Naturalizing Negation

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In *The Threefold Puzzle of Negation and the Limits of Sense*, Jean-Phillipe Narboux offers a picture of negation that is conceptually bound up with the limits of sense and intentionality. Narboux argues that in order to begin thinking about limit and thought's answerability to reality, one must attend to a threefold structure of negation: a fold about intentionality; a fold about intelligibility; and a third fold about the apparent uni- and equivocality of negation. Throughout the essay, Narboux provides blueprints for addressing each aspect of the trilemma by emphasizing the two-way unity and distinction, borrowed from Wittgenstein, between the determination and employment of sense. Narboux explicitly sets out to show that the puzzle of negation is only a pseudo-puzzle and to show that one can begin to unfold the puzzle by transposing it into a sceptical tone. I suggest that this is mistaken. I will argue that Narboux's sceptical treatment of the puzzle of negation stands in the way from a thoroughly ameliorative account of the trilemma. At best, to place the puzzle into the sceptical register allows us to see its complex form; at worst, it prevents us from saying anything about it. I will show that in order to work toward putting the puzzle to rest, we must put it into a naturalized register. The basic idea will be that we ought to see negation as natural to and necessary for thought, something that is bound up with every possible judgment one makes. It is at this point that we can begin to see how negation is bound up with limits of intelligibility and intentionality.

Keywords: Negation, Wittgenstein, Epistemic Naturalism, Intentionality, Intelligibility

Introduction

1. In *The Threefold Puzzle of Negation and the Limits of Sense*, Jean-Phillipe Narboux offers a picture of negation that is conceptually bound up with the limits of sense and intentionality (Narboux, 2022).[1] Narboux argues that in order to begin thinking about limit and thought's answerability to reality, one

must attend to a threefold structure of negation:

1. How can *not-p* so much as negate *p* at all—for if *p* is not the case, then nothing corresponds to *p*?
2. How can *not-p* so much as negate *p* at all when *not-p* rejects *p* not as false, but as unintelligible—for it *p* is unintelligible, then *p* is nothing but scratches and sounds and does not seem apt for negation.
3. How could "*not*" be anything but hopelessly equivocal if it sometimes (per the first puzzle) requires, and sometimes (per the second puzzle) preclude the intelligibility of *p*?

Throughout the essay, Narboux provides blueprints for addressing each aspect of the trilemma by emphasizing the two-way unity and distinction, borrowed from Wittgenstein, between the determination and employment of sense. Narboux explicitly sets out to show that the puzzle of negation is only a pseudo-puzzle and to show that one can unfold the puzzle by transposing it into a sceptical tone.[2]

I suggest that this is mistaken. I will argue that Narboux's sceptical treatment of the puzzle of negation stands in the way from a thoroughly ameliorative account of the trilemma. At best, to place the puzzle into the sceptical register allows us to see its complex form; at worst, it prevents us from saying anything about it. I will show that in order to work toward putting the puzzle to rest, we must put it into a naturalized register. The basic idea will be that we ought to see negation as natural to and necessary for thought, something that is bound up with every possible judgment one makes. It is at this point that we can begin to see how negation is bound up with limits of intelligibility and intentionality. So, the purpose of this essay is to naturalize negation as a basic condition for thinking about judgment at all.

My argument will take the following shape: in (**I.**) I will outline Narboux's account of (and attempts to quiet) the threefold puzzle of negation;[3] in (**II.**) I will offer a naturalized account of negation that answers the three aspects of the trilemma; and in (**III.**) I will indicate that naturalizing negation implies naturalizing the very idea of intelligibility. My hope is to go some distance toward seeing how talk about the limits of sense hinges on[4] negation playing an important role in our capacity for judgment, which, I argue, makes it not so puzzling after all.

I. The Threefold Puzzle of Negation

2. The first fold in the puzzle of negation is about the possibility of negation. The problem is:

How can *not-p* (say, "the books is not on the table") so much as negate *p* at all since if *p* is not the case (as *not-p* claims) then nothing

corresponds to p?

Narboux shows that this fold is quite alive for Frege and Wittgenstein; Irad Kimhi (2018) shows that the problem is as ancient as Parmenides. Frege says:

> It must be possible to negate a false thought, and in order to be able to do that, I need that thought. I cannot negate what is not.

And Wittgenstein repeats:

> It is the mystery of negation: This is not how things are, and yet we can say *how* things are *not.*[5]

Call it the Frege-Wittgenstein problem:

> We are capable of recognizing an implication relation where there should not be one. For if the thought is false (if we can say what is *not*), then it implies *nothing.*

The basic idea is this: iff the book is not on the table is the statement "the book is not on the table" true. But if this is the case, then how can we say something that is not the case? We cannot say anything sensible about the truth-conditions for the proposition because it is a proposition about nothing, so Narboux.

