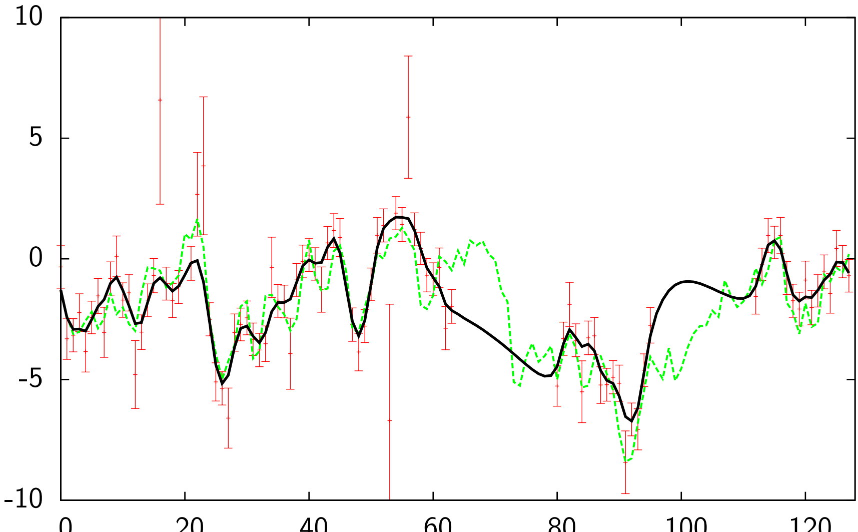
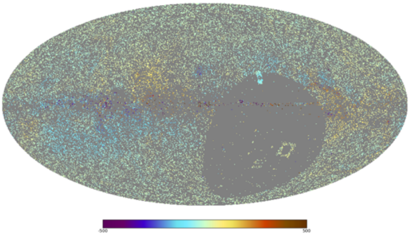
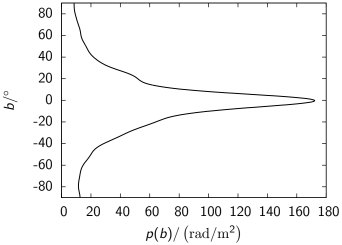
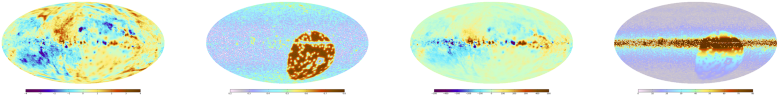


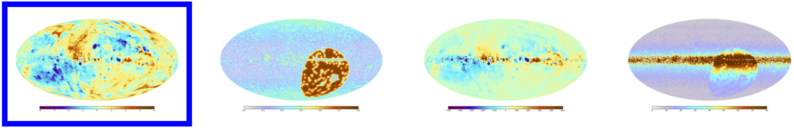




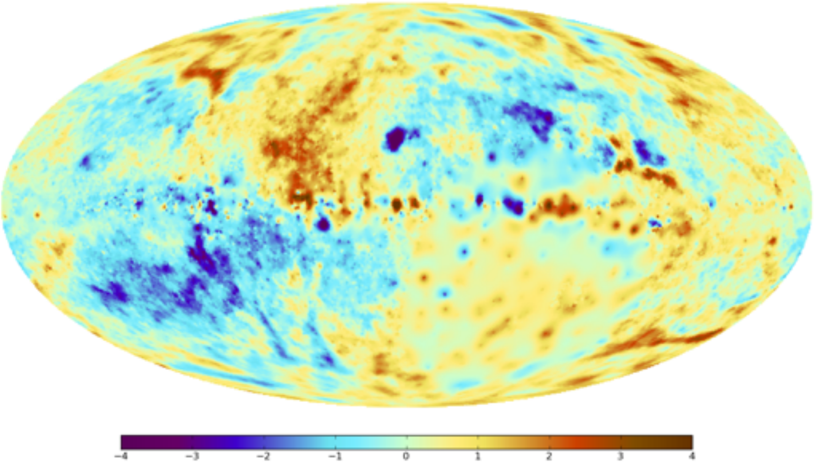
- Approximate $s(b,l) := \frac{\phi(b,l)}{p(b)}$ as a statistically isotropic Gaussian field
- R: multiplication with $p(b)$ and projection on directions of sources
- $N_{ij} = \delta_{ij}\eta_i\sigma_i^2$



Oppermann et al. (2012)

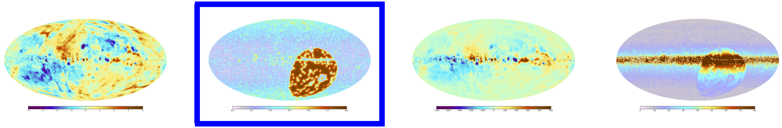


posterior mean of the signal

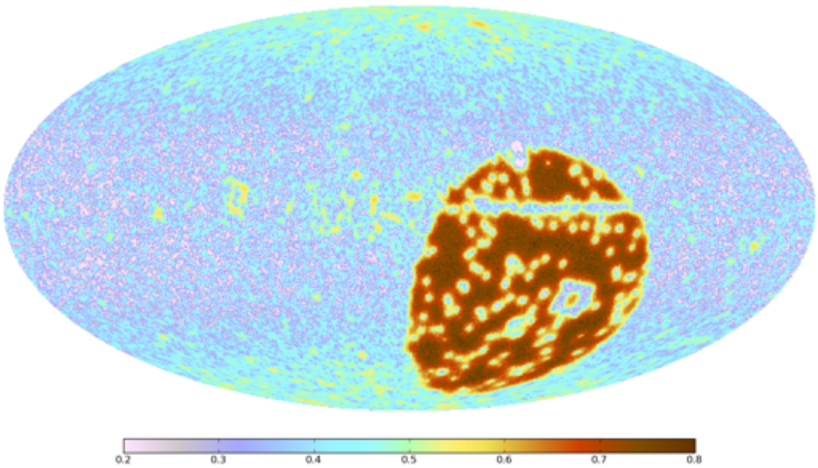


Oppermann et al. (2012)

