CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 13: Introduction to Bayesian Estimation

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Questions

1. **Scene 1:** You are with a friend at a casino in Las Vegas where he is betting on the result of a coin toss. You noticed that out of 50 trials, there were 10 heads and 40 trials. At this point you left the scene to take a walk along the Las Vegas strip.

   Scene 2: When you returned from the walk, your friend tells you that there were 50 more trials that happened in the interim and 40 of those trials resulted in heads.

   For the following questions (a-d), state your prior, derive your likelihood, and show how you determine the posterior distribution clearly.

   a. Assume θ (the probability that the coin flip will result in a head) is a discrete parameter with values 0.2, 0.4, 0.6 and 0.8. Assume that the prior belief on these four states are $[0.1, 0.1, 0.3, 0.5]$. Compute the posterior distribution of θ with the observations from Scene 1. Is the resultant posterior in agreement with the prior? **[5 points]**

   b. Treating θ as a continuous parameter and using a 'flat' prior, derive the functional form of the posterior distribution with the observations from Scene 1. **[5 points]**

   c. Treat θ as a continuous parameter. Using posterior computed from observations in Scene 1 and data available in Scene 2, derive the functional form of the posterior distribution. **[5 points]**

   d. Treat θ as a continuous parameter. Using a flat prior and all the data available from Scenes 1 and 2, derive the functional form of the posterior distribution. Compare the posterior distributions in (c) and (d). Are they the same? **[5 points]**

   e. Generate one plot showing three distributions: the prior, the posterior distributions at the end of Scene 1 (determined in part (b) above), and the posterior at the end of Scene 2 (determined in part (c) above). Use the sample Julia code provided below. **[5 points]**

Bonus question

1. Consider the two scenes in the above question. Assume θ is discrete. Use the posterior distribution at the end of Scene 1 (from Question 1a) as a prior. Use the new observations from Scene 2 to compute the new posterior distribution of θ. Is the new posterior similar to the previous posterior or the original prior?

Sample code

```
using Distributions, Gadfly;
white_panel = Theme(
    panel_fill=colorant"white",
    default_color=colorant"purple",bar_spacing=3mm,
    major_label_font_size=18pt,
    minor_label_font_size=14pt,
    key_title_font_size = 18pt,
    key_label_font_size = 14pt,
    major_label_color=colorant"black",
    minor_label_color=colorant"black"
);

x = collect(0:0.001:1);
prior = ones(length(x));
d1 = Beta(3,9);
d2 = Beta(21,81);
posterior1 = pdf.(d1,x);
posterior2 = pdf.(d2,x);
myplot = plot(
        layer(x=x,y=posterior1,Geom.line,Theme(default_color=colorant"purple")),
        layer(x=x,y=posterior2,Geom.line,Theme(default_color=colorant"blue")),
        layer(x=x,y=prior,Geom.line,Theme(default_color=colorant"black")),
        Coord.Cartesian(xmin=0, xmax=1,ymax=10.2), Guide.ylabel("probability"),
        Guide.xlabel(" "), Guide.manual_color_key("", ["p( |y) NH=2,NT=8",
        "p( |y) NH=20,NT=80", "p( )"], ["purple","blue","black"]), white_panel);
draw(PNG("./figs/beta_posterior2.png", 8inch, 3inch), myplot);
```