MATLAB EXERCISE I

FILTERING AND IDENTIFICATION (SC42025): KALMAN FILTERING

Hand in your answers at the end of the session on 06-12-2017. Hard-copy required.

To fulfil the requirements of this exercise, the student shall:

- *Hand in the answers to the questions below – it is not necessary to hand in a report: use the MATLAB instruction `publish` to output your results (type 'help publish' at the MATLAB command window for more information);*
- *Provide plots comparing the estimated trajectories. Remember to add a legend to make each plot clearly identifiable.*

The given MATLAB data file (`rocket.mat`) contains all the data that are needed for this exercise. You can import the data it contains by using the instruction `load rocket.mat`.

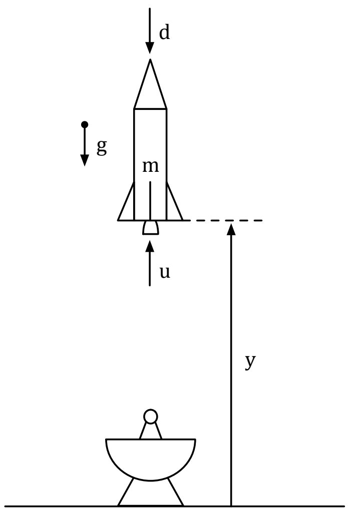

Figure 1: Simplified one-dimensional tracking setup for a rocket being launched vertically.

A group of rocket scientists launches experimental rockets in a vertical direction. Their only means of measuring the trajectory is a ground-based radar. The situation is sketched in Figure 1. The scientists would like to obtain trajectory information such as the velocity and altitude of the rocket. The radar, however, only gives a noisy estimate of the altitude.
In the figure y is the altitude of the rocket, d is the drag force acting on the rocket and u is the thrust of the rocket's engine. The radar system is used to measure the distance y. The rocket has mass m and the gravitational acceleration is g. The parameters are listed in Table 1.

parameter	value
Δt	0.1 s
m	100 kg
g	9.81 m/s^2
$y(0)$	0 m

Table 1: Model parameters

1. A simplified model of the rocket's dynamics based on Newton's 2nd law is given by the following state-space description:

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 & 0 & 0 \\ \frac{1}{m} & -1 & -\frac{1}{m} \end{bmatrix} \begin{bmatrix} u \\ g \\ d \end{bmatrix} \tag{1}$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x \tag{2}$$

where $x = \begin{bmatrix} y & \dot{y} \end{bmatrix}^\top$.
Note that the gravitational acceleration is considered as an external input. The drag force d is unknown, but is estimated to be approximately $d = 1700\,\text{N}$. In reality, the drag force is of course highly nonlinear, depending on the velocity and local atmospheric conditions.
The model can be discretised using a zero-order hold resulting in the discrete-time model

$$x(k+1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{(\Delta t)^2}{2m} & -\frac{(\Delta t)^2}{2} & -\frac{(\Delta t)^2}{2m} \\ \frac{\Delta t}{m} & -\Delta t & -\frac{\Delta t}{m} \end{bmatrix} \begin{bmatrix} u(k) \\ g(k) \\ d(k) \end{bmatrix} \tag{3}$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k). \tag{4}$$

Simulate the given model using the recorded input signal u in `rocket.mat`, the gravity g and assumed drag force d. Does the predicted trajectory $y(k)$ coincide with the measured trajectory?

2. Construct an asymptotic observer to observe the state for the given system. That is, create a system of the form

$$\hat{x}(k+1) = (A - KC)\hat{x}(k) + B \begin{bmatrix} u(k) \\ g(k) \\ d(k) \end{bmatrix} + Ky(k) \tag{5}$$

$$\hat{y}(k) = C\hat{x}(k), \tag{6}$$

where K is such that the poles of $A - KC$ are placed at 0.8 and 0.7 respectively. Note that since we estimate $\hat{x}(k)$, we have obtained estimates of the altitude as well as the vertical velocity of the rocket.

Hint: use the command `K = place(A',C',p)'` *to find such a* K.

Simulate the observer you designed with the signals u, d (again assumed constant), g and y as inputs. Has the estimate of the trajectory (altitude) improved compared to the previous simulation? Also compare the velocity of the rocket to the estimate.
You can use the vectors `ytrue` and `ydottrue` for comparison.

3. You will now construct a Kalman filter to estimate the state. The Kalman filter has the same structure as the asymptotic observer, but K is now chosen based on the covariance matrices of the process and measurement noises. The setup corresponds to the following state-space model.

$$x(k+1) = Ax(k) + B \begin{bmatrix} u(k) \\ g(k) \\ d(k) \end{bmatrix} + w(k) \tag{7}$$

$$y(k) = Cx(k) + v(k), \tag{8}$$

With $Q = \mathbb{E}\{w(j)w(j)^\top\}$ – the covariance of the process noise – we will express that there is a lack of confidence in our model; i.e., the drag force is not exactly known. $R = \mathbb{E}\{v(j)v(j)^\top\}$ represents the covariance matrix of the measurement noise, expressing the uncertainty in the radar measurements $y(k)$.

Find a Kalman gain K based on certain values of Q and R ($S = 0$). Simulate the Kalman filter for these values of K and compare the results to the given true altitudes and velocities. What do you conclude?

Hint: use the Kalman filter for LTI systems (§5.7 in the book). It is known that $R = \sigma_v^2 \approx 1 \cdot 10^3\ m^2$, that is, the standard deviation of the altitude measurement error $v(k)$ is about $30\ m$, corresponding to

1% error when the rocket is at $3\ km$.
Further, choose Q diagonal and tune the diagonal elements to obtain a good estimate. For example, start with $Q = I$.

4. What would be another way to estimate the rocket's vertical velocity? Does that method work in this case?

5. Since the drag force is unknown, the scientists would like to estimate it. It is assumed that the drag varies slowly according to the following *random-walk* model:

$$d(k+1) = d(k) + w_3(k). \tag{9}$$

   $w_3(k)$ is used to express that d is not truly constant, but may vary.
   Append this drag model to the state space model (Eq. 7) used earlier (cf. §5.8 in the book). You should end up with a model that has two inputs (u and g), so that the effect of the drag force is absorbed into the A-matrix. The system should describe the following dynamics:

$$x(k+1) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{(\Delta t)^2}{2m} & -\frac{(\Delta t)^2}{2} & -\frac{(\Delta t)^2}{2m} \\ \frac{\Delta t}{m} & -\Delta t & -\frac{\Delta t}{m} \end{bmatrix} \begin{bmatrix} u(k) \\ g(k) \\ d(k) \end{bmatrix} + w(k) \tag{10}$$

$$d(k+1) = d(k) + w_3(k), \tag{11}$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k) + v(k). \tag{12}$$

   Design a new Kalman filter. Use the same value of R as in question 4. Can you still estimate the rocket's velocity? And the drag force?

6. The true altitude, velocity and drag force are contained in the vectors `ytrue`, `ydottrue` and `dtrue`. For the estimated *altitudes*, *velocities* and *drag forces* compute the errors between the true quantity and the estimate. Determine the root-mean-square value of this error sequence, e.g:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{13}$$

   Do this for the measurements, the asymptotic observer results and the Kalman filter results. What is your conclusion?

Reminder: use the MATLAB *instruction* `publish` *to output your results (type 'help publish' at the* MATLAB *command window for more information).*