CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 17: Multiparameter Models

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Questions

1. A study investigating the utility of a drug *Elixir* in curing patients who contracted Covid-19 enrolled $n_1 + n_2$ patients who tested positive for Covid-19. As part of the study, n_1 patients in the treatment group were given the drug *Elixir*, while the n_2 patients in the control group were given a placebo treatment. It was observed that x_1 is the number of survived patients in the treatment group; while x_2 is the number of survived patients in the treatment group.
   Assume the Binomial model: $x_1 \sim Binomial(n_1, \theta_1)$ and $x_2 \sim Binomial(n_2, \theta_2)$. This model has two parameters θ_1, θ_2.
   The following questions will lead you through the steps involved in Bayesian parameter estimation when multiple parameters are involved.
   a. Assuming a uniform prior $p(\theta_1, \theta_2) \propto 1$, write the expression for the posterior distribution $p(\theta_1, \theta_2 | x_1, x_2)$. **[4 points]**
   b. Assuming θ_2 is a nuisance parameter, derive the expression for the posterior of the variable $p(\theta_1 / x_1, x_2)$ in the traditional way. **[4 points]**
   c. Assuming θ_1 is a nuisance parameter, derive the expression for the posterior of the variable $p(\theta_2 / x_1, x_2)$ in the traditional way. **[4 points]**
   d. Show that the variables θ_1, θ_2 are independent in the posterior. **[4 points]**
2. The temperatures, in fahrenheit, in Cincinnati during the first week of March 2020 are observed as $\{62, 58, 58, 62, 59, 48, 49\}$. Assuming that the data is generated from a Gaussian distribution $\mathcal{N}(\mu, \sigma^2)$, your goal is to estimate the parameter μ of the Gaussian distribution using a Bayesian approach.
   The following questions will lead you through the process of Bayesian parameter estimation in the presence of nuisance parameters.
   a. Write the expression for the likelihood term. **[1 points]**
   b. Individual priors on each parameter are given as $p(\mu) \propto 1$ and $p(\sigma^2) = 1/\sigma^2$. Assume that the variables μ, σ^2 are independent. Write the joint prior. **[1 points]**
   c. Write the expression of the joint posterior. **[2 points]**
   d. Determine the posterior distribution for σ^2 assuming μ is a nuisance parameter. **[2 points]**
   e. Write the steps that you need to take to arrive at a point-estimate for the parameter μ (assuming σ^2 is a nuisance parameter) by starting with the joint posterior in part (c). Be as clear and elaborate as possible for each step. **[3 points]**

Bonus question

1. Consider the problem of performing Bayesian parameter estimation in the case of the logistic regression problem with only one independent variable.
   a. Write the joint posterior distribution. Assume a uniform prior on the parameters.

b. How will you determine point-estimates for the parameters?