Narboux argues that the Frege-Wittgenstein problem is essentially one of negation disguised as intentionality. As such, it is supposed to allow us to bypass talk about truth-makers for varieties of negation in the form of the Frege-Wittgenstein problem. Narboux's point is to argue that "[w]e are led to misinterpret the requirement that what is negated should make sense as the requirement that it should consist in a sense, only to realize that this move does not get us out of trouble" (p. 160). This line of thought is underdeveloped in Narboux's essay; however, I think that we can understand the point like this:

> If we want to see how we can negate something so that the positive proposition would correspond to nothing, then we should consider negation as a special kind of intentional judgment where we deny a state of affairs being the case without needing to specify the conditions which make the judgment true.

Narboux's solution for the first fold is to reiterate Wittgenstein's Tractarian statement that the sentential operator '~' is precisely that: *an operation.* The idea is that the negation of p, as an act of intentionality, contributes nothing to p, just as wishing that p contributes nothing to it. Narboux says:

> "A judges that p" adds nothing to p. Such signs as "~" or "judges" simply have no reference or sense of their own (p. 184).

So, to say that the proposition "the book is not on the table" is true does not require thinking nothing; all it requires, on Narboux's account, is for one to actively reverse the counterfactual proposition and to know that one is doing so. This is where Narboux's treatment of the Frege-Wittgenstein problem basically stops. I think that understanding negation as an operator is correct; but I think

that if we want to put the Frege-Wittgenstein problem to rest, we must see it as an operator that is natural for our capacity for thought. Once we set the first fold as a matter of *doing something*, we must begin to talk about the conditions for *knowing what we are doing.* I expand on this in (**II.**).

3. The second fold of the puzzle is about the intelligibility of negation. The problem is:

> How can *not-p* (say, "there is no square circle") so much as negate *p* at all when *not-p* does not reject *p* as false but instead rejects it as unintelligible, since if *p* is unintelligible (as *not-p* claims) then *p* is nothing but scratches or sounds?

Narboux shows how Wittgenstein contrasts the second fold from the first. Compare the statement "there is no square circle" with the statement "there is no circle on the blackboard". Wittgenstein demonstrates that while the two negations have similar surface-grammars ("there is no *p*"; "nothing is *p*"), they convey profoundly different senses. Wittgenstein's point is to draw out, to use contemporary language, the fact that the terms "square" and "circle" are materially incompatible, whereas "circle" and "being on the blackboard" denote a compatible possibility, but one that simply does not, in a particular situation, obtain. Narboux suggests that this shows that there are logically distinguishable kinds of *nothing* that bring forth different parts of the puzzle of negation. On the one side, there is the nothing that corresponds to material incompatibilities; on the other side, there is the nothing that corresponds to the Frege-Wittgenstein problem.[6] The former constitutes the second fold of the puzzle and is, for Narboux, the most difficult to address. Narboux argues that every time the first fold is alleviated, the second is bound to emerge. Narboux writes:

> Now, however, we are at a loss to account for the intelligibility of not-p (say, "The sweet is not a colour") given that it is evidently required by the truth of not-p, yet the truth of not-p seems (not only not to require but also) positively to rule out the intelligibility of p. Consequently, should we find a way out of the first puzzle, the present [second] puzzle will appear all the more intractable, far from being alleviated (p. 163).

That is, once we see that it is possible to get around the first problem by following the steps of the *Tractatus* (*viz.*, showing the truth-conditions for *not-p* as logical pictures of what is the case), the second problem can be raised by what Narboux sees as negations *without* truth-conditions, i.e., nonsense (e.g., "there is no square circle" '; "there is no reddish-green"). The second fold of the puzzle is: how can one negate nonsense? For Narboux, this puzzle supposedly precludes (while the first supposedly requires) the intelligibility of *p* when making sentential negations. Formulated differently, the basic idea is this:

> iff the statement "there is no square circle" is true, then the statement "there is no square circle" is nonsense. Call it: the incompatibility problem.[7]

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Before continuing, I would like to indicate that for something to be nonsense implies that it corresponds to nothing. To show my cards now: what I will argue later is that the lack of correspondence of the first and second folds of the puzzle hinges on a particular view of truth (and therefore the world). When we modify this view, the puzzle begins to entirely unfold and negative judgment comes into view as a part of *our* world. I return to this in (**II.**).

Narboux's solution to the incompatibility problem is to identify the thought that we ought not cast the judgment that rejects logical nonsense (what I, following Brandom, prefer to label material incompatibilities) as a *single* judgment. For if we do, we risk setting ourselves up with a seemingly intractable problem, as evident in the gravity Narboux gives this particular fold.

I think that there is something to this idea, but it requires two things, given full form below: (1) the distinction between the first and second folds of the puzzle is unnecessary; (2) we need to see negation as having a role along an inferential phylogeny, *not* as a distinct form of judgment.

4. The third fold of the puzzle is the consequence of the first two folds. The problem is:

> How could *"not"* be anything but hopelessly equivocal if it sometimes (per the first puzzle) requires, and sometimes (per the second puzzle) preclude the intelligibility of *p*?

Or, to use Wittgenstein's example:

> Does "negation" have the same meaning in "He did not leave the room" as in "2+2≠5"?

