Budget-Feasible Mechanisms in Two-sided Crowdsensing Markets: Truthfulness, Fairness, and Efficiency

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Abstract—In a crowdsensing platform, users are invited to provide data services, and multiple requesters compete for desired services. Due to users' costs of providing services, it is critical to design incentive mechanisms to incentivize users with (monetary) rewards. Meanwhile, requesters may have individual budgets and compete for services with different procurement abilities. Such a setting falls into the budget-feasible mechanism design. However, most of the existing budget-feasible mechanisms focus on one-sided markets with a single requester rather than the two-sided markets with multiple requesters having different procurement abilities. Moreover, requesters and users can be selfish and strategic with their private information, which requires preventing information manipulation on both requesters' and users' sides. In this paper, we investigate budget-feasible mechanisms in two-sided crowdsensing markets where multiple strategic requesters come with private budgets to obtain services from the strategic users. We also consider the fairness on the requesters' side, *i.e.*, a requester with more budget should obtain more service. We propose budget-feasible mechanisms for two models by distinguishing the types of services, *i.e.*, the homogeneous or heterogeneous services. All proposed mechanisms satisfy fairness, budget feasibility, truthfulness on both users' and requesters' sides, and the constant approximation ratio. Numerical experiment results further demonstrate the efficiency of our proposed mechanisms.

Index Terms—Mobile Crowdsensing; Budget Feasible Mechanism; Incentive Mechanism Design; Two-sided Markets; Fairness.

1 Introduction

With the rapid use of mobile devices, such as phones, smart-glasses and smartwatches that are installed with plethora of on-board sensors (*e.g.*, camera, gyroscope, compass, accelerometer, and GPS), mobile crowdsensing (MCS) has been a novel popular economic paradigm to invite mobile device users to accomplish large-scale sensing tasks, like collecting and sharing sensory data. Mobile crowdsensing systems naturally consist of a platform (*i.e.*, a cloud-based central server), service requesters, and a crowd of mobile device users. A service requester usually wants to recruit the users to provide their service. The platform needs to decide the trades between requesters and mobile device users based on requesters' requests and users' service abilities. Applications like Amazon Mechanical Turks (AMT) and oDesk have allowed exploiting human resources to solve crowdsensing problems.

However, participants' willingness in the platform affects the effectiveness of crowdsensing systems. When providing the crowdsensing services, the mobile users must contribute resource consumption, *e.g.*, the resource of their mobile devices and time, and expose themselves to potential privacy threats by sharing their data. Thus, mobile device users may not be willing to serve requesters unless they receive a satisfying reward. For this reason, it is vital to design an efficient incentive mechanism to stimulate users to contribute to the platform. Many works [3]–[13] model the crowdsensing problems as auctions where the requester works as a buyer and the users act as service sellers who bid for their services. Users achieve monetary rewards after providing crowdsensing services. Moreover, there is a common scenario that the requester comes with a budget, and the designed auction mechanism should satisfy the budget constraint that the total payment from the requester cannot exceed a given budget [14]–[16]. The goal of a requester in this scenario is to maximize the total value of obtained services from users within the budget constraint, which falls into the field of budget-feasible mechanism design.

Most existing works on budget-feasible mechanism design in crowdsensing markets only focus on one-sided markets where there is only one requester (or buyer) with a single budget [6], [14], [15], [17], [18]. However, these works did not provide truthful mechanisms for the two-

- *The earlier conference version of this paper appeared at IJCAI 2018 with the title "Budget-feasible Procurement Mechanisms in Two-sided Markets" [1]. We extend it in the following ways: 1) We apply the proposed mechanisms to the two-sided crowdsensing markets with multiple strategic requesters and strategic mobile device users. 2) We further take the fairness with respect to requesters' budgets into account which is not considered in the earlier paper. 3) We propose a non-trivial fair and budget feasible mechanism for the homogeneous model with much better approximation performance where users' services have identical values for requesters (part of related results for homogeneous model are simply reported in the extended abstract in AAMAS 2020 [2] without showing detailed mechanism description and proofs, and we included all the mechanism details and proofs in this paper). In addition, we further extend the mechanism in [1] for the heterogeneous model to satisfy requesters' fairness.*

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sided crowdsensing markets. Within two-sided crowdsensing markets, multiple strategic requesters (or buyers) come into the crowdsensing system with private *budgets* and want to procure as much value of service as possible from the users with private *costs*. Although some works [19]–[21] consider multiple requesters in mobile crowdsensing, they ignore the budget constraints of requesters. In addition, various requesters would have diverse abilities to procure services from users when they have different budgets in two-sided markets. Therefore, to improve participants' satisfaction and willingness, there is one more natural requirement to consider the *fairness* regarding budgets. Specifically, fairness requires that buyers with more budget procure more value of services from sellers than buyers with less budget in the two-sided markets. A natural question then arises in such scenarios.

Can we design an efficient fair mechanism in two-sided crowdsensing markets that stimulates the desired economic interactions among requesters and users without any requester's payment exceeding her budget?

In the procurement mechanism design problem above, multiple requesters compete to procure more value of service with diverse procurement budgets/abilities. Users compete with each other to sell their services for more payment rewards. The designed mechanism should determine an allocation and a payment scheme to guarantee various desired theoretical properties, like *fairness*, *individual rationality*, *budget feasibility*, *users' truthfulness*, *requesters' truthfulness*, and *budget-balance*. Specifically, we assume that each requester is required to submit the full amount of her claimed budget as a deposit to the platform at the beginning, inspired by the design of mechanisms with verification [22], [23], so that a requester bidding over her true budget would be detected. We show that this assumption is necessary for budget-feasible mechanisms ensuring both requesters' and users' truthfulness in two-sided crowdsensing markets by deriving an impossibility result in Section 3.

Compared to budget-feasible mechanism design in one-sided crowdsensing markets, we attempt to design budget-feasible mechanisms in two-sided markets that satisfy the truthfulness on both sides and guarantee fairness regarding requesters' budgets. There are three main challenges to be addressed. Firstly, while there exists a general guideline (*e.g.*, the monotone allocation rule [24]) on eliciting users' truthfulness in the literature, there exist no general rules on how to guarantee requesters' truthfulness in the budget-feasible procurement setting. Secondly, the payment scheme is implicitly determined by the allocation scheme on the one hand, but on the other hand, it should also satisfy the budgetary constraints of all requesters. Finally, when fairness is considered, the allocation scheme should measure the procurement ability of requesters and allocate the items to buyers fairly.

1.1 Motivation Scenario

We are inspired by the Mobile Crowdsensing System consisting of three parties, multiple requesters [19], a platform, and a crowd of users. Each user usually carries a mobile device and can provide data services. Naturally, each user's service corresponds to a *public* value to requesters, *e.g.*, the amount of sensing data or the result of the environmental sensing task from a single user is identical to requesters. According to real-world applications, each user's service's value may have the following two properties. Firstly, the values of users' services are possibly homogeneous or heterogeneous, *e.g.*, the amount of sensing data provided by users can be the same or different. Furthermore, each user's service value can be indivisible, *e.g.*, the result of the sensing task can be integral [25]. Thus, in our model, we consider the scenario that the value of each user's public service is indivisible (each user's service cannot be divided to allocate to multiple requesters) and identical to all requesters. Moreover, we investigate two models by distinguishing whether users' services have homogeneous or heterogeneous values. Also, each user is associated with a privately known cost when providing the sensing service. The requester should pay a monetary reward to the user who provides her service as compensation. However, requesters cannot pay unlimited rewards and usually have budgets [14]–[16]. Then, it is natural to consider the two-sided budget-feasible mechanisms in such scenarios.

1.2 Our Contribution

We summarize the contributions of this paper as follows.

(1) We are the first to consider the budget-feasible mechanisms in two-sided mobile crowdsensing markets, where there are multiple strategic service requesters with various private budgets and multiple strategic mobile device users owning sensing services with private costs.

(2) To address the truthfulness on both requesters' and users' sides, we introduce a temporal commitment assumption, which is further shown necessary by providing an impossibility result in designing truthful mechanisms in two-sided markets.

(3) According to features of users' services, we investigate two models by distinguishing whether users' services have homogeneous or heterogeneous values, *i.e.*, the service value of users are the same or different. i) In Section 4, we introduce a fair budget-feasible mechanism HomoMech for two-sided markets to solve the homogeneous model. The idea is to first propose a concept of *virtual (unit) price* and use it to measure/match the market participants' demand and supply. Then, we introduce a set of *candidate virtual prices* containing all possible virtual prices from which we try to find a *critical virtual price* to instruct us to determine the final allocation and the payment of our mechanism. ii) In Section 5, we propose Mechanism GRMech to address the heterogeneous model. GRMech randomly combines two sub-mechanisms: one that handles users with *small costs* by applying one-third of each requester's budget and using an appropriate cost-per-value threshold to select users. The other deals with users with *large costs* by using a greedy choosing manner. The proposed mechanisms guarantee desired theoretical properties, like fairness, budget feasibility, individual rationality, budget balance, truthfulness both on the users' and the requesters' sides, and constant approximation to the optimal solution.

(4) Finally, by conducting experiments on the real dataset, we reveal different parameters' impacts on the total value by comparing two benchmark mechanisms. The

results show that, within the requirement of truthfulness on two sides and budget feasibility, our proposed mechanism for the homogeneous model has a superior performance to the benchmark mechanisms, and the proposed mechanism for the heterogeneous model also shows favorable efficiency.

The rest of the paper is organized as follows. Section 2 reviews related works. In Section 3, we introduce the model and formulate problems. In Section 4, we propose a fair and budget-feasible mechanism for the homogeneous model. In Section 5, we further propose a randomized budget feasible mechanism for the heterogeneous model. We then conduct numerical experiments in Section 6 to validate the performance of the proposed mechanisms. Finally, we conclude the paper and discuss the future work in Section 7.

2 Related Work

2.1 Budget Feasible Mechanisms

After the seminal work by Singer [26], many works study budget-feasible mechanisms in the single-buyer setting. Gravin *et al.* [27] propose two budget-feasible mechanisms, both of which obtain tight approximation guarantees. After that, many works [28]–[31] further consider more general scenarios where the buyer may have submodular, sub-additive and symmetric submodular value functions over procured items. Anari *et al.* [32] design a constant-approximation budget feasible mechanism for large markets when sellers' items are divisible. While many works focus on the offline problem, Badanidiyuru *et al.* [33] and Singer *et al.* [34] investigate the online procurement problem where agents arrive in sequential order, and a budget feasible mechanism must make an irrevocable decision whether or not to procure the service as the agents arrive. Dobzinski *et al.* [35] investigate a reversed scenario with multiple budgeted buyers but a single seller with public cost and propose a new truthful mechanism achieving logarithmic approximation. For two-sided markets, most prior works study the double auction mechanisms that ignore the procurement budgets [36]–[38]. The works in [39], [40] address the trade-off between efficiency and incentive compatibility in double auctions, without considering budget feasibility. Hirai and Sato [41] consider two-sided markets, but they only guarantee buyers' truthfulness while sellers are assumed to be truthful.

2.2 Mechanisms in Mobile Crowdsensing

The incentive mechanism for mobile crowdsensing has attracted extensive attention in recent years. Yang *et al.* [4] and Zhang *et al.* [17] focus on the mechanism design for the mobile phone sensing scenarios. Zheng *et al.* [14] propose budget feasible mechanisms to maximize the value of the covered interested regions. Feng *et al.* [7] propose a reverse auction framework by considering the sensing location information. Xiao *et al.* [10] apply the combinatorial multi-armed bandit method to solve the multiple unknown users' recruitment problem in crowdsensing systems. Jiang *et al.* [42] propose a two-sided randomized auction mechanism for mobile crowdsensing markets. Gao *et al.* [43] propose a novel two-sided D2D trading market with a distributed pricing mechanism. Moreover, some works incorporate other essential issues in crowdsensing into the mechanism design, e.g., the privacy of users and the quality of sensing data [44], [45]. Besides the offline mechanism, many works consider the online mechanism design [18], [46]. While most of the above works consider a single requester, Jin, Su, and Nahrstedt [19] consider multiple data requesters in MCS systems and propose a novel integrated framework consisting of the incentive and data aggregation mechanism. Jin *et al.* [20] investigate the pricing mechanism in MCS systems with multiple requesters and dynamically arriving workers. However, none of these works consider multiple requesters coming into the mobile crowdsensing markets with various private budgets.

2.3 Fairness

Being one of the important factors usually addressed in the economic markets, *fairness* illustrates an individual's judgment or evaluation for the appropriateness and rationality of a process or an action. Many efforts show how the fairness affects the behavior of participants in the economic markets. Kauffman *et al.* [47] verify that the incentive mechanism can stimulate consumers' participation by considering the price fairness and procedural fairness. For one-sided markets, Balkanski and Singer [48] consider fair attribution mechanisms where the payments can reflect the sellers' contribution to the buyer's overall utility while sellers' costs are publicly known information. For two-sided markets, Huang *et al.* [49] propose a truthful double auction mechanism for crowdsensing systems by considering the max-min fairness task allocation issue. Previous works rarely consider fairness concerning requesters' procurement abilities/budgets. Our work takes a step forward toward designing mechanisms with fairness and truthfulness in two-sided crowdsensing markets with diverse budgets.

