Solution to Math 481 – Quiz 4

Q1.

For term $x^a y^b z^c w^d$ after expanding $(x+y+z+w)^7$, the coefficient is

$$\binom{7}{a,b,c,d}$$

As $a=b=c=2$ and $d=1$, the coefficient has a maximum value

$$\binom{7}{2,2,2,1}=630$$

Q2.

4^k denotes the number of different ways to repeat selecting from 4 choices for k times.

$3^j\binom{k}{j}=3^j\binom{k}{k-j}$ denotes the number of different ways to select choice-1 for k-j times

while selecting the other three choices for left j times, thus $\sum_{j=0}^{k}3^j\binom{k}{j}$ can cover all ways.

Hence we finally have

$$4^k=\sum_{j=0}^{k}3^j\binom{k}{j}$$

Q3.

The sum is k^n.

Since $n_1+n_2+\cdots+n_k=n$, $\binom{n}{n_1,n_2,\cdots,n_k}$ can be used to represent the coefficient of term

$x_1^{n_1}x_2^{n_2}\cdots x_k^{n_k}$ of $(x_1+x_2+\cdots+x_k)^n$, that is

$$(x_1+x_2+\cdots+x_k)^n=\sum_{n_1+n_2+\cdots+n_k=n}\binom{n}{n_1,n_2,\cdots,n_k}x_1^{n_1}x_2^{n_2}\cdots x_k^{n_k}$$

Let $x_1=x_2=\cdots=x_k=1$, we have

$$(1+1+\cdots+1)^n = \sum_{n_1+n_2+\cdots+n_k=n} \binom{n}{n_1, n_2, \cdots, n_k} 1^{n_1} 1^{n_2} \cdots 1^{n_k}$$

$$\sum_{n_1+n_2+\cdots+n_k=n} \binom{n}{n_1, n_2, \cdots, n_k} = k^n$$

We have finished the proof.

Q4.

Since $n_1 + n_2 + \cdots + n_k = n$, $n_1, n_2, \cdots, n_k \geq 0$, this is a problem of weak compositions of n into k parts, hence the number of terms is $\binom{n+k-1}{n}$.