CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 5 (Continuous Probability Distributions)

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Questions

1. The random variable x of the life lengths of batteries (in hundreds of hours) discussed earlier is associated with a probability density function of the form **[8 points]**

$$f(x) = \begin{cases} \frac{1}{2}e^{-x/2} & x > 0 \\ 0 & otherwise \end{cases}$$

   a. Find the probability that the life of a particular battery of this type is less than 200 hours or greater than 400 hours.
   b. Find the probabiliity that a battery of this type lasts more than 300 hours, given that it already has been in use for more than 200 hours.

2. The weekly demand x, in hundreds of gallons, for propane at a supply station has a density function given by **[5 points]**

$$f(x) = \begin{cases} \frac{x}{4}, & 0 \leq x \leq 2 \\ \frac{1}{2}, & 2 < x \leq 3 \\ 0, & elsewhere \end{cases}$$

   Find the expected weekly demand.

3. The space shuttle has a 2-hour window during which it can launch for an upcoming mission. Launch time is uniformly distributed in the launch window. Find the probability that the launch will occur within 10 minutes of the center of the launch window. **[4 points]**

4. The magnitudes of earthquakes recorded in a region of North America can be modeled by an exponential distribution with a mean of 2.4, as measured on the Richter scale. Find the probabilities that the next earth quake to strike this region will have the following characteristics: **[8 points]**

   a. It will be no more than 2.5 on the Richter scale
   b. It will fall between 2.0 and 3.0 on the Richter scale

Bonus questions

1. Arrivals of customers at a bank follow a Poisson distribution. During the first hour that the bank is open, one customer arrives at the bank.

   a. Find the probability that he arrives during the first 15 minutes that the bank is open.
   b. Find the probability that he arrives after the bank has been open 30 minutes.

2. Let x be an exponential random variable with mean θ. Show that

$$E(x^k) = k!\theta^k$$

3. Explosive devices used in a mining operation cause nearly circular craters to form in a rocky surface. The radii of these craters are exponentially distributed with a mean of 10 feet. Find the mean and the variance of the area covered by such a crater.