Against these previous works, our work considers budget feasible mechanisms in two-sided crowdsensing markets where requesters and users come into the markets with private budgets and costs, and propose effective fair mechanisms that can achieve truthfulness on both requesters' and users' sides with constant approximation ratios.

3 Preliminaries

In this section, we introduce the model of the two-sided budget feasible mobile crowdsensing problem. We begin with a formal description of the interaction between requesters and users in two-sided crowdsensing markets. Then we will set up notations to capture the behavior of participants and the mechanism design problem. Table 1 lists the key notation.

3.1 System Model

A representative process in two-sided crowdsensing markets is shown in Fig. 1. We consider a crowdsensing market with a service platform, a set of n *users* $S = (s_j)_{j=1}^n$ and a set of m service *requesters* $A = (a_i)_{i=1}^m$. Each requester a_i has a *budget* $B_i \in R_+$ to obtain data services from users. Each user s_j can provide the sensing service with a *public value* $v_j \in R_+$ to requesters and has a *private cost* $c_j \in R_+$ [26]. Depending on the categories of users' services, we consider

TABLE 1
Key Notation

Notation	Description
$S=(s_j)_{j=1}^n$	set of users
$A=(a_i)_{i=1}^m$	set of requesters
v_j, c_j, c'_j	value, real cost and reported cost of s_j
V, C	value, real cost profile of users
$C'=(c'_j)_{j=1}^n$	reported cost profile of users
B_i, B'_i	real and reported budget of requester a_i
$B=(B_i)_{i=1}^m$	real budget profile of requesters
B_{min}	minimum threshold of budget
B_{-i}, B'_{-i}	real and reported budget profile except requester a_i
C_{-j}, C'_{-j}	real cost, reported cost profile except s_j
x_{ij}	binary indicator of whether user s_j provides service to requester a_i
p_{ij}	payment paid by requester a_i to user s_j
S_w	winner set
$u_j^s(C', B')$	utility for user s_j under the bid profile b and B'
$u_i^a(C', B')$	utility for requester a_i under reported budget profile B' and the bid profile b

two models: the homogeneous model where users' service values are the same, *i.e.*, $v_{j'} = v_{j''}, \forall j' \neq j''$, and the heterogeneous model where users may have different service values, *i.e.*, $\exists j' \neq j'', v_{j'} \neq v_{j''}$. We consider the *incomplete information* case where the budgets of requesters and costs of users are privately known by themselves. Following the assumption in the classical reverse-auctions [26], [50], we assume that the value v_j is common knowledge, and the objective is to maximize the total value of services obtained by requesters within their budgets [14], [18]. Let $B=(B_i)_{i=1}^m$ denote all budgets of requesters. Let $C=(c_j)_{j=1}^n$ be all costs of users and $V=(v_j)_{j=1}^n$ be all values of users' services. We denote by B_{-i} all budgets except a_i's budget B_i and C_{-j} all costs except s_j's cost c_j, respectively.

This paper addresses buyer-centric procurement mechanisms in crowdsensing markets, where the needs of requesters usually cannot be fulfilled by a single user, and requesters can afford the costs of single users. We assume that all requesters have basic procurement ability $B_i \geq B_{min}$ where B_{min} is a publicly known minimum threshold of budget and no costs of users exceed any requester's procurement ability, *i.e.*, $c_j \leq B_{min}$.

We focus on the strategic scenario where the participants (requesters and users) may act strategically to maximize their utilities. Each user bids a cost c'_j of her service that may be different from the real cost c_j to maximize her utility (or benefit defined below). Let $C' = (c'_j)_{j=1}^n$ denote all the bids of the users and C'_{-j} denote all bids except s_j's bid c'_j. Each requester a_i claims a budget B'_i that may be different from her true budget B_i. Let $B'=(B'_i)_{i=1}^m$ denote all the reported budgets of requesters and B'_{-i} denote all reported budgets except a_i's reported budget. We assume a *temporary commitment* that each requester is required to submit her full amount of claimed budget as a deposit to the mechanism at the beginning so that a requester bidding over her true budget would be detected. Based on such a temporary commitment, a requester cannot claim a budget exceeding her true one. We will show that this assumption is necessary by deriving an impossibility result at the end of this section.

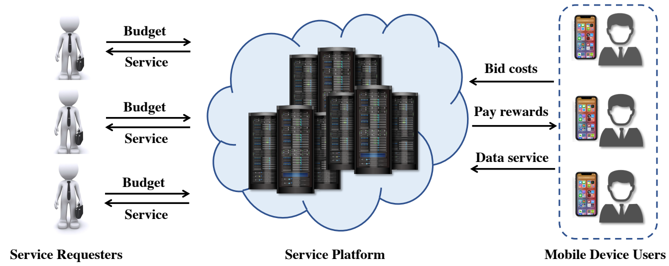

Fig. 1. A Mobile Crowdsensing System

3.2 The Mechanism

Formally, we design a mechanism $M=(f, P)$ which consists of an allocation function f specifying which requester obtains which user's service and a payment function P specifying how much payment is paid. In our work, we consider indivisible services. We use $x_{ij} \in \{0,1\}$ to indicate whether the service of user s_j is allocated to requester a_i and we have $\sum_{i \leq m} x_{ij} \leq 1, \forall j \leq n$. We also use p_{ij} to denote the payment paid by requester a_i to user s_j. Let $X = \{x_{ij}\}_{i \leq m, j \leq n}$ and $P = \{p_{ij}\}_{i \leq m, j \leq n}$ denote the allocation profile and the payment profile. Any user providing service to requesters will be added to the winner set denoted by S_w. The utility of user s_j is the difference between the payment she receives and her true cost, *i.e.*,

$$u_j^s(C', B') = \sum_{1 \leq i \leq m} p_{ij} - c_j \cdot \sum_{1 \leq i \leq m} x_{ij}$$

if s_j is the winning user, and otherwise $u_j^s(C', B') = 0$. The utility of requester a_i is the total value of services obtained from users within her budget, *e.g.*, the sum of the amount of sensing data of selected users [25], *i.e.*,

$$u_i^a(C', B') = \sum_{1 \leq j \leq n} v_j x_{ij}.$$

In this work, we investigate truthful budget-feasible procurement mechanisms in two-sided crowdsensing markets by considering the following properties,

- **Budget feasibility.** The total payment of each requester a_i does not exceed her budget, *i.e.*, $\sum_{1 \leq j \leq n} p_{ij} \leq B_i$.
- **Individual rationality.** The utility of a winning user s_j is non-negative, *i.e.*, $u_j^s(C, B) \geq 0$.
- **Computational efficiency.** The output of the mechanism should be computed in polynomial time.

- **Budget balance (BB).** *Strong budget-balance (SBB)* means that the amount of money paid by requesters is totally and exclusively transferred to users. It is *weak budget-balance (WBB)* if the mechanism does not run a deficit.
- **Users'/requesters' truthfulness.** Any user maximizes her own utility by bidding her true cost $c'_j = c_j$, *i.e.*, $u^s_j((c_j, C'_{-j}), B') \geq u^s_j((c'_j, C'_{-j}), B')$, for any c'_j and bids $C' = (c'_j, C'_{-j})$. Any requester maximizes her own utility when her claimed budget equals her true budget, *i.e.*, $u^a_i(C'_j, (B_i, B'_{-i})) \geq u^a_i(C'_j, (B'_i, B'_{-i}))$, for any B'_i and reported budgets $B' = (B'_i, B'_{-i})$.
- **Fairness.** We want to guarantee the fairness between requesters in the sense that a requester with more budget can procure more values from users. That is, we say a mechanism satisfies *fairness* if $u^a_{i_1}(C', B') \geq u^a_{i_2}(C', B')$ when $B'_{i_1} \geq B'_{i_2}$.
- **Approximation.** We want to maximize the total value of the requesters obtained from the market, *i.e.*, $\sum_{1\leq i\leq n}\sum_{1\leq j\leq m} v_j x_{ij}$. A mechanism is $O(g(n,m))$-approximation if the ratio of the total value between the optimal solution which knows all private information of participants and the solution by the mechanism is $O(g(n,m))$. Ideally, we would like our mechanism to be $O(1)$-approximation.

We now derive that the temporary commitment assumption is necessary by deriving an impossibility result in the following theorem.

Theorem 1. *Even without addressing the fairness, if requesters are allowed to overbid (without resorting to temporary commitments), no truthful budget-feasible mechanisms can guarantee an approximation ratio $\sqrt{n}-\epsilon$ even when all users are homogeneous, i.e., $v_j = 1, \forall j : 1 \leq j \leq n$, under a mild assumption that winning users are equally paid where n is the number of users, and ϵ is a positive constant number.*

Proof. To show the impossibility result above, the main idea is to consider a specific case where there is a requester a and n users holding homogeneous values, *i.e.*, $v_j = 1, \forall j : 1 \leq j \leq n$. Then, the total obtained value of requesters equals the number of winners. Next, we distinguish two kinds of mechanisms: index-based mechanisms that only consider users' indices when determining the allocation and non-index-based mechanisms that consider users' costs.

Index-based Mechanism: For an index-based mechanism, since it does not take users' costs into account when determining the allocation, it can not guarantee an approximation ratio better than $O(n)$ to an optimal solution. The reason is as follows. We assume that the single requester a has a budget $B : B > n$, and the first user's cost is B while the remaining users' costs are 1. Thus, the index-based mechanism chooses users starting from the first user and obtains one value with a payment of at least B. In contrast, the optimal solution obtains $n-1$ value by selecting all users except the first user.

Non-index-based Mechanism: For the non-index-based mechanism $\mathcal{M}$, under the setting where there is a single requester a with budget B and n users holding the same values, we can construct an instance where the optimal solution obtains at most $\sqrt{n}-1$ value under true bidding. Denote by k and k' the obtained value in the true and false bidding of the non-index-based mechanism $\mathcal{M}$, respectively. Accordingly, $k \leq \sqrt{n}-1$. A truthful mechanism $\mathcal{M}$ should guarantee one of the following two cases when requester a overbids $B' = \infty$.

- (1) When requester a reports $B' = \infty$, the obtained value k' of $\mathcal{M}$ will not be higher than that under the true bidding, *i.e.*, $k' \leq k \leq \sqrt{n}-1$. This implies that if requester a has a real budget $B = \infty$, the obtained value of mechanism $\mathcal{M}$ should still be k' where $k' \leq k \leq \sqrt{n}-1$. However, the optimal solution can obtain all users' services with total value n under budget $B = \infty$, which implies that the approximation ratio of $\mathcal{M}$ is at least $\sqrt{n}$.
- (2) When requester a reports $B' = \infty$, the obtained value of $\mathcal{M}$ can be higher than that under the true bidding, *i.e.*, $k' \geq k+1$, but the total payment of requester a will be higher than budget B, *i.e.*, the overbidding of requester a is detected when it is asked to pay $p \cdot k' > B$ where p is the same payment to winners under the mild assumption. Then, there exists at least one non-truthful user, *i.e.*, a non-selected user $s_j, \exists j : 1 \leq j \leq \sqrt{n}$, which could make itself selected and get positive utility by bidding an extremely low cost c'_j close to 0 by applying Myerson's theorem [24] since the non-index mechanisms choose services based on reported costs instead of their indices, which contradicts users' truthfulness.

Thus, no non-index-based mechanism can guarantee an approximation ratio better than $\sqrt{n}$.

Therefore, we conclude that without resorting to temporary commitments, no truthful budget-feasible mechanisms can guarantee an approximation ratio better than $\sqrt{n}$.

□

The impossibility result above shows the necessity of introducing temporary commitment in designing two-sided budget-feasible mechanisms. Therefore, we would resort to such an assumption to propose efficient and fair mechanisms in two-sided markets.

4 Budget-Feasible Mechanism in the Homogeneous Model

In this section, we first consider that users' values are homogeneous and then investigate the heterogeneous model in the next section. In the homogeneous model, as $v_j = 1, \forall 1 \leq j \leq n$, the optimization goal is reduced to maximizing the number of users selected to provide services under the requester fairness constraint. We mainly focus on the competitive scenario when the number of users is at least

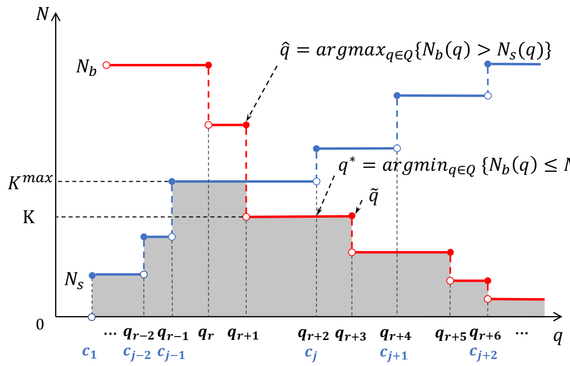

Fig. 2. Demand and supply curves (solid lines) as the virtual price q changes. The red step function ($N_b(q)$) measures how many value requesters could obtain at price q, while the blue one ($N_s(q)$) measures how many value of service users could provide at q. The dark area indicates the amount of obtainable value of requesters at different q. While the maximum obtainable value is K^{max}, the mechanism finally decides to obtain K value.

two times that of requesters ($n \geq 2m$)[1]. We propose the fair and budget-feasible Mechanism HomoMech to address the homogeneous model.