Call it: the equi-univocal problem. Narboux writes:

> In unfolding the first puzzle, we were led to ask whether "red" could assume the same sense in both "This ball is red" and "This ball is not red". By contrast, in articulating the third puzzle, we are driven to ask whether "not" can assume the same sense in both "This ball is not red" and "This ball cannot be red and green at the same time at the same spot" (p. 165).

So, the task is to resolve how sentential negation can be used in different ways. For the proposition *p* standing for "the book is not on the table", we write: ~*p*, just as we would write it if the proposition were standing for "there is no square circle". Narboux thinks that this constitutes an additional fold, one that calls into question the intelligibility of negation itself, because it forces us to make distinct use of "the same class of markers for sentential negation" (p. 164). Narboux emphasizes what he takes to be a foundational similarity between the equi-univocal problem and the verb "to be". His idea is that negation and being have simultaneous unity and distinction that become expressed in varying uses.

Narboux's attempt to alleviate the third fold is to suggest that "the assumption that if the meaning of negation is exhausted by its use, then any change in its

grammar must induce a change in its meaning must be relinquished" (p. 186). Narboux argues that this a commitment to this assumption is to be found in the Tractatus, where, on Narboux's account, the foundation of essence is to be found in the rules governing its use. The problem that Narboux takes with this is that negation is inaccurately categorized as something of absolute simplicity; but as Narboux argues, it is possible to use negation in at least two different ways. So the logical independence of negation cannot be secured, thereby making it unintelligible. By hinting at relinquishing such a commitment, Narboux's idea is to disagree with the thought that use and meaning are indistinct. But this idea is not elaborated.

So far, I have gone through Narboux's threefold puzzle of negation and have touched upon the ways that he thinks it ought to be addressed. We have three folds: the Frege-Wittgenstein problem, the incompatibility problem, and the equi-univocal problem. Narboux only offers partial solutions, all of which, I will argue, are inadequate.[8] Despite the inadequacy of the account, Narboux gives shape to a problem that has been historically complex. In what follows, I will show that we can make use of Narboux's sceptical puzzle by transposing it into a naturalized account of negation. The result will be that the puzzle vanishes.

II. Naturalizing Negation

5. I would like to say straightaway: I do not think that the threefold puzzle of negation is threefold. I do think of Narboux's framing that it provides a useful way of seeing how exercising negation is a natural part of judgment. So, I will offer my solution along the three folds of the puzzle. But it is my hope that this structure will be seen as climbing Narboux's scaffolding, to borrow Wittgenstein's metaphor, in order to do away with it. In fact, I think that seeing negation as natural depends upon treating each fold like falling dominos. In what follows, I outline a way of making the dominos fall.

6. It is important to start by pointing out that Narboux is only able to offer a distinction between the first and the second fold (thereby the existence of the third) as a result of a commitment to a particular view of truth and epistemology about negation. The commitment is to the thought that truth is something that one applies to propositions, as it were, from the outside or sideways-on. For example, we can say *of* a book that it is not on a table or we could say *of* a shape that it is not square and circular. Narboux falls for thinking that the truth-conditions for such propositions are present to judgment in a special, metaphysical way, outside of a space of reasons. In particular, Narboux writes as if this special, non-normative view of books and circles, defines the range of negation. Now, as Rorty (1979) shows, such views run deep in the history of philosophy. But I, following Rorty, would like to point out that they are optional. This is the first domino.

I wrote above that if we transpose negation into the intentional register—seeing

it as something we do—then we are obligated to account for the conditions to know what it is we are doing. Knowing what one does (or, in Stroud's terms, doing something and knowing that it is you who is doing it), implies that one explicitly is making inferences in a shared space of reasons. So, we can truly say that:

> The book is not on the table

without committing a logical error. *That* the book is not on the table hinges on its possibly being there; we in fact could not even recognize things like books and tables without having first mastered the capacity to tell whether or not any object is before us, ostensibly or truly.[9] But the intelligibility of a proposition about a book being on a table (or not) is a result of our having mastered knowing which conditions such a proposition are relevant to and being reliably able to report to others on and in such conditions.

The issue with Narboux tying together negation (as a pragmatic operation) with intention is that it raises the problem of how one knows that they are doing negation (i.e., knowing that one is making a negative proposition). The question, like other questions that rise up when linguistic acts are put into the sceptical register, is this: if negation is bound up with intention, then how is it answerable to the world as an external constraint? This is what makes the second fold of the puzzle seem, to Narboux, most confounding. Stroud (2013) says:

> You come to know in that way what the results of your intentional actions are, but that is not necessarily to know *that* you are intentionally doing what you find you are doing or have done. Knowledge of what is so in the world is helpful in telling an agent to what extent what he has brought about accords with what he intending to bring about. But it can only tell him such a thing only if he knows what he is intending to bring about (p. 7).

