

CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 4 (Discrete Probability Distributions)

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Questions

1. Prove that $E(aX + b) = aE(X) + b$, where X is a random variable, a and b are constants. [2 points]
2. Write the cumulative distribution function for the probability distribution [3 points]

$$\begin{aligned}p(x = 1) &= 0.1 \\p(x = 2) &= 0.3 \\p(x = 3) &= 0.4 \\p(x = 4) &= 0.2\end{aligned}$$

3. In a study of battery life for laptop computers, researchers found that the probability that the battery life (L) will exceed 5 hours is 0.12. Three such batteries are used in independent laptops and we are interested in finding the probability that some x of the three batteries will last 5 hours or more. [10 points]
 - (i) Specify the standard probability distribution you will use to model this scenario.
 - (ii) What is the state space or the set of possible outcomes for this scenario.
 - (iii) Find the probability that only one of the three batteries will last 5 hours or more.
 - (iv) Write Julia code to plot the *true* distribution for all possible values of x .
 - (v) Write Julia code to sample 1000 points from this distribution and plot the *empirical* distribution. [Hint: True distribution will contain the probabilities from the probability function (using `pdf()` in Julia), whereas the empirical distribution will have probabilities computed from the samples of the distribution.]
4. A statistically inclined farmer would like to model the number of grasshoppers per square meter of his rangeland. He is told that typically there are 0.5 grasshoppers per square meter on a rangeland. [10 points]
 - (i) Specify the standard probability distribution you will use to model this scenario.
 - (ii) What is the state space or the set of possible outcomes for this scenario.
 - (iii) Find the probability that there are five or more grasshoppers in a randomly selected square meter region.
 - (iv) Write Julia code to plot the *true* distribution for values of x upto 10.
 - (v) Write Julia code to sample 1000 points from this distribution and plot the *empirical* distribution.

Bonus Questions

1. **Matching coins** A and B agree to the following rules of the coin-matching game: A wins \$1 if the match is tails and \$2 if the match is heads. A loses \$1 if coins do not match. Write the probability distribution of A's winning amount per game. What is A's expected winning ammount? Is this a fair game?
2. Prove that $V(aX + b) = a^2V(X)$, where X is a random variable and a is a constant.
3. Prove that $p(x \geq j + k | X \geq j) = p(x \geq k)$ for a geometric distribution.
4. A large lot of tires contains 5% defectives. Four tires are to be chosen from the lot and placed on a car.

- (i) Find the probability that two defectives are found before four good ones.
- (ii) Find the expected value and variance of the number of selections that must be made to get four good tires. (*Hint:* First find the expected value and variance of the number of defective tires that will be selected before finding the four good ones.)