Exercises Control Theory Set 5 (2017)

Deadline: Thursday 19/10/2016 12:00

Exercise 1

For any pair (A, B) and any feedback gain F show the following facts:

a) If (A, B) is controllable then $(A - BF, B)$ is controllable.

b) If (A, B) is stabilizable then $(A - BF, B)$ is stabilizable.

c) The uncontrollable modes of (A, B) and $(A - BF, B)$ are the same.

Exercise 2

Consider the system

$$\dot{x} = Ax + Bu = \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad x(0) = x_0$$

with cost

$$\int_0^\infty x(t)^T Q x(t) + u(t)^2 \, dt \quad \text{where} \quad Q = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}.$$

a) Does there exist an optimal LQ controller?

b) Compute all real symmetric solutions of the corresponding algebraic Riccati equation.

c) Check that there is indeed a unique stabilizing solution.

d) Let the system be controlled with the LQ optimal gain; for the resulting state- and control-trajectory compute P_1 and P_2 with

$$x_0^T P_1 x_0 = \int_0^\infty x(t)^T Q x(t) \, dt, \quad x_0^T P_2 x_0 = \int_0^\infty u(t)^2 \, dt.$$

e) Find a feedback gain $\hat{F}$ such that $A - B\hat{F}$ has its eigenvalues in -1 and -2.

f) If the system is controlled with $u = -\hat{F}x$ compute $\hat{P}_1$ and $\hat{P}_2$ with

$$x_0^T \hat{P}_1 x_0 = \int_0^\infty x(t)^T Q x(t) \, dt, \quad x_0^T \hat{P}_2 x_0 = \int_0^\infty u(t)^2 \, dt.$$

Compare the matrices with those of the optimal controller and discuss the results.

Exercise 3

Suppose that the matrix

$$M = \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix}$$

is symmetric and that R is positive definite. Show that

a) $\begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} = \begin{pmatrix} I & SR^{-1} \\ 0 & I \end{pmatrix} \begin{pmatrix} Q - SR^{-1}S^T & 0 \\ 0 & R \end{pmatrix} \begin{pmatrix} I & 0 \\ R^{-1}S^T & I \end{pmatrix}.$

b) M is positive semi-definite if and only if $Q - SR^{-1}S^T$ is positive semi-definite.

Exercise 4

Let us consider the LQ-problem as in the lectures for the system $\dot{x} = Ax + Bu$ but now with the cost function

$$\int_0^\infty \begin{pmatrix} x(t) \\ u(t) \end{pmatrix}^T \begin{pmatrix} Q & S \\ S^T & R \end{pmatrix} \begin{pmatrix} x(t) \\ u(t) \end{pmatrix} dt = \int_0^\infty x(t)^T Q x(t) + 2x(t)^T S u(t) + u(t)^T R u(t)\, dt$$

that involves a cross-term defined by S and in which Q is symmetric and R is positive definite.

a) Show that a precompensation $u = -R^{-1}S^T x + v$ reduces this problem to one for a system $\dot{x} = \hat{A}\dot{x} + Bv$ and a cost

$$\int_0^\infty x(t)^T \hat{Q} x(t) + v(t)^T R v(t)\, dt$$

with suitably defined $\hat{A}$ and $\hat{B}$.

b) Determine the hypotheses on the data A, B and Q, S, R in order to make sure that the LQ problem has a solution.

c) Which Riccati equation has to be solve and which control has to be applied in order to solve the LQ problem.

Exercise 5

Continuing Exercise 3 of Set 4 (mini-project), design an optimal LQ state-feedback controller and compare the simulation results with that of the pole-placing controller. Play with the weighting matrices and discuss their influence.