Saying whether or not something is the case depends necessarily upon an array of background context wherein one knows—knows without even needing to think about it—what facets of the environment are salient for a judgment. The point is that our capacity to discriminate between states of affairs is nothing special and certainly nothing puzzling. In fact, we share this capacity with other animals and human infants. A famished cat or human infant are clearly able to tell whether or not there is an edible object before it. And they are disposed to reliably respond accordingly (say, having a fit, crying, etc.) (Rorty 1979; Brandom 2014; 2019). And the same is the case for raw feels, which, I argue should be the model from which we form a dialectical understanding of negation. Rorty (1979) says:

> The existence of raw feels—pains, whatever feelings babies have when looking at colored objects, etc.—is the obvious objection to this doctrine (of Sellars' psychological nominalism). To counter this objection, Sellars invokes the distinction between awareness-as-discriminative-behavior and awareness as what Sellars calls being 'in the logical space of reasons, of justifying and being able to justify

> what one does'. Awareness in the first sense is manifested by rats and
> amoebas and computers; it is simply reliable signaling. Awareness
> in the second sense is manifested only by beings whose behavior we
> construe as the utterance of sentences with the intention of justifying
> the utterance of other sentences. In this latter sense awareness is
> justified true belief—knowledge—but in the former sense it is ability
> to respond to stimuli (p. 182-183).[10]

So, we can view negation as normatively intentional while, at the same time, view
it as possible through our ground-level capacity for discriminative awareness.
The latter addresses the Frege-Wittgenstein problem; the former addresses the
incompatibility problem. And there is nothing puzzling about this (in fact, we
almost always *expect* such reliable responses from those we share language with).
But when we give this or any other process an abstract term (negation, violation
of expectation, disappointment, exasperation, etc.) we make ourselves confused.

Consider the English-language dialogue:

> A: Is there a book on the table?
> B: *No.*
> A: How do you to say what is not the case?
> B: *Well, I just do!*

Now, consider it in ordinary discourse. What should be clear is that A and B
take it for granted that they both know what kinds of things are denoted by the
functional terms "book", "table" and ostensibly quite a bit more to boot. If the
natural linguistic condition for the dialogue to occur had not been mastered by
both language users, then the entire linguistic exchange would be impossible.
Suppose A speaks German and B does not:

> A: Is there a *Buch auf dem Tisch*?
> B: *I don't know.*
> A: How can you not know what is or is not the case?
> B: *Because I don't know what role those words play in my language.*

Now, unless B comes to master basic German sentences, it is hopeless of A to ask
of B anything about books on tables. A's question is, to B, incomprehensible.

The idea is that by judging something to be the case (or not), we are already
engaged in the process of making material inferences expressively recognizable
only in normative language, and that, in certain circumstances, those material
inferences are of material incompatibilities. In fact, I would like to say that one
cannot judge anything at all without having mastered the capacity to judge
material incompatibilities ("this is not my body"; "*you* are something other than
"*I*"; "this is not edible"; "*this* is how these words are *used* (and *not otherwise)*";
etc.), for one would be unable to infer under which circumstances any judgment
would be relevant.

By making reports of our judgments (and by knowing that judgment can be used
in reports at all), it is necessary that we would be able to judge that things *could*

have been otherwise. When Narboux offers up something along similar lines, the picture is limited by understanding it as Wittgenstein's counterfactual trick (*Wie wäre es denn, wenn's anders wäre?*) (p. 168). But the solution, for the entire puzzle, is in recognizing that Wittgenstein's move is not counterfactual, but *subjunctively conditional.* *That* one knows that things like books *could be* on things like tables is what makes the negation "the book is not on the table" possible in a non-puzzling way. The point is that, despite seeming like a reductive movement of judgment, negation generatively instantiates a method of recognizing the limits of our shared space of reasons.

The point that I would like to make is that we *require* negation in order to understand the limits of judgment; but that the idea of a limit of judgment comes by generatively *considering possibilities* for ways of expanding the space of reasons (say, developing a method for cold fusion in the laboratory, to borrow an example from McDowell). I think that this is especially clear in Cora Diamond's warning about ways of thinking about sentences (hence, negation as a part of sentential logic). Diamond (1995) says:

> So, for Wittgenstein, the sign for what is the case (or is not the case) is the sentence, a sign to whose functional character it belongs that no sentence's truth or falsity can rob it of its capacity for comparison with reality; and that logical character is tied to a particular kind of substitution rule, different from that for any referring sign. Possibility and necessity get expressed in the use of ordinary sentences, in inferences from these sentences to more of these sentences. It is a mistake to think that you can in thought catch hold of, mean, that possibility that is reflected in the ordinary sentence you use, and, for example, consider what underlies its being possible, as if that were a characteristic of something (p. 200).

The point is that if negation is divorced from the requisite communicable conditions of inference, it cannot play a role in our discourse. The converse aspect of this point is that once negation comes into view as natural and normative, a wide range of propositional thinking—linguistic behavior—becomes available to us. Brandom (2014) calls a version of this the Kant-Sellars thesis: making reports of experience (and by knowing that experiences can be used in reports at all), requires that we would be able to make other kinds of reports of experience. So, we must have linguistic conceptualization active in experience in order to exercise the criteria for knowing which kinds of reports we ought to make (say, negating a possible location of a book). Negating what is not the case *qua* the first fold depends upon comprehensible conditions in a shared space of reasons.

