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$\boxed{\text{Extra lv2}}$

$$\begin{cases} x = e^s \cos t \\ y = e^s \sin t \end{cases} \qquad f(x,y) = f(x(s,t), y(s,t)) \in C^2$$

Kedjeregeln

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}$$ Analogt för $\frac{\partial f}{\partial t}$

$$\frac{\partial^2 f}{\partial s^2} = \frac{\partial}{\partial s}\left(\frac{\partial f}{\partial s}\right) = \{\text{Produktregeln}\}$$

$$\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial x}\right)\frac{\partial x}{\partial s} + \frac{\partial f}{\partial x}\frac{\partial^2 x}{\partial s^2} + \frac{\partial}{\partial s}\left(\frac{\partial f}{\partial y}\right)\frac{\partial y}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial^2 y}{\partial s^2}$$

Kedjeregeln ger

$$\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}\frac{\partial x}{\partial s} + \frac{\partial^2 f}{\partial y \partial x}\frac{\partial y}{\partial s}$$ & analogt

$$\frac{\partial}{\partial s}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}\frac{\partial y}{\partial s} + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial x}{\partial s}.$$

Symmetri för t & s ger då sammanfattningsvis, genom användande av

$$\begin{cases} \frac{\partial x}{\partial s} = +x & \frac{\partial x}{\partial t} = -y \\ \frac{\partial y}{\partial s} = +y & \frac{\partial y}{\partial t} = +x \end{cases} \qquad \begin{cases} \frac{\partial^2 x}{\partial s^2} = x & \frac{\partial^2 x}{\partial t^2} = -x \\ \frac{\partial^2 y}{\partial s^2} = y & \frac{\partial^2 y}{\partial t^2} = -y \end{cases}$$

$$\frac{\partial^2 f}{\partial s^2} + \frac{\partial^2 f}{\partial t^2} =$$

$$\frac{\partial^2 f}{\partial x^2} \cdot \underbrace{\left(\frac{\partial x}{\partial s}\right)^2}_{x^2} + \cancel{\frac{\partial^2 f}{\partial y \partial x} \cdot \underbrace{\left(\frac{\partial y}{\partial s}\right)}_{y} \underbrace{\left(\frac{\partial x}{\partial s}\right)}_{x}} + \cancel{\frac{\partial f}{\partial x} \underbrace{\left(\frac{\partial^2 x}{\partial s^2}\right)}_{x}} +$$

$$\frac{\partial^2 f}{\partial y^2} \underbrace{\left(\frac{\partial y}{\partial s}\right)^2}_{y^2} + \cancel{\frac{\partial^2 f}{\partial x \partial y} \underbrace{\left(\frac{\partial x}{\partial s}\right)}_{x} \underbrace{\left(\frac{\partial y}{\partial s}\right)}_{y}} + \cancel{\frac{\partial f}{\partial y} \underbrace{\left(\frac{\partial^2 y}{\partial s^2}\right)}_{y}} \Big] +$$

Symmetri i s & t

$$\Big[\frac{\partial^2 f}{\partial x^2} \underbrace{\left(\frac{\partial x}{\partial t}\right)^2}_{(-y)^2} + \cancel{\frac{\partial^2 f}{\partial y \partial x} \underbrace{\left(\frac{\partial y}{\partial t}\right)}_{x} \underbrace{\left(\frac{\partial x}{\partial t}\right)}_{-y}} + \cancel{\frac{\partial f}{\partial x} \underbrace{\left(\frac{\partial^2 x}{\partial t^2}\right)}_{-x}} +$$

$$\frac{\partial^2 f}{\partial y^2} \underbrace{\left(\frac{\partial y}{\partial t}\right)^2}_{(+x)^2} + \cancel{\frac{\partial^2 f}{\partial x \partial y} \underbrace{\left(\frac{\partial x}{\partial t}\right)}_{-y} \underbrace{\left(\frac{\partial y}{\partial t}\right)}_{x}} + \cancel{\frac{\partial f}{\partial y} \underbrace{\left(\frac{\partial^2 y}{\partial t^2}\right)}_{-y}} \Big]$$

$$= (x^2 + y^2)\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right) \blacksquare$$

Cancellation av termer med pilar.

funktionen C^2 så förenklas ty $f''_{xy} = f''_{yx}$