m

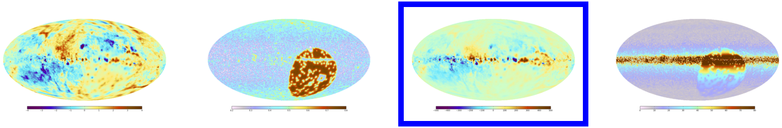


uncertainty of the signal map

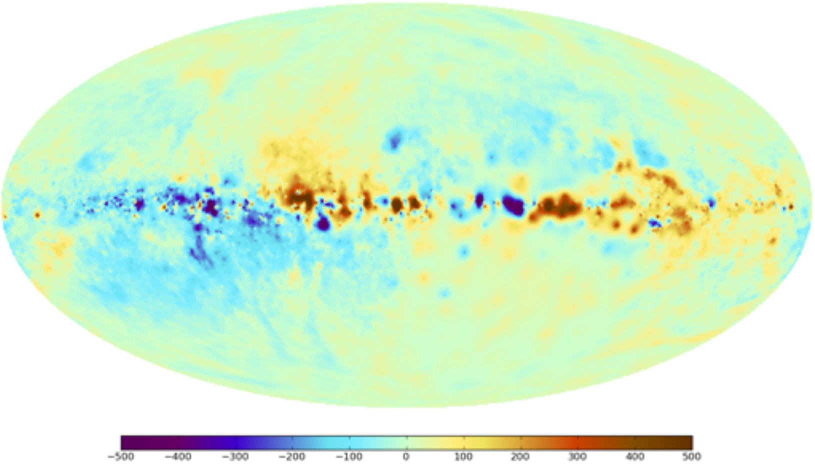


Oppermann et al. (2012)

$$\sqrt{\mathrm{diag}\,(D)}$$

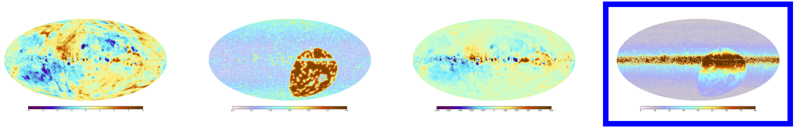


posterior mean of the Faraday depth

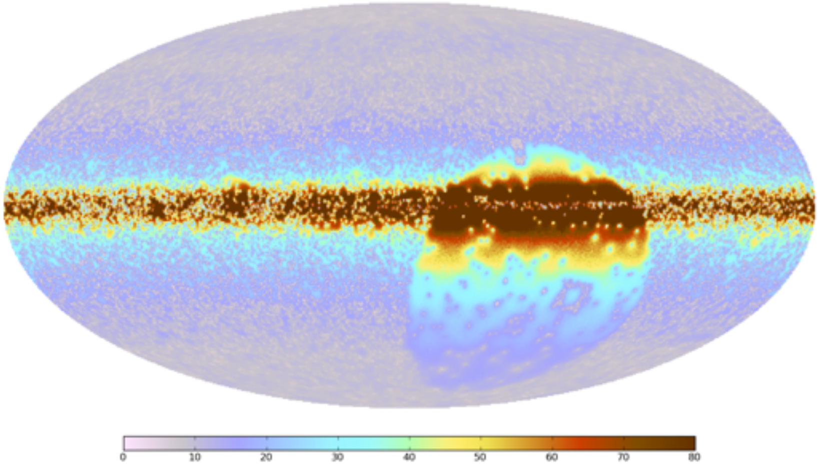


Oppermann et al. (2012)

pm

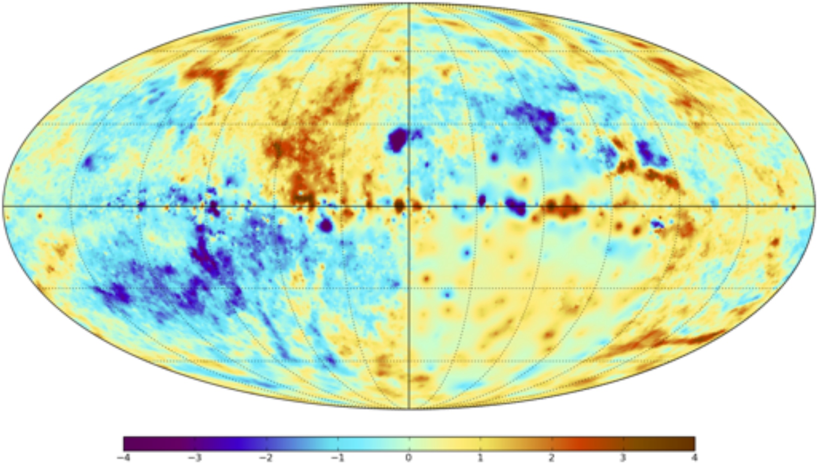


uncertainty of the Faraday depth

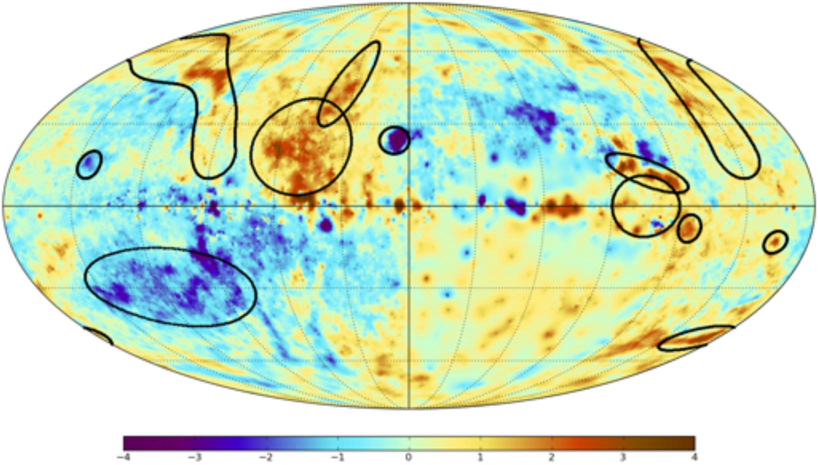


Oppermann et al. (2012)

$$p\sqrt{\mathrm{diag}\,(D)}$$



Oppermann et al. (2012)



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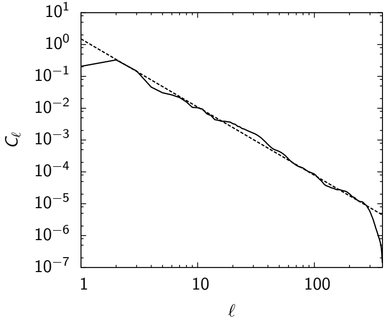
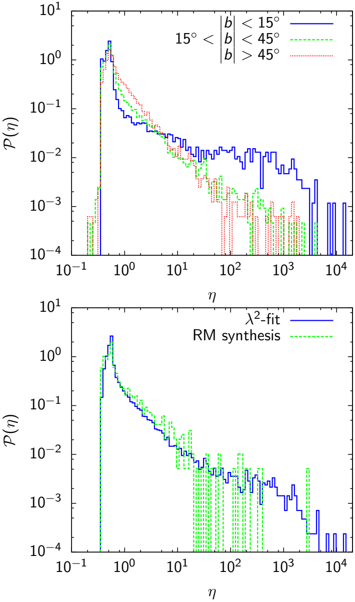


$$C_\ell \propto \ell^{-2.17}$$



$$N_{ij} = \langle n_i n_j \rangle = \delta_{ij} \eta_i \sigma_i^2$$

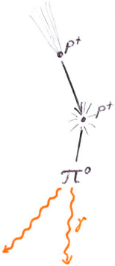
Oppermann et al. (2012)