4.1 Mechanism Design

The high-level idea of the design is as follows. We propose an idea of *virtual (unit) price* and use it to measure the market's demand and supply of participants. Based on the generated demand curve and supply curve as the virtual unit price q changes, we will match the demand with the supply properly to guarantee fairness, truthfulness, and efficiency. Specifically, we will introduce a set of *candidate virtual prices*, denoted as Q. Among these prices, we try to find a *critical virtual price* q^* and correspondingly another value $\tilde{q}$ to instruct us to determine our mechanism's allocation and payment, which can further elicit truthfulness from the users and requesters simultaneously. Fig. 2 illustrates the demand and supply curves.

We first introduce the method we adopt to construct the set Q. We use $\lfloor \frac{B_i}{q} \rfloor$ to measure how many value requesters can obtain from users if the service of each user is set with a *virtual price* $0 \leq q \leq \infty$. We use $N_b(q)$ to denote all requesters' obtainable amount of value when the virtual price is q, *i.e.*, $N_b(q) = \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{q} \rfloor$, and use $N_s(q)$ to denote the total number of candidate users whose costs are no more than q, *i.e.*, $N_s(q) = |\{c_j | c_j \leq q, c_j \in C\}|$. Note that as the virtual price q decreases continuously, $N_b(q)$ is non-decreasing while $N_s(q)$ is non-increasing, hence we can find the set of points where either $N_b(q)$ or $N_s(q)$ changes. Based on the definitions above, we define the set Q of *candidate virtual prices* to be all the possible virtual prices q which may make either $N_b(q)$ or $N_s(q)$ change as q decreases. Note that they can be either the value q that makes the number $N_s(q)$ of candidate users change or the value q that makes the value $\lfloor \frac{B_i}{q} \rfloor$ of requester a_i change. Therefore, to construct set Q, we first add all costs of the users into set Q, and then further add each value q that can divide B_i into set Q, i.e.,

$$Q = \{\frac{B_i}{h} | \forall 1 \leq i \leq m, \forall 1 \leq h \leq n\} \cup \{c_j | \forall 1 \leq j \leq n\}. \quad (1)$$

Now we are ready to introduce our mechanism. First, we generate the candidate set Q of virtual prices. Second, we test the value $q \in Q$ by decreasing its value discretely to find the critical virtual price $q^* = \operatorname{argmin}_{q \in Q}\{N_b(q) \leq N_s(q)\}$. That is, q^* is the smallest value $q \in Q$ among the candidate virtual prices that ensures the obtainable value by the requesters is no greater than the number of candidate users [2]. Third, we compute the total number $K = N_b(q^*)$ of users to be selected and another virtual price $\tilde{q} = \operatorname{argmax}_{q \in Q}\{N_b(q) = K\}$ which will be used in determining the final payment. Here, $\tilde{q}$ is the maximum virtual price $q \in Q$ that achieves the same value $N_b(q) = K = N_b(q^*)$, which is a value that should be utilized in order to elicit truthfulness from the users and requesters. Last, we determine the final allocation and payment scheme. That is, let requester a_i obtain $\lfloor \frac{B_i}{\tilde{q}} \rfloor$ value of service from the first K users arbitrarily and each user is paid $\min\{\tilde{q}, c_{K+1}, B_{min}\}$. Note that here q^* is the value we used to find the number of winning users, but $\tilde{q}$ actually determines the allocation to the requesters and the payment to the users.

4.2 Theoretical Guarantees of Performance

In this part, we analyze the performance of Mechanism HomoMech. Let $\hat{q} = \operatorname{argmax}_{q \in Q}\{N_b(q) > N_s(q)\}$. We will first prepare two basic lemmas on set Q and then conclude the theoretical performance guarantees of the proposed mechanism in Theorem 2.

Lemma 1. *The set Q satisfies the following properties:*

1) *$N_b(q) = N_b(q_{r+1})$, for $q \in (q_r, q_{r+1}]$ and all $q_r \in Q$.*
2) *There does not exist $q \in Q$ such that $\hat{q} < q < q^*$.*

Proof. (1) According to the definition of set Q in (1), $q_r \in Q$ are the first points that make either $N_b(q)$ or $N_s(q)$ change when q decreases continuously. Thus, when $q \in (q_r, q_{r+1}]$, the value $N_b(q)$ does not change, *i.e.*, $N_b(q) = N_b(q_{r+1})$.

(2) Suppose on the contrary that there exists q such that $q \in Q$ and $\hat{q} < q < q^*$. Recall $\hat{q} = \operatorname{argmax}_{q \in Q}\{N_b(q) > N_s(q)\}$ and $q^* = \operatorname{argmin}_{q \in Q}\{N_b(q) \leq N_s(q)\}$. Then, if $N_b(q) \leq N_s(q)$, we have $q^* \leq q$; Otherwise, we have $\hat{q} \geq q$, both leading to a contradiction. □

Lemma 2. *$N_b(\hat{q}) \leq K + m$, where $N_b(q^*) = K$ and m is the number of requesters.*

Proof. Based on Lemma 1, $\hat{q}$ and q^* must be adjacent in the sorted set Q, and $\hat{q} < q^*$. The set Q contains all the possible

1. When the number of users is less than two times that of requesters $n < 2m$, we can propose a simple mechanism that randomly selects $\min\{n, m\}$ users and allocates one user to each requester according to their non-increasing order of budgets with payment B_{min}. As the mechanism randomly selects winners and pays each winner a fixed payment B_{min}, users have no incentive to misreport their costs. Moreover, as we allocate winners to requesters according to their non-increasing order of budgets, requesters with higher budgets are more likely to receive services, and requesters cannot obtain more value by bidding lower budgets, which implies requesters' fairness and truthfulness. Finally, due to $n < 2m$, such a mechanism achieves a 2-approximation.

2. A more intuitive strategy is to set q^* to be $q^* = \operatorname{argmax}_{q \in Q} \min\{N_b(q), N_s(q)\}$, the one that achieves the maximum obtainable value (*i.e.*, $K^{max} = \max_{q \in Q} \min\{N_b(q), N_s(q)\}$). However, while such a strategy satisfies most of the desired theoretical guarantees, it is too aggressive to guarantee requesters' truthfulness.

Algorithm 1 Mechanism **HomoMech**(B, C, V)

Input: B, C, V.

Each requester submits the full amount of her claimed budget as a deposit.
//**Generate candidate set Q and search critical virtual price**
$S_w \leftarrow \emptyset, Q \leftarrow \emptyset$
Add all costs in C into the critical set $Q \leftarrow Q \cup C$
for all $B_i \in B$ **do**
  for all $1 \leq h \leq n$ **do**
    $Q \leftarrow Q \cup \{\frac{B_i}{h}\}$
Resort items in set Q in decreasing order and sort the costs in set C in increasing order.
Let $N_b(q) = \sum_{B_i \in B} \lfloor \frac{B_i}{q} \rfloor$ and $N_s(q) = |\{i : c_i \leq q\}|$
while $q \in Q$ decreases **do**
  Find $q^* = \text{argmin}_{q \in Q}\{N_b(q) \leq N_s(q)\}$.
Set $K = N_b(q^*)$.
//**Winner determination and payment scheme**
Set $N_{max} = K$ and $S_w = \{s_j : 1 \leq j \leq K\}$.
Let $\widetilde{q} = \text{argmax}_{q \in Q}\{N_b(q) = K\}$
Each requester a_i obtains $\lfloor \frac{B_i}{\widetilde{q}} \rfloor$ value of service from the first K users arbitrarily and each user is paid $\min\{\widetilde{q}, c_{K+1}, B_{min}\}$.
return $N_{max}, q^*, \widetilde{q}$

virtual prices which make the obtainable value of requesters change. To prove the lemma, we claim that when q changes from q^* to be a smaller value adjacent to q^*, the value of $N_b(\cdot)$ may increase by at most m. This is because, otherwise, if it increases to be greater than $K+m$, then there is at least one requester B_i who can obtain two more value of service with the changed value q^*, *i.e.*, $\lfloor \frac{B_i}{q^*} \rfloor - \lfloor \frac{B_i}{q^*-\epsilon_1} \rfloor \geq 2$. However, there must be a larger value $q^* - \epsilon_2$ which satisfies that $\lfloor \frac{B_i}{q^*} \rfloor - \lfloor \frac{B_i}{q^*-\epsilon_2} \rfloor = 1$ and $\epsilon_1 > \epsilon_2$. Moreover, $q^* - \epsilon_2 \in Q$ according to the definition of Q. So the value of $q^* - \epsilon_1$ is not the smaller value adjacent to q^* in the sorted set Q. This is a contradiction. Therefore, we have $N_b(\hat{q}) \leq K + m$. □

Theorem 2. *Mechanism HomoMech satisfies budget-feasibility, individual-rationality, computational efficiency, strong budget-balance, users'/requesters' truthfulness, fairness and achieves an approximation ratio of* $2 + \frac{m-1}{K} \leq 3$.

The proof of Theorem 2 is shown in the following subsections. Section 4.2.1 shows budget-feasibility, individual-rationality, computational efficiency, and strong Budget-balance. Section 4.2.2 and 4.2.3 prove users' truthfulness and requesters' truthfulness, respectively. Then, we show the fairness in Section 4.2.4. Finally, we analyze the approximation performance in Section 4.2.5.

4.2.1 Budget-feasibility, Individual-rationality, Computational Efficiency, Strong Budget-balance

First we show budget feasibility. According to Mechanism HomoMech, each requester a_i obtains at most $\lfloor \frac{B_i}{\widetilde{q}} \rfloor$ value and pays a payment $\min\{\widetilde{q}, c_{K+1}, B_{min}\}$. Thus, the total payment of requester a_i is at most $\lfloor \frac{B_i}{\widetilde{q}} \rfloor \cdot \widetilde{q} \leq B_i$. Therefore, the total payment of each requester will not exceed her budget.

Next, we prove the individual rationality. For a user s_j that is not selected, her utility is zero. For a user s_j that is selected, her utility is $p_j - c_j$. Since s_j is a winning user, we have $c_j \leq q^* \leq \widetilde{q}$ and $c_j \leq c_K \leq c_{K+1}$. Hence, the payment $p_j = \min\{\widetilde{q}, c_{K+1}, B_{min}\}$ is no smaller than her cost c_j (since $c_j \leq B_{min}$). Thus, her utility is non-negative.

Then, we prove the computational efficiency. Since each requester buys at most n items, it contributes at most n virtual prices for the set Q. The size of set Q generated is thus at most $O(mn)$. Both the operation for generating Q and searching q^* in Q in the while loop have time complexity $O(mn)$. Thus, the total time complexity of the mechanism is $O(mn)$.

Last, we prove strong budget balance. In Mechanism HomoMech, all requesters totally pay $K \cdot \min\{\widetilde{q}, c_{K+1}, B_{min}\}$ and all winners in S_w receive $|S_w| \cdot \min\{\widetilde{q}, c_{K+1}, B_{min}\}$ payments where $|S_w| = K$. Thus, all payments from requesters will be transferred to the winning users. Therefore, Mechanism HomoMech guarantees strong budget-balance.

4.2.2 Users' Truthfulness

To prove users' truthfulness, we follow the general *Monotone theorem* in [24] for verifying the truthfulness in a single parameter domain.

Theorem 3. ***(Monotone theorem, [24])*** *In single parameter domains with agent's utility as form $u_j(c_j') = p_j - c_j$, a mechanism $M = (f, p)$ is truthful iff:*

- ***f is monotone:*** *$\forall s_i \in S$, if $c_i' \leq c_i$, then $s_i \in f(c_i, c_{-i})$ implies $s_i \in f(c_i', c_{-i})$ for every c_{-i};*
- ***winners are paid threshold payments****: payments to each winning bidder is the critical value $inf\{c_i : i \notin f(c_i, c_{-i})\}$.*

We prove users' truthfulness of the mechanism by showing that it satisfies Theorem 3.

Monotonicity: For any user with $j \leq K$, it is obvious that user s_j is selected, *i.e.*, $s_j \in S_w$. If s_j decreases her bid to be $c_j' \leq c_j$, according to the allocation scheme of Mechanism HomoMech, the function $N_b(\cdot)$ keeps the same since the budgets remain the same. Instead, only the function $N_s(\cdot)$ will be affected when user s_j changes her bid from c_j to a lower bid c_j'. However, in such scenario, it is clear that we still have $N_b(q^*) \leq N_s(q^*)$ since the number of bids lower than q^* does not decrease. Thus, the returned critical virtual price (denoted as $q^{*\prime}$) does not increase, and the total number of users to be allocated satisfies $K' = N_b(q^{*\prime}) \geq N_b(q^*) = K$. This implies that user s_j is still selected. Therefore, Mechanism HomoMech is monotonic.

Threshold payments: In Mechanism HomoMech, all the winning users are paid the same payment, $p = min\{\widetilde{q}, c_{K+1}, B_{min}\}$. The last winner is s_K and $K = min\{N_b(q^*), N_s(q^*)\}$. Let q' be the new value of q^* when one of the users changes her bid. The critical value means that for a user whose bid is greater than p, it will not be selected. A user bidding $c_j' > B_{min}$ is not allowed and selected in the mechanism. Thus, we only need to show that any user bidding higher than $p = \min\{\widetilde{q}, c_{K+1}\}$ will not be selected. We discuss it in the following two cases.

1) $\widetilde{q} \geq c_{K+1}$: In such a scenario, $p = c_{K+1}$ and $N_s(\widetilde{q}) \geq K+1$. Moreover we have $N_b(\widetilde{q}) = K$ since there are K winners. Now assume that a user s_j with $j \leq K$ bids a cost $c'_j > c_{K+1} = p$. We discuss by two sub-cases. Consider the first case that $c'_j > \hat{q}$. If user s_j still wins, then the new value q' under such a bid satisfies $q' \geq c'_j > \hat{q}$. However, at most K users are selected since $N_b(q') \leq K$ with $q' > \hat{q}$ according to Lemma 1. Then, we consider the second case that $\hat{q} \geq c'_j > c_{K+1} = p$. In such a case, the value of $N_b(\hat{q})$ and $N_s(\hat{q})$ will not change under bids of requesters (B'_i, B_{-i}). Thus, we have $N_b(\hat{q}) > N_s(\hat{q})$. This implies that the returned value $q' > \hat{q}$, and again we know that at most K users are selected. Thus, for both sub-cases, any user s_j bidding a cost higher than c_{K+1} would fail to be selected in competing with the other K users. Therefore, c_{K+1} is the critical value/bid that makes a user get allocated and should be the threshold payment for winning bids under the scenario $\widetilde{q} \geq c_{K+1}$.

2) $\widetilde{q} < c_{K+1}$: In such scenario, $p = \min\{\widetilde{q}, c_{K+1}\} = \widetilde{q}$. Obviously $K = N_b(\widetilde{q})$. Moreover, $N_s(\widetilde{q}) = K$ since $\widetilde{q} < c_{K+1}$. Consider that a winning user s_j reports a bid c'_j greater than $\widetilde{q}$. Suppose that c'_j is still selected as the winning bid. Then, the critical virtual price should change to be a value $q' \geq c'_j$ and $N_b(q') \leq N_b(\widetilde{q}) \leq K$. Moreover, $N_s(q') \geq K$ since the other $K-1$ bidders have costs no more than $\widetilde{q}$ and they are still selected in this case. On one hand, we have $N_b(q') \leq N_b(\widetilde{q}) = K$ since $q' \geq c'_j > \widetilde{q}$. On the other hand, we know that $N_b(q') > K-1$ because otherwise at most $K-1$ users win and user s_j under the false bid c'_j would fail to be selected when competing with the first $K-1$ users. Thus, $N_b(q') = K$. However, this contradicts the fact that $N_b(\widetilde{q}) = K$ and $N_b(\widetilde{q} + \epsilon) < K, \epsilon > 0$ by the definition of $\widetilde{q} = \arg\max_{q \in Q}\{N_b(q) = K\}$. Therefore, user s_j fails to be selected in competing with the other bidders. This proves that $\widetilde{q}$ is the threshold payment in such case.

Therefore, all winners are paid threshold payments. Based on the analysis above, we conclude that Mechanism HomoMech satisfies users' truthfulness according to Theorem 3.

4.2.3 Requesters' Truthfulness

Each requester is required to submit the full amount of her claimed budget as a deposit to the mechanism at the beginning. A requester bidding over her true budget would be detected and punished with an infinite cost, e.g., such a user is banned forever from the market or reintegrated after paying a hefty penalty. Thus, each requester cannot claim a budget exceeding her true one.

In contrast to the overbidding case that is detectable, the underbidding case cannot be detected. As a fact, it is pretty involved to show the truthfulness in the underbidding case. Intuitively, a requester has less incentive to bid a lower budget. However, bidding a lower budget may make the system decrease the critical virtual price determined for users and thus give the requester a chance to obtain more services at lower price. In the following, we prove that Mechanism HomoMech guarantees that requesters have no incentives to bid low.

When requester a_i claims her true budget, the mechanism outputs the critical virtual price $q^* = \mathrm{argmax}_{q \in Q}\{N_b(q) \leq N_s(q)\}$, and selects $K = N_b(q^*)$ users. Recall that $\widetilde{q} = \mathrm{argmax}_{q \in Q}\{N_b(q) = K\}, N_b(\widetilde{q}) = K$ and $\widetilde{q} \geq q^*$. We denote by $N_b(q^*, B_i) = \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{q^*} \rfloor$ the total value of service all requesters can obtain and denote by $N_s(q^*, B_i)$ the number of users with costs no greater than q^* when requester a_i bids a budget B_i. We use N_i to denote the value of service that requester a_i can obtain.

Suppose that requester a_i bids a lower budget $\bar{B}_i < B_i$. Let Q' be the new set Q of candidate virtual prices. Denote the returned critical virtual price of the mechanism by $q' = \mathrm{argmin}_{q \in Q'}\{N_b(q, \bar{B}_i) \leq N_s(q, \bar{B}_i)\}$ and $\widetilde{q}' = \mathrm{argmax}_q\{N_b(q, \bar{B}_i) = N_b(q', \bar{B}_i)\}$.

It is easy to see that the value of service that any requester a_i can obtain is the same for q^* and $\widetilde{q}$, *i.e.*, $N_i = \lfloor \frac{B_i}{\widetilde{q}} \rfloor = \lfloor \frac{B_i}{q^*} \rfloor$ since $N_b(q^*) = N_b(\widetilde{q}) = K$. Thus, to simplify the discussion below, when we refer to the value of service requester a_i can obtain, we always refer to the one generated by q^* under truthful bidding case, and the one generated by q' under bid vector (B'_i, B_{-i}). We discuss it by two cases of value q'.

Case 1 $N_b(q', B_i) \leq N_s(q', B_i)$: If $q' > q^*$, the value of service requester a_i can obtain by bidding a lower budget is $\lfloor \frac{\bar{B}_i}{q'} \rfloor \leq \lfloor \frac{B_i}{q^*} \rfloor = N_i$ with $\bar{B}_i \leq B_i$. Otherwise $q' \leq q^*$, then $\hat{q} < q' \leq q^*$ because $N_b(\hat{q}, B_i) > N_s(\hat{q}, B_i)$ and $N_b(q', B_i) \leq N_s(q', B_i)$ (by definition of q'). According to Lemma 1, we have $N_b(q', B_i) = N_b(q^*, B_i)$. It is obvious that $N_b(q', B_i) \geq N_b(q', \bar{B}_i)$ because $B_i > \bar{B}_i$. Thus, the output number of final winning users satisfies:

$$N_b(q^*, B_i) \geq N_b(q', \bar{B}_i) \tag{2}$$

According to (2) and $q' \leq q^*$, the value of service requester a_i can obtain at false bid is at most

$$\begin{aligned} \lfloor \frac{\bar{B}_i}{q'} \rfloor &= N_b(q', \bar{B}_i) - \sum_{B_h \in B \setminus \{\bar{B}_i\}} \lfloor \frac{B_h}{q'} \rfloor \\ &\leq N_b(q^*, B_i) - \sum_{B_h \in B \setminus \{B_i\}} \lfloor \frac{B_h}{q^*} \rfloor = \lfloor \frac{B_i}{q^*} \rfloor = N_i. \end{aligned}$$

Therefore, requester a_i cannot get more services by false bidding when $N_b(q', B_i) \leq N_s(q', B_i)$.

Case 2 $N_b(q', B_i) > N_s(q', B_i)$: We know the value q' is smaller than q^*. the value of service requester a_i can obtain at false bid satisfies that $\lfloor \frac{B_i}{q'} \rfloor \geq \lfloor \frac{B_i}{q^*} \rfloor = N_i$ because $q' < q^*$. Based on the possible value of $\lfloor \frac{\bar{B}_i}{q'} \rfloor$, we discuss further by three sub-cases.

Case 2.1 $\lfloor \frac{B_i}{q'} \rfloor = N_i$: requester a_i will not get more services than reporting a true budget since $\lfloor \frac{\bar{B}_i}{q'} \rfloor \leq \lfloor \frac{B_i}{q'} \rfloor = N_i$.

Case 2.2 $\lfloor \frac{B_i}{q'} \rfloor = N_i + 1$: It is clear that $\lfloor \frac{\bar{B}_i}{q'} \rfloor \leq \lfloor \frac{B_i}{q'} \rfloor = N_i + 1$. We can prove that $\lfloor \frac{\bar{B}_i}{q'} \rfloor \neq N_i + 1$. Otherwise we have

$$\begin{aligned} N_b(q', B_i) &= \sum_{B_h \in B \setminus \{B_i\}} \lfloor \frac{B_h}{q'} \rfloor + \lfloor \frac{B_i}{q'} \rfloor \\ &= \sum_{B_h \in B \setminus \{\bar{B}_i\}} \lfloor \frac{B_h}{q'} \rfloor + \lfloor \frac{\bar{B}_i}{q'} \rfloor = N_b(q', \bar{B}_i) \end{aligned} \tag{3}$$

On the other hand, we have

$$N_b(q', \bar{B}_i) \leq N_s(q', \bar{B}_i) = N_s(q', B_i) < N_b(q', B_i)$$

where the last inequality is the pre-condition, leading to a contradiction to (3). Therefore, we have $\lfloor \frac{\bar{B}_i}{q'} \rfloor \leq N_i$. That is,

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requester a_i also cannot get more items by bidding a lower budget.

Case 2.3 $\lfloor\frac{B_i}{q'}\rfloor \geq N_i+2$: In such case, we can find a value $q \in Q$ such that $\lfloor\frac{B_i}{q}\rfloor = N_i + 1$ and $q' < q < q^*$, based on the definition of set Q. Furthermore, we have $N_b(q, B_i) > N_s(q, B_i)$ by the definition of $q^* = \mathrm{argmin}_{q\in Q}\{N_b(q) \leq N_s(q)\}$. Therefore we have

$$N_s(q, B_i) < N_b(q, B_i), N_s(q, \bar{B}_i) \geq N_b(q, \bar{B}_i) \tag{4}$$

where the second inequality in (4) follows by the definition of q' and $q > q'$. By applying the same deduction as that of the case $\lfloor\frac{B_i}{q'}\rfloor = N_i + 1$ (with q' replaced with q therein), we can show that $\lfloor\frac{\bar{B}_i}{q}\rfloor \leq N_i$. Combining the results above, we have

$$\begin{aligned} N_b(q', \bar{B}_i) &\leq N_s(q', \bar{B}_i) \leq N_s(q, \bar{B}_i) \\ &= N_s(q, B_i) < N_b(q, B_i) \end{aligned} \tag{5}$$

$$\begin{aligned} &= \sum_{B_h\in B\setminus\{B_i\}} \lfloor\frac{B_h}{q}\rfloor + \lfloor\frac{B_i}{q}\rfloor \\ &\leq \sum_{B_h\in B\setminus\{B_i\}} \lfloor\frac{B_h}{q}\rfloor + N_i + 1 \end{aligned} \tag{6}$$

where the first and second inequalities in (5) follow by the definition of q' and $q' < q$, while the third inequality follows by the first inequality in (4). Thus, we have the following inequality,

$$N_b(q', \bar{B}_i) \leq \sum_{B_h\in B\setminus\{B_i\}} \lfloor\frac{B_h}{q}\rfloor + N_i.$$

Accordingly, the value of service that requester a_i can obtain with a lower bid is at most,

$$\begin{aligned} \lfloor\frac{\bar{B}_i}{q'}\rfloor &= N_b(q', \bar{B}_i) - \sum_{B_h\in B\setminus\{\bar{B}_i\}} \lfloor\frac{B_h}{q'}\rfloor \\ &\leq \sum_{B_h\in B\setminus\{B_i\}} \lfloor\frac{B_h}{q}\rfloor + N_i - \sum_{B_h\in B\setminus\{\bar{B}_i\}} \lfloor\frac{B_h}{q'}\rfloor \leq N_i \end{aligned}$$

where the last inequality holds by $q > q'$. Hence, requester a_i has no incentive to bid a lower budget. Therefore, requesters are truthful in claiming their budgets.

4.2.4 Fairness

In Mechanism HomoMech, the total value obtained from users for any requester a_i is $u_i^a(C, B) = \lfloor\frac{B_i}{q}\rfloor$. The value of $u_i^a(C, B)$ is monotonous non-decreasing with the increasing budget B_i. Therefore, HomoMech guarantees fairness.

4.2.5 Constant Approximation

Let OPT be the optimal solution and $V(S) = K$ be the value of service obtained in Mechanism HomoMech. Recall that $\hat{q} = \mathrm{argmax}_{q\in Q}\{N_b(q) > N_s(q)\}$. To analyze the approximation ratio, we will separately discuss the costs no greater than $\hat{q}$ and costs greater than $\hat{q}$. First, we discuss costs no greater than $\hat{q}$. By the definition of value $\hat{q}$, $N_s(\hat{q}) < N_b(\hat{q}) \leq K + m$ where the last inequality holds by Lemma 2. Hence, we have $N_s(\hat{q}) \leq K + m - 1$, which means at most $K + m - 1$ users have costs no greater than $\hat{q}$. The best case for the optimal solution is that it obtains all these $K + m - 1$ users' services with a total cost of 0.

For costs greater than $\hat{q}$, according to the definition of $\hat{q}$, we have $N_b(q) \leq N_s(q)$ for $q > \hat{q}, q \in Q$. For any cost $c_j > \hat{q}$, we prove that $c_j \geq q^*$. Otherwise, we have $\hat{q} < c_j < q^*$ and $c_j \in Q$ leading to a contradiction to third property in Lemma 1. Therefore, $c_j \geq q^*$ for any $c_j > \hat{q}$. Thus, any solution must pay at least q^* to each user and can obtain at most K value of service from users.

According to the above two cases, the optimal solution can obtain at most $2K + m - 1$ value of service. Then, we consider the output of the mechanism $V(S)$. If $q^* \leq B_{min}$, obviously we have $V(S) = K = N_b(q^*) \geq m$. If $q^* > B_{min}$, then we have $\hat{q} \geq B_{min}$ since $\hat{q}$ is adjacent to q^* in sorted set Q and $B_{min} \in Q$ by the definition of set Q. Hence, $N_s(\hat{q}) \geq N_s(B_{min}) \geq n$ by the fact that all users' costs are at most B_{min}. Thus, $V(S) = K = N_b(q^*) \geq N_b(\hat{q}) - m > N_s(\hat{q}) - m \geq n - m \geq m$ where the first inequality holds by Lemma 1, the second last inequality holds by the definition of $\hat{q}$ and the last one holds by $n \geq 2m$. Therefore,

$$\frac{OPT}{V(S)} \leq \frac{(K + m - 1) + K}{K} = 2 + \frac{m - 1}{K} \leq 3.$$

Therefore, Mechanism HomoMech achieves 3-approximation.

5 Budget-Feasible Mechanism in the Heterogeneous Model

In this section, we extend the work in the previous section to the more general model where users have heterogeneous services. We design a general randomized mechanism (GenRanMech, called GRMech for short), which guarantees budget feasibility, individual rationality, budget balance, fairness, and user's/requesters' truthfulness.

5.1 Mechanism Design

Mechanism HomoMech proposed above finds the *critical virtual price* which measures requesters' demand fairly and gives the same payment to the users with homogeneous values. However, such a strategy would fail for the general model with heterogeneous values. The reason is that it is hard to find a *virtual price* to select winning users since, intuitively, a valuable user is worth a higher payment, making it difficult to measure the abilities of requesters. Moreover, requesters have various budgets and, therefore, diverse abilities to obtain services from the users, which adds a significant challenge to designing the mechanism.

To address the challenge above, Mechanism GRMech randomly combines two sub-mechanisms. In general, we try to divide the users into two groups and conquer them separately in two sub-mechanisms. That is, users with *small costs* (no greater than $\frac{B_{min}}{3}$) and users with *large costs* (greater than $\frac{B_{min}}{3}$). The first sub-mechanism, namely, UniformMech, tackles the users with small costs. While for the users with large costs, we design the second mechanism, namely, GreedyMech, that uses a greedy allocation. In detail, Mechanism GRMech returns the results of Mechanism UniformMech with probability $\frac{7}{12}$ and returns the results of Mechanism GreedyMech with probability $\frac{5}{12}$. Note that to slightly improve the performance, we run Mechanism GreedyMech over the original input with all n users.

Algorithm 2 General Random Mechanism **GRMech**(B, C, V)

1: Let $\widetilde{S} = \{s_j : c_j \leq \frac{B_{min}}{3}\}$ be the users with costs no greater than $\frac{B_{min}}{3}$, and denote by $\widetilde{C}$ the corresponding costs of the users and denote by $\widetilde{V}$ the values of their services.
2: With probability $\frac{7}{12}$, **return** UniformMech$(B, \widetilde{C}, \widetilde{V})$
3: With probability $\frac{5}{12}$, **return** GreedyMech(B, C, V)

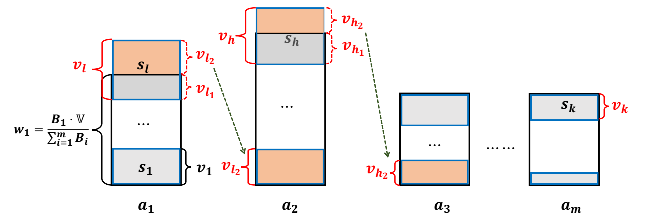


Fig. 3. An example of allocation rule of Mechanism UniformMech.

The high-level idea of Mechanism UniformMech is as follows. Since users have small costs, we take into account a part of each requester's budget, namely, $\frac{B_i}{3}$, and use the virtual total budget $\sum_{1\leq i\leq m}\frac{B_i}{3}$ to select users. Then, we choose the users in the order of non-decreasing cost-per-unit-of-value $\frac{c_j}{v_j}$ successively until a certain threshold is violated. Once the winning users are determined, we introduce a notion based on the obtained threshold, namely, the quota of each requester, to measure the diverse demand of requesters. After determining the winning users and the quota of each requester, we generate a virtual allocation by assigning the services of winning users virtually to requesters, in which a single user's service may be assigned to different requesters. Last, we design a random allocation rule and generate a real allocation based on the virtual allocation.

In detail, it works as follows. Since the users have costs no greater than $\frac{B_{min}}{3}$, we first use each requester's one-third budget $\frac{B_i}{3}$ as a virtual budget, and denote by $B_t = \sum_{1\leq i\leq m}\frac{B_i}{3}$ the sum of each requester's virtual budget. Before selecting winning users, we sort the users with small costs in the order of non-decreasing cost-per-unit-of-value $\frac{c_j}{v_j}$. We test each user successively. If the j-th user s_j satisfies $\frac{c_j}{v_j} \leq \frac{B_t}{\sum_{h\leq j} v_h}$, s_j will be selected as a winning user. Assume that s_k is the last winning user, then we have $\frac{c_{k+1}}{v_{k+1}} > \frac{B_t}{\sum_{h\leq k+1} v_h}$ and total value of winners is $\mathbb{V} = \sum_{1\leq j\leq k} v_j$. Accordingly, we define $q = \min\{\frac{B_t}{\sum_{j\leq k} v_j}, \frac{c_{k+1}}{v_{k+1}}\}$ to be the *virtual critical threshold*.

With virtual budget $\frac{B_i}{3}$, requester a_i can obtain at most $\frac{B_i}{3q}$ value. In order to ensure the fairness among requesters, in the virtual allocation, a_i will be allocated $w_i = \frac{B_i}{\sum_{1\leq l\leq m} B_l}\cdot\mathbb{V}$ value ($w_i \leq \frac{3B_i}{q}$ since $w_i \cdot q \leq \frac{B_i}{\sum_{1\leq l\leq m} B_l}\cdot\mathbb{V}\cdot\frac{B_t}{\mathbb{V}} \leq 3B_i$), called the *quota* of requester a_i. Now we further introduce the detailed implementation of the virtual allocation. We treat the users' services as divisible and allocate winners' services in the non-decreasing order of cost-per-unit-of-value $\frac{c_j}{v_j}$ to the requesters according to their increasing index. Let $\mathbb{X} = (\mathbf{x}_{ij})_{1\leq i\leq m, 1\leq j\leq n}$ denote the virtual allocation of users where $\mathbf{x}_{ij} \in [0, 1]$ is the fraction of s_j's service allocated to requester a_i. Knowing that the first i requesters can obtain at most $\sum_{i'\leq i} w_{i'}$ value, after allocating $\sum_{j'\leq j-1} v_{j'}$ value of the first $j-1$ users, we check whether assigning all v_j value of user s_j would exceed the limit $\sum_{i'\leq i} w_{i'}$. If it does not exceed the limit, *i.e.*, $\sum_{j'\leq j} v_{j'} \leq \sum_{i'\leq i} w_{i'}$, we allocate $\min\{\sum_{j'\leq j} v_{j'} - \sum_{i'\leq i-1} w_{i'}, v_j\}$ value of the service of user s_j to requester a_i, *i.e.*,

$$\mathbf{x}_{ij} = \frac{\min\{\sum_{j'\leq j} v_{j'} - \sum_{i'\leq i-1} w_{i'}, v_j\}}{v_j}.$$

If it exceeds the limit, we allocate $\min\{\sum_{i'\leq i} w_{i'} - \sum_{j'\leq j-1} v_{j'}, w_i\}$ value of the service of user s_j to requester a_i, *i.e.*,

$$\mathbf{x}_{ij} = \frac{\min\{\sum_{i'\leq i} w_{i'} - \sum_{j'\leq j-1} v_{j'}, w_i\}}{v_j}.$$

This would use up the quota of requester a_i and we then consider the next requester a_{i+1}. The process repeats until all services of the first k users are allocated.

As shown in Fig.3, we use black boxes to denote the quotas of requesters by letting their heights equal to requesters' quota respectively, and blue divisible boxes to represent users' services whose heights equal to their values (all boxes have the same width). We allocate services of winners to each requester according to their increasing index. By the virtual allocation, assume that we have allocated services from s_1 to s_{l-1} to requester a_1 and the quota of requester a_1 will be exceeded after allocating the whole value of service v_l, i.e., $\sum_{i=1}^{l} v_j > w_1$. Then, we allocate $w_1 - \sum_{i=1}^{l-1} v_i = v_{l_1}$ value to requester a_1 and the remaining $v_l - v_{l_1} = v_{l_2}$ value to requester a_2 which implies $\mathbf{x}_{1l} = \frac{v_{l_1}}{v_l}, \mathbf{x}_{2l} = \frac{v_{l_2}}{v_l}$. Specifically, value v_{l_2} is smaller than a_2's quota. We then consider the next user s_{l+1}. This process continues until all winners' services are allocated. Similarly, the service of s_h is divided into two parts v_{h_1} and v_{h_2} which are allocated to a_2 and a_3, respectively.

We denote by $V(B_i, B_{-i}, C)$ the amount of value requester a_i obtained in the virtual allocation, which will be written as $V(B_i)$ for short if no ambiguity arises. Note that $\sum_{1\leq i\leq m} w_i = \mathbb{V} = \sum_{1\leq j\leq k} v_j$, which implies that the total value of services to be allocated is not greater than the total value of winners.

Next, based on the virtual allocation $\mathbb{X}$, we generate a real allocation by designing a random allocation rule so that the expected total value obtained by requester a_i is precisely equal to $V(B_i)$, the total value obtained in the virtual allocation. The detailed implementation is as follows. For each winning user $s_j \in S_w$, we allocate her service to requester $a_i, \forall 1 \leq i \leq m$ with probability $\mathbf{x}_{ij}$. Moreover, the requester who gets the allocation rewards user s_j with payment $min\{v_j \cdot q, \frac{B_{min}}{3}\}$, which is the key to guarantee the truthfulness. The process repeats until all the first k services are allocated. As shown in Fig. 3, the service of user s_l will be allocated to requester a_1 with probability $\mathbf{x}_{1l} = \frac{v_{l_1}}{v_l}$, and to requester a_2 with probability $\mathbf{x}_{2l} = \frac{v_{l_2}}{v_l}$. Note that the

Algorithm 3 Mechanism **UniformMech**(B, C, V)

Input: B, C with all $c_j \le \frac{B_{min}}{3}$, V.

Let $B_t = \sum_{i \le m} \frac{B_i}{3}$.
Sort users in the non-decreasing order of $\frac{c_j}{v_j}$.
// Selecting winning users.
$j \leftarrow 1$
while $\frac{c_j}{v_j} \le \frac{B_t}{\sum_{j' \le j} v_{j'}}$ **do**
  $j \leftarrow j+1, S_w \leftarrow S_w \cup \{s_j\}$
Let s_k be the last winning user and $q = min\{\frac{B_t}{\sum_{j \le k} v_j}, \frac{c_{k+1}}{v_{k+1}}\}$.
// Allocation and Payment scheme
Let the quota $w_i = \frac{B_i}{\sum_{l \le m} B_l} \cdot \mathbb{V}$.
Set $i = 1, j = 1$ and generate the allocation as follows.
while $i \le m$, $j \le k$ **do**
  if $\sum_{j' \le j} v_{j'} \le \sum_{i' \le i} w_{i'}$ **then**
    with probability $\mathbf{x}_{ij}$: allocate user s_j to requester a_i where

$$\mathbf{x}_{ij} = \frac{\min\{\sum_{j' \le j} v_{j'} - \sum_{i' \le i-1} w_{i'}, v_j\}}{v_j}.$$

    $j \leftarrow j+1$
  else
    with probability $\mathbf{x}_{ij}$: allocate user s_j to requester a_i where

$$\mathbf{x}_{ij} = \frac{\min\{\sum_{i' \le i} w_{i'} - \sum_{j' \le j-1} v_{j'}, w_i\}}{v_j}.$$

    $i \leftarrow i+1$
  The requester allocated with user s_j pays $\min\{v_j \cdot q, \frac{B_{min}}{3}\}$.

virtual budget is set with one-third of the original budget, which guarantees that even if a requester is allocated with a user exceeding the limit twice (when generating the real allocation, e.g., the services of s_l and s_h, are all allocated to requester a_2 in the real allocation in Fig. 3), the requester can use the remaining two-thirds of her budget to pay the user without violating budget feasibility.

Now, we design the second mechanism GreedyMech, which intends to avoid missing the possible high-value services of users with large costs. The high level idea of GreedyMech is simple, *i.e.*, select the high-value users and pay each selected user a maximum payment B_{min} (it is always possible since all $c_j \le B_{min}$). In detail, it computes the total number $\sum_{1 \le i \le m} \lfloor \frac{B_i}{B_{min}} \rfloor$ of services that can be obtained with payment B_{min}. Then, it allocates the top $K = min\{\sum_{1 \le i \le m} \lfloor \frac{B_i}{B_{min}} \rfloor, n\}$ highest-value users to the requesters based on the non-increasing order of requesters' budget, and accordingly gets a final allocation.

Finally, we have our general Mechanism GRMech by combining UniformMech and GreedyMech randomly.

5.2 Theoretical Guarantees of Performance

In this part, we analyze the performance of Mechanism GRMech. Firstly, we have the following basic property of Mechanism UniformMech according to its allocation rule.

Algorithm 4 Mechanism **GreedyMech**(B, C, V)

Sort the users by non-increasing service values.
$K \leftarrow \min\{\sum_{1 \le i \le m} \lfloor \frac{B_i}{B_{min}} \rfloor, n\}$
Allocate the first K users in highest-value-first order to the requesters based on the non-increasing order of requesters' budgets, where requester a_i buys $\lfloor \frac{B_i}{B_{min}} \rfloor$ services and each user gets payment B_{min}.

Lemma 3. *For Mechanism UniformMech, we have* $\frac{c_k}{v_k} \le \frac{B_t}{\sum_{j \le k} v_j}, \frac{c_{k+1}}{v_{k+1}} > \frac{B_t}{\sum_{j \le k+1} v_j}$.

Theorem 4. *Mechanism GRMech satisfies the budget-feasibility, individual-rationality, computational efficiency, strong budget-balance, users'/requesters' truthfulness, fairness and achieves an approximation ratio of* 12.

The proof of Theorem 4 is shown in the following subsections. Section 5.2.1 shows budget-feasibility, individual-rationality, computational efficiency, and strong Budget-balance. Section 5.2.2 and 5.2.3 prove users' truthfulness and requesters' truthfulness, respectively. Then, we show the fairness in Section 5.2.4. Finally, we analyze the approximation performance in Section 5.2.5.

5.2.1 Budget-Feasibility, Individual-Rationality, Strong Budget-balance

First, we show budget feasibility. For Mechanism GreedyMech, it is easy to see budget feasibility. For Mechanism UniformMech, requester a_i pays $\min\{v_j \cdot q, \frac{B_{min}}{3}\} \le v_j \cdot q$ when a user with value v_j is allocated to her. Since the total value obtained by requester a_i is at most $w_i \le \frac{B_i}{3q}$, her total payment is at most $\frac{B_i}{3q} \cdot q \le \frac{B_i}{3}$ in the virtual allocation generated by Mechanism UniformMech. Furthermore, when the real allocation is generated in the random allocation, requester a_i gets at most two extra users randomly allocated to it (respectively when the limit $\sum_{i' \le i-1} w_{i'}$ and $\sum_{i' \le i} w_{i'}$ are exceeded). Thus, it only needs to reward at most two extra users (which have small bids, and each should be paid at most $\frac{B_{min}}{3}$) using the remaining two-thirds of her budget. Thus, budget feasibility holds. Therefore, Mechanism GRMech is budget feasible.

Next, we prove the individual rationality. For Mechanism GreedyMech, it is easy to see its individual rationality since the payment for each winning user is $B_{min} \ge c_j$. For Mechanism UniformMech, if user s_j is not selected, her utility is zero. If user s_j is selected, her utility is $p_j - c_j$. Since $s_j, \forall 1 \le j \le k$ is a winner, we have $\frac{c_j}{v_j} \le \frac{c_k}{v_k} \le \frac{B_t}{\sum_{j \le k} v_j}$ and $\frac{c_j}{v_j} \le \frac{c_{k+1}}{v_{k+1}}$. Thus, $\frac{c_j}{v_j} \le q$. Moreover, the user has small bid $c_j \le \frac{B_{min}}{3}$. Therefore $p_j = \min\{v_j \cdot q, \frac{B_{min}}{3}\} \ge c_j$. Thus, her utility is non-negative, implying individual rationality.

Last, for strong budget balance, in Mechanism UniformMech, the total payment received by winners is $\sum_{s_j \in S_w}(\min\{v_j \cdot q, \frac{B_{min}}{3}\})$ which is equal to the total payment paid by requesters. Similarly, in Mechanism GreedyMech, the total payment received by winners is $B_{min} \cdot \min\{\sum_{1 \le i \le m} \lfloor \frac{B_i}{B_{min}} \rfloor, n\}$ which is the total payment paid by requesters. Therefore, Mechanism GRMech guarantees strong budget-balance.

5.2.2 Users' Truthfulness

We will show that Mechanism GRMech is truthful. Note that users with cost $c_i \geq \frac{B_{min}}{3}$ would not underbid to participate in Mechanism UniformMech as the payment for the winner in UniformMech is no greater than $\frac{B_{min}}{3}$. Furthermore, as users with costs smaller than $\frac{B_{min}}{3}$ can participate in both Mechanism GreedyMech and UniformMech, they have no incentive to report a higher cost to participate in Mechanism GreedyMech. Thus, it suffices to prove that the two sub-mechanisms are truthful. First, we analyze Mechanism GreedyMech. Clearly, each user cannot get more payment by false bidding because we select users based on the highest-value-first rule, and the payment for each winning user is constant B_{min}. Thus, Mechanism GreedyMech satisfies users' truthfulness.

We prove the users' truthfulness of Mechanism UniformMech by showing that it satisfies Theorem 3.

Theorem 5. *UniformMech satisfies users' truthfulness.*

Proof. Suppose s_k is the last winner in Mechanism UniformMech, and we have $\frac{c_k}{v_k} \leq \frac{B_t}{\sum_{j\leq k} v_j}$.

Monotonicity: For user $s_j, \forall 1 \leq j \leq k$, it is obvious that user s_j is selected when bidding her real cost, *i.e.*, $\frac{c_j}{v_j} < \frac{B_t}{\sum_{h\leq j} v_h}$. If s_j decreases her bid to be $c'_j \leq c_j$, according to the allocation scheme of Mechanism UniformMech, it is clear that $\frac{c'_j}{v_j} \leq \frac{B_t}{\sum_{h\leq j} v_h}$ due to $\frac{c_j}{v_j} \leq \frac{B_t}{\sum_{h\leq j} v_h}$. Thus, user s_j will still be a winner. Therefore, Mechanism UniformMech is monotonic.

Threshold payments: For user $s_j, \forall 1 \leq j \leq k$, the payment is $p_j = \min\{v_j \cdot q, \frac{B_{min}}{3}\}$, where $q = \min\{\frac{c_{k+1}}{v_{k+1}}, \frac{B_t}{\sum_{j\leq k} v_j}\}$. Assume that user s_j increases her bid to be $c'_j > p_j$. Clearly, bidding $c'_j > \frac{B_{min}}{3}$ would make the user fail to be selected in Mechanism UniformMech. Thus, it remains to show that any user bidding higher than $v_j \cdot q$ will not be selected. We discuss two cases of value q:

Case 1 ($\frac{c_{k+1}}{v_{k+1}} \leq \frac{B_t}{\sum_{j\leq k} v_j}$): We have $q = \frac{c_{k+1}}{v_{k+1}} \geq \frac{c_j}{v_j}$ and $\frac{c_{k+1}}{v_{k+1}} > \frac{B_t}{\sum_{j\leq k+1} v_j}$ by Lemma 3. When user s_j with $j \leq k$ bids $b_j > v_j \cdot q$, we have $\frac{b_j}{v_j} > q = \frac{c_{k+1}}{v_{k+1}} > \frac{B_t}{\sum_{j\leq k+1} v_j}$. Assume that s_j is ranked in the t-th place with $t \geq k+1$ after bidding. That is, $\frac{c_{k+1}}{v_{k+1}} \leq ... \leq \frac{b_j}{v_j}$. Suppose on the contrary that s_j is still selected in such case, which implies that $\frac{c'_j}{v_j} \leq \frac{B_t}{\sum_{h\leq j-1} v_h + \sum_{j+1\leq h\leq t} v_h + v_j} \leq \frac{B_t}{\sum_{j\leq k+1} v_j}$. This leads to a contradiction. Thus, user s_j will not be selected as a winning user. Therefore, the value $v_j \cdot q$ is the critical value and the winning users are paid threshold payment.

Case 2 ($\frac{c_{k+1}}{v_{k+1}} > \frac{B_t}{\sum_{j\leq k} v_j}$): We have $q = \frac{B_t}{\sum_{j\leq k} v_j}$. When user s_j with $j \leq k$ bids $b_j > v_j \cdot q$, we have $\frac{c'_j}{v_j} > q = \frac{B_t}{\sum_{j\leq k} v_j}$. Assume that user s_j is ranked in the t-th place with $t \geq k$ after bidding, that is, $\frac{c_k}{v_k} \leq ... \leq \frac{b_j}{v_j}$. Suppose on the contrary that s_j is still selected in such case, which implies that $\frac{c'_j}{v_j} \leq \frac{B_t}{\sum_{1\leq h\leq j-1} v_h + \sum_{j+1\leq h\leq t} v_h + v_j} \leq \frac{B_t}{\sum_{j\leq k} v_j}$. This leads to a contradiction. Thus, user s_j will not be selected as a winning user. Therefore, the value $v_j \cdot q$ is the critical value and the winning users are paid threshold payment.

Based on the analysis of the two cases above, Mechanism UniformMech guarantees users' truthfulness. □

Therefore, we conclude that Mechanism GRMech guarantees users' truthfulness.

5.2.3 Requester's Truthfulness

We will show requesters' truthfulness of Mechanism UniformMech first and then consider Mechanism GreedyMech. As there are no general guidelines on showing requesters' truthfulness in the budget feasible procurement setting, we prove requesters' truthfulness by directly examining the possible change of a requester's utility. For Mechanism UniformMech, since each requester is required to submit the full amount of her claimed budget as a deposit to the mechanism at the beginning, it is easy to detect whether a requester bids over her true budget. To prove the truthfulness of the underbidding case, we first show that the expected total value that requester a_i obtained through the random allocation rule is equal to $V(B_i)$ in the following lemma.

Lemma 4. *The expected total value that requester a_i obtained through the random allocation rule is equal to $w_i = V(B_i)$.*

Proof. In generating the virtual allocation, the services from the first k users are allocated in the increasing order of $\frac{c_j}{v_j}$ to requesters according to their quota, *i.e.*, the first i requesters can obtain $\sum_{i'\leq i} w_{i'}$ value of services where $w_i = \frac{B_i}{\sum_{1\leq l\leq m} B_l} \cdot \mathbb{V}$. Then, the real allocation is generated by allocating the users to requesters with probability proportional to the value allocated to them in the virtual allocation. In detail, after allocating the first $j-1$ users, if assigning all v_j value of user s_j does not exceed the limit $\sum_{i'\leq i} w_{i'}$, then user s_j is allocated to requester a_i with probability $\frac{\min\{\sum_{j'\leq j} v_{j'} - \sum_{i'\leq i-1} w_{i'}, v_j\}}{v_j}$. Thus, according to the random allocation rule, requester a_i will obtain $\min\{\sum_{j'\leq j} v_{j'} - \sum_{i'\leq i-1} w_{i'}, v_j\}$ (which is v_j when $\sum_{j'\leq j-1} v_{j'} - \sum_{i'\leq i-1} w_{i'} > v_j$) in expectation by consuming the corresponding $\min\{\sum_{j'\leq j} v_{j'} - \sum_{i'\leq i-1} w_{i'}, v_j\}$ quota. Then, the next user is considered and requester a_i continues using her remaining quota. In later iterations with some user under consideration, once assigning all the value of the user to requester a_i exceeds her limit, requester a_i will be allocated with $\min\{\sum_{i'\leq i} w_{i'} - \sum_{j'\leq j-1} v_{j'}, w_i\}$ (which is w_i when $\sum_{i'\leq i-1} w_{i'} > \sum_{j'\leq j-1} v_{j'}$) in expectation according to the random allocation rule (with probability $\frac{\min\{\sum_{i'\leq i} w_{i'} - \sum_{j'\leq j-1} v_{j'}, w_i\}}{v_j}$). Meanwhile, this will make the requester use up all her quota and the next requester a_{i+1} will be considered. According to the analysis above, the expected value that requester a_i obtains is exactly $V(B_i)$. □

Then, we show that the expected obtained value of any requester a_i would not increase by bidding a lower budget, *i.e.*, $V(B'_i) \leq V(B_i)$ when $B'_i < B_i$. Assume the one-third of requesters' total virtual budget is B'_t, the virtual critical threshold is q' and the winner set is S'_w when requester a_i underbids a lower budget $B'_i < B_i$. We assume the mechanism selects the first r users as winning users in such case. Let $\mathbb{B} = \sum_{i\leq m} B_i$ and $\Delta B = \mathbb{B} - B_i = \sum_{h\leq m, h\neq i} B_h$ and recall that $\mathbb{V} = \sum_{j\leq k} v_j$.

As requester a_i underbids, the total budget B_t would decrease for Mechanism UniformMech, and the number of winning users would not increase, *i.e.*, $S'_w = \{s_j : j \leq r\} \subseteq S_w$ where $r \leq k$, which implies the total obtained value does not increase, i.e., $\sum_{s_j \in S'_w} v_j \leq \mathbb{V}$. Otherwise, the mechanism could have more winning users when a_i bids her true budget according to the allocation rule. If a_i underbids a lower budget $B'_i < B_i$, the expected total value should have

$$V(B'_i) = \frac{B'_i}{B'_i + \Delta B} \cdot \sum_{j \in S'_w} v_j \leq \frac{B_i}{B_i + \Delta B} \mathbb{V} = V(B_i).$$

Therefore, Mechanism UniformMech satisfies requesters' truthfulness. Next, we prove requesters' truthfulness in Mechanism GreedyMech. requester a_i can obtain at most $\lfloor \frac{B_i}{B_{min}} \rfloor$ services since $B'_i < B_i$. The expected obtained value of requester a_i cannot increase because users are allocated in the order of highest-value-first to requesters. Thus, Mechanism GreedyMech satisfies requesters' truthfulness.

Therefore, Mechanism GRMech satisfies requesters' truthfulness.

5.2.4 Fairness

For Mechanism UniformMech, the expected value obtained from users for requester a_i is $V(B_i) = u^a_i(B_i) = \frac{B_i}{\sum_{1 \leq l \leq m} B_l} \cdot \mathbb{V}$. We have $u^a_{i_1}(B_{i_1}) \geq u^a_{i_2}(B_{i_2})$ when $B_{i_1} \geq B_{i_2}$. Thus UniformMech guarantees fairness. For GreedyMech, we have $\lfloor \frac{B_{i_1}}{B_{min}} \rfloor \geq \lfloor \frac{B_{i_2}}{B_{min}} \rfloor$ when $B_{i_1} \geq B_{i_2}$. According to the allocation rule, the requester with higher budget has priority to be allocated higher value items. Thus, the total value obtained from users for requester a_{i_1} and the total value obtained from users for requester a_{i_2} satisfy $u^a_{i_1}(B_{i_1}) \geq u^a_{i_2}(B_{i_2})$, which guarantees the fairness. Therefore, Mechanism GRMech guarantees *fairness*.

5.2.5 Approximation

Let $V_u(S_1)$ denote the total value obtained in Mechanism UniformMech with the candidate users in set S_1 and similarly let $V_g(S_2)$ denote the total value of services obtained in Mechanism GreedyMech with the candidate users in set S_2 where $S_1 = \{s_j | c_j \leq \frac{B_{min}}{3}\}$, $S_2 = \{s_j | c_j > \frac{B_{min}}{3}\}$ and $S = S_1 \cup S_2$. Correspondingly, we denote by $OPT(S_1)$ and $OPT(S_2)$ the optimal solutions with input of S_1 and S_2, respectively.

Lemma 5. *Mechanism UniformMech achieves an approximation ratio of* $4 + \frac{3}{m-1} \leq 7$, *where* m *is the number of requesters.*

Proof. To analyze the approximation of Mechanism UniformMech, we discuss it by two kinds of costs, the ones with cost-per-unit-of-value less than q and the ones with cost-per-unit-of-value no less than q. First, we consider users with cost-per-unit-of-value less than q. By the definition of q, we have $\frac{c_{k+1}}{v_{k+1}} \geq q$ and all users with cost-per-unit-of-value less than q have at most a total value $\sum_{j \leq k} v_j$. For the optimal solution, the best case is that it obtains all $\sum_{j \leq k} v_j$ value with total cost 0.

Next, we consider the costs with cost-per-unit-of-value no less than q. The best case is that all costs in $\{c_j | j \geq k+1, c_j \in C\}$ are equal to q and the optimal solution pays each user $v_j \cdot q$. In such case, we can obtain at most $\frac{B_t}{q}$ value from these users given a total budget B_t. If $q = \frac{B_t}{\mathbb{V}}$, then $\frac{B_t}{q} = \sum_{j \leq k} v_j$. If $q = \frac{c_{k+1}}{v_{k+1}}$, then $\frac{B_t}{q} = \frac{B_t}{\frac{c_{k+1}}{v_{k+1}}} \leq \frac{B_t}{\frac{B_t}{\sum_{j \leq k+1} v_j}} = \sum_{j \leq k+1} v_j$ since $\frac{B_t}{\sum_{j \leq k+1} v_j} \leq \frac{c_{k+1}}{v_{k+1}}$ by Lemma 3. For both cases, we have $\frac{B_t}{q} \leq \sum_{j \leq k} v_j + v_{k+1}$. However, for the optimal solution, it has the full amount of each requester's budget, which equals $3B_t$. Thus, the optimal solution can get at most $\frac{3B_t}{q} \leq 3 \sum_{j \leq k} v_j + 3v_{k+1}$ value from users with cost-per-unit-of-value no less than q.

Combining the two cases above, the optimal solution can obtain at most $4 \sum_{j \leq k} v_j + 3v_{k+1}$ value from all users. Therefore,

$$\frac{OPT(S_1)}{V_u(S_1)} \leq \frac{4 \sum_{j \leq k} v_j + 3v_{k+1}}{\sum_{j \leq k} v_j} = 4 + \frac{3v_{k+1}}{\sum_{j \leq k} v_j} \tag{7}$$

By Lemma 3, we have $(\sum_{j \leq k+1} v_j) \cdot \frac{c_{k+1}}{v_{k+1}} > B_t$. Thus,

$$v_{k+1} < \frac{c_{k+1} \cdot \sum_{j \leq k} v_j}{B_t - c_{k+1}} \leq \frac{\frac{B_{min}}{3} \cdot \sum_{j \leq k} v_j}{(m-1) \cdot \frac{B_{min}}{3}} = \frac{\sum_{j \leq k} v_j}{m-1} \tag{8}$$

The second inequality holds because each candidate user's bid in Mechanism UniformMech is at most $\frac{B_{min}}{3}$ and the budget for any requester is at least B_{min}. Combining (7) and (8), we have $\frac{OPT(S_1)}{V_u(S_1)} \leq 4 + \frac{3v_{k+1}}{\sum_{j \leq k} v_j} \leq 4 + \frac{3}{m-1} \leq 7$. □

Lemma 6. *Mechanism GreedyMech achieves a 5-approximation.*

Proof. In Mechanism GreedyMech, each winner's payment is B_{min}. Let K denote the number of winners. Thus, the number of winners is $K = \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{B_{min}} \rfloor$. All users will be ordered by their values, $v_1 \geq v_2 \geq \ldots \geq v_n$. The top K values in this order will be allocated to requesters and thus the output total value of Mechanism GreedyMech is $V_g(S) = \sum_{1 \leq j \leq K} v_j$. Since $S_2 \subseteq S$, it is clear that $V_g(S) \geq V_g(S_2)$ where $V_g(S_2)$ is the returned total value of Mechanism GreedyMech with input S_2. For the optimal solution $OPT(S_2)$, it selects the winning users from S_2 and the best case is that the payment for each winning bid is $\frac{B_{min}}{3}$ because $c_j \geq \frac{B_{min}}{3}$ for $c_j \in S_2$. Then, the number of winning bids in the optimal solution is at most $\sum_{1 \leq i \leq m} \lfloor \frac{B_i}{\frac{1}{3} B_{min}} \rfloor$. Moreover, $\lfloor \frac{B_i}{\frac{1}{3} B_{min}} \rfloor \leq 3 \cdot \lfloor \frac{B_i}{B_{min}} \rfloor + 2$. Therefore, we have

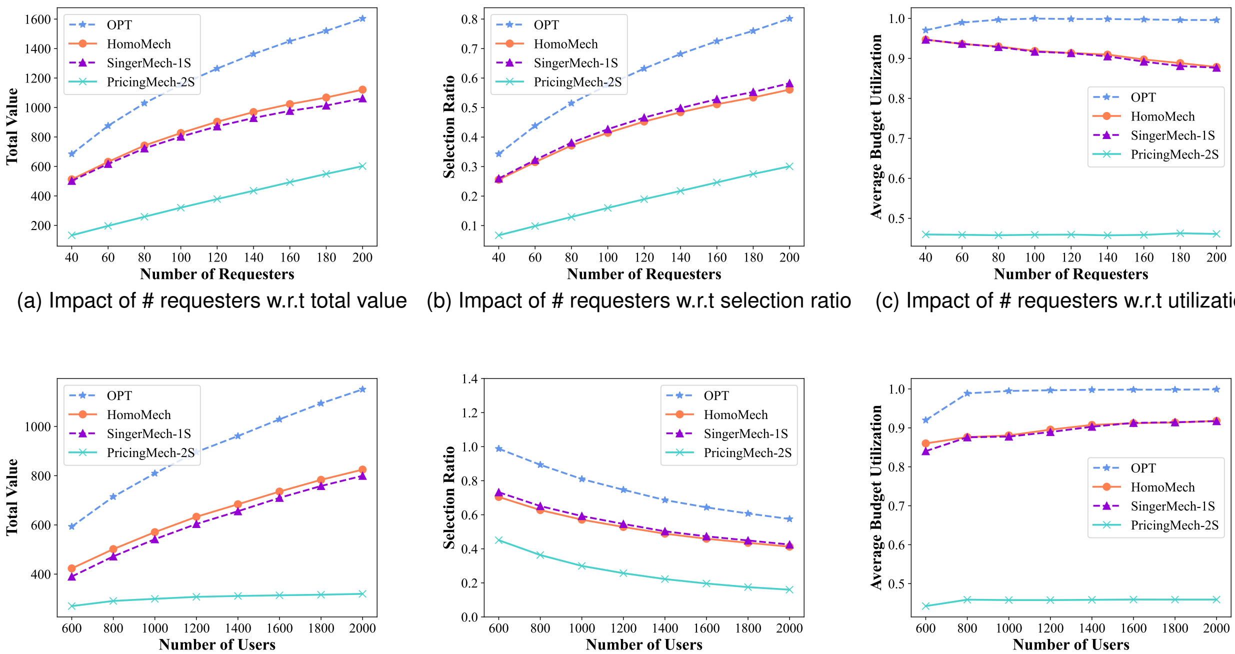
$$\begin{aligned} \sum_{1 \leq i \leq m} \lfloor \frac{3B_i}{B_{min}} \rfloor &\leq 3 \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{B_{min}} \rfloor + \sum_{1 \leq i \leq m} 2 \\ &\leq 3 \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{B_{min}} \rfloor + 2 \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{B_{min}} \rfloor \\ &\leq 5 \sum_{1 \leq i \leq m} \lfloor \frac{B_i}{B_{min}} \rfloor. \end{aligned}$$

Since the optimal solution selects the first $\sum_{1 \leq i \leq m} \lfloor \frac{3B_i}{B_{min}} \rfloor$ highest values, we have

$$\begin{aligned} OPT(S_2) = \sum_{j=1}^{\sum_{1 \leq i \leq m} \lfloor \frac{3B_i}{B_{min}} \rfloor} v_j &\leq 5 \sum_{j=1}^{\sum_{1 \leq i \leq m} \lfloor \frac{B_i}{B_{min}} \rfloor} v_j \\ &\leq 5V_g(S_2) \leq 5V_g(S). \end{aligned}$$

Therefore, Mechanism GreedyMech achieves an approximation ratio of 5. □

For Mechanism GRMech, the total obtained value is $V(S) = \frac{7}{12} \cdot V_u(S_1) + \frac{5}{12} \cdot V_g(S) \geq \frac{1}{12} \cdot (OPT(S_1) + OPT(S_2)) \geq \frac{1}{12} \cdot OPT(S)$ by Lemma 5 and 6.



(a) Impact of # requesters w.r.t total value (b) Impact of # requesters w.r.t selection ratio (c) Impact of # requesters w.r.t utilization

(d) Impact of # users w.r.t total value (e) Impact of # users w.r.t selection ratio (f) Impact of # users w.r.t utilization

Fig. 4. Performance of Mechanism HomoMech under Yelp dataset

6 Experiments

In this section, we conduct experiments and evaluate the performance of the proposed mechanisms. Considering the difficulty of designing truthful mechanism in two-sided markets and no prior works have provided any viable solution, we compare the proposed mechanisms with the following benchmarks modified from existing literature where we use "1s" to indicate that the mechanism is truthful on one side, and "2S" to indicate that it is truthful on two sides.

- ***OPT:*** The optimal solution with full knowledge of all private information (requesters' budgets and users' costs) ignores the incentives and optimizes the total procured value of all reuqesters. Note that it can be reduced to the multiple knapsack problem, which is intractable because of the NP-hardness. However, we utilize an integer programming solver to solve it in the experiments under the small-scale input.
- ***SingerMech-1S:*** A natural mechanism extended from the single-buyer budget-feasible mechanism proposed in Singer [26], which only guarantees truthfulness on the users' side. We extend it to be a two-sided mechanism by assuming a virtual requester with the sum of requesters' budgets and selecting/allocating winners randomly. Such a mechanism can ensure user's truthfulness but fails to requesters' truthfulness.
- ***PricingMech-2S:*** A simple pricing mechanism that admits truthfulness on both sides, which adopts a naive strategy, *i.e.*, deciding a random price within a predefined range for each user, and if the price is no less than this user's bid, select this user as a winner and allocate her to a random requester with enough remaining budget. In addition, each requester only consumes a budget up to B_{min} for requesters' truthfulness. Obviously, Mechanism PricingMech-2S is truthful for both requesters' and users' sides.

We would evaluate these mechanisms in the following metrics:

Total Value is the total contribution of all the selected users, *i.e.*, $\sum_{s_j \in S_w} v_j$. We would measure the total value with respect to some parameters, e.g., the number of users and requesters.

Selection Ratio is the percentage of users who are selected to provide data services, *i.e.*, $\frac{|S_w|}{|S|}$.

Average Budget Utilization is the average percentage of the used budget of requesters, *i.e.*, $\frac{1}{m}\sum_{i \le m} \frac{\sum_{j \le n} p_{ij}}{B_i}$.

We conduct experiments on a real dataset, *i.e.*, the public Yelp dataset [51]. Yelp dataset is a social network dedicated to local business directory services and review websites, including 150,346 companies, 996,275 users, and 6,990,280 reviews. For this dataset, we consider Yelp users as crowdsensing users and companies as requesters. This dataset provides users' average ratings and the number of reviews. We consider this average rating as the values of users and the number of reviews as the user's cost bidding. Yelp dataset also provides the total number of reviews received by the business that is considered as the requester's budget. The details of the dataset are depicted in Table 2.

For the evaluation step, we randomly choose users and requesters from the Yelp dataset. To evaluate the impact of the different ratios between requesters and users, we first fix the number of users as 2000 and change the number of requesters from 40 to 200 with an increment of 20. Then, to evaluate the impact of the number of users, we fix the number of requesters as 100 and change the number of users

TABLE 2
Preliminary Statistics of Dataset

Parameter	Dataset summary
Number of users	996,275
Number of companies	150,346
Number of reviews	6,990,280
Average review of companies	250.97
Average review of users	20.88
Average ranking of users	3.78

from 600 to 2000 with an increment of 200. All results are averaged over 200 rounds.

6.1 Total Value

This section evaluates the total value for requesters with different numbers of requesters and users. Fig. 4a and Fig. 4d show the impact of number of requesters and users in the homogeneous model, respectively. We observe that the total values of all mechanisms increase with the number of requesters or users. This is due to the fact that: 1) the overall procurement ability of requesters grows as more requesters arrive; 2) the competition among users becomes more fierce with the increment of users, and requesters can utilize their budgets more efficiently. Even with the requirement of truthfulness on two sides, Mechanism HomoMech always outperforms Mechanism SingerMech-1S and PricingMech-2S. This is because Mechanism SingerMech-1S uses a total virtual budget to select winners, which does not take each requester's budget into account. This method utilizes the budget less efficiently and leads to total value loss. Mechanism PricingMech-2S determines a random price to ensure the truthfulness on both sides but suffers the inefficiency in payment. In addition, we observe that the ratio between the total value of OPT and Mechanism HomoMech is even smaller than the approximation ratio proved in Section 4.2.5.

Fig. 5a and Fig. 5d show the impact of number of requesters and users for heterogeneous model, respectively. In these two figures, again, the total values of all mechanisms increase with the number of participants. We also note that Mechanism SingerMech-1S is better than Mechanism GRMech, and Mechanism PricingMech-2S performs the worst of all mechanisms. This is because Mechanism SingerMech-1S decides winners and payments using the total budget of requesters, while Mechanism GRMech only uses one-third of the total budget of requesters for budget feasibility. However, although Mechanism SingerMech-1S achieves better total value than Mechanism GRMech, it is vulnerable to the requesters' and users' misreporting strategies, which implies that part of the loss is needed to guarantee truthfulness on two sides and budget feasibility simultaneously. Also, we observe that the ratio between the total value of OPT and GRMech is much smaller than the approximation ratio proved in Section 5.2.5.

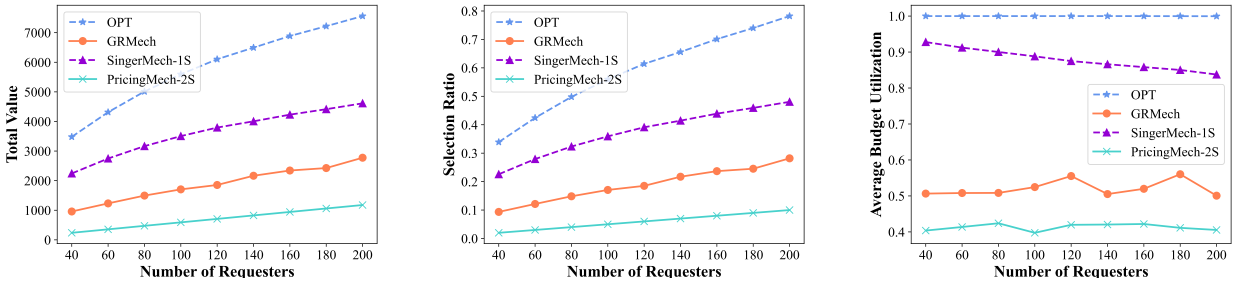
6.2 Selection Ratio

In this part, we evaluate the selection ratio with different number of requesters and users. Fig. 4b and Fig. 4e show the impact of number of requesters and users for the homogeneous model respectively, while Fig. 5b and Fig. 5e show that for the heterogeneous model. The selection ratio of all mechanisms generally increases (decreases) with the increment of requesters (users). The reason is that: 1) more requesters can procure more services from users; 2) although the number of users increases, the selected number of users is limited given the fixed requesters. Specifically, in Fig. 4b and Fig. 4e for the homogeneous model, we also note that the selection ratio of Mechanism SingerMech-1S is always higher than Mechanism HomoMech while the total value of Mechanism SingerMech-1S is always lower than Mechanism HomoMech in Fig. 4a and Fig. 4d. Such phenomenon demonstrates that Mechanism SingerMech-1S may select users with higher cost-per-values which is consistent with the analysis in Section 6.1 that Mechanism SingerMech-1S uses budget inefficiently. Furthermore, the selection ratio of Mechanism PricingMech-2S is always much lower than Mechanism GRMech because PricingMech-2S runs a random pricing method that is more likely to set a user a lower price than her real cost leading to the low selection ratio. For the heterogeneous model, the selection ratio of Mechanism SingerMech-1S is much higher than Mechanism GRMech in Fig. 5b and Fig. 5e. This is because Mechanism SingerMech-1S can use a sum of requesters' budgets as a virtual budget, while Mechanism GRMech only uses one-third of the total budget. However, again, Mechanism SingerMech-1S can only guarantee the truthfulness on the sellers' side while Mechanism GRMech ensures truthfulness on both sides and budget feasibility simultaneously.

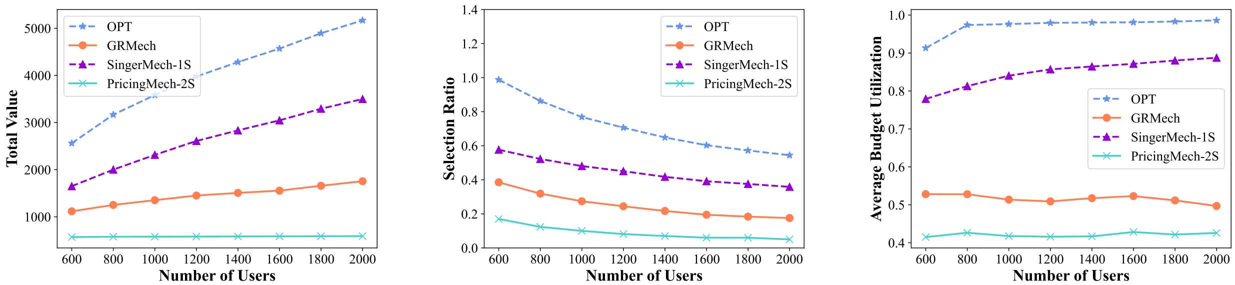
6.3 Budget Utilization

Next, we intend to evaluate the budget utilization with various number of requesters and users. Fig. 4c and Fig. 4f show the impact of number of requesters and users for homogeneous model respectively, while Fig. 5c and Fig. 5f show that for heterogeneous model. In Fig. 4c and 5c, the average budget utilisation for all mechanisms change slightly as the number of requesters increases. That is because, under a fixed set of users in the homogeneous model, more requesters will not significantly affect the efficiency of budget utilization. Furthermore, in Fig. 4f and Fig. 5f, the average budget utilization of all mechanisms except OPT is overall stable or increases at the beginning and then tends to be stable with more requesters participating. That is attributable to the fact that more users can bring more competitive services, thus having more chances to improve the budget utilization efficiency. In 4c and 4f, Mechanism SingerMech-1S and Mechanism HomoMech have the similar average budget utilization, while in 5c and 5f, the average budget utilization of Mechanism SingerMech-1S is much higher than Mechanism GRMech. The reason is that Mechanism GRMech selects winners by using one-third of each requester's budget to ensure participants' truthfulness and budget feasibility.

Moreover, to check requesters' budget feasibility, we show the budget utilization of requesters when GRMech



(a) Impact of # requesters w.r.t total value (b) Impact of # requesters w.r.t selection ratio (c) Impact of # requesters w.r.t utilization



(d) Impact of # users w.r.t total value (e) Impact of # users w.r.t selection ratio (f) Impact of # users w.r.t utilization

Fig. 5. Performance of Mechanism GRMech under Yelp dataset

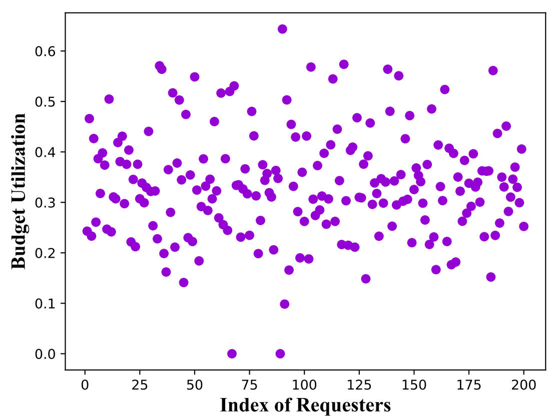


Fig. 6. Budget utilization in Mechanism UniformMech

runs sub-mechanism UniformMech as shown in 6. In Fig. 6, we fix the number of requesters and users as 200 and 2000, respectively. We observe that the budget utilization of each requester does not exceed one, which implies budget feasibility. In addition, most of the requesters' budget utilizations are close to $\frac{1}{3}$, which is consistent with the design of Mechanism UniformMech where each requester's expected budget consumption is at most one-third of her budget.

7 Conclusion and Future Work

In this work, we consider the budget-feasible mechanisms in two-sided mobile crowdsensing markets. We consider both the requesters' and users' truthfulness and fairness with respect to requesters' budgets when designing budget-feasible mechanisms. In such a scenario, we design efficient, fair, and budget-feasible mechanisms for the homogeneous and the heterogeneous setting, respectively. The designed fair mechanisms can guarantee the desired theoretical properties, like budget feasibility, individual rationality, truthfulness both on the users' side and the requesters' side and constant approximation to the optimal total procured value, and, more importantly, the fairness that requesters with more budgets procure more values of services from users than the requesters with lower budgets.

There are some natural extensions for future work. Here, we considered the additive utility function for requesters. It would be a future direction to extend our approach to more complex utility functions, such as submodular and sub-additive value functions, which, however, bring more challenges, *e.g.*, the difficulty of measuring the cost-per-value of each user, which is essential to guide the mechanism in selecting winners. The reason is that the value of an individual user will depend on the service received by the requester assigned to her due to the requesters' submodular or sub-additive utility function.

Acknowledgments

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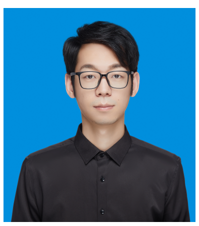

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