CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 16: Fisher Information, Jeffreys Prior

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Question

1. Number of crimes reported in the Clifton neighborhood in Cincinnati during each of the first 10 days of October 2018 are shown in the table below:

Date	# Crimes
Oct 1, 2018	2
Oct 2, 2018	6
Oct 3, 2018	2
Oct 4, 2018	2
Oct 5, 2018	0
Oct 6, 2018	6
Oct 7, 2018	1
Oct 8, 2018	0
Oct 9, 2018	1
Oct 10, 2018	2

Data Source: PDI (Police Data Initiative) Crime Incidents http://data.cincinnati-oh.gov
The following questions will lead you through the process of constructing Jeffreys prior and doing Bayesian parameter estimation.

   a. Derive the Fisher Information for the random variable y that represents number of crimes reported in the Clifton neighborhood. **[8 points]**

   b. Derive the expression for Jeffreys prior. **[3 points]**

   c. Derive the expression for the posterior. **[3 points]**

   d. Write Julia code to plot Jeffreys prior and the posterior. How is the procedure for plotting posterior in this case different from when you were plotting posteriors that map to a standard distribution? **[5 points]**

   [*Hint:* This computing strategy (shown in the sample code) is called 'brute-force' method. In the case where the posterior distribution is not a familiar functional form, then one simply computes values of the posterior on a grid of points and then approximates the continuous posterior by a discrete posterior that is concentrated on the values of the grid.]

   e. Write the Jeffreys prior for a parameter ϕ, assuming that $\lambda = \phi^2$. **[3 points]**

   f. Derive the expression for the posterior for ϕ. **[3 points]**

Bonus question

1. For Question 1 above,
   a. Write Julia code to plot Jeffreys priors for λ and ϕ. Visually compare the two priors and determine if these priors are different?

2. How different is your posterior with a Jeffrerys prior different from that constructed in Exercise 14 using a Uniform prior?

Sample code

1. Plotting Jeffreys prior and resultant posterior

```
using Distributions, Gadfly;

#plotting Jeffreys prior
x=0:0.0015:1;
y = zeros(length(x));
for i=1:length(y)
    y[i] = 1/sqrt(x[i]*(1-x[i]));
end
y = y./sum(y[y.!=Inf]);
p_theta = y;
myplot1 = plot(x=x, y = y, Geom.line,Guide.ylabel("p( )"),
               Guide.xlabel(" "),Guide.title("Jeffreys Prior  "));

#plotting posterior
x=collect(0:0.01:1);
y = zeros(length(x));
for i=1:length(y)
    y[i] = (x[i]^2)*((1-x[i])^8)*p_theta[i];
end
y = y./sum(y[!isnan.(y)]);
myplot2 = plot(x=x, y = y, Geom.line,Guide.ylabel("p( |y)"),
               Guide.xlabel(" "),Guide.title("Jeffreys Posterior  "));
myplot = hstack(myplot1,myplot2);
draw(PNG("./figs/jeffreys_prior.png", 10inch, 4inch), myplot);
```