CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 19: Accept-Reject Method; Monte Carlo Integration

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Questions

1. Draw samples from a density $f(x) \propto x^{\alpha-1}(\frac{1-x}{1+x})^{\beta-1}$ ($\alpha = 2.7$ and $\beta = 6.3$) using Accept-reject method using a Beta distribution with ($\alpha = 2$ and $\beta = 2$) as a candidate.

   [15 points]

   a. List the *three criteria* you verified to ensure that the candidate density is suitable. Provide results from your verification.
   [*Hint:* To test compatibility in terms of support, use visualization. Use Julia code to determine the value of M.]
   b. Write psuedo code for the Accept-reject method in this context.
   c. Write Julia code to generate samples from $f(x)$.
   d. Plot the histogram for samples generated from target and candidate densities.
   e. What fraction of the total samples were accepted?

2. Draw samples from a density $f(x) \propto x^{\alpha-1}(\frac{1-x}{1+x})^{\beta-1}$ ($\alpha = 2.7$ and $\beta = 6.3$) using Accept-reject method using a Beta distribution with ($\alpha = 2$ and $\beta = 5$) as a candidate.

   [6 points]

   a. List the *three criteria* you verified to ensure that the candidate density is suitable. Provide results from your verification.
   [*Hint:* To test compatibility in terms of support, use visualization. Use Julia code to determine the value of M.]
   b. Write Julia code to generate samples from $f(x)$.
   c. What fraction of the total samples were accepted?

3. Between the candidate distributions used in (1) and (2), which of them results in a tighter envelope?

   [4 points]

   a. How do you determine this using the value of M in both cases?
   b. How do determine this using the *fraction of accepted samples*?
   c. What is the advantage of using a tighter envelope?

Bonus question

1. Use Accept-reject method to generate samples from a Poisson distribution with $\lambda = 2$ using a Uniform distribution as a candidate.
   a. List the three criteria and the results from your verification.
   b. Write psuedo code for the Accept-reject method in this context.
   c. Write Julia code to generate samples.
   d. Compare histogram of these samples with that of random samples generated using 'rand(Poisson(2),10000)'

Sample code

1. Julia code for visualizaing target and candidate densities

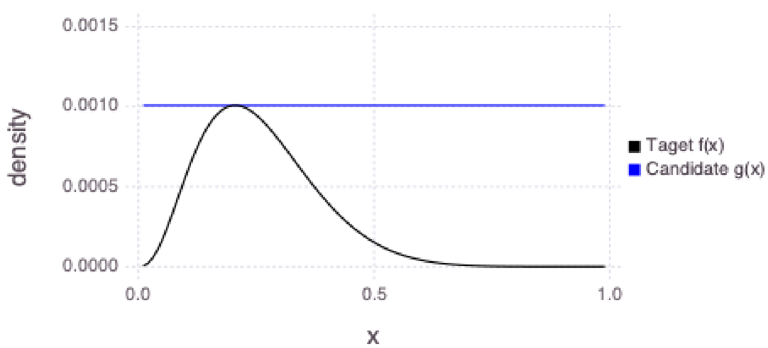
```
x = collect(0.01:0.01:0.99);
d = Uniform(0,1); # candidate density
```

```
## Distributions.Uniform{Float64}(a=0.0, b=1.0)
```

```
f(x) =  x.^2.7 .* (((1.-x)./(1.+x)).^6.3);
g(x) = pdf(d,x);
M = maximum(f.(x)./g.(x))
```

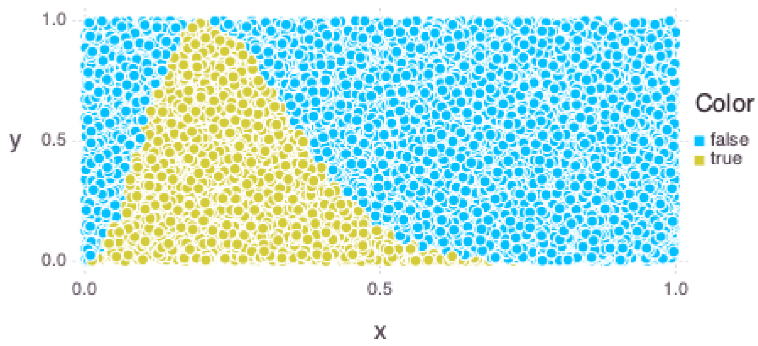
```
## 0.0010080798592428059
```

```
myplot = plot(
        layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
        layer(x=x,y=M.*g.(x),Geom.line,Theme(default_color=colorant"blue")),
        Guide.ylabel("density"),Guide.xlabel("x"),
        Guide.manual_color_key("", ["Taget f(x) ", "Candidate g(x)"], ["black","blue"]));
draw(PNG("./figs/viz_f_g.png", 5inch, 2.5inch), myplot);
```



2. Accept-reject method

```
n = 10000;
y = rand(d,n);
u = rand(Uniform(0,1),n);
x_samples = y[u.<f.(y)./(M*g.(y))];
myplot = plot(x=y,y=u,color = u.<f.(y)/M,Geom.point);
draw(PNG("./figs/accept_reject_samples.png", 5inch, 2.5inch), myplot);
```



3. Plotting histograms of samples

```
samples1 = y;
hist1 = [fit(Histogram,samples1,x).weights; 0]./96500;
samples2 = x_samples;
hist2 = [fit(Histogram,samples2,x).weights; 0]./99000;

myplot = plot(
        layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
        layer(x=x,y=M.*g.(x),Geom.line,Theme(default_color=colorant"blue")),
        layer(x=x,y=hist2, Geom.bar,
              Theme(default_color=colorant"SteelBlue")),
        layer(x=x,y=hist1, Geom.bar,
              Theme(default_color=colorant"orange")),
        Coord.Cartesian(xmin=0, xmax=1),
        Guide.ylabel("density"),Guide.xlabel("x"),
        Guide.manual_color_key("", ["Taget f(x) ", "Candidate Mg(x)", "Samples g(x)",
                "Samples f(x)"], ["black","blue","orange","SteelBlue"]));
draw(PNG("./figs/hist_target_candidate_samples.png", 5inch, 2.5inch), myplot);
```