In [2]:
```
using Distributions;
using Gadfly;
using StatsBase;
```

1

a)

i) f(x) and g(x) have compatible support
ii) There is a constant M such that f(x)/g(x) <= M for all X
iii) We want M to be as small as possible without violatiing ii)

b)

i) Generate x ~ beta(2,2)

ii) independently Generate u ~ uniform(0,1)

iii) check u <= ((x.^2.7 .* (((1.-x)./(1.+x)).^6.3)/(0.00116 * pdf(beta(2,2),x)))

iv) if iii) is true accept x as sample else reject and start again with step 1

c)

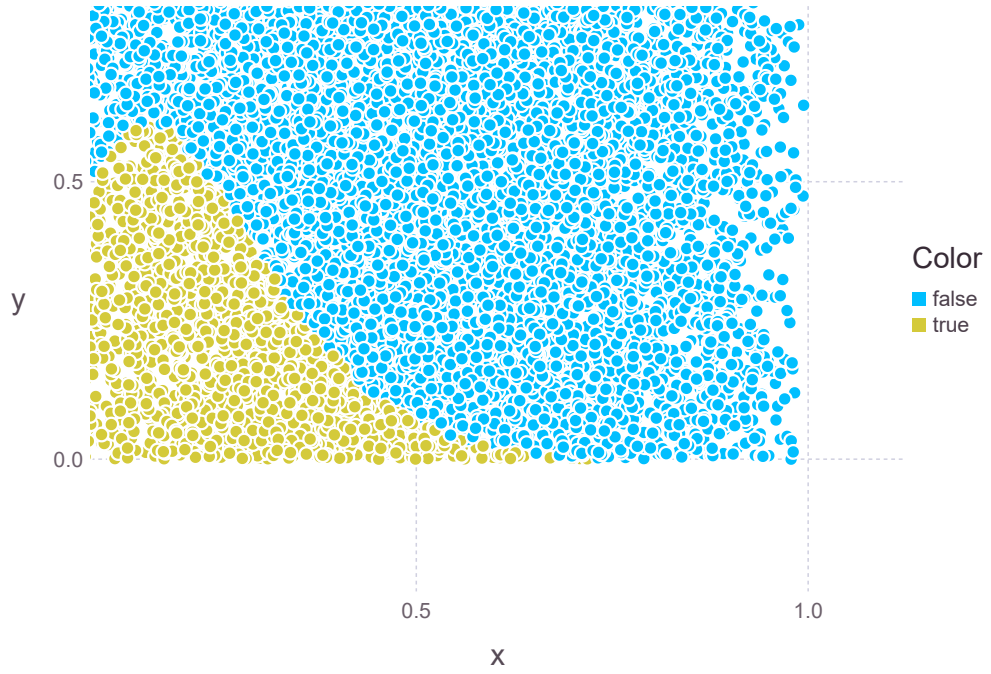
In [3]:
```
x = collect(0.01:0.01:0.99);
d = Beta(2,2);
```

In [4]:
```
f(x) = x.^1.7 .* (((1.-x)./(1.+x)).^5.3);
g(x) = pdf(d,x);
M = maximum(f.(x)./g.(x))
```

Out[4]: 0.013248296363157712

In [5]:
```
n = 10000;
y = rand(d,n);
u = rand(Uniform(0,1),n);
x_samples = y[u.<f.(y)./(M*g.(y))];
plot(x=y,y=u,color = u.<f.(y)/M,Geom.point)
```

Out[5]:



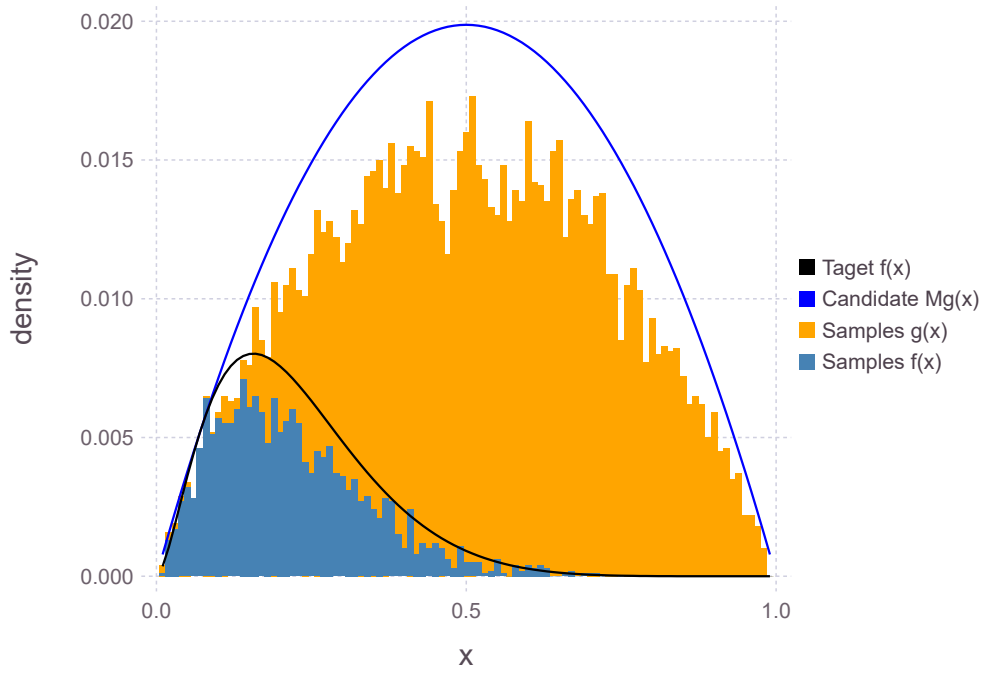
d)

