Assignment - 14

1)

a)

$$p(y|\theta) = p(y_1|\theta)\, p(y_2|\theta) \cdots p(y_{10}|\theta).$$

$$= \prod_{i=1}^{10} p(y_i|\theta).$$

$$= \prod_{i=1}^{10} \frac{e^{-\lambda} \lambda^{y_i}}{y_i!}$$

$$= \frac{e^{-10\lambda} \lambda^{\sum_{i=1}^{10} y_i}}{\prod_{i=1}^{10} y_i!}$$

K & $\prod_{i=1}^{10} y_i$ are constants

some remove from the equation.

b)

$p(\lambda) = K$

$$\int_0^\infty p(\lambda)\, d\lambda$$

~~always constant~~

$$\int_0^\infty K\, d\lambda = [K\lambda]_0^\infty$$

$= K(\infty) - 0$

$= \infty$

No it's not a proper prior.

c)

$p(\lambda|y) \propto p(y|\lambda)\, p(\lambda)$

$$\propto \frac{e^{-10\lambda} \lambda^{\sum_{i=1}^{10} y_i}}{\prod_{i=1}^{10} y_i!} K.$$

$p(\lambda|y) \propto e^{-10\lambda} \lambda^{22}$

$p(\lambda|y) \propto \lambda^{23-1} e^{\lambda/10}$

$p(\lambda|y) \propto \text{Gamma}(23, 1/10)$

e).

$p(\lambda) = \text{Gamma}(\alpha, \beta)$

$$= \frac{1}{\Gamma(\alpha) \beta^\alpha} \lambda^{\alpha-1} e^{-\lambda/\beta}$$

$$p(\lambda|y) = \frac{p(y|\lambda)\, p(\lambda)}{p(y)}$$

~~ignoring constants~~

$p(\lambda|y) \propto p(y|\lambda)\, p(\lambda)$

$$\propto \frac{e^{-10\lambda} \lambda^{22}}{\prod_{i=1}^{10} y_i!} \frac{1}{\Gamma(\alpha) \beta^\alpha} \lambda^{\alpha-1} e^{-\lambda/\beta}$$

$p(\lambda|y) \propto e^{-10\lambda} \lambda^{22} \lambda^{19} e^{-\lambda/0.1}$

$\propto \lambda^{41} e^{-20\lambda}$

$\propto \text{Gamma}(42, 1/20)$

f).

Using the information in the Gamma prior we can intrepret that the data has 19 crimes in 10 days which is very close to the true 21 crimes in 10 days.

g).

$p(\theta|y) \propto \lambda^{41} e^{-20\lambda}$

$\log p(\theta|y) \propto \log(\lambda^{41} e^{-20\lambda})$

$\propto 41 \log \lambda + \log e^{-20\lambda}$

$\propto 41 \log \lambda - 20\lambda$

$\frac{d \log(p(\theta|y))}{d\lambda} \propto \frac{41}{\lambda} - 20$

$\cancel{\frac{41}{\lambda} + \frac{\lambda^{2}}{20} = 0}$

$\cancel{\frac{40 \times 41 + \lambda^3}{40\lambda} = 0}$

$\cancel{\lambda^3 = -40 \times 41}$

$\cancel{\lambda = -11.7927}$

$\frac{41}{\lambda} - 20 = 0$

$20\lambda = 41$

$\lambda = 41/20$

$\lambda = 2.05$

$P(X \geq 4) = 1 - P(X \leq 3)$.

$= 0.152$

d)

In [1]:
```
using Distributions, Gadfly;
```

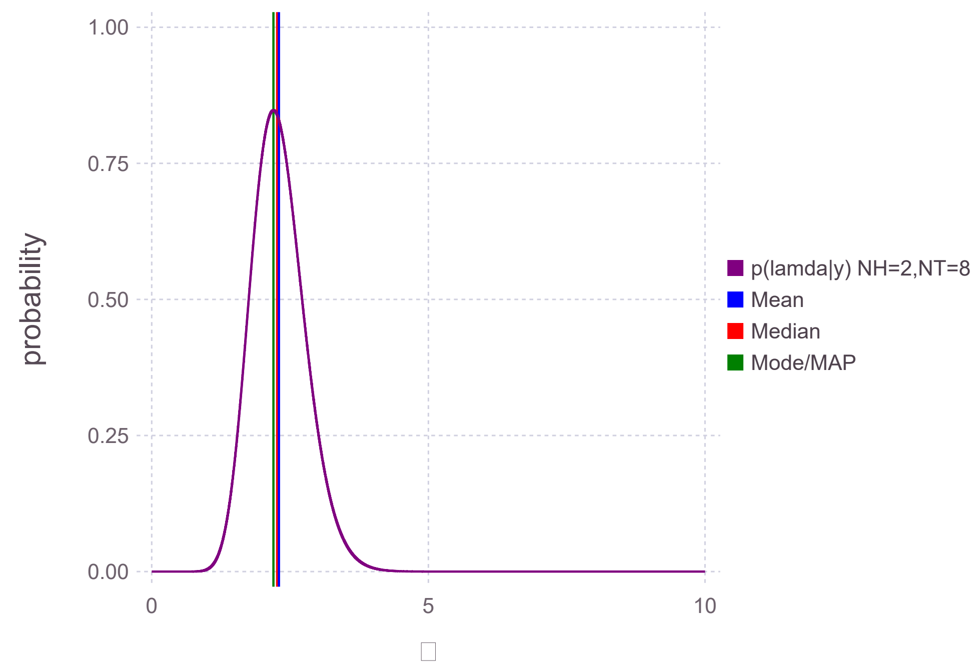
In [17]:
```
d = Gamma(23,1/10);
```

In [18]:
```
median_val = Base.median(d);
mean_val = Base.mean(d);
mode_val = Distributions.modes(d);
print("Median: ")
print(median_val)
print("\n")
print("Mean: ")
print(mean_val)
print("\n")
print("Mode: ")
print(mode_val)
print("\n")
```

```
Median: 2.2667539177878635
Mean: 2.3000000000000003
Mode: [2.2]
```

In [20]:
```
x = collect(0:0.0001:10);
prior = ones(length(x));
posterior1 = pdf.(d,x);
Gadfly.plot(
layer(x=x,y=posterior1,Geom.line,Theme(default_color=colorant"purple")),
layer(xintercept=[mean_val],Geom.vline(color=colorant"blue")),
layer(xintercept=[median_val],Geom.vline(color=colorant"red")),
layer(xintercept=[mode_val[1]],Geom.vline(color=colorant"green")),
 Guide.ylabel("probability"),
Guide.xlabel("□"), Guide.manual_color_key("", ["p(lamda|y) NH=2,NT=8",
"Mean", "Median", "Mode/MAP"], ["purple","blue","red","green"]))
```

Out[20]:



In [24]:
```
1 - cdf(Poisson(2.05),3)
```

Out[24]: 0.15200978018134315