Assignments: EM algorithm

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The second assignment topic is the EM algorithm. If you draw the topic "EM algorithm" at the oral exam, you will have to present a solution of one of the two assignments below.

Remember the five points:

- How can you test that your implementation is correct?
- Can you implement alternative solutions?
- Can the code be restructured e.g. by modularization, abstraction or object oriented programming to improve readability?
- How does the implementation perform (benchmarking)?
- Where are the bottlenecks (profiling), and what can you do about them?

For the EM-algorithm assignments performance is less important. Test of convergence, experiments with convergence criteria and experiments with different ways of organizing the code is of greater value. Robustness towards the initial choice of parameters is important, and testing of the implementation on several (simulated) data sets is a good idea.

Assignment 1: Poisson mixture

Consider the problem formulated in Exercise 4.2 in CS. Implement the EM-algorithm for the model described in this problem and use it to fit the model to the data from Table 4.2. You can use the formulas from Problem 4.2a, but it is a good idea to show how they follow from the theory. However, you should not use time on that during the exam.

Then implement one or more ways for computing the observed Fisher information and thus standard errors of the parameter estimates.

Assignment 2: Gaussian mixture

The Gaussian mixture model is given by the density

$$p\varphi(x;\mu_1,\sigma_1) + (1-p)\varphi(x;\mu_2,\sigma_2)$$

for parameters $p \in (0,1)$, $\mu_1, \mu_2 \in \mathbb{R}$ and $\sigma_1, \sigma_2 > 0$. Here $\varphi(\cdot;\mu,\sigma)$ denotes the density for the Gaussian distribution with mean μ and standard deviation σ. This is the distribution of

$$X = ZY_1 + (1-Z)Y_2$$

where Y_1, Y_2, Z are independent, $P(Z=1) = 1 - P(Z=0) = p$, and Y_i is $\mathcal{N}(\mu_i, \sigma_i)$ distributed.

Implement the EM-algorithm for the Gaussian mixture model and fit it to the ψ-angle data from the lecture on density smoothing (in the file `psi.txt`).

Then implement one or more ways for computing the observed Fisher information and thus standard errors of the parameter estimates.