In [8]:
```
samples1 = y;
hist1 = [fit(Histogram,samples1,x).weights; 0]./10000;
samples2 = x_samples;
hist2 = [fit(Histogram,samples2,x).weights; 0]./10000;
myplot = plot(
layer(x=x,y=f.(x),Geom.line,Theme(default_color=colorant"black")),
layer(x=x,y=M.*g.(x),Geom.line,Theme(default_color=colorant"blue")),
layer(x=x,y=hist2, Geom.bar,
Theme(default_color=colorant"SteelBlue")),
layer(x=x,y=hist1, Geom.bar,
Theme(default_color=colorant"orange")),
Coord.Cartesian(xmin=0, xmax=1),
Guide.ylabel("density"),Guide.xlabel("x"),
Guide.manual_color_key("", ["Taget f(x) ", "Candidate Mg(x)", "Samples g(x)",
"Samples f(x)"], ["black","blue","orange","SteelBlue"]))
```

```
WARNING: Default for keyword argument "closed" has changed from :right to :le
ft.
To avoid this warning, specify closed=:right or closed=:left as appropriate.
Stacktrace:
 [1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
 [2] _check_closed_arg at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/
hist.jl:12 [inlined]
 [3] #fit#108(::Symbol, ::Function, ::Type{StatsBase.Histogram{Int64,N,E} whe
re E where N}, ::Tuple{Array{Float64,1}}, ::Tuple{Array{Float64,1}}) at /user
s/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:282
 [4] (::StatsBase.#kw##fit)(::Array{Any,1}, ::StatsBase.#fit, ::Type{StatsBas
e.Histogram{Int64,N,E} where E where N}, ::Tuple{Array{Float64,1}}, ::Tuple{A
rray{Float64,1}}) at ./<missing>:0
 [5] fit(::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Array{Floa
t64,1}, ::Array{Float64,1}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBas
e/src/hist.jl:225
 [6] #fit#112(::Array{Any,1}, ::Function, ::Type{StatsBase.Histogram}, ::Arra
y{Float64,1}, ::Vararg{Array{Float64,1},N} where N) at /users/PES0801/nifaull
ah/.julia/v0.6/StatsBase/src/hist.jl:340
 [7] fit(::Type{StatsBase.Histogram}, ::Array{Float64,1}, ::Array{Float64,1})
at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:340
 [8] include_string(::String, ::String) at ./loading.jl:522
 [9] include_string(::Module, ::String, ::String) at /users/PES0801/nifaulla
h/.julia/v0.6/Compat/src/Compat.jl:84
 [10] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/s
ite/v0.6/IJulia/src/execute_request.jl:180
 [11] (::Compat.#inner#6{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socke
t,IJulia.Msg}})() at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.j
l:125
 [12] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/
eventloop.jl:8
 [13] (::IJulia.##15#18)() at ./task.jl:335
while loading In[8], in expression starting on line 2
WARNING: Default for keyword argument "closed" has changed from :right to :le
ft.
To avoid this warning, specify closed=:right or closed=:left as appropriate.
Stacktrace:
 [1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
 [2] _check_closed_arg at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/
hist.jl:12 [inlined]
```

```
 [3] #fit#108(::Symbol, ::Function, ::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Tuple{Array{Float64,1}}, ::Tuple{Array{Float64,1}}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:282
 [4] (::StatsBase.#kw##fit)(::Array{Any,1}, ::StatsBase.#fit, ::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Tuple{Array{Float64,1}}, ::Tuple{Array{Float64,1}}) at ./<missing>:0
 [5] fit(::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Array{Float64,1}, ::Array{Float64,1}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:225
 [6] #fit#112(::Array{Any,1}, ::Function, ::Type{StatsBase.Histogram}, ::Array{Float64,1}, ::Vararg{Array{Float64,1},N} where N) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:340
 [7] fit(::Type{StatsBase.Histogram}, ::Array{Float64,1}, ::Array{Float64,1}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:340
 [8] include_string(::String, ::String) at ./loading.jl:522
 [9] include_string(::Module, ::String, ::String) at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.jl:84
 [10] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/execute_request.jl:180
 [11] (::Compat.#inner#6{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socket,IJulia.Msg}})() at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.jl:125
 [12] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/eventloop.jl:8
 [13] (::IJulia.##15#18)() at ./task.jl:335
while loading In[8], in expression starting on line 4
```