Faraday rotation

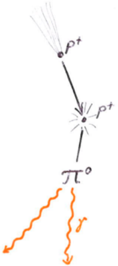
Gamma rays

CMB foregrounds

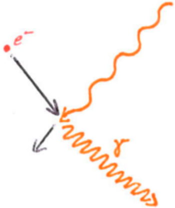
Pion decay



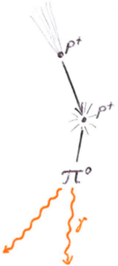
Pion decay



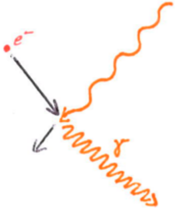
Inverse-Compton



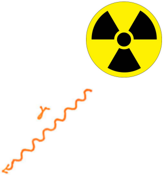
Pion decay



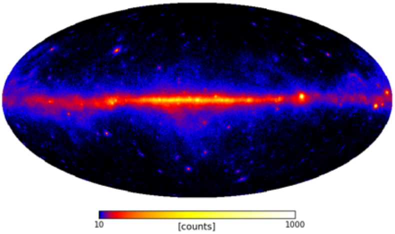
Inverse-Compton



Radioactive decay



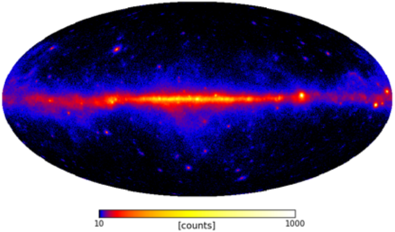
Gamma-ray photon counts:



FERMI data

- non-Gaussian
- always positive
- varying over several orders of magnitude

Gamma-ray photon counts:

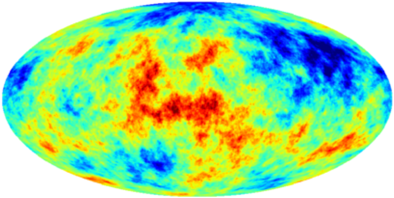


FERMI data

- non-Gaussian
- always positive
- varying over several orders of magnitude

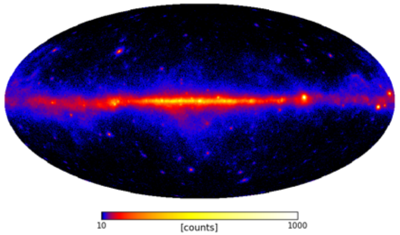
The log-normal model

- use logarithm of photon flux density as signal
- model this as Gaussian random field



s

Gamma-ray photon counts:



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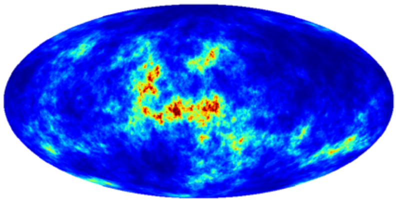
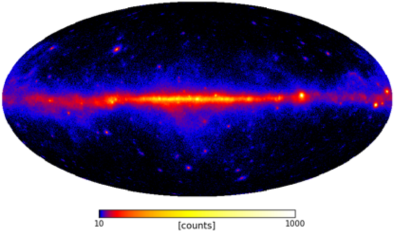


e^s

Gamma-ray photon counts:



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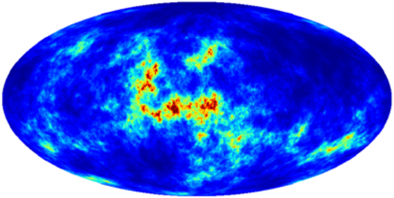
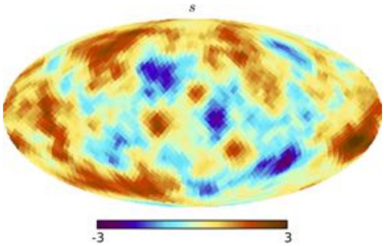


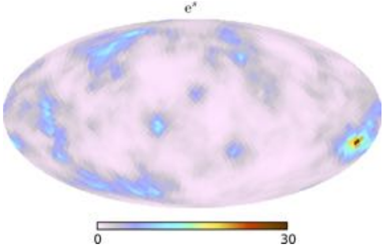
e^{s}

$$d = R\mathrm{e}^{s} + n$$

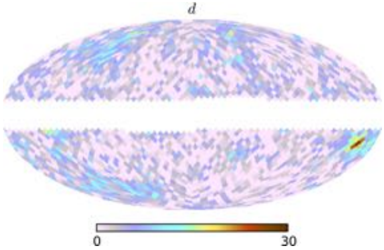
signal



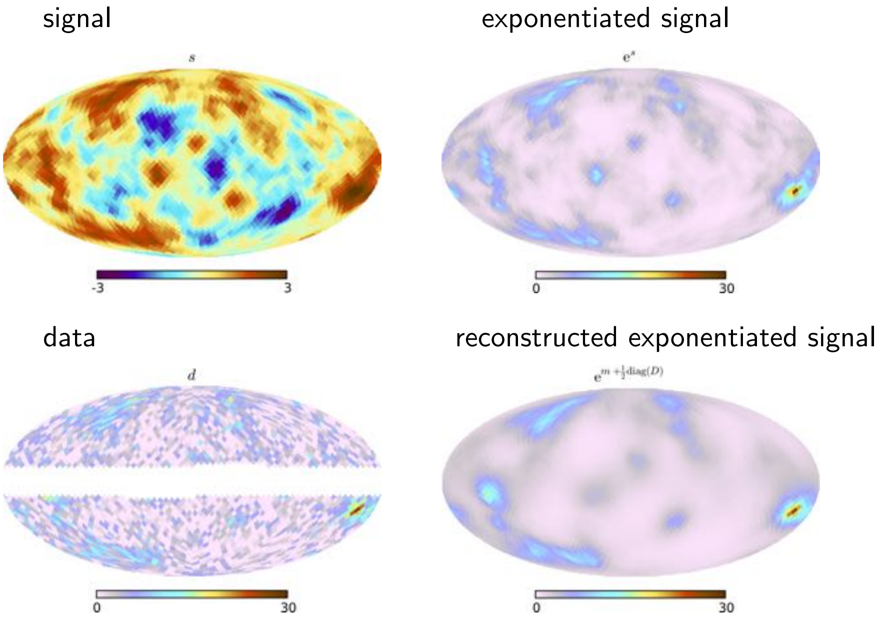
exponentiated signal



data

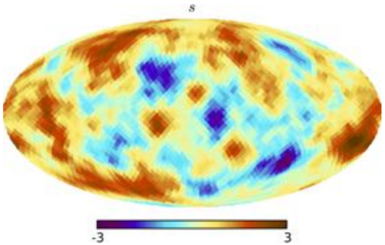


Oppermann et al. (2013)

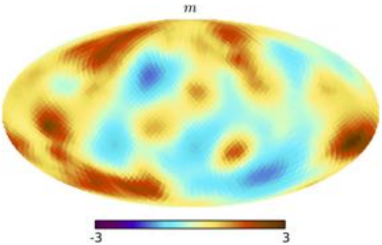


Oppermann et al. (2013)

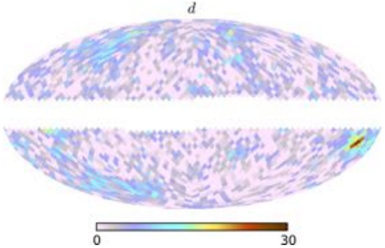
signal



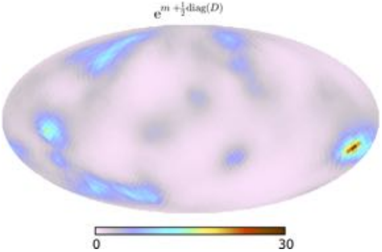
reconstructed signal



data

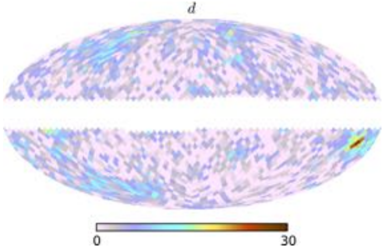


reconstructed exponentiated signal

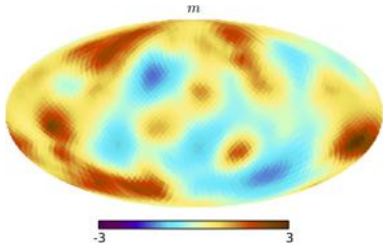


Oppermann et al. (2013)

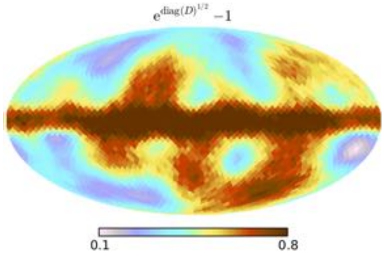
uncertainty



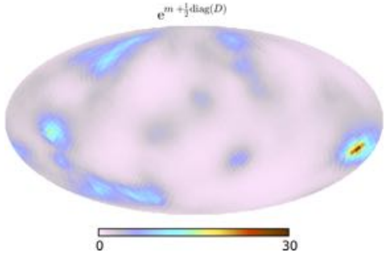
reconstructed signal



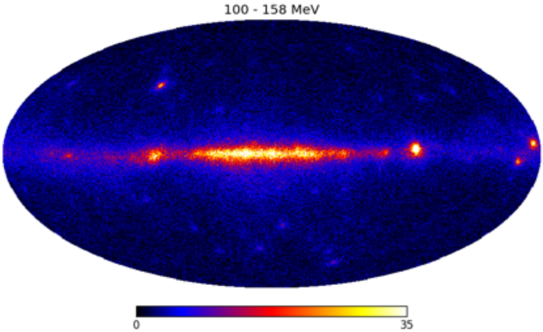
fractional uncertainty

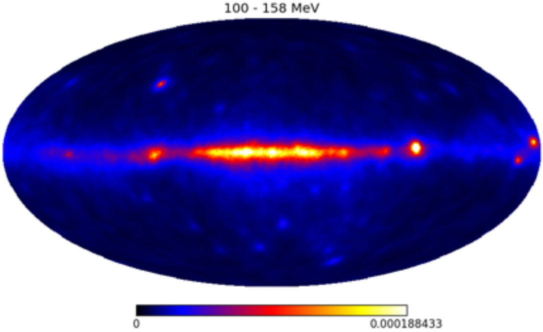


reconstructed exponentiated signal

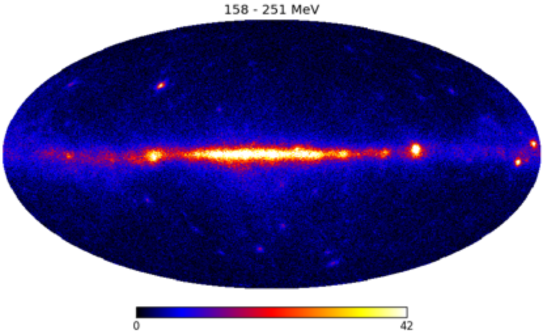


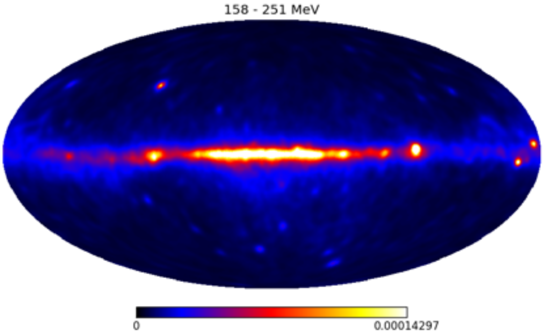
Oppermann et al. (2013)





Selig et al. (in prep.)





