MATLAB EXERCISE II

FILTERING AND IDENTIFICATION (SC42025): SYSTEM IDENTIFICATION

Hand in your solutions at the end of the session of 13-12-2017 (only hard-copy solutions are accepted)

To fulfil the requirements of this exercise, the student shall:

- *Provide answers to the questions and hand those in before the deadline – it is not necessary to prepare a report;*
- *Provide the Bode plots of question 3.5. Remember to add a legend to make each plot clearly identifiable.*
- *Append your code to your answers using the function* publish *from* MATLAB

A given MATLAB *data file (*`homework.mat`*) contains all the data that are needed for this exercise. You can import the data it contains by using the instruction* `load`*.*
Important*:*

- *The first two questions of this exercise require the preparation of two* MATLAB *functions. This task can take some time, especially for those who are not quite familiar with* MATLAB *programming. Please start working on those sections before the session on 13-12-2017! You should have already come up with something when the session starts.*
- *You are not allowed to use the system identification toolbox from* MATLAB.

1. An ARX model of a noiseless system of order n is described by a transfer function of a structure like:

$$y(k) = \frac{B(q)}{A(q)}u(k) = \frac{b_0 + b_1 q^{-1} + b_2 q^{-2} + \ldots + b_n q^{-n}}{a_0 + a_1 q^{-1} + a_2 q^{-2} + \ldots + a_n q^{-n}}u(k)$$

where a_i, b_i are coefficients of the denominator and numerator ($a_0 = 1$, $b_0 = 0$ always), and q is the time-shift operator.

If you are given a set of N inputs $u(k)$ and a set of N outputs $y(k)$, the problem of finding an ARX model of order n can be solved by minimizing the difference between the actual output y and the predicted output $\hat{y}$ that would be generated by a generic ARX model predictor:

$$\text{find } a_i, b_i \quad \text{such that} \quad J = \|y - \hat{y}\|_2^2 = \sum_{k=1}^{N} \left(y(k) - \hat{y}(k|k-1)\right)^2 \text{ is minimal,}$$

with:

$$\hat{y}(k|k-1) = b_1 u(k-1) + b_2 u(k-2) + \ldots + b_n u(k-n) - a_1 y(k-1) - a_2 y(k-2) - \ldots - a_n y(k-n)$$

Due to the fact that $\hat{y}$ is linear with respect to a_i and b_i, the minimisation problem shown above can be cast into a least-squares problem and easily solved.

Write your own MATLAB function for identification of a single-input single-output discrete-time ARX model. The prototype of the function shall be the following:

```
[aest, best] = myarx(y, u, n)
```

The function will return the numerator (`best`) and denominator (`aest`) coefficients of the discrete-time SISO ARX model from a given input (column) vector `u` and its corresponding output vector `y`. The user should also specify the desired order (`n`) for the model.
Hint: look at Example 8.2 on page 271 in the textbook.

2. Write your own MATLAB function for subspace identification of a single-input single-output state-space model. The prototype of the function shall be the following:

```
[At, Bt, Ct, Dt, x0t, S] = mysubid(y, u, s, n)
```

The function will return the state-space matrices (`At`, `Bt`, `Ct` and `Dt`) and the initial state `x0t` from a given input (column) vector `u` and its corresponding output vector `y`. The user should also specify the desired order (`n`) for the model and the parameter `s` (the number of rows of the block-Hankel matrices $Y_{0,s,N}$ and $U_{0,s,N}$ used in subspace identification). An additional output (`S`) is a vector containing the singular values of $Y_{0,s,N}\Pi^{\perp}_{U_{0,s,N}}$.
Hint: if you find it too difficult to calculate `Bt`, `Dt` and `x0t`, you can use the function that is provided in the file `subidhelp.m`.

3. The file `homework.mat` contains four couples of inputs and outputs of an unknown dynamic system (the acoustical duct): (`u1`, `y1`), (`u2`, `y2`), (`u3`, `y3`) and (`u4`, `y4`). We would like to use such sets for identifying a 6^{th} order model.

   3.1 Of the four given sets, only two are useful for identification. Which ones? Why?
   Hint: look at Definition 10.1 on page 358 in the textbook.

   3.2 When using `mysubid` for estimating a model of order n, how should s be chosen?

   3.3 Why is it useful to have also the singular values of $Y_{0,s,N}\Pi^{\perp}_{U_{0,s,N}}$ as an output (the `S`) for `mysubid`?

   3.4 Use both your functions to identify a 6^{th} order model of the system from the two suitable sets of data.

   3.5 The variable `tfse` contained in `homework.mat` is the transfer function of the original system. Draw the Bode plots of the 6^{th} order transfer functions obtained with the identification process and compare them to the plot of the given transfer function. What are your considerations about these Bode plots? Do you see any differences between the models obtained with subspace identification and the ARX ones?
   Note that you can obtain a bode diagram of the identified model as follows (the sample time is unspecified):

   ```
   sys_id = tf(best,aest,-1)
   bode(sys_id)
   ```

4. Consider the vectors `u0` and `y0` in `homework.mat`.

   4.1 Use `mysubid` to estimate the order of the system, and identify a model for it. Which is your estimate of the model order? How can you say this?

   4.2 Now suppose that the measures have a bias error of 0.2. Compute a vector `y00` $=$ `y0` $+\,0.2$. Use again `mysubid` to estimate the order and identify a model, this time with `y00` instead of `y0`. Do you notice any relevant differences with what was found at 4.1?
   Hint: the answer is yes...

   4.3 Is the system found at 4.2 reachable? Which are its poles? Give an explanation.
   Hint: do not use the instruction `rank` to compute the rank of a matrix.