To respond to the question: *How can one negate what is not the case?*, one could reply: *Because we are speaking like normal persons who have learned how to make inferential reports of what we perceive, ostensibly or not.* We should not follow Narboux in trying to have cake and eat it, too. We cannot say, on the one hand, that negation is bound up with intentionality, and, on the other, demand

a special kind of universal, extra-linguistic, non-normative quality to *what* it is. I would accept only a modified version of the first part of that problem; it needs to be modified to not have it bound up with *intentionality*, which comes later, but with the *shared linguistic instantiation of natural perceptual capacities. That* we call normative functions *intentional* brings nothing to bear on our capacity to differentiate situations from one another. I cannot see what remains puzzling about the Frege-Wittgenstein problem. And now we've started talking about how to address the second fold.

7. Now, the main part of the puzzle—the incompatibility problem—is already partially fixed by Narboux addressing the first fold through making it intentional. The unexplored corollary to this move, though, is that it becomes explicitly pragmatic (doing something) *and* implicitly normative (doing something with words in a shared linguistic practice). The way to leverage the intentional framing of the first fold into a natural normative practice for putting the second to rest is by recognizing that negation is not something extra-linguistic. It has no foundation *outside* of grammar. There is no need for a sideways-on view of our puzzle.

The incompatibility problem has, on Narboux's account, been put backwards. We need to recognize that negation of material incompatibilities ("there is no square circle"; "there is no reddish-green"; "the table is not a sweet", etc.) *is* the external constraint to intentional action. It is something that we know before negating any other propositions. Stroud continues:

> When I have finished my calculations [of multiplying 79x45 'in my head'], someone could report, with amused incredulity, 'He multiplied 79 by 45 in his head and look what he got!'. Nonetheless, that is what I was doing, and I knew that I was doing it. This shows that what I actually do does not serve as a ground or basis of my knowledge of what I am intentionally doing. I think my knowledge that I am intentionally multiplying 79 by 45 has no *ground* or *basis* in that sense. It is something that I know, but not in any derivative or indirect way, just as my perceptual knowledge that there is a red tomato on a white plate before me is not *based* on or *derived* from what I see. In that case, what I see *is* what I know to be so (p. 8).

I think that we can apply this line of thought to all three folds of Narboux's puzzle, but that it comes out especially clearly when addressing the incompatibility problem. The point is that one makes (or is capable of making) distinctions non-intentionally. That we are capable of doing so *and knowing that it is we who do so* shows that the origins of the intelligibility of negating material incompatibilities are grounded in a shared space of reasons. When Wittgenstein asks[11] about what kind of reality could hold the statement "there is no reddish-green", he is not lodging an intractable problem. Wittgenstein is saying that we are capable of knowing that the statement "there is no reddish-green" is true if and only if our way of getting about our world prevents us from perceiving reddish-green items. The kind of reality that the statement holds is *our* reality, one that we

are naturally a part of. That is the point.

I wrote above that seeing the first and second folds of the puzzle as distinct rests on a particular view of truth. In particular, Narboux is only able to formulate the threefold puzzle of negation with a kind of correspondence theory of truth. It comes about by standing on premises like:

> P1: If there is no book on this table, then I cannot negate it. For it is not the case to begin with.

and:

> P2: There are no square circles or objects which are reddish-green at the same location and time. So, I cannot negate such objects. For it is not the case that things like square circles exist, other than as nonsense.

To be able to formulate P1 and P2 as *puzzles* requires a commitment to truth (and so negation) as something that descriptively mirrors states of affairs in a dichotomous manner. Put differently, truth is something for Narboux that *exists outside* and is eligible for us to apply onto situations. Truth, and so negation, for Narboux, are metaphysically real; accordingly, they delimit, quite unnecessarily, what we can say and know about how we use negative judgment. Narboux fails to consider that truth, and so negation, can be naturally *and* normatively real at once, with the external constraining conditions on thought being objective without being mysteriously metaphysical. For we can confidently say that the natural conditions for our normative frameworks—the ways in which we speak—permit and preclude certain kinds of discourse. But that which precludes is simply out-of-reach: we cannot speak where we *cannot speak*.

If we commit ourselves to a different view of truth along these lines, the puzzle begins to unfold. I think that we can assemble a perspective of negation (and so work toward a natural but non-metaphysical account of truth) built out of a disquotational theory of truth and a modest form of direct realism. Doing so allows one to view negative judgment as something natural to *our* world. This is the second domino.

