CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 18: Random Sampling

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Questions

1. In the lecture, we talked about performing three tests to make sure the random numbers generated using 'rand(10000)' in Julia are infact random. **[5 points]**
   a. If you generate random samples from an exponential distrbution using 'rand(Exponential(1),10000)', do you expect them to pass the same three tests? Provide a reason.
   b. Perform the three tests using the provided sample Julia code and list your observations.

2. Use inverse transform method to generate samples from a Poisson distribution with $\lambda = 2$. **[10 points]**
   a. Describe the steps involved.
   b. Write Julia code to generate Poisson samples.
   c. Compare this with random samples generated using 'rand(Poisson(2),10000)'
   d. Evaluate (using the three tests) if these are infact random numbers.

3. *Box-Mueller Method* is for generating samples from a Gaussian distribution using two independent random samples from a Uniform distribution. If $u_1, u_2 \sim U(0,1)$, then $x_1, x_2 \sim \mathcal{N}(0,1)$, where **[10 points]**

$$x_1 = \sqrt{-2\ln u_1}\ cos(2\pi u_2) \quad \text{and} \quad x_2 = \sqrt{-2\ln u_1}\ sin(2\pi u_2)$$

   a. Describe the steps involved in generating Gaussian samples, using above functions.
   b. Write Julia code to generate samples.
   c. Compare histogram of these samples with that of random samples generated using 'rand(Normal(0,1),10000)'
   d. Evaluate (using the three tests) if these are infact random numbers.

Bonus question

1. Use inverse transform method to generate samples from a mixture of Gaussian distributions.

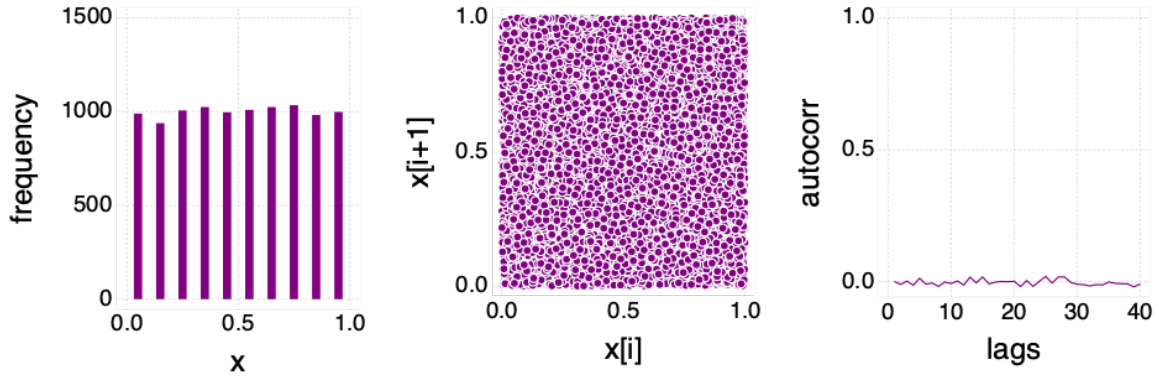
$$f(x) = 0.34\mathcal{N}(0,1) + 0.33\mathcal{N}(-1,1) + 0.33\mathcal{N}(1,1)$$

   a. Describe the steps involved.
   b. Write Julia code to generate samples.
   c. Compare histogram of these samples with that of random samples generated using 'rand(Poisson(2),10000)'
   d. Evaluate (using the three tests) if these are infact random numbers.

Sample code

1. Three tests to evaluate randomness

```
x = rand(10000);
myplot1 =  plot(x=x, Geom.histogram(bincount=10),
                Guide.xlabel("x"),Guide.ylabel("frequency"),white_panel);
myplot2 =  plot(x=x[1:end-1],y=x[2:end], Geom.point,
                Guide.xlabel("x[i]"),Guide.ylabel("x[i+1]"),white_panel);
myplot3 =  plot(x=1:40,y=autocor(x,1:40), Geom.line,
                Coord.Cartesian(ymax=1),Guide.xlabel("lags"),
                Guide.ylabel("autocorr"),white_panel);
myplot = hstack(myplot1,myplot2,myplot3);
draw(PNG("./figs/mt_rv_test.png", 10inch, 3.5inch), myplot);
```



2. Plotting the distribution of random samples along with the pdf for comparison.

```
u = rand(10000,3);
x = -log.(u);
myplot =  plot(layer(x=collect(0:0.1:10),
                     y=21000.*pdf.(Exponential(1),collect(0:0.1:10)),
                     Geom.line,Theme(default_color=color("blue"))),layer(x=x[:],
                     Geom.histogram(bincount=10)), Guide.xlabel("x"),
                     Guide.ylabel(""),white_panel);
draw(PNG("./figs/exp.png", 10inch, 3.5inch), myplot);
```