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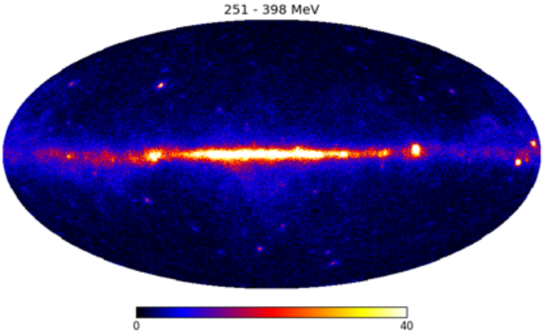

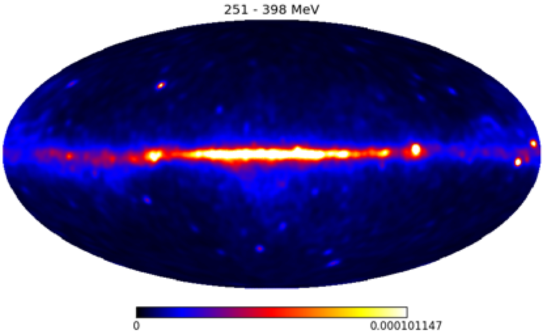

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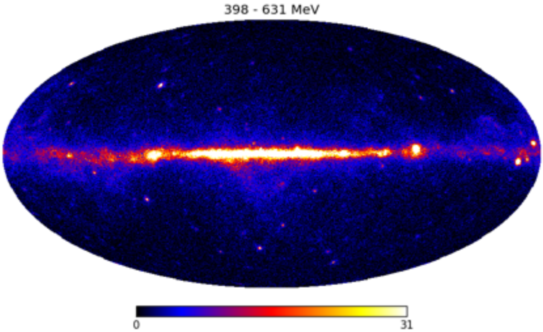

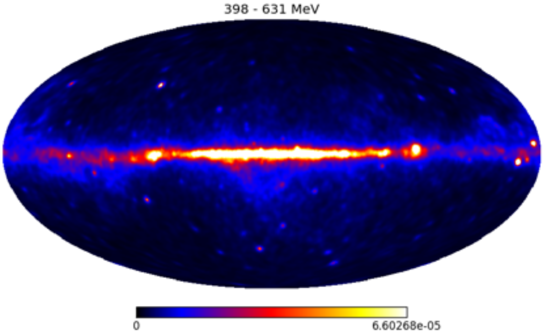

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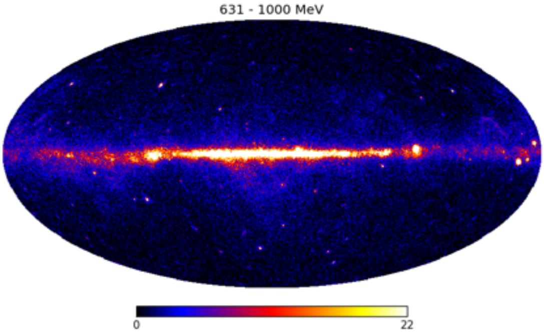

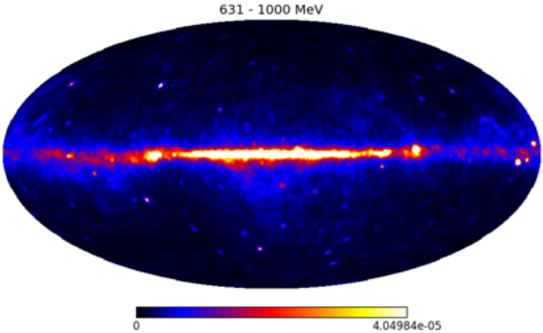

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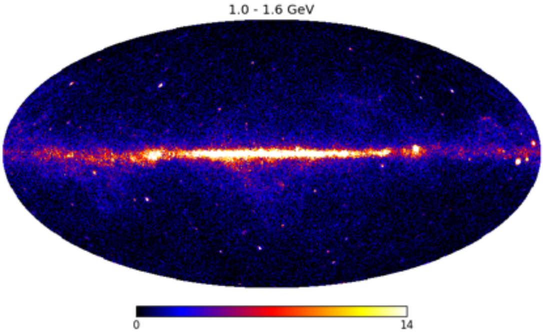

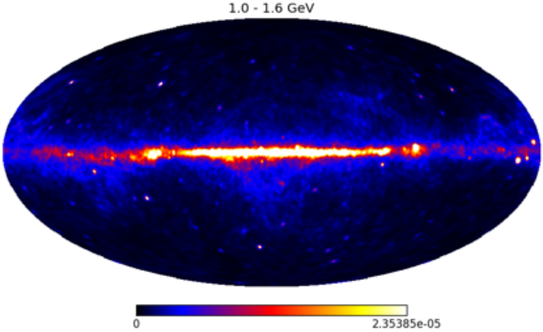

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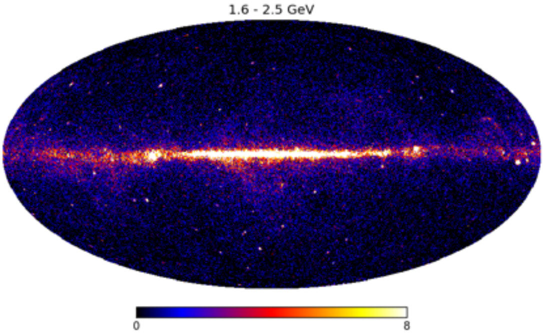

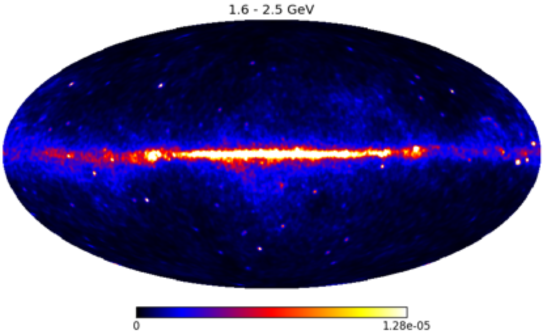

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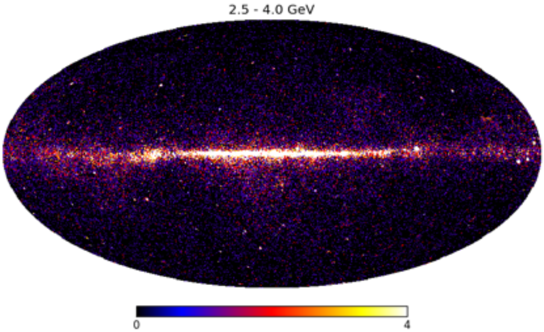

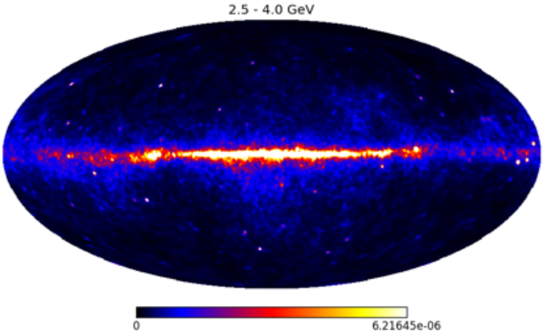

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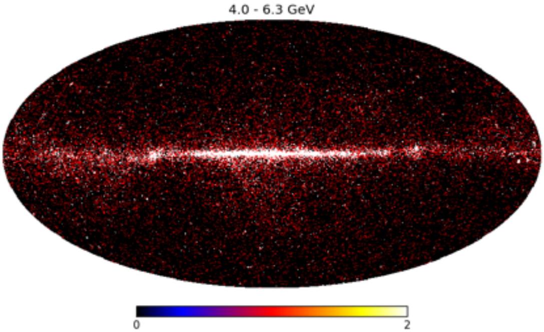