Consider G. Spencer-Brown's example about drawing circles (Spencer-Brown 2014). If the world were such that every visible surface were a torus, our capacity to draw circles would hinge on the capacity to make oblique angular cuts through surfaces; doing so makes the idea of Villarceau circles intelligible. But if Villarceau (or anyone else) had never done this with a torus, then we could reasonably say: *there is no such thing as a circle.* This is where Narboux's second fold enters the discourse of a torus world. But the fact is that, despite persons believing otherwise prior to Villarceau's discovery, drawing circles on tori turned out to be mathematically possible. That is, with the discovery of Villarceau circles, a theoretically postulated particular became a concrete part of the world. The point is that one's capacity, at the turn of the nineteenth century, carried with it the possibility to truly negate the proposition that *there*

is no such thing as a circle in a world comprised exclusively of torus surfaces.
This particular negation made the discovery possible in the first place.

I am trying to convey that there is something natural—so, intelligible—about our capacity to recognize—so, negate—material incompatibilities (or to put it in Hegelian terms: consequences from determinate negations).[12] And what is natural about the recognition of material incompatibilities is that we are able to instantiate them as natural linguistic items that have emerged between language users interacting with the world. Accordingly, material incompatibilities are eligible for us to take up in our shared space of reasons. And they become eligible when we frame them in terms of disquotational truths expressed by norm-governed employment. In responding to the same Rorty text I noted above, McDowell (2005) outlines a view of this. Take the proposition:

> Cold fusion has not been achieved in the laboratory.

or its alternative:

> It is not the case that *cold fusion has been achieved in the laboratory.*

Despite the fact that what is negated is of a piece with "there is no square circle" or "there is no reddish-green", there is nothing puzzling about this. We (ordinary persons, physicists, or mathematicians of geometry) just know that the negated proposition is true by the lights of disquotability. That this is simultaneously and isomorphically *normative* comes about by the fact that it may very well be possible for persons or other animals to evolve to see reddish-green surfaces, to model square circles, and to achieve cold fusion in laboratories. We must see the incompatibility problem of negation in a twofold manner: we can only make negative propositions that are true through disquotation because we are capable of considering modal, subjunctive conditional, or counterfactual alternatives; we cannot make negative propositions in cases where our conceptual capacities cannot consider such alternatives.

So, Narboux is correct in the hypothetical case that one could not negate *nothing in itself*, but the very idea of a *nothing* having an *in itself* is metaphysically outlandish. We simply and matter of factually cannot imagine a state of affairs where the second puzzle would apply. If we *could*, we could intelligibly negate it. And we cannot because it is necessarily the case that something that could not figure into our space of reasons simply *doesn't*. It is *we* who instantiate the second fold when we insist, in quite an unfounded manner, that statements like "there is no square circle", "there is no reddish-green", etc., cannot be negated without rendering negation unintelligible.

8. But here one must take caution. For, without a non-monotonic view of negation, we risk slipping back into making egregious metaphysical claims (say, *it is not possible for one to draw circles on a torus*). We need to see that it is possible—in fact, I urge this view—to have external constraints on knowledge (and thereby intentional acts like negation) while nevertheless discarding the idea that we have some kind of privileged access to the "logical picture" of the

world because we are capable of intentional acts. The point is that we can say of the proposition "the book is not on the table" that it is true, as the atomic elements of the proposition logically depict a particular state of affairs, but we *cannot* say that our logical pictures take place outside of normative commitments to others in our space of reasons. Such a view of truth avoids both Givenness, as destroyed by Sellars, as well as a bare constructivist relativism that says of truth that it is meaningless *qua* being built socially. Instead, we can embrace the thought that our propositions are materially isomorphic with the natural world only because *they are a part of the natural world.*[13]

So, we can save the intelligibility of negation without committing ourselves to a view of truth that cannot be revised and from a view of truth that is relative, in both scientific and discursive practice. The point is that if we remain fixed in keeping a distinction between the first and the second fold of the puzzle, we *prevent* more adequate views of the world from forming. That is, if we say and believe that *there is only three-dimensional space* and make the negation of that statement incomprehensible, it would not be possible to identify the parameters for modeling four-dimensional space (or any other unobservable theoretically postulated item which may turn out to be true in just this normative-natural dialectic that I have been urging we adopt).

9. Now that the first and second folds have been put to rest, there is no reason that we need to hold onto the third, as it is framed as a consequence of the others. But I think that it is worth ending this section with a final treatment of the entire puzzle.

A way of addressing the third fold in light of my above treatment is to say that the uni- or equivocality of negation is besides the point. What *does* matter is that we are able, by the lights of the Kant-Sellars thesis, to distinguish just what sorts of statements we negate in relevant circumstances. McDowell (2005) says:

> It is the thought that things really are a certain way: for instance, that cold fusion really has not occurred. To insist on this distinction is not to try to think and speak from outside our practices; it is simply to take it seriously that we can really mean what we think and say from within them (p. 140).

