Assignments: Univariate Simulation

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The third assignment topic is univariate simulation. If you draw the topic "Univariate Simulation" at the oral exam, you will have to present a solution of one of the two assignments below.

Remember the five points:

- How can you test that your implementation is correct?
- Can you implement alternative solutions?
- Can the code be restructured e.g. by modularization, abstraction or object oriented programming to improve readability?
- How does the implementation perform (benchmarking)?
- Where are the bottlenecks (profiling), and what can you do about them?

When comparing simulation algorithms it may be necessary to use the same pseudo random numbers (set the seed) or to compute results of comparable accuracy as measured by e.g. the standard error.

Assignment 1: Rejection sampling

The purpose of this assignment is to sample from the probability distribution on $[0, \infty)$ with density

$$f(y) \propto \exp(-y^3 + y), \quad y \geq 0.$$

Find a Gaussian envelope of f and implement rejection sampling from the distribution with density f using this envelope. Then implement an adaptive rejection sampling algorithm and compare it with the one based on the Gaussian envelope.

You are welcome to invent other envelopes and try to optimize the choice of envelope. However, for the exam the focus should be on the implementations and their comparisons and not on theoretical derivations of envelopes.

Assignment 2: Importance sampling

Let $X_1, \ldots, X_n$ be i.i.d. Poisson random variables with mean λ. In this assignment you will investigate a test of the hypothesis $H_0 : \lambda \leq 2$ using the test statistic

$$Z = \frac{\overline{X} - 2}{\sqrt{2/n}},$$

where $\overline{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$ denotes the average. Rejecting the hypothesis for $Z \geq 1.645$ gives – using the CLT – a nominal level of 0.05.

Implement an importance sampling algorithm based on simulating Poisson variables with mean λ_0. Use it to estimate the *power curve* defined as the function

$$\beta(\lambda) = P_\lambda(Z \geq 1.645)$$

for $n = 25$, different choices of λ_0 and using unstandardized as well as standardized weights. Compare the results. Pay attention to how the pointwise standard errors for the power curve depends on the choice of λ_0. Some suggested choices of λ_0 are λ, 2, 2.5 and 3. Is there an optimal choice?