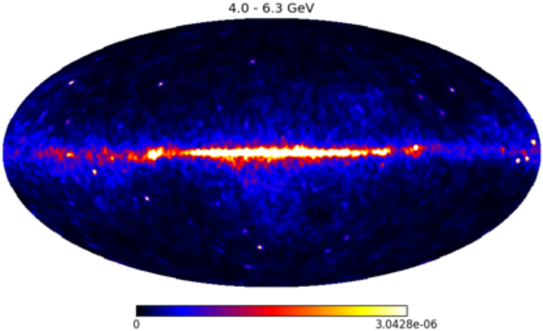

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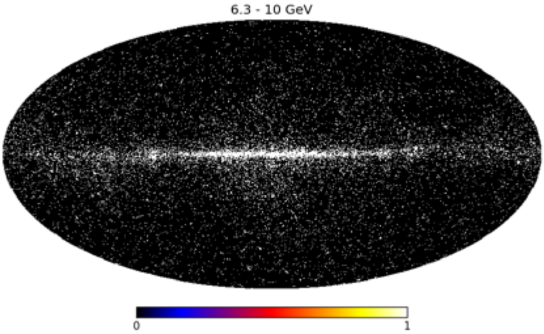

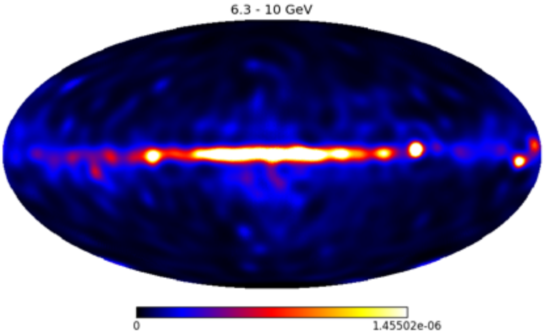

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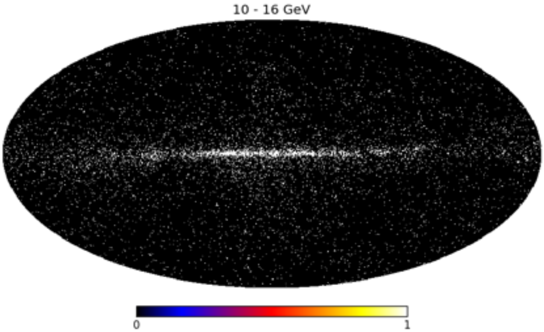

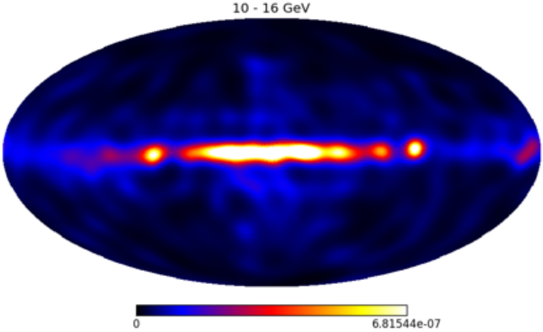

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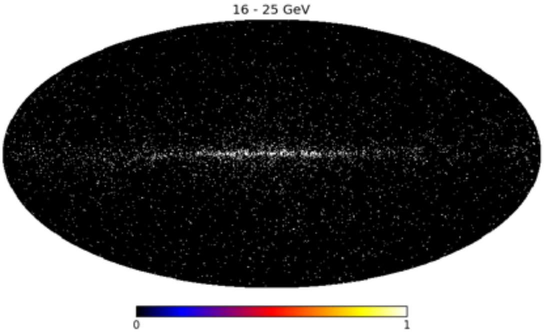

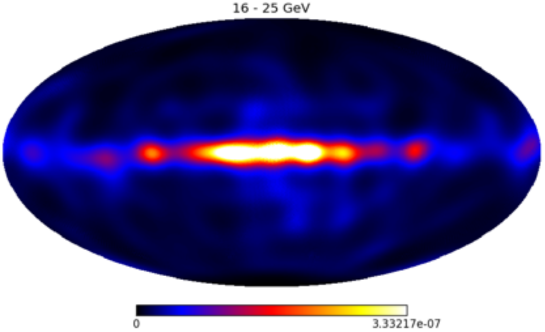

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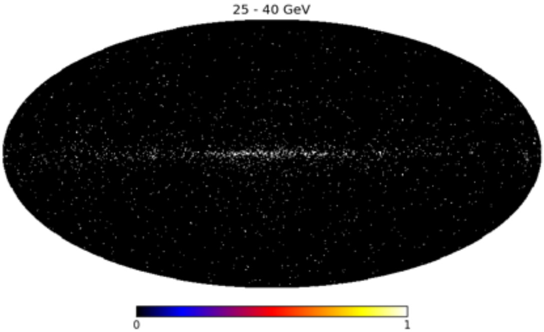

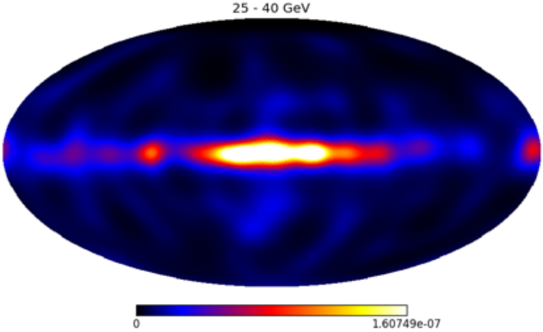

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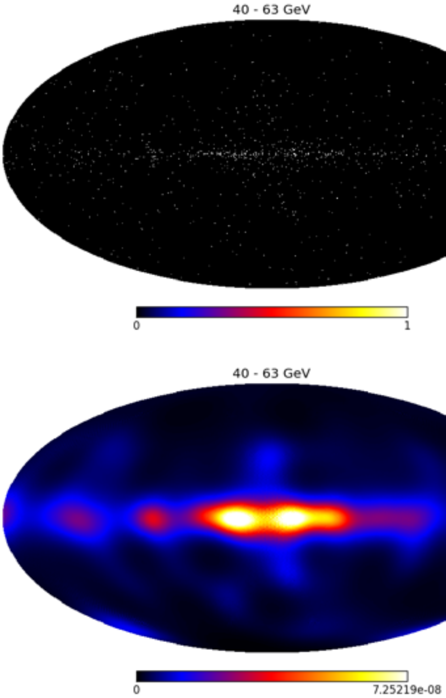

