Assignment - 13

1)

a)

$\Theta \in \{0.2, 0.4, 0.6, 0.8\}$

Prior Probabilities

$P(\Theta = 0.2) = 0.1$

$P(\Theta = 0.4) = 0.1$

$P(\Theta = 0.6) = 0.3$

$P(\Theta = 0.8) = 0.5$

Likelihood.

$$P(y|\theta) = \prod_{i=1}^{n} P(y_i|\theta)$$

$$= \prod_{i=1}^{50} P(y_i|\theta)$$

where.

$$P(y_i|\theta) = \theta^{I(y_i = H)} (1-\theta)^{I(y_i \neq H)}$$

$\therefore$

$$P(y|\theta) = \prod_{i=1}^{n} \theta^{I(y_i = H)} (1-\theta)^{I(y_i \neq H)}$$

$= \theta^{\sum_{i=1} I(y_i = H)} (1-\theta)^{\sum_{i=1} I(y_i = T)}$

$= \theta^{N_H} (1-\theta)^{N_T}$

N_H = # of Heads in 50 trials

N_T = # of tails in 50 trials

$= \theta^{10} (1-\theta)^{40}$

Calculating Posterior

$p(\theta|y) \propto p(y|\theta)\, p(\theta)$

$P(\theta=0.2|y) \propto p(y|\theta=0.2)\, p(\theta=0.2)$

$\propto 0.2^{10}\, 0.8^{40}\, 0.1$

$P(\theta=0.4|y) \propto 0.4^{10}\, 0.6^{40}\, 0.1$

$P(\theta=0.6|y) \propto 0.6^{10}\, 0.4^{40}\, 0.3$

$P(\theta=0.8|y) \propto 0.8^{10}\, 0.2^{40}\, 0.5$

$c = p(y|\theta=0.2)\, P(\theta=0.2) + P(y|\theta=0.4)\, P(\theta=0.4) + p(y|\theta=0.6)\, p(\theta=0.6) + p(y|\theta=0.8)\, p(\theta=0.8)$

$P(\theta=0.2|y) = \dfrac{0.13 \times 10^{-11}}{c}$

$P(\theta=0.4|y) = \dfrac{0.14 \times 10^{-13}}{c}$

$P(\theta=0.6|y) = \dfrac{0.21 \times 10^{-18}}{c}$

$P(\theta=0.8|y) = \dfrac{0.59 \times 10^{-29}}{c}$

No the resultant posterior is not in agreement with prior.

b) Let θ be a continuous parameter.

flat prior (1,0)

$P(\theta) = 1 \quad \forall\, \theta \in [0,1]$.

$\therefore$ the posterior equation now changes to:

$p(\theta|y) \propto P(y|\theta) \cdot P(\theta)$

$\propto P(y|\theta)$

$\propto \theta^{10} (1-\theta)^{40}$

$P(\theta|y) = \frac{1}{c}\theta^{10}(1-\theta)^{40}$

This takes the functional form of Beta distribution.

with

$\alpha = 11$ $\beta = 41$.

(1,0) $p(\theta|y) \sim Beta(11, 41)$.

$p(\theta|y_1,y_2) \propto \theta^{40}(1-\theta)^{10}\theta^{10}(1-\theta)^{40}$

$\propto \theta^{50}(1-\theta)^{50}$

this again takes functional form of beta distribution.

$p(\theta|y_1,y_2) \sim Beta(51, 51)$

where $\alpha = 51$ & $\beta = 51$

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c)

Let y_1 & y_2 be the observations recorded from scene 1 & scene 2 independently from the same distribution using same 'θ'.

w.r.t.

$p(\theta|y_1,y_2) \propto p(y_2|\theta)\, p(\theta|y_1)$.

$\propto \theta^{40}(1-\theta)^{10}\theta^{10}(1-\theta)^{40}$.

d).

Using observations from Scene 1 & scene 2 we get

$N_H = 50$ $N_T = 50$.

$P(\theta|y) \propto p(y|\theta)\,p(\theta)$

$\propto \theta^{50}(1-\theta)^{50}$

$= \frac{1}{c}\theta^{50}(1-\theta)^{50}$

=).

$p(\theta|y) \sim Beta(51, 51)$

where $\alpha = 51$ & $\beta = 51$

* yes c & d are same.

e)

In [1]:
```
using Distributions, Gadfly;
white_panel = Theme(
panel_fill=colorant"white",
default_color=colorant"purple",bar_spacing=3mm,
major_label_font_size=18pt,
minor_label_font_size=14pt,
key_title_font_size = 18pt,
key_label_font_size = 14pt,
major_label_color=colorant"black",
minor_label_color=colorant"black"
);
```

In [4]:
```
x = collect(0:0.001:1);
prior = ones(length(x));
d1 = Beta(11,41);
d2 = Beta(51,51);
posterior1 = pdf.(d1,x);
posterior2 = pdf.(d2,x);
plot(
layer(x=x,y=posterior1,Geom.line,Theme(default_color=colorant"purple")),
layer(x=x,y=posterior2,Geom.line,Theme(default_color=colorant"blue")),
layer(x=x,y=prior,Geom.line,Theme(default_color=colorant"black")),
Coord.Cartesian(xmin=0, xmax=1,ymax=10.2), Guide.ylabel("probability"),
Guide.xlabel("□"), Guide.manual_color_key("", ["p(x|y) NH=10,NT=40",
"p(x|y) NH=50,NT=50", "p(x)"], ["purple","blue","black"]), white_panel)
```

Out[4]: