FAIRNESS IN KNAPSACK

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1 Fairness

We consider a setting in one-sided markets (similar to knapsack setting) where there are a buyer with budget constraint B and n sellers $S = \{s_1, s_2, ..., s_n\}$. Each seller owns a single item with public value v_i (first assume $v_i = 1, \forall i \in S$) and private cost c_i. Let $V\{v_1, v_2, ..., v_n\}$ and $C = \{c_1, c_2, ..., c_n\}$ denote the value vector and cost vector. These items can be divided into m disjoint groups. Assume that there are m groups $G = \{G_1, G_2, ..., G_m\}$ and $\uplus_{j\leq m} G_j = S$.

1.1 Auction Model

Each seller needs to report their private information cost b_i to the platform when he entering the markets. We assume that each seller is strategic that he reports false information to achieve more utilities, i.e., $b_i \neq c_i$. Let $b = \{b_1, b_2, ..., b_n\}$ denote the bid profile. After bidding their costs, the platform needs to decide winning sellers $S_w \subseteq S$ and pay each winning seller a reward $p_i > 0 (i \in S_w)$. The utility of each seller is its payment minus its cost, $u_i = p_i - c_i$. The revenue of buyer is total value procured from sellers $u_b = \sum_{i\in S_w} v_i$. Our goal is to maximize the revenue (or the social welfare) that the total value procured from sellers.

1.2 Desired Properties

We want to design the mechanism which guarantees following properties:

- Fairness: We want to maximize the revenue with the consideration of fairness in each group. A natural fairness objective function is:
$$\max \min_{j\leq m} \frac{|S_w \cap G_j|}{|G_j|} \tag{1}$$
Inspired by the method in previous work, we can solve this fairness problem by transferring it to a submodular function:
$$\max \sum_{j\leq m} \sqrt{|S_w \cap G_j|} \; s.t. \; \sum_{i\in S_w} p_i \leq B \tag{2}$$
or
$$\max \sum_{j\leq m} \log(|S_w \cap G_j|) \; s.t. \; \sum_{i\in S_w} p_i \leq B \tag{3}$$
However, for the additive value function, the revenue may perform bad in (2) with the budget constraint. (It is a possible reason why we consider fairness from budget) Thus, from another aspect, we hope that the group with more sellers and higher average cost should consume more budget than others which is also helpful for the fairness in (1).
- Truthfulness: Each seller can get maximum utility by bidding truthful cost, i.e., $u_i(c_i, b_{-i}) \geq u_i(b_i, b_{-i}), b_i \neq c_i$.

- Budget Feasibility: The sum of payments of all the sellers does not exceed the budget constraint, i.e., $\sum_{i \in S} p_i \leq B$.
- Individual Rationality: The utility of each seller is non-negative, i.e., $u_i \geq 0$.

1.3 Possible Mechanism

At the beginning, we need to divide all the budget B into different groups based on the number of sellers in each group and sellers' costs. Assume that, the budget for group G_j is B_j. We consider this problem with the large markets assumption where the cost of each seller satisfies $c_i \ll B$ and there are adequate sellers in each group. We divide the budget into groups by the proportion of costs:

$$B_j = \frac{B}{\sum_{i \in S} b_i} \cdot \sum_{i \in G_j} b_i \tag{4}$$

In this way, the seller can manipulate the auction by bidding false cost and effect the budget allocation. Thus, in group G_i, we use the sample method to divide sellers into two partitions G_i^A and G_i^B, randomly. Thus, we calculate the budget B_i by the proportion of total costs, i.e.,

$$B_j = \frac{B}{\sum_{j \leq M} \sum_{i \in G_j^A} b_i} \cdot \sum_{i \in G_i^A} b_i \tag{5}$$

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Within each group G_j, we use the greedy algorithm to select winning sellers among G_j^B and adopt budget feasible mechanism to decide the final payment for each winning seller with the budget constraint B_j.

2 Another Idea

Last time, we also talk about another problem that additional requirement for group G_i is that a fixed number N_i of sellers (items) should be selected, $|S_w \cap G_i| \geq N_i$.***

References

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