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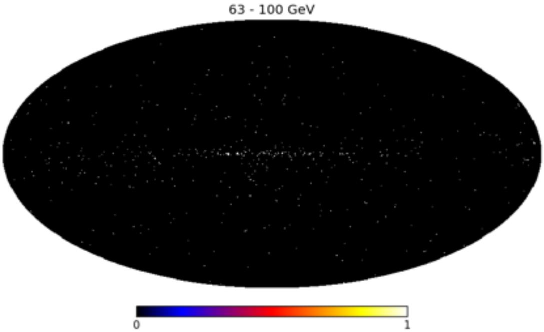

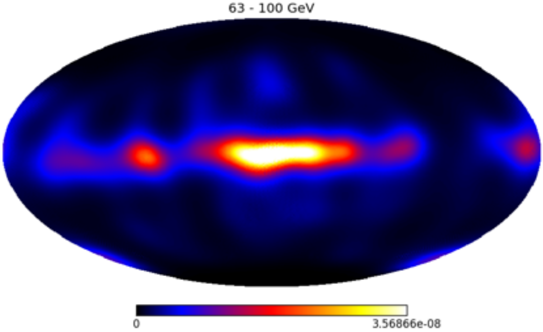

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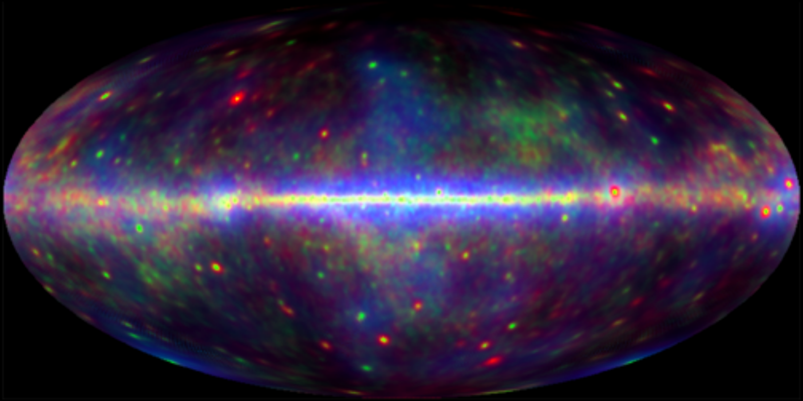


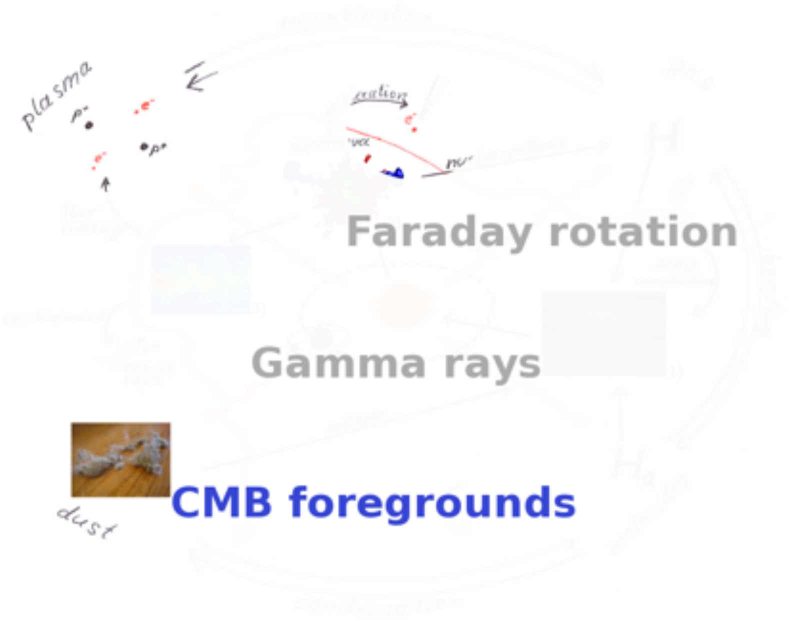
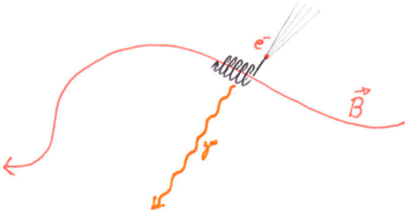
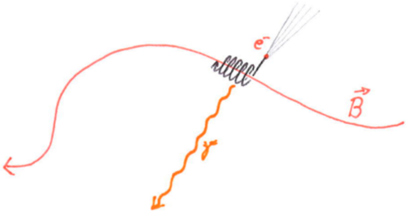


Image credits: 1) D. Darling; 2) N.J. Hammer/MPA; 3) C. Fukushima/TUDelft

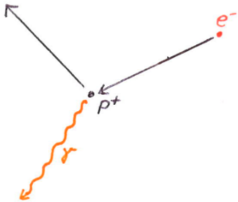
Synchrotron radiation



Synchrotron radiation



Bremsstrahlung (free-free)



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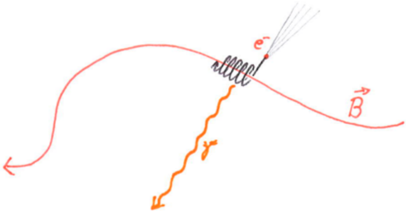

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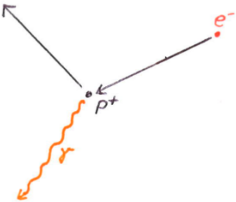
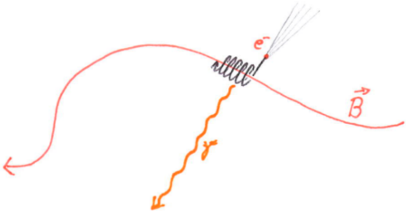
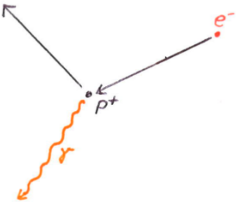


Thermal dust radiation

Synchrotron radiation



Bremsstrahlung (free-free)



Thermal dust radiation

Radiation from rotating dust grains

data:

- measurements at different frequencies
- inhomogeneous noise

response:

- mixing matrix according to frequency spectra of components

signal:

- different emission mechanisms
- Gaussian (CMB) and log-normal (foregrounds)
- cross-correlated

$$\begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \end{pmatrix} = \begin{pmatrix} f^{\nu_1}_{\mathrm{CMB}} & f^{\nu_1}_{\mathrm{synch}} & f^{\nu_1}_{\mathrm{ff}} & f^{\nu_1}_{\mathrm{dust}} \\ f^{\nu_2}_{\mathrm{CMB}} & f^{\nu_2}_{\mathrm{synch}} & f^{\nu_2}_{\mathrm{ff}} & f^{\nu_2}_{\mathrm{dust}} \\ f^{\nu_3}_{\mathrm{CMB}} & f^{\nu_3}_{\mathrm{synch}} & f^{\nu_3}_{\mathrm{ff}} & f^{\nu_3}_{\mathrm{dust}} \\ f^{\nu_4}_{\mathrm{CMB}} & f^{\nu_4}_{\mathrm{synch}} & f^{\nu_4}_{\mathrm{ff}} & f^{\nu_4}_{\mathrm{dust}} \\ f^{\nu_5}_{\mathrm{CMB}} & f^{\nu_5}_{\mathrm{synch}} & f^{\nu_5}_{\mathrm{ff}} & f^{\nu_5}_{\mathrm{dust}} \end{pmatrix} \begin{pmatrix} s_{\mathrm{CMB}} \\ s_{\mathrm{synch}} \\ s_{\mathrm{ff}} \\ s_{\mathrm{dust}} \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \end{pmatrix}$$

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- measurements at different frequencies
- inhomogeneous noise

response:

- mixing matrix according to frequency spectra of components

signal:

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1. Determine mixing matrix.
2. “Invert” equation.

data:

- measurements at different frequencies
- inhomogeneous noise

response:

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- different emission mechanisms
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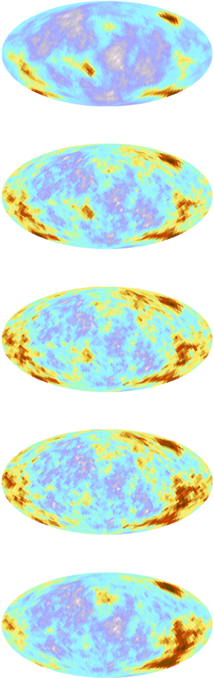
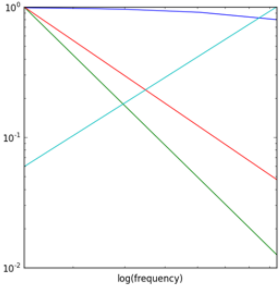
2. "Invert" equation.

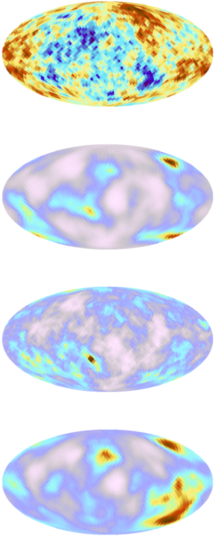
data:



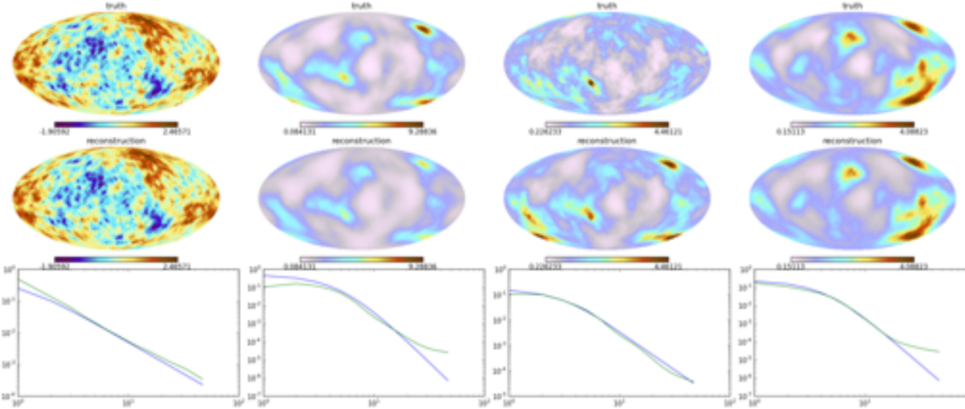
response:



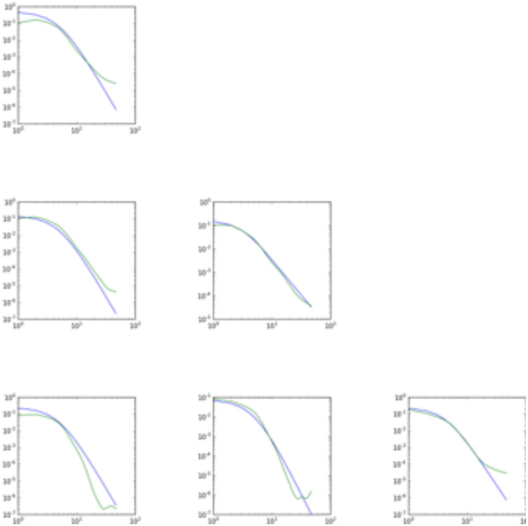
signal:



WORK IN PROGRESS

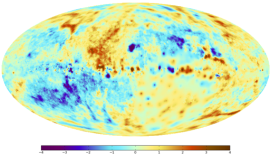


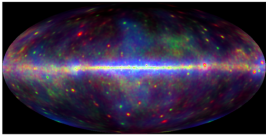
WORK IN PROGRESS

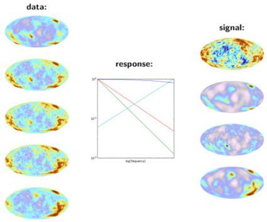


Summary

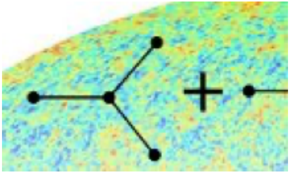
- Probabilistic inference problems
- Use correlation structure to interpolate
- Probabilistic method for dealing with outliers
- Non-linear response / Non-Gaussian signals can be dealt with







Information Field Theory



Numerical IFT for python

NIFTY

http://www.mpa-garching.mpg.de/ift/

http://www.mpa-garching.mpg.de/ift/nifty/

- Lecture on IFT next Wednesday (July 10th)
- NIFTy tutorial next Thursday (July 11th)