Out[8]:



e)

In [9]:
```
length(x_samples)/10000
```

Out[9]:
```
0.1777
```

2)

a)

i) f(x) and g(x) have compatible support
ii) There is a constant M such that f(x)/g(x) <= M for all X
iii) We want M to be as small as possible without violatiing ii)

b)

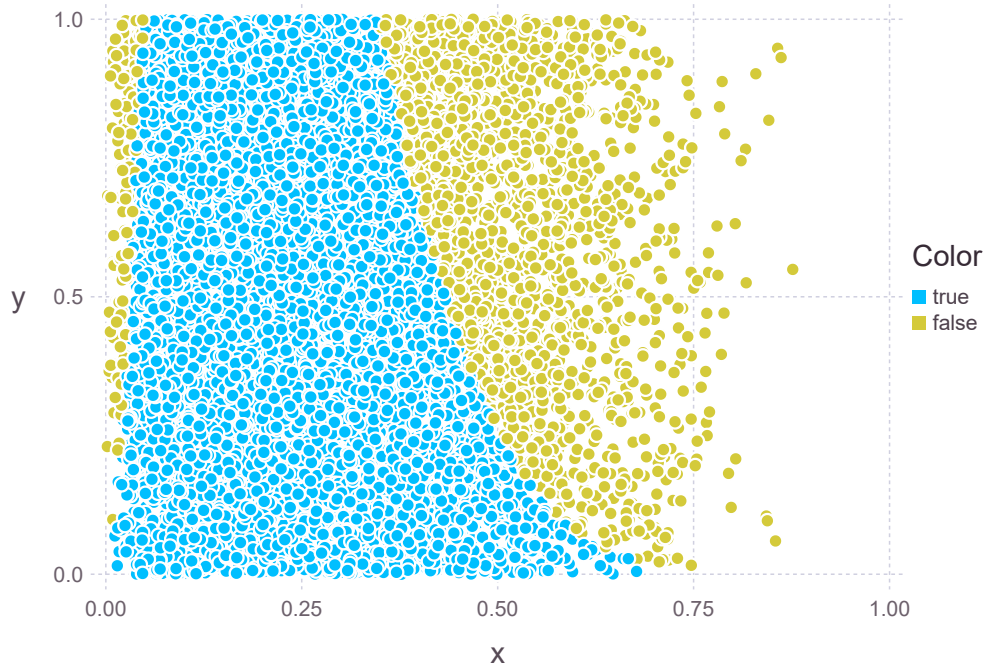
In [11]:
```
x = collect(0.01:0.01:0.99);
d = Beta(2,5);
```

In [12]:
```
f(x) = x.^1.7 .* (((1.-x)./(1.+x)).^5.3);
g(x) = pdf(d,x);
M = maximum(f.(x)./g.(x))
```

Out[12]:
```
0.0035143810295186345
```

In [13]:
```
n = 10000;
y = rand(d,n);
u = rand(Uniform(0,1),n);
x_samples = y[u.<f.(y)./(M*g.(y))];
plot(x=y,y=u,color = u.<f.(y)/M,Geom.point)
```

Out[13]:



In [14]:
```
length(x_samples)/10000
```

Out[14]:
```
0.6524
```

3)

a) The probability of acceptance = 1/M. Greater M means a loose envelope where more points are not accepted and the probability of acceptance is low, smaller M means a tighter envelope and the probability of acceptance is high. M is nothing but the ratio of max(f(x)/g(x))

Based on this description Beta(2,5) is a more tighter envelope.

b) Higher Proportion of accepted samples means the candidate generation is tightly enveloping the target distribution, whereas a small proportion of accepted samples suggest that the candidate distribution is loosely enveloping the target distribution hence large number of samples are gettinng rejected.

Based on this description Beta(2,5) is a more tighter envelope.

C) The main advantage of a tighter envelope is computational efficiency, where you don't have to simulate for a large number of samples that are bound to be rejected