

1)

a)

$$f(\theta, n, x) = \left( \frac{n!}{x!(n-x)!} \right) \left( \frac{T(10)}{T(5)T(5)} \right) (\theta^{x+4}) ((1 - \theta)^{n-x+4}) ((e^{-4})^4 / n!)$$

b) Accept-Reject method: Average out the nuisance parameters to marginalize to  $f(x)$

challenge -

1. picking candidate distribution is difficult in 3 dimensions
2. Closed form analytical solutions may not be possible

c) I would use metropolis-hastings algorithm to sample from  $f(x)$

challenge -

1. Choosing proposal density in 3 dimensions
2. Difficulty in tuning

$$d) f(x | \theta, n) = \left( \frac{n!}{x!(n-x)!} \right) \theta^x (1 - \theta)^{n-x}$$

$$e) f(\theta | x, n) = \left( \frac{T(10)}{T(5)T(5)} \right) (\theta^4) ((1 - \theta)^4)$$

$$f) f(n | \theta, x) = \left( \frac{n!}{x!(n-x)!} \right) \theta^x (1 - \theta)^{n-x} ((e^{-\lambda})^4 (\lambda^n / n!)) = [\lambda (1 - \theta)]^x / (n-x)! = \text{poisson}(4(1 - \theta))$$

g)

1. initialize  $x_0, \theta_0, n_0 \sim q$
2. for iteration  $i = 1, 2, \dots$  do
  3.  $x_{<i>} \sim \text{Binomial}(n_{<i-1>}, \theta_{<i-1>})$
  4.  $\theta_{<i>} \sim \text{Beta}(a + x_{<i>}, b + (n_{<i-1>} - x_{<i>}))$
  5.  $n_{<i>} \sim x_{<i>} + \text{poisson}(4(1 - \theta_{<i>}))$
6. Repeat Until convergence

h)

```
In [1]: using Distributions;
using Gadfly;
using StatsBase;
```

```

In [2]: N = 10000;
n = 20;
a = 5;
b = 5;
lambda = 4;
theta = zeros(N);
x = zeros(N);
n_s = zeros(N);
x[1] = rand(Binomial(n,theta[1]));
theta[1] = rand(Beta(a,b));
n_s[1] = x[1] + rand(Poisson(lambda*(1-theta[1])))
for i=2:N
    x[i] = rand(Binomial(n_s[i-1],theta[i-1]));
    theta[i] = rand(Beta(x[i]+a,n_s[i-1]+b-x[i]));
    z = rand(Poisson(lambda*(1-theta[i-1])))
    n_s[i] = x[i] + z
end

```

**WARNING:** Binomial(n::Real, p) is deprecated. Please use Binomial(n::Integer, p) instead.

Stacktrace:

```

[1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
[2] Distributions.Binomial(::Float64, ::Float64) at /users/PES0801/nifaullah/.julia/v0.6/Distributions/src/deprecates.jl:11
[3] macro expansion at ./In[2]:13 [inlined]
[4] anonymous at ./<missing>:?
[5] include_string(::String, ::String) at ./loading.jl:522
[6] include_string(::Module, ::String, ::String) at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.jl:84
[7] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/execute_request.jl:180
[8] (::Compat.#inner#6{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socket,IJulia.Msg}})() at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.jl:125
[9] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/eventloop.jl:8
[10] (::IJulia.##15#18)() at ./task.jl:335
while loading In[2], in expression starting on line 12

```

i)

```
In [4]: hist_x = [fit(Histogram,x,collect(0:1:10)).weights; 0]./10150;
plot(x=collect(0:1:10),y=hist_x, Geom.bar,Guide.ylabel("probability"
),Guide.xlabel("x"),
Coord.Cartesian(xmin=0,xmax=10))
```

**WARNING:** Default for keyword argument "closed" has changed from :right to :left.

To avoid this warning, specify closed=:right or closed=:left as appropriate.

Stacktrace:

```
[1] depwarn(::String, ::Symbol) at ./deprecated.jl:70
[2] _check_closed_arg at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:12 [inlined]
[3] #fit#108(::Symbol, ::Function, ::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Tuple{Array{Float64,1}}, ::Tuple{Array{Int64,1}}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:282
[4] (::StatsBase.#kw##fit)(::Array{Any,1}, ::StatsBase.#fit, ::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Tuple{Array{Float64,1}}, ::Tuple{Array{Int64,1}}) at ./<missing>:0
[5] fit(::Type{StatsBase.Histogram{Int64,N,E} where E where N}, ::Array{Float64,1}, ::Array{Int64,1}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:225
[6] #fit#112(::Array{Any,1}, ::Function, ::Type{StatsBase.Histogram}, ::Array{Float64,1}, ::Vararg{Any,N} where N) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:340
[7] fit(::Type{StatsBase.Histogram}, ::Array{Float64,1}, ::Array{Int64,1}) at /users/PES0801/nifaullah/.julia/v0.6/StatsBase/src/hist.jl:340
[8] include_string(::String, ::String) at ./loading.jl:522
[9] include_string(::Module, ::String, ::String) at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.jl:84
[10] execute_request(::ZMQ.Socket, ::IJulia.Msg) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/execute_request.jl:180
[11] (::Compat.#inner#6{Array{Any,1},IJulia.#execute_request,Tuple{ZMQ.Socket,IJulia.Msg}})() at /users/PES0801/nifaullah/.julia/v0.6/Compat/src/Compat.jl:125
[12] eventloop(::ZMQ.Socket) at /usr/local/julia/0.6.4/site/v0.6/IJulia/src/eventloop.jl:8
[13] (::IJulia.##15#18)() at ./task.jl:335
while loading In[4], in expression starting on line 1
```

Out[4]:

