HOMEWORK EXERCISE III

FILTERING AND IDENTIFICATION (SC42025)

Hand in your solutions at the end of the lecture on **06-12-2017** (only hard-copy solutions are accepted).

Please highlight your final answer!

1 Exercise 1: Kalman filtering and RLS

Consider the following state-space system:

$$\begin{aligned} x_{k+1} &= x_k \\ y_k &= C_k x_k + v_k \end{aligned}$$

where $x_k \in \mathbb{R}$, $y_k \in \mathbb{R}$, $v_k \sim (0, 1)$. Furthermore, we will use the following definitions:

$$\begin{aligned} P_{k|k-1} &= E[(x_k - \hat{x}_{k|k-1})(x_k - \hat{x}_{k|k-1})^T] \\ P_{k|k} &= E[(x_k - \hat{x}_{k|k})(x_k - \hat{x}_{k|k})^T] \\ R_k &= P_{k|k}^{-1} \end{aligned}$$

where $\hat{x}_{k|k-1}$ and $\hat{x}_{k|k}$ are the one-step-ahead predicted state estimate and filtered state estimate of x_k respectively. R_k is often referred to as the information matrix.

1. Assume that we are given the unbiased estimate $\hat{x}_{k|k-1}$ and corresponding covariance $P_{k|k-1}$. Find K_k in

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \left(y_k - C_k \hat{x}_{k|k-1} \right)$$

   such that $P_{k|k}$ is minimized and $E[\hat{x}_{k|k}] = E[x_k]$.

2. Using the estimate $\hat{x}_{k|k}$ and its error covariance matrix, give the new one-step-ahead predicted state estimate $\hat{x}_{k+1|k}$ and covariance $P_{k+1|k}$.

3. Give a recursive expression for R_k in terms of R_{k-1} and C_k.

4. Express the Kalman gain K_k in terms of R_k and C_k.

5. Compare the derived Kalman filter with the *Recursive Least Squares (RLS)* algorithm (slide 20 of Lecture 3).

Exercise 2: Input Reconstruction

We use the theory of Kalman filtering to examine the following unknown input reconstruction problem of the system given as:

$$
\begin{aligned}
x(k+1) &= Ax(k) + Bb(k) \\
y(k) &= Cx(k) + e(k) \quad E[e(k)e(\ell)^T] = R\Delta(k-\ell)
\end{aligned}
$$

with $x(k) \in \mathbb{R}^n, y(k) \in \mathbb{R}$ and the unknown input $b(k) \in \mathbb{R}$. You may assume that $CB = 1$.

1. To get information on the unknown input $b(k)$ in the output measurement we consider the output at time instant $k+1$. Express $y(k+1)$ in terms of the state $x(k)$ and the unknown input $b(k)$.

2. When an unbiased estimate of the state $x(k)$ is given by $\hat{x}(k|k)$ with error covariance matrix $E[(x(k) - \hat{x}(k|k))(x(k) - \hat{x}(k|k))^T] = P(k|k)$ and satisfying $E[(x(k) - \hat{x}(k|k))e(k+1)^T] = 0$, then determine the matrices M and N of the linear estimator:

$$
\begin{bmatrix} \tilde{x} \\ \tilde{b} \end{bmatrix} = My(k+1) + N\hat{x}(k|k)
$$

such that $E[\tilde{x}] = E[x(k)] = E[\hat{x}(k|k)]$ and $E[\tilde{b}] = E[b(k)]$.

3. Determine the variance $E[(\tilde{b} - b(k))^2]$.

With the unbiased estimate $\tilde{b}$ of $b(k)$ we can estimate $x(k+1)$ and continue the recursive procedure to get an estimate of $b(k+1)$ etc. We will however not further explore the estimation of the state vector $x(k+1)$