That questions which solicit negative assertoric responses have different kinds of shapes (*Do square circles exist?*; *Is the book on the table?*) brings nothing to bear on our capacity to correctly reply: *No.* It is about the question and patently not about the possible assertoric response. That we are able to exercise the capacity to correctly reply to such questions is a consequence (or achievement) of our having taken up a justified role in a shared space of reasons. Across the range of possible *yes-no* questions is the fact that we are normatively governed by saying what can be justified. And saying "*no*" to any proposition-soliciting question requires only that we have mastered the norms required for being able to give any answer at all. So when we say "the book is not on the table" or "there is no square circle", *all* that we are doing is providing a response to a

proposition-soliciting question or situation.[14] This is the third domino.

III. Conclusion: Naturalized Negation & Asymmetry in Propositions

10. So far, I have outlined Narboux's formulation of the threefold puzzle of negation. I have shown that the main issue with the puzzle comes from Narboux's sceptical register in tandem with a particular sideways-on, correspondential view of truth. I have quieted the puzzle by demonstrating that it is not actually threefold: the problem is about accounting for objectively constrained normative practices and is *not* about something mysterious about the pragmatic process that we call *negation*, or truth more broadly. We can now recognize that *naturalized negation* is unproblematically simple: our ground-level ability to discriminate situations cooperates with our capacity for conceptualization and results in normative sequences that get intentionally taken up in shared linguistic practices.

11. I would like to close this treatment of the puzzle with a provocation: *Could there be any kind of judgment without negation?* And I think that the answer is *No*.

While walking through Parmenides' gate, Irad Kimhi says that positive and negative assertoric acts are a part of the same two-way capacity: the *same* determining conceptualization is at work when we judge something's being the case or not (Kimhi 2018). But Kimhi says of the two-way capacity that it is *asymmetrical* and chooses to prioritize positive over negative assertion as being logically foundational. I agree that the judgment is the same two-way capacity; I also agree that it is an asymmetrical capacity; but I disagree with the prioritization of positive over negative assertion.[15]

The purpose of turning Kimhi's asymmetry around here is that, in its original shape, it comes from an attempt to answer the second fold of negation in its own way. Kimhi (2018) says:

> The puzzle of the meaningfulness of a negated predicate (which, as we have indicated above, is not distinct from the puzzle of veridical being) will disappear once we recognize that the difference between the predicate and the negation of the predicate is only syncategorematic— or, to put the same point differently: that the contradictory difference between combination and separation is not a predicative difference (p. 19).

Kimhi's elegant argument for this asymmetry is built along the following lines:

> I wish to propose that the contradictory judgments "S is *F*" and "S is not-*F*" are to be understood as the positive and negative acts of the a single two-way logical capacity—which, as we shall see, can be

specified through its positive act: "S is *F*". The capacity is *asymmetrical*, since the positive act is prior to the negative. This means that the only predicative determination in a simple contradictory pair is the *positive* predication. Yet even the positive case is essentially one of a *pair* of acts.

[. . .]

Between the positive and negative acts of predication, there is only ever a single determination in play. Therefore, the difference between the two cases is not a predicative difference. Rather, it is an example of what I shall call *syncategorematic difference* (p. 20).

Kimhi's idea is that *p* and *~p* are a contradictory pair of assertions and uses the term *unit* to name the pair. In light of the unit, *p* exemplifies the assertion it makes by standing on its own without a sentential operator. By contrast, *~p* does not exemplify the assertion it makes but only gestures to it, as it requires a sentential operator to be what it is. That the unit contains the exemplifying *p* in both of its branches, while the non-exemplifying negation of *p* cannot, allows Kimhi to urge that the former takes priority within the unit. The positive proposition entails combination and the negative proposition entails separation in that it removes something from the former. This is what Kimhi bases his distinction between predicative and syncategorematic difference upon.[16]

12. I think that Kimhi is correct to demonstrate that there is asymmetry between the unit *p* and *~p*. I think that the direction of the asymmetry is backwards.

The problem with Kimhi's asymmetry is that it assumes that the act of distinction which recognizes material incompatibilities—those which enable ordinary inferences—is a positive move. But this cannot be. If it were, it would be materially impossible to discriminate between different kinds of objects, and would thereby render negation impossible.

This is a simple point. Consider the point of view provided by Tarskian sentences. The T-sentence:

"*X*" is true iff *p*

hinges on one's capacity to be able to put the conceptual capacity of determinate difference to work. In order to assert, in any language (formal or otherwise), that *"snow is white" is true iff snow is white*, one must already be able to negate the possibility of snow being white. Positive predication is contingent upon negative judgment. Otherwise T-sentences would be unintelligible. For, how is the description *white*, the object *snow*, or the name *"snow is white"* possible without relying on a contrast case for each aspect of the statement?

