Assignment - 11

1).

a) $p(y|\lambda) = \frac{e^{-\lambda}\lambda^{y}}{y!}$

$$I_y(\lambda) = -E_{y|\lambda}\left[\frac{\partial^2}{\partial\theta^2}\log f(y|\lambda)\right]$$

$$= -E_{y|\lambda}\left[\frac{\partial^2}{\partial\lambda^2}\log\left(\frac{e^{-\lambda}\lambda^{y}}{y!}\right)\right]$$

$$= -E_{y|\lambda}\left[\frac{\partial^2}{\partial\lambda^2}\left[-\lambda\log e + y\log\lambda - \log y!\right]\right]$$

$$= -E_{y|\lambda}\left[\frac{\partial^2}{\partial\lambda^2}\left[-\lambda + y\log\lambda - \log y!\right]\right]$$

$$= -E_{y|\lambda}\left[0 - y/\lambda^2 - 0\right]$$

$$= \frac{E_{y|\lambda}(y)}{\lambda^2} = \frac{\lambda}{\lambda^2}$$

$$= 1/\lambda$$

[redacted]

$$I_y(\lambda) = \frac{10}{\lambda}$$

b)

$$p(\lambda) \propto \sqrt{I_y(\lambda)}$$

$$\propto \sqrt{\frac{10}{\lambda}}$$

c)

$$p(\lambda|y) \propto p(y|\lambda)\, p(\lambda)$$

$$\propto \frac{e^{-10\lambda}\lambda^{22}}{2^4 (6!)^2} \sqrt{\frac{10}{\lambda}}$$

$$\propto \frac{e^{-10\lambda}\lambda^{22}\, (10)^{1/2}}{2^4 (6!)^2\, \lambda^{1/2}}$$

$$\propto \frac{e^{-10\lambda}\, \lambda^{43/2}\, \sqrt{10}}{2^4 (6!)^2}$$

$$\propto \frac{e^{-10\lambda}\, \sqrt{\lambda^{43}}\, \sqrt{10}}{2^4 (6!)^2}$$

d)

$$\lambda = \phi^2$$

[redacted]

$$I_y(\lambda) = n/\lambda$$

$$I_y(\phi) = 10/\phi^2$$

$$p(\phi) \propto \sqrt{I_y(\phi)}$$

$$\propto \sqrt{10/\phi^2}$$

$$\propto 1/\phi \sqrt{n}$$

$$p(\phi) = \frac{\sqrt{n}}{\phi} \left| \frac{d(\phi^2)}{d\phi} \right|$$

$$= \frac{\sqrt{n}}{\phi}\, 2\phi$$

$$= 2\sqrt{n}$$

$$= 2\sqrt{10}$$

f).

$$P(\phi|y) \propto p(y|\phi)\, p(\phi).$$

$$P(y|\lambda) = \frac{e^{-10\lambda}\,\lambda^{22}}{\prod_{i=1}^{10} y_i!}$$

$$P(y|\phi) = \frac{e^{-10\phi^2}\,\phi^{44}}{\prod_{i=1}^{10} y_i!}$$

$$P(\phi|y) \propto \frac{e^{-10\phi^2}\,\phi^{44}}{\prod_{i=1}^{10} y_i!} * 2\sqrt{10}.$$

d)

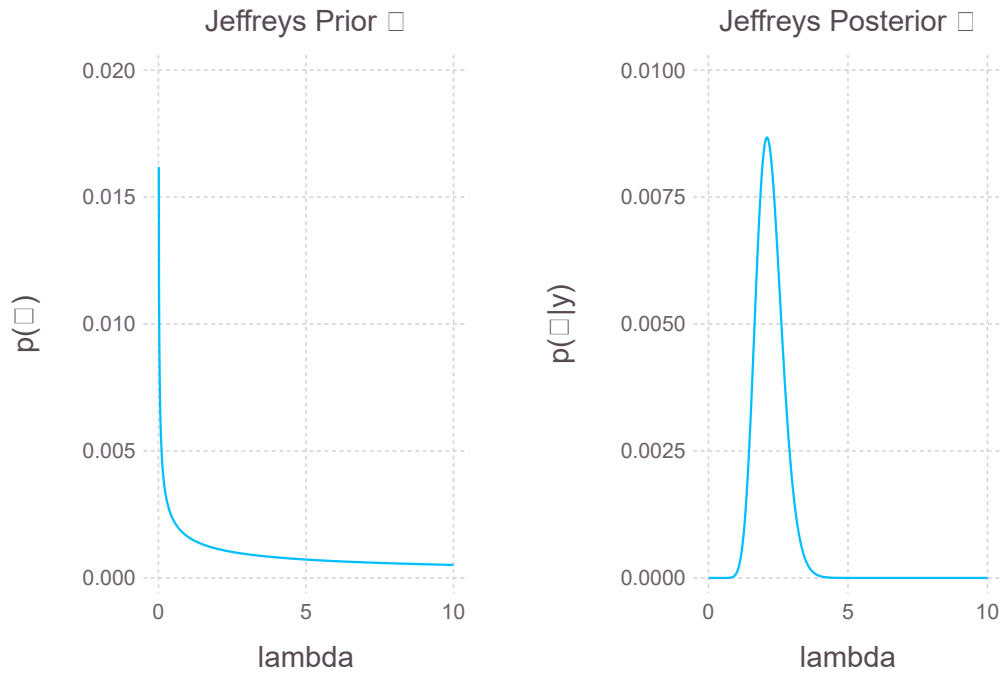
In [1]:
```
using Distributions;
```

In [2]:
```
using Gadfly;
```

In [14]:
```
x=collect(0:0.01:10);
y = zeros(length(x));
for i=1:length(y)
    y[i] = sqrt(1/x[i]);
end
y = y./sum(y[y.!=Inf]);
p_theta = y;
myplot1 = plot(x=x, y = p_theta, Geom.line,Guide.ylabel("p(□)"),
Guide.xlabel("lambda"),Guide.title("Jeffreys Prior □"));
#plotting posterior
#x=collect(0:0.01:1);
y = zeros(length(x));
for i=1:length(y)
    y[i] = (sqrt(x[i]^43))*(exp(-(10 *x[i])))*p_theta[i];
end
y = y./sum(y[!isnan.(y)]);
myplot2 = plot(x=x, y = y, Geom.line,Guide.ylabel("p(□|y)"),
Guide.xlabel("lambda"),Guide.title("Jeffreys Posterior □"))
myplot = hstack(myplot1,myplot2)
myplot
```

```
WARNING: !(B::BitArray) is deprecated, use .!(B) instead.
Stacktrace:
 [1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
 [2] !(::BitArray{1}) at ./deprecated.jl:57
 [3] include_string(::String, ::String) at ./loading.jl:522
 [4] include_string(::Module, ::String, ::String) at /users/PES0801/nifaulla
h/.julia/v0.6/Compat/src/Compat.jl:84
 [5] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/si
te/v0.6/IJulia/src/execute_request.jl:180
 [6] (::Compat.#inner#6{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socke
t,IJulia.Msg}})() at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.j
l:125
 [7] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/e
ventloop.jl:8
 [8] (::IJulia.##15#18)() at ./task.jl:335
while loading In[14], in expression starting on line 17
```

Out[14]:



Here we're approximating to a continous function instead of getting pdf on set of thetas.