

# CS 5135/6035 Learning Probabilistic Models

Exercise Questions for Lecture 7 Parameter estimation, Maximum Likelihood Estimation

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## Questions

1. The lifetime (in years) of a certain brand of light bulbs,  $l$ , follows an exponential distribution ( $f(x) = \lambda e^{-\lambda x}$ ) with (an unknown) parameter  $\lambda$ . A sample of five bulbs were tested and the lifetimes were found to be 2, 3, 1, 3, and 4 years, respectively. **[4+2+4+2 points]**
  - a. Write the likelihood function.
  - b. Write the log-likelihood function.
  - c. Derive the equation for estimating  $\lambda$  using MLE?
  - d. What is the estimate  $\hat{\lambda}_{MLE}$  for the sample given above?
2. A particular gene occurs as one of two alleles ('A' and 'a'), where allele 'A' has frequency  $\theta$  in the population. That is, a random copy of the gene is 'A' with probability  $\theta$  and 'a' with probability  $1 - \theta$ . Since a diploid genotype consists of two genes, the probability of each genotype is given by: **[6+2+3+2 points]**

genotype	AA	Aa	aa
probability	$\theta^2$	$2\theta(1 - \theta)$	$(1 - \theta)^2$

When a random sample of 100 people are tested, 'AA' genotype was found in 20 people, 'Aa' genotype was found in 10 people, and 'aa' genotype was found in 70 people.

- a. Write the likelihood function.
- b. Write the log-likelihood function.
- c. Derive the equation for estimating  $\theta$  using MLE?
- d. What is the estimate  $\hat{\theta}_{MLE}$  for the sample given above?

## Bonus Question

1. Derive equations for estimating parameters of a Gamma distribution, assuming  $n$  samples  $\{x_1, x_2, \dots, x_n\}$  are sampled (i.i.d.) from it.