I am saying that we could not do anything without first having mastered how to make distinctions of determinate difference, the original distinction being simple, ground-level negative cognitive judgments (e.g., learning that one's body is not the body of another person). The point is that one could not recognize a difference between *p* and *~p* without first knowing that *each member of the*

pair of propositions designates difference. I think that teaching and learning arithmetic demonstrates this.When we teach children to count, we teach them to take contradictory pairs (say, 3 and 333) and perform functional operations with them. We can teach a child to perform the calculation, but their ability to do so must come from first knowing that 3 and 333 are distinct items.[17] So it is with *p* and *~p.*

We can only differentiate, say, colours—even one colour by itself—insofar as we are able to exercise the conceptual capacity to make distinctions at all. If we accept the above treatment of the threefold puzzle, we must concede that the ability to token different colours with different terms is present in the first cognitive distinction between two items. In other words, if I want to normatively label a patch of something as red, I can only do so if it naturally appears distinct to me from other patches. By the same lights, if I want to say of something that it is either *red* or *not-red,* I must have first made the distinction—have mastered the linguistic (i.e., articulable) conceptual ability to discriminate—between those two possibilities. There can be no *p* without *~p.* There can be no judgment without naturalized negation.

For how could one know that one is judging without judging *against something?*

The consequence of naturalized negation, however, is that we must come to see negation as an inference like any other. That it is logically prior to positive inference brings nothing to bear upon this. So, rather than take negation to be its own *thing,* something with a peculiar nature, we ought to see it as occupying a primary point along a *phylogeny of inference.*

13. By recognizing that negation is natural to thought—that our capacity to *use* it is what permits our participation in a language game—I have put the threefold puzzle to rest. I have also indicated that we cannot make inferences without first being able to make negative distinctions. We have come to see negation as natural to and necessary for thought, something that is bound up with every possible judgment one makes. It is at this point that we can begin to see how negation is bound up with limits of intelligibility and intentionality: where one cannot make a distinction, one cannot judge. By urging a phylogeny of inference—as opposed to isolated object-like intentional operators—I have tried to indicate that all of our inferential movements are tied together. I have also tacitly indicated that there might be a processual ontogeny in the order of judgment. A more substantial treatment of this thought would take us too far away, so I close this account by only gesturing toward this direction.

—AP. September 2024.

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1. Narboux defines intentionality as "thought's capacity to engage with reality, be answerable to it" (p. 154).
2. Narboux: "This essay belongs almost entirely to the aporetic register" (p. 155).

3. My rehearsal of Narboux's argument will be concise. There are two reasons for this: 1. it is not my task to repeat more than the essential aspects of the argument; 2. Narboux goes too far afield in his discussion of, in particular, the second fold of the puzzle by speaking about Freud and denial. In my view, this has no place here or anywhere else in contemporary philosophy. Rather than focus on why I think this is at best superfluous, I avoid discussion altogether and leave the reader to determine its sense and worth.

4. In Wittgenstein's sense of the word. See: *On Certainty*, §341.-§344. The point I will argue along these lines is that we cannot begin to show anything without recognizing negation as necessary for thought.

5. See: *Notebooks, 1914-1916*, 15.11.14., p. 30e.

6. Though it goes unmentioned, this idea was made explicit by Brandom's account of Hegelian determinate negation. I will return to this below to argue that Brandom's machinery offers part of the solution to the second-fold of the puzzle.

7. By using this name, I am straightaway suggesting that the problem is not as drastic (or even so different from the first) as Narboux thinks. I return to this in (II.).

8. For no other reason, because they are dramatically sparse. I think that Narboux spends more time on tangential matters than the puzzle itself; the consequence is an incomplete exposition of and solution to the puzzle. See Footnote 3 above.

9. cf. McDowell's *Having the World in View* for a complete dissolution of the distinction between ostensible seeings and true seeings.

10. For a concise discussion of Sellars' psychological nominalism, see Rorty's *Philosophy and the Mirror of Nature*, Part Two, Chapter IV, §2-3 (p. 173-192).

11. In *Lectures on the Foundations of Mathematics* and taken up by Narboux.

12. cf. Brandom (2014; 2019)

13. I suggest that this is how one ought resolutely to read Wittgenstein's Tractarian isomorphism. The point is that the isomorphism is not between our *thoughts* and *the world* but as recognizing that our thoughts are objects of the world, natural to our judgment and conceptually articulated in a natural normative framework with others who share in this capacity. So, we can resolutely say of the propositions 1.1-1.13 from the Tractatus that it is our language-game, our shared logical space of reasons, which are the "totality of facts" but that such a totality is determinate (1.11) only insofar as what is eligible to play a role in our space of reasons is what would already be natural to our linguistic capacities. *That* we cannot sensibly perceive or know square circles is a *natural*—hence determining—fact of our language game. Again, the isomorphism is about thinking *as* language and *is not* about some needless metaphysical canyon between thinking and the world.

14. I think the idea of making a distinction between *question* and *situation* is problematic, but that is not the point of this essay.

15. I would like to say that I altogether disagree with Kimhi's reliance on a foundation of judgment, outside of a space of reasons. I do not have the space and it is not the point of the present writing to develop this thought here.
16. Kimhi says: "A categorematic expression can be a component of a predicative proposition, whereas a syncategorematic expression cannot be a component of a predicative proposition [. . .] and do not correspond to any bit of reality" (p. 16-17). I think that this distinction deserves more elaboration than Kimhi provides, given the importance to his argument.
17. Items in the dynamically processual and pragmatic sense of being things that one uses to perform functions.

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